

Project number: 035/99
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Activity-based Education in Basic Electricity and Circuit Theory

Abstract

Education research regarding electricity and circuit theory have shown that students, even at university or college level, have difficulties in acquiring a functional understanding of and to distinguish between fundamental concepts such as current, voltage, energy and power. Student's lack of qualitative understanding results in difficulties solving quantitative problems correctly.

In this project an introductory course in electric circuit theory for electrical engineering students are reformed using active engagement methods. Students take this course during the second semester in their first year. This course contains fundamental circuit theory for DC- and AC-circuits including matrix-methods and the jw-method, coupled circuits, some theory for three-phase AC-circuits in the first part of the course. In the second part of the course Fourier-series, Fourier-transforms and Laplace-transforms are introduced for treating stationary and non-stationary circuit problems. Some system theory and some feedback theory are also introduced in the second part of the course.

We have developed "tutorials" to be used in the recitation sections and a series of labs, which are focused on helping the students to develop a better functional understanding. In the labs low-tech (batteries, bulbs) and high-tech (computers, MBL-interfacing, simulations) are combined using active engagement methods. MBL-interfacing allows the use of more advanced tools such as FFT (Fast Fourier Transforms) and the modelling of transient response using experimental data. We have also developed some interactive lecture demonstrations (ILD) using MBL-interfacing for use in the lectures allowing real-time display of experimental data.

During the 2001/2002 academic year students actions and communications during the labs have been videotaped by a doctoral student from the Swedish National Graduate School for Science and Technology Education Research.

In the 2002/2003 academic year the recitation sections and the lab sessions will be merged into a joint experimental problem solving session. Also a doctoral student will study this implementation of a reformed curriculum.

Final report
(Preliminary version)

Activity based education in electricity and circuit theory

Project no: 035/99

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ABSTRACT

Education research regarding electricity and circuit theory have shown that students, even at university or college level, have difficulties in acquiring a functional understanding of and to distinguish between fundamental concepts such as current, voltage, energy and power. Student's lack of qualitative understanding results in difficulties solving quantitative problems correctly.

In this project an introductory course in electric circuit theory for electrical engineering students have been reformed using active engagement methods. Students take this course during the second semester in their first year. This course contains fundamental circuit theory for DC- and AC-circuits including matrix-methods and the $j\omega$ -method, coupled circuits, some theory for three-phase AC-circuits in the first part of the course. In the second part of the course Fourier-series, Fourier-transforms and Laplace-transforms are introduced for treating stationary and non-stationary circuit problems. Some system theory and some feedback theory are also introduced in the second part of the course.

In the first implementation of the reformed electric circuit a series of conceptual labs were developed. In a second implementation these conceptual labs were merged with the problem solving sessions into problem solving labs.

We have videotaped students' actions and communication during these labs. The videorecordings from the labs dealing with AC-electricity and with transient response have been analysed in more detail.

Our analysis show that the conceptual labs have been good at fostering conceptual understanding. By taken these conceptual labs one step further by merging problem solving into the labs and systematically develop the task using the theory of variation we show that we have been even more successful.

INTRODUCTION AND RATIONALE FOR CHANGE

“What do you *really* do using these complex numbers [in alternating current problems]?” This exclamation one of us (J. B.) heard from his co-instructor when preparing an alternating current lab for a university-level electricity course about 15 years ago. The person behind this statement had, at that time, recently got a Ph D in physics.

This experience points to two conjectures from a rich body of research in physics and engineering education (Bernhard, 2000; McDermott, 1997; Thornton, 1997; Arons, 1995; Arons, 1997):

- A functional understanding (in this case an understanding why complex representation and phasors are used in theory of AC-electricity) is not typically an outcome of traditional instruction. Qualitative reasoning and the ability to make verbal explanations must specifically be addressed in teaching.
- Even faculty members, graduate students and students at high ranking institutions have problems with their conceptual understanding.

Learning electric circuit theory is important in engineering education. For an engineer it's important to know not only DC-circuit theory but also AC-circuit theory since AC-electricity is much more common in technological practise. Students specialising in electrical engineering or engineering physics typically need to study not only AC-circuits but methods for handling more complex circuits and are usually requested to learn to apply various transform methods (phasor, Fourier and Laplace) and Fourier-series in circuit analysis. Understanding of concepts from circuit theory, and specially AC-electricity, periodic signals and transients, is important for understanding of for example electronics, telecommunication and system theory.

However research on student learning and understanding of electric circuit theory is still in its infancy. Student's conceptions in circuit theory and electricity are not as well investigated as those in mechanics. To our knowledge very little research have been done on student understanding of more advanced topics in DC-theory such as superposition, source transformation, mesh-current and node-voltage methods or on student understanding of AC-electricity, periodic signals and on transients. Most studies have dealt with pre-university students understanding of simple resistive DC-circuits or simple circuits with a few bulbs and a battery. Little research has been done on university level students understanding.

OVERVIEW OF EARLIER RESEARCH

Learning and teaching of physics and engineering

Research into the learning and teaching of physics in general have been summarised into the following points (Bernhard, 2000; McDermott, 1997; Thornton, 1997):

- Facility in solving standard quantitative problems is not an adequate criterion for functional understanding. *Questions that require qualitative reasoning and verbal explanation are essential.*
- A coherent conceptual framework is not typically an outcome of traditional instruction. Rote use of formulas is common. *Students need to participate in the process of constructing qualitative models that can help them understand relationships and differences among concepts.*

- Certain conceptual difficulties are not overcome by traditional instruction. *Persistent conceptual difficulties must be explicitly addressed by multiple challenges in different context.*
- Growth in reasoning ability does not usually result from traditional instruction. *Scientific reasoning skills must be expressly cultivated.*
- Connections among concepts, formal representations, and the real world are often lacking after traditional instruction. *Students need repeated practice in interpreting physics formalism and relating it to the real world.*
- Teaching by telling is an ineffective mode of instruction for most students. *Students must be intellectually active to develop a functional understanding.*

The late Arnold Arons (1995) have stressed “The pre- and mis-conceptions found to be widely prevalent among students in introductory physics courses extend to students in upper division courses, to secondary school teachers, to graduate students, and even to some university faculty members. The *proportion* of individuals exhibiting such difficulties decreases significantly but does not drop to zero discontinuously beyond introductory level.” [Emphasis in original text]

Understanding of electric circuits

As mentioned in the introduction most of the research done on electric circuits are in the domain of pre-university students understanding of DC-circuits.

According to this body of research (Duit & Rhöneck, 1997; Cohen *et al*, 1983; McDermott & Shaffer, 1992; Shaffer & McDermott, 1993; Stocklmayer & Treagust, 1996; Shipstone *et al*, 1988) students tend to “cluster” together concepts such as voltage, current, power and energy. This means that students do not clearly distinguish between these concepts and from this “clustering” view follows conceptions such as:

- *Current consumption.*
- *Battery as constant **current** supply.*
- *No current – no voltage.*
- *Voltage is a part or a property of current.*

Research have also shown that it is very difficult for students to see a circuit as an system and to understand that local changes in a circuit results in global changes and that all voltages and currents in a circuit are affected. One can see both:

- *Local reasoning.* Students focus their attention upon one point in the circuit. A change in the circuit is thought on as only affecting current and/or voltages in the circuit there the change is made.
- *Sequential reasoning.* If something is changed in the circuit this is thought on as only affecting current and/or voltages in elements coming after the place there the change was made, not before.

The research which have targeted university students (even electrical engineering students) or secondary school teachers understanding of electrical circuits indicates that these groups reveals very much the same difficulties as found among younger students.

DESIGNING A REFORMED COURSE

– results and discussion

FIRST IMPLEMENTATION

The project was implemented in an electric circuit course for students studying electrical engineering. In the first years of the project experience from a previously successfully developed innovative course in engineering mechanics (Bernhard, 2000b; Bernhard, 2003) using conceptual labs were used. Conceptual labs in electric circuit theory were developed. This series of labs consisted of nine two hour labs utilising a computerbased measurement system.

Our aim was to help students relate electric circuit phenomena and their representational means (mathematical and graphical). We are convinced, and the finding of us and other researches support this claim, that this must expressively and extensively cultivated to make the process transparent to students.

Student's communications and actions during labs were videotaped. Typically two lab-groups were videotaped each time. Thus about one-third of the total number of students were videotaped each time. On two occasions, for technical reasons, only one camera was used. We have also supplemented the videotaping of labs with interviewing selected students inside and outside this course. The videorecordings have not yet been fully analysed. Below we will discuss the results from the labs dealing with AC-electricity.

We see in the videorecordings from the lab, and in from in-class observations, very much the same problems with conceptual understanding of AC-electricity as have been reported before with DC-circuits. A typical example is from different tasks in the lab there students are asked to measure AC-voltages and AC-currents in a circuit using voltage and current sensors. Students who have not conceptualised the difference between voltage and current struggle very much with how to connect these sensors. Typically some students would try to connect the current sensor to the circuit in the same way as a voltage sensors. This means that they are connecting the current sensor parallel to the circuit elements instead of connecting it in series as shown in figure 1.

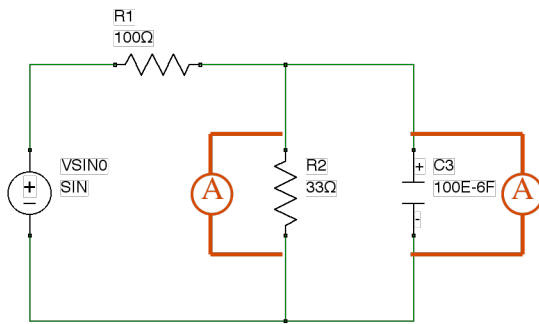


Figure 1. Task: Measure the current delivered by the source and the current through the 33Ω resistance and through the $100\ \mu\text{F}$ capacitor using current-sensors. Many students struggle with how to connect the sensors. The connection above was not uncommon indicating voltage - current confusion

We also see that many students struggle with the interpretation of mathematics in this context. Although some students have problems with their understanding of “pure” mathematics this is not our main point. Our conjecture is that the students have problems with the translation back and forth between the “real” world and the mathematical representation of the observed data. This means that we focus on first arrow (Real world \rightarrow Mathematical Representation) and third arrow (Mathematical Representation \rightarrow Real world). When mathematics is re-interpreted in a physics or in an engineering context many things change: The role of symbols, the conventions for interpreting the symbols and the way equations are interpreted.

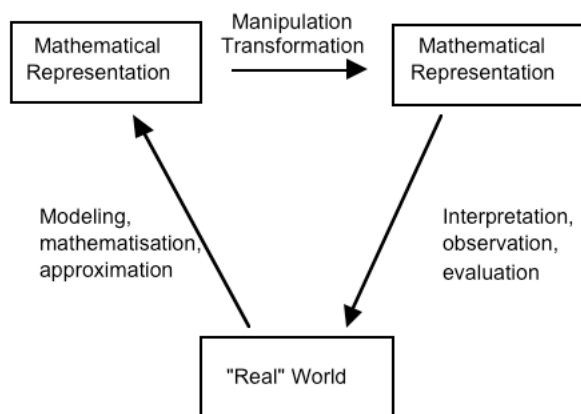


Figure 2. Steps involved in modelling or in the use of mathematics in for example problem solving.

Physics and engineering is not just applied mathematics. The way one thinks about mathematics differs from what is taught in the subject of mathematics. This process is not transparent for students.

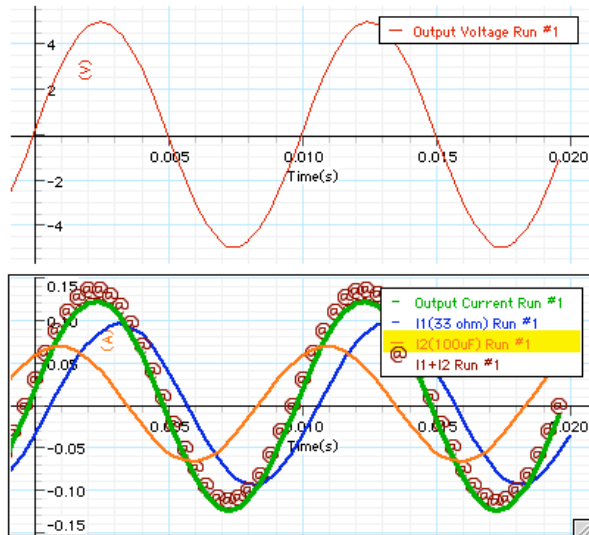


Figure 3. Measurement of AC-currents in one of the task in the laboratory aiming to give students an understanding of Kirchoffs' current law in the context of AC-electricity. The circuit is arranged in such that "output current" should equal the sum of the currents I1 and I2. Included is also this summation (I1+I2) made by the software.

One task in the laboratory was to measure the AC-current in different circuit configurations (Figure 3). We were aiming at enhance student understanding of Kirchoffs' current law. When students were asked to also represent the addition of currents $i_1(t) + i_2(t)$ with the corresponding complex phasor representation many students were lost. Although they had learned complex numbers in mathematics and although the complex representation had been discussed in lecture and in the textbook students struggled with the translations of the measurements similar to the ones displayed in figure 3 to the corresponding complex mathematical representation.

The observations we have made are very similar to the observations of Roth and Bowen (2001) in the context of graph interpretation:

"Our research shows that competent readings are related to understanding of both the phenomena signified and the structure of the signifying domain, familiarity with the conventions relating the two domains, and familiarity with the translating between the two domains. Graphs are not significant (signifying!) signs on their own. /.../ Finally, only through the continuous movement between the experiential and expressive domains do we expect students to begin to dissociate the features of the two, which lead, without familiarity in translating, to iconic errors.

To deal with all these issues will require much more than traditional instruction in graphing has allowed for. To read a graph competently, one needs more than instruction in graphing has allowed for. To read a graph competently, one needs more than instruction on the mechanical aspects of producing graphs. *One's extensive interaction with the phenomena and representational means seems to be prerequisite for competent graphing practises.*" [Our emphasis]

If one substitute mathematical representation into graphing into the writings of Roth and Bowen above this well be very much in agreement with what we see in our preliminary analysis.

$$\begin{aligned}
 y(t) &= y_1(t) + y_2(t) = \\
 &Y_{1m}\sin(\omega t + \phi_1) + Y_{2m}\sin(\omega t + \phi_2) = \\
 &\text{Im}(Y_{1m} \cdot e^{j(\omega t + \phi_1)}) + \text{Im}(Y_{2m} \cdot e^{j(\omega t + \phi_2)}) = \\
 &\text{Im}(Y_{1m} \cdot e^{j\phi_1} \cdot e^{j\omega t}) + \text{Im}(Y_{2m} \cdot e^{j\phi_2} \cdot e^{j\omega t}) = \\
 &\text{Im}(Y_1 \cdot e^{j\omega t}) + \text{Im}(Y_2 \cdot e^{j\omega t}) = \\
 &\text{Im}(Y_1 \cdot e^{j\omega t} + Y_2 \cdot e^{j\omega t}) = \text{Im}((Y_1 + Y_2) \cdot e^{j\omega t}) = \\
 &\text{Im}(Y \cdot e^{j\omega t}) = Y \cdot \sin(\omega t + \arg(Y))
 \end{aligned}$$

There

$$\begin{aligned}
 \mathbf{Y} &= \mathbf{Y}_1 + \mathbf{Y}_2 = \\
 Y_{1m}(\cos\phi_1 + j\sin\phi_1) + Y_{2m}(\cos\phi_2 + j\sin\phi_2) &= \\
 = Y_{1m}\cos\phi_1 + Y_{2m}\cos\phi_2 &+ \\
 + j(Y_{1m}\sin\phi_1 + Y_{2m}\sin\phi_2) &= \\
 = Y_{1x} + Y_{2x} + j(Y_{1y} + Y_{2y}) &
 \end{aligned}$$

$$\mathbf{Y} = [Y] = \sqrt{(Y_{1x} + Y_{2x})^2 + (Y_{1y} + Y_{2y})^2}$$

$$\arg(\mathbf{Y}) = \arctan\left(\frac{Y_{1y} + Y_{2y}}{Y_{1x} + Y_{2x}}\right)$$

$$\begin{aligned}
 Y_{1x} &= Y_{1m} \cdot \cos\phi_1 \\
 Y_{1y} &= Y_{1m} \cdot \sin\phi_1 \\
 Y_{2x} &= Y_{2m} \cdot \cos\phi_2 \\
 Y_{2y} &= Y_{2m} \cdot \sin\phi_2
 \end{aligned}$$

FIGURE 4. Some mathematics behind complex representation: Addition of two sines (such as in figure 3) with the same frequency. When relating to observations several translations back and forth, as discussed in figure 2 and in the text, is needed.

SECOND IMPLEMENTATION

As mentioned above the first implementation of the reformed course had 9 lab-sessions lasting two hours, except the last one which lasted four hours. All these were videotaped. The students also had lectures 2 hours/week and classroom-sessions 2 hours/week. The videotapes were preliminary analysed. The questions raised by the students were in focus of the analysis, and the question we tried to answer was whether it would be possible to include the teacher inventions into the new revised course.

In the second implementation the course were further revised and the lab-sessions and the classroom-sessions have been integrated, resulting in 13 weekly four hour "problem-solving labs":

- Voltage and current – PSpice
- Voltage and current – MATLAB
- AC-electricity – Complex (phasor) representation
- AC-electricity – Circuit analysis
- AC-electricity – Frequency dependency
- AC-electricity – Power
- Magnetic circuit
- Transient response I
- Transient response II
- Fourier I
- Fourier II
- Mathematical methods for circuit analysis
- Summary – further problem solving

The labs which are the focus of this analysis are the last ones – Transient Response. In the former course this lab lasted four hours and the classroom-sessions 2x2, i.e. four hours, which in the new course transformed into two four hour sessions. Thus the same amount of time was appropriated for this part in both courses.

Method for analysis of the lab-instructions

The lab-instructions were revised by using the Theory of variation to promote learning. One of the changes made was to integrate the theory and practice by integrating the problemsolving-sessions into the labs. The other was to vary the problems in such a way that the students would discern critical aspects of the problems, such as that the roots of the polynomial, called the transfer function, could be used to determine the most dominant property of the graph that they also later would measure, see appendix. To find the critical aspects we had looked at the videotapes from the lab course 2002, and as described by Runesson, Mok et. al. (Morris et.al. 2003) discussed those aspects that were critical to the teachers intended object of learning, which did not appear as critical to, or were at all dealt with by the students: the lived object of learning.

What has to be dealt with in the analysis is what is critical to learning, which is not always the same as what is critical to the profession, and analyse how these critical features may be varied in such a way that the student may discern them. It is thus important to analyse which features to vary, and which to keep invariant, as is especially dealt with in the comparisons shown by the founders of the Variation Theory (see eg. examples in Chapter 3 in Marton, Tsui et. al.). One of the more important questions dealt with is how the focus of awareness changes due to which aspect is the one varied and which is the one kept invariant.

It is also important to vary some aspects simultaneously, either synchronously or diachronically. Diachronic simultaneity means that critical aspects are experienced at different

times but are in the awareness simultaneously, and synchronic simultaneity means that they are experienced at the same time. Regarding lab-work it is very often supposed that students should work with diachronic simultaneity, although it is not explicitly said in the instructions, as we will show later in this paper.

Method for analysis of the activity during the labs

Learning is to experience the world in new ways (Marton & Booth, 1997), and to analyse learning is to analyse the new ways students experience their world. One way to analyse the learning in the lab is to observe the students conversation and actions that takes place during a lab-session. Typically very long sequences of videotapes concern the same area of learning, and the students seldom talk about their learning explicitly. Thus it was earlier considered a non-fruitful way to learn about students learning through observation of students when studying (Marton & Säljö 1986, p. 58).

A method to study when learning takes place is to use the method suggested by P-O Wickman and Leif Östman at Uppsala University. It is concisely presented in Wickman (in press):

“Wickman and Östman [2002 – discourse...] have suggested a theoretical mechanism for the learning process that can be used to analyse students’ practical epistemologies. Inspired by the later Wittgenstein(1967;1969) it views talk and actions as inextricably entwined in socially shared language-games. In a language-game certain things *stand fast*. What stands fast is used as point of departure in *encounters* with the world in speech and in action. If they were questioned the practice would stop, and we would no longer be able to act and to communicate with each other. When we say ‘give me a towel’ or when we reach for a towel we do not ask what a towel is or the meaning of these acts. The encounters between persons and towels gain meaning from the language-game, which may be that of ‘getting dry’ (Wittgenstein, 1969,§510).

When people encounter something during talk or in action (utterances, artifacts, natural phenomena etc.) a *gap* occurs. They then establish what is and occurs in the encounter by establishing *relations* to what stands fast, which may fill the gap. However, sometimes people cannot fill the gap immediately. When this happens people typically stage additional encounters to fill the gap with relations to what stand fast in these additional encounters. Such staged encounters typically involve relations from earlier and current experiences and the purpose of the practice. Relations of similarities and differences, sometimes of details, sometimes of wholes are what gives speech and actions sense and what fills the gaps and go on with our doings (cf. Wittgenstein, 1967, §66). If a gap is not filled eventually, the current activity or theme of discourse stops, and a gap will linger. However, before a gap can be filled, it must be noticed. The progress and direction of learning thus depends on the gaps noticed and filled.”

The intended object of learning – Transient Response

Transient response is referred to as one of the more difficult parts of electric circuits, and skipped in many engineering curricula especially at college level. What makes it difficult is that the mathematics used is rather advanced, using the Laplace Transform to solve differential equations. Very often the mathematics is handled in the maths course and in the problem solving sessions, the graphs in the lab course and the conceptual understanding of the transients in the lectures, and still it is expected that the students should make links between them.

The idea of conceptual labs that are carried out in physics courses (Bernhard 2003) has been used in the development of the lab instructions. In this kind of labs as well physical phenomena as their mathematical and graphical representations are elaborated. Working with the Laplace transform to solve the differential equations the intention is to learn and reflect on the chains from the real circuit through the mathematics onto the graph derived mathematically, to compare this graph with the measured graph and thus relating back to the real circuit again, see the chain below. Relations are sought for as well clockwise as counter-clockwise.

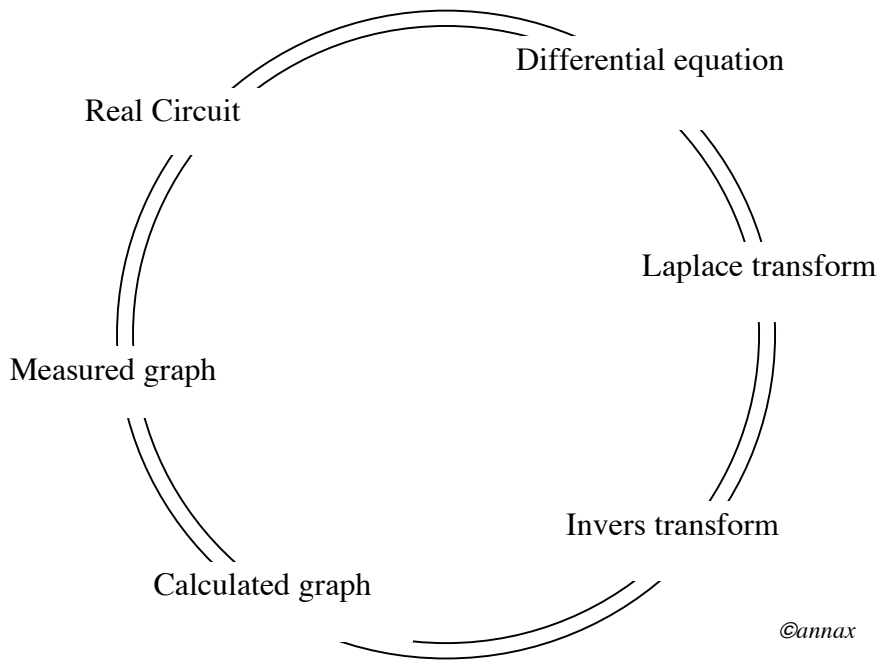


Figure 5: *The intended object of learning*

The students are measuring the current through an electric circuit in which a varying resistor is put in series with a capacitor and an inductor:

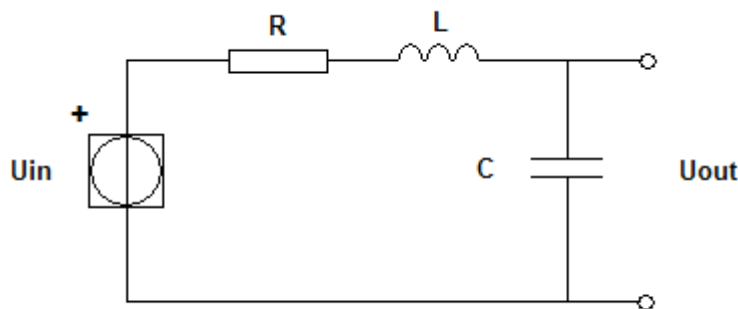


Figure 6: Circuit analysed in the transient response lab

The input voltage is in this lab a step (practically achieved by a square wave with low frequency).

Basically there are two qualitatively different graphs that may be expected from that kind of input(see appendix). Depending on the value of the resistor the graph will show one or the other of the two different curves. The equations that will render the two different types of curves (the solutions to the differential equation, that depend on the denominator polynomial) are either

$$i(t) = ae^{bt} \sin(ct + d)$$

or

$$i(t) = ae^{bt} + ce^{dt}$$

Only one of these equations is given in the lab-instruction, since one aim is to make the students aware of the significant differences of the solutions of differential equations in the context of electric circuits. This ought to be possible since it is supposed that the students

prepare themselves by some problem solving, where both of these equations are elaborated. In the MBL-environment it is possible to get both the measured and the calculated graph in the same diagram, so one task is to put in "the right formula" and change the parameters a, b, c and d, until the calculated and the measured curves coincide. In the report the students are asked to reflect on what difference the change of resistance makes both mathematically and physically.

Results

The results will consist of three parts. First some significant examples from the video recordings from the labs carried out before changes were made, and then a discussion about the lab instructions and the changes made will follow. The third part will give some examples from the video recordings from the new course. (All video recordings are not yet transcribed.)

Analysis of the videotapes before changes

First all groups open the lab instructions which are available via the course page on the Internet. For some groups it only takes a minute, while for others it can take as long as 10 minutes.

Group one 2002

Group one starts immediately and already after two minutes they have some measurements documented on their screen. The first gap that occurs for the students is when they encounter the word "step response":

2002_Group_1_Tape_1 3:44

Anne: Record the step response Reads from instruction

Betty: the step response? questioning, looks towards
the screen

Anne: step response looks in the instruction,
turns towards the

screen

Betty: what? how is it supposed
to look then

The discussion goes on for another two minutes before they ask the teacher what the step response is, and teacher 1 answers that it is the output from a circuit when the input is a step, and that a step can be achieved by a square wave of low frequency. They do some measurements which takes about a quarter of an hour. They study the instructions thoroughly, but stop at the point where they read:

2002_Group_1_Tape_1 17:18

Anne: Here it says: "Your task is to
make fittings to the graph
showing the measured current,
when L and C are kept constant

(20 s)
instructions and

Anne looks at the
the lab board

alternately

Anne: Is that what we have done?

Reviews her notes from the
measurements

(30 s)

Anne: I don't understand

(5s)

Betty: Let's start with one, then
Anne: We are to add some kind of
curve onto the other onto
those we have saved, that
is. OK, it's just to do it

Turns to the page where settings
for the measurements

begin

Betty: Mm

Anne: Let's open the first one then.

They start with the first. They open the user defined fit, but have not entered any function, so they get a straight line at zero. They ask teacher 1 if this is the right curve. He reenters the curve fit (not looking at their window), and he shows them how to receive only one step (which they did not ask), by doing it for them, and after that he tells them to enter the function. He then walks away. They are back at the same point as before the teacher came, so the gap is still not filled.

2002_Group_1_Tape_1 24:39

Anne: Are we supposed to calculate
it first

Betty: But we have no idea about what
formula this is

(30s)

Anne: Do you think this is the formula
to put in

Betty: But it is hardly so, We probably
have something different, other
parameters

Betty starts to add the function which is in the instructions, Anne looks
around. Teacher 1 comes up to them:

26:39

Anne: Mister, this formula here, is
this the one we should enter?

Points to the instruction

T1: Yeah, it is is it is

Anne: Is it exactly this one or what is it?

T1: It is a damped sine now yes, it is this one then, it looks like a
damped sine, but then it generally concerns then what formula it is
supposed to be

Anne: And one is supposed to know that?

Betty: But that's difficult to know

T1: What did you say

Betty: But how do you know?

T1: Yeah, but you get a tip from
calculating the current as a
function of R, which gives you
different kinds of poles.

(4s)

Betty: I don't understand

T1: If you express the current by means
of the Laplace transform

The discussion goes on for another minute or so, but ends with Anne's question:

2002_Group_1_Tape_1 28:15

Anne: But are we supposed to enter this
one

T1: Yes

T1 leaves

Again here is a lingering gap. The students did not make any relations to what they had learned in the lectures or problem solving sessions, but again repeated the very same question as they had started this conversation with. They again get the straight line, and ask the teacher why. He explains that they need to try to find out what the parameters mean. He also talks

about the internal resistance in the inductor, something they didn't ask for, and this is left by the students. They now start to explore the parameters and find the best fit within 20 minutes. Nest graph is not obviously a damped sine, which the students discuss, but since they cannot find out what kind of curve it might be instead, they try to fit a damped sine again. It takes about 20 minutes, and when they are satisfied they ask the teacher to come:

2002_Group_1_Tape_2 36:16

Anne: We can't get it any better now

T1: No, which one are you doing

Anne: We are doing the one with the 10Ω resistance

T1: 10Ω

Anne: It is if we change here, but then it happens some change there

T1: Yeah, but that may be due to the zero there

Anne: But is it OK?

After this they discuss if they should carry out some calculations, but decide to postpone that. They start their third measurement. After a couple of minutes:

2002_Group_1_Tape_3 5:19

Anne: This is the hard thing, one doesn't understand anything

Betty: No, exactly

Anne: Do you find this to be a damped sine?

Both start to laugh

Betty: Yes!

Anne: No, it can't be

They discuss the differences between this new curve and the old ones, especially the "sharp peak" at the top of the curve. They still try to fit it with a damped sine, return twice to the "sharp peak". After about a quarter of an hour they ask the teacher about their problems:

2002_Group_1_Tape_3 17:19

Betty: It looks so strange

Anne: Yes, it does

T1: Which one are you doing

Anne: R33

T1: Is it obvious that it is a damped sine

Anne: No, but we didn't have any other guess.

T1: What alternatives are there?

Anne: We don't know

After a couple of minutes they have come to the conclusion that it is two exponential functions added. For 10 minutes they are now using trial and error to make the curve look somewhat like the measured graph. They change the parameters randomly. Between each statement they do, there is a 2-3s pause. But suddenly they get something more like the graph, they also found out that a and c are of opposite signs. From now the conversation changes and the testing is not random any more. Still they don't get the right curve, and they ask the teacher for more help. He asks if they have done any calculations yet, which they have not. First they say to him that they will do that later, but he insists that they should do some calculations now, in order to find out what kind of values that may be possible, eg. a and c are of opposite signs, and also that both b and d have to be negative.

After 2½ hours they have done the calculations on the third example together with another group, and also received a satisfactory curve fit to both the third and the fourth graphs. Betty reviews their saved material, and Anne continues to do some more calculations, but leaves here place after a while, which results in video-recordings without conversations. They leave after 3:45.

Group three 2002

Group three on the other hand have trouble getting the instructions, and after ca 10 minutes they start to connect the components and the measurement cables to the interface. They ask the teacher:

2002_Group_3_Tape_1 11:45

Mike: John, it says here connect across the whole circuit

After a while they receive a curve on the screen, a sinewave, but are not satisfied, they discuss inaudibly. Their hands show that they had expected a step response, but got a sinewave. After about two and a half minutes they ask another group:

2002_Group_3_Tape_1 17:51

Mike: Should it be set to square wave or

They continue the measurements and save the graphs from using R_{inductor} , R_{10} and R_{33} . After about 40 minutes students from another group ask this group whether they have done any fittings of the curves, which they have not. They review the lab instructions, and utter:

2002_Group_3_Tape_2 0:06

0:06 Mike: What the h_ are we doing?

1:54 Mike: But what curve fit. Are you supposed to just test it
some minutes later

2002_Group_3_Tape_2 4:37

Mike: This is not at all like theirs turns towards the teacher
This is just a bunch of errors loudly across the room
Now he's gonna have to explain
for once

Silence until teacher 1 comes

Pete: We've done this

Mike: We've opened a user defined looks towards the
screen

The teacher shows how to open the right window (the conversation is almost inaudible) Continues at 6:34

T1: So now you can continue

Mike: But we got error there too

T1: But you haven't defined anything yet

Mike: But what am I supposed to define then?

T1: But it tells in the instruction

Mike: And we were supposed to understand this?

T1: Mm, Now it is about unders' now you have got a function an' then
it's jus'to (.) well it's measurements that you're to try to
model mathematically an' it is ab' try to recognise what it is
can be which function it is

Mike: Well, I wouldn't have guessed that one

T1: Hmm now you are on another one refers to
 R_{ind} and

the students

measured R_{33}

Mike, Pete and T1 says something simultaneously

T1: Like that damped one it is most obvious in the first measurement

Mike: =Mm

T1 =with the inductor then for the other also it is then to (.)
think about what (.) what it is can be which type of function
You can also find out which function it is by looking at the
Laplace Transform of the current and

Mike: =Mm

T1: =the poles are

Mike: Now this became all too advanced. I would never have figured
that out

T1 leaves, Pete browses the instruction and Mike giggles
Mike: Alright, let's Laplace-transform and check the pole-values then.

This last quote is reoccurring at least three times more during the lab, and can be considered a lingering gap. The students reflect on the appearance of the curve, and say that they don't agree on the teacher's suggestion of the function as being the damped sine-wave, but although they tried to do the fitting of the third curve as their first one, the teacher does not notice this, but just tells them that the first curve is of this kind.

The group goes on with the fitting of the first curve, but has difficulties, so they repeat:

2002_Group_3_Tape_2 14:37

Mike: let's run the Laplace-transform and check the poles, hey. ironically

They ask for help once more and the tip is to try to figure out which parameter changes what, and they try to find the relation between the constants a, b, c and d and the real world, i.e. c stand for the frequency of the sinewave, a is the amplitude, and b the damping. But, again the students fail. They start with parameter b, which they have found to be the damping, but since they use too low frequency the curve seems to "jump" randomly when they try to change parameter a. Again they ask for help and they find it easier to start with parameter c, which easily can be calculated by measuring the period of the sinusoidal oscillation.

The first task, to fit the first curve, was thus finished after about two hours of the lab-session.

During this last half hour Mike has also started to do some calculations. By the time Mike has finished the calculations of the poles for the first transfer function, Pete has fitted the second curve after about 2½ hours.

When they come back to curve three again they still consider it to be a damped sine, but there is no possibility to make the curve fit to the function. They ask the group next to them and ask about what value they have for a, b, c and d, they tell their values:

2002_Group_3_Tape_4 25:10 (ca 2hours 45 minutes)

Pete: But there we ought to have 4000

Mike: No, maybe not, 'cause this isn't complex like last time.

Charles: No, that's right

Mike: Here we just have ordinary

David: You have to change the function, you know.

Charles: You can't use the sine on that one because=

Mike: =then it will just be two e:s, won't it

They change the formula and the fitting is finished after about three hours. The fourth curve is done in just some few minutes, but now the two groups work together. They continue with some calculations, check if they have saved all the graphs that are needed for the report. The extra task was to look at the change of the curve if an iron core would be inserted into the inductor. They work with the curve fit for a while but conclude (after 3½ hours):

2002_Group_3_Tape_4 25:10 (ca 2hours 45 minutes)

Mike: Now this doesn't work at all.

Pete leaves

I'm tired of all this testing

Mike leaves

Analysis of the lab instructions before and after changes

The most obvious problem in the first course was that the students did not recognise the graphs as showing either a damped sine-wave or a function of two added exponential functions it seemed very important to highlight this. So one of the changes in the lab-instruction was to make the students draw graphs from the solutions to the differential equations, solutions they received through inverse Laplace transformation of examples that

could represent transfer functions of the kind they would be able to measure in the lab. One way of doing this was to make the students work on the inverse Laplace transforms in mathematical terms, by hand, and another to let them elaborate the graphs through Matlab's Simulink, where transfer functions are evaluated numerically, and graphs achieved directly. By using systematically chosen transfer functions that would show the two significantly different curve types, with reference to the two different kinds of poles to the denominator polynomial, as well as some other critical features such as limit value, it was argued that it would become easier for the students to identify the curves they measured. It was also argued that not until the students had begun to do some mathematical work on the Laplace transforms would they possibly be able to fit the measured curve to the user defined function. The normal text books would offer transfer functions with randomly chosen constants, and many of the resulting time-domain-functions that are calculated would never occur in the real world.

The changes in the instructions were thus to

- 1) Include a part where the students elaborated the six transfer functions (suggested in the appendix) in Matlab, Simulink, drawing conclusions about how the graphs were related to the transfer functions
- 2) Make the students do the calculations intertwined with measurements.

The problem with the step response was not considered to remain as a problem after the simulations in Simulink, since the input block would have the name STEP, and the step response would be discussed during that new part of the lab.

Since we have found that students seldom want to, what they call, "waste time" by doing calculations during the lab sessions, even when they are asked to, we inserted the calculations at a point in the lab-instruction where the students would try the most difficult example during the lab session (and also get some hints from the white board on how to do it) and then be asked to do the rest of the examples at home, between the two sessions. By this we would possibly gain as well that the students would study more continuously during the course as would they bring materials from lectures and home work to the lab room.

Analysis of the videotapes after changes

Group one 2003

The students start with the simulations in Matlab. Tess almost immediately starts with the calculations, while Benny tries to figure out what transfer functions they are supposed to simulate. Benny takes some help from the group sitting next to them. After about 10 minutes he has set up the transfer function for the RCL-circuit, calculates the different constants (depending on the possible combinations of R, L and C) to use in the transfer function. They start with $R=100$ $L=100\text{mH}$ and $C=10\mu\text{F}$ and then they change to $C=1\mu\text{F}$

2003_Group_1_Tape_1 25:18

Chris: Yeah, and we should explain this mathematically and physically

Benny: Bu', how easy is this to explain

Chris: I think it ought to be the other way around, almost

Benny: Difficult to tell

Chris: Smaller capacitor, makes less resistance so it ought to go down faster

Benny: =Yeah

Chris: Less resistance so then the voltage will drop faster and since it is

the voltage across the capacitor
that we measure

Benny: =Yeah the voltage across the
capacitor becomes faster, you may say

Chris: =Yeah it runs away faster

Benny: Yeah, here it is charging,
and it does that faster when
it's small=

Chris: =Yes=

Benny: It's pretty obvious

Chris: Then it's the question why there
is a peak=

Benny: Yes

Chris: It has to some exchange between
the capacitor and the inductor there

Benny: The inductor gives a push here
in some way, but the inductor tries
to counteract, not to forget,
here 't is.

Chris: The inductor tries to hinder the
charging of the capacitor

Benny: You can't say that the inductor
sucks out the capacitor?

Chris: But now, wait a second, the
capacitor is charged, the there
is current through the inductor,
and then when it is full, the
capacitor , the current still
continues to come, since the
inductor wants it to keep on for
another while

Benny: =Yes=

Chris: Then it becomes even more charges
in the capacitor than it wants,
so the voltage raises a little more=

Benny: =Yea since the inductor=

Chris: =Then it falls back since the
capacitor throws that voltage
overcharge back out because it
can't keep it.

Benny: Yeah, 'cause when the inductor
has evened out 'cause the current
decreases

Already here in the beginning of the lab there are vivid discussions on subject matter.

They now go on with the examples from the appendix, but add:

2003_Group_1_Tape_1 25:18

Benny: Alright, it's just to do as it
says here then, but have we've
gotto calculate, What's the use
of calculating when the computer
has already done it?

Chris: You can't sit here and calculate
what it will look like

Tess: What?

from her calculations

Chris: Calculate the step response

Tess: Yes you can

Chris: We have a differential equation
for how this circuit will behave
over time

Tess: Yes, but only when you have
inverse-transformed it you will

raises

get it back

The discussion goes on for a while where the guys consider it to render in too much work and Tess concludes "so you might as well get started!" Next Chris and his fellow student start to calculate the inverse transform, using Maple to do the partial fractions, and then do the rest by hand. Benny fetches the lab-board and wires up the circuit and starts DataStudio (the program which shows the measured graphs) Tess continues to calculate the inverse transforms for the six examples. After about one hour of the lab session the teacher is asked to do one of the examples on the white board, which he does. This takes about 20 minutes. After about two hours Benny has made some measurements and is trying to fit the damped sine-function to his graph. He turns to his neighbors:

2003_Group_1_Tape_3 22:49

Benny: Do you know what we'll get from this?

Chris: Sort of

Benny: Well, here I am now points to the screen
To make it raise I have to
increase

Chris: Think like this: This is a sine
wave that rolls away, and here points to the screen
well (.) let's see (.) here is
the amplitude

Benny: Ok, I can see that

Chris: The damping

Benny: Yeah

Chris: How fast it declines (.) Here it
declines too little You have
too damp it harder.

This discussion goes on for another minute or so, and after this Benny has no problems to finish this first curve fit. After half an hour both Benny and the neighbors have finished both of the two first measurements and have problems when fitting the third. They call for the teacher who asks if this is the right function, and the students answer that they don't know. The teacher asks what they think the curve looks like, but the students don't know. They start guessing, but do not suggest exponential functions even if the teacher tries to get them to. Thus the teacher asks them if they have done any calculations, which none of them have. Tess who has been calculating the whole lab session now takes a calculator and gets a graph calculated from one of the exponential functions that she has received through the inverse transform, and shows the guys. The guys hopes for a simpler way to get the right curve than to have to do all the work Tess has done. After a 5 minutes discussion Benny utters:

2003_Group_1_Tape_4 13:02

Benny: Can't we just calculate what it should look like?

Chris: Of course we can

Benny: But that's gotta be the simplest way

They start to do some calculations but decide that they can as well do it at home before the next lab session.

A week later they are back for the second session. The session starts with a discussion:

2003_Group_1_Tape_1_Session2 00:00

Tess: I think we are supposed to process this curve.

Benny: Add a curve?

Tess: Mm..

Benny: Yea, then we'll have to do that.

Tess: But do you think we should go back to the 10[□]:er first, and do
it on that first, and then measure each one again?

Benny: Yea, that's what we'll have to do. Connect the 10[□]:er.
I didn't save anything.

Tess: It doesn't matter.

Benny: We can as well erase the graphs and do them again.
Tess: Mm..
Benny: Oh, yea, and I thought it would be so simple
(Benny starts measuring, and Tess studies the instructions)
Tess: But what else have you processed?
Benny: Ehh (.)I did it on (.)
Tess: I mean this with "fit" and such (.)
Benny: I did it with this one. (Points at the instruction)
The first one I did with this (points at the instructions
again). This one(.) an' then next one (.)'twas much more
difficult to fit 'cause then you need another formula to fit to,
an' it's not so easy to know which one to use.
(Looks through the instructions)
Benny: Here one has to take the one he used.
Tess: OK
Benny: I guess this is the one. Let's see (.)
(Enters the formula into the computer)
Tess: Which formula are you using?
Benny: OK, this is (.)
Tess: Well, I don't want to interrupt, but I don't think that's the
right formula.
Benny: You never know.
(continues writing on the keyboard to see what happens, about
one minute later Tess tries to interrupt again)
Tess: See, This here, this is for the damped sine-wave (points at the
instructions), It looks like this
Benny: Yea.
Tess: And that's not our curve!
Benny: Nop, (looks at the computer) It's not! (starts suddenly turning
the pages in the instructions)
I was perhaps (.) I thought it would be this simple (.) Ehh
(2s)
Tess: We could (.) (starts turning pages again, reviewing the whole
instruction)
Benny: But what can we (.) (scratches his head, looks alternately
towards Tess and the computer, mumbles)
Benny: I don't know what to do.
Benny: Let's see (.) What kind can this be?
Tess: (Difficult to hear) z is not in here, so I don't believe
this is it.
Benny: But later it works, when we do this, but (.) yea, then the
inductor is also in here yea, so, that's another story, then you
get the "sinedamped story" again, yea, But this isn't (.)
but(.). But isn't this an exponential graph maybe?!
Tess: But look at this, it just looks as one of those upgoing ones
from an inductor, (.), At the lecture he showed one like this.
Benny: Wait now, just a resistor and an inductor, hey? Or a resistor
and a capacitor?
(Tess reads her lecture notes and Benny his Lab-instructions)
Tess: No, this was the step-response.
Benny: But, just a second,
Tess: (interrupts) but that's what we have!
Benny: But we have an inductor in this. (wonders)
Tess: But test this one, this is the transient for an inductor. Just
this part.
Benny: But this is only for an inductor (.) Nop, it's not, is it?
Tess: And I think this is for the capacitor. Yes this is the one for a
capacitor (paus) But try something with an exponential function.
Benny: Yea, but where does it say anything about the exexpexponential
function ehh (.) $a \cdot \exp * a$ (.)
Tess: No, no, Not like that
Benny: Is it plus some phase shift ehh, do you think we can just
add (.) ab, ab plus c comma zero (.) well, it

(writes the formula onto the computer)

Tess: It doesn't seem to work

Benny: Nop (.) Oh well, (.) But (.)
(continues to write on the keyboard)

Benny: Well, it's no use in just keeping on guessing.

Tess: No, it isn't. But if we make the rest of the measurements,
and try to figure out the mathematics later.

Benny: But the measurements are made very quickly, it's rather
automatic. There will be no measurements that we will save. It
seems useless to record them. I will lose them anyway. They are
so easy to make.

Tess: Mmm

Benny: I think, I think it's ,that we'd as well go ahead with the math.

Benny: We'd better do it straight away, now when we can get some help.
(Both students look at their notes and instructions)

Benny: What's this?
(They are looking at one page in their notes for about one minute)

Benny: Which capacitor did we use? 100?
And the last one was the (.) 10?
Well, this has to be the fourth one that we've got here!

Tess: But this is a little confusing. I don't see where it ends.
(Looks at calculations made in here notes)
We are supposed to fit the measured current.

Benny: Yea, that's it. I knew that. I didn't think about it.
(Starts writing on the computer again)

Tess: But I don't know if we are supposed to do this for all of them.
I'll check that.

Benny: Yes I think it is, but only the current.

Tess: What do you mean. current?

The discussion goes on

Discussion and Conclusions

(I will call the groups old and new referring to the groups from 2002 and 2003 respectively)

It is obvious from both years that students have difficulties connecting the mathematical representation to the measured graphs and the circuit they use. Especially this is seen in the first half of the lab. As an example Tess has been doing all the calculations, and Benny has worked on the simulations, and when they after about 40 minutes are supposed to wire up the circuit they read:

Tess: "Wire up the circuit" (reads from instruction)
(turns her head
towards B)
It seems taken for granted what circuit he talks about

Benny: Yea, we'd better read this again

Even though Benny had worked with the circuit in order to find the equation to work on, he has now forgotten which circuit he was working with. It does not take very long before they know which circuit to work on, but the stop here is typical of the gap that has not been filled yet.

The gap may also be illustrated by the circle which shows the intended relations to make:

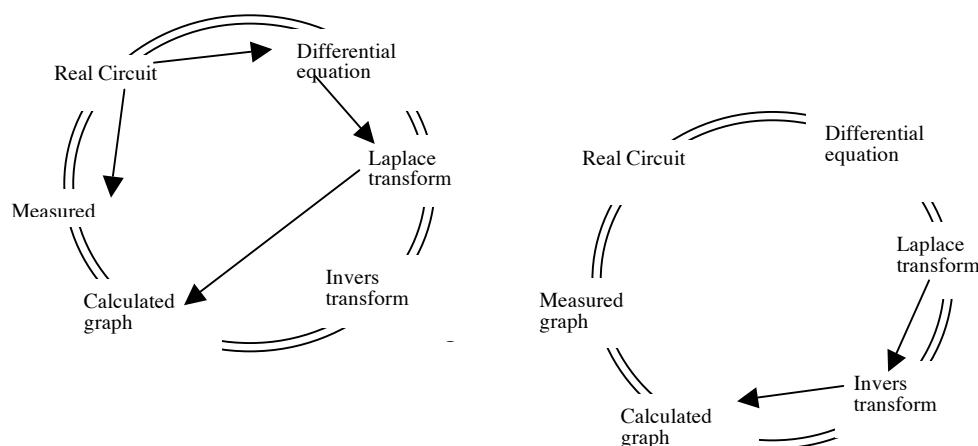


Figure 7: a) Benny's lived object of learning in this first part of the lab
 b) Tess' lived object of learning in this first part of the lab

Tess and Benny have here encountered different objects of learning, and in order to fill the gap they have to make relations to what they know, what is standing fast. None of them is now thinking about the real circuit, because in order to do so they have to make links back, Benny from the graph and Tess from the mathematics.

That the students do not connect what is done in other previous sessions, especially lectures, is evident. For instance the comment that group three in the old course makes several times: "let's Laplace-transform an' check the pole-values" is a comment which shows that they have heard of the Laplace transform, but have no idea what it is. That this comment is constantly recurring is also a sign of a lingering gap. In the new course, where the lab sessions and problem solving sessions are integrated, the students are used to the fact that they need to bring their notes from lectures. In the old course this was only explicitly asked for in the problem solving sessions, and very seldom in the labs, although the teachers had expected the students to bring both books and notes to all sessions of the course.

To integrate the lab sessions and the problem solving session thus gives some important changes in the students' ways to handle the subject matter

- 1) They bring their knowledge from the mathematical context into the lab-room, but can also use the graphs when elaborating the mathematical context. And as a consequence they also bring their materials from the different sessions to all sessions.
- 2) When simultaneously working from as well the real world as the mathematical worlds, the students make the two meet, so that the gaps between the two worlds, may be filled

In the old course one of the questions asked, and asked several times by all groups, was: "Is this curve good enough for the report?" This question is never asked in the new course. The question seems to be stated because the students are not quite sure of what they have been doing, and have thus no idea of what to expect. It also shows that the students' expectations of the lab-work is most of all to pass the course. Of course the students were asked to do homework on problem solving (several examples were recommended in the course information) also in the old course, but they did not do that until late in the course. Forcing the students to work continuously on the mathematical models during the course make them keep up with the course and thus learn more. The change is thus that:

- 3) The focus of the lab work is changed. Instead of focusing on what to report, the students now focus on what is to be learned, i.e. to make links between all the components of the circle:

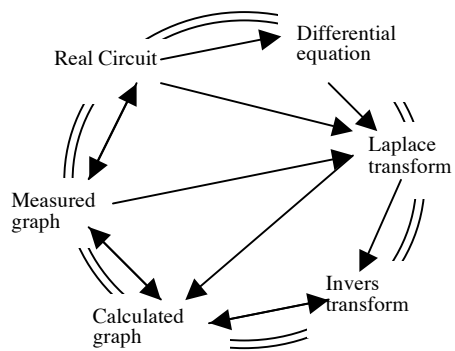


Figure 8: *Links made at the end of the lab-work in the new course*

At the end of the lab-session the Tess and Benny have made all the links described in figure 8. Their discussion simultaneously covers two or more of the links, and the others are figural to their awareness so that they draw conclusions from what they see.

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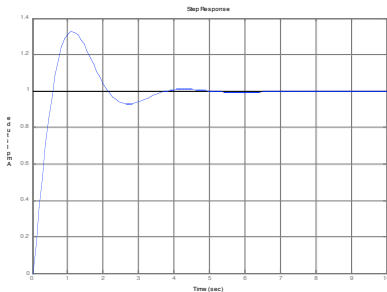
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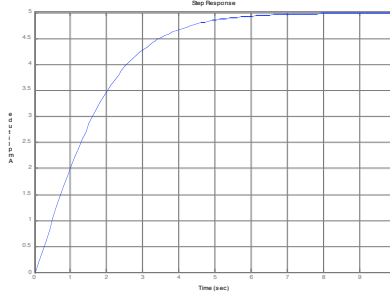
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Appendix: Examples of systematically varied Laplace-functions to analyse, mathematically and graphically

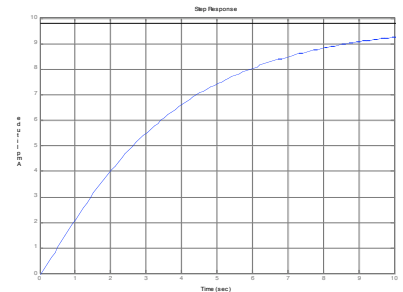
$$G(s) = \frac{2s + 5}{s^2 + 2s + 5}$$



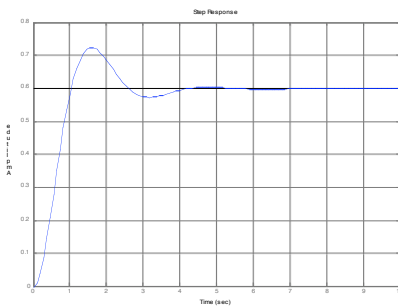
$$G(s) = \frac{2s + 5}{s^2 + 2s + 1}$$



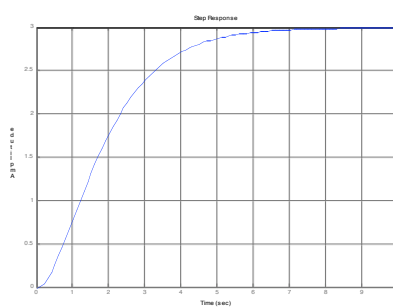
$$G(s) = \frac{2s + 5}{s^2 + 2s + 0.51}$$



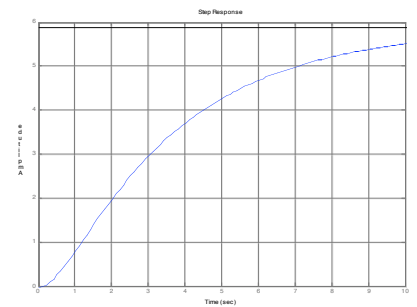
$$G(s) = \frac{3}{s^2 + 2s + 5}$$



$$G(s) = \frac{3}{s^2 + 2s + 1}$$



$$G(s) = \frac{3}{s^2 + 2s + 0.51}$$



Important characteristics:

1) Solutions to the characteristic polynomial, i.e. the poles to the transfer function give different shapes to the curves:

$$s = -1 \pm \sqrt{1-5}$$

$$s_1 = -1 + 2j$$

$$s_2 = -1 - 2j$$

gives under-critically damped behavior

$$s = -1 \pm \sqrt{1-1}$$

$$s_{1,2} = -1$$

gives critically damped behavior

$$s = -1 \pm \sqrt{1-0.51}$$

$$s_1 = -1 + 0.7 = -0.3$$

$$s_2 = -1 - 0.7 = -1.7$$

gives over-critically damped behavior

2) Note the different start behavior that depend on the difference in degree of powers in the nominator and denominator polynomials

3) The Steady-State value depends on the transfer-function's limit-value when s approaches zero.