

On Integral Representation with Weights on Complex Manifolds

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Abstract

We present a method for finding weighted integral representation formulas for differential forms on a complex manifold X for which there exists a vector bundle $E \rightarrow X \times X$ of rank $\dim X$, and a holomorphic section η of E that defines the diagonal of $X \times X$.

The method is applied to Stein manifolds, where we look at some examples of the uses of weights. Most of our applications, however, are to compact manifolds, such as Grassmannians, where we find weights which allow representations of forms with values in any holomorphic line bundle as well as in the tautological vector bundle and its dual. As a consequence we obtain some vanishing theorems of the Bott-Borel-Weil type. We also relate the projection part of our formulas to the Bergman kernels associated to the line bundles. We treat the special case of complex projective space \mathbb{P}^n in some detail, as well as applying the method to $\mathbb{P}^m \times \mathbb{P}^m$.

We also find new integral representations of solutions to division problems in \mathbb{C}^n involving matrices of polynomials. We find estimates of the polynomial degree of the solutions by means of careful degree estimates of the so-called Hefer forms which are components of the representations.

Keywords: integral representation, Bochner-Martinelli formula, Grassmannians, complex projective space, residue currents, effective Nullstellensatz

AMS 2000 Subject Classification: 32A26; 32L20; 32M10; 32M05; 32Q99; 13P10

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