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POSITIONAL PREFERENCES IN TIME AND SPACE: IMPLICATIONS FOR OPTIMAL INCOME TAXATION**

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ABSTRACT

This paper concerns optimal nonlinear taxation in an OLG model with two ability-types, where people care about their own consumption relative to (i) other people's current consumption, (ii) own past consumption, and (iii) other people's past consumption. We show that intertemporal consumption comparisons affect the marginal income tax structure in the same qualitative way as comparisons based on other people's current consumption. Based on available empirical estimates, comparisons with other people's current and previous consumption tend to substantially increase the optimal marginal labor income tax rates, while they may either increase or decrease the optimal marginal capital income tax rates.

Keywords: Optimal income taxation, asymmetric information, relative consumption, status, habit formation, positional goods.

JEL Classification: D62, H21, H23, H41.

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1. INTRODUCTION

A rapidly growing body of evidence suggests that people have positional preferences in the sense that they derive utility from their own consumption relative to that of others.¹ Alongside this development, a corresponding literature dealing with optimal policy responses to positional concerns has evolved,² showing that such concerns may have a substantial effect on the incentive structure underlying public policy. There is also a large literature suggesting that various forms of habit formation can explain several empirical patterns that are difficult to reconcile with conventional preferences.³ Yet, all earlier studies on optimal second-best policy responses to positional concerns that we are aware of assume that people only make “atemporal” consumption comparisons, by valuing their own current consumption relative to the current consumption by other people. A much more general approach has recently been presented by Rayo and Becker (2007): according to their evolutionary model,⁴ selfish genes would prefer that the humans they belong to are simultaneously motivated by their own current consumption relative to (i) their own past consumption, (ii) other people’s current consumption, and (iii) other people’s past consumption. In the macroeconomic literature of dynamic consumption behavior, (i) corresponds to what is typically denoted *Habit formation* (sometimes denoted *Internal habit formation*), (ii) corresponds to *Keeping up with the*

¹ For happiness research evidence, see, e.g., Easterlin (2001), Blanchflower and Oswald (2005), Ferrer-i-Carbonell (2005), and Luttmer (2005). Stevenson and Wolfers (2008) constitute a recent exception in the happiness literature, claiming that the role of relative income is overstated. For questionnaire-based approaches, see, e.g., Johansson-Stenman et al. (2002), Solnick and Hemenway (2005), and Carlsson et al. (2007). Various kinds of physiological and health-related evidence are provided by Marmot (2004); for a more recent example, see Daly and Wilson (2009) who found that suicide rates seem to depend on relative concerns. There is also recent evidence from brain science, e.g., Fliessbach et al. (2007).

² Earlier studies address a variety of issues such as optimal taxation, public good provision, social insurance, growth, environmental externalities, and stabilization policy; see, e.g., Boskin and Sheshinski (1978), Layard (1980), Ng (1987), Tuomala (1990), Blomquist (1993), Corneo and Jeanne (1997, 2001), Ireland (2001), Brekke and Howarth (2002), Abel (2005), Aronsson and Johansson-Stenman (2008, forthcoming), and Wendner and Goulder (2008). Clark et al. (2008) provide a good overview of both the empirical evidence and economic implications of relative consumption concerns; see also Frank (1999, 2005, 2007, 2008) for extensive and illuminating informal discussions of relative consumption concerns and how the society should deal with them.

³This includes various kinds of asset pricing puzzles, such as the equity premium puzzle; see, e.g., Abel (1990), Constantinides (1990), Campbell and Cochrane (1999), Chan and Kogan (2003), and Díaz et al. (2003).

⁴ See Saad (2007) for a more general treatment of the evolutionary basis for consumer behavior, including conspicuous consumption.

Joneses, while (iii) corresponds to *Catching up with the Joneses* (sometimes denoted *External habit formation*).⁵ The present paper takes these three types of consumption comparisons as a point of departure in a study of optimal income taxation in a dynamic economy.

The study of optimal taxation in economies where relative consumption matters for individual utility is typically based on static models with linear tax instruments. The present paper, in contrast, is based on an overlapping generations (OLG) model, where individuals differ in ability and the set of available tax instruments consists of nonlinear taxes on labor and capital income. This means that the tax instruments considered here are based on informational limitations; not on any other *a priori* restriction (such as linearity). Therefore, our framework enables us to capture that the optimal income tax responses to positional concerns may involve purely corrective as well as redistributive elements. Furthermore, a dynamic model allows us to explore intertemporal aspects of consumption comparisons as well as provides a natural framework for studying capital income taxation. The latter is important not least due to the difficulties in explaining the widespread use of capital taxes with conventional public economics models. Earlier research shows that relative consumption concerns may motivate such taxes (Aronsson and Johansson-Stenman, forthcoming), and one might perhaps conjecture such concerns to be particularly important when the concept of relative consumption has more than one dimension, as we assume here.

Only a few earlier studies deal with optimal nonlinear income taxation in the context of positional preferences, and almost all of them have in common that they use static models.⁶ To our knowledge, the only exception is Aronsson and Johansson-Stenman (forthcoming), who consider optimal income taxation in an OLG model where each consumer exhibits positional preferences for consumption in the sense of comparing his/her own current consumption with other people's current consumption both when young and when old. However, it is important to emphasize that although their study is based on a dynamic model,

⁵ This literature rarely analyzes the optimal policy responses related to the externalities induced by relative consumption concerns. Ljungqvist and Uhlig (2000) constitute a noteworthy exception. They analyze, in a first best representative consumer economy with external shocks, how the externalities due to relative consumption concerns call for an optimal tax policy that affects the economy counter-cyclically. Gomez (2006) is another example, using a representative consumer model of endogenous growth with external habit formation.

⁶ See, e.g., Oswald (1983), Tuomala (1990), Ireland (2001), and Aronsson and Johansson-Stenman (2008).

the consumption comparisons still remain atemporal in the sense that the measure of reference consumption facing each individual solely depends on other people's current consumption, i.e., it is solely based on a Keeping up with the Joneses framework.

The present paper, in contrast, addresses the implications of such atemporal comparisons for optimal income taxation simultaneously with the implications of relative consumption comparisons over time. These extensions are important. In addition to the empirical evidence for between-people comparisons mentioned above, there is evidence suggesting that people also make comparisons with their own past consumption (e.g., Loewenstein and Sicherman, 1991; Frank and Hutchens, 1993). It also makes intuitive sense that old people compare their own consumption with several different reference levels, including what they recall about their own and others' consumption when they were young. When growing up, most people are also likely to receive information from parents and grandparents about the consumption (and other living conditions) characterizing earlier generations. Such comparisons are also consistent with the empirical pattern of some financial puzzles such as the equity premium puzzle (e.g., Campbell and Cochrane, 1999) and, as mentioned above, they are in line with recent research based on evolutionary models.

Our results show that relative comparisons with one's own past consumption (Internal habit formation) do not directly affect the policy *rules* for marginal income taxation (although they may, of course, influence the *levels* of marginal income tax rates). The intuition is that such comparisons do not generate any externalities. However, positional concerns governed by comparisons with other people's current and past consumption give rise to externalities and will, therefore, also directly affect the incentive structure underlying marginal income taxation. Specifically, we show that optimal tax responses are associated with two distinct motives for public policy: the government wants to (i) internalize positional externalities, and (ii) relax the self-selection constraint by exploiting that a potential mimicker may either be more or less positional than the mimicked agent. The former mechanism works to increase the marginal labor income tax rates, independently of whether individuals compare their own current consumption with other people's current or past consumption (or use a combination of these two reference measures). This is so because both types of comparisons imply that each individual imposes negative externalities on others; either at present or in the future. We also show how the marginal capital income tax structure is governed by differences in

positionality over the individual life-cycle, where the relevant measure of reference consumption is again based on both the current and past consumption of others.

In general, positional concerns governed by other people's past consumption give rise to much more complex policy responses than comparisons based on other people's current consumption. This is due to the fact that consumption comparisons over time give rise to an intertemporal chain reaction with welfare effects in the entire future, whereas comparisons with other people's current consumption only lead to "atemporal externalities." We can nevertheless derive strong results for a natural benchmark case, implying that relative consumption comparisons over time (based on the Catching up with the Joneses preferences) give rise to the same qualitative marginal labor and capital income tax rate responses as comparisons with other people's current consumption (based on the Keeping up with the Joneses preferences). Moreover, we illustrate with a particular Cobb-Douglas functional form and show, based on parameter estimates from the literature, that positional preferences of both the Keeping up with the Joneses and Catching up with the Joneses types substantially increase the optimal marginal labor income tax rates for both ability-types.

The outline of the study is as follows: Section 2 presents the model and the outcome of private optimization, while Section 3 presents the optimal tax problem faced by the government. The results are presented for the most general formulation of the model in Section 4, and for the somewhat more restricted version in Section 5. Section 6 illustrates the results based on a Cobb-Douglas functional form, whereas Section 7 summarizes and concludes the paper; proofs are presented in the appendix.

2. CONSUMERS, FIRMS, AND MARKET EQUILIBRIUM

We start this section by describing the OLG framework and people's preferences, followed by the definition of some useful measures of the extent to which people care about relative consumption. We then present the individual optimality conditions for labor supply and savings, followed by the corresponding profit maximization conditions for the firms and the condition for market equilibrium.

2.1 The OLG framework and positional preferences

Consider an OLG model where each individual lives for two periods and works during the first but not during the second. Since each individual only works during the first period of life, there is no evolution of productivity over time for a single individual, as in Kocherlakota (2005), although we allow for technical progress (discussed subsequently) that makes labor productivity increase over time. There are two types of individuals in each time period, where the low-ability type (type 1) is less productive than the high-ability type (type 2). The number of individuals of ability-type i who were born at the beginning of period t is denoted n_t^i . Each individual cares about his/her consumption when young and when old, c_t^i and x_{t+1}^i , and his/her leisure when young, z_t^i , given by a time endowment, H , less the hours of work, l_t^i (when old, all available time is leisure). For further use, we define the average consumption in the economy as a whole in period t as $\bar{c}_t = \left[\sum_i n_t^i c_t^i + \sum_i n_{t-1}^i x_t^i \right] / \sum_i \left[n_t^i + n_{t-1}^i \right]$.

People also care about their own consumption relative to that of others.⁷ In accordance with the bulk of earlier comparable literature, we focus on *difference* comparisons, where relative consumption is defined by the difference between the individual's own consumption and a measure of reference consumption.⁸ The appropriate measure of reference consumption at the individual level is, of course, an empirical question; yet, as indicated above, there is very little information available. Our approach is to follow the recent contribution by Rayo and Becker (2007), who argue in the context of an evolutionary model of happiness that the reference point of an individual might be determined by three components: (i) other people's current consumption, (ii) his/her own past consumption, and (iii) other people's past

⁷ We do not attempt to explain *why* people care about relative consumption. Therefore, while we share the view that signaling of some attractive characteristic constitutes a likely important reason for why people tend to care about relative consumption (see, e.g., Ireland, 2001, and also Sobel, forthcoming, for a more general treatment of signaling games), we choose to follow the considerably simpler modeling strategy where people's preferences depend directly on relative consumption. We also follow earlier comparable literature in assuming that people do not care about their relative leisure; see Aronsson and Johansson-Stenman (2009) for an analysis of the case where also relative leisure matters.

⁸ See, e.g., Akerlof (1997), Corneo and Jeanne (1997), Ljungqvist and Uhlig (2000), Bowles and Park (2005), and Carlsson et al. (2007). Alternative approaches include ratio comparisons (Boskin and Sheshinski, 1978; Layard, 1980; Abel, 2005; Wendner and Goulder, 2008) and comparisons of ordinal rank (Frank, 1985; Hopkins and Kornienko, 2004, 2009). Dupor and Liu (2003) consider a specific flexible functional form that includes the difference comparison and ratio comparison approaches as special cases. All of these social comparison models belong to the more general class of models with interdependent preferences; cf., e.g., Sobel (2005).

consumption. In the context of our model, we interpret these three components such that people care about three different kinds of relative consumption: their own current consumption relative to (i) the current average consumption when young and when old, i.e., $c_t^i - \bar{c}_t$ and $x_{t+1}^i - \bar{c}_{t+1}$; (ii) their own consumption one period earlier, i.e., $x_{t+1}^i - c_t^i$; and (iii) the average consumption one period earlier when young and when old, i.e., $c_t^i - \bar{c}_{t-1}$ and $x_{t+1}^i - \bar{c}_t$.⁹

The utility function of ability-type i born in the beginning of period t can then be written as

$$\begin{aligned}
 (1) \quad U_t^i &= V_t^i(c_t^i, z_t^i, x_{t+1}^i, c_t^i - \bar{c}_t, x_{t+1}^i - \bar{c}_{t+1}, x_{t+1}^i - c_t^i, c_t^i - \bar{c}_{t-1}, x_{t+1}^i - \bar{c}_t) \\
 &= v_t^i(c_t^i, z_t^i, x_{t+1}^i, c_t^i - \bar{c}_t, x_{t+1}^i - \bar{c}_{t+1}, c_t^i - \bar{c}_{t-1}, x_{t+1}^i - \bar{c}_t) \\
 &= u_t^i(c_t^i, z_t^i, x_{t+1}^i, \bar{c}_{t-1}, \bar{c}_t, \bar{c}_{t+1})
 \end{aligned}$$

The first line of equation (1) is expressed in terms of the five consumption differences described above, as well as in terms of leisure and private consumption when young and when old, respectively. However, since c_t^i and x_{t+1}^i are decision variables of the individual, we can without loss of generality rewrite this utility formulation as the "reduced form" function on the second line, although the partial derivatives will now have a more complex interpretation than on the first line. For instance, the partial derivative of $v_t^i(\cdot)$ with respect to c_t^i reflects both the direct utility effect of increased absolute consumption when young and the (presumably negative) utility effect due to lower relative consumption when old compared to when young.¹⁰ Therefore, all analytical results derived in a model where individuals do not compare their own current and past consumption will continue to hold also in the case where people make such comparisons. Intuitively, people will internalize such comparisons perfectly.

⁹ Although one can easily imagine that each individual compares himself/herself more with some people than with others, we follow the bulk of earlier comparable literature by using the average consumption as a basis for the reference points. Aronsson and Johansson-Stenman (forthcoming) also consider alternative measures of reference consumption based on within-generation and upward comparisons, respectively, and find policy responses that are qualitatively similar to those that follow if the reference point is based solely on the average consumption; yet with a modified interpretation to reflect the type of comparison underlying the analysis.

¹⁰ On the second line, the effect of $x_{t+1}^i - c_t^i$ on utility is hence embedded in the effects of c_t^i and x_{t+1}^i .

The third line contains the most general utility formulation and resembles a classical externality problem. Here, we do not specify anything regarding the structure of the social comparisons, beyond that others' consumption levels cause negative externalities. As will be demonstrated, for some results we do not need any stronger assumptions regarding the preference structure. Yet, we need the more restrictive utility formulation based on the function $v_t^i(\cdot)$, where we specify that people care about additive comparisons, to establish a relationship between, on the one hand, the optimal tax policy and, on the other, the degree to which the utility gain from higher consumption is associated with increased relative consumption. The definition of such measures is the issue to which we turn next.

2.2 The degree of current versus intertemporal consumption positionality

Since much of the subsequent analysis is focused on the relative consumption concerns, it is useful to introduce measures of the degree to which such concerns matter for each individual. By using $\Delta_t^{i,c} = c_t^i - \bar{c}_t$, $\Delta_{t+1}^{i,x} = x_{t+1}^i - \bar{c}_{t+1}$, $\Omega_t^{i,c} = c_t^i - \bar{c}_{t-1}$, and $\Omega_{t+1}^{i,x} = x_{t+1}^i - \bar{c}_t$ as short notations for the four differences in the function $v_t^i(\cdot)$ in equation (1), we can define the *degree of current consumption positionality* when young and when old, respectively, as

$$(2a) \quad \alpha_t^{i,c} = \frac{v_{t,\Delta^c}^i}{v_{t,\Delta^c}^i + v_{t,\Omega^c}^i + v_{t,c}^i},$$

$$(2b) \quad \alpha_{t+1}^{i,x} = \frac{v_{t,\Delta^x}^i}{v_{t,\Delta^x}^i + v_{t,\Omega^x}^i + v_{t,x}^i},$$

where the subindex indicates partial derivative, i.e. $v_{t,c}^i = \partial v_t^i(\cdot) / \partial c_t^i$ and similarly for the other terms. The variable $\alpha_t^{i,c}$ can be interpreted as the fraction of the overall utility increase from an additional dollar spent when young in period t that is due to the increased consumption relative to the average consumption in period t , whereas $\alpha_{t+1}^{i,x}$ has a corresponding interpretation when old in period $t+1$. By analogy, we can define the *degree of intertemporal consumption positionality* when young and when old, respectively, as

$$(3a) \quad \beta_t^{i,c} = \frac{v_{t,\Omega^c}^i}{v_{t,\Delta^c}^i + v_{t,\Omega^c}^i + v_{t,c}^i},$$

$$(3b) \quad \beta_{t+1}^{i,x} = \frac{v_{t,\Omega^x}^i}{v_{t,\Delta^x}^i + v_{t,\Omega^x}^i + v_{t,x}^i}.$$

The variables $\beta_t^{i,c}$ and $\beta_{t+1}^{i,x}$ reflect the fraction of the overall utility increase from an additional dollar spent in period t and $t+1$ (i.e., when young and when old), respectively, that is due to the increased consumption relative to other people's past consumption. We assume that $0 < \alpha_t^{i,c}, \alpha_{t+1}^{i,x}, \beta_t^{i,c}, \beta_{t+1}^{i,x} < 1$ for all t .

Let us next define the notions of *the average degree of current consumption positionality* and *the average degree of intertemporal consumption positionality*, which are given by

$$(4a) \quad \bar{\alpha}_t = \sum_i \alpha_t^{i,x} \frac{n_{t-1}^i}{N_t} + \sum_i \alpha_t^{i,c} \frac{n_t^i}{N_t} \in (0,1),$$

$$(4b) \quad \bar{\beta}_t = \sum_i \beta_t^{i,x} \frac{n_{t-1}^i}{N_t} + \sum_i \beta_t^{i,c} \frac{n_t^i}{N_t} \in (0,1),$$

respectively, where $N_t = \sum_i [n_{t-1}^i + n_t^i]$. Note that both $\bar{\alpha}_t$ and $\bar{\beta}_t$ are measured among those alive in period t .

2.3 The optimal conditions for individuals and firms and market equilibrium

The individual budget constraints are given by

$$(5a) \quad w_t^i l_t^i - T_t(w_t^i l_t^i) - s_t^i = c_t^i,$$

$$(5b) \quad s_t^i (1 + r_{t+1}) - \Phi_{t+1}(s_t^i r_{t+1}) = x_{t+1}^i,$$

where w_t^i is the before-tax wage rate, implying that $w_t^i l_t^i$ is the before-tax labor income; s_t^i is savings, r_{t+1} is the market interest rate, and $T_t(\cdot)$ and $\Phi_{t+1}(\cdot)$ denote the payments of labor income and capital income taxes, respectively. Thus, consumption levels when young are given by gross labor income net of labor income taxes and savings, whereas consumption

levels when old are given by the sum of savings and capital income net of capital income taxes.

We assume that each individual treats the average consumption as exogenous. To be more specific, and with reference to equation (1) above, this means that ability-type i of generation t treats \bar{c}_{t-1} , \bar{c}_t , and \bar{c}_{t+1} as exogenous. The first order conditions for the hours of work and savings can then be written as

$$(6) \quad u_{t,c}^i w_t^i \left[1 - T_t'(w_t^i l_t^i) \right] - u_{t,z}^i = 0,$$

$$(7) \quad -u_{t,c}^i + u_{t,x}^i \left[1 + r_{t+1} \left[1 - \Phi_{t+1}'(s_t^i r_{t+1}) \right] \right] = 0,$$

in which $u_{t,c}^i = \partial u_t^i / \partial c_t^i$, $u_{t,z}^i = \partial u_t^i / \partial z_t^i$, and $u_{t,x}^i = \partial u_t^i / \partial x_{t+1}^i$, while $T_t'(w_t^i l_t^i)$ and $\Phi_{t+1}'(s_t^i r_{t+1})$ are the marginal labor income tax rate and the marginal capital income tax rate, respectively.

The production sector consists of identical competitive firms producing a homogenous good with constant returns to scale; the number of firms is normalized to one for notational convenience. The production function is given by

$$(8) \quad F(L_t^1, L_t^2, K_t; t) = g(\theta^1 L_t^1 + \theta^2 L_t^2, K_t; t),$$

where $L_t^i = n_t^i l_t^i$ is the total number of hours of work supplied by ability-type i in period t , and K_t is the capital stock in period t ; θ^1 and θ^2 are positive constants. The direct time-dependency implies that we allow for exogenous technological change. The firm obeys the necessary optimality conditions

$$(9) \quad F_{L^i}(L_t^1, L_t^2, K_t; t) = \frac{\partial g}{\partial (\theta^1 L_t^1 + \theta^2 L_t^2)} \theta^i = w_t^i \quad \text{for } i=1, 2,$$

$$(10) \quad F_K(L_t^1, L_t^2, K_t; t) = \frac{\partial g}{\partial K_t} = r_t.$$

Note that equation (9) implies that the relative wage rate between the two ability-types is constant both within each period and over time, i.e. $w_t^1 / w_t^2 = \theta^1 / \theta^2 = \phi$, where ϕ is a constant.¹¹

3. THE SOCIAL OPTIMIZATION PROBLEM

In this section, we begin by specifying the social objective function. Then we will characterize the self-selection constraint, i.e., that the high-ability type should be prevented from mimicking the low-ability type in each period, as well as the overall resource constraint. Finally, we form the Lagrangean corresponding to the optimization problem and present the associated first order conditions for an interior solution.

We assume that the government faces a general social welfare function as follows:

$$(11) \quad W = W(n_0^1 U_0^1, n_0^2 U_0^2, n_1^1 U_1^1, n_1^2 U_1^2, \dots),$$

which is increasing in each argument. Since the optimum conditions are expressed for any such social welfare function, they are necessary optimum conditions for a Pareto efficient allocation.¹²

Following the convention in earlier literature on optimal nonlinear taxation, we assume that the government is able to observe income, that ability is private information, and that the government wants to redistribute from the high-ability to the low-ability type. Therefore, one would like to prevent the high-ability type from pretending to be a low-ability type in order to gain from the redistribution. The self-selection constraint that may bind then becomes

$$(12) \quad U_t^2 = u_t^2(c_t^2, z_t^2, x_{t+1}^2, \bar{c}_{t-1}, \bar{c}_t, \bar{c}_{t+1}) \geq u_t^2(c_t^1, H - \phi l_t^1, x_{t+1}^1, \bar{c}_{t-1}, \bar{c}_t, \bar{c}_{t+1}) = \hat{U}_t^2,$$

¹¹ This simplifying assumption is made solely for analytical convenience, as endogenous relative wage rates are not particularly important for the qualitative results derived below.

¹² A similar formulation is used by Pirttilä and Tuomala (2001), although they additionally assume that the social welfare function is utilitarian within each generation.

where $\phi = w_t^1 / w_t^2$ is the wage ratio, which is a constant by the assumptions made earlier. The expression on the right-hand side of the weak inequality in (12) is the utility of the mimicker. Although the mimicker enjoys the same consumption as the low-ability type in each period, he/she enjoys more leisure (as the mimicker is more productive than the low-ability type).¹³

Since $T_t(\cdot)$ is a general labor income tax that can be used to implement any desired combination of l_t^1 , c_t^1 , l_t^2 , and c_t^2 , given the savings chosen by each ability-type, we will use l_t^1 , c_t^1 , l_t^2 , and c_t^2 , instead of the parameters of the labor income tax function, as direct decision variables in the social resource allocation problem. Similarly, the capital income tax, $\Phi_{t+1}(\cdot)$, can be used to implement any desired combination of c_t^1 , x_{t+1}^1 , c_t^2 , x_{t+1}^2 , and K_{t+1} , given the labor income of each individual. Therefore, instead of deciding upon the parameters of the capital income tax function, we formulate the social optimization problem such that x_{t+1}^1 , x_{t+1}^2 , and K_{t+1} also become direct decision variables.

The resource constraint implies that output in each time period is used solely for private consumption and net investment, i.e.,

$$(13) \quad F(L_t^1, L_t^2, K_t; t) + K_t - \sum_{i=1}^2 [n_t^i c_t^i + n_{t-1}^i x_t^i] - K_{t+1} = 0.$$

The Lagrangean corresponding to the social optimization problem, with the restrictions given by equations (12) and (13), can then be written as

$$(14) \quad \begin{aligned} \mathcal{L} = & W(n_0^1 U_0^1, n_0^2 U_0^2, n_1^1 U_1^1, n_1^2 U_1^2, \dots) + \sum_t \lambda_t [U_t^2 - \hat{U}_t^2] \\ & + \sum_t \gamma_t \left[F(L_t^1, L_t^2, K_t; t) + K_t - \sum_{i=1}^2 [n_t^i c_t^i + n_{t-1}^i x_t^i] - K_{t+1} \right]. \end{aligned}$$

¹³ Given the set of available policy instruments in our framework, it is possible for the government to control the present and future consumption as well as the hours of work of each ability-type (this is discussed more thoroughly below). As a consequence, in order to be a mimicker, the high-ability type must mimic the point chosen by the low-ability type on each tax function (both the labor income tax and the capital income tax), and thus consume the same amount as the low-ability type in both periods.

Let $\hat{u}_t^2 = u_t^2(c_t^1, H - \phi l_t^1, x_{t+1}^1, \bar{c}_{t-1}, \bar{c}_t, \bar{c}_{t+1})$ denote the utility of the mimicker based on the third utility formulation in equation (1). The direct decision-variables relevant for generation t are $l_t^1, c_t^1, x_{t+1}^1, l_t^2, c_t^2, x_{t+1}^2$, and K_{t+1} , and the social first order conditions are given by¹⁴

$$(15) \quad -\frac{\partial W}{\partial(n_t^1 U_t^1)} n_t^1 u_{t,z}^1 + \phi \lambda_t \hat{u}_{t,z}^2 + \gamma_t n_t^1 w_t^1 = 0,$$

$$(16) \quad \frac{\partial W}{\partial(n_t^1 U_t^1)} n_t^1 u_{t,c}^1 - \lambda_t \hat{u}_{t,c}^2 - \gamma_t n_t^1 + \frac{n_t^1}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = 0,$$

$$(17) \quad \frac{\partial W}{\partial(n_t^1 U_t^1)} n_t^1 u_{t,x}^1 - \lambda_t \hat{u}_{t,x}^2 - \gamma_{t+1} n_t^1 + \frac{n_t^1}{N_{t+1}} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} = 0,$$

$$(18) \quad -\left[\frac{\partial W}{\partial(n_t^2 U_t^2)} n_t^2 + \lambda_t \right] u_{t,z}^2 + \gamma_t n_t^2 w_t^2 = 0,$$

$$(19) \quad \left[\frac{\partial W}{\partial(n_t^2 U_t^2)} n_t^2 + \lambda_t \right] u_{t,c}^2 - \gamma_t n_t^2 + \frac{n_t^2}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = 0,$$

$$(20) \quad \left[\frac{\partial W}{\partial(n_t^2 U_t^2)} n_t^2 + \lambda_t \right] u_{t,x}^2 - \gamma_{t+1} n_t^2 + \frac{n_t^2}{N_{t+1}} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} = 0,$$

$$(21) \quad \gamma_{t+1} [1 + r_{t+1}] - \gamma_t = 0,$$

where we have used $w_t^i = F_{L^i}(L_t^1, L_t^2, K_t; t)$ for $i=1,2$, and $r_t = F_K(L_t^1, L_t^2, K_t; t)$ from the first order conditions of the firm. For notational convenience, we have written equations (16), (17), (19), and (20) such that the right-hand side contains the derivative of the Lagrangean with respect to the appropriate measure of reference consumption, i.e., the measure of reference consumption that is affected by a change in c_t^1, x_{t+1}^1, c_t^2 , and x_{t+1}^2 , respectively. The derivative $\partial \mathcal{L} / \partial \bar{c}_t$ will be referred to as the *positionality effect* in period t and will play a crucial role in the subsequent analysis of optimal taxation.

¹⁴ Note that there is a potential time-inconsistency problem involved here since the government may have incentives to modify the second period taxation facing each generation once the individuals have revealed their true ability-types. Although we acknowledge this potential problem, we follow the bulk of earlier comparable literature on optimal nonlinear taxation in dynamic economies by considering a situation where the government commits to its tax policy. See Brett and Weymark (2008) for a recent study of (time-consistent) optimal nonlinear income taxation without commitment.

4. GENERAL RESULTS

In this section, we present the optimal marginal labor income and capital income tax rates derived from the model set out above. We start with a general characterization of optimal taxation and then examine the positionality effects mentioned above in greater detail. Section 5 in contrast derives results under a more restrictive formulation where the degrees of positionality are constant over time.

4.1 Labor Income Taxation

By defining the marginal rate of substitution between leisure and private consumption for ability-type i and the mimicker, respectively, as

$$MRS_{z,c}^{i,t} = \frac{u_{t,z}^i}{u_{t,c}^i} \text{ and } \hat{MRS}_{z,c}^{2,t} = \frac{\hat{u}_{t,z}^2}{\hat{u}_{t,c}^2},$$

we obtain the following expressions for the marginal labor income tax rates by combining equations (6), (15), (16), (18), and (19):

$$(22) \quad T_t'(w_t^1 l_t^1) = \frac{\lambda_t^*}{w_t^1 n_t^1} \left[MRS_{z,c}^{1,t} - \phi \hat{MRS}_{z,c}^{2,t} \right] - \frac{MRS_{z,c}^{1,t}}{\gamma_t w_t^1 N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t},$$

$$(23) \quad T_t'(w_t^2 l_t^2) = - \frac{MRS_{z,c}^{2,t}}{\gamma_t w_t^2 N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t},$$

where $\lambda_t^* = \lambda_t \hat{u}_{t,c}^2 / \gamma_t$. If consumption were completely non-positional, i.e., $\partial \mathcal{L} / \partial \bar{c}_t \equiv 0$, our model would reproduce the marginal labor income tax formulas derived from the conventional two-type model (e.g., Stiglitz 1982). In this case, therefore, the marginal labor income tax rate of the low-ability type reduces to the first part on the right-hand side of equation (22) – which is positive if all individuals share a common utility function – and the marginal labor income tax rate of the high-ability type becomes equal to zero. Therefore, the terms proportional to the positionality effect in each tax formula summarize how the marginal labor income tax structure is modified as a consequence of positional preferences. We can

also observe that the terms reflecting positional concerns can simply be added to the term reflecting optimal taxation in the absence of such concerns.¹⁵

4.2 Capital Income Taxation

Let us now turn to the marginal capital income tax rates. We define the marginal rate of substitution between consumption in periods t and $t+1$ for ability-type i and the mimicker as

$$MRS_{c,x}^{i,t} = \frac{u_{t,c}^i}{u_{t,x}^i} \text{ and } \hat{MRS}_{c,x}^{2,t} = \frac{\hat{u}_{t,c}^2}{\hat{u}_{t,x}^2},$$

respectively. The optimal marginal capital income tax rates in period $t+1$ are obtained by combining equations (7), (16), (17), (19), and (20):

$$(24) \quad \Phi'_{t+1}(s_t^1 r_{t+1}) = \frac{\lambda_t \hat{u}_{t,x}^2}{\gamma_{t+1} n_t^1 r_{t+1}} [MRS_{c,x}^{1,t} - \hat{MRS}_{c,x}^{2,t}] + \frac{1}{\gamma_{t+1} r_{t+1}} \left[\frac{\partial \mathcal{L}}{\partial \bar{c}_t} \frac{1}{N_t} - MRS_{c,x}^{1,t} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} \frac{1}{N_{t+1}} \right],$$

$$(25) \quad \Phi'_{t+1}(s_t^2 r_{t+1}) = \frac{1}{\gamma_{t+1} r_{t+1}} \left[\frac{\partial \mathcal{L}}{\partial \bar{c}_t} \frac{1}{N_t} - MRS_{c,x}^{2,t} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} \frac{1}{N_{t+1}} \right].$$

The first term on the right-hand side of equation (24), which does not directly depend on positional concerns, is due to the self-selection constraint and is well understood and explained in earlier research (Brett, 1997; Pirttilä and Tuomala, 2001). The final part of each tax formula shows how the policy incentives are modified by the relative consumption concerns. As the marginal capital income tax rates reflect a desired tradeoff for society between present and future consumption, each such term is decomposable into two parts. The basic intuition is that each individual generates positional externalities both when young and when old. Therefore, whether positional concerns lead to a higher or lower marginal capital income tax rate in period $t+1$ depends on the difference between the positionality effect in

¹⁵ Equations (22) and (23) correspond to Equations (17) and (18) in Aronsson and Johansson-Stenman (forthcoming) in a model without intertemporal consumption comparisons. The positionality effect, as represented by the derivative $\partial \mathcal{L} / \partial \bar{c}_t$, takes a different form here, as it reflects both the effect of between-people comparisons in the same period and the effect of intertemporal consumption comparisons. This will be described in more detail below.

period t and the discounted positionality effect in period $t+1$, where the discount factor is given by the marginal rate of substitution between present and future consumption.

Note that the marginal income tax results presented so far rely on the most general specification of the utility function, i.e., the function $u_t^i(\cdot)$ in equation (1), meaning that equations (22)-(25) hold for all possible functional forms of the social comparisons, as long as these comparisons are based on the measures of average consumption described above. To be able to say more about the relation between the relative consumption concerns and the optimal marginal income tax rates, we must explore the positionality effect in more detail. This is the task to which we turn next.

4.3 Exploring the positionality effect

The positionality effect measures the welfare effect of an increase in the reference consumption, *ceteris paribus*. This welfare effect is due to direct consumption comparisons between people currently alive, as well as to comparisons with the average consumption in the previous period. It also reflects the self-selection constraint in the sense that increased reference consumption may affect the incentives to become a mimicker.

For convenience, we denote the difference in the degree of current and intertemporal positionality between the mimicker and the low-ability type in period t

$$\alpha_t^d = \frac{\lambda_{t-1} \hat{u}_{t-t,x}^2}{\gamma_t N_t} [\hat{\alpha}_t^{2,x} - \alpha_t^{1,x}] + \frac{\lambda_t \hat{u}_{t,c}^2}{\gamma_t N_t} [\hat{\alpha}_t^{2,c} - \alpha_t^{1,c}]$$

$$\beta_t^d = \frac{\lambda_{t-1} \hat{u}_{t-t,x}^2}{\gamma_t N_t} [\hat{\beta}_t^{2,x} - \beta_t^{1,x}] + \frac{\lambda_t \hat{u}_{t,c}^2}{\gamma_t N_t} [\hat{\beta}_t^{2,c} - \beta_t^{1,c}],$$

respectively, where the symbol “ \wedge ” denotes “mimicker” (as before), while the superindex “ d ” stands for “difference.” Note that α_t^d and β_t^d reflect positionality differences between the young mimicker and the young low-ability type and between the old mimicker and the old low-ability type, respectively. Then, by using the short notations

$$A_{t+k} = \frac{N_{t+k} \gamma_{t+k} [\alpha_{t+k}^d - \bar{\alpha}_{t+k}]}{1 - \bar{\alpha}_{t+k}},$$

$$B_{t+k} = \frac{N_{t+k+1} \gamma_{t+k+1} [\beta_{t+k+1}^d - \bar{\beta}_{t+k+1}]}{1 - \bar{\alpha}_{t+k}},$$

we obtain

$$(26) \quad \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = A_t + B_t + \sum_{k=1}^{\infty} [A_{t+k} + B_{t+k}] \prod_{j=1}^k \frac{\bar{\beta}_{t+j}}{1 - \bar{\alpha}_{t+j-1}}.$$

We will refer to equation (26) as the *positionality effect* in period t . This is clearly a rather complex expression based on several mechanisms. Consider the first variable, A_t , which in itself encompasses two components. We can interpret $-N_t \gamma_t \bar{\alpha}_t / (1 - \bar{\alpha}_t) < 0$ as measuring the direct welfare loss in period t of an increase in \bar{c}_t ; the intuition is that an increase in \bar{c}_t , ceteris paribus, leads to lower utility for all consumers via the argument $c_t^i - \bar{c}_t$ in the function $v_t^i(\cdot)$ in equation (1). This effect depends on the average degree of current positionality without any consideration of positionality differences between the mimicker and the low-ability type. The other component, $N_t \gamma_t \alpha_t^d / (1 - \bar{\alpha}_t)$, reflects the self-selection constraint (in periods $t-1$ and t) and arises because the mimicker and the low-ability type typically differ with respect to the degree of positionality both when young and when old. If the low-ability type has a higher degree of current positionality than the mimicker in both generations alive in period t (i.e., generations t and $t-1$), then $\alpha_t^d < 0$, meaning that increased reference consumption gives rise to a greater utility loss for the low-ability type than for the mimicker. As such, it becomes more attractive to become a mimicker, implying an additional welfare loss. On the other hand, if the mimicker is more positional than the low-ability type for both generations alive in period t , increased reference consumption will, instead, contribute to relax the self-selection constraint, implying that $\alpha_t^d > 0$. Although it seems intuitively plausible that the direct effect dominates, in which case we have $A_t < 0$, we cannot a priori rule out that a positive self-selection effect may dominate the negative direct effect.

The variable B_t is analogous to A_t ; yet with the modification that it refers to intertemporal rather than current positionality. It also encompasses two distinct components. The term $-N_{t+1} \gamma_{t+1} \bar{\beta}_{t+1} / (1 - \bar{\alpha}_t) < 0$ is interpretable as the direct welfare in period $t+1$ of an increase in \bar{c}_t , and the underlying mechanism here is that \bar{c}_t affects individual utility negatively via the argument $x_{t+1}^i - \bar{c}_t$ in the function $v_{t+1}^i(\cdot)$. Again, this is a pure externality that is characterized

by the average degree of positionality without any reference to positionality differences across agents. The component $N_{t+1}\gamma_{t+1}\beta_{t+1}^d/(1-\bar{\alpha}_t)$ reflects the corresponding welfare effects through the self-selection mechanism at time $t+1$. If the low-ability type has a higher degree of intertemporal positionality than the mimicker in both generations alive in period t , then $\beta_{t+1}^d < 0$. Therefore, an increase in the reference consumption means a larger utility loss for the low-ability type than for the mimicker and, as a consequence, an additional welfare loss. If, instead, the mimicker has a higher degree of intertemporal positionality than the low-ability type, i.e., $\beta_{t+1}^d > 0$, then an increase in the reference consumption contributes to relax the self-selection constraint, which leads to higher welfare. Again, we cannot a priori rule out that a positive self-selection effect may dominate the negative direct effect.

The third part of equation (26) reflects an intertemporal chain reaction. The intuition is that the intertemporal aspect of the consumption comparisons, i.e., that other people's past consumption affects utility, means that the welfare effects of changes in the reference consumption are not time-separable (as they would be without intertemporal consumption comparisons). This is so since a change in the reference consumption today means behavioral adjustments in the future, which in turn influence the reference consumption relevant for future generations. Finally, note that in the absence of relative comparisons over time, a case analyzed by Aronsson and Johansson-Stenman (forthcoming), the right-hand side of equation (26) collapses to A_t .

From equation (26), we obtain the following result regarding the sign of the positionality effect:

LEMMA 1. *If, from period t and onwards, the low-ability type is at least as positional as the mimicker on average, or if the positionality differences are sufficiently small, in any of the following senses:*

- (i)
$$\frac{N_t\gamma_t\alpha_t^d + N_{t+1}\gamma_{t+1}\beta_{t+1}^d}{1-\bar{\alpha}_t} + \sum_{k=1}^{\infty} \frac{N_{t+k}\gamma_{t+k}\alpha_{t+k}^d + N_{t+k+1}\gamma_{t+k+1}\beta_{t+k+1}^d}{1-\bar{\alpha}_{t+k}} \prod_{j=1}^k \frac{\bar{\beta}_{t+j}}{1-\bar{\alpha}_{t+j-1}} \leq 0,$$
- (ii)
$$\alpha_{t+k}^d < \bar{\alpha}_{t+k} \text{ and } \beta_{t+k+1}^d < \bar{\beta}_{t+k+1} \quad \forall k \geq 0,$$
- (iii)
$$\alpha_{t+k}^d \leq 0 \text{ and } \beta_{t+k+1}^d \leq 0 \quad \forall k \geq 0,$$

then increased reference consumption in period t reduces the welfare.

Given the assumption that the individual degrees of positionality (both in the current and intertemporal dimensions) are always between zero and one, (i) gives a sufficient condition for when increased reference consumption in period t leads to lower welfare. Yet, condition (i) is not necessary, since the terms in equation (26) that solely reflect the average degrees of positionality (i.e., the pure externality terms) contribute to lower welfare as well. Condition (ii) is not necessary either, since $\partial\mathcal{L}/\partial\bar{c}_t$ can clearly be negative even if (ii) does not hold for some k . Note finally that condition (iii), which we refer to due to its straightforward interpretation, is actually redundant since it implies condition (ii).

By combining Lemma 1 with equations (22) and (23), we obtain the following result:

PROPOSITION 1. *If any of the conditions in Lemma 1 holds, so that increased reference consumption leads to lower welfare, ceteris paribus, then the positionality effect in period t contributes to increase the marginal labor income tax rates for both ability-types in period t .*

The interpretation of Proposition 1 is straightforward. If the low-ability type is at least as positional as the mimicker on average, or if loosely speaking the positionality differences are sufficiently small, and given the assumption that the individual degrees of positionality are always between zero and one, then we obtain from equation (26) that $\partial\mathcal{L}/\partial\bar{c}_t < 0$.

Similarly, by combining Lemma 1 with equations (24) and (25), and then using $1+n_{t+1} = N_{t+1}/N_t$ to denote the population growth factor in period $t+1$, we can derive the following result for how positional concerns contribute to the marginal capital income tax rates:

PROPOSITION 2. *Suppose that any of the conditions in Lemma 1 holds, so that increased reference consumption leads to lower welfare, ceteris paribus. Then, if the preferences become less (more) positional over time in the sense that*

$$\left| \frac{\partial\mathcal{L}}{\partial\bar{c}_t} \right| > (<) \frac{MRS_{c,x}^{i,t}}{(1+n_{t+1})} \left| \frac{\partial\mathcal{L}}{\partial\bar{c}_{t+1}} \right|,$$

i.e., the positionality effect in period t dominates (is dominated by) the positionality effect in period $t+1$, then the joint contribution of the positionality effects in periods t and $t+1$ is to decrease (increase) the marginal capital income tax rate for ability-type i in period $t+1$.

The intuition behind Proposition 2 is straightforward. If an increase in the average consumption (in any period), *ceteris paribus*, leads to lower welfare, and if the positionality effect in period t dominates the corresponding effect in period $t+1$, there is an incentive for the government to discourage the consumption in period t relative to the consumption in period $t+1$. The opposite policy incentive arises if the positionality effect in period $t+1$ dominates. An interesting implication of the proposition is that it would be optimal with increasing marginal capital income taxation over time in an economy where the preferences become more positional over time (*i.e.*, if we tend to attach a higher value to relative consumption increases than to absolute consumption increases as time passes). Such a pattern is actually broadly consistent with some empirical evidence: Clark et al. (2008) analyze the impact of relative income on happiness and conclude that the concern for relative income tends to increase as the average income in a country increases. Note also that we can interpret the component $MRS_{c,x}^{i,t}/(1+n_{t+1})$ as the effective discount factor for ability-type i , which is used to discount the positionality effect in period $t+1$ to period t .

To go further, one would also like to express the marginal income tax rates in terms of the degrees of positionality defined in Section 2.2. However, the expressions that can be obtained for the general case turn out to be very complex and do not add much to the results derived above. In the next section we will, therefore, make some more restrictive assumptions that simplify the analysis considerably.

5. FURTHER RESULTS UNDER MORE RESTRICTIVE ASSUMPTIONS

In this section, we consider a special case of the model analyzed above. To be more specific, we add the assumptions that the population is constant¹⁶ and that the degrees of positionality (the average degrees as well as the indicators of positionality differences between the mimicker and the low-ability type) are constant over time in the sense that $\bar{\alpha}_t = \bar{\alpha}$, $\bar{\beta}_t = \bar{\beta}$,

¹⁶ This assumption is not necessary for the analysis to hold. The same qualitative results as those derived below would also follow with a constant population growth rate, yet at the cost of slightly more notation.

$\alpha_t^d = \alpha^d$, and $\beta_t^d = \beta^d$ for all t . In addition, to simplify the calculations further (yet with little loss of generality), we also add the assumption that the interest rate is fixed and equal to r . This implies from equation (21) that $\gamma_{t+k} = \gamma_t / [1+r]^k$. This special case is either interpretable in terms of a steady state¹⁷ – provided that a steady state exists – or may follow as a consequence of adding additional assumptions about the preferences and technology (see below).

Although these assumptions of course reflect a more restrictive model, similar (or stronger) assumptions are typically made in the “catching up with the Joneses” literature; see, e.g., Campbell and Cochrane (1999) and Díaz et al. (2003). It should also be noted that the model is still general enough to reflect different preferences between types, including different positionality degrees.

Equation (26) then reduces to a geometric series, such that

$$(27) \quad \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = \frac{N\gamma_t}{1-\bar{\alpha}} \left[\alpha^d - \bar{\alpha} + \frac{\beta^d - \bar{\beta}}{1+r} \right] \sum_{i=0}^{\infty} \left[\frac{\bar{\beta}}{[1-\bar{\alpha}][1+r]} \right]^i = N\gamma_t \frac{\alpha^d - \bar{\alpha} + [\beta^d - \bar{\beta}]/[1+r]}{1-\bar{\alpha} - \bar{\beta}/[1+r]},$$

where in the last step we have implicitly assumed that $0 < \bar{\beta} < [1-\bar{\alpha}][1+r]$ so that the series converges.

5.1 The Time-Inclusive Degree of Consumption Positionality

Let us now aggregate the current and intertemporal degrees of consumption positionality into a single measure, which will be referred to as the *time-inclusive degree of consumption positionality*,

$$\rho_t^{i,c} = \alpha_t^{i,c} + \beta_t^{i,c} = \frac{v_{t,\Delta^c}^i + v_{t,\Omega^c}^i}{v_{t,\Delta^c}^i + v_{t,\Omega^c}^i + v_{t,c}^i}$$

¹⁷ This requires that the preferences and technology do not change over time, and that the economy approaches a stationary equilibrium in which l_t^i , c_t^i , x_t^i (for $i=1, 2$), and K_t all remain constant over time.

$$\rho_{t+1}^{i,x} = \alpha_{t+1}^{i,x} + \beta_{t+1}^{i,x} = \frac{v_{t,\Delta^x}^i + v_{t,\Omega^x}^i}{v_{t,\Delta^x}^i + v_{t,\Omega^x}^i + v_{t,x}^i}.$$

The variable $\rho_t^{i,c}$ then reflects the fraction of the overall utility increase from an additional dollar spent when young in period t that is due to the increased relative consumption compared to either the average consumption in period t or the average consumption in period $t-1$. $\rho_{t+1}^{i,x}$ can be interpreted correspondingly when old in period $t+1$.

We can then define *the average degree of time-inclusive consumption positionality*, in present value terms, as follows:

$$(28) \quad \bar{\rho} = \bar{\alpha} + \frac{\bar{\beta}}{1+r}.$$

Intuitively, $\bar{\rho}$ reflects the overall social loss in a first best world of consuming an additional dollar today. The first term, $\bar{\alpha}$, reflects the part of this loss that will occur through current consumption positionality, whereas the second term, $\bar{\beta}/[1+r]$, reflects the loss due to intertemporal consumption positionality; the reason why the latter loss is discounted is, of course, that it will occur in the next period. By analogy, the *difference in the time-inclusive degree of consumption positionality between the mimicker and the low-ability type* (also in present value terms) can be written as

$$(29) \quad \rho^d = \alpha^d + \frac{\beta^d}{1+r}.$$

By substituting equations (28) and (29) into equation (27), we have derived the following result:

LEMMA 2. *If the population is constant, and if $\bar{\alpha}_t = \bar{\alpha}$, $\bar{\beta}_t = \bar{\beta}$, $\alpha_t^d = \alpha^d$, $\beta_t^d = \beta^d$, and $r_t = r$ for all t , then the positionality effect reduces to read*

$$(30) \quad \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = -N\gamma_t \frac{\bar{\rho} - \rho^d}{1 - \bar{\rho}}.$$

Consequently, the positionality effect can here be written as the sum of two terms. The term $-N\gamma_t \bar{\rho} / (1 - \bar{\rho}) < 0$ is interpretable as the direct welfare loss of an increase in \bar{c}_t , and arises because a higher \bar{c}_t leads to lower utility for all consumers via the arguments $c_t^i - \bar{c}_t$ and $x_{t+1}^i - \bar{c}_t$ in the utility function. In other words, this component is a pure externality and depends on the average degree of time-inclusive consumption positionality. The term $N\gamma_t \rho^d / (1 - \bar{\rho})$ captures the differences in the degree of time-inclusive consumption positionality between the mimicker and the low-ability type (when young and when old). The intuition is, of course, that an increase in \bar{c}_t may either tighten ($\rho^d > 0$) or relax ($\rho^d < 0$) the self-selection constraint.

5.2 Labor Income Taxation

With equation (30) at our disposal, we can relate the marginal income tax rates to the average degree of time-inclusive consumption positionality and to differences in this measure of positionality between the mimicker and the low-ability type. Starting with the marginal labor income tax rates, we combine equations (22) and (23) with equation (30). Using the short notations σ_t^1 and σ_t^2 for the optimal marginal labor income tax rates without relative consumption concerns, i.e.

$$\sigma_t^1 = \frac{\lambda_t^*}{w_t^1 n_t^1} \left[MRS_{z,c}^{1,t} - \phi \hat{MRS}_{z,c}^{2,t} \right] \text{ and } \sigma_t^2 = 0,$$

we can then rewrite the formulas for the marginal labor income tax rates as follows:

PROPOSITION 3. *If the positionality effect is given by equation (30) in Lemma 2, the optimal marginal labor income tax rate for each ability-type can be written in the following additive form (for $i=1, 2$):*

$$(31) \quad T_t^i(w_t^i l_t^i) = \sigma_t^i + [1 - \sigma_t^i] \bar{\rho} - [1 - \sigma_t^i] [1 - \bar{\rho}] \frac{\rho^d}{1 - \rho^d}.$$

To interpret equation (31),¹⁸ note that if the resource allocation were first best (i.e., in the absence of any informational asymmetry between the government and the private sector), we have $\lambda_t = \sigma_t^1 = \sigma_t^2 = 0$ for all t and $\rho^d = 0$, hence $T_t'(w_t^1 l_t^1) = T_t'(w_t^2 l_t^2) = \bar{\rho}$. In this case, therefore, the optimal marginal labor income tax rate is simply an externality-correcting Pigouvian tax, i.e., each individual is taxed for the negative positional externality that he/she imposes on others. As a consequence, if we were to (erroneously) neglect the positional externality generated by the Catching up with the Joneses motive for consumption, we would also underestimate this corrective tax.

Returning to our second best economy, the first term on the right-hand side of equation (31) is the expression for the marginal labor income tax rate that would follow in the standard optimal income tax model without any positional concern. The second term measures the marginal external cost of consumption as reflected by the average degree of time-inclusive consumption positionality, although its contribution to the marginal labor income tax rates is modified compared to the first best formula. The intuition is that the fraction of an income increase that is already taxed away does not give rise to positional externalities. Therefore, if $\sigma_t^1 > 0$ (as one would expect if all agents have the same utility function; see Stiglitz, 1982), this “second best modification” tends to reduce the externality-correcting component in the formula for the low-ability type.

The third term on the right-hand side of equation (31) reflects self-selection effects of the positional concerns. To provide intuition, suppose first that $\rho^d > 0$, meaning that the mimicker has a higher degree of time-inclusive positionality than the low-ability type. In this case, increased reference consumption gives rise to a larger utility loss for the mimicker than it does for the low-ability type, and the government may relax the self-selection constraint by implementing policies that lead to increased reference consumption. This provides an incentive for the government to implement a lower marginal labor income tax rate than it would otherwise have done, which means that the third term contributes to decrease the

¹⁸ The formulas in Lemma 2 and Proposition 3 take the same general form as equations (16) and (19) in Aronsson and Johansson-Stenman (forthcoming); the difference is that $\bar{\rho}$ and ρ^d are here based on the time-inclusive positionality concept introduced above, i.e., that each individual compares his/her consumption with both other people’s current consumption *and* other people’s past consumption.

marginal labor income tax rate. Consequently, if ρ^d is positive and sufficiently large, then this effect may (at least theoretically) dominate the externality-correcting component, implying that relative consumption concerns contribute to reduce the marginal labor income tax rates. If $\rho^d < 0$, on the other hand, then increased reference consumption tightens the self-selection constraint, meaning that the third term on the right-hand side contributes to increase the marginal labor income tax rate. In this case, therefore, the positionality effect as a whole leads to increased marginal labor income taxation (see Proposition 1).

5.3 Capital Income Taxation

Turning to the capital income tax structure, we can similarly combine equations (24) and (25) with equation (30), and use the short notation δ_t^i for the optimal marginal capital income tax rate for ability-type i that would follow in the absence of relative consumption concerns, where

$$\delta_t^1 = \frac{\lambda_t \hat{u}_{t,x}^2}{\gamma_{t+1} n_t^1 r_{t+1}} \left[MRS_{c,x}^{1,t} - \hat{MRS}_{c,x}^{2,t} \right] \text{ and } \delta_t^2 = 0.$$

We can then derive the following result:

PROPOSITION 4. *If the positionality effect is given by equation (30) in Lemma 2, the optimal marginal capital income tax rate for each ability-type can be written as follows (for $i=1, 2$):*

$$(32) \quad \Phi_{t+1}^i(s_t^i r_{t+1}) = \frac{1 - \bar{\rho}}{1 - \rho^d} \delta_t^i.$$

It can immediately be observed that there is no direct effect of relative consumption concerns reflected in equation (32). Intuitively, since the positionality degrees are constant over time, the current and future aspects of relative consumption concerns largely cancel out. The remaining effect is, however, not necessarily unimportant, and implies that positional concerns still modify the marginal capital income tax rate implemented for the low-ability type.

Consider first the situation where $MRS_{c,x}^{1,t} > \hat{MRS}_{c,x}^{2,t}$, in which the mimicker values an additional dollar today in terms of consumption tomorrow less than does the low-ability type, implying that $\delta_t^1 > 0$ and $\Phi_{t+1}^1(s_t^1 r_{t+1}) > 0$. This is so because, if $\delta_t^1 > 0$, we may relax the self-selection constraint by implementing a positive marginal capital income tax for the low-ability type. The term $1 - \bar{\rho}$ serves to modify this effect, i.e. to reduce the effect that δ_t^1 would otherwise have on the marginal capital income tax rate. The intuition is, of course, that capital income taxation leads to an increase in \bar{c}_t and, therefore, gives rise to positional externalities. This policy incentive is, in turn, either counteracted or further strengthened by the component $1 - \rho^d$, as increased reference consumption may either relax ($\rho^d > 0$) or tighten ($\rho^d < 0$) the self-selection constraint. Analogous results and interpretations hold for the case where $MRS_{c,x}^{1,t} < \hat{MRS}_{c,x}^{2,t}$.

Let us finally relate to a classical result by Ordober and Phelps (1979), regarding when it is optimal not to use capital taxation at all. From Proposition 4, it is straightforward to derive such conditions also in our model. The following result is an immediate consequence of Proposition 4:

COROLLARY 1. *If leisure is weakly separable from private consumption in the sense that $U_t^i = q_t^i(f_t(c_t^i, x_{t+1}^i, \Delta_t^{i,c}, \Delta_{t+1}^{i,x}, \Omega_t^{i,c}, \Omega_{t+1}^{i,x}), z_t^i)$ describes the utility function, then both optimal marginal capital income tax rates are zero.*

This result follows from acknowledging that the mimicker and the low-ability type differ only with respect to preferences and use of leisure. Given the separability assumption and that the consumers share a common sub-utility function $f_t(\cdot)$, it follows that $MRS_{c,x}^{1,t} = \hat{MRS}_{c,x}^{2,t}$, and hence that the possibility for relaxing the self-selection constraint by capital taxation vanishes.

Note finally again that $\bar{\rho}$ reflects the average degree of time-inclusive consumption positionality in present value terms, i.e., $\bar{\rho} = \bar{\alpha} + \bar{\beta}/(1+r)$. Therefore, all qualitative results in this section hold in the special case without consumption comparisons over time, i.e., where $\bar{\rho} = \bar{\alpha}$, which is the case addressed by Aronsson and Johansson-Stenman

(forthcoming). Similarly, all qualitative results hold in the other extreme situation where $\bar{\rho} = \bar{\beta}/(1+r)$, in which there are no comparisons with other people's current consumption.

6. RESULTS BASED ON A COBB-DOUGLAS UTILITY FUNCTION

In order to more clearly illustrate some implications of the relative consumption comparisons for optimal income taxation, let us use the same assumptions as in Section 5 with respect to a constant population size and interest rate, but in addition consider the following Cobb-Douglas utility function:

$$(33) \quad U_t^i = k^i (z_t^i)^{k_z^i} (c_{net,t}^i)^{k_c^i} (x_{net,t+1}^i)^{k_x^i}$$

where $k^i, k_z^i, k_c^i, k_x^i > 0$ are constants and $k_z^i + k_c^i + k_x^i < 1$; $c_{net,t}^i$ and $x_{net,t+1}^i$ reflect what we may think of as consumption net of relative consumption concerns, when young and when old, for an individual of ability-type i born in period t , as defined below:¹⁹

$$(34a) \quad c_{net,t}^i = [1 - a - a']c_t^i + a[c_t^i - \bar{c}_t] + a'[c_t^i - \bar{c}_{t-1}] = c_t^i - a\bar{c}_t - a'\bar{c}_{t-1}$$

$$(34b) \quad x_{net,t+1}^i = [1 - b - b']x_{t+1}^i + b[x_{t+1}^i - \bar{c}_{t+1}] + b'[x_{t+1}^i - \bar{c}_t] = x_{t+1}^i - b\bar{c}_{t+1} - b'\bar{c}_t.$$

By substituting equations (34a) and (34b) into equation (33), we obtain:

$$(35) \quad U_t^i = k^i [z_t^i]^{k_z^i} [c_t^i - a\bar{c}_t - a'\bar{c}_{t-1}]^{k_c^i} [x_{t+1}^i - b\bar{c}_{t+1} - b'\bar{c}_t]^{k_x^i}.$$

Although the utility functions are allowed to differ between the ability-types, through the parameters k^i, k_z^i, k_c^i , and k_x^i , the individual degrees of current as well as intertemporal consumption positionality are clearly the same between types, and also constant over time.

¹⁹ For analytical simplicity, we abstract from the possibility that each individual also compares his/her own current consumption with his/her own previous consumption. As mentioned before, allowing for such comparisons would not directly affect the optimal tax formulas, since the individuals would internalize these effects themselves.

The degrees of current consumption positionality when young and when old for each type are equal to a and b , respectively, whereas the corresponding degrees of intertemporal consumption positionality are given by a' and b' .

From equation (31), we then have that the optimal marginal labor income tax rate for type i is given by:

$$(36) \quad T'_i(w_i^i l_i^i) = \sigma_i^i + [1 - \sigma_i^i] \left[\frac{a+b}{2} + \frac{a'+b'}{2(1+r)} \right],$$

where the first expression in brackets thus represents the average current degree of consumption positionality and the second the (one period discounted) average intertemporal degree of consumption positionality. As before, σ_i^i reflects the optimal marginal labor income tax rate for ability-type i without relative consumption concerns.

Regarding optimal capital income taxation, it is easy to see that the separability assumption in Corollary 1 above is fulfilled by the utility function in Equation (35). Therefore, we know that the optimal marginal capital income tax rate is zero for each ability-type and in all time periods, irrespective of the parameter values of the utility function.

6.1 Orders of Magnitude

Let us now briefly discuss possible orders of magnitude of the optimal marginal income taxes. A couple of studies have attempted to measure the average degree of current consumption positionality, corresponding to $(a+b)/2$ in equation (36). According to the survey-experimental evidence of Solnick and Hemenway (1998), Johansson-Stenman et al. (2002), Alpizar et al. (2005), and Carlsson et al. (2007), the average degree appears to be in the order of magnitude of 0.5. Wendner and Goulder (2008) argue, based on the existing empirical evidence, for a value between 0.2 and 0.4, whereas evidence from happiness studies such as Luttmer (2005) suggests a much larger value in the order of magnitude of 0.8.

There is less direct evidence regarding the average intertemporal degree of consumption positionality, corresponding to $(a'+b')/2$ in equation (36). Alvarez-Cuadrado et al. (2004)

refer to a benchmark value used by Carrol et al. (1997), with a value of a parameter that can be interpreted as an intertemporal degree of consumption positionality equal to 0.5.²⁰ As a sensitivity analysis, they use a value of 0.8, based on Fuhrer (2000).

As an illustrative example, consider the case where the optimal marginal labor income tax rate in the absence of relative consumption concerns equals 0.3, and where both the average degree of current consumption positionality and the average degree of intertemporal consumption positionality are also 0.3, i.e., $\sigma_t^i = (a+b)/2 = (a'+b')/2 = 0.3$. Then, if the real interest rate between the periods is given by $r = 1$,²¹ it follows that the optimal marginal labor income tax rate is equal to $T_t'(w_t^i l_t^i) = 0.3 + 0.7[0.3 + 0.3/2] = 0.615$. In other words, the optimal marginal labor income tax rate would be above 60%, instead of 30% as in the absence of relative consumption comparisons. While the underlying estimates of the current and intertemporal degrees of positionality presented above are highly uncertain, and can hardly be interpreted as completely independent of each other,²² it nevertheless seems as if their joint effect on the marginal labor income tax rates may be substantial.

7. CONCLUSION

The present paper simultaneously recognizes three mechanisms behind relative consumption concerns: comparisons with (i) other people's current consumption (Keeping up with the Joneses), (ii) own past consumption (Habit formation), and (iii) other people's past consumption (Catching up with the Joneses). We are not aware of any previous normative economic analysis in such a setting. The model here considers optimal nonlinear income taxation in an OLG model with asymmetric information between the government and the

²⁰ In Carrol et al. (1997), the reference consumption is not others' average consumption one period earlier (since their study is not based on an OLG model), but a weighted average of others' average consumption where the weight is larger the closer to the present the consumption takes place.

²¹ This corresponds to an annual real interest rate of slightly less than 2 percent if we assume 40 years between the periods.

²² We are not aware of any study that simultaneously attempts to estimate the average degree of current and intertemporal consumption positionality.

private sector, where people compare their own current consumption with the three measures of reference consumption mentioned above.

We show that comparisons with one's own past consumption do not affect the optimal policy rules, since such comparisons are internalized by each individual (although the interpretations become slightly modified). However, comparisons with other people's past consumption generate positional externalities. In addition, such comparisons give rise to considerably more complex policy responses than comparisons solely based on other people's current consumption. While some results were possible to derive and interpret based on the most general setting, considerably stronger results are obtained for the somewhat more restrictive case where the population size, the interest rate, and the degrees of current and intertemporal consumption positionality are constant over time, e.g., if the economy has reached a steady state. The optimal tax policy is then derived in terms of the average degree of time-inclusive consumption positionality, which is essentially the sum of the average degree of current consumption positionality and the average degree of intertemporal consumption positionality. The tax policy also depends on differences between the mimicker and the low-ability type with respect to the degree of time-inclusive consumption positionality.

We then show that the optimal marginal labor income tax rates become larger, *ceteris paribus*, the more positional people are on average, in terms of the average degree of time-inclusive consumption positionality. It is also demonstrated that this modifying effect can be substantial. Yet, the net effects of relative consumption concerns also depend on whether the low-ability type is more or less positional (broadly speaking) than the mimicker. The reason is that this determines whether an increase in the reference consumption works to relax or tighten the self-selection constraint. There are no direct effects of relative consumption concerns on the marginal capital income tax rates; instead, positional concerns enter the marginal capital income tax rate implemented for the low-ability type as a scale factor, which is based on the degree of time-inclusive consumption positionality. We are then also able to generalize the well-known result of Ordober and Phelps (1979) for when there should be no capital income taxes, to a much more complex model where people compare their own current consumption with several different measures of reference consumption. Specifically, if, in addition to the other assumptions mentioned, leisure is weakly separable from the other goods in the utility function, then the marginal capital income tax rates should be zero.

Finally we illustrate with a Cobb-Douglas functional form and show, based on parameter estimates from the literature, that positional preferences of both the Keeping up with the Joneses and Catching up with the Joneses types substantially increase the optimal marginal labor income tax rates for both types. Since the leisure separability conditions are fulfilled for this form, the optimal marginal capital income tax rates are consequently zero for both types.

We believe that the research area consisting of normative economic analysis when relative consumption matters is still underexplored. Examples of issues that remain to be analyzed include a multi-country setting, public provision of private (non-positional) goods, public good provision in a dynamic economy, and long-term social discounting.

APPENDIX

Derivation of equations (22) and (23)

Consider the tax formula for the low-ability type. By combining equations (15) and (16), we obtain

$$(A1) \quad \frac{u_{t,z}^1}{u_{t,c}^1} \left[\lambda_t \hat{u}_{t,c}^2 + \gamma_t n_t^1 - \frac{n_t^1}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} \right] = \lambda_t \phi \hat{u}_{t,z}^2 + \gamma_t n_t^1 w_t^1.$$

By substituting $T_t'(w_t^1 l_t^1) w_t^1 = w_t^1 - u_{t,z}^1 / u_{t,c}^1$ from equation (6) into equation (A1) and rearranging, we obtain equation (22). The marginal labor income tax rate of the high-ability type, equation (23), is derived in a similar way.

Derivation of equations (24) and (25)

Let us consider the marginal capital income tax rate of the low-ability type. By combining equations (16) and (17), we obtain

$$(A2) \quad MRS_{c,x}^{1,t} \left[\lambda_t \hat{u}_{t,x}^2 - \gamma_{t+1} n_t^1 + \frac{n_t^1}{N_{t+1}} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} \right] = \lambda_t \hat{u}_{t,c}^2 + \gamma_t n_t^1 - \frac{n_t^1}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t}.$$

We then use equations (7) and (21) to derive $MRS_{c,x}^{1,t} = 1 + r_{t+1} - r_{t+1} \Phi'_{t+1}(s_t^1 r_{t+1})$ and $\gamma_t = \gamma_{t+1} [1 + r_{t+1}]$, respectively. Substituting into equation (A2) and rearranging, we obtain equation (24). Equation (25) is derived in a similar way.

Derivation of equation (26)

By definition, we have

$$(A3) \quad \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = \sum_{i=1}^2 \frac{\partial W}{\partial (n_{t-1}^i U_{t-1}^i)} n_{t-1}^i u_{t-1, \bar{c}_t}^i + \sum_{i=1}^2 \frac{\partial W}{\partial (n_t^i U_t^i)} n_t^i u_{t, \bar{c}_t}^i + \sum_{i=1}^2 \frac{\partial W}{\partial (n_{t+1}^i U_{t+1}^i)} n_{t+1}^i u_{t+1, \bar{c}_t}^i \\ + \lambda_{t-1} [u_{t-1, \bar{c}_t}^2 - \hat{u}_{t-1, \bar{c}_t}^2] + \lambda_t [u_{t, \bar{c}_t}^2 - \hat{u}_{t, \bar{c}_t}^2] + \lambda_{t+1} [u_{t+1, \bar{c}_t}^2 - \hat{u}_{t+1, \bar{c}_t}^2]$$

From equation (1), we have

$$u_{t,c}^i = v_{t,c}^i + v_{t,\Delta_t^c}^i + v_{t,\delta_t^c}^i = \frac{v_{t,\Delta_t^c}^i}{\alpha_t^{i,c}} = \frac{v_{t,\delta_t^c}^i}{\beta_t^{i,c}}, \\ u_{t,x}^i = v_{t,x}^i + v_{t,\Delta_t^x}^i + v_{t,\delta_t^x}^i = \frac{v_{t,\Delta_t^x}^i}{\alpha_{t+1}^{i,x}} = \frac{v_{t,\delta_t^x}^i}{\beta_{t+1}^{i,x}}, \\ u_{t,\bar{c}_t}^i = -v_{t,\Delta_t^c}^i - v_{t,\delta_t^x}^i, \\ u_{t,\bar{c}_{t-1}}^i = -v_{t,\delta_t^c}^i, \\ u_{t,\bar{c}_{t+1}}^i = -v_{t,\Delta_t^x}^i,$$

so

$$(A4) \quad u_{t,\bar{c}_t}^i = -\alpha_t^{i,c} u_{t,c}^i - \beta_{t+1}^{i,x} u_{t,x}^i,$$

$$(A5) \quad u_{t,\bar{c}_{t-1}}^i = -\beta_t^{i,c} u_{t,c}^i,$$

$$(A6) \quad u_{t,\bar{c}_{t+1}}^i = -\alpha_{t+1}^{i,x} u_{t,x}^i,$$

which substituted into equation (A3) imply

$$(A7) \quad \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = -\sum_{i=1}^2 \frac{\partial W}{\partial (n_{t-1}^i U_{t-1}^i)} n_{t-1}^i \alpha_t^{i,x} u_{t-1,x}^i - \sum_{i=1}^2 \frac{\partial W}{\partial (n_t^i U_t^i)} n_t^i [\alpha_t^{i,c} u_{t,c}^i + \beta_{t+1}^{i,x} u_{t,x}^i] \\ - \sum_{i=1}^2 \frac{\partial W}{\partial (n_{t+1}^i U_{t+1}^i)} n_{t+1}^i \beta_{t+1}^{i,c} u_{t+1,c}^i + \lambda_{t-1} [-\alpha_t^{2,x} u_{t-1,x}^2 + \hat{\alpha}_t^{2,x} u_{t-1,x}^2] \\ + \lambda_t [-\alpha_t^{2,c} u_{t,c}^2 - \beta_t^{2,x} u_{t+1,x}^2 + \hat{\alpha}_t^{2,c} \hat{u}_{t,c}^2 + \hat{\beta}_{t+1}^{2,x} \hat{u}_{t,x}^2] + \lambda_{t+1} [-\beta_{t+1}^{2,c} u_{t+1,c}^2 + \hat{\beta}_{t+1}^{2,c} \hat{u}_{t+1,c}^2]$$

From equations (16), (17), (19), and (20), we have

$$(A8) \quad \frac{\partial W}{\partial (n_t^1 U_t^1)} n_t^1 u_{t,c}^1 = \lambda_t \hat{u}_{t,c}^2 + \gamma_t n_t^1 - \frac{n_t^1}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t},$$

$$(A9) \quad \frac{\partial W}{\partial (n_t^2 U_t^2)} n_t^2 u_{t,c}^2 = -\lambda_t u_{t,c}^2 + \gamma_t n_t^2 - \frac{n_t^2}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t},$$

$$(A10) \quad \frac{\partial W}{\partial (n_{t-1}^1 U_{t-1}^1)} n_{t-1}^1 u_{t-1,x}^1 = \lambda_{t-1} \hat{u}_{t-1,x}^2 + \gamma_t n_{t-1}^1 - \frac{n_{t-1}^1}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t},$$

$$(A11) \quad \frac{\partial W}{\partial(n_{t-1}^2 U_{t-1}^2)} n_{t-1}^2 u_{t-1,x}^2 = -\lambda_{t-1} u_{t-1,x}^2 + \gamma_t n_{t-1}^2 - \frac{n_{t-1}^2}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t}.$$

By substituting equations (A8)-(A11) into equation (A7), and collecting terms, we obtain

$$(A12) \quad \begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} &= \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} \frac{\bar{\beta}_{t+1}}{1-\bar{\alpha}_t} - N_t \gamma_t \frac{\bar{\alpha}_t}{1-\bar{\alpha}_t} - N_{t+1} \gamma_{t+1} \frac{\bar{\beta}_{t+1}}{1-\bar{\alpha}_t} \\ &+ \frac{\lambda_{t-1} \hat{u}_{t-1,x}^2}{1-\bar{\alpha}_t} [\hat{\alpha}_t^{2,x} - \alpha_t^{1,x}] + \frac{\lambda_t \hat{u}_{t,c}^2}{1-\bar{\alpha}_t} [\hat{\alpha}_t^{2,c} - \alpha_t^{1,c}] + \frac{\lambda_t \hat{u}_{t,x}^2}{1-\bar{\alpha}_t} [\hat{\beta}_{t+1}^{2,x} - \beta_{t+1}^{1,x}] + \frac{\lambda_{t+1} \hat{u}_{t+1,c}^2}{1-\bar{\alpha}_t} [\hat{\beta}_{t+1}^{2,c} - \beta_{t+1}^{1,c}], \\ &= \frac{1}{1-\bar{\alpha}_t} \left[\bar{\beta}_{t+1} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} + N_t \gamma_t [\alpha_t^d - \bar{\alpha}_t] + N_{t+1} \gamma_{t+1} [\beta_{t+1}^d - \bar{\beta}_{t+1}] \right] \end{aligned}$$

where we have used the short notations α_t^d and β_t^d as defined earlier. Using the short notations

$$\begin{aligned} A_t &= \frac{N_t \gamma_t [\alpha_t^d - \bar{\alpha}_t]}{1-\bar{\alpha}_t}, \\ B_t &= \frac{N_{t+1} \gamma_{t+1} [\beta_{t+1}^d - \bar{\beta}_{t+1}]}{1-\bar{\alpha}_t}, \\ \varphi_t &= \frac{\bar{\beta}_{t+1}}{1-\bar{\alpha}_t}, \end{aligned}$$

the recursive equation (A12) can more conveniently be rewritten and expanded as follows:

$$(A13) \quad \begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} &= A_t + B_t + \varphi_t \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} = A_t + B_t + \varphi_t \left[A_{t+1} + B_{t+1} + \varphi_{t+1} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+2}} \right] \\ &= A_t + B_t + \varphi_t \left[A_{t+1} + B_{t+1} + \varphi_{t+1} \left[A_{t+2} + B_{t+2} + \varphi_{t+2} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+3}} \right] \right] \\ &= A_t + B_t + [A_{t+1} + B_{t+1}] \varphi_t + [A_{t+2} + B_{t+2}] \varphi_t \varphi_{t+1} + [A_{t+3} + B_{t+3}] \varphi_t \varphi_{t+1} \varphi_{t+2} \dots \\ &= A_t + B_t + \sum_{i=1}^{\infty} [A_{t+i} + B_{t+i}] \prod_{j=1}^i \varphi_{t+j-1} \end{aligned}$$

Substituting back $\varphi_t = \bar{\beta}_{t+1}/[1-\bar{\alpha}_t]$ into equation (A13) implies equation (26).

Derivation of equation (31)

By combining equations (22) and (30), we obtain

$$(A14) \quad T_t'(w_t^1 l_t^1) = \frac{\lambda_t^*}{w_t^1 n_t^1} \left[MRS_{z,c}^{1,t} - \phi M\hat{R}S_{z,c}^{2,t} \right] - \frac{MRS_{z,c}^{1,t}}{w_t^1} \frac{\rho^d - \bar{\rho}}{1-\bar{\rho}}.$$

Then, by using $MRS_{z,c}^{1,t} / w_t^1 = 1 - T_t'(w_t^1 l_t^1)$ and rearranging, we obtain equation (31) for the low-ability type. The corresponding tax rate for the high-ability type is derived in a similar way.

Derivation of equation (32)

Substituting equation (30), for period t and period $t+1$, into equations (24) and (25), we obtain

$$(A15) \quad \Phi'_{t+1}(s_t^i r_{t+1}) = \delta_t^i + \frac{1}{r} \frac{\rho^d - \bar{\rho}}{1 - \bar{\rho}} \left[\frac{\gamma_t}{\gamma_{t+1}} - MRS_{c,x}^{i,t} \right]$$

for $i=1, 2$, where we have used the short notations δ_t^1 and δ_t^2 as defined earlier. Using $MRS_{c,x}^{i,t} = 1 + r - r\Phi'_{t+1}(s_t^i r_{t+1})$ together with $\gamma_t / \gamma_{t+1} = 1 + r$ in equation (A15) and rearranging, we obtain equation (32).

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