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An Attempt to Combine Mathematics and Visual Arts

A Research on Islamic Geometric Patterns

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ABSTRACT

An Attempt to Combine Mathematics and Visual Arts

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In this thesis work I try to explain the ways in which I attempted to combine mathematics and visual arts by utilizing Islamic geometric patterns.

The thesis starts with the reasons why I chose to work with Islamic geometric patterns. It continues with a chapter about the mathematical aspects of Islamic geometric patterns in order to give the reader a technical view on Islamic geometric patterns. In the following chapter, I explain my first two projects at C:Art:Media, *Trifinity* and *Earthly*. Afterwards I discuss in detail my degree project *Iduk*. Finally, the thesis ends with a conclusion about my utilization of Islamic geometric patterns in my attempt to combine mathematics and visual arts.

My primary reasons for working with Islamic geometric patterns are two-fold. First, they include both mathematical and artistic aspects, and second, they contain a vast amount of symbolism that I can utilize for my projects. During my projects the symbolic aspect of my works has changed; the symbolic language in my projects evolved from an abstract and ancient language of symbols to a language of more dynamic, current and mundane symbols.

To utilize Islamic geometric patterns in my investigation on how to combine mathematics and visual arts, my methodology was to use these patterns in my projects. In each of my projects, I used Islamic geometric patterns in different ways and observed the results. Moreover, I tried both digital and traditional media to create my projects.

I explained my first two projects *Trifinity* and *Earthly* by discussing my inspirations for the projects, my intentions with the projects and the project's mathematical and technical background. For my degree project *Iduk*, I explained in detail my inspirations, the change of symbolic aspects in my work and the creation process of the project.

Keywords: Islamic geometric patterns, mathematics, visual arts, symbolism

PREFACE

I am a person of many interests. They vary from psychology to botany, from philology to fuzzy logic. However, two of my interests are the most determining for my life: Mathematics and Visual Arts. My enthusiasm for those fields led me to have a B.Sc. in mathematics and to choose Islamic geometric patterns as my research topic in my Master's studies.

Ever since I was a little boy, I enjoyed going to mosques in Istanbul and observing the beauty of Islamic art on the walls of those splendid buildings. Being in the middle of those pleasant geometric patterns always had a mollifying and charming effect on me. With a B.Sc. degree in mathematics and current efforts for finishing my Master's degree, I am convinced that Islamic geometric patterns are at the intersection of my interests in mathematics and visual arts.

Even though I have been interested in arts, mathematics and specifically geometry as separate subjects, my interests began to merge when I took a geometry course at SUNY Binghamton University, when I was an exchange student in USA as part of my Bachelor's degree education. The course was about the construction of geometries from the axioms, and as an artist I was fascinated by the opportunities that geometry offers to art. Geometry, as a visual application of mathematics, provided me with abundance of possibilities to combine mathematics and art. After my exchange semester in USA, I continued to pursue my interest in geometry. For my Bachelor's thesis I researched the Erlanger program, which suggests an algebraic approach for the construction of geometries instead of the axiomatic approach. The axiomatic approach constructs the geometries from postulates, which are arguments accepted without any proof. Then by using these postulates, the axiomatic approach creates the theorems and builds the geometries in that way. However, the algebraic approach considers the construction of geometries as an algebraic operation on a set, e.g. real numbers. Thus, this approach does not need any postulates for creating a particular geometry (Rédei, 1968).

In my view, mathematics is the underlying order of all natural phenomena. To understand the nature deeply we have to first comprehend the mathematical relations among substances. For instance, the distribution or arrangement of leaves on a stem is governed by the Fibonacci number sequence, which is a number sequence where every number of the sequence is the sum of the two preceding numbers 1,1,2,3,5,8,13,21,34,55... (Adam, 2003, p. 216). Specifically, if we are to understand the principles of harmony and beauty, whether it is visual or not, we need to consider the notion of proportion. Pythagoreanism was one of the most influential and early teachings which tried to understand beauty and harmony through mathematical reflections. For instance, Pythagorean music scale was completely constructed upon the ratios of whole-numbers, e.g. ratios of 2:1 or 3:1 (Fauvel, Flood, & Wilson, 2004, p. 14-15). From a mathematician's perspective, music is nothing else but a sequence of numbers (frequencies) which are related to each other according to some certain proportions. However, I believe that this does not reduce the value of a musical score in any way. Instead, it shows that the notion of proportion is essential for euphony, and more generally, that mathematics is the key to understand harmony and beauty.

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1 INTRODUCTION

Since the beginning of my studies at C:Art:Media, I am engaged with Islamic geometric art, and Islamic geometric patterns has been the research topic of my Master's studies. In this master thesis, I will discuss my attempt to combine mathematics and visual arts via Islamic geometric patterns with an emphasis on my degree project and explain my related background and motivations leading me to create my projects. By explaining my background and motivations and by discussing my former projects, I aim to give the context in which I created my last project, and at the same time the progression of my exploration of the question of how to combine mathematics through the use of Islamic patterns.

Islamic geometric art consists of complex geometric patterns which demonstrate us the mathematical level of the Islamic world. Though its roots are in Byzantine and Roman geometric decoration, Islamic geometric art has created a complex and authentic language of its own. Through these patterns, the ancient Islamic artists raised the visual expression of geometric figures to a very high level (Burckhardt, 1976).

On the other hand, visual appeal was not the only purpose of this art form. Since iconography was forbidden in Islam, the elements of calligraphy and geometry were the principal media for artists to express the precepts of Islam (Burckhardt, 1976). Thus, the artists produced these patterns not only to create visual beauty but also to convey the divine notions of Islam (Critchlow, 2001). They usually followed esoteric teachings and used visual symbolism to place these divine notions in the patterns. For instance, they used five pointed stars to symbolize the five teachings of Islam. Furthermore, following Pythagorean teachings, most of the Islamic geometric artists made use of the symbolism of the numbers and geometric figures while creating their art (Critchlow, 2001). However, usually the symbolism was determined in the context of the piece. For example, number eight may symbolize the duality of four elements or the duality of four seasons depending on the context (Critchlow, 2001). Below is one of the typical eight pointed stars which are used excessively in Islamic art.



Figure 1 - Eight pointed Islamic star

1.1 RESEARCH OBJECTIVE

My research objective during my studies at C:Art:Media was to investigate the question of how to combine mathematics and visual arts through the utilization of Islamic geometric patterns.

1.2 RESEARCH BACKGROUND

There are several reasons why I chose Islamic geometric patterns as an instrument in my efforts to explore how to combine mathematics and visual arts. Some of these reasons were apparent to me in the beginning of my research, and some became more apprehensible during the course of my investigation of my research question.

To begin with, I believe that Islamic geometric patterns are one of the most beautiful instances of the combination of mathematics and visual arts. For instance, the mathematical properties of those patterns are examined by crystallographers, since these patterns have similar constructions as the crystals. Emil Makovicky is one of the crystallographers who made extensive research on the crystallographic properties of Islamic patterns (Makovicky, 2008). In addition, modern mathematicians studied these patterns under the category of tessellations (Grünbaum & Shephard, 1989). Even though ancient artists created Islamic patterns for mainly artistic reasons, modern science is recently discovering many other features of these patterns, and these studies are abundant resources in my exploration of the question how to combine visual arts and mathematics through Islamic geometric patterns.

Second, Islamic geometric art contains a vast amount of symbolism and divinity that I can utilize for my own art practice and for conveying my own ideas. However, there is the problem of delivering these symbolic notions to the viewer. Although they are already contained in the Islamic patterns, without any prior knowledge about them, it is almost impossible for the viewers to perceive the symbolic notions. In order to overcome this problem, I usually added a description of the symbols which I use in the project. However, I think that it was not very effective in conveying the meanings of the symbols. Even though the viewers read the description, they are usually not so familiar with the symbolism of Islamic art, and this creates a major problem for conveying the symbolic notions fully. Therefore, for my last project I decided to use a simpler symbolism, which was more down-to-earth. The symbolism in ancient Islamic art works were meaningful in their own ancient historical context, however if one is to use it in today's world then the symbolic meanings lose their context and become too difficult to understand for a viewer who is not acquainted with the ancient Islamic symbolism. Thus, instead of following the old symbolism of Islamic art, I decided to create my own symbols out of much mundane figures, such as a dish-washing detergent bottle or computer to convey and symbolize the ideas of water and metal, respectively.

Third, since I am from Istanbul Turkey, I find a cultural familiarity in Islamic geometric patterns. Istanbul, where I grew up and lived until I was 24 years old, is a city which has substantial Islamic art heritage. To name a few, Blue Mosque, Suleymaniye Mosques and Topkapi Palace are all very fine examples of Islamic art, which contain an abundant amount of Islamic geometric patterns on their walls and doors.

In addition, since my high school was situated in the old city, where most of finest examples of Ottoman mosques and buildings are, I became especially familiar with Islamic geometric art, seeing it every day around my high school. Moreover, in Istanbul Islamic geometric patterns can be found everywhere. For instance, on Galata bridge, which connects Sirkeci and Karaköy in the Golden Horn area, a typical Islamic pattern can be seen on the fences of the bridge.



Figure 2 - Galata Bridge and the Islamic geometric pattern on its fence (TiogaAdventures, 2008)

However, my familiarity with Islamic geometric patterns is not limited to Istanbul. Anatolia has been home to both great Seljuk and Ottoman empires, and the heritage of these Islamic empires can be seen everywhere in Anatolia. Mosques, caravansaries and old residences from the era of these empires are all over Anatolia, and they contain plentiful examples of Islamic geometric art.

Moreover, Dutch graphic artist M.C. Escher's art has always been a big inspiration for me. His interlocking patterns were for me intricate puzzles to solve. I was amazed by the possibility of such patterns, and was wondering how it was possible to create such patterns. After I began to study at C:Art:Media, I extensively studied M.C. Escher's art to understand his techniques and inspirations more deeply and learned that his starting point for his patterns were his observations of Islamic patterns at Alhambra. About his inspirations Escher wrote: "The Moors were masters in the filling of surface with congruent figures and left no gaps over. In the Alhambra, in Spain, especially, they decorated the walls by placing congruent multicolored pieces of majolica together without interstices." (Abas, 2002, p. 105) To comprehend M.C. Escher's work more fully, I understood that I should have studied Islamic patterns more deeply.

Besides the utilization of Islamic patterns, Escher's art was appealing me in other ways as well. In addition to their graphical qualities, Escher's patterns bore a deeper level, which was stimulating me to contemplate about how such patterns could exist. Every time I was studying his patterns, I was

discerning something new and was thinking how he was able to create such puzzling images. One of the most eminent followers of Escher, István Orosz was stating this peculiarity of Escher's art in an interview with Marjoire Senechal: "... the picture is not important anymore, the important thing is the thinking, the mysterium." (Senechal, 2002, p. 85)



Figure 3 - An interlocking pattern of M.C. Escher (Schattschneider, 2004, p.169)

Finally, I found a spiritual fulfillment in those patterns while I was working with them. As I mentioned earlier, Islamic art, specifically Islamic geometric patterns, had always a mollifying and charming effect on me. To work with them was, in a sense, worship for me. As I studied sacred geometry and symbolism of Islamic art, I felt that I was able to comprehend much deeper levels of this art form. The geometric figures were not created only to please the eye, but also to address the human spirits. While I was studying and examining them I had a sense that as if I was part of a greater unity, and behind the visual forms, there were more things to explore. When the ancient designers created their patterns for the holy places, such as mosques and shrines, they were not only concerned about the looks of their patterns; they also wanted to create a spiritual environment for the people who would come to these places. In this spiritual fulfillment, the symbolism of these patterns played a crucial role. Having said this, during the course of my research, my ideas about and usage of symbolism in the patterns have changed. I will discuss this shift in my understanding in more detail in the following chapters.

2 ISLAMIC GEOMETRIC PATTERNS IN BRIEF

Before going any further, I would like to first explain some mathematical characteristics of Islamic geometric patterns in order to give a better and technical picture of these patterns.

The base of Islamic geometric patterns is tessellations. According to the website *Wolfram Mathworld*, “A regular tiling of polygons (in two dimensions), polyhedra (three dimensions), or polytopes (n dimensions) is called a tessellation” (Wolfram MathWorld, 2008), where by regular tiling we mean that a regular polygon (an n-gon with equal sides, e.g. a triangle, a square or a hexagon) fills up the whole 2-dimensional space (which is basically just a plane) without any gaps. However, there are only three polygons which can fill a plane without any gaps: triangles, squares and hexagons (Wolfram MathWorld, 2008).

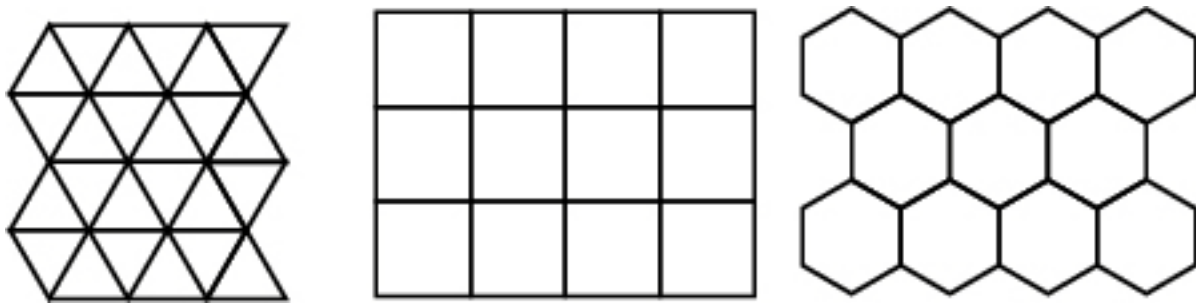


Figure 4 - Regular tilings of triangles, squares and hexagons (Wolfram MathWorld, 2008)

Islamic geometric art employs these tessellations as an underlying grid and develops its own patterns.

Main content of the Islamic geometric patterns is Islamic stars. The typical construction of an Islamic geometric pattern consists of a major star in the middle and accompanying stars or other geometric elements around this central star.

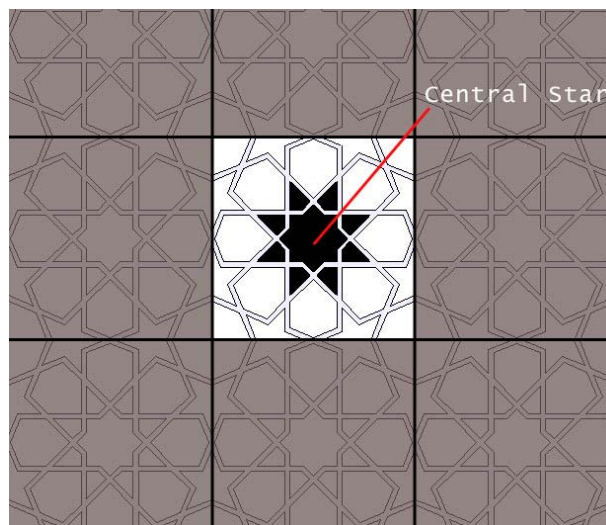


Figure 5 - Islamic pattern and its central star

According to El-Said and Parman (El-Said & Parman, 1976), there are 3 main classes of Islamic geometric patterns based on the proportions in the geometric figures:

2.1 THE CLASS OF $\sqrt{2}$ RELATIONS

These patterns are based on the square tiling of a plane.

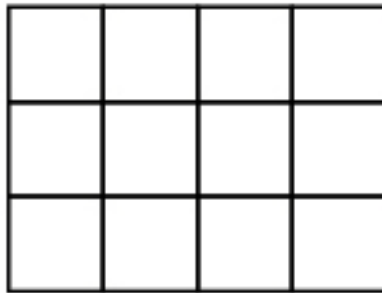


Figure 6 - Square tiling of a plane (Wolfram MathWorld, 2008)

The name derives from the fact that the proportion of a square's diagonal to its side is $\sqrt{2}$. In other words, if the length of a side of a square is 1 cm, then the length of its diagonal is $\sqrt{2}$ (~ 1.414) cm.

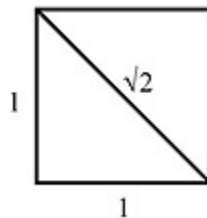


Figure 7 - Proportions in a square

The most common examples of this class are the patterns with eight pointed stars. The pattern in Figure 5 is a typical instance of this class.

2.2 THE CLASS OF $\sqrt{3}$ RELATIONS

These patterns are based on the hexagonal tiling of a plane.

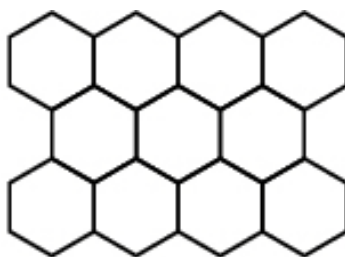


Figure 8 - Hexagonal tiling of a plane (Wolfram MathWorld, 2008)

This class is named after the proportion of a hexagon's height to its side. Put differently, if the length of a side of a hexagon is 1 cm, the length of the hexagon's height is $\sqrt{3}$ (~1.732) cm.

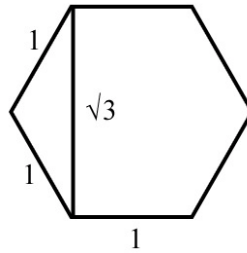


Figure 9 - Proportions in a hexagon

The most common examples for this class are the patterns with 6 or 12 pointed stars.

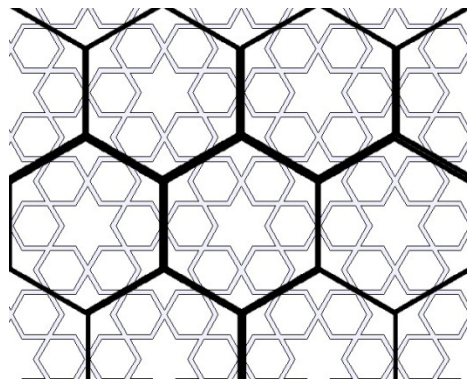


Figure 10 - A pattern of $\sqrt{3}$ relations class

2.3 THE CLASS OF $\sqrt{5}$ RELATIONS

The patterns of $\sqrt{5}$ relations class are based on ten pointed stars and ten pointed stars are based on pentagons. This class is named after the ratio of a pentagon's diagonal to its side $(1 + \sqrt{5})/2$ (~1.618). The ratio of a pentagon's diagonal to its side is also known as golden ratio since the antiquity (Lawlor, 1982).

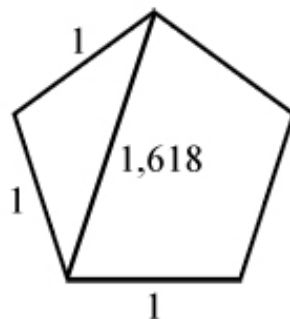


Figure 11 - Proportions in a pentagon

However, since pentagons cannot fill the plane regularly, these patterns were not commonly used to fill big planes like walls. Rather, they were used to decorate more limited and smaller planes, such as doors or windows.



Figure 12 - A door decoration with ten pointed star

After having discussed the mathematical properties of Islamic geometric patterns, now I would like to talk about my projects at C:Art:Media and the ways in which I tried to investigate the question of combining visual arts and mathematics through the utilization of Islamic geometric patterns.

3 PREVIOUS PROJECTS AT C:ART:MEDIA

Before explaining my final project *Iduk*, I want to first discuss my previous works in which I utilized Islamic geometric patterns. In this way, I hope to elucidate the progression process of my exploration of my research question and give the context in which I created my final project.

3.1 TRIFINITY

My first project at C:Art:Media was *Trifinity*, which was a video animation loop. It was played on an LCD screen, and in front of the screen two conjunctive mirrors were placed. The LCD screen and mirrors were altogether placed in a dark box. The viewer could only look through a slight hole on this box facing towards the mirrors. Thus, what the viewer saw was the repetitive reflections of the animation on the mirrors. (For a visual explanation see the figure below)

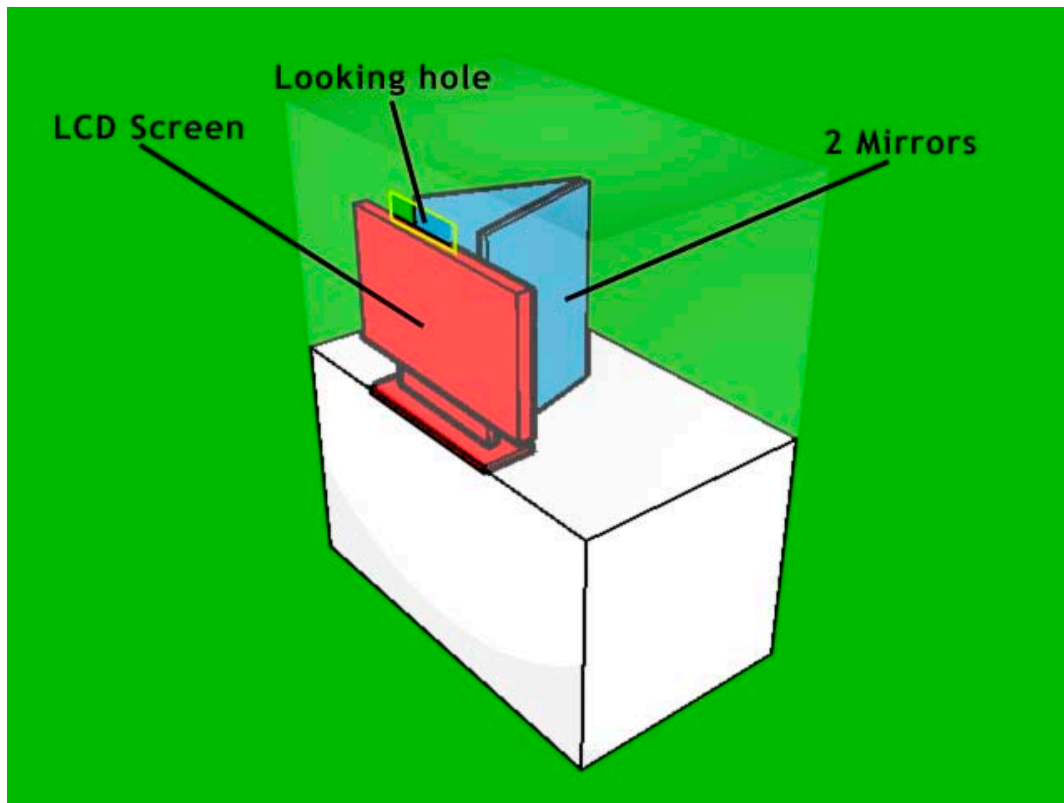


Figure 13 - Placement of the LCD screen and the mirrors in the box

The content of the animation was the progression of three tessellations, in which each tessellation morphed into the next one in a loop, and an accompanying musical tune. The tessellations were based on triangles, hexagons and the combination of triangles and hexagons.

3.1.1 BACKGROUND AND INSPIRATIONS

Trifinity was my first project, in which I tried to combine mathematics and visual arts. As I researched about animations which utilized mathematical aspects, I discovered works of John Whitney. The notions of proportion and periodical repetition, which are fundamental in Islamic geometric patterns, were well observable in his works. He used consecutive progressions of geometrical shapes, and also employed musical tunes by synchronizing the images with the music. I enjoyed the synchronization of music with visuals in his works. I thought that this synchronization created a really strong effect due to the combined concordant usage of images with music and also wanted to have a similar trait in my project. On the other hand, most of his works visually consisted of curves, which are one dimensional geometric shapes. However, Islamic geometric patterns consist of two dimensional geometric shapes, and in my project I wanted to use geometrical figures which had some volume to them and could cover a plane.

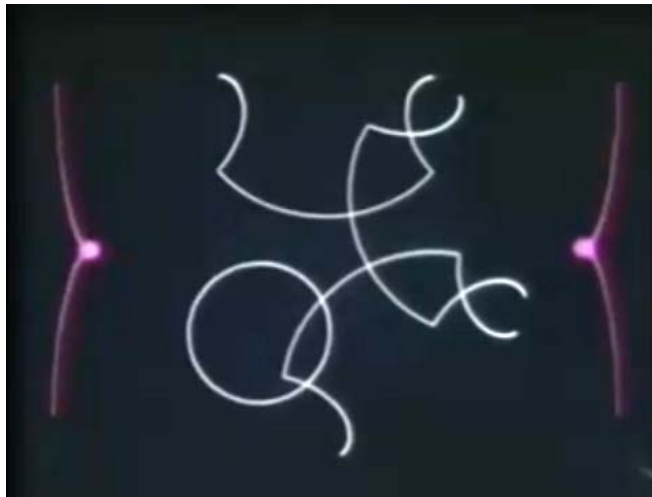


Figure 14 - Arabesque by John Whitney (Whitney, 1975)

Before the project, I was making extensive research on the mathematical properties of tessellations and of Islamic geometric art. As I researched, I became more aware of the importance of tessellations in the Islamic patterns. Tessellations were the starting point of everything; thanks to them, it was possible for a shape to fill the whole plane without leaving any gaps. An Islamic geometric pattern was able to fill a plane without any gap due to its fundamental tessellation based construction. Bearing these in mind, I decided to utilize tessellations for my start at C:Art:Media.

For the animation, I chose three tessellations which were my favorite at that time. These tessellations appealed to me very much, since while they are very simple, they have a strong sense of motion and harmony, and their composition is firmly knitted. The tessellations consist of triangles, hexagons and the combination of triangles and hexagons. In addition, each of them has their mathematical names: 3.3.3.3.3.3, 6.6.6, and 3.6.3.6, respectively (See figure below)

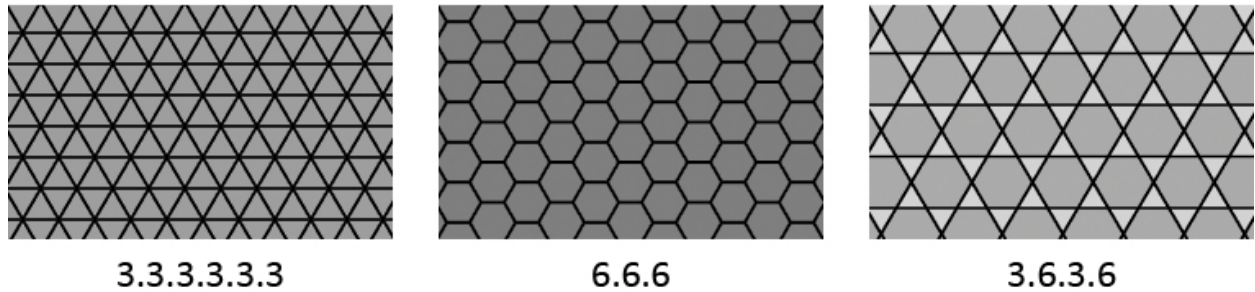


Figure 15 - Tessellations used in *Trifinity*

Having been inspired by Whitney's synchronization of music and images, I used music in my project in a similar way as well. I composed a tune depending on the mathematical names of the tessellations in order to emphasize the project's connection to mathematics. For example, in the case of 3.3.3.3.3.3 the accent of the tune was on every third beat. Thus, each tessellation was accompanied by a tune in which the accent was on the beats that the tessellation's mathematical name indicates.

3.1.2 INTENTIONS

My intention with this project was to start my exploration about how mathematics and visual arts could be combined through the usage of Islamic geometric patterns, in the practice. I had my mathematical knowledge from my Bachelor's education already, but to apply my knowledge in the new language and context of visual arts was a challenge for me. I had worked with visual arts before my Master's studies; however, it was the first time that I was trying to combine both mathematics and visual arts. I had to learn the language of visual arts more deeply, and had to comprehend its ways of expressions so that I could use these expressions in my exploration of my research question.

As I mentioned earlier, after some research about the mathematical aspects of Islamic patterns I became aware of the importance of the tessellations in the foundation of these patterns. This awareness led me to decide on tessellations as the fundamental element in my work.

The reason I used the mirrors was to enhance the notion of infinity of tessellations. The patterns are by definition shapes which can fill a plane infinitely. Therefore, when I used the tessellations as patterns, I wanted to strengthen the feeling of infinity by putting two conjunctive mirrors to create a kaleidoscopic effect. (See figure below)

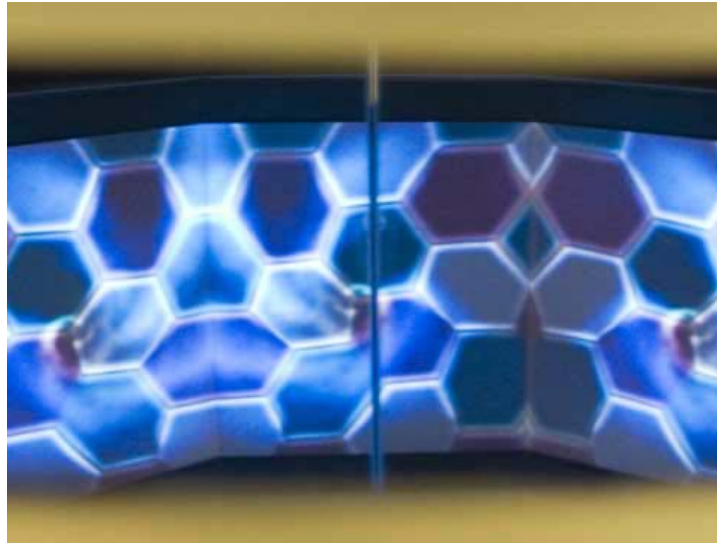


Figure 16 - Kaleidoscopic effect in *Trifinity* as seen through the looking-hole (Valand, 2008)

In addition to my goal of creating a strong effect with the synchronized use of music and images, my aim in the musical tune that I composed for the project was to refer to the mathematical roots of the project. By composing the tune according to the mathematical names of the tessellations, I wanted pay my tribute to mathematics. When the tessellation 3.3.3.3.3.3 was on the screen, the accents of the tune were on every third beat. When 6.6.6 was on the screen, the accents were on every sixth beat. Finally, when 3.6.3.6. was on the screen, the accents were on every third and sixth beat alternatively.

Besides referring to project's mathematical roots, I also wanted to utilize the musical tune for creating a tense atmosphere for the viewers. When I made the visual part of the project, it reminded me of the strange mirrors in amusement parks. In my view, the most important characteristic of those mirrors are their ability to create a tense and strange atmosphere, and I also wanted to create a somewhat similar atmosphere in my project via the musical tune. Thus, I used repeated linear drum patterns in an effort to create a strange and tense atmosphere in *Trifinity*. The reason I chose these linear drum patterns were that, in my observations, this type of drum patterns were most commonly used to create such atmospheres in movies.

3.1.3 MATHEMATICAL BACKGROUND

In *Trifinity*, I used the tessellations, which consist of only triangles and hexagons. The first two, 3.3.3.3.3.3 and 6.6.6. are regular tessellations. Regular tessellations are defined as tessellations consisting of only one type of regular polygons. On the other hand, the last tessellation, 3.6.3.6. is a semiregular tessellation. Semiregular tessellations are defined as tessellations consisting of a combination of two or more regular polygons. (ThinkQuest, 1998)

In my choice of the tessellations, the determining factors were that all these tessellations have the same 6-fold symmetries, which are 6-fold rotational and reflection symmetries, and the commonality of elements they consisted of, namely triangles and hexagons. I chose to work particularly with triangles,

since they are polygons with least number of corners. In addition, a hexagon can be divided into six equilateral triangles, and so, hexagons can be considered as derivatives of triangles.

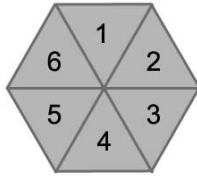


Figure 17 - A hexagon divided into 6 equilateral triangles

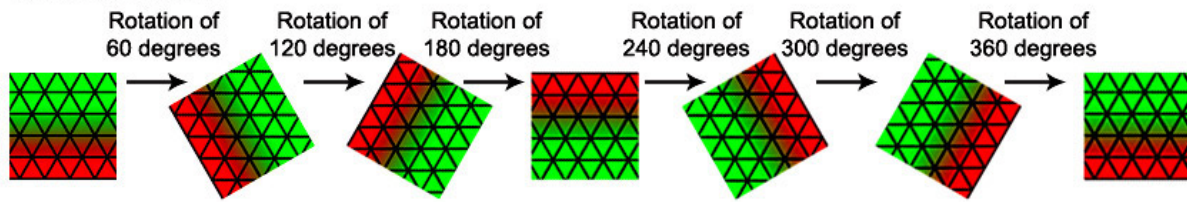
Furthermore, as I will explain in the following paragraphs, both the tessellations 6.6.6. and 3.6.3.6. can be derived from 3.3.3.3.3.3. Thus, although the tessellations I used in the project are different, they can be regarded as derivatives or descendents of 3.3.3.3.3.3.

Thus, in *Trifinity*, I was only using triangles and the tessellation 3.3.3.3.3.3. and their derivatives. This provided me with the opportunity to benefit from the simplicity and the convenience of using different but closely related tessellations and aided me in my effort to reach a sense of unity in the project.

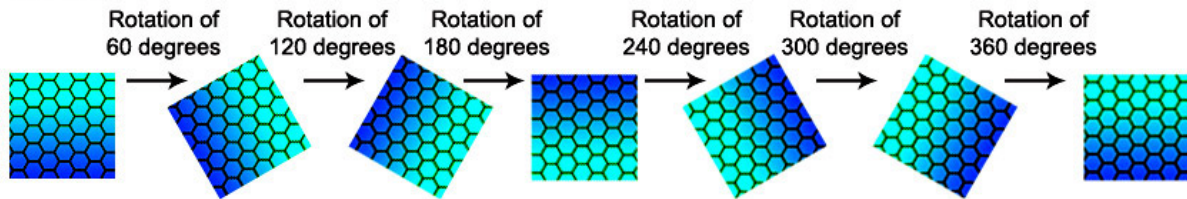
Now, I want to discuss more about the mathematical properties of the tessellations.

The 6-fold symmetry can be observed on all tessellations as 6-fold rotational symmetry and as 6-fold reflection symmetry. By rotational symmetry, we mean that the figure remains unchanged under a rotation of some certain degree (Jacobs, 2003). In the case of the tessellations at hand, a rotation of 60 degrees, 120 degrees, 180 degrees, 240 degrees, 300 degrees and 360 degrees would keep the original tessellations unchanged. (See figure below)

3.3.3.3.3.3.



6.6.6.



3.6.3.6.

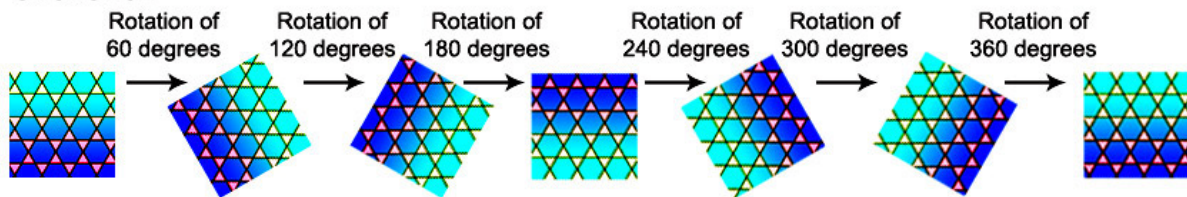


Figure 18 - Rotational symmetries of the tessellations used in *Trifinity*

By reflection symmetry, we mean that there exists an axis of symmetry (basically a line) which intersects the figure into two parts which are the reflections (mirror images) of each other with respect to this axis (Jacobs, 2003). In the case of the tessellations I used in *Trifinity*, there are six axes of symmetry, which are labeled as R1, R2, R3, R4, R5 and R6 in the figure below. For example in the case of tessellation 6.6.6., the axis R1 divides the tessellation into left and right sides which are the mirror images of each other with respect to R1.

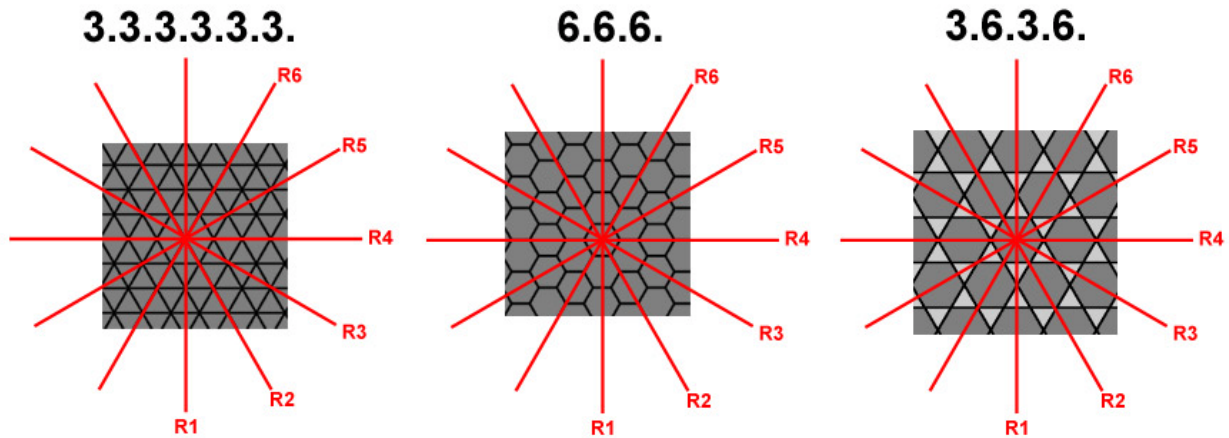


Figure 19 - Reflection symmetries of the tessellations used in *Trifinity*

In addition to their common symmetries, the tessellations consist of common shapes. In fact, both the tessellations 6.6.6. and 3.6.3.6. can be derived from the tessellation 3.3.3.3.3.3. by keeping some corners visible and neglecting other corners. In the case of 6.6.6., at any vertex if we combine the six triangles around this vertex and only leave the perimeter of these six triangles visible, the resulting image would be a hexagon. Thus, by applying this process to necessary vertices, we can derive the tessellation 6.6.6. from 3.3.3.3.3.3. This process is illustrated by the figure below.

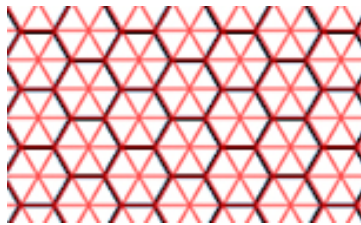


Figure 20 - Derivation of 6.6.6. from 3.3.3.3.3.3.

Similarly, 3.6.3.6. can be derived from 3.3.3.3.3.3. Around some certain vertices, we can create hexagons again using the method above, and we can leave the remaining triangles as they are. Thus, the resulting image will be 3.6.3.6. (See figure below)

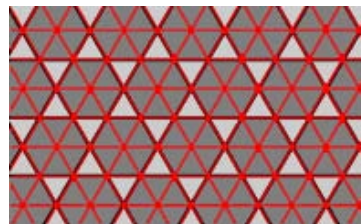


Figure 21 - Derivation of 3.6.3.6. from 3.3.3.3.3.3.

3.1.4 TECHNICAL BACKGROUND

For the animation, I mainly used the 3D modeling and animation program 3ds Max, since it was the 3D software I was most familiar and comfortable with. In addition, the modeling and duplication process of geometric figures were relatively easier compared to other 3D software, such as Maya. Besides 3ds Max, I used the visual particle effects application Particle Illusion and the motion graphics and compositing program After Effects, since again they were the software, which I was most familiar with for creating particle effects and compositing, respectively.

To make the animation, I first created the shapes in 3ds Max as 2D and then turned them into 3D shapes. Afterwards, I created a small animation in which the camera was moving through the three tessellations. I also added a warping effect to the 3D tessellations in an effort to create a feeling of dizziness in the project. I also tried to strengthen this feeling by a whirling wind, which I first created in Particle Illusion and then combined with the animation in After Effects. I tried to create a feeling of dizziness in the project, since the kaleidoscopic effect of the mirrors I used was reminding me of amusement parks. When I was a child, strange mirrors were always my favorite in amusement parks, and I was always feeling a little dizzy whenever I was among them. Therefore, I wanted to create a similar effect for my project.

For the musical tune I used the digital audio application FL Studio, since I was very familiar with the software, and its interface was much more user-friendly compared to other popular music software, such as Qbase. In addition, in FL Studio it was especially easy to work with percussion synthesizers, since the main interface of FL Studio is based on rhythm and beats. To every beat in the tune, one can assign a particular sound and in this way construct a rhythm pattern. This function facilitated my experiments with different types of percussion synthesizers and different rhythms.

3.2 EARTHLY

My second project at C:Art:Media was *Earthly*. The project consisted of four wooden tiles which were handcrafted by me. I based the design of the tiles on eight pointed Islamic stars.

3.2.1 BACKGROUND AND INSPIRATIONS

After using digital media for my first project, I wanted to try traditional media for my next project with patterns. I wanted to have concrete pieces that existed in real world. To decide to make the tiles out of wood required me many trials and errors. At the beginning of the project I was considering to use several other media such as big paper stickers. However, I wanted to have something close to what the ancient tiles were. Since I was not able to build the tiles out of ceramic, I decided to make them instead out of wood, which was easy to shape and to paint however required too much effort to work with.

For this project, instead of the basics of Islamic patterns, which are tessellations, I wanted to make something closer to the real geometric patterns and created my patterns with the inspiration of the ancient works. As I researched contemporary artists and designers, who work with Islamic geometric

patterns, I became familiar with Jay Bonner and Tim Backhouse. Especially, Jay Bonner was a distinguished designer who worked with Islamic geometric patterns professionally and used them in many architectural projects and restorations in addition to his publications on Islamic geometric patterns. To design their geometric patterns, both Bonner and Backhouse were using mainly straight lines as in the classic Islamic geometric patterns. However, I wanted to create a new pattern which would be based on classic Islamic geometric patterns but would still not exactly follow the ways ancient designers used. I thought that instead of only lines and geometric shapes with straight sides, using a combination of lines, curves and geometric shapes with curvy sides could be a good way to create a new pattern. Moreover, I wanted to design my pattern in a way that it would have some natural flavor in it. Therefore, instead of straight lines, I used predominantly curves in my design. I tried to make my Islamic stars look more like flowers than stars without using any floral figures.

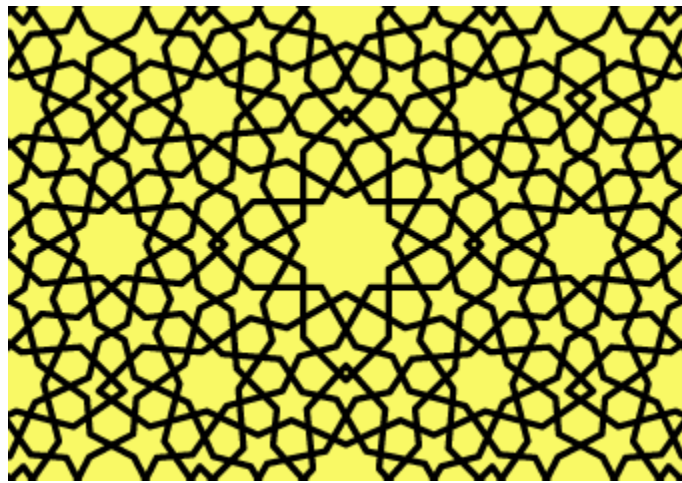


Figure 22 - A geometric pattern by Jay Bonner (Bonner, 1990)

I wanted to symbolize the element *earth* in my project, since I believe that earth is where the human life begins. Moreover, according to Quran the humans are created from clay, “And certainly We have created man from an extraction of clay” (Quran 23: 12). Both in ancient Greece and Islamic thought number four was the number which was closely associated with earth (Critchlow, 2001, p.150). Thus, I chose the square, which is the geometric representation of number four, to work with. The patterns in $\sqrt{2}$ relations class were the ones which were created using the square, and I based my own pattern on the Islamic geometric patterns from the $\sqrt{2}$ relations group. Furthermore, the project was presented on the floor to strengthen the symbolization of the earth, and four tiles were used to strengthen the symbolization of number four.

In addition to the symbolic meanings of square and number four, I added extra curves to the pattern in order to symbolize the vibrations in the earth. Through my studies of sacred geometry, I became familiar with the fundamental idea that everything in the universe was result of vibrations. Even an atom basically exists through vibrations of atomic particles (Lawlor, 1982). This idea was so intriguing for me that I decided to include it in my project. Therefore, I tried to express this idea in connection to earth through the curves that I added to my pattern.



Figure 23 - *Earthly*

3.2.2 INTENTIONS

My main aim in this project was to utilize Islamic geometric patterns in traditional media and investigate the results. Since I used digital media in my first project, I wanted to explore the results of traditional media in my work and how this media would aid my efforts to combine mathematics and visual arts. However, after exhibiting the project, I felt that the traditional media was not that convenient for my efforts. I thought that the patterns were to fill a plane fully. However, if the pattern did not fill a plane fully, it seemed that something was undone or unfinished with the project. In principle, by producing enough tiles to fill a plane this obstacle could be overcome. However, since I was only one person handcrafting all the tiles, to produce that many tiles was not that feasible.

Another aim was to integrate symbolic meanings into the project. In my first project I did not use any symbolism. However, as I researched more about Islamic geometric patterns and became aware of their symbolic properties I wanted to use this property also in my work. Thus, I decided on the symbols of earth. On the other hand, I also realized the problems of using an ancient symbolic language in this project. Even though the symbolic meanings were apparent to me or to other people who are familiar with ancient symbolism, the symbolic language that I used was not compatible with the current context they were exhibited. Hence, I had major problems of conveying the symbolic meanings to the people who were not familiar with the ancient symbolism. I tried to overcome this problem by adding an explanatory paper about the symbolic meanings of number four and square to the project. However, I observed that it has been not that successful in conveying the symbolic meanings.

3.2.3 MATHEMATICAL BACKGROUND

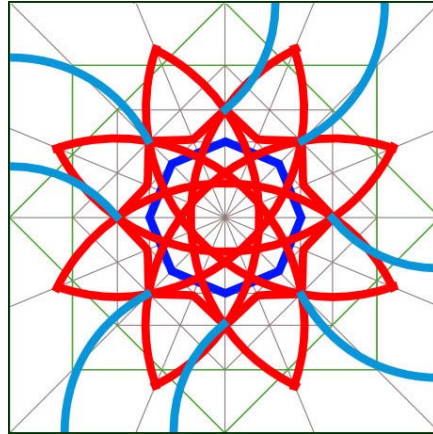


Figure 24 - *Earthly pattern*

Since I chose to symbolize the element earth through number four and its geometrical representation square, the design heavily depended on the square and $\sqrt{2}$ relations. I began the design with nesting two additional squares in the base square. If we assume one side of the base square is 1, then $\sqrt{2}$ relations, a side of the nested squares is $1/\sqrt{2}$. By having the first nested squares, I based the design on one of the most commonly used square based Islamic geometric pattern.

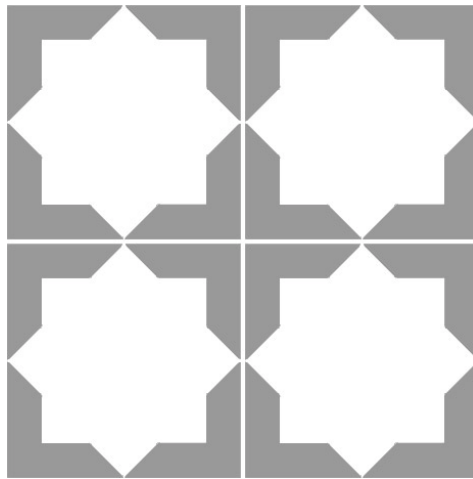


Figure 25 - *Square based Islamic geometric pattern*

I continued the design by nesting more squares in the base square. By $\sqrt{2}$ relations, in every step the side of the square was decreasing to $1/\sqrt{2}$ times the side of the preceding square. I first nested two more additional squares with side $1/2$ and afterwards two additional ones with side $1/2\sqrt{2}$ in the squares with sides $1/2$. Thus, I had three pairs of additional squares nested in the base square. Subsequently, using the circle which is contained in the intersection of the last nested squares and has the diameter $1/2\sqrt{2}$, I drew the octagon which is inscribed in this circle.

Generally to create the Islamic stars, straight lines were used. Most of the stars were created by superposing polygons onto each other and then by keeping some certain parts of the resulting shape visible and omitting the rest (Critchlow, 2001). However for my project, I wanted to make my stars more natural-looking and therefore, decided to use curves instead of straight lines. In my opinion, one of the biggest advantages of curves were that their combinations were looking much more like flowers rather than stars, which was giving the combination of the curves a more natural flavor. Thus, I drew the circle with the center where the right corner of the oblique square of the second nested square pair meets with the right side of the upright square of the first nested square pair, and with the radius which extends from this center to the upper corner of the oblique square of the last nested square pair. I continued to use this method for all the remaining corners, and created a flower-like star.

In an effort to create a smooth transition between the octagon and the flower-like star, I placed an eight pointed star in the design, such that the tips of the star were at the intersections of the curves. Again I did not want to have a star consisting of straight lines. Therefore, I created an eight pointed star with curvy sides in the vector graphics program Inkscape. The reason I used Inkscape was the easiness of creating stars with sides as curves in this software.

Finally, I added the curves, which represented vibrations in the earth, to the design in a way that they would append to each other in the composition of the tiles.

This creation process is illustrated by the following figure.

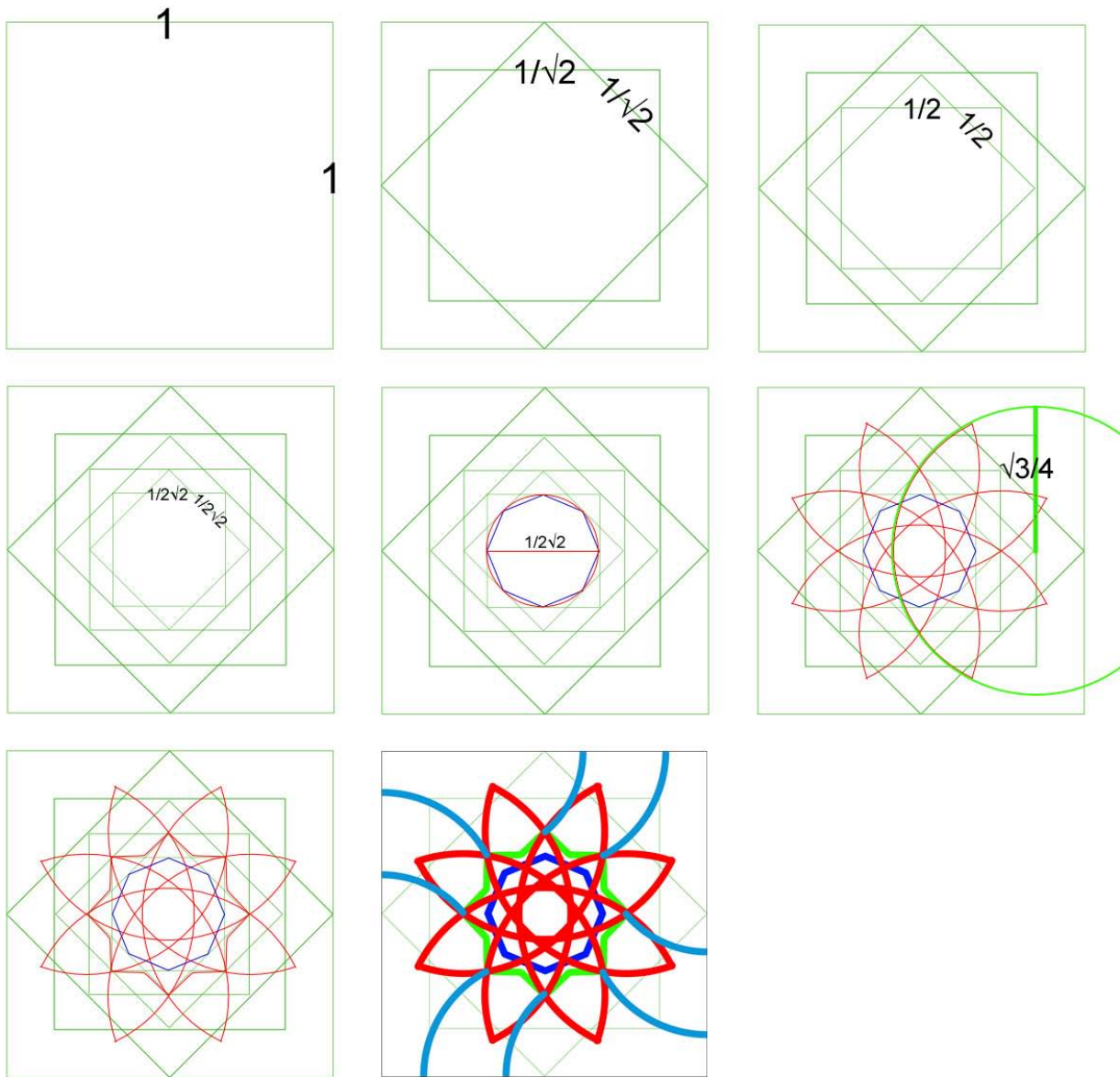


Figure 26 - Creation process of *Earthly*

In this project, the main pattern (without the curves representing vibrations in the earth) has 8-fold rotational and reflection symmetries. A rotation of 45 degrees, 90 degrees, 135 degrees, 180 degrees, 225 degrees, 270 degrees, 315 degrees and 360 degrees would keep the pattern unchanged. In addition, the eight axes of reflection symmetry can be observed in Figure 24.

In mathematics, the algebraic group consisting of the rotational and the reflection symmetries of a figure is called a *dihedral group*, and denoted with D_i where i indicates the number of symmetries (Feyzioglu, 1990). The main pattern I designed for *Earthly* has D_8 as its symmetry group, which consists of the eight rotations and eight reflections I mentioned above. By saying D_8 is the symmetry group of a

geometric figure, we mean that applying any rotation or reflection of D_8 or their composition on the figure will keep the figure unchanged.

By having these symmetry properties and D_8 as the symmetry group of the figure, I tried to strengthen the notion of the square mathematically in the project. Since, all these properties are very closely related to $\sqrt{2}$ relations, and $\sqrt{2}$ relations stem from the usage of square.

3.2.4 TECHNICAL BACKGROUND

In my efforts to create the pattern, I mainly utilized the vector graphics program Inkscape, since it has a special function called “Create tiled clones”, which clones a figure according to symmetry groups. This function was a great help for me, since whenever I had a pattern I could check how a combination of several duplicates of the pattern would look like, or if I designed only a part of the pattern rather than the whole pattern itself, I was able to try different symmetries and check how the resulting whole pattern would look.

After creating my patterns in Inkscape, I exported them to another vector graphics program Illustrator, in order to make some additional vector graphical corrections. To finish the design, I exported the resulting pattern to the graphics editing program Photoshop, to make the final touches, such as coloring and sizing. I chose to work with Illustrator and Photoshop, since they were the software I was most familiar with in vector design and image editing, respectively.

After these computer processes, to make the concrete tiles, I first carved several wood blocks according to the shapes in the pattern and cut four wooden squares to use them as bases for the tiles. Afterwards, I glued those shapes on the four base squares. Finally, I painted the wooden tiles elaborately with acrylic color.

4 IDUK: DEGREE PROJECT

My final project at C:Art:Media was a video animation loop, in which five of my patterns were morphing into each other. I also used a musical tune in the animation, which I composed and played with the guitar.

4.1 BACKGROUND AND INSPIRATIONS

In my final project I did not want to use the Islamic patterns as they were, rather I wanted to create something mine which was based on Islamic patterns. In other words, this time I did not want to use the typical Islamic stars or other typical elements of Islamic patterns. I wanted to keep the roots of the project in Islamic geometric patterns, but tried to develop a distinct design which could be easily associated with me.

After much consideration, I decided to replace the elements in Islamic geometric patterns. This was possible since most of the Islamic patterns consist of very small number of elements. Even though Islamic geometric patterns seem very intricate and complicated, they are actually built with the repetition of very small number of elements. However, the repetition is done in several geometric operations, such as rotation or reflection, which creates the rhythm and complexity in the patterns.

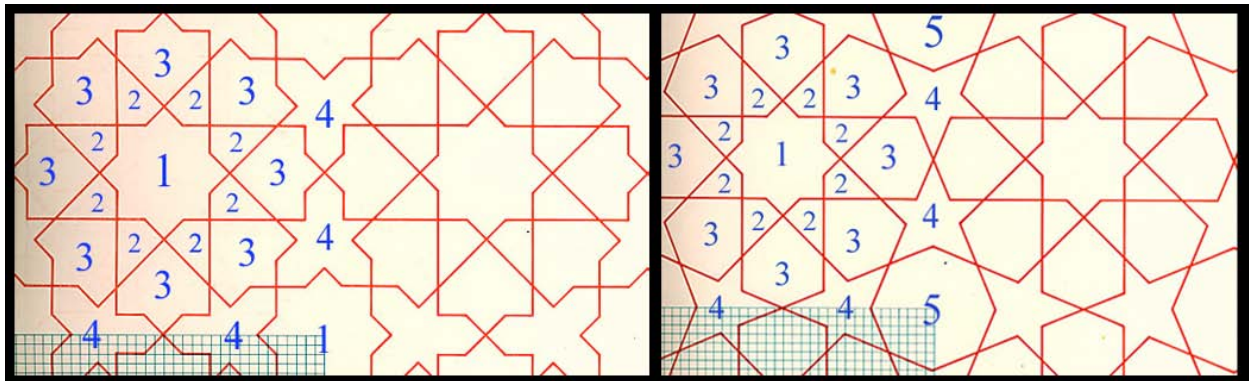


Figure 27 - Elements in Islamic geometric patterns (El-Said, 1993, p.34, 41)

How to make this replacement was a tough challenge for me. There were many ways to replace the elements in an Islamic geometric pattern, such as replacing them with another geometric figures or converting them into 3D objects, and I had to choose a way among many possibilities. First I thought about making a transition between patterns which had one or more common elements (e.g. elements 1 and 2 in the above figure), in an animation by keeping their common elements and replacing the different ones. If I would have used the patterns in the figure above for making a transition from the left pattern into the right pattern, this would mean that I would keep elements 1 and 2 in the central star in their original places and change the elements 3, 4 and 1 (the one outside the central star) of the left pattern into the elements 3, 4 and 5 of the right pattern.

After several tries, the results were not that satisfactory for me, and I felt the lack of “me” in the trials. In my opinion, only using the Islamic patterns as they were, excluded me from the project to a certain extent. Islamic geometric patterns existed for many centuries, and I was using them again just as they were without making any particular changes on them. This sort of utilization of Islamic patterns could have been basically made by anyone, who is capable of using some certain software. Thus, such a project would have no particular characteristic, which could be attributed easily back to me. However, I wanted to create a project which would bear my sign on it. Therefore I began to contemplate about how I could create the project so that it could refer back to me.

Since drawing was main passion in visual arts, I decided to use my own drawings for replacements. I would use the Islamic patterns as a base, but I would still be able to include myself as my drawings in the project. I already had tried this before the project, while I experimented with Islamic patterns in order to utilize them in my projects and had some idea about how to do it. But my main concern was, at the same time, not to repeat Escher, since replacing the elements of Islamic patterns with his drawings was his start in the route of designing more complicated patterns. (See figure below)

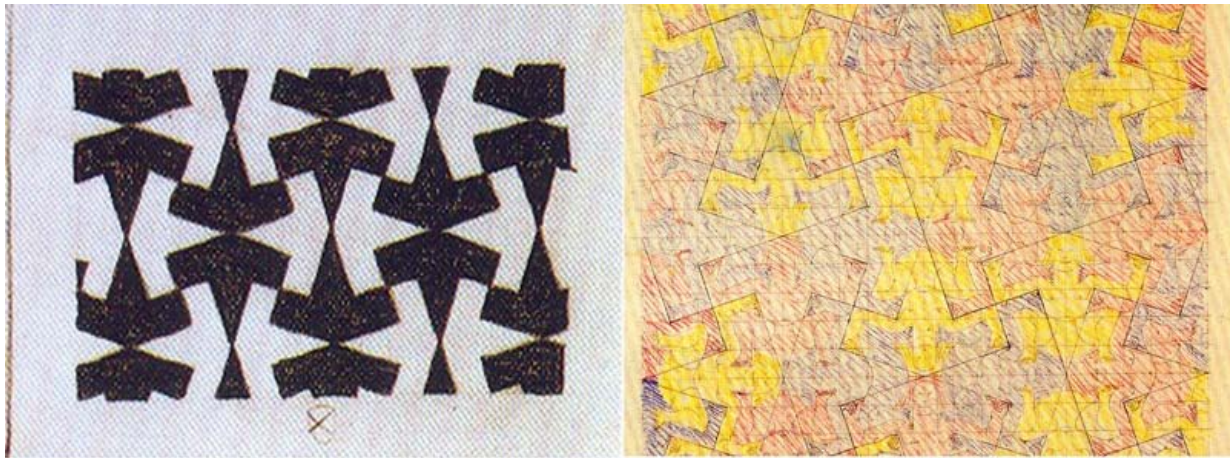


Figure 28 - Drawings of M.C. Escher, in which his replacement of elements of Islamic geometric patterns with his drawings can be observed (Schattschneider, 2004, p.17-18)

However, after discussion with other people and observing other artists’ works, such as Marian Bantjes and Hop David, I was convinced that repeating Escher was anyways not possible. Even if my methodology of creating the patterns was similar to his that would not be a problem, since I would replace the elements with my own drawings which would make the project totally mine. For instance, in the works of Hop David, his inspiration from Escher was evident. He was using interlocking figures to create his compositions and was employing predominantly animal figures as Escher did in most of his patterns. Nevertheless, the works were clearly his, since the choices of figures and their compositions in the pieces belonged to him. His goat and bird figures were distinctly his own drawings, and even though his compositions were similar to Escher’s, they were not the same. One could still feel the tessellation background in David’s works. However, they were far from being precise tessellations of the plane consisting of repetition of the same figure in contrast to most of Escher’s compositions being tessellations consisting of repetition of one or more figures.



Figure 29 - Rams by Hop David (David, 2005)

After having decided about how I would replace the elements of the Islamic geometric patterns, I needed a theme for the project. Since it was my final project, I wanted my theme to be something which meant a lot to me. Thus, I decided to represent the ancient Turkish elements in my work, since I was very much interested in ancient Turkish cosmology. This theme also determined the name of the project: *Iduk*, which means *holy* in ancient Turkish. In addition, using ancient Turkish elements would also make it possible for me to utilize symbolic aspects in the project. However, this time I wanted make a shift in my usage of symbols in my patterns.

4.2 CHANGE OF SYMBOLIC ASPECTS

According to German perceptual psychologist and art theorist Rudolf Arnheim, “an image acts as a symbol to the extent to which it portrays things which are at a higher level of abstractness than the symbol itself.” (Arnheim, 1969, p. 138) In Islamic geometric patterns, it is easy to observe that the geometric figures are referring to concepts which are at a higher level of abstraction than the geometric figures themselves. For instance, as I mentioned before, a pattern with eight pointed stars can symbolize the duality of four elements (Critchlow, 2001).

When I began first to study the symbolic meanings of Islamic geometric patterns, I considered them rather as fixed meanings, which are put in the patterns by the ancient designers and are not changeable. Besides, there was also the problem of conveying these symbolic meanings. Even though the geometric figures can be used as symbols, in my opinion, the main obstacle in elucidating an association of a particular geometric figure to a particular concept is the high abstractness of the geometric figures. As Arnheim states about the usage of geometric figures as symbols, “... highly abstract concepts, although narrow in extension, are broad in extension, that is, they can refer to many things” (Arnheim, 1969, p. 142). For instance, an upward triangle may symbolize the element fire in a textbook about ancient

Greece and or danger if used in vehicle traffic. Thus, for a highly abstract design, an explanatory context becomes the determining factor for its symbolic meaning (Arnheim, 1969).

On the other hand, the symbolic meanings of Islamic geometric patterns also rely heavily on an historical context. Without prior knowledge about Islamic symbolism, it would be very difficult to comprehend the symbolic meanings of the Islamic geometric patterns. By quoting a term from Arnheim, Islamic geometric figures are not “full-time symbols” (Arnheim, 1969). An eight pointed star may represent the duality of four elements in the Islamic historical context; however, not in every other possible context. Therefore, an explanatory context becomes again a necessity for conveying symbolic meanings.

Considering the dependence on explanatory context of symbolic meanings of Islamic geometric patterns, I began to regard the symbolic meanings of Islamic patterns not as a property of patterns themselves, rather as a property that is attached to patterns by the viewers within the context that the patterns are in. Even though the designer may have used a certain geometric figure to symbolize a certain concept, without its explanatory context, the perceived symbolic meaning of the pattern would heavily depend on viewers’ associations with the geometric figure.

This shift in my understanding of the symbols made a great impact on my usage of symbolic meanings in my patterns. In *Earthly*, I tried to use the old symbolic meanings, regardless of the new context in which they were in now. Even though number four or square were referring to the element earth in an ancient context, in our recent world number four or square did not necessarily refer to earth. This awareness of the importance of the context for symbols liberated my utilization of symbols. As Arnheim wrote, “In principle, any specimen ... can serve as a symbol, if somebody chooses to use it in that way. But in such cases, the image leaves the effort of abstracting entirely to the user.” (Arnheim, 1969, p. 138) Thus, instead of using an ancient invariable symbolism, I decided to attach my own symbolic meanings to my designs.

In *Iduk* I tried to develop my own symbolism. Instead of using very abstract designs, such as geometric shapes, I employed mundane figures, which I derived from my own connotations, as my symbols. For instance, to symbolize the element earth, I used a walking man figure instead of number four or a square. Furthermore, by using daily objects as symbols, I wanted to facilitate the associations of the figures for the viewers. Taking into account the current context the project is exhibited, I thought that a walking man figure would be easier for viewers to associate with earth than a square. On the other hand, developing my own symbols also greatly aided me to include myself in the project. Although I utilized Islamic geometric patterns as base for my own patterns in *Iduk*, I created my own figures and used them as my own symbols. Thus, I was able to create something which bore my sign on it.

4.3 THE CREATION PROCESS

In this part, I would like to discuss in more detail how I created *Iduk*.

In order to represent the ancient Turkish elements through my patterns, my methodology was as follows: First, I determined the connotations of each element in my life. Second, I decided the objects

and figures which would visualize these connotations. In the next step, I found a suitable Islamic geometric pattern, whose components can be replaced with these objects and figures. Then I filled the particular suitable pattern with my own drawings of the objects and figures. Consequently, I created my own pattern to represent the element. To explain this process fully, I will discuss each step in detail.

4.3.1 ANCIENT TURKISH ELEMENTS

In the early times of Turkish people's history, they had a cosmological view which was reminiscent to the one of the ancient Chinese. In this cosmological view, the universe was constituted of five elements, namely: earth, wood, fire, metal and water. (Esin, 2001, p.24-25)

In my work, I wanted to represent these elements through my own patterns. To reach this goal, my first step was to personalize these elements for my patterns by determining the connotations of each ancient element in my life.

4.3.2 CONNOTATIONS

To personalize the elements, first of all, I wanted to associate them with my own experiences. For example, in the case of *earth* instead of the general and standard connotations, such as soil or fields, I wanted to find a particular connotation, which would associate the notion of earth to me in a distinctive manner. In other words, I wanted to find the connotations of earth, which would explain what earth meant to me.

Following again the example of earth, the first thing I did, to personalize the element earth, was to write down all the connotations of earth in my life. While I was contemplating about earth, I was thinking about my own experiences with it. My connotations were: the place that I walk on, where the flowers grow, mud, crack and wet.

After having decided the connotations, I wanted to choose the one which suited me and my recent situation the best. After some more consideration, I concluded that the main and forefront meaning of earth was 'the place that I walk on' for me.

4.3.3 OBJECTS AND FIGURES

The next step after determining my own connotations was to find objects and figures, which would represent my decided connotation. Again in the case of earth, I was looking for the objects and figures, which would represent the connotation 'the place that I walk on'. The objects and figures which I came up with were shoes, road and walking man. However in this step of the process, I did not try reduce my objects and figures to one, since for creating a pattern I may have needed to use more than one object or figure.

4.3.4 FINDING A SUITABLE PATTERN

Having my objects and figures at hand, the next step was to find a suitable Islamic pattern, whose elements I could replace with my objects and figures.

Again in the case of earth, having in mind the notions of shoes, road and walking man, I checked an ample amount of Islamic geometric patterns and tried to visualize my objects and figures as the elements of the patterns. When I found a suitable pattern, I tried to replace its elements with my drawings of the objects and figures. This process consisted of mainly trials and errors. If I saw an element of an Islamic geometric pattern that could be replaced by one of my objects and figures, I tried to draw my object or figure in a way that my drawing would resemble this element. If the resulting drawing was good enough for me, I tried to replace the remaining elements of the pattern, if necessary. If I did not like the result, I moved to another suitable pattern and began the same process of drawing my objects and figures all over again.

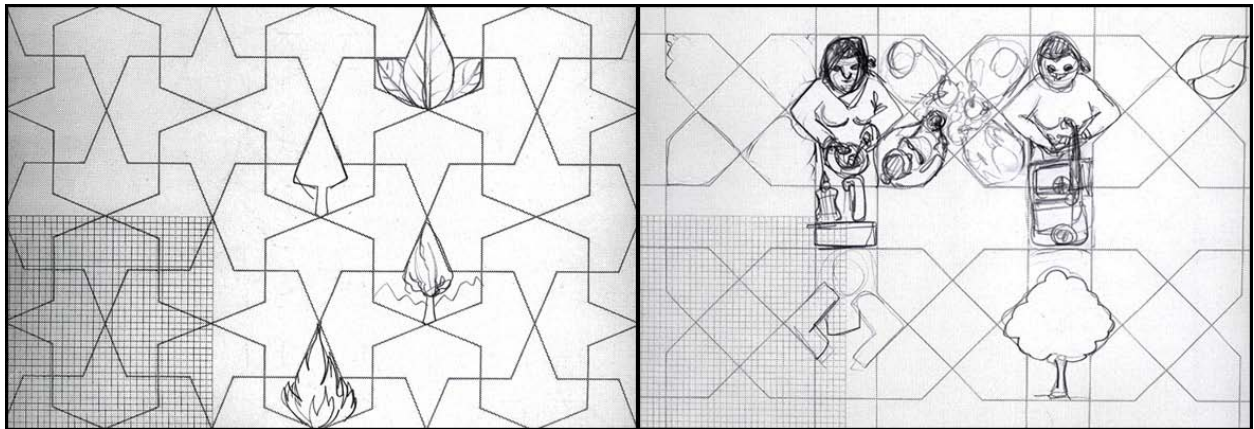


Figure 30 - Examples from my sketches of trying to find suitable patterns for the elements wood (left) and water (right)

After several tries I decided on an Islamic geometric pattern whose components I could replace with the figure of a *walking man*. (See figure below)

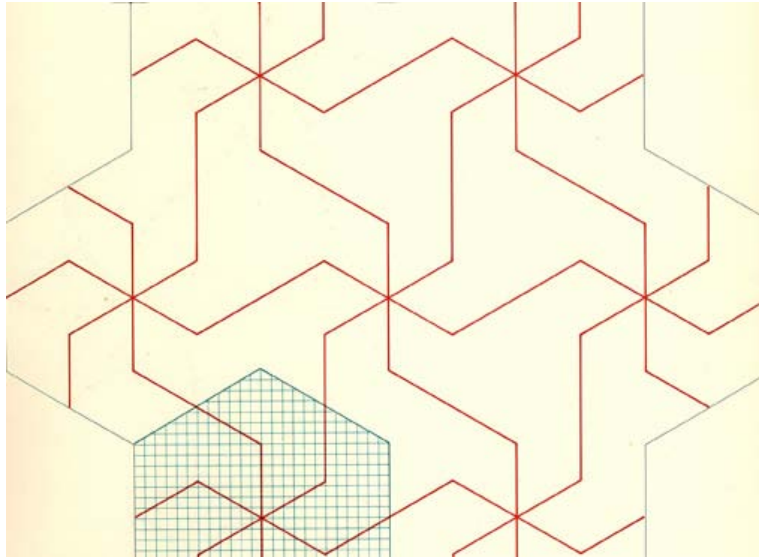


Figure 31 - Islamic geometric pattern which I utilized for the figure of *walking man* (El-Said, 1993, p. 78)

4.3.5 REPLACEMENT OF GEOMETRIC ELEMENTS WITH MY OWN DRAWINGS

After I decided on the pattern, I drew my objects and figures with a PC tablet. Using a PC tablet had a two-fold benefit for me; I could keep the flavor of hand-drawing, and at the same time, I was able to utilize the competences of computer software, such as ease of duplication of a figure or to be able to easily apply geometric operations on a figure, such as rotation or reflection.

In the example of earth, after I decided on the Islamic pattern, I drew a walking man figure in the way that it resembled the elements of the pattern in the graphics editing program Photoshop via a PC tablet. (See figure below)

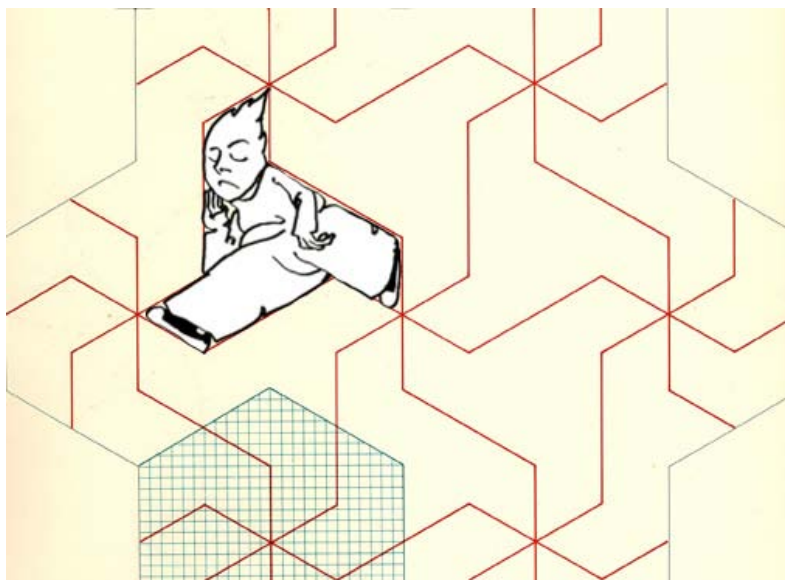


Figure 32 - Replacement of the element of the suitable pattern with the figure of *walking man*

Afterwards, I duplicated this figure according to the composition of the pattern. (See figure below)

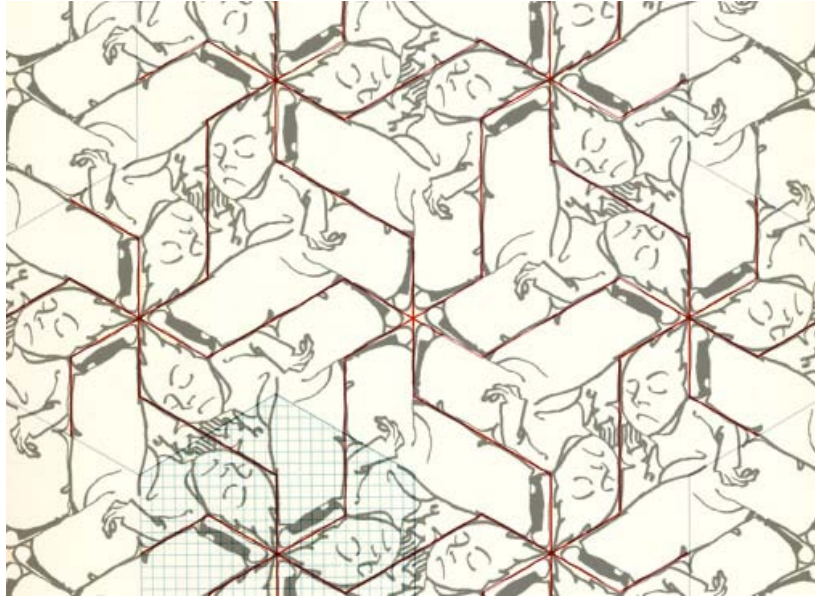


Figure 33 – Duplication of the figure according to the composition of the Islamic geometric pattern

As a result, I replaced the geometric elements which constitute the Islamic pattern, with my own drawings.

Finally, after all these steps, I had my final pattern which represents the ancient Turkish element.

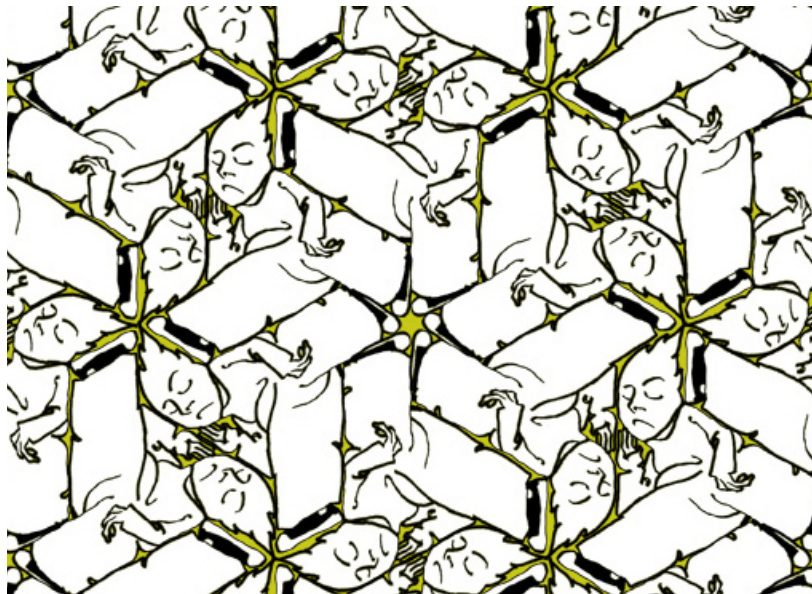


Figure 34 - Finished pattern

4.3.6 ISLAMIC PATTERNS USED IN *IDUK*

In this part, I want to discuss the Islamic geometric patterns I used in *Iduk*, and how I created my compositions in my patterns utilizing the Islamic patterns as a base.

4.3.6.1 EARTH

To create the pattern for the element *earth*, I wanted to have a pattern consisting of only one element. Therefore, as I mentioned in the preceding chapters, I chose a pattern from $\sqrt{3}$ relations class.

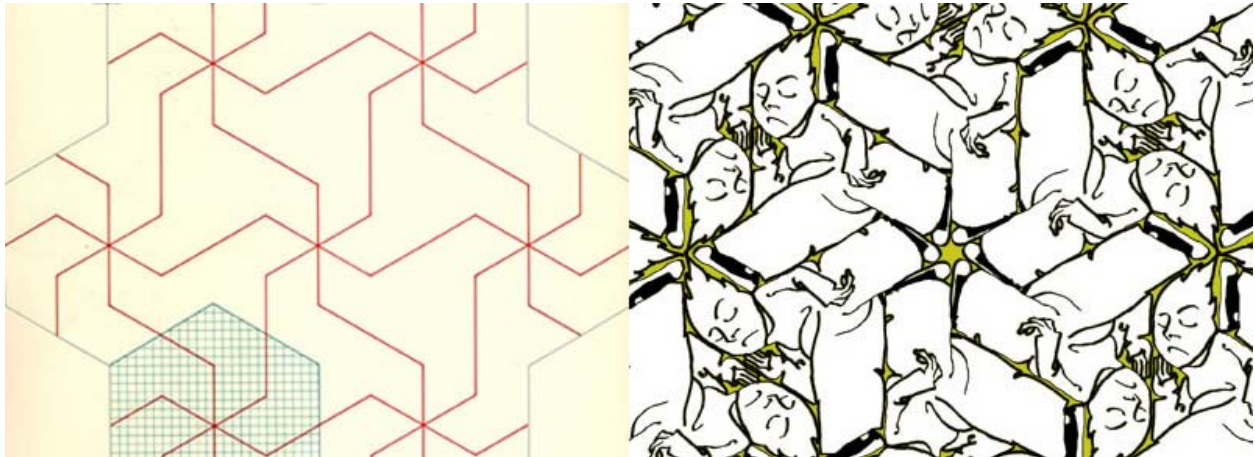


Figure 35 - Pattern used to represent the element *earth*

In an effort to create movement in the design, I placed my walking man figure in the center of the composition by duplicating and rotating it through 60 degrees for six times. Afterwards, next to every figure in the middle, I placed a 180 degrees rotated version of itself, and continued to place the rest of the figures following this method.

4.3.6.2 WOOD

To create the pattern for the element *wood*, I used three different figures. However, I chose an Islamic geometric pattern from $\sqrt{2}$ relations class which consists of only two elements. Whereas I replaced the kite shapes with leaf figures, I replaced the squares alternatively by bird and flower figures.

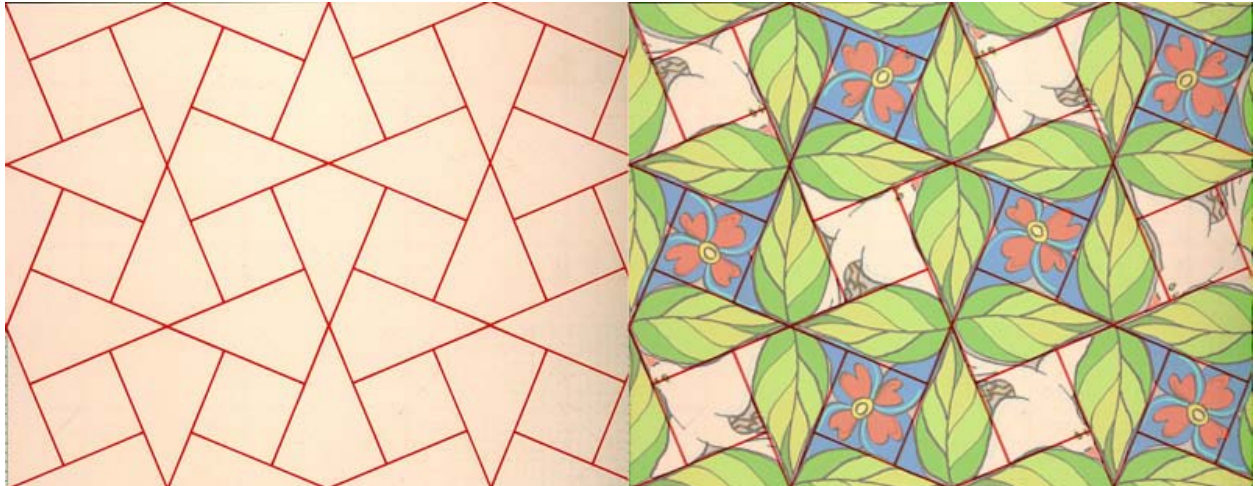


Figure 36 - Pattern used to represent the element *wood*

Again to give a sense of movement in the pattern, I utilized rotations. Throughout the pattern I rotated the elements through 90 and 180 degrees.

4.3.6.3 FIRE

For the element *fire*, I utilized a pattern with three elements from $\sqrt{2}$ relations class. Again, to place my figures in the pattern, I used rotations. However, this time I used a simpler rotation methodology compared to previous patterns. I employed the grills as centers and then duplicated and rotated the candle figure through 90 degrees around this center. While duplicating the grill and candles, I rotated the grills through 90 degrees at every progression. Moreover, I placed the word “ATES” (which means *fire* in Turkish) through 90 degree rotations in the pattern.

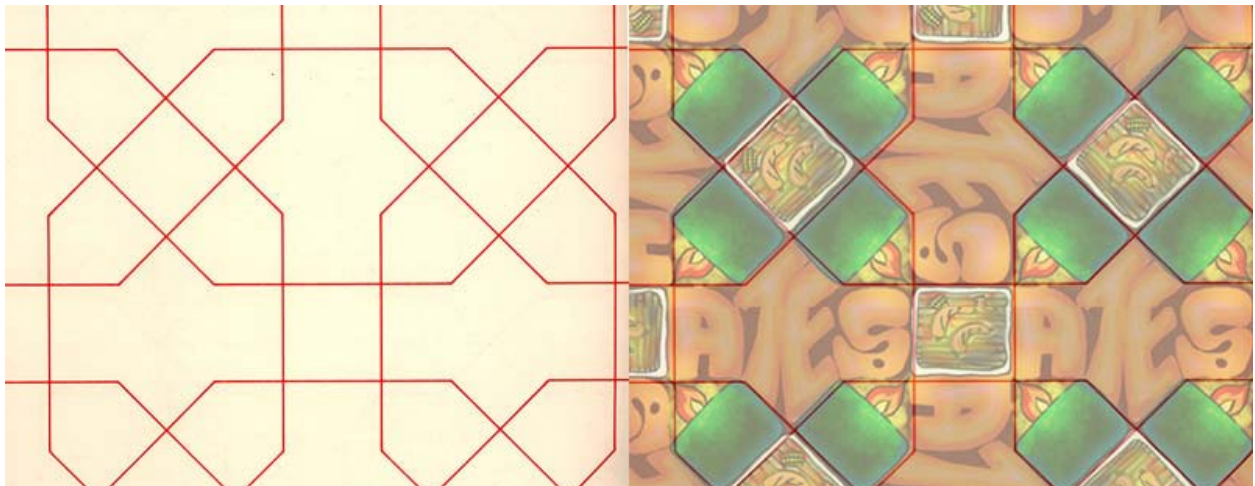


Figure 37 - Pattern used to represent the element *fire*

4.3.6.4 METAL

To represent the element *metal*, I employed one of the most commonly used Islamic geometric patterns from $\sqrt{3}$ relations class with 3 elements, which is a semi-regular tessellation at the same time. (ThinkQuest, 1998)

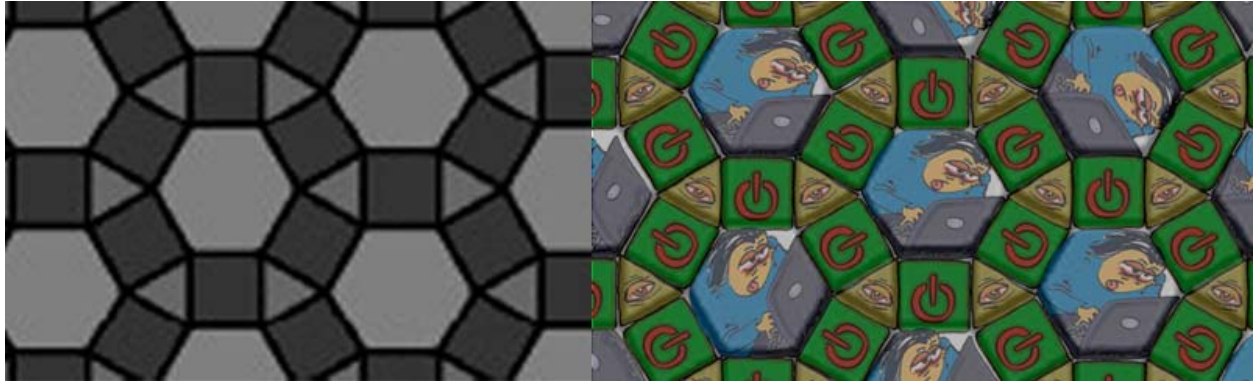


Figure 38 - Pattern used to represent the element *metal*

For this pattern, I used both rotation and reflection to place my figures in the pattern. To place the person with computer and the on-off sign, I utilized rotations through 60 degrees, and to place the eyes, I only used vertical reflections.

4.3.6.5 WATER

Finally, to represent the element *water*, I used the same Islamic pattern, which I used for the element *wood*, as a base. However this time I had two figures to replace the two elements of the pattern. Following the design of the pattern, I employed rotations again to allocate my figures.

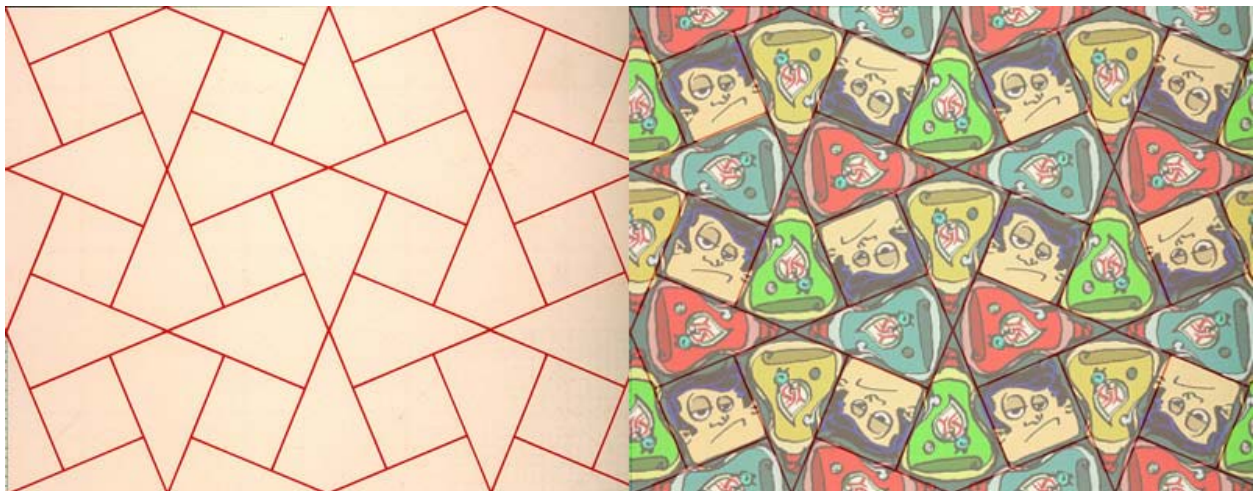


Figure 39 - Pattern used to represent the element *water*

This time, I also utilized color as a distinguishing trait for the rotations. For every rotation of the Yes bottle figure, I used another color. Thus, I started with green, turned the figure 90 degrees clockwise and colored it red, and then continued with yellow and blue. To place the head figures, I rotated them through 90 and 180 degrees.

4.3.7 ANIMATION

After having all my patterns for ancient Turkish elements, I used the compositing software After Effects to compose an animation in which all the elements were shown one after another, and the transition of patterns was made through a slow fade-in of the new pattern and its gradual covering of the whole screen. Each of the patterns was visible for fifteen seconds and then was proceeded by the next pattern.

4.3.8 MUSICAL TUNE

After having completed my animation visually, to strengthen the Turkish essence, I wanted to add a musical tune which was composed in a typical Turkish scale. Thus, I composed a musical tune by improvising in a typical Turkish scale and added this tune to the project.

4.4 PRESENTATION

How to present my patterns was another concern for me. While exhibiting my former project *Earthly* I felt that the way that I exhibited my pattern did not really work. I thought the problem was that the patterns were to fill a plane fully, but to limit them in a certain place seemed to leave their objective of filling the plane unfulfilled. Thus, when I limited my pattern in *Earthly* to four tiles, they seemed undone. When I looked at my project, I felt as if the pattern had to fill the whole floor that it was exhibited on.

Thus, I decided to fill a plane fully with my patterns in *Iduk*. But how I would fill the plane depended greatly on the way I would exhibit the patterns. If I wanted to use them, for instance, on a wall, I could have printed and then stuck them on the wall so that the whole wall would be covered with the pattern.

However, I did not want to make my presentation in that way. I wanted the presentation of the patterns to be rather lively than static. Therefore, I decided to use computer animation again. This was more convenient for me, since I could have used the competences of computers, and filling a plane fully in an animation meant basically to fill the whole screen, on which the animation was played, with patterns. Consequently, I created my animation, and it was exhibited in a dark room as a projection on a white screen which was mounted on a wall.

5 CONCLUSION

Through my projects, my aim was to investigate the question of how to combine mathematics and visual arts through the use of Islamic geometric patterns. To reach this aim, I extensively researched the mathematical and artistic aspects of Islamic geometric patterns as well as their history, and I utilized my background in mathematics in the field of visual arts.

The reasons why I chose to work with Islamic geometric patterns were several. First of all, in my view, they are one of the most beautiful examples of the combination of mathematics and visual arts. Second, they contain an abundant amount of symbolism, which I could have utilized in my efforts. Moreover, I am culturally familiar with them, and one of my biggest inspirations, M.C. Escher has utilized them to create his own fascinating patterns. Finally, working with Islamic patterns always contained a spiritual aspect for me.

To present my patterns I tried both traditional and digital media, and during the course of creating my projects, I made many trial and errors. Islamic geometric patterns were usually designed in ancient times as decorations of walls and doors and were usually presented as tiles or wood carvings. However, to put the Islamic geometric patterns in the framework of digital media and in the current context, for my first project *Trifinity* I changed the media to computer animation. For my second project *Earthly*, I wanted to go back to fundamentals and tried traditional media, such as wooden tiles for my patterns. However after having completed the project, I felt that something was not working in my project. Through this experience, I became aware of the constraints and the drawbacks of using traditional media for my work. After these trial and errors, for my degree project *Iduk* I used computer animation again and wanted to utilize the opportunities and advantages of computers and digital media.

The symbolic aspect of my works has also evolved. In the beginning, I regarded the symbols as static and pre-defined meanings and associations. I followed the symbolism of ancient times and employed the symbolic language that was used back then. However, this actually led me to disregard the current context that I was presenting my patterns. Even though number four was regarded as a symbol of the element earth in the ancient times, in the current context this was not an immediate association of number four. After realizing the importance of an explanatory context for the meaning of symbols, I changed the usage of symbols in my work and regarded the symbolic meanings as a property that is attached to a piece or figure by the viewers in the context they are presented. Moreover, in order to make the associations for symbolic meanings easier in my projects, I tried to create a rather down-to-earth symbolic language by using ordinary and daily objects as my symbols instead of abstract geometric figures or numbers. I also tried to personalize the symbols that I used by utilizing certain items as symbols in association to their role in my life.

I believe that for my efforts in exploring how mathematics and visual arts can be combined through the employment of Islamic patterns, digital media and computer animation served as the best means. Compared to traditional media, digital media is easier to work with and offers more opportunities for working with the geometric aspects of the patterns. For instance, even the simple process of duplication requires an immense effort in traditional media, whereas duplicating a shape is one of the simplest

functions of computer software. On the other hand, animation has the important competences of time and motion. For example, by showing figures through same time intervals, one can try to create a sense of order and periodicity in a project, which could be later related to a mathematical property. In addition, being able to resize or rotate patterns in an animation makes it easier to emphasize the geometrical aspects of a project.

In retrospect, the investigation of trying to find possible ways to combine mathematics and visual arts through Islamic geometric patterns was more of a journey without a certain end for me. The ways in which mathematics and visual arts can be combined are infinite, and my attempts cover only a very small portion of these ways. No one can define or distinguish distinctively where these two fields intersect and disjoin, and thus, all the attempts to combine these two will be bound to be incomplete.

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