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To develop the ability of teacher students to reason mathematically

Abstract

The aim of the project
The project aims at developing the ability of teacher students to reason mathematically and to understand pupils' reasoning in mathematics.

The problem in focus
For a teacher to be able to talk to pupils in a way that suits the learning of the individual pupil she has a need to be capable of expressing herself in many different ways and to give arguments and to reason in such a way that the pupil understands. This aim is rarely reached by the ways teacher courses are structured today. A stronger focus on mathematical reasoning and communication is necessary.

What is the ability to reason mathematically?
Mathematics as a science is characterised among other things by the method to work with deductions and proofs which means that starting from certain given conditions you can with full certainty prove propositions and theorems. To be able to follow and develop such ways of thinking and arguing you have to develop your own ability to reason mathematically. In the school curriculum deductions and proofs have been given new importance. It is necessary for the teacher to have developed her own ability to reason mathematically in order to be able to work with the pupils to reach the aims of the curriculum in this area. The teacher must in conversations with the pupils be flexible and be able to understand different ways of thinking and reasoning. This makes it necessary for the teacher to be able to reason, to follow multiple ways of thinking, and to compare them and to value how viable they are.

What do we mean by ability to reason mathematically?
A good ability to reason mathematically means that you can reformulate questions and propositions in different ways, make and test conjectures, reject them or verify them, formulate counterexamples, specialise, generalise, draw conclusions, find alternative ways when you are stuck, decide if a solution is reasonable and judge the validity of arguments, describe, explain, and convince others about your arguments and to prove your claims. Mason, Burton and Stacey (1982) have demonstrated how to encourage, develop and foster the processes mentioned above, which seem to come naturally to mathematicians.

They suggest a method of working that is highly practical by starting from exemplary questions and problems and involving the learner in the discussions. In this way a deeper awareness of the nature of mathematical thinking and reasoning can grow.

Methods

We want to take a departure in research results from mathematics education and offer the student teachers opportunities to develop their own ability to reason mathematically in a more conscious way. We want to develop the mathematics parts of the courses. Instead of starting in a traditional way by presenting the theory and then continuing with problem solving exercising the application of the theory, we want to start with open or exploratory problem solving that give the students an experiential background for the introduction of theoretical concepts. The work will be done in small groups and the focus on reporting the work will be on presenting their findings and on convincing others about the reasonableness of the conclusions they have drawn. To work with different pieces of mathematical argument, both from former teacher students and pupils, will be another type of task. Alongside they will work with a selected number of more conventional mathematical problems, but be stimulated to continue to use the group as a resource, discussing different difficulties and explaining to each other different ways of solving the problems. Comparing and valuing the viability of alternative solutions is vital here. We will construct open problems and exercises that foster reasoning and try them out in group-work with student teachers. After evaluation and reconstruction if necessary the problems will be used in future mathematics teacher training courses.

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Studenterna – en försummad resurs i det akademiska utvecklingsarbetet

Vid Högskolan Kristianstad har vi under läsåret 2001/02 drivit ett projekt i syfte att stödja utveckling av studenternas förmåga att resonera matematiskt. Fyra av de 27 studerande har medverkat i projektets förberedelser, planering, genomförande, utvärdering och rapportering och många av de förslag studenterna förde fram kunde förverkligas. Studenterna är en försummad resurs i det akademiska utvecklingsarbetet.

Rådet för högskoleutbildning, vid Högskoleverket, stöder utvecklingsarbeten, som syftar till en pedagogisk förnyelse av grundutbildningen. Vid Högskolan Kristianstad har vi under läsåret 2001/02 drivit ett projekt i syfte att stödja utveckling av studenternas förmåga att resonera matematiskt. Studenterna förväntas medverka i det arbetet. De informerades både skriftligt och muntligt om projektets syfte. Tre kvinnliga och en manlig student ställde upp som frivilliga för att arbeta tillsammans med lärarna.

Aktiva studenter

I förberedelseskedet träffades projektgruppen ungefär varannan vecka för planering och diskussion. I ett tidigt skede var det möjligt för lärarna att få studenternas synpunkter på tänkta arbetsmetoder, uppgiftsförslag, dokumentation och examination. De fyra studenterna deltog aktivt i alla möten. I vissa fall tog de frågor med sig tillbaka till hela studentgruppen och hämtade synpunkter från dem. Under de femton veckor våren 2002 då projektet genomfördes träffades projektgruppen i stort sett en gång per vecka för att stämma av arbetet och göra justeringar och utvärdering.

Genom de fyra studenterna kanaliserades snabbt hela studentgruppens reaktioner. Det gjorde att lärarna relativt omgående kunde modifiera arbetet med att utveckla studenternas förmåga att resonera matematiskt så att det låg på lämplig nivå och var av rätt omfång för den avsatta tiden.

Ömsesidig respekt uppstod

De fyra studenter som deltog i lärarnas planering fick en tydlig bild av hur hårt lärarna anstränger sig för att skapa en bra inlärningssituation och för att det ska gå bra för studenterna. En uppenbar ömsesidig respekt uppstod mellan lärargruppen och studenterna. Studenterna såg tydligt att lärarna verkligen lyssnade till deras åsikter och gärna gick deras önskemål till mötes. Många av de förslag studenterna förde fram kunde förverkligas. Deras medverkan i utvärderingen var givetvis väsentlig.

Lärarna i kursen var imponerade över studenternas konstruktiva arbete. Det skulle vara både värdefullt och trevligt att i alla kurser få arbeta så nära samman med studenterna. Studenterna är en försummad resurs i det akademiska utvecklingsarbetet. Kanske är det även så i det normala vardagsarbetet?

För mer information om projektet se
http://www.hgur.se/activities/projects/financed_projects/f-h/grevholm_barbro_00.htm

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To develop the ability of teacher students to reason mathematically

The aim of the project

The project aims at developing the ability of teacher students to reason mathematically. In their future work as teachers they have to be able to present and discuss mathematics in a fruitful, flexible way and to understand pupils reasoning in mathematics. This implies that they must possess a good ability to reason mathematically, which is not always reached in the mathematics courses in teacher training today.

The problem in focus

For a teacher to be able to talk to pupils in ways that suits the learning of the individual pupil she has a need to be capable of expressing herself in many different ways and to give arguments and to reason in such a way that the pupil understands. Teachers need to be prepared to talk about mathematical concepts in multiple ways and on different levels adjusted to the pupil's knowledge and understanding. This aim is rarely reached by the ways teacher courses are structured today. A stronger focus on mathematical reasoning and communication is necessary. Such learning situations must be part of teacher training.

Background

In most courses in mathematics for teacher students there is a need for further work with components that support the students in their development of the ability to reason mathematically. In the curriculum for the compulsory school and the upper secondary school the importance of communication in mathematics and of the opportunity for the pupils to speak and write in mathematics is underlined. In a research study at Kristianstad University (Grevholm, 1998, 2000, 2002, in press a, b) it is shown that teacher students lack a sufficient and professional language (i. e. a language suitable for teachers of mathematics for use in their profession). During the studies they start to develop a professional language. This development needs support and structure in order to help the students to reach a higher level before they start teaching. In the education of teachers of mathematics in Kristianstad University writing and communication as part of the learning process is

stressed through work in study-groups and seminars. The students carry out written tasks, oral presentations and multimedia-presentations.
The research study by Grevholm also shows that students have vague, limited or incomplete concept formation in mathematics. One important way for students to develop their mathematical concepts is to communicate in a problem-situation with fellow students. They discover differences in their concept formation and question their own or others images of the concepts. Such situations can initiate a change of concept image or a development towards a more differentiated and complete concept perception. See Grevholm (in press a).
The course description for the first course in the education program for 4-9-teachers contains the aim that students develop the ability to communicate in mathematics with the aid of different forms of expressing oneself. The second and third courses have as one aim that the students develop their own ability to talk and write in mathematics and understand how important this is for the learning of mathematics.
Thus students already work with their ability to communicate, but the above mentioned research project has made it clear that the students do not develop as far as wanted. There is a need for deeper changes and a renewal of the ways of working in order to reach the goals. This project is a means for development in the wanted direction.

What is the ability to reason mathematically?

Mathematics as a science is characterised among other things by the method of working with deductions and proofs, which means that starting from certain given conditions you can with full certainty prove propositions and theorems. To be able to follow and develop such ways of thinking and arguing you have to develop your own ability to reason mathematically. In the school curriculum deductions and proofs have been given a new importance. It is necessary for the teacher to have developed her own ability to reason mathematically in order to be able to work with the pupils to reach the aims of the curriculum in this area. The teacher must in conversations with the pupils be flexible and be able to understand different ways of thinking and reasoning. This makes it necessary for the teacher to be able to reason, to follow multiple ways of thinking, and to compare them and to value how viable they are.
With a departure in research results from mathematics education the student teachers are offered opportunities to develop their own ability to reason mathematically in a more conscious way.
What is meant by ability to reason mathematically? A good ability to reason mathematically means that you can reformulate questions and propositions in different ways, make and test conjectures, reject them or verify them, formulate counterexamples, specialise, generalise, draw conclusions, find alternative ways when you are stuck, decide if a solution is reasonable and judge the validity of arguments, describe, explain, and convince others about your arguments and to prove your claims. Mason, Burton and Stacey (1982) have demonstrated how to encourage, develop and foster the processes mentioned above, which according to the authors seem to come naturally to mathematicians. They suggest a method of working that is highly practical by starting from exemplary questions and problems and involving the learner in

the discussions. In this way a deeper awareness of the nature of mathematical thinking and reasoning can grow. The authors claim that

> *Experiencing working with students of all ages has convinced us that mathematical thinking can be improved by*
> - *Tackling questions conscientiously;*
> - *Reflecting in this experience;*
> - *Linking feelings with action;*
> - *Studying the process of resolving problems; and*
> - *Noticing how what you learn fits with your own experience.* (p. ix)

John Mason (professor in mathematics education at Open University, London) took part of the project plans in June 2001 and he said ”I have long wanted to get someone to incorporate generality and conjecturing in their classroom and see what happened over a year or more. I suspect you would see excellent gains by the students.” (Mason, 2001, personal mail communication)
Earlier experience shows that teacher students often are quite unaware of their own ways of reasoning. They see neither merits nor shortcomings in them.

Student learning in focus

Research has shown that it is necessary to expose students to explicit teaching and discussions of a certain area or phenomenon to be sure to reach good learning outcomes in that area (Niss, 2001). Knowledge and skills that are expected to be acquired incidentally will not for sure be lasting in a majority of the students. By tradition the ability to reason mathematically has been one of the aspects that students are expected to acquire en passé. The aim is to make this ability visible, teach about it and discuss it and make the students work with it and become aware about their own learning of reasoning. The students will have an active role in this part by trying out ways of thinking and reasoning on each other, by acting as ”critical friends” to each other and by evaluating their own and others mathematical reasoning. In traditional mathematics education the individual, written problem solving is the dominating student activity. A conscious and student-centred work with the development of the ability to reason mathematically means a far-reaching renewal of undergraduate mathematics education.

Comparable research or development projects in Sweden and internationally

No other development project similar to this in Sweden has been undertaken as far as known. There are some similar ideas in the project run by Andrejs Dunkels in 1995 (Dunkels, 1996). He used student activity in groups and communication in mathematics in a way similar to what we want to do. But the new idea in our project is to have the students in groups observe and focus on their own reasoning in a concrete learning-situation, comparing, analysing and valuing it and learning from this meta-cognitive activity. They relate the learning exposed both to themselves and to their future students in school.
In the international context the interest for reasoning in mathematics is evident. Already used as course books in the teacher-training program are

Mason et al (1982) and Stacey (1988). Students find the way of working suggested in these books rewarding. Research literature supports deeper work in this direction. Hanna and Jahnke (1996) discuss the role of proof and proving in different levels of the educational system.

> *Proof is an essential characteristic of mathematics and as such should be a key component in mathematics education. Translating this statement into classroom practice is not a simple matter, however, because there have been and remain differing and constantly developing views on the nature and role of proof and proving and on the norms to which it should be adhere. (p. 877)*

They strongly advocate that one of the key tasks of mathematics educators at all levels is to enhance the role of proof in teaching. Ellerton and Clarkson (1996) review recent research on mathematics and communication and present a framework relating language, mathematics and mathematics education. They give evidence for the importance of writing and problem posing in mathematics education and also discuss language and assessment in mathematics. Support can be found for the importance of focusing on mathematical reasoning and thinking in different forms. Developing mathematical reasoning in grades K-12 is the title of the yearbook 1999 from the National Council of Teachers of Mathematics (NCTM, 1999). One quotation:

> *Mathematical reasoning must stand at the centre of mathematics learning. Mathematics is a discipline that deals with abstract entities and reasoning is the tool for understanding abstraction.*

The book contains many practical examples of problems and questions that can be used as starting points for the reasoning of teacher students. In the book Lynn Steen raises questions as

- ”Can teachers teach about mathematical reasoning?”
- ”Is the ability to reason mathematically a suitable goal in school mathematics?”
- ”Can one improve one’s ability to reason mathematically through learning proofs?”

These questions seem to be similar to the ones we are discussing for this project.

Yusof and Tall (1994) report from a course, which encourages co-operative problem-solving coupled with reflection on the thinking activities involved, significant changes in students’ attitudes and perception of mathematics. Before the course most students saw mathematics as a fixed body of knowledge rather than learning to think for themselves. Half of the students declared that mathematics did not make sense. A majority declared negative attitudes to mathematics as abstract facts and procedures to be memorised, reporting anxiety, fear of new problems and lack of confidence. After the course all measures investigated improved confirming that it is possible to alter students’ perception of mathematics to see it as an active thinking process. In this research study student work was based on the ideas from Thinking mathematically.

A major overview of research on learning to think mathematically has been published by Schoenfeld (1992). He writes that learning to think mathematically means

- Developing a mathematical point of view - valuing the process of mathematization and abstraction and having predilection to apply them

- Developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure - mathematical sense-making

Theoretical background and relevant literature

It is crucial to achieve that the student learning is meaningful and this means among other things that concept development is important. There are several arguments for the decision to focus on concept development and communication.

Knowledge construction is a complex product of the human capacity to build meaning, cultural context, and evolutionary changes in relevant knowledge structures and tools for acquiring new knowledge (Novak, 1998). Novak claims that concepts play a central role in both the psychology of learning and in epistemology. He defines concept as a perceived regularity in events or objects, or records of events or objects, designated by a label. Propositions are two or more concepts linked to form a meaningful statement. Novak has used concept mapping as a tool to represent conceptual/propositional frameworks derived from clinical interviews or constructed by learners. Concept mapping has proved to be a useful tool in planning instruction and helping students learn how to learn. The ability to state knowledge propositions is central for the learning.

According to Vygotskij concepts are central carriers of meaning in our linguistic repertoire (Özerk, 1998). Concepts indicate to us how things around us are grouped, how they are connected, how they are interrelated and what characteristics they share. Development of concepts facilitates further learning and concepts connect different experiences.

The hypothesis here is that it is important for student teachers to create meaning in learning by developing concepts and structures of concepts for themselves that can be useful in future professional work. They need to be offered opportunities to communicate the personal meaning they have of the concepts and compare them with peers. To be able to understand pupils' learning and support it teachers need well developed concept structures that allow flexible thinking, interpreting and understanding. A well-developed network of links between interrelated concepts facilitates the teachers associations and opportunities to follow a student's way of thinking. This can be compared with the skills that are expected from teachers in their actions in the classroom.

Both Novak's theory of human constructivism and Vygotskij's ideas about concept development and language support this view. Cooney found that teacher students' thinking was not flexible enough (Cooney, 1994). Ausubel (1963) contrasts meaningful learning to rote learning, where meaningful learning results in the creation and assimilation of new knowledge structures. In many of the theories on learning there seems to be a continuum from a lower quality learning to a higher quality learning, where higher quality often includes concept development. The theories can be seen as different ways to model the quality of learning and how it evolves.

The constructive nature of learning demands that the individual actively creates his own conceptions. The two research traditions, the cognitive research and the sociocultural research have inspired each other and partly converged in the end of last century. In the later tradition it is central that the thinking develops trough communication in speech and writing about a meaningful content (Madsén, 2002).

A useful source in the creating of tasks have been a Danish work-card book for the teacher training, Matematik i laereruddannelsen, Undersöge, konstruere og argumentere (Beck, Hansen, Jörgensen Örsted Petersen & Bollerslev, 2001). The book is meant to give the reader opportunities to investigate mathematical situations, construct mathematical knowledge and argue about and with mathematics and mathematics education. In focus is the aim that the reader builds a competence in developing mathematics and didactical ways of thinking.
Other resources for inspiration of new problems have been the NCTM Yearbook and the literature from Open University mention above.

Project realization

The overall situation
Teacher education is at present under reform in Sweden. With a start in the autumn 2001 students meet a totally changed education. Teacher education programs are therefore in the middle of a fundamental restructuring and development of the courses, also in mathematics. The project has opened an opportunity to develop the kind of student-centred courses focusing on creativity that has long been asked for in the mathematics education literature (e.g. Schoenfeld, 1992).

Phases of the project
The project consists of different phases that all have demanded time to work out. There is a planning phase where also some of the students have been important active participants. This part contains deeper reading of the literature and selecting possible material in the form of examples, problems and investigations. A discussion about the choice of material, the amount of work with reasoning, forms of working, assessing, documenting and reporting had to take place with the teachers and students. For the documentation and evaluation preparations have been made and work carried through, the pre- and post-tests had to be constructed and carried through, and the actual teaching and student activities had to be acted out. All this demanded time for work and reflection, writing and evaluation.

The participants
The project leader, five other mathematics teachers and four students have formed the working group that planned, implemented, documented and evaluated the new ideas. Students in two teacher training courses have carried through the group problems. One was GS6, that is term six in the earlier education. The other was a new course with students aiming for year 4-9 or upper secondary school.

The teaching and learning situation for students

The courses consist of lectures, seminars, laboratory work, problem-solving sessions and natural study groups. The latter are sessions where the students work in groups of four or five without a teacher. They get a group task and spend 90 minutes together to discuss, solve and answer the problem and construct a written group report. The report is handed in and read and commented by the teacher. Findings in the reports, that the teachers judge as important, are discussed in following lectures. The natural study group sessions were found most suitable for the kind of work the project could build on. Natural study groups have been used since 1996 in the program and this concept is inspired by the research of Tresman (1992), who showed the importance of students' work in groups without the teacher.
The group of teachers in the project together with four selected students in the course group worked during autumn 2001 and until the course start in February 2002 with creating group task that could suit the aims of the project. The course content was analysed and the most important concepts focused. This was made with the aid of concept maps (Novak, 1998). (See one example in Appendix 1). Relevant literature was read and used to create ideas. For each course part a number of group tasks were constructed, written down, discussed in the planning group, refined and finalised. (See one example in Appendix 2). The work in the whole student group was done in small groups, that changed from one day to the next (a wish by the students). The focus on reporting the work was on presenting their findings and on convincing others about the reasonableness of the conclusions they have drawn.
In order to bridge the gap between old and new courses (the new mathematics courses started in the spring 2002), the start in autumn 2001 was made by planning and creating material for reforming one of the earlier courses that has three parts dealing with statistics, function theory and geometry. Such material could be implemented also in one of the new courses during Spring 2002. The courses have two parallel tracks: one major dealing with mathematics and one minor in mathematics education. The aim was to develop mainly the mathematics part.

Instead of starting in a traditional way by presenting the theory and then continuing with problem solving exercising the application of the theory, the start was often taken by using open or exploratory problem solving, that gave the students an experiential background for the introduction of theoretical concepts. To work with different pieces of mathematical argument, both from former teacher students and pupils, was another type of task. Data in the form of such examples was taken from the research study by Grevholm. It could be for example written solutions, reports, essays and similar things. Alongside students worked with a selected number of more conventional mathematical problems, but stimulated to continue to use the group as a resource, discussing different difficulties and explaining to each other different ways of solving the problems. Comparing and valuing the viability of alternative solutions was vital here.

The importance of student participation in the working group
During the whole project four students have taken part in the planning, implementation and the development of the courses, and have continually evaluated the work and expressed their opinion about the way the project preceded.

The student participation in the planning and development of the project has been of high importance. Students, that earlier have graduated from teacher education at Kristianstad University, have been able to assist with actual learning situations from their practices. Data from their lessons have been collected and used in the group work. These situations have served as starting points for reasoning, where younger students consider how they could explain difficult concepts to pupils in different situations. During practice they can try their ideas and discuss them with more experienced teachers.
The teachers participating in the project have different and complementing experience. The more experienced teachers have knowledge about teacher education in mathematics from different systems and the younger ones see the situation with fresh eyes as fairly new teacher educators. This can create a stimulating discussion where both the history of Swedish teacher education and new ideas can contribute. For both of these groups it is highly valuable to listen to student teachers' perspectives on their education.

The creation of tasks
The teachers created tasks for the work in natural study groups with the aim to create learning situations that could open for stimulating and challenging discussions and explorations in the student groups. Central concepts in the courses were chosen as content of the tasks. The tasks consisted of both mathematical parts for students to work with and investigate and mathematics education tasks for them to reflect on and consider as future teachers. See example in Appendix 2. In all about 40 tasks were created and tested. We intend to collect all the tasks in a booklet after they have been revised according to experiences from the first project year. This booklet will be available free.

Documentation of the project

Planning process recorded
In the planning phase written notes that often were disseminated to the whole group documented the meetings with students and teachers. The teachers took notes from interesting parts in the read literature and their reflections about it. The concept maps (see appendix 1) were disseminated to all and discussed in the group. The students found the overview given by the concept maps especially valuable. They suggested that the maps could also be used in the course to help students structure their knowledge.

Observations and video-taping
The project tasks where documented while carried through by the student groups by video-taping at least one group and one or two teachers taking observation notes in other groups. Thus it was possible to make an immediate comparison and evaluation of how the task worked. The processes in different groups were often similar and gave the teachers a good picture of the level of knowledge and reasoning among the students.

Student records
Each group handed in their written common report and it was read and commented on by the teacher. Before handed back to the student it was copied as part of the documentation. The students' written answers at the

examination were collected and copied. The teachers sometimes used ideas from student reports as the basis for discussion in following lectures.

Questionnaire on attitudes
The questionnaire by Yusof and Tall, mentioned above, was translated to Swedish and slightly changed. It was given to the students in class before and after the course.
See appendix 4.

Students' oral and written report
The four students, that took part in the working group, gave oral comments in each meeting (often once a week) and written comments after the work was finished.

Teachers' comments and experiences
The participating teachers made continuous observations and took notes of their reflections. After the respective course was over they were asked to write down their comments on the experiences made on each group task. The experiences could be related back at once to the work in the student group and followed up according to findings.

Results

An instrument for measuring the ability to reason is not easy to find or construct. Thus we have to rely on observations and on the students' own judgement of the learning effect of the work in the project.
First of all the students agree that it is very important for them to try to develop their ability to reason mathematically. They are aware of their lack of professional language and varied ways of explaining their thinking. This conforms the observations of Cooney (1998). The students can see that the experiences gained in the group activities are highly useful for them as new teachers in school. They even claim that the special tasks in the project have given them a base of good teaching ideas to start with in the first position as teacher. The focus on central concepts and the use of concept maps was highly appreciated by the students (Novak, 1998).
The students report that they have developed a greater courage to formulate questions and ask them both to fellow students and teachers. They feel that their ability of reasoning has developed and they find words to express their mathematical thoughts easier. Normally two to three group project activities took place each study week, which means that around 50 hours were spent on reasoning and discussion. For the students one risk was that the traditional problem solving, that could have taken place during this time, was not done as homework and thus a lower degree of practising common procedures was achieved. However the students had plenty of time for home- work and the practise of traditional problem-solving.
The students find that the learning by listening and explaining to each other is strong and understand that this is something they must allow their own pupils to experience. Students felt more safe and secure in the lessons. They developed a better collegiate climate and improved collaborative work. The students will take the ideas they met into their own work in school. They

emphasised the importance of mathematical conversations in the classroom (Özerk, 1998).

Students report other positive effects of the project that were not expected. As they learnt to co-operate with all fellow students through the changing groups the social climate in the group improved much. They claim that everybody was included in the collegiate work, they met more often out of lessons and it became more natural to ask questions about not yet understood parts of the course to anyone in the group. A climate of willingness to assist and help each other grew. Attitudes are important as an influencing factor for the learning (Yusof and Tall, 1998).
The four students that took part in the teachers working group got a good picture of how teachers plan and prepare a course. They saw that the teachers made a lot of efforts to make the learning as good as possible for the students and worked hard to create rewarding learning situations. This picture obviously was transferred to the whole study group and they became aware of the teachers' wish and willingness to work hard to support student learning.
For the teachers the project became a competence development. The working group meetings opened an opportunity to take part in deep discussions about the mathematical content of the course and important concepts to focus on.
To do this together with students was a new and rewarding experience.

Students' attitudes to mathematics
In order to find out if and how the students' attitudes to mathematics and problem solving were influenced during the project we used a questionnaire developed by Yusof and Tall (1998). The questionnaire was translated into Swedish and adjusted to the needs here. A full report on the investigation is given in Appendix 4. The report is written in cooperation with Kristina Juter.
As in Yusof's and Tall's investigation the students achieved more positive attitudes to mathematics.
A comparison of the results (for Gs6) of the questionnaire before and after the course shows for example that a larger part of the students think that mathematics is a collection of facts and processes to remember before the course than after. More students think that mathematics is meaningful for them after the course than before. No change has taken place regarding rote learning or ability to link ideas together. The attitudes to problem solving have not changed much.

Teachers' comments and reflexions after the project
The participating teachers have reflected on the experiences gathered from evaluations made by the students and from the reports. Many observations could be used at once and carried back to students in the lectures after the reports were handed in. They served as motivating facts for deeper discussions on the learning in the groups. Other observations have been used by the teachers to revise and adjust the problems to become more clear and better concerning the level of difficulty. Especially in the sub course on functions many of the problems first given were too hard for the groups. An explanation for that can be that this course is normally considered to be harder than the other two. But it can also be due to the fact the teacher responsible for this part of the course was new to the course and not able to judge what could be expected from the teacher students. All problems are going to be revised

according to discussions with the participating students and in the teacher group.

Evaluation and dissemination of results

As a part of the project new parts for the examination have been developed. Since one of the most determining factors for student learning is the examination used, these will naturally have to support the kind of reasoning skills that are fostered here. The students have worked with exploratory tasks on their own (an example of problem-posing) and similar tasks have been used as examination tasks.

One evaluation method was the use of a pre- and post-test focusing on attitudes to mathematics that are related to the reasoning skills aimed for. From research reported by Yusof and Tall (1998) we developed an instrument to meet this end. The result from the questionnaire is not obviously connected to the project work. Many other factors are influencing students attitudes. Continually undertaken interviews with some of the students have taken place in order to follow the development of the project. The interviews have been documented and analysed. Students were more satisfied with the work than the teachers had expected. The group tasks were challenging and demanding for students but still they subjectively report a good learning outcome from them.

The methods, problems and tasks used and developed as part of the project will be published through the Council for use by anyone and dissemination of the results through presentations at conferences have started and will continue. Articles in journals are planned. The examples, created problems and pieces of teaching that are used have been documented and evaluated. The parts that are judged useful can be exposed on the Internet for everyone to use. When the project period is over the working methods and ways of viewing reasoning must be so well integrated in the courses that they will last without extra financial support. The results will be possible to generalise and will be lasting after the project is closed and be useful for others. It is desirable that all mathematics teacher education include parts of this kind of work in order to meet the needs of the teacher students.

Contact has been taken with the project run by Kerstin Ekstig in Uppsala. A two-day meeting for all the teachers was arranged in August 19-20 where we compared experiences and investigated similarities and differences in the two projects. This is highly interesting and will be discussed in the report of the second year of this project.

During the summer Gerd Brandell reported on the project on a conference on research on teacher education in Park City Institute in USA. She was allowed to borrow one video (students agreed). The transcription and translation in Appendix 3 is made by her from the video and then checked, corrected and adjusted by me. This group session is on problem 1 in Appendix 2 and deals with statistical measures. It gives a good picture of student reasoning and the level of the discussions. The participants of the conference found the project highly interesting and want to continue the cooperation.

In September 2002 I will give a lecture together with Kerstin Ekstig on our projects at the yearly national conference for teacher educators in mathematics, LUMA. It is also possible to try to present the project at CERME3, The Third European Congress on Mathematics Education.

Continuation of the work

In the spring 2002 the reform project was carried through in the mathematics courses dealing with geometry, functions and statistics in term six, GS6. Alongside with that the first new course in the new reformed teacher training was given. The project required special developmental efforts. But without any extra resources the project material could be used in the new course. The developmental work will continue in the autumn 2002, and then in the spring 2003 the improved group tasks will be refined and methods for the work in the basic courses will be consolidated (courses of all together 40 points). In the autumn 2003 a last step will be taken in developing a specialised course (20 points) building on the basic courses. After that hopefully the created material and methods can be used by any mathematics teacher, who wants to use it.

Acknowledgements

The students and teachers participating in the project thank the Council for the Renewal of Higher Education for supporting this work. It has been highly rewarding for all the participants and the methods and material developed can be used in future student groups. We also thank professor John Mason, Open University, UK and professor Gilah Leder, La Trobe University, Melbourne for support and discussions during the work.

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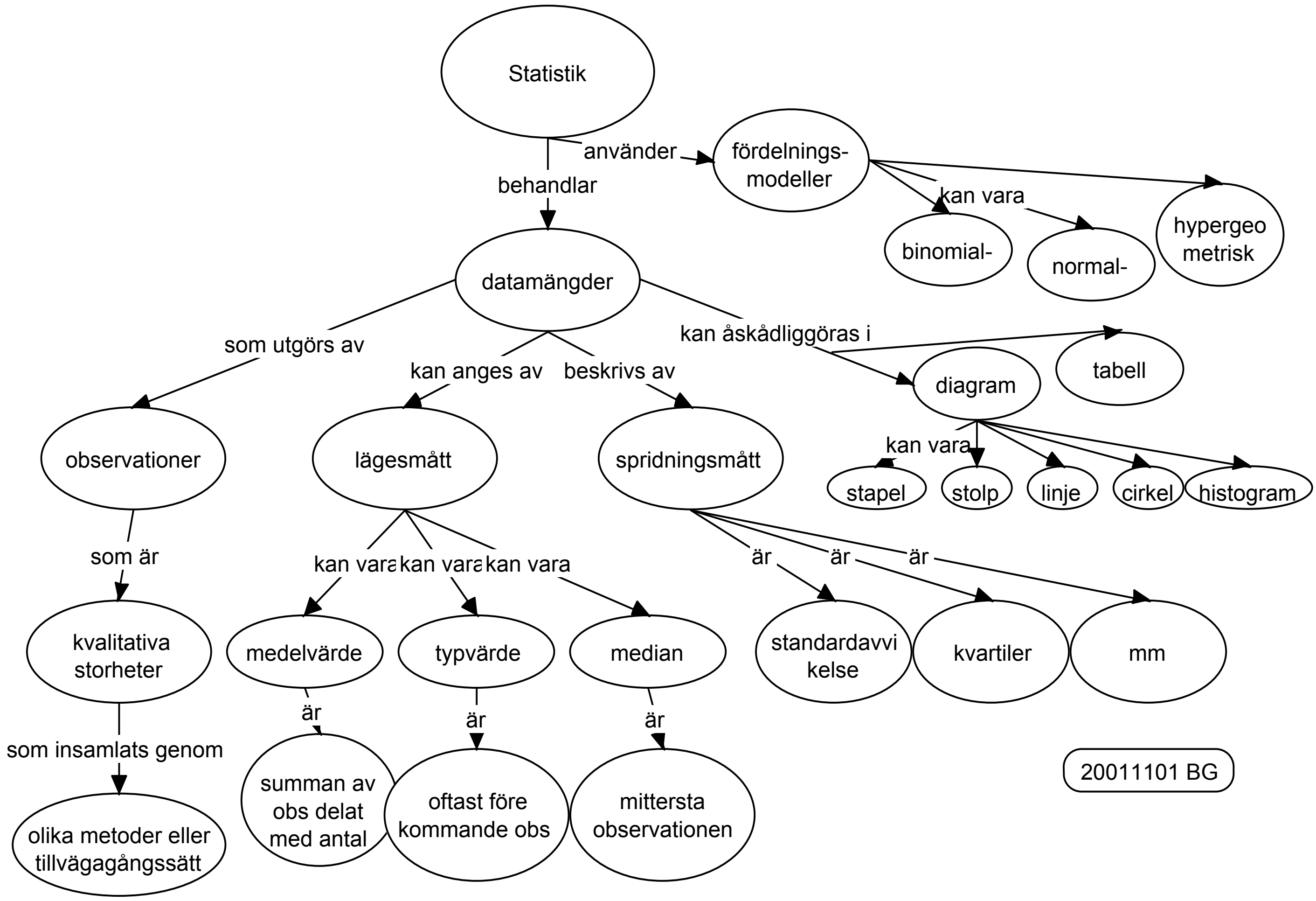
Remark Participating students were Caroline Olofsson, Elin Olsson, Camilla Persson and Fredrik Söderberg.
Participating teachers were Barbro Grevholm (project leader), Thomas Dahl, Inger Holmberg, Kristina Juter and Örjan Hansson. (Lena Hansson took part in some of the meetings and the examination, but was not responsible for project tasks.)

Appendix 1 En begreppskarta över innehållet i kursen beskrivande statistik



Appendix 2 Examples of group tasks from the project (Statistics)

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Uppgift 1 Talar direktören i High-Tec sanning om lönerna?

Direktören Birger Jonasson i IKT-företaget High-Tec intervjuas i TV och berättar att företagets lönenivå är hög. De tretton anställda har en medellön på 166555 kr per månad. Typvärdet för månadslönen är 1 miljon kronor.

Reportern frågar hur stor medianlönen är.
”Ja, den är 16 000 kr per månad men det är ju inte så intressant i det här sammanhanget”, hävdar direktören.

Fråga 1 Talar direktören sanning? Kan uppgifterna om lönerna verkligen stämma? Hur skulle lönebilden kunna se ut?

Fråga 2 Tre olika statistiska lägesmått nämns i texten. När är det ena eller andra måttet relevant att använda? Hur valde direktören mått och varför kan han tänkas ha gjort så?

Fråga 3 Hur vill du planera ett undervisningsavsnitt om lägesmått för elever i år 5 respektive år 9? Gör ett utkast till en uppläggning som du tror är bra och motivera varför du valt denna modell. Vilken kunskap om lägesmått anser du är viktig för eleverna?

Fråga 4 Vad lärde du dig av denna övning? Hur skiljer den sig från tidigare uppgifter du löst som handlat om lägesmått? Kan elever i grundskolan lösa denna typ av uppgifter? Finner du sådana uppgifter i läroböckerna?

Material: Utdrag ur boken Matematikterminologi i skolan, utdrag ur kursplan i matematik för grundskolan.

Gruppens gemensamma skriftliga redovisning lämnas den 27 februari 2002.

Kommentarer till uppgift 1

Syftet med uppgiften är att låta studenterna möta lägesmått från ett annat perspektiv än det vanliga, där data är givna och lägesmåttet ska beräknas. Förhoppningen är att den omvända frågeställningen ska inbjuda till resonemang och argument när de försöker hitta en tänkbar löneprofil. Situationen är också ovanlig för dem genom att de måste förstå att det inte kan finnas ett enda svar på uppgiften. Situationen bör även inbjuda till att lära sig använda en sunt skeptiskt hållning till statistiska uppgifter som ges i media eller på annat sätt.
Syftet är också att de ska få tillfälle att tänka igenom och resonera med varandra om de olika lägesmåttens egenskaper och hur de kan brukas och missbrukas.
I den naturliga följduppgiften att planera en undervisningssekvens får studenterna tillfälle att ta ställning till vad är verkligt viktig kunskap för elever i år 5 respektive 9.

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Uppgift 2 Missbruk av statistik

Studera exemplen som du får nedan på hur statistik kan användas i press och media. Diskutera i gruppen vad ni speciellt lägger märke till. Finns det inslag som man bör förhålla sig särskilt kritisk till? Har ni lagt märke till några speciellt vanliga sätt att försöka vilseleda med statistik?

Notera alla de punkter där ni i gruppen vill rikta kritik mot det som presenteras i exemplen. Motivera er kritik.

Exempel 1 *Antalet varsel ökade*. Vilka diagramtyper har man använt här? Är de lättolkade och tydliga? Risk för missförstånd på någon punkt? Andra kommentarer?

Exempel 2 *Sydkraft*. Kommentarer till utformningen av diagrammen? Vinklat?

Exempel 3 *Bonden får inte hälften*. Är det vanligt att man försöker använda bilder som diagram? Synpunkter på lämplighet och utformning? Kritiska aspekter?

Exempel 4 *De nya siffrorna om kärnkraften*. Hur är diagrammet utformat? Risk för misstolkning? Vilken skala använder man? Kommentarer och kritik?

Exempel 5 *Fälldin har krympt*. En illustration för att liva upp har använts men hur brukas den? Är bilden missvisande och i så fall på vilket sätt?

Exempel 6 *Ingen minskning av antibiotika*. Studera diagrammet. Vad är det som visas? Kan genomsnittet märkt Skånesnitt beräknas ur diagrammets uppgifter? Rimligen borde det som sticker upp över snittnivån till vänster om snittet vara lika mycket som det som ligger under snittnivån till höger om snittet, eller hur? Är det det? Om inte, vad beror det på? Hur är snittet beräknat?

Diskutera frågan om när elever i grundskolan kan vara mogna att lära sig att kritiskt ta ställning till statistiska presentationer? Tycker du att det är viktigt att eleverna övar sig i att kritiskt granska material? Varför? Hur kan du som lärare ta upp detta med eleverna och erbjuda dem erfarenhet av kritisk granskning av bruk av statistik? Vilken typ av material eller exempel vill du använda?

Hur kan eleverna aktiveras under arbete med detta avsnitt? Ge exempel.

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Uppgift 3 Hur många spel?

Tävlingskommittén i en idrottsorganisation ska planera två turneringar. I båda fallen ska 32 lag deltaga.

Lös först uppgiften 1 respektive 2 individuellt så långt du kan. Diskutera därefter era lösningar i gruppen och förklara för varandra hur ni tänkt.

1. I den första turneringen ska alla lag möta alla andra lag en gång. Hur många spel ska genomföras?

2. I den andra turneringen ska man spela så kallat cup-spel. Lagen möts två och två efter lottdragning. Vinnaren går vidare, förloraren är ute ur turneringen. Vinnarna möts två och två enligt samma princip. Spelen fortgår tills ett enda lag är kvar. Hur många spel ska genomföras innan man har en slutlig vinnare?

Diskutera därefter båda uppgifterna parallellt i gruppen och förklara tillvägagångssätt och resultat för varandra. Om någon har använt formler från tidigare matematikstudier bör den förklara hur det fungerar för de andra gruppmedlemmarna.

Lös nu uppgiften på nytt men med 30 deltagande lag i tävlingen. Vad upptäcker du som skiljer från det tidigare fallet?

Pröva nu om ni kan generalisera resultaten så att de kommer att gälla för ett godtyckligt antal lag i tävlingarna. Kom ihåg vad ni lärde termin 1 i kursen om problemlösning genom boken Thinking mathematically. Har du kört fast? Pröva att lösa ett enklare specialfall först. Kan du därifrån gå till ett mera allmänt fall? Eller lös uppgiften i några olika enkla fall och försök lista ut genom en tabell över de olika fallen vad som gäller generellt.

Diskutera slutligen om den här typen av problem lämpar sig för elever i grundskolan. Är de vanligt förekommande i läroböckerna i matematik?

Uppgift 4 På hur många sätt kan kulorna läggas i glasen? Ett samtal från klassrummet

Eleverna arbetar med problemet:
På hur många sätt kan man lägga tre kulor – en röd, en gul och en grön – i tre glas så att minst ett glas förblir tomt?

Så här säger elev nummer 1
Det var då lätt. Först väljer du ett tomt glas. Det kan göras på tre sätt. Så kan du fördela de tre kulorna fritt mellan de andra två glasen på $2 \cdot 2 \cdot 2 = 8$ sätt. Totalt är det alltså möjligt att få $3 \cdot 8 = 24$ sätt att göra det på.

Så här säger elev nummer 2
Nej, nej, nej! Så här ska man göra: Först placerar vi den första kulan - tre möjligheter. Så placerar vi den andra kulan - också tre möjligheter. För den sista kulan finns det nu endast två möjligheter eftersom minst ett av glasen ska vara tomt. Alltså $3 \cdot 3 \cdot 2 = 18$ möjligheter totalt.

Elev nummer 3 säger då
Ni är helt förvirrade båda två! Nä, nu ska ni få höra. Först räknar vi möjligheterna att få precis ett tomt glas. Vi kan placera en kula fritt på tre sätt. Den andra placerar vi i ett av de andra glasen. Det kan göras på två sätt. Den tredje kulan läggs nu i samma som en av de två andra: två möjligheter. Totalt finns det alltså $3 \cdot 2 \cdot 2 = 12$ möjligheter med precis ett tomt glas.
Med två tomma glas måste vi välja ett glas att lägga alla kulorna i. Det kan göras på tre sätt. Alltså finns det $12 + 3 = 15$ olika sätt att göra det på.

Här står du nu som lärare och alla eleverna ser frågande på dig. Du förväntas ge expertens avgörande.

Din första reaktion blir kanske att säga: Gå hem och tänk över problemet. Det var bra att ni tog upp det för jag hade tänkt att det skulle vara läxuppgift till i morgon.
Då har du räddat dig för stunden och fått lite tid till att tänka igenom vad eleverna sagt. Men du måste till slut svara på:

- Vad är det rätta svaret?

- Vad är det som gör att det skiljer sig mellan de olika svaren? Det vill säga vilka kombinationer räknas för många respektive för få gånger i de olika svaren jämfört med det rätta?

Diskutera i gruppen hur ni skulle göra om ni var i lärarens ställe och vilka lösningar ni skulle erbjuda eleverna och hur de kan förklaras. Hur anser ni att den rätta lösningen ska utformas och bäst presenteras? Redovisa er egen rätta

lösning med motiveringar. Kan eleverna själva reda ut läget? På vilket sätt kan du hjälpa dem till en sådan situation?
Gruppens gemensamma skriftliga redovisning inlämnas senast den 6 mars.

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Uppgift 5 Bilen och getterna

På sidan 3 i din lärobok beskrivs problemet Bilen och getterna. Det är hämtat från en TV-show i USA och ingår i en tävling. Den tävlande ställs framför tre dörrar. Bakom två av dörrarna finns en get och bakom en dörr finns en bil. Den tävlande får välja en dörr varefter programledaren öppnar en av de andra dörrarna. Programledaren öppnar alltid en dörr med en get bakom, vilket alltid är möjligt. Därefter får den tävlande frågan om han/hon vill byta dörr. Frågan är nu om den tävlande ökar sina vinstchanser genom att byta dörr.

Vad svarar ni i gruppen på frågan om den tävlande ökar sina vinstchanser genom att alltid byta dörr vid valet jämfört med att hålla fast vid sitt ursprungliga val? Diskutera igenom situationen tills ni är eniga.

Problemet fick stor publicitet då Marilyn vos Savant i tidskriften Parade Magazin påstod att chansen att vinna fördubblades om den tävlande använde strategin att alltid byta dörr jämfört med att hålla fast vid ursprungsvalet.

Hade Marilyn vos Savant rätt i sitt påstående? För att testa hennes påstående kan ni genomföra tävlingen några gånger med ett modellmaterial som ni konstruerar. Ni får lämpligt material att arbeta med.

Hur kan du med hjälp av statistisk teori som du nu lärt dig bevisa vad som gäller i den beskrivna situationen?

Använd eventuellt lärobokens tips att simulera problemet på någon hemsida, sökord Monty Hall.

Kan man finna detta problem i svenska matematikläroböcker?

Kan problemet användas i grundskolan och i så fall hur?

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Uppgift 6 Slumpvariation i en stickprovspopulation

Media rapporterar regelbundet från SIFOs mätningar av väljarsympatierna. Tänk dig att vi har en medelstor ort i Sverige där väljarna just nu fördelar sina sympatier jämt mellan blocken. Om SIFO tillfrågar ett stickprov av 400 väljare kan vi förvänta oss att 200 stöder det borgerliga blocket och 200 de socialistiska. Nu vet vi att om vi gör ett sådan stickprov flera gånger får vi lite olika resultat på grund av slumpvariationen. Men hur mycket varierar resultatet egentligen? Om vi gör ett stort antal stickprov hur fördelar sig andelen väljare på respektive block? Hur nära kommer vi andelen 0,5 som är det sanna värdet om det väger jämt mellan blocken? Hur stor blir variationen mellan de olika andelar vi får fram?
Nu är det dyrt och tidsödande att gå ut och fråga väljare så vi kan simulera försöket genom att kasta mynt istället.
Andelen borgerliga i ett stickprov på 400 av väljarna på denna ort skulle alltså bli densamma som andelen krona upp vid n = 400 kast med ett symmetriskt mynt.
Vi kan få en tillräckligt bra illustration av slumpvariationen genom att kasta 40 mynt i stället för 400 och att upprepa det försöket många gånger. Om alla i hela gruppen medverkar med sina resultat är det rimligt att göra 100 sådana kastserier.
Syftet med laborationen är att

- undersöka hur fördelningen av resultaten i kastserien ser ut,
- undersöka vad medelvärdet för de olika utfallen blir jämfört med det förväntade
- undersöka hur långt från det förväntade värdet 0,5 vi måste gå för att fånga in 95 % av alla utfall

Gör så här: Arbeta i par. Läs igenom hela instruktionen först. Utse en som gör sammanfattande protokoll för hela gruppen. Varje par gör först 40 myntkast och noterar antalet krona upp. Använd 4 mynt som du kastar samtidigt 10 gånger. Antalet krona i de 40 kasten noteras och rapporteras till den som utsetts att vara protokollförare och som gör ett protokoll på tavlan. Gör därefter om kastserien och rapportera resultaten. Försöket pågår tills grupperna sammanlagt har gjort 100 sådana kastserier.
I paret gör ni ett protokoll över kastserierna. Det kan till exempel se ut så här:
Arbetstabell 1 Antal krona upp vid samtidigt kast av 4 enkronor

Kastserie nr	1	2	3	4	5	6	7	8	9	10	11
Kast nr											
1											
2											
3											
4											
5											
6											
7											

8
9
10
Summa ....

På tavlan görs följande sammanställning av den som utsetts att göra det. Rita i ditt eget anteckningsblock en tabell som den nedan och gör den komplett med plats för alla de 40 möjliga utfallen. Beräkna och fyll i resten av p-värdena. Anteckna era erhållna frekvenser när alla de 100 kasten är klara.

Arbetstabell 2 Antalet och andelen krona upp i 100 serier om n = 40 myntkast

Antal krona (x)	Andelen krona (p = x/40)	Antal observationer i klassen (f)	Relativ frekvens f/100
<6	0,150		
7	0,175		
8	0,200		
9	osv		
...	...		
...	...		
...	...		
30			
31			
32			
33			
>34			
		Summa 100	Summa ≈ 1 (Kontrollera)

Uppgifter

1. Försök formulera en hypotes för hur ni tror att fördelningen kommer att se ut med alla resultaten markerade i ett diagram över frekvensen.

2. Beräkna medelvärdet av de hundra värden ni fått för x/40 (dvs antal krona genom 40).

3. Beräkna hur långt bort från mitten man måste gå för att fånga in 95 % av resultaten.

4. Rita ett histogram över alla resultaten. Klassindela först materialet i arbetstabell 2 genom att till exempel föra samman värdena tre och tre.

5. Diskutera vilka slutsatser ni kan dra med anledning av undersökningen. Har ni tidigare stött på en fördelning som den ni fått fram i försöket? I vilka andra situationer kan man vänta sig att få en fördelning av detta slag?

6. Lämpar det sig att i grundskolan behandla den fördelning ni undersökt i laborationen? Hur skulle man i så fall kunna göra det? Tar grundskolans böcker upp något om fördelningar i statistik?

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Uppgift 7 Simulering

Hypotestprövning i skolan

Till din klass i grundskolan tar du med en spann med tusen röda och vita kulor. Eleverna får röra runt i spannen och frågar om där är flest röda eller vita. Att räkna alla verkar alltför tidsödande så någon förslår att man ska göra ett stickprov.

Ni tar ett stickprov med 20 kulor och det visar sig innehålla 8 röda. Eleverna får diksutera vad detta innebär. Vad vet man nu om hur kulorna fördelar sig på vita och röda i spannen?

Med en enkel modell kanske vi vill uppskatta antalet kulor till 400 i spannen. Hur kan en elev som helst vill tänka konkret komma fram till den slutsatsen?

Några elever kommer att ifrågasätta resultatet att där är 400 röda kulor. Gör ett nytt stickprov. Vad visar det?
Låt klassens alla elever göra ett stickprov med 20 kulor. Anteckna resultaten.

Som lärare vet du att du kan simulera den hypergeometriska modellen på Excell. Skriv i Excell =HYPERGEO(A10;20;400;1000) så får du sannolikheten att ett utfall ger det antal röda kulor som står i cell A10 när det finns 1000 kulor varav 400 är röda och du tar ett stickprov på 20 kulor. Låt cellerna A2 till A22 innehålla talen 1 till 20 och placera den uträknade sannolikheten i cellerna B2 till B22.

Hur stämmer den teoretisk fördelning du får fram här med den eleverna fått i sina stickprover. Spela rollen av elever själva och gör jämförelsen.

Hur kan du diskutera med eleverna vad de kan lära av denna övning med stickprov?

Tillägg till Laboration nummer 3

Simulering av ett klassiskt problem

De följande två uppgifterna skapar ofta diskussion

Uppgift 1
En kvinna träffar en gammal vän, som hon inte sett på många år. Hon har fått veta att vännen har två barn och att det ena barnet är en flicka. Hur stor är sannolikheten att det andra barnet också är en flicka?

Uppgift 2
En kvinna träffar en gammal vän, som hon inte sett på många år. Hon har fått veta att vännen har två barn och att det äldsta barnet är en flicka. Hur stor är sannolikheten att det andra barnet också är en flicka?

Diskutera:
Är det skillnad på sannolikheterna för att det andra barnet är en flicka i de två situationerna? Om inte, vad är då sannolikheten? Om ja vad är i så fall de två sannolikheterna?

Du kan simulera försöken genom att kasta mynt ett stort antal gånger. Om du tycker det tar för lång tid använd då datorn. Gör till exempel en Excellutskrift med 600 myntkast och dela upp dem i grupper på två (tvåbarnsfamiljer). Undersök resultatet.
Kan du med hjälp av simuleringen ge en kombinatorisk förklaring till resultatet?
Vilken kritik kan man rikta mot den matematiska modell som används i simuleringen?

Hur många försök måste man utföra i en simulering för att vara säker på att försöket ska visa det rätta resultatet? Diskutera frågan i gruppen.

Vad betyder *De stora talens lag*?

En tänkbar uppgift för tentamen. Någon variant av denna uppgift kan användas. Den prövar då i vilken mån studenten lärt sig något från laboration 1 *Hundkapplöpning*.

Högskolan Kristianstad
Matematik
Barbro Grevholm, 2001-12-17

Förslag till problem inom sannolikhetsläran

Britney Spears utmanar Arn Magnusson i ett tärningsspel. De kastar två tärningar. Om tärningarnas summa är 4 eller 10 får Britney ett poäng. Hon har valt sina lyckotal. Om tärningarnas summa är 7 får Arn ett poäng.
Är spelet rättvist?
Vem vinner om de kastar minst 100 gånger?

Kommentar:
1. Vad ska man mena med ett rättvist spel? Diskutera det och ge förslag.
2. Ställ upp en hypotes om resultatet av spelet.
3. Undersök om din hypotes är rimlig genom att spela spelet.
4. Kan du bevisa hur det förhåller sig med sannolikheten att ett kast ger summan 7 respektive 4 eller 10? Genomför beviset och övertyga dina kamrater.
5. Om du vill låta elever i år 9 arbeta med detta experiment, vilka olika sätt vill du visa dem för att reda ut hur det förhåller sig med sannolikheterna?
6. Konstruera några problem som handlar om kast med tärningar och lämpar sig för högstadiet.

Appendix 3 Transcription of parts of a group work session at Kristianstad University, January 2002

Participants are five teacher students in year 3. The whole session was video filmed and is between 60 and 70 minutes long. Seven parts of about 22 minutes in all is shown on an edited version of the film. The goal with the shortened version was to show students' work with all four questions in the task, and to show different aspects and results of the group work. The edited film has been transcribed. The parts that were left lout are indicated in the transcription as well as their length.

The students are called, from left to right on the film:
Peter, John, Mary, Ann and Brian

(2 minutes to get started, John is a little late)

Ann I don't know what mode means.
John No I have no idea either – I will check in the book.
Mary Lucky that you brought the book.
Ann There is an index in the back of the book.
John What did you say – mode, page 52 and 211. Mode *(reads aloud)* "The scale for which no measure of location is indicated is a nominal scale. The measure of location that is used in that case is the mode."
There is a little more. Let us see. Here is one example on page 29. About ice creams. It is always easier with examples. Here is a scale with different ice creams. We can see the different choices. Then it says: *(reads aloud)* "In the data material with the types of ice creams that was presented in example 3.1, page 29 the mode will be magnum classic." It is the one with most numbers, so it must be the highest salary.
Brian The highest salary.
John Yes, that's it.
Brian I wonder what the lowest is called then?
Peter Yes.
(Laughs)

(7 minutes during which the students agree on what the median is, start to do some computations from the median and the average value, and start to discuss the mode again)

John It is to be found here in the terminology of mathematics *(a booklet),* all concepts are there.
Peter It also says – you are not allowed to use the calculator.
John Here it is. The mode. *(Reads aloud)* "The number or numbers with the largest frequency is called the mode. Median is the middle observation."
Ann What did you say – the largest frequency – did you say that?
John Yes.
Ann Well, then it is the one, which most persons have. The largest frequency is the one with the largest number.
John Is it? Perhaps. Let's see.
Peter So the one with the largest number.
Ann So we have done wrong?
John We have done wrong.
Peter It can't be.
Ann There are thirteen employed persons so then seven must...
John Then two must have...
Peter But have we found out about any salary?
Brian The picture of the salaries would be interesting.
Ann That means that seven persons, at least seven persons must have a salary of one million per month.
Peter No, but it can't be.
John Not seven.
Peter No, because if thirteen employed have an average salary of 166 000 per month it must be...

Ann	Oh, how stupid I am. No they do not mean half – most of – exactly. Now I am making mistakes again – but we are getting nearer.
Peter	Then two may have…
Ann	And then there may not be two others who have the same salaries.
Peter	The other six may have sixteen, fifteen, fourteen, thirteen…
Brian	Yes, they may.
Mary	They may have 10 also.
Brian	They may have ten, easily.
John	But in all their salaries add up to 2 millions 165 000 crowns.
Brian	Yes, that's it.
Ann	Two persons have two millions.
Peter	Two persons have one million each.
John	So there is 165 000 crowns left – for eleven persons.
Mary	Why eleven?
John	There were thirteen.
Ann	Dud we use that the median is sixteen?
John	That means that all have about sixteen.
Mary	Then I start all over again.
John	At least – well only two can have one million. The mode…
Brian	Two millions – how did you do that?
John	Well it says. The mode for the salaries is one million crowns and the mode is the value with the largest frequency.
Brian	The largest salary – it is the largest salary.
John	No. The largest number. Two persons have one million – the others do not have the same salary.
Ann	Let's check the ice creams.
John	The frequency – it is the number of times a specific result appears. So the salary one million appears twice. And there is no other salary that is alike – one has 16 000, no one else has 16 000, they may have 16 100, so all other salaries differ.
Brian	So that is what is meant by mode?
John	Yes, here it says what mode is…
Ann	I do not find the ice creams again.
John	The number with the largest frequency is called the mode.

(8 minutes. The students sum up the discussion about question number one and start working with question number two, first about when the average value is appropriate to use)

John	More about when to use the different things?
Ann	I have the impression that the average is most commonly used. It is what you hear mostly. But that does not mean that it is the best.
John	But I think that is what he uses here. Most common people know what average value is – one adds and divides. Median is a word that is a bit more difficult - not many know what it is. I think he uses that a little, all think ohhh, average value ohhh, directly without thinking
Mary	that it is the true, the real…
John	Exactly, since it is used so often I daily life. And then mode…
Brian	It is not common, well when you look in the newspapers they don't mix the managers with the so called workers, take the average value for a whole company. It is customary to use groups within the working place, workers, officials etc. That is when they use average value, not median. But here one puts together the whole company from officials to workers to cleaning staff, but I haven't seen that in the papers. There they divide into groups according to what they do and what is relevant in that professional group.
Ann	And then one uses the average.
Brian	Yes, then the average comes in. That is what one mostly says. It is not often they are grouped together in this way.
Mary	That they are grouped together…
Brian	That is why I was so surprised in the beginning when I looked

John (*Reads aloud from the task)* "How did the manager choose measure and what could his motives be?" Well, he has chosen the mode one million crowns since most people do not know what mode is and it sounds good with one million – it sounds high. If one just reads it like that, the mode is one million many would say – ohhh that high.

Brian Yes if the manager had said in the media that "Well the average salary for managers is about one million and for workers down there it is sixteen thousand. Whoops it doesn't sound so good in the papers, then it is better to go for this model.

John It is a little hidden, but he still speaks the truth.

John One has to really try to understand.

Brian He gets away somehow.

Ann Then if you are talking about salaries – the mode is not, someone may have sixteen-two, sixteen-three, someone sixteen and four. So in this case the mode is most uninteresting.

Peter It is interesting for someone who has…

John If someone knows about this then he will see through his bluff or whatever it may be called.

Brian Yes if one thinks about it that far.

John Yes that is what I mean. If one sits down and analyses.

Brian But not many would do that.

John No, they just have a quick look. Mode of one million. Wow.

Brian That is what I did – It took me a quarter of an hour to realize what it said – I was cheated.

Ann We were all cheated.

Mary Well but mode sounds a bit like a typical salary *(Mode is called "typvärde" in Swedish, from the same stem as "typisk" meaning typical)*

John Then one does not think about how that fits with the 165 000.

Mary That comes in afterwards, that all ought to be added.

Peter Well it sounds great that the …

(9 minutes. The students continue their discussion about measures of location in question two)

John Well what about the third question?

Peter What does measures of location mean?

John That is what we have been discussing, median, average and mode.

Ann Statistical measures of location.

John Do we have the curriculum?

Peter Average value – that is important.

Brian One has to show all models.

Ann I think you have to learn them – one has to show what is average value, what is median and then also mode – the one we did not know.

Brian And who is using them, who uses average, who uses median.

Peter But it says here we should do this for year five and year nine, and in year five it is probably just…

John Yes, just the average.

Ann Show how they complement each other and that you need more than one.

John Here it says what goals students should reach by the end of year five *(reads aloud)* know how to read and interpret given data in tables and in graphs and know how to use elementary measures of location." What is *elementary* measures of location, that is a question of interpretation – it may be those we have used.

Peter In year five then the average value.

John Yes, that is what I think.

Ann Eh why did you choose that one?

Peter It is most commonly used.

Ann Most commonly yes.

John It is the one that is most often used in newspapers – I agree with Peter.

Ann But one may still show how wrong it may work out. They have to learn to be critical also.

Peter Yes but do they have to be that in this context?

Ann	You don't have to go deep into it, just show what it is and show that it can get very wrong, that is enough. Take a result form a test, compute the average and show what a wrong impression it may give.
Brian	Or ten lakes, the average depth, nine lakes that are eight meters deep and one lake that is five hundred meters and then you take the average – then they will react – what is this?
Mary	One could actually make laboratory work – take boxes for ice cream and one big bucket, then eight boxes and one bucket of five litres, or bigger, and they can measure.
John	The book for year seven. Tables and diagrams, yes measures of location comes in next. First the average.
Peter	They don't have median or mode?
John	First they take the average and then well the median.
Ann	Does it say anything about being critical to the result, do they show anything?

(10 minutes, they study the curriculum in more detail and continue the planning of the teaching for pupils in school)

Mary	Now I am a bit behind, I started on question four, we learnt what mode is.
Ann	You are ahead of us, we are waiting for you.
John	We learnt about mode.
	Yes we did.
	Yes that is true.
Ann	And about open questions, that is one can discuss and still learn something.
John	And while we were discussing we realized that we had been wrong in question one and we could change that.
Ann	In fact really good.
Peter	Looking back one could say. Reflection is the mother of knowledge. P-O would have been happy.
Ann	*(Reads aloud)* How does this exercise differ from others you have done earlier?
John	Well I have done this type of excercises that you find in this book.
Peter	You have, have you?
John	Well during my school time, only such mechanical exercises. That book is not especially good. It is just compute the average value.
Ann	It is extremely boring that book.
John	Compute the average salary, it is the same exercise all the way.
Brian	Routine.
Ann	I did never that. I did almost nothing in lower or upper secondary about statistics or probability. In lower secondary our teacher said "This is extremely boring but we have to do it" – it took one lesson. When we entered the upper secondary school our teacher said "This is extremely boring, but we have to do it." - it took one half lesson. Then it becomes boring.
John	Positive attitude towards the subject then.

(5 minutes. They discuss how books are used in school.)

Ann	When we did our practice in school, I and Linda went to the same school and we did a small booklet of our own, with some of these tricky things, a guy in year nine, that big tough guy who does not care about anything, he come forward and said to us: "Great that you make an effort." What does this say to us – it means a lot.
John	It is true. In elementary school the kids love – ohh may we work in the math book, they really love it, then when they get to secondary school they hate the math book, so if you have other material, it is really worth it.
Brian	After 7, 8 years they may be tired of working that way.
Ann	That is not strange.
Mary	One can calculate how many exercises they have done by that.
Brian	It is obvious that they become more and more.
Ann	Since all exercises are alike. You won't find this kind of exercise in the books – maybe in some book?

John	Okay. (Reads aloud) “Can students in compulsory school solve this type of exercise?“
Peter	Yes – that is what I think.
Ann	Absolutely.
Brian	Which year is that – year one?
John	Since we are in the course for teachers in later years.
Ann	With this type of tasks, one can vary it somewhat, if it is used in year five one can remove the mode and change the figures a little, but the type works.
Mary	Yes, there are a lot of possibilities. But what I believe s important is that they have to practise first. If one would do this in one of the classes that I have been to well they would have been silent for a quarter of an hour and then one have been forced to ask then, well is there something that you do not understand, like this.
Brian	Then one has to go through the concepts somehow.
Peter	One has to practise.
Mary	Well they have to get used too.
Peter	The method.
Ann	But they have to know a little.
Mary	You can’t just throw it on to them.
John	But if one focus on this type of tasks it is probably easier if one starts with a group in year five. They will get used to it. It may be difficult in a group in year nine who are used to very closed exercises, then one has to start on a somewhat lower level, maybe the way you would do it with a year five group
Brian	Well that is true for all concepts, start out cautiously. The tragic thing is that so many teachers say, we skip that now and save it for year eight
John	I was not thinking of the concepts, I was thinking more about the type of task. Even if they are in year nine one may not be able to do this with a completely open task, if they are not used to that kind of exercises. But if you start in year five then in year seven you may use completely open tasks, since they are used to it.
Brian	Yes one has to adapt to the group, and has to know where they stand also .

(4 minutes, discussion about the books and the lack of true problems in many of the books)

Mary	Do you wish me to read what I have written. Pupils in year 4 – 9 can clearly solve this task. We agree on that don’t we? It is important that the pupils know the important concepts and are used to open ended tasks, if not you have to let them get used. We are very positive to open ended tasks, but it is difficult to find those in the books. We found some in Encounter with mathematics.
Brian	Open tasks are that laboratory work.
Mary	No but discussions.
John	Like this task, here was not much to discuss, everything is clear, but one learns a lot. It is not the classical compute the average of fifteen numbers.
Ann	Here is some. They are supposed to make lottery wheels and a chocolate wheel and with the same chance to win on all numbers. It is slightly better. They may work in groups, but it is up to the teacher, how they work, they may work individually.
Brian	This very task, with director Birger Johansson, I can understand if a kid in year seven or eight or six becomes a bit confused by this text. I myself was confused.
Mary	But isn’t that the whole idea. You are not supposed to get the whole meaning directly, you must go deeper.
Brian	Yes that’s it.
Mary	This is the goal of this task, it is not a question of finding the right answer and write it down. That is what they do mostly, they have the book and work through an exercise, answer five, answer ten. It is a question of working like we did, sit and discuss.
Brian	But do you think they would.
Mary	Definitely.
John	Yes, maybe not this task, but a task of this type.
Mary	This is adapted for our level, but that depends on what students you have, you can not start by giving them one such exercise where they are supposed to

	reason with a group. You must build this slowly but safely and in the end you can give them a task like this, tjofff. Here is an exercise and they do the task because they know already how to work with it and understand what they are supposed to do.
Brian	Yes, but they must know the basic.
Mary	Yes, absolutely.
Ann	Well you have to start some time.
John	One has to give them some exercises of the usual type, compute the average value, to do a couple of those but not three pages with exactly the same.
Mary	Especially not if it works the way it does now. Do these exercises and if you know them you may continue. Oh, no no you do not need to do the C-exercises. It becomes so clear that everything looks alike. Of course you also have to compute, not just talk, you have to get it also from the hand into the brain.
John	Variation.
Ann	Not mechanical.

Appendix 4

Report on questionnaire on attitudes to mathematics and problem-solving

Questionnaire for GS3 autumn 2001

A questionnaire was used that was originally created by David Tall and Yudariah B. M. Yusof to reveal the students' attitudes to mathematics and problem solving. It was adjusted to fit our needs and translated it to Swedish. A pilot investigation took place in the group that started a year after the group where the main study was made. 30 teacher students participated, where about half of them are going to become teachers in mathematics and science and the rest are going to be teachers in sports and mathematics. 27 of the students answered the questionnaire.
The students answered the questionnaire anonymously and it does not regard gender which can be a weakness. The result of the questionnaire is rather inconsistent, but on some questions there are trends.
Most of the students think that mathematics is a collection of facts and processes to remember (20). 25 think that mathematics is about problem solving. Many (21) thought that mathematics at universities and university colleges is abstract, but this investigation was done at the students' third lecture, so they would not be able to make such judgements if they had not studied mathematics before. One reason for the answer can be that the students just had been dealing with proof by with induction. 16 do not understand a new mathematical idea fast. 19 have to work very hard to understand mathematics. About half of the group feels secure about their ability to solve mathematical problems, the other half fell the opposite (6 in the middle). 17 feel a great pleasure from solving mathematical problems (8 in the middle). 7 solve problems to be able to manage the course. The questionnaire implies some anxiety and fear connected to mathematical problem solving. 3 think that the most important thing in mathematics is to find the answers. 1 is unwilling to look for other angles if an attempt fails. 6 give up if a problem is hard. The full results are presented below.

Questionnaire result **31**

Attitudes to mathematics and problem solving

Put an X by the sentences A1-A9 and B1-B8 in the appropriate box.
J means: Totally agree.
j means: Agree partially
- means: Indifferent
n means: Do not agree totally
N means: Do not agree at all.
Part A: Attitudes to mathematics

	J	**j**	**-**	**n**	**N**
1. Mathematics is a collection of facts and processes to remember.	0	20	2	5	0
2. Mathematics is about problem solving.	12	13	2	0	0
3. Mathematics is about bringing out new idea	4	9	6	5	3
4. Mathematics at universities and university colleges is very abstract.	8	13	1	3	0

5. I usually understand a new mathematical id fast.	1	7	4	13	3
6. The mathematics we study at the university college is meaningful for me.	6	13	2	9	3
7. I have to work very hard to understand mathematics.	6	13	2	5	1
8. I learn mathematics by rote learning	0	5	4	14	3
9. I can connect mathematical ideas that I hav learnt.	0	16	6	5	0

10. Describe your feelings for mathematics with a few sentences.____
____
____
____

Part B: Attitudes to problem solving.

	J	j	-	n	N
1. I am confident about my ability to solve mathematical problems.	2	10	6	7	2
2. I find great pleasure in solving mathematica problems.	3	14	8	2	0
3. I only solve mathematical problems to mana the course.	0	7	5	11	3
4. I feel anxiety when I am asked to solve mathematical problems.	2	7	3	11	4
5. I often fear unexpected mathematical proble	1	7	5	9	5
6. I think that the most important thing in mathematics is to find the right answers.	0	7	5	9	5
7. I am willing to try a new approach if an attempt fails.	7	16	2	1	0
8. I give up quite easily if a problem is hard.	1	5	3	11	6

After the ten week course was finished, the students filled out the questionnaire again. This time, gender and group was considered, i.e. if it is mathematics and science or mathematics and sports the students study. 25 students participated. Task number 10 was on all questionnaires but it is only displayed on the first result presentation. It will be discussed separately. The results on the other tasks were as follows.

Questionnaire result **(Gs3 second)**

Attitudes to mathematics and problem solving

Mark the appropriate row:

Gender: Woman____ Man_____

Put an X by the sentences A1-A9 and B1-B8 in the appropriate box.

J means: Totally agree.
j means: Agree partially
- means: Indifferent
n means: Do not agree totally
N means: Do not agree at all.
Part A: Attitudes to mathematics

	J	**j**	**-**	**n**	**N**
1. Mathematics is a collection of facts and process to remember.	2	11	4	6	2
2. Mathematics is about problem solving.	5	20	0	0	0
3. Mathematics is about bringing out new ideas.	2	8	7	8	1
4. Mathematics at universities and university coll is very abstract.	9	13	1	2	0
5. I usually understand a new mathematical idea f	1	9	4	6	4
6. The mathematics we study at the university col is meaningful for me.	0	3	2	15	4
7. I have to work very hard to understand mathem	4	9	2	9	0
8. I learn mathematics by rote learning	3	7	2	10	3
9. I can connect mathematical ideas that I have le	4	14	4	2	1

Part B: Attitudes to problem solving

	J	j	-	n	N
1. I am confident about my ability to solve mathematical problems.	1	12	2	9	1
2. I find great pleasure in solving mathematica problems.	3	13	7	1	1
3. I only solve mathematical problems to mana the course.	3	9	2	8	3
4. I feel anxiety when I am asked to solve mathematical problems.	2	3	4	14	2
5. I often fear unexpected mathematical proble	0	4	3	13	5
6. I think that the most important thing in mathematics is to find the right answers.	1	8	3	11	3
7. I am willing to try a new approach if an attempt fails.	8	13	4	0	0
8. I give up quite easily if a problem is hard.	1	5	5	9	5

The attitude to mathematics has changed during the course. There are more students who think that mathematics is a collection of facts and processes to remember before the course. The students' perception of the abstraction level has not changed. More students think that they understand a new mathematical idea fast after the course than before. In the second questionnaire more students think that the mathematics they study at the university college is meaningless. More students say that they have to work very hard to understand mathematics before than after the course.
The ability to solve mathematical problems has not changed. Neither has the pleasure in solving them. A few more solve problems to manage the course on answers in the second questionnaire than on the first one. Anxiety and fear for mathematical problems have decreased.

The questionnaire was handed out in the spring 2002. Students filled it out at a session at the end of the course before the mathematics course starts.

Questionnaire result 61 **(Gs6 first)**

Attitudes to mathematics and problem solving
Mark the appropriate row:
Gender: Womman____ Man_____

Put an X by the sentences A1-A9 and B1-B8 in the appropriate box.
J means: Totally agree.
j means: Agree partially
- means: Indifferent
n means: Do not agree totally
N means: Do not agree at all.
Part A: Attitudes to mathematics

	J	**j**	**-**	**n**	**N**
1. Mathematics is a collection of facts and processes to remember.	0	9	2	8	0
2. Mathematics is about problem solving.	8	11	0	0	0
3. Mathematics is about bringing out new idea	2	13	2	2	0
4. Mathematics at universities and university colleges is very abstract.	7	8	3	1	0
5. I usually understand a new mathematical id fast.	2	8	5	4	0
6. The mathematics we study at the university college is meaningful for me.	2	5	2	9	1
7. I have to work very hard to understand mathematics.	2	6	3	8	0
8. I learn mathematics by rote learning	0	3	3	9	4
9. I can connect mathematical ideas that I hav learnt.	2	12	3	2	0

Part B: Attitudes to problem solving

	J	j	-	n	N
1. I am confident about my ability to solve mathematical problems.	2	13	0	3	0
2. I find great pleasure in solving mathematica problems.	2	12	1	2	0
3. I only solve mathematical problems to mana the course.	0	4	3	7	4
4. I feel anxiety when I am asked to solve mathematical problems.	1	2	0	7	8
5. I often fear unexpected mathematical proble	1	3	1	5	8
6. I think that the most important thing in mathematics is to find the right answers.	1	6	1	3	7
7. I am willing to try a new approach if an	10	8	0	0	0

attempt fails.					
8. I give up quite easily if a problem is hard.	0	3	2	10	3

23 has filled out the questionnaire at the end of semester 6 and this is the result..

Questionnaire result (Gs6 second)

Part A: Attitudes to mathematics

	J	**j**	**-**	**n**	**N**
1. Mathematics is a collection of facts and processes to remember.	1	5	5	10	1
2. Mathematics is about problem solving.	14	8	1	0	0
3. Mathematics is about bringing out new idea	7	11	4	1	0
4. Mathematics at universities and university colleges is very abstract.	0	15	3	5	0
5. I usually understand a new mathematical id fast.	2	11	3	7	1
6. The mathematics we study at the university college is meaningful for me.	3	10	4	5	1
7. I have to work very hard to understand mathematics.	3	3	8	5	4
8. I learn mathematics by rote learning	0	2	6	9	5
9. I can connect mathematical ideas that I hav learnt.	3	12	6	2	0

Part B: Attitudes to problem solving

	J	j	-	n	N
1. I am confident about my ability to solve mathematical problems.	2	13	3	5	0
2. I find great pleasure in solving mathematica problems.	7	12	3	1	0
3. I only solve mathematical problems to mana the course.	1	1	4	6	11
4. I feel anxiety when I am asked to solve mathematical problems.	2	1	3	6	11
5. I often fear unexpected mathematical proble	1	2	2	6	12
6. I think that the most important thing in mathematics is to find the right answers.	2	3	4	5	9
7. I am willing to try a new approach if an attempt fails.	12	10	1	0	0
8. I give up quite easily if a problem is hard.	0	3	5	10	5

A comparison of the results of the questionnaire before and after the course shows for example that a larger part of the students think that mathematics is a collection of facts and processes to remember before the course than after. More students think that mathematics is meaningful for them after the course than before. No change has taken place regarding rote learning or ability to link ideas together. The attitudes to problem solving have not changed much.

Task 10 indicated that the students had a more positive attitude to mathematics after the course was done.

The investigation that Tall and Yusof did showed that the students that got the questionnaire before a mathematics course had a different attitude to mathematics in some aspects compared to our students (Questionnaire result 62) On question one for example, most of the American students had answered j or J (together 34 of 42) while the distribution in our class was 9 j (no J) and 8 n (no N). All of our students thought that mathematics is about solving problems while 27 of 43 of the American students thought so. Half of the Americans thought mathematics is about bringing out new ideas. Among our students it was 2 who wrote n and 15 wrote J or j. Our students thought to a higher degree that mathematics is abstract at university colleges, but it is a much bigger part of the American students that say that they do not understand a mathematical idea fast. The ability to link mathematical ideas together also differs. 18 of the 43 American students' claims not to be able to do it and 2 of the 19 Swedish students say that they can not. 30 of 42 of the Americans claim to learn by rote while the corresponding figure among the Swedes is 3 of 19. It is hard to compare these questionnaire answers since the students have different backgrounds and it is so few participating in the study, but it can nevertheless indicate differences and similarities. We do not compare the results after the course is done because the courses are different.