

## **Teaching and Learning in Undergraduate Mathematics, Computer Integration and Better Learning via New Organizational Methods**

### **Final report**

Our project is a complex teaching experiment in that it consists of many components. The changes we intend to implement in our first year math courses are geared at:

- adapting the teaching of mathematics to individual needs
- a greater level of computer integration, both in regards to computer-aided problem solving and algebraic computation.

In order to carry out these changes we must:

- change the organization of our courses
- prepare for the use of computers in the classroom setting where it is expected that teachers will use the computer to illustrate and demonstrate mathematical concepts.

Our hope is that knowledge gained from this experiment will eventually lead to a higher passing rate among our first-year students. We feel that this goal may be achieved through a greater understanding of mathematical concepts, more exposure to applications, and greater emphasis on verbal and written communication. By upgrading our course on computer-aided mathematics (both problem-solving and algebraic computation) via the introduction of more involved and meaningful assignments, more extensive instruction, and greater emphasis on presentation of results, we hope that the computer part of our curriculum will be more profitable for the students.

We aim to achieve our goals through instruction geared to individual needs, and through the creation of a positive atmosphere, which nurtures development. Some of the more important changes the students will experience under the fall term of 99 are:

- the introduction of a web-site specially designed for students to meet their fellow classmates and teachers, even before they begin their studies in Umeå
- an individual evaluation of the results from the math skills test which each first-year student writes just prior to the beginning of the fall term
- delegation of first-year students into groups of 4 each, allowing students

free discussions in the small group setting

- individual help for students who fail an exam on their first attempt. This takes place under the duration of one week in preparation for the make-up exam. There may be up to 3 such weeks in one term while instruction for the make-up exam takes place.

Those who pass an exam on their first try will be engaged in our computer course on computer-aided problem solving.

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Thanks go moreover to the Department of Mathematics and to Umeå University in general. It is a great place for development of new thoughts on studying and teaching.

Attachments located at the end of the report:

Statistics (comments in English)

Pictures (produced by students, with Swedish words) describing the project.

Course evaluation in three steps.

## **Teaching and Learning in Undergraduate Mathematics, Computer Integration and Better Learning via New Organizational Methods.**

### **Aim of the Project**

The purpose of the project was to improve the educational process in mathematics and thereby achieve good academic results. By good results, we mean to understand mathematical concepts, definitions, and theories, to be able to apply these, and to solve problems using computers in mathematical activities.

We assumed that the goal would be achieved by increased student adaptation, individualization and interaction with computers.

If we achieved results that were in line with the above-mentioned goals, our perception was that the outcome would be an increase in the flow of students through our classes.

### **Execution**

Introduction:

To be able to achieve the above objectives and goals we started with four simple principles, a)-d) below.

- a) Educating and learning become successful and effective through cooperation and cooperation is easier if both parties know each other.

On this basis, our thought was that, as much as possible, both student and teacher would be able to have the answers to questions such as: Who are you? Who am I? What are you capable of? How can I help you? These questions, and others like them, should be answered in part by sending out a welcome letter, where the University, institution, and teachers are presented. They should also be answered by establishing an inter-active website with extensive information and presentations. In addition to this, a face-to-face introduction should also take place upon the students' arrival to Umea.

b) Individual logical mathematical knowledge is created through carefully building on the individual's existing knowledge.

This cannot be achieved by feeding the student large amounts of mathematical information without considering what the student already knows. Therefore, the student must be aware of his/her competence and the teacher should equally be aware of what level the student is at in order to be able to adjust accordingly. Because of this, we planned that the students should start their mathematical studies in Umea with a diagnostic test. This test should be graded and afterwards reviewed by both the teacher and student together and used to indicate the starting point for the student's university education. The same thought process should be used as a guide in the event of a reexamination, i.e. after a failed test the student and teacher discuss the result together and the student derives a study plan, which is also reviewed and thereafter followed.

c) The work that is involved in creating an individual's logical mathematical knowledge from the material presented in a class has its origin in the student's existing knowledge and is a complicated, dynamic process. It is not only mathematics that needs to be practiced but also good study habits and self-reflection.

The development of knowledge usually starts with some type of lecture or reading assignment. For the average student, it is important that the material is dissected during discussion with fellow students, through problem solving, and in dialogue with the teacher. Because students face mathematics in a certain chronological order, one might be tempted to believe that mathematical knowledge is built linearly, however, that is not the case. To establish a dynamic mathematical knowledge a great deal of work is required. The new mathematical material needs to be related to the already established knowledge through a number of steps and along many intricate, connecting paths in a complex structure. The least developed setting where one can reflect on mathematics, which also does not require faculty time, is the study group. Here students can work on mathematical theory and applications together. Through well-chosen mathematical exercises or even word problems that do not directly relate to mathematics, the students can tap into this unique resource to form their individual knowledge.

d) Students must become accustomed to using modern tools such as computers.

In this case, teachers should show the benefits by providing good examples.

Facing the project's first year, we worked out a detailed plan in accordance with the above-mentioned principles.

In this plan we described how we imagine the students and teachers would work together and the different courses of action were classified as follows:

- **The Welcome Letter**
- **The Math Skills Test**
- **Small Groups**
- **Reorganization of Courses**
- **Computer Algebra and the Use of Computers**
- **Five Laptop Computers**
- **Preparing for Reexamination**
- **Composition and Coordination**

Below we describe the execution and related results step by step. In some cases, we save the results for a special paragraph (Results). However, before the implementation is described, we start with a short description of the environment at the mathematical institution because it has influenced our work during the project.

The project environment at the Department of Mathematics at Umeå University is favorable to educational experimentation. Worth mentioning is that some of the Lecturers have a background as teachers in both the middle and high school levels, and one of the institution's most experienced lecturers was the Head of the University's educational center. In addition, one of the country's most successful research groups in didactics in mathematics is located in Umeå, the Department has held classes at the Teacher's University, and the department's Senior Professor has always taken a great interest in the quality of education at the Department. Above mentioned factors have meant that it is easy to maintain an educational dialogue that has risen above the customary coffee-room philosophizing and grumbling (of the therapeutic kind). It is relatively common at the mathematics department, that these discussions are systemized and organized. A series of short seminars in education issues, called Pedagogical Pizza, can be mentioned as an example.

### **Execution and corresponding results described step by step**

Lecturers, the department, the University, the city of Umeå and the first class for the fall were presented in **the welcome letter that was sent out** to the accepted students. In the letter the students were encouraged to solve 100 specific repetition problems. In addition to this, the students received an individual password to an interactive web site where each lecturer had posted a personal introduction of him or herself and the students were encouraged to do likewise. When the students arrived in Umeå this letter was supplemented with a face-to-face introduction.

The fore-mentioned welcome procedure produced a very good response, according to both lecturers and students:

Almost everyone took part in the information exchange (only one student did not), and no one said that the information seemed excessive. On the contrary, there were even requests for more information. For example, some suggested that it would be useful to have information from older students about what it is like to study at the program.

At the face-to-face session that took place upon the students' arrival, the personal presentations were so exciting and pleasant that they consumed a third of the day. The project manager reflected that these successful initial presentations laid the groundwork for good cooperation within the work groups.

One of the Lecturers felt that the web presentations, which were partially built around a number of questions, were too short and that in the future we should include more questions. Before the fall semester of 2001 the number of questions was increased and at the same time they were made less demanding. The outcome of this was negative and has allowed us to draw the conclusion that it is probably more important to have fewer questions that are formulated so that they distinctly require an answer.

### **The math skills test**

The first week of the students' math studies started with a diagnostic test. This test was extensive and lasted most of the first day. It was corrected immediately and then each student, with the help of a lecturer, reviewed the results. Based on the outcome of that assessment, it was discussed what the student needed to review and the rest of the first week was dedicated to this task. The test results were carefully noted for statistical purposes and they were also compared with grades from high school.

In the discussion that followed, as well as in the survey of the students, everyone was pleased with the system of taking a diagnostic test that is afterwards followed by a dialog with a lecturer. The diagnostic test has been performed since the fall semester of 1998, but the careful follow-up was an addition from this project.

### **Small groups**

Besides traditional lectures and problem solving classes, the students were assembled into small groups, usually numbering four in each. Within these groups, the students were encouraged to discuss mathematical theory, solve problems and exchange general experiences regarding their studies. The discussions were encouraged and supported by problems given to the groups that were to be discussed and solved. Some examples of such problems include:

- 1) Write a diary about your studies and discuss it with the others in your group. Summarize your experiences. What have you learned? How do you

- study effectively? How do you make the studies interesting? Could the lecturer contribute to the latter, or possibly your fellow students?
- 2) Start with a chapter of the textbook that you have read. Sketch a structure of the contents and the different theories, definitions and concepts.
  - 3) Give examples that illustrate the different definitions and theorems.
  - 4) Is it possible to change the conditions in the theorems? Why not, why and how?
  - 5) Draw illustrations regarding the mathematics and your studies.
  - 6) Solve certain problems. Work out a solution that the group as a whole thinks is best. Present the solution to the class.

The majority of both the students and the teachers responded positively to the work in small groups. Several lecturers and many students were enthusiastic about the benefits that resulted from the organization and work of the small groups. The students were particularly happy about the fact that they were “forced”, from the first day to work as a group and that it promoted very good study habits. One of the few negative effects was that students with children did not like it. They felt that the other students (without children) were not focused enough. Thanks to a very active dialog regarding the organization during the course we became better at solving this problem. A Lecturer reflected that math students must learn to work individually and that time must be given so that this skill also can be developed. The work taking place in small groups can sometimes be a little undisciplined. However, from class 2, which starts in week 5, the students claimed that they had learned that they could switch groups and work in a way that they felt more comfortable with.

### **Reorganization of courses**

The four 5-point courses during the fall were organized so that they were taught during 4 consecutive weeks. The fourth course, computer algebra with experiments, was divided up into three parts that were each placed immediately after one of the three non-divided parts. A student that did not pass one 5-point course could then prioritize and omit the computer algebra course and instead study for the re-exam. The purpose behind this was that this student could then have a good starting point for the next course and also have a good chance to pass the re-exam. We decided that a reorganization of this kind, or one like it, was necessary to improve on the previous system of re-exams that had very poor results (less than 1 of 20 students passed). It clearly shows in the attachment that this change produced the desired result of more students passing the re-exams. However, the students were very displeased with the computer course being split up and the lecturers acknowledged this complaint. The teachers also thought that it was too hard to administer. Therefore, there was a need to try to make changes but still maintain the positive effects of improved results in the re-examinations and make a better computer algebra course. While we wanted to make changes, we also realized that we needed to cut the contents of the course. The reason for this was that the extent of the students’ previous knowledge was declining at an alarming rate.

Our changes to the project before phase in year (third year) was to:

- 1) Decrease the contents of course
- 2) Increase the emphasis of understanding concepts
- 3) Turn computer algebra into its own course that could be taken the second semester
- 4) Establish two levels, one for students who had little to no prior knowledge that included a preparation course and a second level with out any preparation

At the time, with the present critical level, the changes turned out to be good. The negative side was that we, in some sense, had to give up on the goal of having computer use become an integral part of the students' studies since the class had been isolated to its own course. On the other hand, this class was possible to execute in a very successful way. Both students and teachers have expressed great satisfaction with the new system (see attachment). The number of students passing the first semester courses has also significantly increased as shown in the attached statistics.

### **Computer algebra, the use of computers and the five lap top computers**

We have already commented on this in the above text.

The interactive part of the introduction process was kept and has in some sense been refined. However, we did not succeed in integrating computer use in the normal part of the courses, as we had planned from the beginning. This was partly because we underestimated the problems, and partly because other problems took priority. Those problems, as we see it, required solutions that could not be combined with our vision for integrated computer education. The portable computers were most useful during the summertime when we could use them to follow the development of introductions on the website. We had also promised to answer questions from the students that were posted on the interactive website which was easily accomplished through the mobile computers. Further, our technical expert could also correct errors in the software running the site regardless of where he was located.

### **Preparing for the re-exam and composition and coordination of courses**

Prior to the fall semester of 1999, with the exception of some experiments, we had one exam for every 5-point course, which contained both theory (25%) and problem solving (75%). Within this project we planned to have two exams, one with only theory (42%) and one with problem solving (58%), thereby giving the theory more emphasis. The theory exam was scheduled to take place a week before the problem-solving exam. After the problem-solving exam we planned to correct the tests immediately and thereafter notify the students through email. The students that passed the exam could then spend a week taking the computer algebra class, while the ones that failed would spend their time studying for the re-exam.



Those were our plans and we followed through on them successfully. If, for instance, the problem-solving exam was held on a Friday we corrected it during the weekend and sent out the results through email on Sunday evening.

Those that failed were called to a meeting on Monday for a review of the test. After the review each student received an assignment for the next day; to plan their course of study for the re-exam based on their math knowledge as well as the result on the exam. They also set up time for a meeting with the lecturer for the next day. At that meeting the student presented his/her plan to the lecturer and possibly to other students in the same position. The plan was discussed and in most cases small adjustments were made. Afterwards, the preparation work started. This was done in small part through lecturing of all students in one group (10-15) and in larger part through semi-individual work where students worked in small groups (1-3). When problems arose, students were able to get a hold of one available lecturer who was on call approximately 4 hours a day. The results from the re-examinations were encouraging and significantly better than from those seen in previous years. In short, the individual support before the re-exam was a success (see attached statistics). In preparation for the third year, the so-called phase-in year, when all good ideas were to be combined, we were faced with the task of combining incompatible ideas. Since our class at the A-level contained a rather large amount of material and a great number of our students had gaps in their knowledge, we had an extensive review of the AB-level. The main purpose was to reduce the amount of material covered at the A-level to benefit the quality of understanding.

The result of this assessment was that we reduced the material covered by approximately 25% at the A-level by moving some parts to the B-level. One action that was taken was that the computer algebra course was entirely moved to the B-level.

The arriving students' previous knowledge level has continued to decrease with a consistently large spread. It is primarily this very large spread that has made it difficult to find an appropriate level to start the education at. This has caused an even stronger need to adjust to each individual in a form called Adjusted Studies (see special attached report). The result of this has been that the students with a poor knowledge level (approx. 40% of the students) have had to take a preparation course before the actual A-level started. By doing this we could, by the project's last year, offer both students with good prior knowledge as well as those with poorer knowledge a sensible education, in a much-improved manner. The result has been very positive, both regarding academic performance (for all categories of students, see attachment) as well as the students' general appreciation for their educational environment. Further, almost all ideas from the A-project were able to be implemented, which we think is very pleasing. After completing the third year, the so-called phase-in year, the final year, we can see that the number of students completing the program has increased radically (see attached statistics). The

faculty has accepted the increase in cost that the two levels has caused and has provided the extra funds. However, we would like to emphasize that we do not think that the problem with the students deteriorating knowledge of math when arriving to the university is solved, even though we think that we have found a number of actions that allows us to handle the situation. A statistical analysis indicates that the combined effect of downsizing the material as well as the introduction course helps the students with the lowest level of knowledge to achieve a grade G in mathematics.

### **Special results, effects on other courses at the department, other departments within the faculty and at other universities**

The achieved results from this project are generally described by the *italic* text in previous sections. Extending beyond those there are also results that are more difficult to describe through quantitative terms, as well as those that are documented using pure statistics with comments. We will now describe some of those results.

As mentioned previously, the students were asked to keep dairies, noting their work and habits. The purpose of this was to make them aware of the use of self-reflection as well as to give them some understanding of self. The individual reflections were to evolve into discussions with their fellow students and to be summarized. This “exercise” has its origin in research that one of our “matematikdidaktiker” had done that showed that those that fail math are simply not spending enough time with their studies. For example, it was at one of these sessions that the problems that single mothers experience created a rather serious discussion.

Another rather difficult assignment was to have the students sketch a structure of the material covered in a chapter of the book. The students were to sketch this by illustrating important concepts, definitions and theories along with how they are connected. It is important that the lecturer does not influence the students on how to do this, but allows them to figure this out themselves. This assignment is designed to help increase understanding of what connections in mathematical theory are important.

The third assignment was to have the students illustrate concepts, definitions and theories through their own examples along with discussing changes in the formulating of mathematical theories, both possible and impossible. The latter can lead to very interesting discussions that are closely related to what a mathematical researcher works with.

A more playful (but still important) exercise that was tried during a lunch-break consisted of the project leader revealing to the students the project that they were a part of and afterwards having them draw illustrations according to their impressions. Some pictures are attached.

As mentioned earlier, we strived towards making math both fun and interesting. For example, this goal was achieved through the so-called “mathematical evenings”. We had a lot of those during 2000 (“matematikens år”). During a standard year we usually only have two per semester. The evening typically consisted of a main seminar, which was preceded by a short-seminar, mingling, snacks, games and discussions.

### **General reflections regarding the evaluation**

During their studies, the students are expected to participate in a number of evaluations. They are under the impression that these evaluations are done because “the rules say that an evaluation has to be done”, and not because “in this particular case an evaluation is justified and very valuable.” Sometimes they question how, and even if, the results of an evaluation will be used. The latter uncertainty stems from the fact that the evaluations are almost always done at the end of a course. In short, among many students, the word evaluation has a bad ring to it, and this is a dilemma.

In our project we have tried to avoid evaluations that look traditional and also have attempted to vary the appearance. We will now describe some of them. One is called “course dialogue.” Ten days into the course, when the students have become more familiar, we put aside 30 minutes for a discussion. The students discuss in small groups what is working well and what can be improved. They touch on the topics of lecturers, themselves, fellow students as well as general issues. At this time the lecturer also does the same. Afterwards, a representative from each group as well as the teacher writes down their thoughts on the classroom board, so that they can then discuss them. There is always something that everyone can agree on and this is experienced as very positive. Furthermore, it is important and constructive to discuss in small groups. In a small group thoughts that are not very well thought through are filtered out and at the same time the students can maintain a certain level of anonymity and dare to bring up sensitive issues. The representative is not held responsible for what has been brought up in the group. The students appreciate the fact that they have a chance to collectively correct things early, long before the class is over. Another type of non-traditional evaluation, which we call the three-step-method, consists of three discussion meetings with recorded comments. The purpose behind this type of course evaluation is that it will work as a long conversation with time for reflection, discussion and a possible exchange of opinions amongst a larger group (one example is submitted). In step I, and II a group is formed that consists of student volunteers and lecturers with the students making up the majority. During the first meeting the group discusses what might be important to evaluate. After the meeting, one teacher notes the most important issues discussed and might also comment on them. After this copies are distributed to the assembled group and they all add their comments. Finally, the material is made available to the entire student group, all those participating in the course, and they will have the final say. Through this they actually get the chance to follow how the discussion went and also

note what they think. The advantage with this type of discussion is that it is well thought-out and that it can be a little more interesting compared to dry statistics from a traditional evaluation.

### **Evaluations of the project's effect on the education within the department, other departments, the faculty and at other universities**

#### I) Effects of the education in the department

The diagnostic test has been introduced for all beginning students, i.e. in all categories of programs: Master of Science, Bachelor of Science, Teachers Programs and all separate courses. Therefore, almost all the lecturers at the department have been involved in this work. By doing this the teachers have experienced a more accurate image of the new students' level of knowledge. The education at the entry levels has therefore been able to be adjusted towards the actual knowledge level of the students, which was the main purpose of the diagnostic test. It is also clear that the test has brought with it an interest of revitalizing the pedagogical discussions among the faculties teachers. The part of the project that contained the introduction of a diagnostic test can therefore be said to have had great significance for the new students.

#### II) Effects at other departments within the faculty

Another important effect of the new knowledge level of the students leaving high school is that the faculty has had to do a major review and revision of math courses, primarily within the science programs, but also within the Masters of Science program that makes up a large volume of the faculty. This work has been conducted in cooperation with the people responsible for the different programs after a decision from the faculty's base education council, on advice from the departments' teachers. An increased awareness and understanding of the difficulties of math education has therefore been spread throughout the faculty. As a result of this, cooperation with other departments has increased and a solid foundation for further cooperation between math and other subjects has been laid.

#### III) Effects at other universities

Information about the A-project has spread to other universities in a number of different ways. It has been extended through the yearly study director's conference (where representatives from the universities and the larger colleges' mathematical departments participate). Also, the information has been exchanged through other conferences such as the ministry of education's conference called "A Good Start in Mathematics". The information has also been passed along through normal cooperation between the universities. An example of the latter would be the seminar that the department arranged in the spring of 2001 regarding educational issues at the A-level, where

participants from Karlstad, Uppsala and Linköping attended. Reports, for instance the Ministry of Education's national evaluation of the math subject, have also helped the information to spread. The department's work with pre-university knowledge and results were part of the basis for a report called "Räcker kunskaperna i matematik?" (The advisory group for students' pre-university knowledge in mathematics, Ministry of Education)

It is worth mentioning that the diagnostic test is now used at Uppsala University, Linköping University and at Mitthögskolan, and other schools have shown a great interest in trying individualized programs.

#### IV) Information outside the academic world

We would like to mention that the A-project has aroused interest outside the academic world. The project has been mentioned in different newspapers, both locally and nationally. For instance, in the coming number of "Aktuell Forskning & Utveckling" there is an article on individualized programs. We have started collaboration with high school teachers here in Umea that has resulted in a joint project in algebra.

#### **The Students' Participation at the Development of the Project Plan (1) and at the Execution of the Project (2).**

We realized that student participation in our discussions regarding study and educational improvements was of great importance years before the project started. As a direct result of the decision of the Council to grant us the opportunity to submit a complete application, we worked extra hard to achieve a high level of student participation in the project.

During the two-year time period before the start of the project, we had a good amount of practice in shaping discussions with students where the objective was to use their ideas. So, for example, we created the groups

*Matematik A på sikt* (committee on long range planning of first term studies in mathematics) and *studentutvärderingsgruppen* (student evaluation committee)

Later a group of five teachers and nine students played a key role in the project planning, in giving the project its final form, and in determining the description for the project application. The group's numerous meetings decidedly impacted if certain ideas were "approved" or "disapproved". These decisions were primarily regarding the implementation of the concepts but also to some extent the actual concept. The fact that the students formed a majority at the meetings and that the group gathered for long periods of time, created a very good atmosphere and setting for a free exchange of ideas.

2) Now we will move on to the description of the students' participation at implementation of the project. At this time it is important to make note of several special actions taken to increase the cooperation between the student groups and teachers. Examples of these actions include

- The WEB-based part of the welcoming procedure
- The dialogue between student and teacher regarding the initial diagnostic testing
- The extensive "tell me about yourself" session

- The organized activities for small work groups
- The careful design of the “individual plan” for each student who is preparing for a re-exam
- The formalized course dialogue that was preformed two weeks into each five-point segment

In addition to this, a number of polls were taken and a number of evaluations were performed, to truly create different forums where the students had the opportunity to air opinions and ideas before everything was summed up in a larger evaluation. One of the more extensive evaluations was called ”the three steps method”, ( trestegsmetoden attached).

Step I: This first step involved a group of volunteer students and a couple of teachers discussing matters (a teacher kept notes where value adding information was recorded).

Step II: The notes from Step I were distributed to participating students (and teachers). After some time had passed the students turned in their thoughts on matters recorded in the notes. The final document that contained thoughts from both students and teachers was then in

Step III distributed to the small work groups. From there, it was examined and their comments were added. The material was then consolidated and distributed to the students.

In addition to the above-mentioned method many short discussions and inquiries were performed. These had influence on the final modifications of the project when the “third year” was planned.

### **Attachment**

Academic results for Mathematics Level A. A comparison over the years 1998-2001

The mathematical department has done a large amount of work to follow up and analyze academic results from 1998 through 2001. The tables shown below are based on statistics from this follow up, see attachment 9 in “Diagnostiska prov I gymnasimatematiken hösten 1998 vid Umeå universitet” and attachments 2, 3 and 11 in “Anpassade studiegångar inom Matematik A – Utvärdering av genomförandet ht 2001”. The very last report mentioned is attached.

Table 1 below shows the percentage of students that passed the level A exam in Math. The compilation contains all starting students who were registered in Level A Mathematics during each year (i.e. applied mathematics, high school teaching, computer science, physics and free courses) and that have taken the diagnostic test. The number of students during the fall semester of 2001 was significantly lower than in other years. This is mostly due to the fact that the students from computer science (approx 35) did not take Level A Mathematics.

Table 1 Academic results, all students:

Fall year	No of students	Exam	Re-exam	Trailing*
1998	113	43.5	48.2	7.9
1999	75	49.0	63.0	27.5
2000	97	37.3	52.4	24.1
2001	38	67.8	80.3	38.8

\* This column indicates the percentage of students that passed the re-exam of those that failed the original exam. For fall 1998:  $(48.2-43.8)/(100-43.8)*100=7.9$ .

Table 2 and 3 below are based on the same figures as Table 1, but the students are divided into those who had “good previous knowledge”, A2 (table 2), and those who had “poor previous knowledge”, A1 (table 3). Note that for the fall of 1998 the numbers only reflect the 47 students that had math grades from the new Program High School (of a total of 113).

Table 2 Academic results, A2 students (good previous knowledge)

Fall year	No of students	Exam	Re-exam	Trailing*
1998	29	63.8	68.1	11.9
1999	41	57.9	72.0	33.5
2001	22	84.1	90.9	42.8

Table 3 Academic results, A1 students (poor previous knowledge)

Fall year	No of students	Exam	Re-exam	Trailing*
1998	18	16.9	22.2	6.4
1999	34	38.2	52.2	22.7
2001	16	45.3	65.6	37.1

From the tables above it is possible to see that the A-projects’ transformation of re-exams from the fall semester of 1999 has been very successful, see the last column in each table. Further, it is also possible to see that after the introduction of individual programs (A1 and A2) in the fall of 2001, we can see significant academic improvement, both for students with poor previous knowledge (A1-students, see table 3) as well as for those with good previous knowledge (A2-students, see table 2).