UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

WORKING PAPERS IN ECONOMICS

No 377

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September 2009

ISSN 1403-2473 (print)
ISSN 1403-2465 (online)

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SEASONAL UNIT ROOT TESTS FOR TRENDING AND BREAKING SERIES WITH APPLICATION TO INDUSTRIAL PRODUCTION *

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September 10, 2009

Abstract

Some unit root testing situations are more difficult than others. In the case of quarterly industrial production there is not only the seasonal variation that needs to be considered but also the occasionally breaking linear trend. In the current paper we take this as our starting point to develop three new seasonal unit root tests that allow for a break in both the seasonal mean and linear trend of a quarterly time series. The asymptotic properties of the tests are derived and investigated in small-samples using simulations. In the empirical part of the paper we consider as an example the industrial production of 13 European countries. The results suggest that for most of the series there is evidence of stationary seasonality around an otherwise nonseasonal unit root.

Keywords: Seasonal unit root tests, Structural breaks, Linear time trend, Industrial production.

JEL Classification: C12, C22.

*Westerlund would like to thank the Jan Wallander and Tom Hedelius Foundation for financial support under research grant W2006–0068:1.

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1 Introduction

The persistence of macroeconomic shocks is one of the most investigated issues within the field of empirical economics. In a seminal paper, Nelson and Plosser (1982) argue that, in contrast to the tradition view, most shocks have permanent effects. Using the Dickey and Fuller (1979) test they found that the null hypothesis of a unit root cannot be rejected for 13 out of the 14 annual macroeconomic variables considered. Their study triggered the development of several unit root tests as well as numerous simulation studies directed at comparing their small-sample performance.

At the same time, many of the shortcomings of these tests became apparent. Perron (1989) questioned the preference of Nelson and Plosser (1982) to only consider the case of a linear time trend when actually their data cover periods of major economic events such as the oil crisis of the 1970's and the Great Depression, which may well have affected the slope of the trend. The problem is that the presence of such structural breaks induces serial correlation properties that are akin to those of a random walk, and conventional tests such as the Dickey and Fuller (1979) test may therefore incorrectly accept the null hypothesis of a unit root when the data are in fact stationary around a broken trend. To account for this possibility Perron (1989) developed a procedure to formally test the null hypothesis of a unit root in the presence of a structural break, which he then applied to the same 14 variables considered by Nelson and Plosser (1982) with very different results. This is important because the finding that macroeconomic variables do not have unit roots would make it necessary to reconsider much of the previous empirical work.

With quarterly data there is not only the occasionally breaking trend but also pronounced seasonal movements, which are just as problematic, and much effort has therefore gone into the development of unit root tests that are robust against such movements. Here the focal issue has been whether the seasonality varies in a non-stationary way or whether the seasonality is stationary. In the latter case, season-specific intercepts are usually enough to capture the seasonality, whereas in the former case, annual differencing is required.

1.1 Limitations of earlier studies

Hylleberg *et al.* (1990) were among the first to analyze the issue of seasonal unit roots. They consider a quarterly time series y_t, observable for $t = 1, ..., T$, whose seasonal properties can be analyzed by using the following auxiliary regression:

$$\Delta_4 y_t \;=\; \sum_{s=1}^{4} \mu_s D_{s,t} + \sum_{s=1}^{3} \rho_s y_{s,t-1} + \rho_4 y_{3,t-2} + \varepsilon_t, \tag{1}$$

where $D_{s,t}$ equals one if t is in season s and zero otherwise, and ε_t is a serially uncorrelated error term. The variables $y_{1,t-1}$, $y_{2,t-1}$ and $y_{3,t-1}$ are given by

$$y_{1,t-1} \;=\; \sum_{s=1}^{4} y_{t-s}, \quad y_{2,t-1} \;=\; \sum_{s=1}^{4} (-1)^s y_{t-s}, \quad y_{3,t-1} \;=\; -y_{t-1} + y_{t-3}.$$

The authors show that the hypothesis of a nonseasonal, or zero frequency, unit root corresponds to $\rho_1 = 0$, that a seasonal unit root at the biannual frequency corresponds to $\rho_2 = 0$, and that seasonal unit roots at the annual frequency corresponds to $\rho_3 = \rho_4 = 0$. The first two hypotheses are tested using a conventional t-test, while third is tested using an F-test. The seasonal intercept dummies are irrelevant under the null but are there in order to make the test robust against the alternative that the series is stationary around a seasonal mean.

One problem with the Hylleberg *et al.* (1990) approach is that it does not account for the fact that certain shocks may cause the seasonal fluctuations to shift permanently, see for example Ghysels (1991) who argue that many postwar macroeconomic variables have been subject to seasonal means shifts. If this is the case, then the tests based on (1) are likely to be misleading in the sense that they are biased towards accepting the null hypothesis, see for example Lopes and Montañés (2005), and Smith and Otero (1997).

As a response to this Franses and Vogelsang (1998) propose an alternative model, which allows for an unknown break in one or more of the seasonal means. This break may be instant but it may also be gradual, reflecting the fact that even major breaks, such as the stock market crash of 1929 or the oil price shocks of the 1970's, usually do not display their full impacts immediately. The resulting test statistics are therefore very general, and widely applicable.

The problem with the Franses and Vogelsang (1998) approach is that it does not allow the series to be trending, which we have argued to be one of the key features of most macroeconomic variables. In other words, while potentially very promising and general when it comes to the seasonal variation, the Franses and Vogelsang (1998) approach cannot handle series that are trending. The problem is that, as in the case of an unattended break, if the test regression is fitted with seasonal dummies but the data contain a trend, then the ensuing unit root test will be biased in favor of the null.

1.2 A motivating example and the main results of this study

In recent years, there has been a great deal of research focusing on the persistence of industrial production. This is an important and relevant question because industrial production is oftentimes used as a measure of output, which is a key variable in many economic models, whose validity hinges critically on whether output is stationary or not. There is also a large body of empirical work based cointegration that relies on industrial production being nonstationary. Take for example the study of Fernandez (1997), who uses industrial production as a measure of economic activity in order to study the long-run relationship between output and money supply. Similar approaches have been used by Nasseh and Strauss (2000) and Binswanger (2004) to study the relationship between stock prices and macroeconomic activity among western industrialized countries.

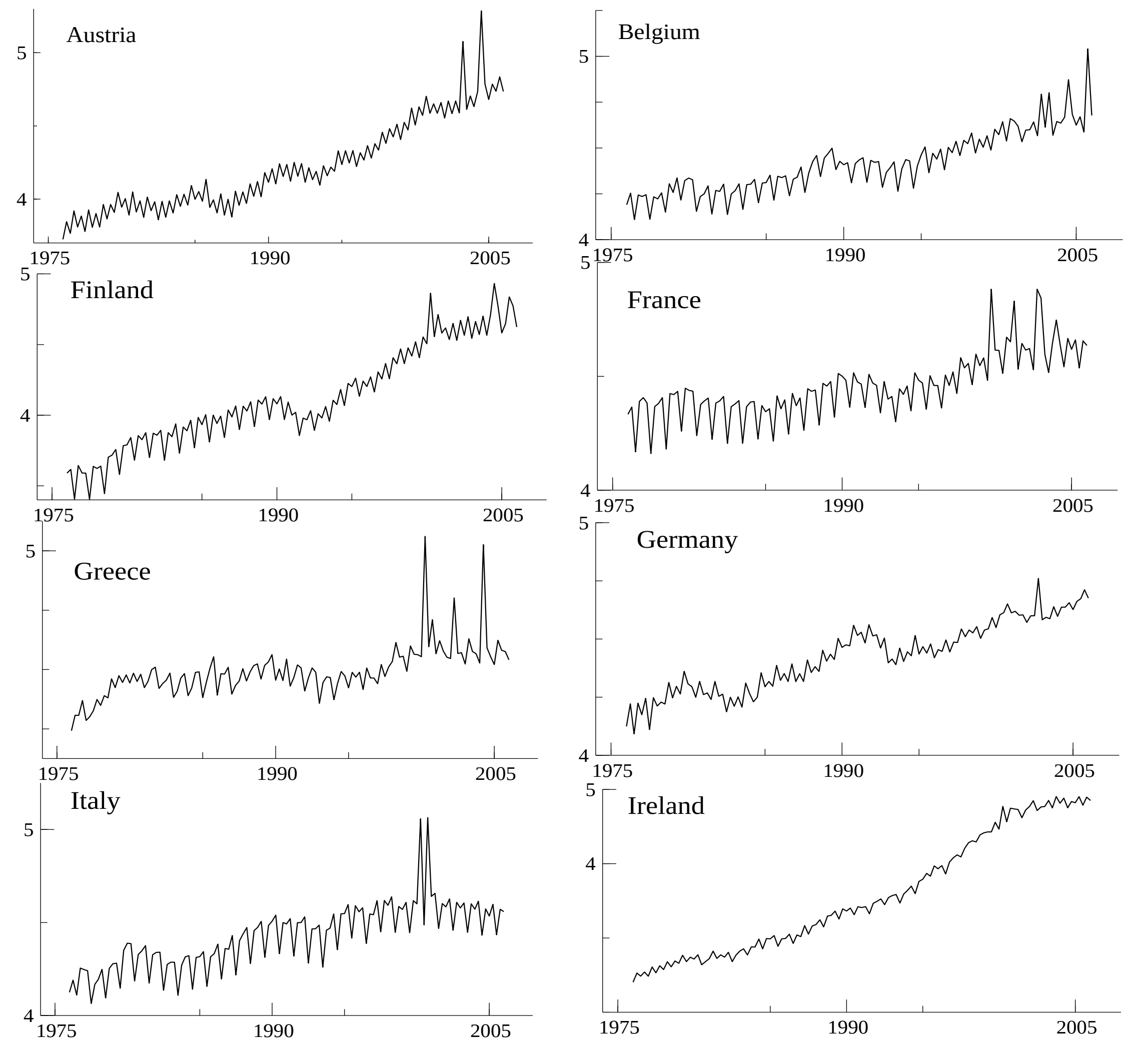
But the persistence of industrial production is interesting not only because of its use as a measure of output or economic activity, but also in its own right. In fact, ever since the provocative study of Nelson and Plosser (1989) researchers have been obsessed with trying to revaluate their findings, see Hylleberg *et al.* (1993) and Osborn *et al.* (1999) for some examples using industrial production.

The 13 series considered in this paper are the log of the industrial production index for Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Spain, Sweden and the United Kingdom. The series are quarterly and seasonally unadjusted. All data are taken from the OECD Statistical Compendium 2007, and cover the period 1976:1–2006:1.

One implication of using quarterly rather than annual data is that although the series have more observations they cover a shorter time span. We therefore loose some information about the long-run behavior of the series. On the other hand, quarterly data are richer in the sense that they provide more information about the short-run behavior of the series. This is illustrated in Figure 1 and 2, which plot the log of the 13 series considered.

The first thing to notice is the obvious seasonal variation. At the one end of the scale we have France, Italy, Norway, Spain and Sweden, where the seasonality is very pronounced, while at the other end of the scale, we have countries such as Austria, Ireland and the United Kingdom, where the series are much smoother. In fact, for most of the series the trending pattern is just as regular and pronounced as the seasonal pattern. All countries have experienced substantial growth in industrial production, which seem to have lasted throughout

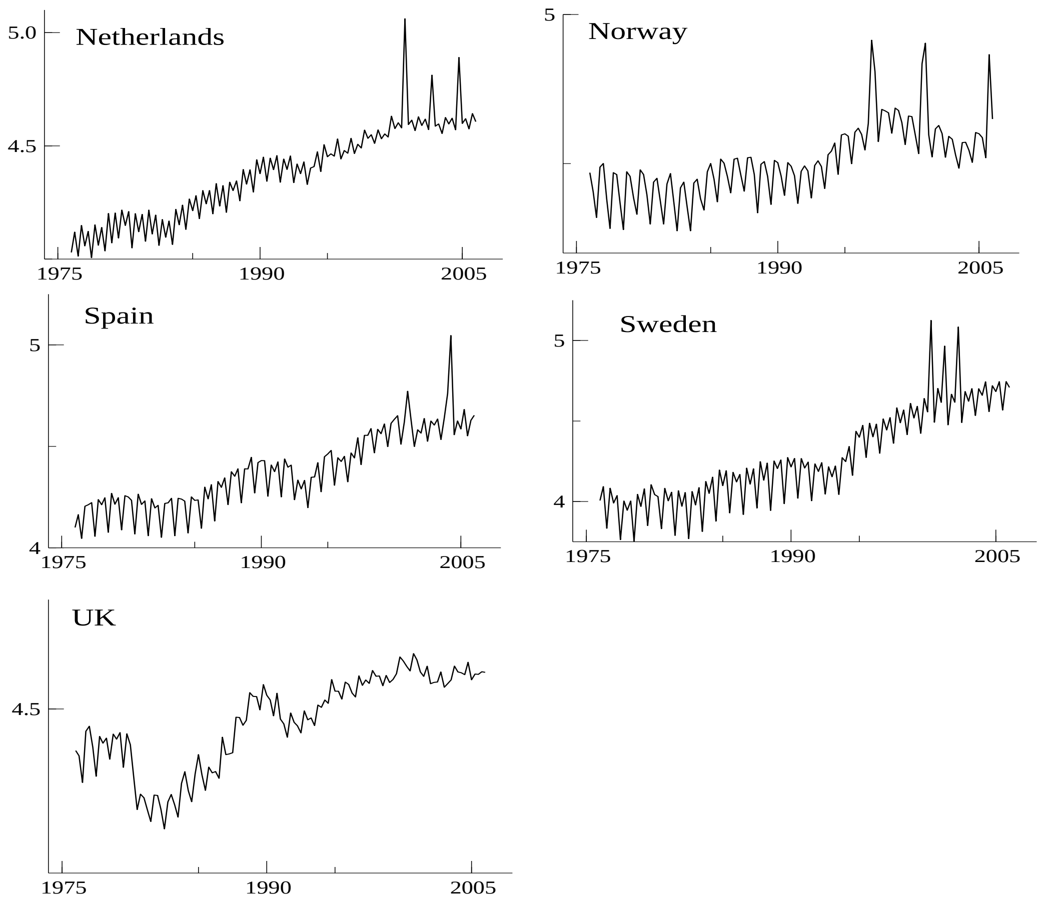
Figure 1: Log of industrial production.



the sample.

These observations clearly illustrates the need of allowing for both season-specific means and linear time trends. But this is not all. We also see that most of the series display a clear break in both the seasonal mean and trend slope halfway into the sample. Consider for example the industrial production of Finland, which display a very clear-cut change in the seasonal pattern from the mid-1990's and onward, becoming less pronounced but still very regular. At the same time we also observe an increase in the growth of the series, which lasts for about 10 years, but then it falls back again.

Figure 2: Log of industrial production.



In other words, there is not only the need to allow for season-specific means and linear trends, but there also a need to allow for the possibility of breaks in their coefficients, and the current paper therefore makes an attempt in this direction. Three new seasonal unit root tests are proposed, which are general enough to allow for a gradual structural break in both the seasonal mean and linear trend of the series. As for the location of the break, we consider two cases. In the first we take the breakpoint as given, while in the second, the breakpoint is treated as an unknown parameter to be estimated from the data. The asymptotic distributions of the new test statistics are derived, and verified in small samples using simulations.

When we apply the new tests to our industrial production data we find that the null hypothesis of a nonseasonal unit root must be accepted for 11 out of the 13 series considered, suggesting that most shocks have permanent effects. On the other hand, the null of a biannual unit root is rejected much more frequently, nine times, and the null of annual unit roots is rejected even more often, 11 times. Thus, most series can be characterized as conventional unit root processes with stationary seasonality.

The paper is organized as follows. The next section describes the model under consideration and the tests that will be used to test it. Section 3 reports the asymptotic results, whose accuracy in small samples is examined in Section 4. Section 5 concern itself with the empirical application, whereas Section 6 concludes. Proofs of important results are provided in the appendix.

2 The seasonal unit root tests

We consider a component model, in which the observed series y_t is decomposed into a deterministic part d_t and a stochastic part s_t,

$$y_t = d_t + s_t, \tag{2}$$

$$s_t = \rho s_{t-4} + e_t \tag{3}$$

with $e_t = \Phi(L)u_t$, where $\Phi(L)$ is a polynomial in the lag operator L and u_t is independently and identically distributed with mean zero and variance σ^2. As for the deterministic component d_t, we consider two models, henceforth denoted by $m \in \{1,2\}$. Model 2 is the most general one and allows for a break in both the trend and seasonal mean of the series. Specifically,

$$d_t = \sum_{s=1}^{4} \mu_s D_{s,t} + \lambda t + \Phi(L)\left(\psi DT_t + \sum_{s=1}^{4} \phi_s DU_{s,t}\right), \tag{4}$$

where $D_{s,t}$ again equals one if t is in season s and zero otherwise, $DU_{s,t} = 1(t > T^\circ)D_{s,t}$, $DT_t = 1(t > T^\circ)(t - T^\circ)$, $1(x)$ is the indicator function and T° denotes the date of the break, which is such that $T^\circ = \tau^\circ T$ with $\tau^\circ \in (0,1)$. Model 1 is obtained by setting $\psi = 0$, which leads to a model with a break in the seasonal mean but not in the trend.

In both models, because of the multiplication by the lag polynomial $\Phi(L)$, the break is assumed follow the same dynamic path as the innovations to y_t.[1] Suppose for example that $\psi = \phi_2 = ... = \phi_4 = 0$ so that it is only the mean of the first season that changes, then it is

[1]Of course, assuming that the shifts have the same dynamics as the innovations is by no means the only way to model the gradual impact of the mean shifts. But it is convenient.

not difficult to see that the immediate impact of the break is given by ϕ_1 and the long-term impact is $\Phi(1)\phi_1$. Of course, on could also consider the case when the full effect of the break takes place immediately, but a gradual effect seem more consistent with the data at hand.[2]

The testing approach that we use is taken from Hylleberg *et al.* (1990), which is very convenient as it enables us to test for unit roots at all seasonal frequencies and at the zero frequency. After deriving the reduced form of the component model in (1) and (2), and then nesting and approximating it in the spirit of Perron and Vogelsang (1992), we obtain the following test regression for model 2:

$$\begin{aligned}\Delta_4 y_t &= \sum_{s=1}^{3}\rho_s y_{s,t-1} + \rho_4 y_{3,t-2} + \beta t + \delta DT_{t-4} + \sum_{s=1}^{4}(\alpha_s D_{s,t} + \theta_s \Delta_4 D_{s,t} + \pi_s DU_{s,t-4}) \\ &+ \sum_{s=1}^{p}\gamma_s \Delta_4 y_{t-s} + \varepsilon_t. \end{aligned} \tag{5}$$

where β, δ, α_s, θ_s and π_s are derived from the coefficients in (4), ε_t is a serially uncorrelated error term that comprises an unexplained regression error plus the error that comes from the approximation, and $y_{1,t-1}$, $y_{2,t-1}$ and $y_{3,t-1}$ are as before. The corresponding test regression for model 1 is obtained by imposing $\delta = 0$.

Equation (5) is similar to a two-step procedure, in which (1) is fitted to the residuals of a first-step regression of y_t onto the elements of the deterministic component in (4). The approach considered here is more convenient though, as it involves only a one-step regression, wherein the coefficients of both the deterministic and stochastic components of y_t are estimated simultaneously.

Following Hylleberg *et al.* (1990) we consider three different null hypotheses:

1. $H_0^1 : \rho_1 = 0$, corresponding to a non-seasonal unit root;

2. $H_0^2 : \rho_2 = 0$, corresponding to a seasonal unit root at the biannual frequency;

3. $H_0^3 : \rho_3 = \rho_4 = 0$, corresponding to seasonal unit roots at the annual frequency.

The first two hypotheses can be tested by using the conventional t-statistic for testing the significance of ρ_s, which is henceforth denoted $t_s^m(T^\circ)$ with the superscript m indicating the model under consideration. The reason for writing t_s^m as a function of T° is to indicate that the statistic has been computed for a particular choice of breakpoint, and that its limiting

[2]Note also that although in the current paper we only consider the case of a single break, this is probably not necessary. Indeed, intuition suggests that our results can be extended to the case of multiple breaks.

distribution depends on it. For testing the third hypothesis we use the F-statistic for the joint significance of ρ_3 and ρ_4, which is written in an obvious notation as $F_{34}^m(T^\circ)$.

As we just pointed out, the results reported so far are based on the assumption that T° is known. When it is unknown, a natural approach is to treat the estimation problem as a model selection issue, and to estimate T° by minimizing an information criterion. The particular estimator used in this paper is very similar in spirit. Specifically, the proposal of Popp (2007, 2008) is adopted, which focuses on the significance of the coefficient of impulse dummy $\Delta_4 DU_{s,t}$ in (5). Specifically, let us denote by $F_\theta(T^*)$ the F-statistic for testing the joint significance of $\theta_1, ..., \theta_4$ when the breakpoint is $T^* = \tau^* T$, where $\tau^* \in (0, 1)$. The breakpoint estimator is defined as

$$\hat{T}^\circ = \arg \max_{T^* \in [qT, (1-q)T]} F_\theta(T^*),$$

where $q \in (0, 1)$ is a trimming factor that eliminates the endpoints. Given $\hat{T}^\circ$, feasible versions of our test statistics can be computed as $t_s^m = t_s^m(\hat{T}^\circ)$ and $F_{34}^m = F_{34}^m(\hat{T}^\circ)$, where the dependence upon $\hat{T}^\circ$ is henceforth suppressed.

It is worth taking a moment to discuss the rationale that underlies the above estimation procedure. Note how the dummy variables DT_t and $DU_{s,t}$ appear in lagged form in (5), which is different from the regression considered by Franses and Vogelsang (1998). This discrepancy arises naturally from our choice of a component model, which is different from the data generating process considered by Franses and Vogelsang (1998). The main advantage of using our model is that the interpretation of the regression coefficients does not change depending on whether we are under the null or not, see Schmidt and Phillips (1992) for a more detailed discussion. In particular, it implies that the hypothesis of no break can be implemented as a test of the restriction that $\theta_1 = ... = \theta_4 = 0$.

By contrast, Franses and Vogelsang (1998) adopt the approach of Perron and Vogelsang (1992), which is based on the significance of the slope coefficient of $DU_{s,t}$, whose meaning depend critically on the integratedness of y_t, see Popp (2007). As we demonstrate in Section 4, this difference can have a substantial impact on the accuracy of the estimated breakpoint.

3 Limiting distribution

To fix ideas, suppose that the overall null hypothesis of $\rho_1 = ... = \rho_4 = 0$ is true. In this case it is possible to show that as long as the order of the lag augmentation p is large enough to capture the serial correlation in e_t, the least squares estimator of ρ_s in (5) is asymptotically invariant with respect to the other coefficients of the model. Therefore, we do not loose generality by setting them equal to zero. It follows that if we in addition assume that $y_{-3} = ... = y_0 = 0$, then (5) reduces to

$$\Delta_4 y_t \;=\; u_t, \tag{6}$$

which simplifies the asymptotic analysis considerably. It should be pointed out, however, that these assumptions are not necessary, and that they are only for convenience, see for example Franses and Vogelsang (1998) for a further discussion.

In the theorem that follows we report the asymptotic distribution of the new test statistics under the null hypothesis given by (6). However, before we come to the theorem we need to introduce some notation. In particular, let us define

$$M_{D^*u} = \frac{\sigma}{2}\begin{bmatrix} \int_0^1 dW \\ \int_{\tau^\circ}^1 dW \end{bmatrix}, \quad M_{D^{**}u} = \frac{\sigma}{2}\begin{bmatrix} \int_{\tau^\circ}^1 rdW \\ \int_{\tau^\circ}^1 (r-\tau^\circ)dW \end{bmatrix},$$

where dW is the increment of $W = (\, W_1,\, W_2,\, W_3,\, W_4\,)'$, a four-dimensional standard Browninan motion on $r \in [0,1]$.[3] Moreover, defining

$$\begin{aligned} B_1 &= \frac{1}{2}\sum_{s=1}^{4} W_s, \quad B_2 = \frac{1}{2}\sum_{s=1}^{4}(-1)^s W_s, \quad B_3 = \frac{1}{2}(W_1 - W_3), \\ B_4 &= \frac{1}{2}(W_2 - W_4), \end{aligned}$$

then $B = (\, B_1,\, B_2,\, B_3,\, B_4\,)'$,

$$\begin{aligned} M_{Yu} &= -\sigma^2 \begin{bmatrix} -\int_0^1 B_1 dB_1 \\ \int_0^1 B_2 dB_2 \\ \int_0^1 (B_3 dB_3 + B_4 dB_4) \\ \int_0^1 (B_4 dB_3 - B_3 dB_4) \end{bmatrix}, \quad M_{YY} = \sigma^2 \operatorname{diag} \begin{bmatrix} 4\int_0^1 B_1^2 dr \\ 4\int_0^1 B_2^2 dr \\ 2\int_0^1 (B_3^2 + B_4^2) dr \\ 2\int_0^1 (B_3^2 + B_4^2) dr \end{bmatrix}, \\ M_{YD^*} &= \sigma \begin{bmatrix} \int_0^1 G dr \\ \int_{\tau^\circ}^1 G dr \end{bmatrix}, \quad M_{YD^{**}} = \sigma \begin{bmatrix} \int_0^1 rG dr \\ \int_{\tau^\circ}^1 (r-\tau^\circ) G dr \end{bmatrix}, \\ M_{D^*D^*} &= \frac{1}{4}\begin{bmatrix} I_4 & (1-\tau^\circ) I_4 \\ \cdot & (1-\tau^\circ) I_4 \end{bmatrix}, \\ M_{D^{**}D^{**}} &= \frac{1}{3}\begin{bmatrix} 1 & (1-(\tau^\circ)^3) - \frac{3}{2}\tau^\circ(1-(\tau^\circ)^2) \\ \cdot & (1-(\tau^\circ)^3) - 3\tau^\circ(1-(\tau^\circ)^2) + 3(\tau^\circ)^2(1-\tau^\circ) \end{bmatrix}, \\ M_{D^{**}D^*} &= \frac{1}{8}\begin{bmatrix} \iota_4' & (1-(\tau^\circ)^2)\iota_4' \\ (1-(\tau^\circ)^2 - 2\tau^\circ(1-\tau^\circ))\iota_4' & (1-(\tau^\circ)^2 - 2\tau^\circ(1-\tau^\circ))\iota_4' \end{bmatrix} \end{aligned}$$

[3]Here and throughout all Brownian motions such as $W(r)$ will be written as W, with the measure of integration omitted. Integrals such as $\int_0^1 W(r)dr$ and $\int_0^1 W(r)dW(r)$ will be written $\int_0^1 Wdr$ and $\int_0^1 WdW$, respectively.

with $\iota_4 = (1, 1, 1, 1)'$ and

$$G = \frac{1}{4}\begin{bmatrix} B_1 & B_1 & B_1 & B_1 \\ -B_2 & B_2 & -B_2 & B_2 \\ B_3 & B_3 & -B_3 & -B_3 \\ B_4 & -B_4 & -B_4 & B_4 \end{bmatrix}.$$

Theorem 1. *Under (6) as $T \to \infty$,*

$$\begin{aligned}
t_s^1(T^\circ) &\to_w \bar{t}_s^1(\tau^\circ) = \frac{R_s^1(JM_{XX}J')^{-1}JM_{Xu}}{\sigma\sqrt{R_s^1(JM_{XX}J')^{-1}R_s^{1\prime}}}, \\
t_s^2(T^\circ) &\to_w \bar{t}_s^2(\tau^\circ) = \frac{R_s^2 M_{XX}^{-1}M_{Xu}}{\sigma\sqrt{R_s^2 M_{XX}^{-1}R_s^{2\prime}}}, \\
F_{34}^1(T^\circ) &\to_w \bar{F}_{34}^1(\tau^\circ) = \frac{1}{\sigma^2}M_{Xu}'J'(JM_{XX}J')^{-1}R_{34}^{1\prime}\left(R_{34}^1(JM_{XX}J')^{-1}R_{34}^{1\prime}\right)^{-1} \\
&\quad \cdot \; R_{34}^1(JM_{XX}J')^{-1}JM_{Xu}, \\
F_{34}^2(T^\circ) &\to_w \bar{F}_{34}^2(\tau^\circ) = \frac{1}{\sigma^2}M_{Xu}'M_{XX}^{-1}R_{34}^{2\prime}\left(R_{34}^2 M_{XX}^{-1}R_{34}^{2\prime}\right)^{-1}R_{34}^2 M_{XX}^{-1}M_{Xu},
\end{aligned}$$

where $\to_w$ denotes weak convergence, R_s^m and R_{34}^m are the restriction matrices corresponding to $\rho_s = 0$ and $\rho_3 = \rho_4 = 0$ in model m, respectively, J is the identity matrix with row 10 removed, $M_{Xu} = (M_{D^*u}', M_{D^{**}u}', M_{Yu}')'$ and

$$M_{XX} = \frac{1}{4}\begin{bmatrix} M_{D^*D^*} & M_{D^{**}D^*} & M_{YD^*} \\ M_{D^{**}D^*}' & M_{D^{**}D^{**}} & M_{YD^{**}} \\ M_{YD^*}' & M_{YD^{**}}' & M_{YY} \end{bmatrix}.$$

It is important to realize that the limiting distributions of t_s^m and F_{34}^m do not depend on σ^2. That is, σ^2 cancels out in the numerators and denominators. Thus, the new tests are asymptotically invariant not only with respect to the coefficients of the equation driving y_t but also with respect to variance of u_t. Moreover, although the asymptotic distributions are for the case in which e_t is serially uncorrelated, as we pointed out earlier this assumption is only for convenience, and can be relaxed at the cost of some extra notation. The only thing that is needed for this to hold is that the order p is sufficiently large.

One problem with Theorem 1 is that it assumes that the true breakpoint T° is known, as indicated by the dependence of the limiting distributions on τ°. The asymptotic distributions of the feasible test statistics are provided in the following corollary.

Corollary 1. *Under (6) as $T \to \infty$,*

$$t_s^m \to_w \bar{t}_s^m(\tau_m^*), \qquad F_{34}^m \to_w \bar{F}_{34}^m(\tau_m^*),$$

where

$$\tau_1^* = \frac{1}{\sigma^2} M_{Xu}' J' (J M_{XX} J')^{-1} R_\theta^{1\prime} \left(R_\theta^1 (J M_{XX} J')^{-1} R_\theta^{1\prime} \right)^{-1} R_\theta^1 (J M_{XX} J')^{-1} J M_{Xu},$$
$$\tau_2^* = \frac{1}{\sigma^2} M_{Xu}' M_{XX}^{-1} R_\theta^{2\prime} \left(R_\theta^2 M_{XX}^{-1} R_\theta^{2\prime} \right)^{-1} R_\theta^2 M_{XX}^{-1} M_{Xu},$$

with R_θ^m being the restriction matrix corresponding to $\theta_1 = ... = \theta_4 = 0$ in model m.

As in Theorem 1, although it appears in the formula for τ_m^*, there is no real dependence on σ^2, which cancels out when forming $\bar{t}_s^m(\tau_m^*)$ and $\bar{F}_{34}^m(\tau_m^*)$. The limiting distributions of the feasible test statistics are therefore completely free of nuisance parameters. Note in particular how the dependence on $\tau_\circ$, the true break fraction, is now gone. In the next section we use simulations to obtain the critical values of t_s^m and F_{34}^m.

4 Simulations

4.1 Critical values

The critical values are obtained by making 5,000 draws of length T from the data generating process in (6), where $u_t \sim N(0,1)$.[4] The computation of the test statistics requires two choices. The first is how many lags of $\Delta_4 y_t$ to use in the test regression, here denoted by p. The second is how much to trim when estimating the breakpoint, that is, how to pick q. As for the choice of p, we consider two approaches. One is to set $p = 0$, while the other is to follow Franses and Vogelsang (1998) and to set p according to the general-to-specific procedure of Hall (1994) with a maximum of five lags. As for the choice of q, we follow the usual convention and set $q = 0.1$, so that 10% of the observations in both beginning and end of the sample are trimmed away. All computational work is performed in GAUSS. The results are reported in Table 1.

4.2 Size and power

The size and power comparisons are based on 5,000 draws from the data generating process given by (2) to (4), where $\Phi(L) = 1$ and $\mu_1 = ... = \mu_4 = \lambda = 0$. As for the mean break coefficient ϕ_s we consider two cases. In case 1, $\phi_s = \phi$ for all s, so that all the seasons are

[4] As before, the test statistics are asymptotically invariant with respect to the parametrization of the data generating process. Therefore, we do not loose generality when generating the data from (6), at least not asymptotically.

affected by the same break, while in case 2, $\phi_1 = \phi_3 = -\phi_2 = -\phi_4 = \phi$, so that the effect of the break is allowed to change with the season. In both cases we assume that the break is located in the middle of the sample, that is, $\tau^\circ = 0.5$. The test statistics are constructed in exactly the same way as described in Section 4.1.

Franses and Vogelsang (1998) develop three tests, denoted t_1^{FV}, t_2^{FV} and F_{34}^{FV}, that are designed to test the null of a seasonal unit root when there is a break in the mean but the data are not allowed to be trending. As with t_1^m and t_2^m, t_1^{FV} and t_2^{FV} are constructed as simple t-tests of the null hypotheses H_0^1 and H_0^2, respectively, while F_{34}^{FV}, in similarity to F_{34}^m, is constructed as an F-test of the joint null of H_0^3. In terms of construction the tests are therefore very similar. The main difference is that Franses and Vogelsang (1998) presume that the researcher can be confident that the data generating process does not include a linear time trend. Thus, not only is it assumed that the researcher has full certainty over the trend, but also that there is no trend, which is of course highly unlikely to hold in practice. It is therefore interesting to see how these tests perform in the presence of an unattended trend, which in addition may be subject to a structural break.

Table 2 summarizes the results from the size and power of the t_1^{FV}, t_2^{FV} and F_{34}^{FV} tests at the 5% significance level. Some results of the correct selection frequency, and of the mean and standard deviation of the estimated breakpoint are also reported.

The first thing to notice is the size, which increases considerably with the size of the break, as measured by ϕ. As an extreme example, consider case 1 when $T = 152$, in which an increase in ϕ from zero to 10 causes the size of t_1^{FV} to go from 5% to almost 100%. The distortions do get smaller as T increases but the size is still severely distorted, even when T is as large as 500. The results for case 2 are more favorable. However, the tendency for the size distortions to increase with ϕ still remains. The t_2^{FV} test suffers the same problem but with this test the distortions are more pronounced in case 2. The F_{34}^{FV} test has some distortions in both cases and is therefore more robust in this sense.

Moreover, looking now at the results from the estimated breakpoint, in agreement with the discussion of Section 2, we see that the estimation procedure of Franses and Vogelsang (1998) is unable to pinpoint the location of the break.[5] However, since the performance is roughly the same in the two break cases, this is probably not the reason behind the size

[5]Similar results have been documented by for example Harvey *et al.* (2002), Lee and Strazicich (2001), and Popp (2007, 2008).

distortions in the unit root tests.

The results obtained by applying our tests to the same data are reported in Table 3 for model 1 and in Table 4 for model 2. As expected, we see that the size accuracy for all three tests is almost perfect, and that the performance is unaltered by the size of the break. The results from the power of the tests are also quite encouraging. Specifically, we see that although the tests can sometimes have difficulties in discriminating between the null and alternative hypotheses, as expected, the power increases quickly as T grows. The overall best performance is obtained by using F_{34}^{m}, which is to be expected since it is a joint hypothesis test.

We also see that the breakpoint estimator seems to perform very well with almost perfect accuracy in a majority of the experiments, which of course stands in sharp contrast to the overall poor performance of the Franses and Vogelsang (1998) estimator. As expected, the accuracy increases slightly with T and also with ϕ, which seems reasonable as a larger break is more easy to discern.

5 The motivating example continued

5.1 Preliminary results

As a complement to the graphical evidence of Figures 1 and 2, Table 5 reports the average and standard deviation for the percentage change of each series, which are computed as $100 \cdot \Delta y_t$, where y_t is the log of industrial production. Ireland has experienced the most rapid growth by far with an average growth rate of about 2% per quarter, while in Norway and the United Kingdom the average growth is much lower, only 0.15%. Sweden stands out as having the most volatile series, which is partly due to a relatively strong season. This is seen in the rightmost column, which reports R^2-measure from a regression of Δy_t onto $D_{1,t}, ..., D_{4,t}$. The lowest R^2 is obtained for Austria, which is consistent with its relatively weak seasonal pattern, as can be seen in Figure 1.

Before applying the new seasonal unit root tests, in interest of comparison we first consider some results from applying the conventional Dickey and Fuller (1979) and Hylleberg *et al.* (1990) tests. The former is denoted by t_1^{DF}, while the latter are denoted by t_1^{HEGY}, t_2^{HEGY} and F_{34}^{HEGY} to indicate their close connection with the tests proposed here. All four tests are computed while allowing for the presence of a linear trend.[6] Thus, the main difference here is that while our tests permit for the possibility of a break, the other tests do not. The t_1^{DF} test is even more restrictive and does not allow for seasonality either. The results are reported in Table 6.

So far our findings suggest that a majority of the series exhibit both strong seasonal variation and permanent shifts, which in turn implies that the tests of Dickey and Fuller (1979) and Hylleberg *et al.* (1990) are likely to be biased in favor of the unit root null. In agreement with this result we see that the t_1^{DF} and t_1^{HEGY} tests are unable to reject the null of a nonseasonal unit root. This inability to reject is also observed when testing null hypothesis of a biannual unit root, in which case we count five rejections at the 10% level and one rejection at the 5% level. The results for the null of annual unit roots are different. In this case we count five rejections at the 1% level, nine rejections at the 5% level, and 12 times at the 10% level. Thus, while weak at the two lowest frequencies, the evidence against the null is stronger at the highest frequency.

Of course, these results should not be taken too seriously, as the possibility remains that they have been spuriously induced by the presence of seasonality and breaks in the case of the Dickey and Fuller (1979) test, and by the presence of breaks in the case of the Hylleberg *et al.* (1990) tests.

5.2 The results of the new tests

Table 7 reports the results of the new tests, which are implemented in the same way as described in Section 4. We begin by looking at the t_1^m test, which tests the null of a nonseasonal unit root. The results for the two models are very similar. If we allow for a break in the seasonal mean but not in the trend, then we count two rejections at the 5% level, whereas if there is a break in the trend, then we count only one rejection at the same level of significance.

The t_2^m test, which tests the null of a biannual unit root, results in more rejections. For model 1 we count 11 rejections at the 5% level, whereas for model 2 we count nine rejections. At the 10% level there is evidence against the null for all countries but Austria, Finland, the Netherlands and Sweden. The fact that the evidence against the null is so much stronger now in comparison to Table 6 suggests the presence of a break, which is not accounted for

[6] As in Section 4, the appropriate number of lags to use is determined by using the general-to-specific approach of Hall (1994) with the maximum lag length set to five.

when using the Hylleberg *et al.* (1990) t_2^{HEGY} test.

Next, we consider the results from the F_{34}^{m} test and the null hypothesis of annual unit roots. Looking at the 1% level we see that the hypothesis is refuted for all countries but two, for Belgium and Finland, which seem largely consistent with the graphical evidence reported in Section 1.

Finally, we take a look at the estimated breakpoints that come out as a bi-product in the testing procedure. Focusing on the most general model with a break in both the seasonal mean and trend slope we see that there is a predominance of breaks occurring in the late 1990's, which is in agreement with the graphical evidence. The estimated breaks in the mid-1980's for Belgium and the United Kingdom are also clearly visible in the figures.

6 Conclusions

This paper is inspired by the large amount of empirical research that has gone into the testing for unit roots in output, and in particular industrial production, which at a quarterly basis is typically characterized by strong seasonality and an upwards trend. Then there is also the presence of breaks that permanently shift both the seasonal regularity and the rate of growth. However, most studies based on examining the persistence of industrial production fail to account for these features, and use tests that are invalid in their presence.

In this paper we take these observations as our point of origin. The purpose is to device a test procedure that is able to handle all the major features of this kind of data. In particular, three new seasonal unit root tests are proposed that allow not only for seasonal and trending behavior, but also for a break of unknown timing in both the seasonal mean and trend slope. The relevant asymptotic theory and critical values are provided. Some simulation results are also reported to suggest that the tests perform well with very high size accuracy and good power in most experiments considered.

In our empirical application we consider the industrial production for a sample of 13 European countries that cover the period from 1976:1 to 2006:1. There are two main results. Firstly, all series seem to have a clear nonseasonal unit root. Secondly, for a majority of the series there do not seem to be any seasonal unit roots at all. Hence, for these series it seem reasonable to assume a stationary season, in which the season-specific means reflect the seasonal cycle.

Appendix: Mathematical proofs

Lemma A.1. *Under the above conditions, as $T \to \infty$*

$$\begin{aligned} \frac{1}{T^2}\sum_{t=1}^{T} Y_{t-1}Y_{t-1}' &\to_w M_{YY}, \\ \frac{1}{T}\sum_{t=1}^{T} Y_{t-1}u_t &\to_w M_{Yu}, \end{aligned}$$

where $Y_{t-1} = (\,y_{1,t-1},\, y_{2,t-1},\, y_{3,t-2},\, y_{3,t-1}\,)'$.

Proof of Lemma A.1.

Consider first the results for $y_{1,t}$. Letting $N = T/4$ and $S_{s,j} = \sum_{n=1}^{j} u_{4n-(4-s)}$, where $j = \lfloor t/4 \rfloor$ with $\lfloor x \rfloor$ denoting the integer part of x, then by a functional central limit theorem,

$$\frac{1}{\sqrt{N}} S_{s,j} \to_w \sigma W_s \tag{A1}$$

as $T \to \infty$. Thus, since

$$y_{1,t} = \sum_{n=1}^{t} u_n = \sum_{s=1}^{4} S_{s,j} + O_p(1).$$

it follows that

$$\frac{1}{\sqrt{T}} y_{1,t} = \sum_{s=1}^{4} \frac{1}{\sqrt{4N}} S_{s,j} + o_p(1) \to_w \sigma B_1 \tag{A2}$$

as $T \to \infty$, and by further application of the continuous mapping theorem,

$$\frac{1}{T^2}\sum_{t=1}^{T} y_{1,t}^2 \to_w \sigma^2 \int_0^1 B_1^2 dr.$$

Moreover,

$$\frac{1}{T}\sum_{t=1}^{T} y_{1,t-1}u_t = \frac{1}{4N}\sum_{j=1}^{N}\left(\sum_{s=1}^{4} S_{s,j}\right)\sum_{s=1}^{4} u_{4j-(4-s)} + o_p(1) \to_w \sigma^2 \int_0^1 B_1 dB_1.$$

Next, consider $y_{2,t}$, for which it holds that

$$y_{2,t} = \sum_{s=1}^{4}(-1)^s \begin{cases} -S_{s,j} + O_p(1) & t \bmod 2 = 1 \\ S_{s,j} + O_p(1) & t \bmod 2 = 0 \end{cases}. \tag{A3}$$

Hence,

$$\frac{1}{\sqrt{T}} y_{2,t} \to_w \sigma \begin{cases} -B_2 & t \bmod 2 = 1 \\ B_2 & t \bmod 2 = 0 \end{cases},$$

which in turn suggests that $\frac{1}{T^2}\sum_{t=1}^{T} y_{2,t}^2 \to_w \sigma^2 \int_0^1 B_2^2 dr$. Also,

$$\frac{1}{T}\sum_{t=1}^{T} y_{2,t-1}u_t = -\frac{1}{4N}\sum_{j=1}^{N}\left(\sum_{s=1}^{4}(-1)^s S_{s,j}\right)\sum_{s=1}^{4}(-1)^s u_{4j-(4-s)} + o_p(1) \to_w -\sigma^2\int_0^1 B_2 dB_2.$$

A similar calculation reveal that as $T \to \infty$

$$\frac{1}{\sqrt{T}} y_{3,t} \to_w \sigma \begin{cases} B_3 & t \bmod 4 = 1 \\ B_4 & t \bmod 4 = 2 \\ -B_3 & t \bmod 4 = 3 \\ -B_4 & t \bmod 4 = 0 \end{cases} \tag{A4}$$

giving

$$\begin{aligned} \frac{1}{T^2}\sum_{t=1}^{T} y_{3,t}^2 &\to_w \frac{\sigma^2}{2}\int_0^1 (B_3^2 - B_4^2) dr, \\ \frac{1}{T}\sum_{t=1}^{T} y_{3,t-2}u_t &\to_w -\sigma^2\int_0^1 (B_3 dB_3 + B_4 dB_4), \\ \frac{1}{T}\sum_{t=1}^{T} y_{3,t-1}u_t &\to_w -\sigma^2\int_0^1 (B_4 dB_3 - B_3 dB_4). \end{aligned}$$

By combining the results we obtain $\frac{1}{T}\sum_{t=1}^{T} Y_{t-1}u_t \to_w M_{Yu}$ as $T \to \infty$. The proof of the second result is made complete by noting that $y_{1,t-1}$, $y_{2,t-1}$, $y_{3,t-2}$ and $y_{3,t-1}$ are asymptotically orthogonal, ensuring that M_{YY} is a diagonal matrix, see Appendix A of Ghysels *et al.* (1994). ■

Proof of Theorem 1.

The test regression can be written in matrix format as

$$\Delta_4 y_t = X_t'\gamma + u_t, \tag{A5}$$

where $X_t = (D_t^{*\prime}, D_t^{**\prime}, Y_{t-1}')'$, $D_t^* = (D_t', DU_t')'$ and $D_t^{**} = (t, DT_t)'$ with D_t and DU_t being the vectors stacking $D_{s,t}$ and $DU_{s,t}$, respectively. The one-time dummy variable $\Delta_4 DU_{s,t}$ is asymptotically negligible and is therefore omitted from (A5).

Using $\hat{\gamma}$ to denote the least squares estimator of γ,

$$H^{-1}(\hat{\gamma} - \gamma) = \left(H^{-1}\sum_{t=1}^{T} X_t X_t' H^{-1}\right)^{-1} H^{-1}\sum_{t=1}^{T} X_t u_t, \tag{A6}$$

where

$$H = \sqrt{T}\begin{bmatrix} I_8 & 0 & 0 \\ \cdot & T\,I_2 & 0 \\ \cdot & \cdot & \sqrt{T}\,I_4 \end{bmatrix}.$$

Consider $H^{-1}\sum_{t=1}^{T} X_t u_t$. Define $N^\circ = T^\circ/4$. Clearly, since $\frac{N^\circ}{N} = \frac{T^\circ}{T} \to \tau^\circ$ as $T \to \infty$,

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T} DU_{s,t}u_t = \frac{1}{\sqrt{T}}\sum_{t=T^\circ+1}^{T} D_{s,t}u_t = \frac{1}{\sqrt{4N}}\sum_{j=\lfloor N^\circ \rfloor+1}^{N} u_j \to_w \frac{\sigma}{2}\int_{\tau^\circ}^{1} dW_s,$$

and by the same arguments, $\frac{1}{\sqrt{T}}\sum_{t=1}^{T} D_{s,t}u_t \to_w \frac{\sigma}{2}\int_0^1 dW_s$. Therefore,

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T} D_t^* u_t \to_w M_{D^*u},$$

and in view of this result it is not difficult to see that

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T} DT_t u_t = \frac{1}{\sqrt{T}}\sum_{t=T^\circ+1}^{T} (t-T^\circ)u_t \to_w \frac{\sigma}{2}\int_{\tau^\circ}^{1} (r-\tau^\circ)dW_1,$$

from which we deduce $\frac{1}{\sqrt{T}}\sum_{t=1}^{T} D_t^{**}u_t \to_w M_{D^{**}u}$. These results, together with Lemma A.1, yield

$$H^{-1}\sum_{t=1}^{T} X_t u_t = \sum_{t=1}^{T}\begin{bmatrix} \frac{1}{\sqrt{T}}D_t^* u_t \\ \frac{1}{T^{3/2}}D_t^{**}u_t \\ \frac{1}{T}Y_{t-1}u_t \end{bmatrix} \to_w \begin{bmatrix} M_{D^*u} \\ M_{D^{**}u} \\ M_{Yu} \end{bmatrix}. \tag{A7}$$

Next, consider the denominator of $H^{-1}(\hat{\gamma}-\gamma)$, which is given by

$$H^{-1}\sum_{t=1}^{T} X_t X_t' H^{-1} = \sum_{t=1}^{T}\begin{bmatrix} \frac{1}{T}D_t^* D_t^{*\prime} & \frac{1}{T^2}D_t^{**}D_t^{*\prime} & \frac{1}{T^{3/2}}Y_{t-1}D_t^{*\prime} \\ \frac{1}{T^2}D_t^* D_t^{**\prime} & \frac{1}{T^3}D_t^{**}D_t^{**\prime} & \frac{1}{T^{5/2}}Y_{t-1}D_t^{**\prime} \\ \frac{1}{T^{3/2}}D_t^* Y_{t-1}' & \frac{1}{T^{5/2}}D_t^{**}Y_{t-1}' & \frac{1}{T^2}Y_{t-1}Y_{t-1}' \end{bmatrix}, \tag{A8}$$

where

$$\frac{1}{T}\sum_{t=1}^{T} D_t^* D_t^{*\prime} = \frac{1}{T}\sum_{t=1}^{T}\begin{bmatrix} D_t D_t' & D_t DU_t' \\ DU_t D_t' & DU_t DU_t' \end{bmatrix} \to \frac{1}{4}\begin{bmatrix} I_4 & (1-\tau^\circ)I_4 \\ (1-\tau^\circ)I_4 & (1-\tau^\circ)I_4 \end{bmatrix} = M_{D^*D^*}$$

as $T \to \infty$. Furthermore,

$$\begin{aligned} \frac{1}{T^2}\sum_{t=1}^{T} D_t^{**}D_t^{*\prime} &= \frac{1}{T^2}\sum_{t=1}^{T}\begin{bmatrix} tD_t' & tDU_t' \\ DT_t D_t' & DT_t DU_t' \end{bmatrix} \\ &\to \frac{1}{8}\begin{bmatrix} \iota_4' & (1-(\tau^\circ)^2)\iota_4' \\ \big(1-(\tau^\circ)^2-2\tau^\circ(1-\tau^\circ)\big)\iota_4' & \big(1-(\tau^\circ)^2-2\tau^\circ(1-\tau^\circ)\big)\iota_4' \end{bmatrix} \\ &= M_{D^{**}D^*}, \end{aligned}$$

where we have used that

$$
\begin{aligned}
\frac{1}{T^2}\sum_{t=1}^{T} tD_{s,t} &= \frac{1}{16N^2}\sum_{j=1}^{N} 4j \;\to\; \frac{1}{16}\int_0^1 4rdr \;=\; \frac{1}{8}, \\
\frac{1}{T^2}\sum_{t=1}^{T} tDU_{s,t} &= \frac{1}{16N^2}\sum_{j=\lfloor N^\circ\rfloor+1}^{N} 4j \;\to\; \frac{1}{16}\int_{\tau^\circ}^1 4rdr \;=\; \frac{1}{8}(1-(\tau^\circ)^2), \\
\frac{1}{T^2}\sum_{t=1}^{T} DT_t DU_{s,t} &= \frac{1}{16N^2}\sum_{j=\lfloor N^\circ\rfloor+1}^{N} 4(j-N^\circ) \;\to\; \frac{1}{4}\left(\int_{\tau^\circ}^1 rdr - \tau^\circ(1-\tau^\circ)\right) \\
&= \frac{1}{8}\left(1-(\tau^\circ)^2 - 2\tau^\circ(1-\tau^\circ)\right),
\end{aligned}
$$

where the last result also applies to $\frac{1}{T^2}\sum_{t=1}^{T} DT_t D_{s,t}$. A similar calculation reveals that $\frac{1}{T^3}\sum_{t=1}^{T} D_t^{**}D_t^{**\prime} \to M_{D^{**}D^{**}}$.

As for $\frac{1}{T^{3/2}}\sum_{t=1}^{T} Y_{t-1}D_t^{*\prime} = \frac{1}{T^{3/2}}\sum_{t=1}^{T} Y_{t-1}(\,D_t',\, DU_t'\,)$ note that

$$
\frac{1}{T^{3/2}}\sum_{t=1}^{T} Y_{t-1}D_t' \;=\; \frac{1}{T^{3/2}}\sum_{t=1}^{T}\begin{bmatrix} y_{1,t-1}D_{1,t} & y_{1,t-1}D_{2,t} & y_{1,t-1}D_{3,t} & y_{1,t-1}D_{4,t} \\ y_{2,t-1}D_{1,t} & y_{2,t-1}D_{2,t} & y_{2,t-1}D_{3,t} & y_{2,t-1}D_{4,t} \\ y_{3,t-2}D_{1,t} & y_{3,t-2}D_{2,t} & y_{3,t-2}D_{3,t} & y_{3,t-2}D_{4,t} \\ y_{3,t-1}D_{1,t} & y_{3,t-1}D_{2,t} & y_{3,t-1}D_{3,t} & y_{3,t-1}D_{4,t} \end{bmatrix},
$$

where

$$
\frac{1}{T^{3/2}}\sum_{t=1}^{T} y_{1,t-1}D_{s,t} \;=\; \frac{1}{8N^{3/2}}\sum_{j=1}^{N} y_{1,4j-(4-s)} \;\to_w\; \frac{\sigma}{4}\int_0^1 B_1 dr
$$

as $T \to \infty$. But the same result applies to all s and so we have that the first row of the matrix $\frac{1}{T^{3/2}}\sum_{t=1}^{T} Y_{t-1}D_t'$ converges to the first row of $\sigma\int_0^1 Gdr$, which in turn is the first element of M_{YD^*}. Let us now consider $\frac{1}{T^{3/2}}\sum_{t=1}^{T} y_{2,t-1}D_{s,t}$. As $T \to \infty$

$$
\begin{aligned}
\frac{1}{T^{3/2}}\sum_{t=1}^{T} y_{2,t-1}D_{s,t} &= \frac{1}{8N^{3/2}}\sum_{j=1}^{N} y_{2,4j-(4-s)} \;=\; \frac{1}{8N^{3/2}}\sum_{j=1}^{N}\begin{cases} -S_{s,j}+O_p(1) & s \bmod 2 \;=\; 1 \\ S_{s,j}+O_p(1) & s \bmod 2 \;=\; 0 \end{cases} \\
&\to_w \;\frac{1}{4}\int_0^1 \begin{cases} -B_s & s \bmod 2 \;=\; 1 \\ B_s & s \bmod 2 \;=\; 0 \end{cases}.
\end{aligned}
$$

Similar calculations for $y_{3,t-2}$ and $y_{3,t-1}$ across seasons yield $\frac{1}{T^{3/2}}\sum_{t=1}^{T} Y_{t-1}D_t' \to_w \sigma\int_0^1 Gdr$ as $T \to \infty$. The results for $\frac{1}{T^{3/2}}\sum_{t=1}^{T} Y_{t-1}DU_t'$ and $\frac{1}{T^{5/2}}\sum_{t=1}^{T} Y_{t-1}D_t^{**\prime}$ are immediate consequences of this, and so the proof is complete. ∎

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Table 1: Critical values for the new tests.

		1%			5%			10%		
p	T	t_1^m	t_2^m	F_{34}^m	t_1^m	t_2^m	F_{34}^m	t_1^m	t_2^m	F_{34}^m
Model 1, $m = 1$										
0	52	−4.766	−4.137	14.51	−4.001	−3.407	10.09	−3.645	−3.034	8.46
	100	−4.535	−4.037	12.75	−3.860	−3.397	9.56	−3.548	−3.054	8.11
	152	−4.328	−3.978	12.56	−3.790	−3.397	9.48	−3.501	−3.073	8.03
	300	−4.375	−3.967	11.62	−3.825	−3.331	9.14	−3.500	−2.992	7.87
	500	−4.361	−3.964	11.85	−3.820	−3.344	9.18	−3.525	−3.032	7.88
GTS	52	−5.419	−4.506	16.43	−4.363	−3.641	11.77	−3.933	−3.218	9.56
	100	−4.786	−4.141	13.79	−4.074	−3.474	10.06	−3.693	−3.146	8.38
	152	−4.669	−3.995	12.28	−3.942	−3.421	9.49	−3.616	−3.133	8.23
	300	−4.497	−3.974	11.85	−3.905	−3.383	9.29	−3.584	−3.062	7.99
	500	−4.370	−3.922	12.23	−3.865	−3.391	9.23	−3.556	−3.058	8.03
Model 2, $m = 2$										
0	52	−5.114	−4.163	13.75	−4.413	−3.446	10.08	−4.046	−3.099	8.45
	100	−4.979	−3.891	12.20	−4.274	−3.397	9.46	−3.929	−3.060	8.11
	152	−4.973	−4.070	12.27	−4.251	−3.400	9.28	−3.932	−3.055	7.97
	300	−4.777	−3.986	11.75	−4.222	−3.302	9.31	−3.911	−3.007	7.93
	500	−4.698	−3.869	11.65	−4.164	−3.334	9.01	−3.864	−3.027	7.77
GTS	52	−5.773	−4.350	16.26	−4.840	−3.566	11.96	−4.387	−3.204	9.63
	100	−5.228	−4.123	13.49	−4.469	−3.443	10.13	−4.111	−3.130	8.60
	152	−5.022	−4.055	12.81	−4.345	−3.417	9.62	−4.024	−3.108	8.24
	300	−4.827	−3.945	11.85	−4.287	−3.399	9.24	−3.944	−3.089	7.92
	500	−4.815	−3.946	11.84	−4.201	−3.368	8.99	−3.910	−3.028	7.73

Notes: p refers to the number of lag augmentations, where GTS indicates that p has been determined according to the general-to-specific procedure of Hall (1994) with the maximum number of lags set to five.

Table 2: Simulation results for the Franses and Vogelsang (1998) tests.

ϕ	λ	ψ	t_1^{FV}	t_2^{FV}	F_{34}^{FV}	Corr	Mean	SE
				$T = 152$				
0	0	0	0.050	0.050	0.050	0.009	78.7	30.2
3	0	0	0.991	0.028	0.092	0.000	72.0	1.1
5	0	0	1.000	0.185	0.663	0.000	72.0	0.0
10	0	0	1.000	0.998	1.000	0.000	72.0	0.0
0	3	0	0.001	0.057	0.061	0.012	78.7	31.0
0	5	0	0.001	0.065	0.060	0.013	78.4	30.8
0	10	0	0.001	0.065	0.057	0.008	77.7	31.1
0	0	3	0.000	0.012	0.009	0.155	75.1	1.1
0	0	5	0.000	0.011	0.014	0.257	75.4	1.0
0	0	10	0.000	0.008	0.020	0.318	75.4	0.9
				$T = 300$				
0	0	0	0.050	0.050	0.050	0.005	154.6	61.6
3	0	0	0.937	0.013	0.038	0.000	145.8	10.8
5	0	0	1.000	0.034	0.202	0.000	146.0	0.0
10	0	0	1.000	0.912	1.000	0.000	146.0	0.0
0	3	0	0.000	0.052	0.045	0.005	153.2	62.7
0	5	0	0.001	0.055	0.053	0.004	152.9	63.2
0	10	0	0.000	0.052	0.053	0.007	154.1	63.5
0	0	3	0.000	0.007	0.008	0.119	149.0	1.0
0	0	5	0.000	0.006	0.008	0.203	149.2	0.9
0	0	10	0.000	0.006	0.010	0.292	149.3	0.9
				$T = 500$				
0	0	0	0.050	0.050	0.050	0.003	255.8	103.2
3	0	0	0.812	0.020	0.031	0.000	245.2	33.1
5	0	0	0.999	0.010	0.049	0.000	246.0	0.1
10	0	0	1.000	0.641	0.997	0.000	246.0	0.0
0	3	0	0.000	0.051	0.053	0.003	256.6	105.7
0	5	0	0.001	0.047	0.050	0.004	254.7	105.9
0	10	0	0.000	0.052	0.047	0.003	254.0	105.8
0	0	3	0.000	0.005	0.006	0.083	249.0	1.0
0	0	5	0.000	0.003	0.006	0.152	249.2	0.9
0	0	10	0.000	0.004	0.008	0.262	249.3	0.8

Notes: ϕ refers to the break in the intercept, λ refers to the trend slope, and ψ refers to the break in the trend. Corr, Mean and SE refer to the correct selection frequency, the mean and the standard deviation of the estimated breakpoint.

Table 3: Simulation results for the new tests in model 1.

		$\rho = 1$						$\rho = 0.8$					
T	ϕ	t_1^1	t_2^1	F_{34}^1	Corr	Mean	SE	t_1^1	t_2^1	F_{34}^1	Corr	Mean	SE
		Case 1: $\phi_1 = \phi_2 = \phi_3 = \phi_4 = \phi$											
152	0	0.050	0.050	0.050	0.012	78.3	26.1	0.123	0.179	0.281	0.010	76.6	26.5
	3	0.035	0.046	0.046	0.828	76.0	3.1	0.085	0.158	0.251	0.798	75.9	3.6
	5	0.047	0.044	0.043	0.959	76.0	0.3	0.098	0.154	0.261	0.922	75.9	0.3
	10	0.048	0.046	0.051	0.972	76.0	0.2	0.112	0.161	0.270	0.952	75.9	0.3
300	0	0.050	0.050	0.050	0.007	152.4	51.6	0.336	0.483	0.786	0.007	152.2	52.3
	3	0.037	0.049	0.048	0.888	150.1	6.8	0.279	0.467	0.778	0.871	149.9	6.3
	5	0.034	0.044	0.049	0.996	150.0	0.1	0.283	0.460	0.775	0.991	150.0	0.1
	10	0.037	0.044	0.048	1,000	150.0	0.0	0.290	0.465	0.773	0.999	150.0	0.0
500	0	0.050	0.050	0.050	0.003	253.2	86.8	0.775	0.911	0.997	0.003	249.7	88.1
	3	0.045	0.050	0.045	0.914	249.8	11.1	0.751	0.902	0.997	0.887	249.8	12.0
	5	0.036	0.047	0.047	0.998	250.0	0.0	0.761	0.906	0.997	0.997	250.0	0.1
	10	0.042	0.045	0.045	1.000	250.0	0.0	0.754	0.899	0.999	1.000	250.0	0.0
		Case 2: $\phi_1 = \phi_3 = -\phi_2 = -\phi_4 = \phi$											
152	0	0.050	0.050	0.050	0.010	78.3	26.2	0.137	0.184	0.272	0.012	76.8	26.3
	3	0.030	0.043	0.037	0.861	76.0	3.0	0.096	0.152	0.239	0.845	76.0	3.2
	5	0.034	0.041	0.042	0.976	76.0	0.2	0.087	0.151	0.227	0.976	76.0	0.2
	10	0.029	0.045	0.046	0.992	76.0	0.1	0.092	0.159	0.234	0.987	76.0	0.2
300	0	0.050	0.050	0.050	0.006	152.0	51.5	0.342	0.513	0.799	0.006	151.3	51.8
	3	0.036	0.047	0.050	0.900	150.0	5.5	0.295	0.475	0.782	0.881	150.1	6.5
	5	0.034	0.050	0.056	0.997	150.0	0.1	0.296	0.478	0.778	0.995	150.0	0.1
	10	0.038	0.049	0.051	1.000	150.0	0.0	0.290	0.481	0.784	1.000	150.0	0.0
500	0	0.050	0.050	0.050	0.003	253.7	87.2	0.750	0.919	0.998	0.004	248.4	87.0
	3	0.036	0.054	0.050	0.913	250.0	10.0	0.725	0.908	0.997	0.890	250.5	11.2
	5	0.036	0.053	0.051	0.999	250.0	0.0	0.726	0.915	0.996	0.997	250.0	0.1
	10	0.035	0.055	0.052	1.000	250.0	0.0	0.740	0.915	0.999	1.000	250.0	0.0

Notes: See Table 2 for an explanation of the features of the table.

Table 4: Simulation results for the new tests in model 2.

			$\rho = 1$						$\rho = 0.8$					
T	ϕ	ψ	t_1^2	t_2^2	F_{34}^2	Corr	Mean	SE	t_1^2	t_2^2	F_{34}^2	Corr	Mean	SE
Case 1: $\phi_1 = \phi_2 = \phi_3 = \phi_4 = \phi$														
152	0	0	0.050	0.050	0.050	0.012	79.7	25.8	0.099	0.165	0.325	0.009	77.8	25.9
		5	0.036	0.043	0.055	0.982	76.0	0.1	0.076	0.147	0.283	0.985	76.0	0.1
		10	0.047	0.049	0.055	0.638	75.6	0.5	0.090	0.146	0.290	0.604	75.6	0.5
	5	0	0.042	0.041	0.051	0.998	76.0	0.1	0.082	0.150	0.279	0.994	76.0	0.1
		5	0.036	0.041	0.051	0.999	76.0	0.0	0.077	0.146	0.278	0.997	76.0	0.1
		10	0.043	0.043	0.048	1.000	76.0	0.0	0.084	0.142	0.282	0.999	76.0	0.0
	10	0	0.037	0.044	0.054	1.000	76.0	0.0	0.077	0.132	0.293	1.000	76.0	0.0
		5	0.037	0.043	0.052	1.000	76.0	0.0	0.073	0.142	0.274	1.000	76.0	0.0
		10	0.041	0.040	0.049	1.000	76.0	0.0	0.079	0.149	0.280	1.000	76.0	0.0
300	0	0	0.050	0.050	0.050	0.007	155.0	51.1	0.250	0.461	0.823	0.006	150.9	52.3
		5	0.048	0.038	0.057	0.648	150.4	0.5	0.227	0.431	0.786	0.839	150.2	0.4
		10	0.043	0.042	0.057	0.999	150.0	0.0	0.231	0.431	0.808	0.993	150.0	0.1
	5	0	0.044	0.044	0.058	1.000	150.0	0.0	0.219	0.445	0.819	1.000	150.0	0.0
		5	0.043	0.041	0.062	1.000	150.0	0.0	0.211	0.439	0.802	1.000	150.0	0.0
		10	0.037	0.039	0.058	1.000	150.0	0.0	0.214	0.447	0.764	1.000	150.0	0.0
	10	0	0.043	0.044	0.055	1.000	150.0	0.0	0.222	0.455	0.758	1.000	150.0	0.0
		5	0.044	0.043	0.063	1.000	150.0	0.0	0.215	0.438	0.768	1.000	150.0	0.0
		10	0.047	0.042	0.052	1.000	150.0	0.0	0.215	0.456	0.759	1.000	150.0	0.0
500	0	0	0.050	0.050	0.050	0.004	259.3	84.6	0.652	0.910	0.998	0.005	250.0	86.3
		5	0.059	0.046	0.055	0.042	251.0	0.2	0.612	0.863	0.995	0.186	250.8	0.4
		10	0.055	0.046	0.054	0.992	250.0	0.1	0.641	0.907	0.999	1.000	250.0	0.0
	5	0	0.061	0.046	0.053	1.000	250.0	0.0	0.651	0.903	0.999	1.000	250.0	0.0
		5	0.055	0.046	0.049	1.000	250.0	0.0	0.649	0.903	0.997	1.000	250.0	0.0
		10	0.060	0.050	0.056	1.000	250.0	0.0	0.659	0.900	0.996	1.000	250.0	0.0
	10	0	0.054	0.051	0.055	1.000	250.0	0.0	0.651	0.907	0.999	1.000	250.0	0.0
		5	0.057	0.053	0.054	1.000	250.0	0.0	0.641	0.904	0.999	1.000	250.0	0.0
		10	0.059	0.047	0.058	1.000	250.0	0.0	0.655	0.905	0.997	1.000	250.0	0.0

Continued overleaf

Table 4: Continued.

			$\rho = 1$						$\rho = 0.8$					
T	ϕ	ψ	t_1^2	t_2^2	F_{34}^2	Corr	Mean	SE	t_1^2	t_2^2	F_{34}^2	Corr	Mean	SE
						Case 2: $\phi_1 = \phi_3 = -\phi_2 = -\phi_4 = \phi$								
152	0	0	0.050	0.050	0.050	0.012	79.5	26.0	0.104	0.197	0.309	0.013	78.0	26.1
		5	0.042	0.047	0.047	0.985	76.0	0.1	0.084	0.169	0.265	0.980	76.0	0.1
		10	0.047	0.052	0.047	0.632	75.6	0.5	0.082	0.170	0.274	0.610	75.6	0.5
	5	0	0.038	0.044	0.039	1.000	76.0	0.0	0.081	0.177	0.263	0.999	76.0	0.0
		5	0.039	0.042	0.039	1.000	76.0	0.0	0.074	0.170	0.267	1.000	76.0	0.0
		10	0.036	0.047	0.042	0.999	76.0	0.0	0.073	0.159	0.262	1.000	76.0	0.0
	10	0	0.039	0.045	0.044	1.000	76.0	0.0	0.083	0.165	0.263	1.000	76.0	0.0
		5	0.035	0.043	0.043	1.000	76.0	0.0	0.081	0.163	0.256	1.000	76.0	0.0
		10	0.038	0.041	0.044	1.000	76.0	0.0	0.088	0.163	0.276	1.000	76.0	0.0
300	0	0	0.050	0.050	0.050	0.007	155.2	51.6	0.224	0.497	0.777	0.006	150.3	52.7
		5	0.058	0.050	0.049	0.646	150.4	0.5	0.224	0.466	0.755	0.839	150.2	0.4
		10	0.052	0.049	0.052	0.998	150.0	0.1	0.215	0.479	0.757	0.994	150.0	0.1
	5	0	0.052	0.048	0.055	1.000	150.0	0.0	0.204	0.471	0.762	1.000	150.0	0.0
		5	0.052	0.049	0.049	1.000	150.0	0.0	0.214	0.489	0.761	1.000	150.0	0.0
		10	0.049	0.051	0.053	1.000	150.0	0.0	0.199	0.485	0.770	1.000	150.0	0.0
	10	0	0.054	0.054	0.053	1.000	150.0	0.0	0.210	0.476	0.771	1.000	150.0	0.0
		5	0.050	0.050	0.053	1.000	150.0	0.0	0.204	0.490	0.778	1.000	150.0	0.0
		10	0.050	0.047	0.050	1.000	150.0	0.0	0.206	0.480	0.768	1.000	150.0	0.0
500	0	0	0.050	0.050	0.050	0.004	257.1	85.8	0.644	0.905	0.998	0.004	251.1	86.6
		5	0.047	0.050	0.042	0.040	251.0	0.2	0.604	0.866	0.990	0.189	250.8	0.4
		10	0.048	0.044	0.043	0.993	250.0	0.1	0.620	0.912	0.997	1.000	250.0	0.0
	5	0	0.050	0.050	0.044	1.000	250.0	0.0	0.622	0.913	0.996	1.000	250.0	0.0
		5	0.050	0.050	0.049	1.000	250.0	0.0	0.635	0.909	0.996	1.000	250.0	0.0
		10	0.049	0.044	0.051	1.000	250.0	0.0	0.626	0.903	0.996	1.000	250.0	0.0
	10	0	0.052	0.046	0.054	1.000	250.0	0.0	0.630	0.902	0.997	1.000	250.0	0.0
		5	0.053	0.048	0.053	1.000	250.0	0.0	0.618	0.905	0.997	1.000	250.0	0.0
		10	0.050	0.042	0.050	1.000	250.0	0.0	0.626	0.909	0.996	1.000	250.0	0.0

Notes: ψ refers to the size of the break in the trend. See Table 2 for an explanation of the remaining features of the table.

Table 5: Descriptive statistics.

Country	Mean	SE	R^2
Austria	0.842	13.784	0.591
Belgium	0.405	10.940	0.683
Finland	0.862	13.391	0.673
France	0.252	12.937	0.798
Germany	0.460	5.878	0.724
Greece	0.249	10.960	0.624
Ireland	2.038	9.071	0.755
Italy	0.361	15.539	0.747
Netherlands	0.481	11.538	0.667
Norway	0.150	10.206	0.705
Spain	0.460	12.351	0.809
Sweden	0.585	20.816	0.822
United Kingdom	0.149	3.751	0.739

Notes: Mean and SE refer the mean and standard deviation of $100 \cdot \Delta y_t$, while R^2 refers to the R^2-measure in a regression of Δy_t onto four quarterly dummy variables.

Table 6: Empirical results from the Dickey and Fuller (1979) and Hylleberg *et al.* (1990) tests.

	DF		HEGY			
Country	t_1^{DF}	$\hat{p}$	t_1^{HEGY}	t_2^{HEGY}	F_{34}^{HEGY}	$\hat{p}$
Austria	-1.667	4	-1.662	-1.281	23.298^{***}	4
Belgium	-2.093	3	-0.601	-1.601^{*}	4.768^{***}	4
Finland	-2.034	4	-1.335	-1.147	2.377^{*}	4
France	-2.045	5	-1.531	-1.609^{*}	3.877^{**}	4
Germany	-1.857	3	-3.086	-1.948^{**}	8.503^{***}	1
Greece	-3.076	3	-1.035	-1.765^{*}	4.230^{**}	4
Ireland	-1.757	4	-1.773	-1.450	3.356^{**}	4
Italy	-2.563	4	-1.123	-1.637^{*}	1.507	4
Netherlands	-3.325	3	-0.532	-1.555	5.412^{***}	4
Norway	-2.997	4	-2.977	-1.558	2.610^{*}	1
Spain	-2.134	3	-1.671	-0.862	2.373^{*}	4
Sweden	-2.546	4	-1.728	-0.835	3.169^{**}	4
United Kingdom	-1.586	3	-1.575	-1.811^{*}	4.944^{***}	1

Notes: $\hat{p}$ refers the number of lag augmentations as estimated by the general-to-specific procedure of Hall (1994). The maximum lag length is set to five. The superscripts *, ** and *** denote significance at the 10%, 5% and 1% level, respectively. DF and HEDG refer to the tests of Dickey and Fuller (1979) and Hylleberg *et al.* (1990), respectively. t_1^{DF} and t_1^{HEGY} test the null of a nonseasonal unit root, t_2^{HEGY} tests the null of a biannual unit root, and F_{34}^{HEGY} tests the null of annual unit roots.

Table 7: Empirical results for the new tests.

Country	t_1^m	t_2^m	F_{34}^m	Rejections	$\hat{p}$	$\hat{T}^\circ$	$\hat{\tau}^\circ$
			Model 1, $m = 1$				
Austria	-3.063	-4.717^{***}	22.654^{***}	H_0^2, H_0^3	0	1985:4	0.33
Belgium	-3.078	-3.441^{*}	16.106^{***}	H_0^2, H_0^3	0	1991:4	0.53
Finland	-1.376	-3.807^{***}	13.861^{***}	H_0^2, H_0^3	0	1990:4	0.49
France	-2.632	-4.671^{***}	21.656^{***}	H_0^2, H_0^3	0	1992:2	0.54
Germany	-2.388	-5.519^{***}	45.267^{***}	H_0^2, H_0^3	0	1992:2	0.54
Greece	-3.820^{**}	-5.638^{***}	33.369^{***}	H_0^1, H_0^2, H_0^3	0	1992:3	0.55
Ireland	-2.783	-5.675^{***}	47.634^{***}	H_0^2, H_0^3	0	1996:3	0.69
Italy	-3.623	-5.177^{***}	80.514^{***}	H_0^2, H_0^3	0	1996:4	0.69
Netherlands	-4.247^{**}	-4.462^{***}	47.459^{***}	H_0^1, H_0^2, H_0^3	0	1995:4	0.66
Norway	-2.774	-3.887^{**}	19.171^{***}	H_0^2, H_0^3	0	1996:3	0.69
Spain	-3.355	-5.131^{***}	33.307^{***}	H_0^2, H_0^3	0	1992:2	0.54
Sweden	-3.273	-3.559^{*}	15.265^{***}	H_0^2, H_0^3	0	1994:2	0.61
United Kingdom	-3.083	-5.371^{***}	65.741^{***}	H_0^1, H_0^2, H_0^3	0	1986:3	0.36
			Model 2, $m = 2$				
Austria	-2.326	-2.519	26.863^{***}	H_0^3	1	1999:3	0.79
Belgium	-2.275	-8.158^{***}	5.668	H_0^2	4	1987:4	0.39
Finland	-2.405	-0.553	1.506	–	5	1999:4	0.79
France	-1.588	-3.756^{**}	31.206^{***}	H_0^2, H_0^3	2	1999:2	0.78
Germany	-2.542	-5.563^{***}	50.258^{***}	H_0^2, H_0^3	0	1992:2	0.54
Greece	-3.410	-7.498^{***}	15.585^{***}	H_0^2, H_0^3	3	1992:3	0.55
Ireland	0.499	-6.064^{***}	30.378^{***}	H_0^2, H_0^3	0	1999:4	0.79
Italy	-4.264^{**}	-12.574^{***}	77.089^{***}	H_0^1, H_0^2, H_0^3	2	1999:4	0.79
Netherlands	-2.298	-1.593	13.943^{**}	H_0^3	5	1999:4	0.79
Norway	-3.244	-5.622^{***}	22.378^{***}	H_0^2, H_0^3	0	1996.1	0.67
Spain	-3.301	-5.021^{***}	31.008^{***}	H_0^2, H_0^3	0	1992:2	0.54
Sweden	-1.706	-2.816	11.369^{***}	H_0^3	1	1999:4	0.79
United Kingdom	-3.853	-3.619^{**}	16.608^{***}	H_0^2, H_0^3	1	1986:1	0.34

Notes: $\hat{\tau}^\circ$ refers to the estimated break fraction with $\hat{T}^\circ$ being the associated break date. See Table 6 for an explanation of the remaining features.