Abstract

Let $P(z, \varphi)$ denote the Poisson kernel in the unit disc. Poisson extensions of the type $P_\lambda f(z) = \int_{\mathbb{T}} P(z, \varphi)^{\lambda+1/2} f(\varphi)\, d\varphi$, where $f \in L^1(\mathbb{T})$ and $\lambda \in \mathbb{C}$, are then eigenfunctions to the hyperbolic Laplace operator in the unit disc. In the context of boundary behaviour, $P_0 f(z)$ exhibits unique properties.

We investigate the boundary convergence properties of the normalised operator, $P_0 f(z)/P_0 1(z)$, for boundary functions f in some function spaces. For each space, we characterise the so-called natural approach regions along which one has almost everywhere convergence to the boundary function, for any boundary function in that space. This is done, mostly, via estimates of the associated maximal function.

The function spaces we consider are $L^{p,\infty}$ (weak L^p) and Orlicz spaces which are either close to L^p or L^∞. We also give a new proof of known results for L^p, $1 \leq p \leq \infty$.

Finally, we deal with a problem on the lack of tangential convergence for bounded harmonic functions in the unit disc. We give a new proof of a result due to Aikawa.

Keywords: Square root of the Poisson kernel, approach regions, almost everywhere convergence, maximal functions, harmonic functions, Fatou theorem.

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