

# Generalized Patterns in Words and Permutations

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## Abstract

Recently, Babson and Steingrímsson introduced *generalized permutation patterns* (GPs) that allow the requirement that two adjacent letters in a pattern must be adjacent in the permutation. The present thesis is dedicated to studying such patterns in permutations and words.

The thesis consists of nine papers. In the first one we investigate simultaneous avoidance of two or more GPs with three letters, all adjacent. In the next three papers we study permutations avoiding GPs with additional restrictions such as beginning or ending with certain patterns. It is shown that there is a bijection between  $n$ -permutations that avoid the GP 132 and begin with the GP 12 and *increasing rooted trimmed trees* with  $n + 1$  nodes. Papers IV and V introduce *partially ordered generalized patterns* (POGPs). A POGP is a GP some of whose letters are incomparable. As an application of POGPs, we find the generating function (g.f.) for the entire distribution of the maximum number of non-overlapping occurrences in permutations of a GP  $\tau$  with adjacent letters, provided we know the g.f. for the number of permutations that avoid  $\tau$ . The last three papers deal with counting occurrences of patterns in certain words that are finite approximations of certain sequences. These sequences are sequences generated by a certain morphism, the *Peano curve*, and the *sigma-sequence* (related to the *Dragon curve*) used by Evdokimov in construction of chains of maximal length in the  $n$ -dimensional unit cube.

**Keywords and phrases:** Generalized pattern avoidance, partially ordered generalized patterns, occurrence of a pattern in a word or permutation, iterated morphism, Peano curve, sigma-sequence, Dragon curve

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