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Black-Scholes Option Pricing Formula

An empirical study

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Industrial and Financial Management
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Abstract

Title: The Black and Scholes Option Pricing Model – An Empirical Study

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Keywords: Black and Scholes, call option, put option, option pricing, volatility, price difference, pricing error, moneyness, at-the-money, in-the-money, out-of-the-money, deep-in-the-money, deep-out-of-the-money, dividend, risk free interest rate, time to expiry, standard deviation, correlation coefficient, Least-Squares Linear Regression Analysis.

Purpose: The purpose of this study is to empirically test the accuracy of the Black and Scholes model by examining the difference between theoretical prices predicted by the model and actual market prices. We will also try to determine whether the accuracy of the model varies with the time left to expiration or the *moneyness* of an option.

Method: In order to examine the accuracy of the model we will compare the theoretical option prices of the model to the actual prices observed on the market. We will also examine how the differences in price relates to time left to expiration and moneyness, meaning the degree in which the options are in- or out-of-the-money, of the options.

The stocks chosen for this study are the five Swedish stocks on the OMX Nordic Exchange whose options had the largest total trading volume during 2007. The stock options in question are ABB, Astra Zeneca, Boliden, Ericsson and Hennes & Mauritz.

Conclusion: When conducting the study we found that approximately 70% of all our observed options had less than 90 days to expiration, approximately 60% of all observations were out-of-the-money and the center of gravity for all observations was shifted towards being out-of-the-money and underpriced by the Black and Scholes model.

The conclusion of the study is that the relative pricing error is generally larger for observations out-of-the-money than for observations in-the-money. And secondly, the relationship between relative pricing error and time left to expiry suggests that options with little time left to expiry are priced slightly less accurately than options with longer time left to expiry.

Glossary

This short glossary contains a list of abbreviations and terms that we have used in this study. Most of the abbreviations are only used in the headings of the figures in the Appendixes for space purposes.

General Abbreviations

ATM	At-the-money
ITM	In-the-money
OTM	Out-of-the-money
STD	Standard deviation
DTE	(Number of) Days to expiry
OMX	The Nordic exchange
C_M	Market price of a call-option
C_{BS}	Theoretical Black and Scholes Price of a call-option
D	The relative pricing error, $D = \frac{C_M - C_{BS}}{C_{BS}}$
M	The Moneyness of an option, $M = \frac{S - Ke^{-rT}}{Ke^{-rT}}$

Short names for stocks

ABB	Asea Brown Boveri
AZN	AstraZeneca
BOL	Boliden
ERICB	Ericsson B
HMB	Hennes & Mauritz B

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Chapter One: Introduction to the Study

This thesis is an empirical study of the Black and Scholes Option pricing Model; a more than thirty year old option model still widely used in financial markets around the world. In this study we will examine the model's applicability on Swedish financial markets. An introduction of the study will follow below. In this chapter will introduce the reader to the background, purpose, group of interested parties and delimitations of the study.

1.1 Background

Before we present the problem discussion and the Black and Scholes model we want to start off by presenting the theoretical background and terminology of financial option contracts. We believe it is important to be familiar with the theoretical terminology of financial options before continuing in order to fully apprehend this study and its purpose.

A financial option contract is a contract that gives the holder the right, but not the obligation, to sell or buy an asset in the future at a fixed price. There are two basic contract types, *call options* and *put options*. A call option gives the holder the right to buy an asset and a put option gives the holder the right to sell an asset at a specified price at a specified time in the future (Berk & DeMarzo, 2007). The contracted price at which the asset to be bought or sold is known as the *exercise price* or *strike price*. The date when the option contract expires is known as the *expiration date*. Options are divided into two main groups depending on when they can be exercised. American options are the most common kind and can be exercised at any time between initiation date and expiration date. European options on the other hand can only be exercised on the expiration date (Hull, 2003).

Options can be *at-the-money*, *in-the-money* or *out-of-the-money*. When an option's exercise price is equal to the current price of the underlying asset the option is said to be at-the-money. This goes both for call options and put options. When a *call option's* exercise price is less than the current price of the underlying asset the option is said to be in-the-money. And finally, when a *call option's* exercise price is higher than the current price of the underlying asset the option is said to be out-of-the-money (Berk & DeMarzo, 2007). The opposite goes for put options, i.e. when a *put option's* exercise price is higher than the current price of the underlying asset the option is said to be in-the-money and when a *put option's* exercise price is less than the current price of the underlying asset the option is said to be out-of-the money. The term in-the-money therefore refers to a situation where the holder of the option would make a profit had the option been exercised under the current market conditions. Out-of-the-money on the contrary is a situation where the holder would lose money had the option been exercised under current market circumstances. Whether an option is in, at or out-of-the-money is often measured on a scale ranging of five values: deep-out-of-the-money,

out-of-the-money, at-the-money, in-the-money and deep-in-the-money. A measurement of where an option resides on this scale is often referred to as its *moneyness*.

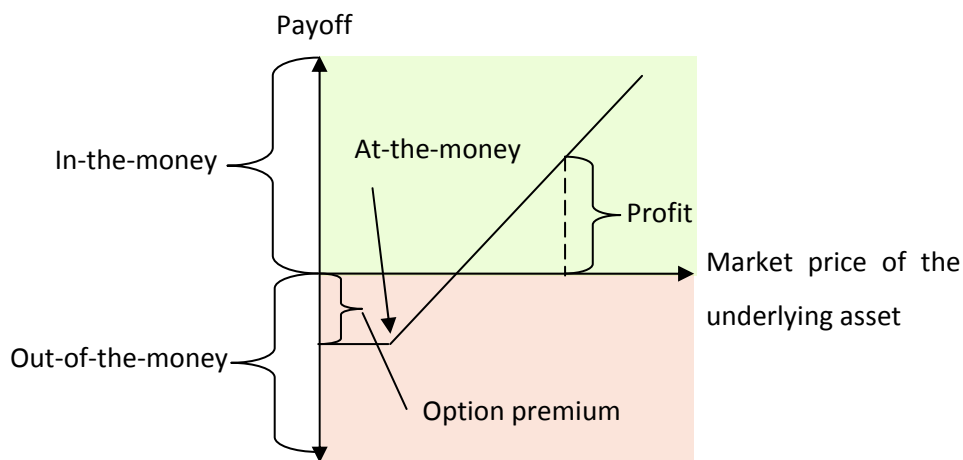


Figure 1

Options can be traded both on exchange markets and over-the-counter markets. Exchange markets are organized markets where standardized contracts are bought and sold without risk of default. In over-the-counter markets, trades are normally large and contracts are not standardized. Over-the-counter markets consist of network of dealers who quote prices at which they are prepared to sell or buy an asset. The disadvantage with over-the-counter markets is the credit-risk involved because of the risk of default (Hull, 2003).

The markets participants can generally be divided into three categories: hedgers, speculators and arbitrageurs. Hedgers use financial derivatives such as options to reduce risk that stems from market fluctuations. Speculators are the opposite of hedgers who use derivatives as a means to take on more risk in order to gain from the fluctuations in the market. The third group is the arbitrageurs. They are not interested in speculating nor, they are looking for market discrepancies in market prices which they use to make riskless profits (Hull, 2003).

So far we have introduced the theoretical background of this study, in the next section we will go on to the main subject of this study and discuss the problems regarding the Black and Scholes Model. We will also mention the group of interested parties, present the purpose and the delimitations made in the study.

1.2 Problem Discussion

The Black and Scholes model was first published in the Journal of Political Economy in year 1973 a few years after the trade with forwards and futures had started to blossom with Chicago as its financial center (Black & Scholes, 1973). Many analyses have been made since then and more and more additions have been made to the original model to enable calculations with options on new assets like stocks with dividend yield, currencies and so on. The accuracy of the model is still not perfect and the difficult part in the model is how to predict the future volatility of the underlying asset, in order to determine a correct option price.

The most common way to estimate the future volatility of an asset is to make a measurement of its historical volatility and assume that the volatility of the asset will be the same in the future as it was in the past thus using the historical volatility as an estimate of the future volatility. The problem is deciding how far back one should measure to obtain a volatility as close to the actual volatility as possible.

In the Black and Scholes model five values are imputed to calculate the option price. The values inserted are: the price of underlying asset, the exercise price of the option, time to expiration, the risk free interest rate and the estimated volatility of the underlying asset. Out of these five, four are easily obtained market statistics. It is only the volatility of the underlying asset that has to be estimated. This means that the volatility is the only uncertain factor when calculating the market price of an option. Consequently, when the theoretical option prices suggested by the Black and Scholes model do not coincide with the market prices it is because the market has made its own implicit estimate of the future volatility of the underlying asset. This implicit volatility can be determined simply by trying different volatility values and calculating the theoretical price of the option until you obtain the same value as the market price.

The pricing of options is very important for the actors on the financial markets who are exchanging assets, hedging and speculating. Many of them use the Black and Scholes model as a tool to price options and would benefit from information on how accurate the model is.

1.3 Purpose

Our purpose is to empirically test the accuracy of the model by examining the difference between theoretical prices and real market prices. We will also determine if the accuracy of the model varies with the time left to expiration or the moneyness of the option.

1.4 Delimitation

The data material of the study is delimited to the period 2007/01/01 – 2007/12/31. We wanted to examine a whole year, from January to December. We therefore chose 2007 since it is the closest continuous historical year at the time. Furthermore the study is limited to examining the five Swedish stocks whose options were most traded on the Swedish stock option market during this period. The stocks whose options were the most trade during 2007 was: ABB, AstraZeneca, Boliden, Ericsson B and H&M B. The Black and Schoels model differs for call resp. put options. Mixing the two would mean that the results could not be deduced exclusively to one of the two models, making it harder to draw conclusions. We therefore also chose to limit ourselves to only examining call options.

1.5 Disposition

The disposition and content of this study will be presented in the following order.

Chapter One: Introduction to the Study. *Here we present the background, problem discussion, purpose and delimitation of the study.*

Chapter Two: Theoretical Framework. *In this chapter we will illustrate the Black and Scholes model, earlier studies and discuss the assumptions in the model.*

Chapter Three: Methodology. *The research process of this study will be discussed in this chapter, what research method was chosen, validity, reliability and research critique.*

Chapter Four: Data Processing and Calculations. *This chapter will present the data material processing and the calculations made in order to examine the model.*

Chapter Five: Results and Analysis. *Here we will discuss the calculations and see how we can analyze the results.*

Chapter Six: Results and Conclusions. *A presentation of the results and conclusion will follow in this chapter.*

Chapter Seven: Closing Discussion. *We will end this study with a closing discussion and give further suggestion on subjects for research.*

At the end of the study you will find a list of references and appendixes where we illustrate the results graphically in the form of diagrams.

Chapter Two: Theoretical Framework

We start this chapter by introducing earlier research and studies made in the field of option pricing. Then we continue by illustrating the Black and Scholes model and the assumptions being made in the model. Finally this chapter ends with an illustration of the most important parameter in the Black and Scholes model which is the volatility.

2.1 Earlier Studies

In this section we will introduce some of the relevant studies that have been made in the field of option pricing and their result. When Black and Scholes published their option pricing model in 1973 their study were pioneering. Many studies have been published since then some of which are developments of the Black and Scholes model and some new competing models. Many previous studies of the Black and Scholes model show conflicting results and we will present such results from a couple of authors. The results from the authors who will be presented are: Macbeth and Merville, Merton, Hull and White and finally Byström.

In 1979 the two researchers Macbeth and Merville tested the Black and Scholes empirically on call options. They found that the Black and Scholes model tends to overprice out-of-the-money options and underprice in-the-money options with a remaining duration of less than ninety days. Furthermore, they came to the conclusion that the more in-the-money an option is the more the model underprices and vice versa for out-of-the-money options. Macbeth and Merville relate to the results of Black and Merton in this study and points out the fact that Black on the other hand came to the conclusion that deep-out-of-the-money options are underpriced by the model while deep-in-the-money options are overpriced by the model. This is not the only conflicting empirical result made by researchers. The results of Merton's study conflict with the result of the previous mentioned authors Macbeth, Merville and Black. Merton suggests the Black and Scholes theoretical option prices are lower than market option prices for both deep-in-the-money and deep-out-the-money options.

Later in 1987 Hull and White made an empirical study of the Black and Scholes model using random (stochastic) volatility instead of assuming constant volatility. This is a wide spread adaptation of the model today, but was news when Hull and White made their study. Their result showed that the theoretical prices of options in-the-money are underpriced and options out-of-the-money are overpriced. These results show that the overpricing increase with the remaining time of duration and also points out the more out-of-the-money the higher the overpricing.

A study was made on OMX-index call options in 2000 by a Swedish researcher, H. Byström. He showed that regardless of whether using a constant or a stochastic volatility the Black and Scholes

model more accurately prices options at-the-money and in-the-money than options out-of-the-money.

This is a small selection of the conflicting results in studies that have been published during the years. We will now continue and present the Black and Scholes model theoretically as a whole and illustrate the assumptions being made in the model.

2.2 The Black and Scholes Option Pricing Model

The Black and Scholes option pricing formula is presented in the equations below.

$$C = S \times N(d_1) - Xe^{-rt} \times N(d_2)$$

$$P = Xe^{-rt} \times N(-d_2) - SN(-d_1)$$

$$d_1 = \frac{\ln[S/X] + (r + \sigma^2/2)t}{\sigma \times \sqrt{t}}$$

Where

C Price of a call option

P Price of a put option

S Price of underlying asset

X The strike price

R The riskless rate

T Time to maturity

σ^2 Variance of underlying

σ Standard deviation of underlying

N The cumulative normal distribution

$$d_2 = d_1 - \sigma \times \sqrt{t}$$

There are five values that need to be inserted into the Black and Scholes option pricing model in order to calculate a theoretical option price. These are *S*; the current price of the underlying asset and *X*; the strike price of the option and *r*; the riskfree interest rate and *t*; the time left to expiry measured in years and finally σ ; the estimated volatility of the underlying asset. Out of these five, *S*, *T* and *X* are usually directly observable from option data itself. The riskfree interest rate needs to be approximated since there is no rate that is truly riskfree but good approximations such as treasury bonds and interbank rates are readily available. And last the most important input variable is the volatility of the underlying asset as this is the only variable that is directly tied to the underlying asset (Hull, 2003).

Since the volatility of the underlying asset is the only unknown variable in the equation we can quickly deduce that any deviations between the market price and the theoretical Black and Scholes price must be a result of the market having set its own implicit volatility that differs from the one used to calculate the theoretical price.

2.3 Assumptions in the Model

There are a number of assumptions in the Black and Scholes model. Even though some of the assumptions made in the model do rarely reflect real market conditions the model does not lose its popularity as there are ways to circumvent the discrepancies between the assumptions in the model and real life. The assumptions are as follows:

- *European options.* Black and Scholes assume the options being priced in the model are European options. There are two kinds of options depending on when they can be exercised. European options can only be exercised on the expiration date. American options are however the most common kind and they can be exercised at any time between initiation date and expiration date.
- *No dividends occur.* The Black and Scholes model does not take into consideration that dividends occur in the financial markets. However you can compensate for this by reducing the observed price of the underlying stock with the net present value of the dividends payment discounted with the riskfree interest rate (Hull, 2003)
- *No transaction cost.* In reality there are transaction costs when buying or selling the underlying asset or option.
- *There are no penalties when selling short.*
- *The risk-free interest rate is constant.* The risk-free interest rate is the interest rate to which it is possible to borrow money without any risk of default, i.e. one can be positive the money will be paid back. The model assumes the risk-free interest rate is known and constant over time. Risk-free interest rate only exists in theory. In reality there is no risk-free interest rate. In general treasury bonds are used as an approximation of the risk-free interest rate in the Black and Scholes model.
- *The volatility is constant.* The volatility of the underlying asset's price is according to Black and Scholes also known and constant. This is an incorrect assumption since in reality it changes over time.

2.4 Volatility

The volatility of the underlying asset is the most important variable when calculating theoretical option prices since it is the only variable that is directly tied to the underlying instrument.

The general equation for calculating volatility is presented below.

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (u_i - \bar{u})^2} ; u_i = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

Where

- σ The standard deviation
- u_i The lognormal return
- \bar{u} The mean of the lognormal return
- n The number of observations

(Körner & Wahlgren, 2002)-

Since the volatility of an asset changes over time the measurement of historical volatility is merely an estimate of the future volatility of the asset. It is therefore hard to decide on how many historical days to base your calculations. Hull (2003) discusses this issue in his book "Options futures and other derivatives" and he suggests that a good rule of thumb is to set the number of observations, n , to the same amount of days that the volatility is to be applied to. In other words when setting the price of an option with 120 days left to expiration one should base the historical volatility measurement on 120 days alike.

Chapter Three: Methodology

In this chapter we will discuss the research process of this study. We begin by introducing the chosen research method followed by a short paragraph illustrating the relation between empirics and theory. We then continue by describing how we carried out the study in practice and present the chosen theory. The chapter will end with a discussion of the importance of validity, reliability and research critique.

3.1 Choice of Method

Within the field of scientific research there are two main methods for collecting and analyzing data. They are described as qualitative and quantitative. The choice of method mostly depends on the nature of the study and the result wanted (Patel & Davidsson, 2003).

The aim of qualitative studies is to describe a certain phenomenon on a deeper level. Qualitative data is therefore normally based on text rather than on numbers, for example: written stories, transcribed interviews, observations and so on (Holme & Solvang, 1997). Quantitative studies on the other side aims to process and compare large quantities of data, statistical selections and populations. The result of a quantitative study should be possible to reproduce several times, generating the same outcome as long as the source of data is the same.

In our study we will process a large number of daily closing prices on options, in total approximately 16000 observations. Our calculations will then be used to analyze and draw conclusions about the Black and Scholes pricing model for options. Our study is therefore a quantitative study.

3.2 Inductive and Deductive

The relation between theory and empirics can be described through two concepts, induction and deduction. Induction can be described as a process of making conclusions based on experience i.e. a child who has burnt itself will keep its distance from a hot stove in the future. This process of making conclusions based on experience is described as inductive. However, if you start with already existing theories, and from those draw conclusions of the collected data, the study is called to be deductive (Patel & Davidsson, 2003).

In our study we will be using the deductive process, using an existing theory with already set preconditions to test on our empirics (Lindblad I, 1998). In our case this means we will examine the Black and Scholes pricing model for options and from our empirical data and calculations see if we can draw any conclusions on how well the model performs.

3.3 Choice of Theory

There is a big interest in option pricing on the market and through the years a number of competing and acknowledged option pricing models have been developed from science research. Some of the more renowned options pricing models are Black and Scholes, the Binominal Option Pricing Model and the Monte Carlo Option Pricing Model. We chose to study Black and Scholes since it is the most used and widely known option pricing model in the financial markets.

3.4 Conducting the Study

The purpose of this study is to examine to which extent the theoretical price on stock options suggested by the Black and Scholes model concur with market prices. We will also examine if there is any relation between the degree of miss pricing, time to expiry and moneyness of the options.

The data material in this study is represented by the five stocks whose options were most traded during 2007 on the Swedish exchange, OMX. The stock options in question are ABB, AstraZeneca, Boliden, Ericsson and H&M. The source data material is secondary data consisting of approximately 83 000 observations of daily closing prices collected from the OMX Nordic Exchange during the time period 2007/01/01 - 2007/12/31. We decided that we wanted to perform the study on a full calendar year, hence the chosen time period as 2007 is the closest historical continuous year at the time. To be able to make all the necessary calculations we used MS Excel as a tool for analyzing the data. We had to use several advanced formulas and VBA-macros (Visual Basic) to automate the processing of observations since there were so many.

To start with we had a large number of observations of option prices. A lot of the observations were however made on days where no trade had occurred with the option in question so no price was therefore set for the option on that day. We thought about setting the price of the option to the same value as when it was last traded in order to obtain more observations for our study. But doing so would be assuming that the reason that the option was not traded was that the market price of the option had not changed. We concluded that such an assumption was wrong as there could be other reasons to why an instrument is not traded on a specific day such as lack of liquidity. We therefore eliminated approximately 80% of all observations, leaving approximately 16 000 observations left to conduct the study on.

The Black and Scholes model assumes that dividend yield occurs. Since dividends are paid on the underlying asset we had to choose between excluding all observations where dividends was paid or compensate for the effect of the dividend payment in some way. We chose to compensate for the dividend effect by adjusting the price of the underlying asset for those observations where dividend

is paid during the remaining lifespan of the option. Apart from this we have not made any further adjustments to the source material.

3.5 The Validity and Reliability of the Study

There are certain factors to consider when performing a scientific study in order for the study to be considered qualitative and trustworthy. The factors affecting the quality and the trustworthiness in a study are the validity, the reliability and the objectivity (Eriksson & Wiedersheim, 2006). Since our study is based on quantitative data it is easier to stay objective than if the study would have been based on qualitative data in the form of personal meetings, interviews, texts, stories and so on. Consequently we will put the main focus in this study on the validity and the reliability.

Validity is a term that defines if the study really measures what it intends to measure. The term also includes the relevance of the result. One has to consider if the outcome and result of the study is of relevance to the group of interested parties? (Eriksson & Wiedersheim, 2006). We believe the validity in this study is high as we are studying an established option pricing model and the relation between the market prices on options and the theoretical option suggested by the Black and Scholes model.

Reliability is a question of the ability to show trustworthy results in a scientific study. As mentioned earlier Eriksson & Wiedersheim (2006) also states that independent researcher should be able to conduct quantitative studies using the same data material and still show the same or similar results. Since our study is built up by quantitative data anyone who wishes should be able to perform this study all over again and still get the same or similar results as us. According to Esaiasson et al (2007) high reliability can be obtained if the researcher aim to be objective, uses data sources with high credibility and performs the study systematically and precisely. We have been working with the criteria recommended by Esaiasson et al and feel that our study has a high level of reliability. What would even more increase the reliability of this study would be if an independent researcher would do the same study resulting in the same conclusions.

3.6 Research Critique

When calculating the theoretical price of an option using the Black and Scholes model the volatility is the only variable that is directly tied to the underlying instrument. Therefore the volatility variable is of great importance to theoretical value of the option and thus in our case critical for the conclusion of our study. The historical volatility can be calculated in many different ways using different models. We have chosen to calculate the historical volatility with an unbiased estimate of the n most recent observations of the stocks movement where n is set to same number of observations as there are trading days left to expiration of the option.

We could have chosen other ways of calculating the historical volatility of the stock such as EWMA, ARCH or GARCH. They might have given us a better estimate of the historical volatility and thus theoretical option prices closer to market prices. However, using these more advanced ways of calculating historical volatility for such a large number of observations that we are dealing with would pose a problem. We would require many times the computing power using much more advanced tools than Excel. Also, using more advanced ways of predicting future volatility does not guarantee better results, as mentioned above it *might* give better results. We have therefore chosen to use the more straightforward, simple, way of calculating historical volatility mentioned above.

Chapter Four: Data Processing and Calculations

In this chapter we present how we processed the data material and made the calculations that we will later use in the analysis. We illustrate how we calculated on all parameters used in the Black and Scholes model in order to examine the theoretical option prices. The parameters we had to take into consideration in order to examine the model were: dividends, time to expiry, implied and historical volatility, moneyness and price differences, standard deviation from zero of D , least-squares linear regression analysis and the correlation coefficient.

4.1 The Source Data

We received our source data directly from OMX Nordic Exchange. The stocks chosen for this study are the five Swedish stocks on the OMX Nordic Exchange whose options had the largest total trading volume during 2007. The stocks chosen are ABB, Astra Zeneca, Boliden, Ericsson and Hennes & Mauritz. For the remainder of this study the stocks will be named by their respective short names on the stock market meaning; ABB, AZN, BOL, ERICB and HMB.

Option	Obs Date	TYPE	S	X	Expire date	C_M	Vol	rf	TTM	$I\sigma$	$H\sigma$	C_{BS}	M	D
ABB7B100	2007-01-15	CALL	123.75	100.00	2007-02-16	25.25	40.00	3.20%	0.1000	57.88%	19.83%	24.070	0.241	0.049

Table 1

Table one above is an example observation made on 15th of January 2007 on a call option on ABB with the exercise price of 100 SEK and expiry date 2007-02-16. The market price of the option on the observation day was 25.25 SEK, the traded volume was 40 contracts, the riskfree interest rate was 3.20% and the time to expiry was 0.1 years or 25 days. Given this information we calculated the implied volatility to 57.88%, the historical volatility to 19.83% and the theoretical Black and Scholes market price to 24.07 SEK. The two parameters M and D in table one above stands for *moneyness* and *price difference*.

All of the observations will be divided into three groups depending on time to expiry for each observation. The three groups are options with time to expiry between 1 - 89 days, 90 – 180 days and observations with time to expiry in excess of 180 days. The reason for doing this is that we hope to come to some conclusions about the relationship between the accuracy of the model and time left to expiration.

The stocks and some general data about them are presented in table 2 below.

	Number of valid Observations	Observations with 1 - 89 days to expiry	Observations with 90 - 179 days to expiry	Observations with 180+ days to expiry	Trading Volume	Return 2007
ABB	2 237	1 826	297	114	629 748	46.83%
AZN	2 387	1 614	446	327	925 581	-25.54%
BOLI	2 262	1 925	181	156	634 438	-53.97%
ERICB	5 034	3 472	611	951	18 919 640	-46.17%
HMB	2 694	1 958	465	271	850 403	13.07%
TOTAL	14 614	10 795	2 000	1 819	21 959 810	---
MEAN	2 922.80	2 159.00	400.00	363.80	4 391 962	-13.15%
OMXS30	250.00	---	---	---	---	-7.10%

Table 2

4.2 Dropout

During our study there were quite a substantial number of the initial observations that had to be excluded from the study. We started with about 50 000 observations and ended up using about 13 000 of them in our study. We therefore feel that it is important to the reliability of our study to thoroughly explain why these observations were excluded from the study.

When we started we had on average 10 000 observations of option prices *per stock*, meaning we had almost 50 000 observations in total. A lot of the observations were however options that had been offered for sale on the market but that had never been bought during its lifetime. In other words there had never been any trade with those options what so ever. These options accounted for roughly half of our total number of observations.

After removing never traded options a total of about 25 000 observations remained. Out of those, yet again about half were observations where no trade had occurred on the observed day. During their lifetime, the remaining options had been traded during at least one day but not during all days. After examining the options that had been traded only during one or a few days we noticed that those options were generally deep-out-of-the-money options with little chance of ever being exercised, often traded in small volumes. We suspect that a lot of the trading with these options was of a speculative nature where the buyer was hoping for a sudden substantial price change in the underlying stock.

We now had to determine how to handle the remaining observations that regarded days when an option had not been traded. These “no trade” –observations can be divided in to two categories. The first category concerns observations where no trade has occurred on the observed day and where the option has not been traded during its lifetime prior to the observation date. In this case there is

no market price available for the options and therefore observations like these must be omitted. This category amounted to about 7 000 observations so after removing them we had about 18 000 observations left.

The second category concerns observations where no trade has occurred on the observation date but the option has been traded prior to the observation date. These observations can be handled in one of two ways. Either we eliminate all these observations or we substitute the observed market price for these observations with the price at which the option was last traded. As we discussed earlier, these options were generally only traded during one or a few days during their lifetime in low volumes and were also generally deep-out-of-the-money. We therefore reasoned that substituting the observed price for these observations with the price at which the option was last traded would be wrong as the options are presumably very illiquid and could probably not be sold on the market even if the holder wished to do so. We therefore decided to eliminate also these observations bringing the remaining number of observations to about 16 000.

Finally, for some of the observations, approximately 1300, the Black and Scholes model returned theoretical prices so small that they were tangent to zero. All of these observations were invariably deep-out-of-the-money with little chance of ever being exercised. The theoretical prices being so low caused some problems later on in the study when calculating the relative price difference between the theoretical Black and Scholes prices and market prices. After careful consideration we have chosen to exclude all observations where the theoretical Black and Scholes were too small to enable a useful comparison with market prices. As we will show later in this study, this will have no effect on our conclusions.

After carefully reviewing the observations and excluding those that we do not deem fit for our study we end up with 14 614 valid observations to work with. Although heavily reduced we still believe that the remaining number of observations is sufficient for the purposes of this study.

4.3 Dividends

A common problem when valuing options with the Black and Scholes option pricing model is that the model makes an assumption that no dividends occur during the duration of the option. It is possible to compensate for this by reducing the observed stock price by the present value, discounted with the riskfree interest rate, of any dividend paid between the observation date and the expiration date (Hull 2003).

We have adjusted all of our stock observations according to this procedure and therefore feel that we have properly compensated for the effects of any dividends paid on the stock. The dividends paid for each stock can be seen in table 3 below. Dividend data has been collected from the respective homepage of each company and the internet journal *Privata affärer*.

STOCK	Dividend date	SEK
ABB	2007-05-15	1.34 SEK
AZN	2007-03-19	8.60 SEK
AZN	2007-09-17	3.49 SEK
BOLI	2007-05-04	3.69 SEK
ERICB	2007-04-12	0.50 SEK
HMB	2007-05-04	11.50 SEK

Table 3

4.4 The Risk free Interest Rate

For this study we have chosen to use the closing rate of three month Swedish treasury bills as the risk free interest rate. The rate data has been gathered from the Swedish central bank's homepage and the values are updated daily in our calculations.

4.5 Time to Expiry

In order to correctly price an option with the Black and Scholes option pricing model we have to determine the time left to expiration for each option on each observation. We have done this by gathering information about public holidays and determining on exactly which days the exchange was open and on which days it was closed. This data was then confirmed by the stock data that we received from OMX Nordic Exchange since the dates on which the exchange was closed were missing in these data series. We determined that the exchange was open exactly 250 days during 2007 and for each observation we have used an excel formula called *net working days* to determine the exact number of exchange days left until expiration for each observation. Time to expiry and the historical volatility was then calculated only based on days when the exchange was open i.e. based on 250 days rather than 365.

4.6 Historical Volatility

We have calculated the historical volatility individually for each observation. When calculating historical volatility one must solve for the optimal number of days or observations, n , to base the volatility calculation on. Hull (2003) suggests that a good rule of thumb is to set n equal to the number of days to which the volatility is to be applied, i.e. when valuing an option with 120 days to expiry one should calculate the historical volatility on 120 days backwards. We have however added a constraint in that we never calculate the historical volatility on less than 90 days backwards. Our historical volatility is in other words calculated on the same number of days left until expiration unless that value is less than 90 days in which case 90 days is used.

To calculate the volatility of the stock we first calculate the daily return of the stock using the following equations:

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{t=1}^n (u_t - \bar{u})^2} ; u_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

This gives us the average daily volatility for the time period and we then multiply this value with the square root of 250 to obtain the annual volatility.

Table 4 below shows the historical volatility during 2007 for each of the stocks that we examined.

ABB	AZN	BOL	ERICB	HMB	OMXS30
29.65%	19.90%	40.27%	40.36%	23.34%	20.28%

Table 4 Historical volatility based on all values during 2007

4.7 Moneyness and Price Differences

M and D are calculated as shown by the equations below. M shows how far in- or out-of-the-money the option is as a percentage of the present value of the exercise price discounted with the risk free interest rate. A negative value of M means that the option is out-of-the-money and a positive value means that the option is in-the-money.

D shows the relative pricing error made by the Black and Scholes option pricing model as a percentage of the theoretical price, giving positive values where Black and Scholes underprices an option and vice versa. Since the market value of an option can never be less than zero, D can never be less than -1. Plotting these two values as coordinates in a table will show us the relationship between the relative pricing error and moneyness. The equations for calculating m and d are:

$$m = \frac{S - Xe^{-rt}}{Xe^{-rt}} ; d = \frac{C_M - C_{BS}}{C_{BS}}$$

The values of m and d will be plotted in a system of co-ordinates as shown below. D will be plotted against the y-axis and m will be plotted against the x-axis.

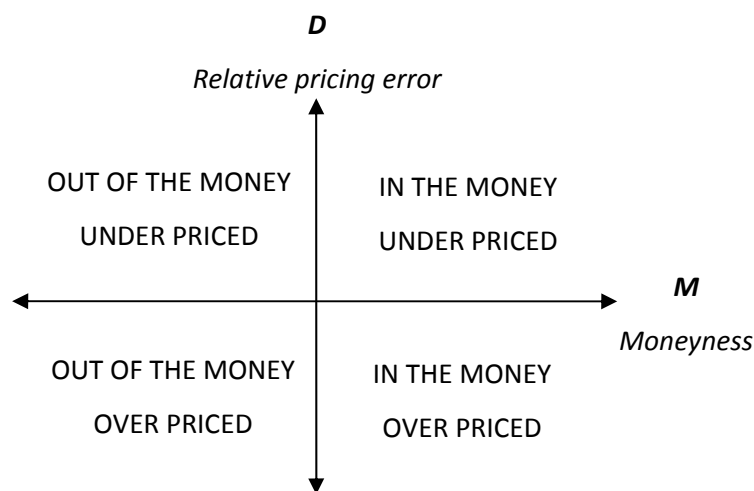


Figure 2

We have created charts of the results of these calculations which can be seen in Appendix A. We have created one chart per stock and time interval. In table 5 below we have summed up the total number of observations per time interval.

ALL Observations				1 - 89 Days to expiry			
	ITM	OTM	SUM		ITM	OTM	SUM
$C_M > C_{BS}$	33.30%	23.74%	57.05%	$C_M > C_{BS}$	32.84%	23.46%	56.30%
$C_{BS} > C_M$	26.85%	16.10%	42.95%	$C_{BS} > C_M$	26.15%	17.55%	43.70%
SUM	60.15%	39.85%		SUM	58.99%	41.01%	

90 - 179 Days to expiry				180+ Days to expiry			
	ITM	OTM			ITM	OTM	
$C_M > C_{BS}$	43.35%	23.10%	66.45%	$C_M > C_{BS}$	25.01%	26.11%	51.13%
$C_{BS} > C_M$	25.35%	8.20%	33.55%	$C_{BS} > C_M$	32.66%	16.22%	48.87%
	68.70%	31.30%			57.67%	42.33%	

Table 5

4.8 Standard Deviation from Zero of D

After completing the calculations of M and D we went on to calculate the standard deviation from zero of D to show how large the mispricing of the options was. The standard deviation is normally a measurement of how much a population of values deviates from their own mean value. The equation used to calculate the standard deviation is normally:

$$S = \sqrt{\frac{\sum(X - \bar{X})^2}{n - 1}}$$

(Körner och Wahlgren, 2002)

We however modified the equation deleting the part \bar{X} that represents the mean value. With this part of the equation gone the result will instead show the standard deviation from zero for the population which is exactly what we want to measure. The equation we used was this:

$$S = \sqrt{\frac{\sum X^2}{n - 1}}$$

To illustrate how this works consider the numbers -15, -8, 7 and 16. The mean of these numbers are zero. If we were to calculate the standard deviation with the original equation we would obtain a standard deviation of approximately 14. The same result is obtained using our modified equation. This shows that altering the equation as we have done has the same effect as measuring the population's standard deviation from zero.

The results of our calculations are presented in Table 6 below.

ALL Observations				1 - 89 Days to expiry			
	ITM	OTM	ALL		ITM	OTM	ALL
$C_M > C_{BS}$	35.92%	15.76%	29.27%	$C_M > C_{BS}$	38.66%	17.31%	31.56%
$C_{BS} > C_M$	30.46%	12.72%	25.31%	$C_{BS} > C_M$	31.22%	12.71%	25.46%
ALL	33.59%	14.61%	27.81%	ALL	35.55%	15.51%	29.05%
90 - 179 Days to expiry				180+ Days to expiry			
	ITM	OTM	ALL		ITM	OTM	ALL
$C_M > C_{BS}$	26.70%	10.68%	22.46%	$C_M > C_{BS}$	28.40%	10.28%	21.16%
$C_{BS} > C_M$	31.43%	16.45%	28.49%	$C_{BS} > C_M$	25.60%	10.22%	21.73%
ALL	28.53%	12.44%	26.64%	ALL	26.84%	10.25%	21.44%

Table 6

4.9 Least-squares Linear Regression Analysis

To examine if there is a relationship between relative pricing error and time to expiry respectively moneyness we will use the least-squares linear regression model to calculate linear regression lines for each of the quadrants of our data series. We will be looking for individual linear relationships in each of the four quadrants as shown in figure 3 below. A strong linear correlation in for example the upper left quadrant would imply that the pricing error made by the Black and Scholes option pricing model follows a certain pattern which can be estimated. The regression lines will be plotted in charts and enclosed in Appendix C.

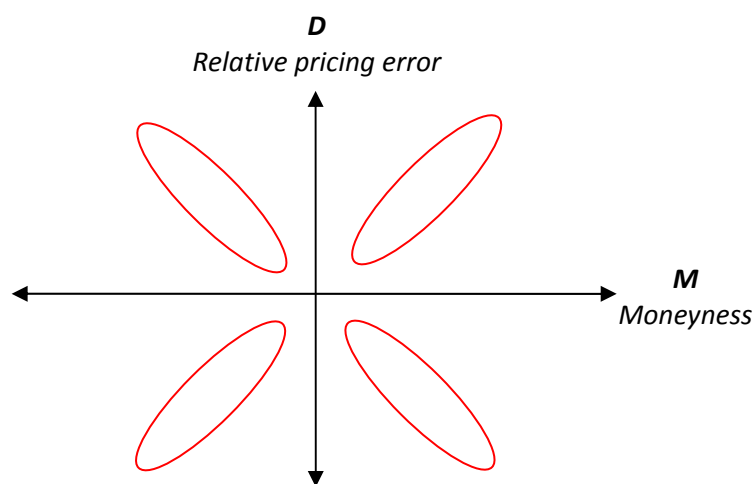


Figure 3

The equation of the regression lines is:

$$y = a + b \times x$$

Where:

$$a = \bar{y} - b \times \bar{x}$$

$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

4.10 The Correlation Coefficient

To test if the relative pricing error is related to either time left to expiration or the moneyness of the option we will calculate the correlation coefficient for our data series using the following formula.

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2 \sum(y - \bar{y})^2}}$$

Table 7 below shows some of the general correlations that we calculated when looking for linear correlation between the values of M and D .

ALL Observations		ITM	OTM	1 - 89 Days to expiry		ITM	OTM
$C_M > C_{BS}$		-0.14	-0.29	$C_M > C_{BS}$		-0.10	0.17
$C_{BS} > C_M$		0.30	0.25	$C_{BS} > C_M$		0.26	0.03

90 - 179 Days to expiry		ITM	OTM	180+ Days to expiry		ITM	OTM
$C_M > C_{BS}$		-0.24	-0.37	$C_M > C_{BS}$		-0.34	-0.29
$C_{BS} > C_M$		0.52	0.26	$C_{BS} > C_M$		0.33	0.24

Table 7

Table 8 below shows the R^2 residuals from the regression above, as can be seen in table 7 the correlation is not strong to begin with and the values in table 8 below also indicates a poor fit of the regression model on the data. Similar correlation and R^2 tests was done for all of the individual stock but no evidence of any correlation between M and D could be found.

ALL Observations		ITM	OTM	1 - 89 Days to maturity		ITM	OTM
$C_M > C_{BS}$		1,96%	8,48%	$C_M > C_{BS}$		0,99%	3,00%
$C_{BS} > C_M$		8,93%	6,46%	$C_{BS} > C_M$		6,52%	0,08%

90 - 179 Days to maturity		ITM	OTM	180+ Days to maturity		ITM	OTM
$C_M > C_{BS}$		5,80%	13,64%	$C_M > C_{BS}$		11,65%	8,67%
$C_{BS} > C_M$		27,54%	6,81%	$C_{BS} > C_M$		10,73%	5,66%

Table 8

4.11 The Black and Scholes Option Formula

The Black and Scholes option pricing formula is presented in the equations below.

$$C = S \times N(d_1) - X e^{-rt} \times N(d_2)$$

$$P = X e^{-rt} \times N(-d_2) - S N(-d_1)$$

$$d_1 = \frac{\ln[S/X] + (r + \sigma^2/2)t}{\sigma \times \sqrt{t}}$$

Pricing

presented

Where

C Price of a call option

P Price of a put option

X The strike price

R The riskfree rate

T Time to expiry

σ^2 Variance of underlying

σ Standard deviation of underlying

N The cumulative normal distribution

$$d_2 = d_1 - \sigma \times \sqrt{t}$$

The Black and Scholes option pricing model makes the following assumptions about reality:

- Stock prices follow a geometric Brownian motion with constant drift μ and volatility σ and the returns are log-normally distributed.
- It is possible to short sell the underlying stock.
- There are no transaction costs or taxes.
- There are no dividends during the life of the derivative.
- There are no riskfree arbitrage opportunities.
- Security trading is continuous. (I.e. it is possible to buy or sell any fraction of a share)

(Hull, 2003)

We have calculated the theoretical Black and Scholes option value with a VBA script in excel which can be seen below. The script works as a function in excel and enables us to easily calculate large numbers of theoretical prices for all of our observations. The script consists of two parts, one provides a function that calculates the $d1$ parameter and the other calculates the option price with use of the $d1$ function.

```

Function dOne(Stock, Exercise, Time, Interest, sigma)
  dOne = (Log(Stock / Exercise) + Interest * Time) / (sigma * Sqr(Time)) + 0.5 * sigma * Sqr(Time)
End Function
Function BSCallOption(Stock, Exercise, Time, Interest, sigma)
  BSCallOption = Stock * Application.NormSDist(dOne(Stock, Exercise, Time, Interest, sigma)) - Exercise *
  Exp(-Time * Interest) * Application.NormSDist(dOne(Stock, Exercise, Time, Interest, sigma)) - sigma *
  Sqr(Time))
End Function

```

As we discussed in section 1.4 we had a problem with calculating the theoretical prices of the options. Many out-of-the-money options with a short time to expiry had theoretical prices that were so low that they were tangent to zero. When this occurs the relative price difference that we calculated earlier becomes infinitely large even though the money-difference of the prices is not even a thousand of a Swedish crown (SEK). After careful consideration we chose to address this issue by eliminating all observations which produced a D value over 1 which meant removing 1 345 values.

The values that we excluded all have a short time left to expiry and are invariably out-of-the-money. As the equation returns the value D as a percentage of the theoretical Black and Scholes model price, D can never be less than -1 no matter how much larger the theoretical price is than the market price. Positive values of D however can be infinitely large.

All of the large values of D that were excluded are a result of out-of-the-money options with short times to expiration creating a theoretical Black and Scholes value that is close to zero. With such small theoretical option prices it is enough that 0.01 SEK is paid for an option for the D value to be out “of the charts” high. So although many of the excluded values of D are very big the price difference in real money terms is low, on average 0.7 Swedish Crowns (SEK). As we will show later removing these values has no effect on the validity of our conclusions. Also 91.57% of our observations have a D value between -1 and 1 so the excluded values represent about 8% of our total number of populations.

Chapter Five: Results and Analysis

The charts from plotting the different M and D values for our different stocks are for practical reasons presented in Appendix A. In this section we will present and analyze the results of this study. We will first illustrate some general results and then we will discuss the relative pricing error, moneyness and time to expiry. We will finish the chapter by referring back to our purpose that we set up for the study in chapter one.

5.1 General Results of the Study

The equation that we used to produce the value D is $d = \frac{C_M - C_{BS}}{C_{BS}}$. The equation in other words gives

us the over resp. under pricing of the option expressed as a percentage of the theoretical Black and Scholes model price. Positive values of D mean that the Black and Scholes model price is less than that of the market and vice versa. A more intuitive way to interpret the value of D could be to see it as if positive values of D means that the market over prices the option relative to the Black and Scholes model and vice versa.

When plotting the values of D and M against each other D is assigned to the y-axis and M to the x-axis. Table 9 below shows in percent how many of our total number of observations ended up in which quadrant of the co-ordinates system. 60.15% of the observations were out-of-the money and 39.85% were in-the-money. 57.05% of the observations were underpriced by the Black and Scholes model whereas 42.95% were overpriced. 33.30% of the observations were out-of-the-money and underpriced by the Black and Scholes model. 26.85% of the observations were out-of-the-money and overpriced by the model. 23.74% of the observations were in-the-money and underpriced by the Black and Scholes model. And finally 16.10% of the observations were in-the-money and overpriced by the model. These figures hint that the center of gravity for all the observations is out-of-the-money and underpriced.

ALL Observations				1 - 89 Days to expiry			
	ITM	OTM	SUM		ITM	OTM	SUM
$C_M > C_{BS}$	33.30%	23.74%	57.05%	$C_M > C_{BS}$	32.84%	23.46%	56.30%
$C_{BS} > C_M$	26.85%	16.10%	42.95%	$C_{BS} > C_M$	26.15%	17.55%	43.70%
SUM	60.15%	39.85%		SUM	58.99%	41.01%	
90 - 179 Days to expiry				180+ Days to expiry			
	ITM	OTM	SUM		ITM	OTM	SUM
$C_M > C_{BS}$	43.35%	23.10%	66.45%	$C_M > C_{BS}$	25.01%	26.11%	51.13%
$C_{BS} > C_M$	25.35%	8.20%	33.55%	$C_{BS} > C_M$	32.66%	16.22%	48.87%

SUM	68.70%	31.30%	SUM	57.67%	42.33%
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Table 9

5.2 The Relative Pricing Error and Moneyness

When looking at Figures 4 to 22 in Appendix A the most striking resemblance that is common for all the stocks we have examined is that the pricing error is grater for out-of-the-money options than for in-the-money options. This can be seen in all the M and D figures as a funnel shaped dispersion of the plotted values as we move from the right to the left along the x-axis. This is also confirmed when we calculate the standard deviation of all observations that are in- respectively out-of-the-money as can be seen in Table 7. A f-test was also performed to test whether the mispricing difference between in-the-money and out-of-the-money options is statistically significant. The null hypothesis is that the variance for in-the-money and out-of-the-money are the same; $H_0 = \sigma_{ITM}^2 = \sigma_{OTM}^2$. The p-value returned by the test was 0 which firmly rejects the null hypothesis, i.e. the mispricing difference between in-the-money and out-of-the-money is statistically significant. The Results from the f-test can be seen in table 10 in appendix D.

The standard deviation for all in-the-money observations is 14.61%. Before we excluded all observations with values of D over one, the same measurement for the out-of-the-money observations returned a standard deviation of approximately 8.25×10^{51} . After excluding all observations with D values above 1 the standard deviation now has a value of 33.59%. It might seem arbitrary to exclude values with such a distinct impact on the calculation itself. However, as we wrote earlier, over 91.57% of the values are less than one anyway so most of them are taken into account by this measurement. Also, the calculation shows that the standard deviation is higher for options out-of-the-money than in-the-money so removing those higher values and still obtaining a higher standard deviation only emphasizes the conclusion.

The same relationship between relative pricing error and moneyness are consistent for all groups of time to expiry. The charts in Appendix A show no sign of M and D having any strong linear correlation. We sampled the correlation between the two in some sections of the data that looked promising i.e. the out-of-the-money negative D values in Figure 10 in Appendix A but the highest correlation coefficient that we could find returned values around 0.6 and only as in this case in selected parts of the data. We therefore conclude that at least no *linear* relationship between the values of M and D exists in the data as a whole.

The only consistent relationship that we have found is that the pricing error made by the Black and Scholes option pricing model on average is greater for out-of-the-money options than for in-the-

money options. This conclusion is also consistent with much of the prior research that has been done on the subject.

5.3 Relative Pricing Error and Time to Expiry

Figure 28 and Figure 29 in Appendix C graph the relative pricing error, D , against days left to expiry on the options. The data has been divided onto two charts for out-of-the-money respectively in-the-money observations. Again we see the characteristic funnel shaped dispersion of the plotted values in both charts. When looking at these charts it is easy to be misguided to assume that the average relative pricing error of the model is higher than it really is for options with shorter times to expiration. But remember that approximately 70% of all of our observations had less than 90 days left to expiration. The reason for the funnel shaped dispersion of the values is simply that the area to the left is more densely plotted. To confirm this we calculated the standard deviation from zero for all values of D for each day left to expiry.

In our case the standard deviation from zero can be seen as an average measurement of how much the Black and Scholes model misprices options. A low value indicates that the model is accurate and a high value that the model is inaccurate. We can observe a funnel shaped dispersion of the plotted values also in Figure 30 and 31 but this time expanding from the left to the right. The funnel shape this time is a result of the fact that the weight of the observations is concentrated to options with less than 90 days to expiration i.e. to the left of the figure. When calculating standard deviations, more observations lead to a better estimate of the deviation. Thus the vertical concentration to the left of the figure is merely a result of the calculations being based on more observations and thus giving a more accurate value. The vertical dispersion of the plotted values has nothing to do with the accuracy of the model but with the standard deviation calculation.

We calculated the least-squares linear regression line for both sets of data which gave us the following regression lines for figure 30 respectively Figure 31.

$$y = -0.0003x + 0.17 ; y = -0.0005x + 0.3746$$

This shows that both sets of data return regression lines that have a slightly negative inclination. The inclination may seem small but given that it stretches over 350 days this small inclinations gives a non dismissible increase in the relative pricing error over time, i.e. $0,0003 \times 350 = 0,105$ which indicates an average increase of 10.5% of the average relative pricing error over 350 days.

This gives an indication that the relative pricing error is greater for options with a short time to expiry both for in-the-money options and out-of-the-money options. Also, it is important to point out for this conclusion that the values of D over 1 that we excluded earlier in the study were all out-of-the

money observations with a short time to expiration. Therefore the conclusion that we have just drawn would not have been weakened but rather strengthened had those values not been excluded.

The regression lines also tell us that the standard deviation from zero is approximately 17% on average for in-the-money options and 37% for out-of-the-money options which is approximately what we concluded in Table 6. This also enhances the validity of our calculations indication that they are right.

The coefficient of determination of the regression line, R^2 , was however too low to provide a good fit of the plotted values. The coefficient of determination was only 0.12 for out-of-the-money options and 0.24 for in-the-money options. The plotted regression line might provide a general indication of how the standard deviation from zero changes with days left to maturity of the option but does not offer an exact description.

We therefore decided to further test the hypothesis that the standard deviation from zero is higher for options with fewer days until maturity. In order to do this we divided all measurements of average standard deviation from zero per day into two groups, values with less than 175 days until maturity and values with more than 175 days until maturity. We then performed an F-test on the two groups to see if the difference in standard deviation from zero was statistically significantly higher for the two groups with fewer days to maturity. The null hypothesis is that the standard deviation from zero would be the same for both groups $H_0: \sigma_{>175 DTM} = \sigma_{<175 DTM}$. The alternative hypothesis being that the standard deviations from zero is larger for options with less than 175 days to maturity. The p-value gives the probability that the difference in standard deviation between the two groups is just a random occurrence given that the null hypothesis holds (Lee, Lee, Lee).

The p-values for the f-tests were 0.0000088379 for in-the-money options and 0.000000298 for out-of-the-money options. These low p-values soundly reject the null hypothesis and therefore statistically significantly confirm our earlier conclusion that the standard deviations from zero are greater the less days to maturity the option has left. The Results from the f-test can be seen in table 11 and 12 in appendix D.

5.4 Results

In order to properly conclude this study we would like to refer back to our purpose that we set up for the study in chapter one. This is done to clearly present the results of the study and answer the questions posed in the purpose section in chapter one.

Our purpose with this study as stated in section 1.3 is:

Our purpose is to empirically test the accuracy of the model by examining the difference between theoretical prices and real market prices. We will also determine if the accuracy of the model varies with the time left to expiration or the moneyness of the option.

In section 5.1 we accounted for the general differences between the Black and Scholes prices and the market prices. We also showed this graphically in the plotted graphs in Appendix A.

In section 5.2 we examined the relationship between the relative pricing error and the moneyness of the options and concluded that the relative pricing error is generally larger for out-of-the-money options than for in-the-money options but that no linear correlation seems to exist between the two.

Finally, in section 5.3 we related the number of days left to expiry of the observations to the relative pricing error where we conclude that the Black and Scholes model tends to make larger relative pricing errors for options with a short time to expiry than for options with a longer time to expiry.

After reviewing our results above we feel that we have achieved the purpose of this study.

Chapter Six: Closing Discussion

We will finish this study with a closing discussion below. We intend to bring up some of the issues already discussed in the study that we want to emphasize and some issues not mentioned earlier in the study because they were not of relevance to the results but still interesting thoughts regarding the subject of option pricing. Finally we will end this study with suggestion on further subjects for research.

6.1 Closing Discussion

Our results show that the Black and Scholes option pricing model tends to misprice out-of-the-money options at a larger extent than in-the-money options. This is in accordance with much of the earlier research that has been done on the subject and seems to be the only conclusion to be persistent in all studies. However there seems to be no consensus on how the model misprices those options. We speculate that the reason for this is that there are many more factors that influence the exchange than can never be inputted into a mathematic model such as hopes and fears for the future of the collective market psyche. Another factor that influences the behavior of the model is the actual traders and their relationship to the model. If traders respect and believe in the model then the model will become a sort of self-fulfilling prophecy in that trades will only be willing to pay for an option what the model stipulates. This is discussed by author Donald MacKenzie (2003) in his paper *Constructing a Market, Performing Theory: The Historical Sociology of a Financial Derivatives Exchange*.

After reviewing our results and the lack of consensus among earlier studies we come to thinking that it is only possible to describe the behavior of the Black and Scholes model to no more than a certain general extent as the behavior of the model changes over time. The relatively greater mispricing of out-of-the-money options than in-the-money options is probably one of few general behaviors that have been persistent in all studies. More specific behaviors like pricing trends that follow a regression line seems to be changing with time, market circumstances and many other intangible factors.

As an example of this we have decided to include the below quotation from a study conducted in 1984 by Robert Geske and Rihard Roll.

...Black [1] reported that the model systematically underpriced deep out-of-the money options and near-maturity options while it overpriced deep in-the-money options during the 1973-1975 period. MacBeth and Merville [8] confirmed the existence of a striking price bias for the 1975-1976 period, but found that it was the reverse of the bias reported by Black. Rubinstein [10] also found that the striking price bias had reversed in the 1976-1977 time period. Both Rubinstein [10] and Emanuel and MacBeth [7] reported that the original striking price bias observed by Black had reestablished itself in late 1977 and 1978...

Even though the model is unpredictable it must be said that it performs remarkably well even in its most basic form in which we have tested it. Even though many of the assumptions that the model makes does not hold in reality and despite the fact that we only used a most simple way to determine the historical volatility the model still predicted the market price quite accurately in most cases. In our opinion the Black and Scholes gives a good approximation of the market price of an option. And doing that is often good enough since no model will ever be able to set an exact value of an option as that would require telling the future.

6.2 Suggestion to Further Subjects of Research

Because of limitations in time and computing power we chose to only process options of the five stocks whose options were the most traded in the Swedish exchange during 2007. If it were not for these limitations we would have processed more options with common denominators such as options in a specific field of business or options with ties to specific geographical regions. In doing so one could solve for systematic differences and that perhaps were specific for these different business areas and geographic areas. This could be an interesting subject to investigate further in the future.

Another interesting idea for a study would be examining the performance of the Black and Scholes model with regard to the complexity of the variables that you put into it. I.e. what is the marginal gain in accuracy of using advanced volatility measurements instead of the more basic ones that we used in this study. Is it worth the extra time and money and for whom is it worth the whole to put in the extra effort?

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http://en.wikipedia.org/wiki/Robert_C._Merton **2008-05-08**

Appendix A

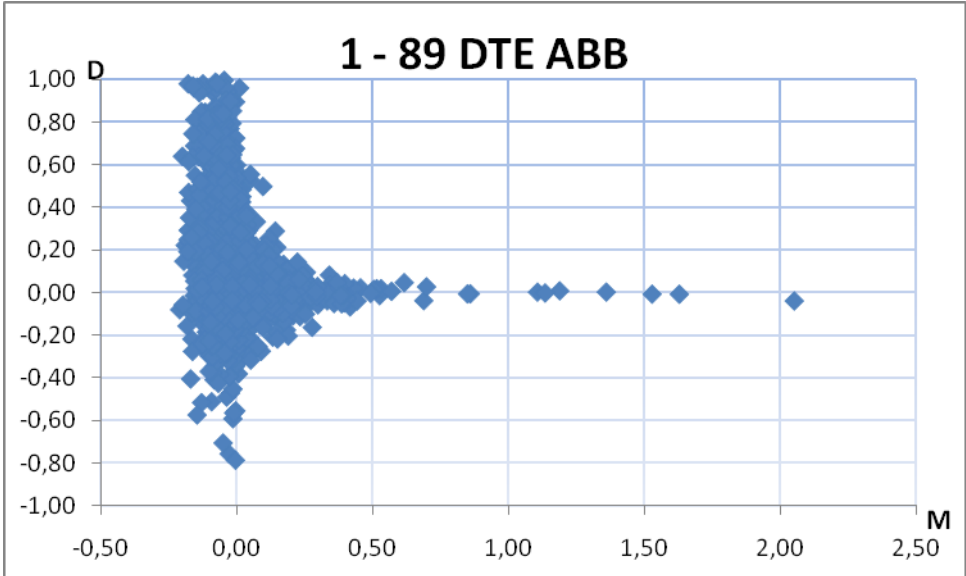


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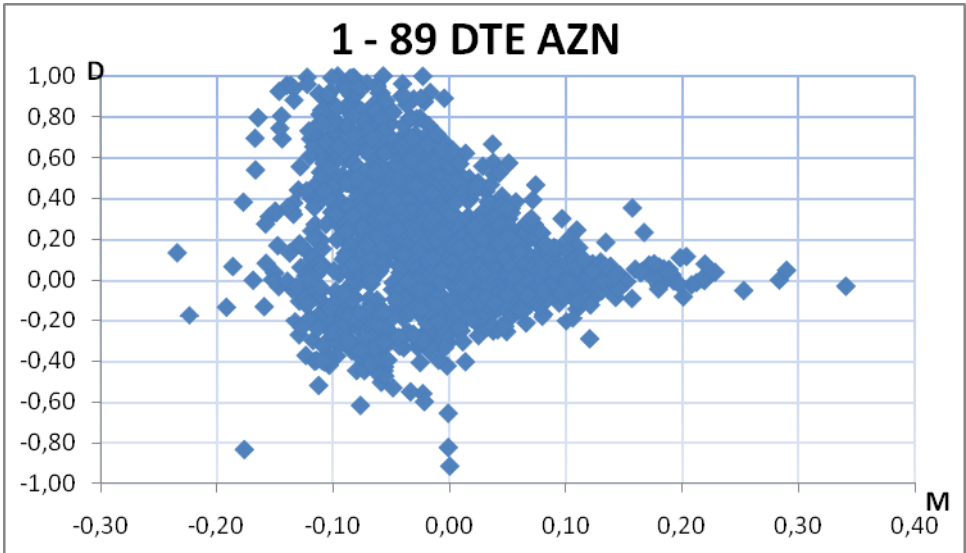


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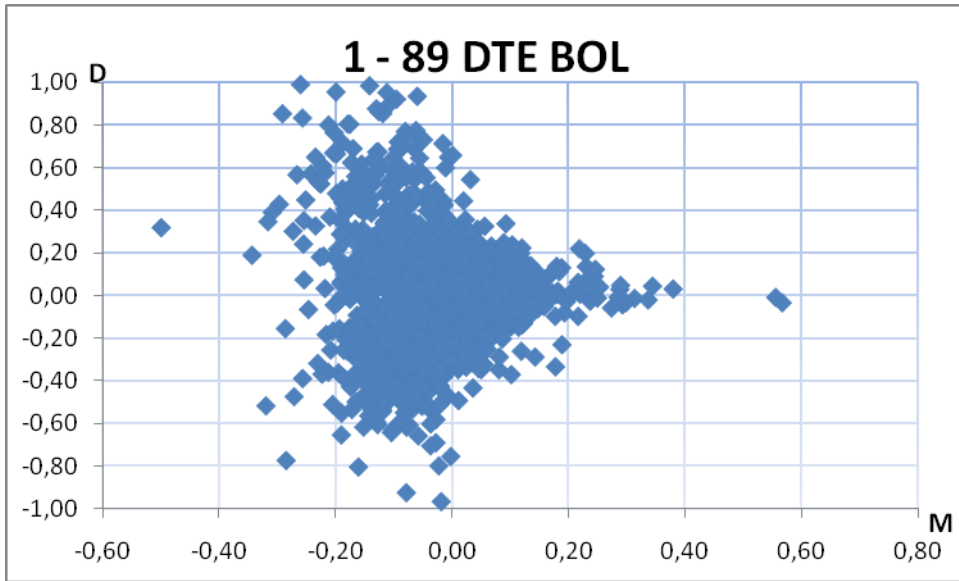


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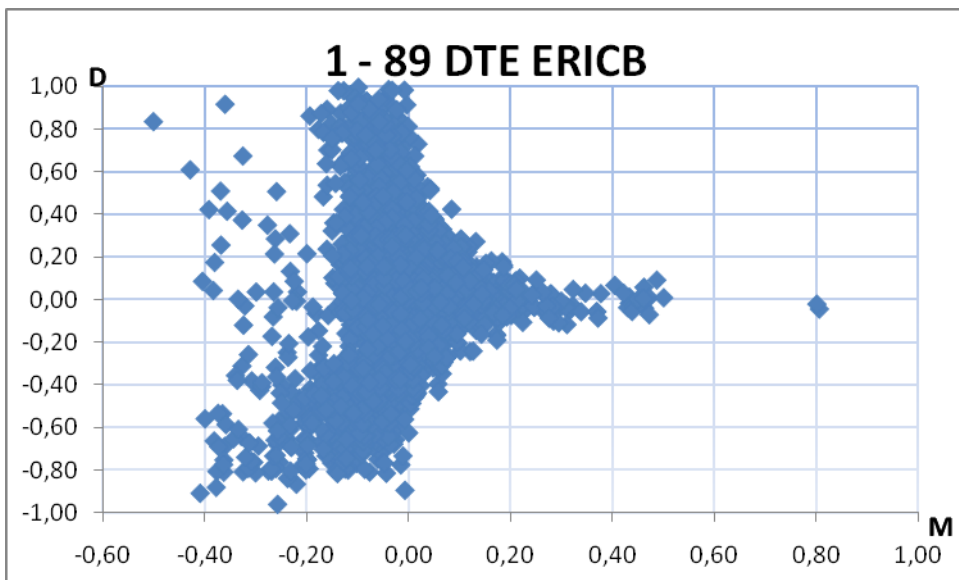


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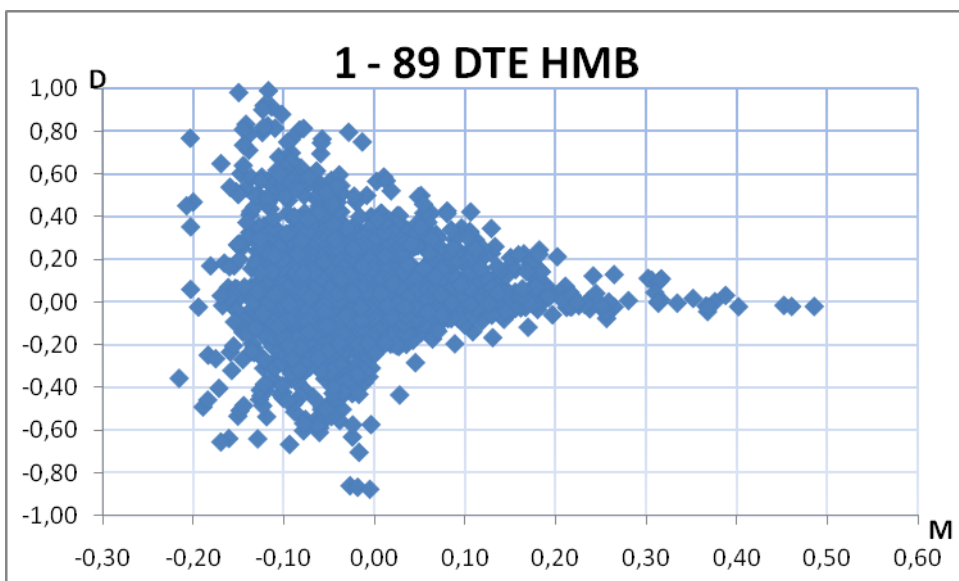


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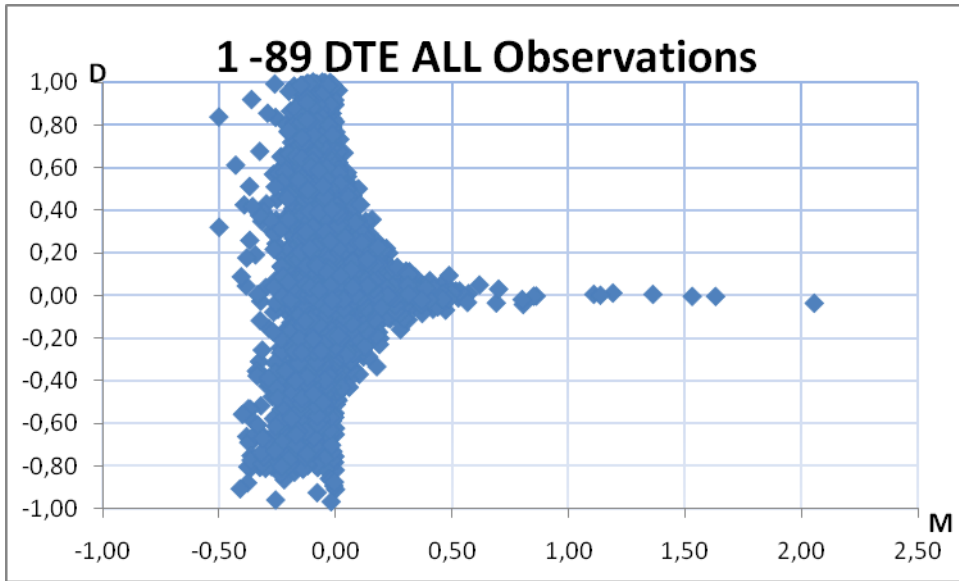


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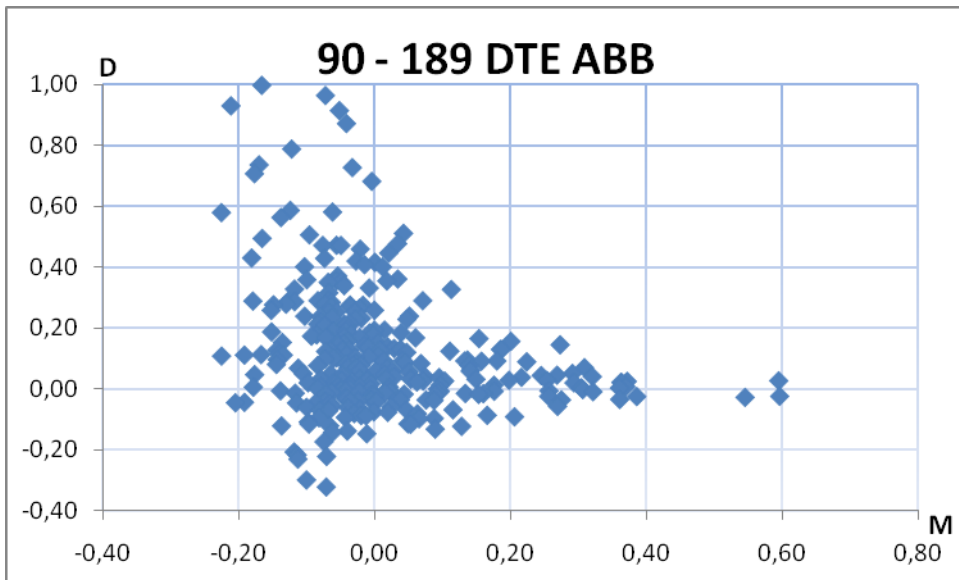


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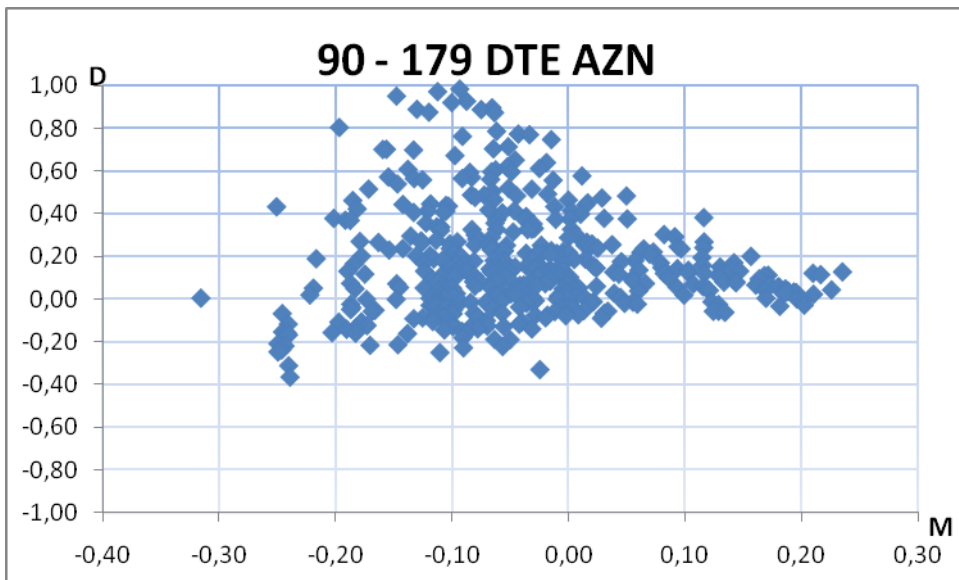


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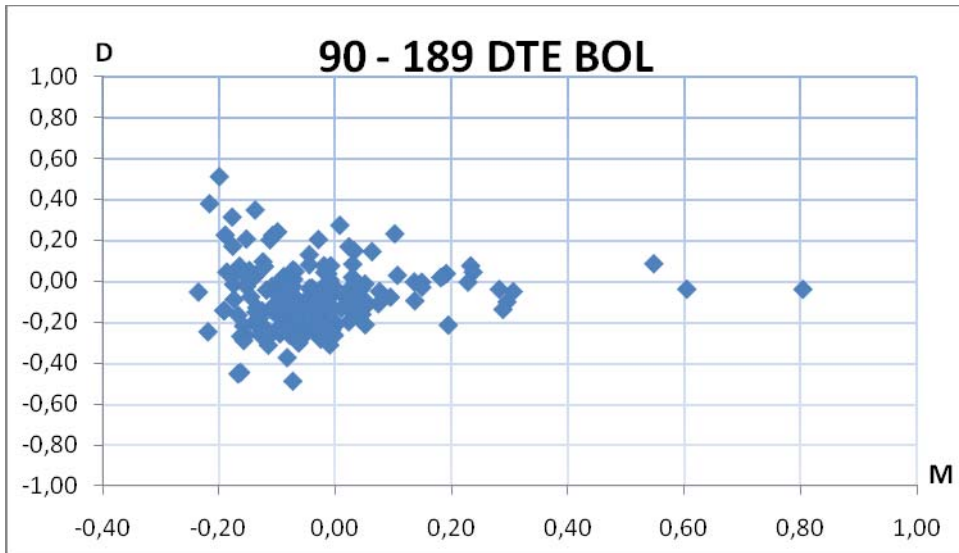


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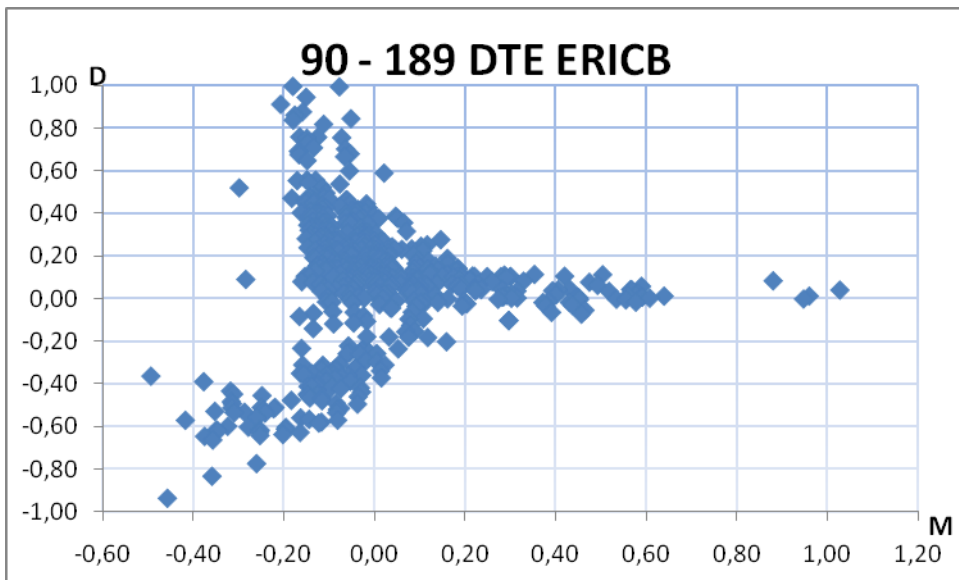


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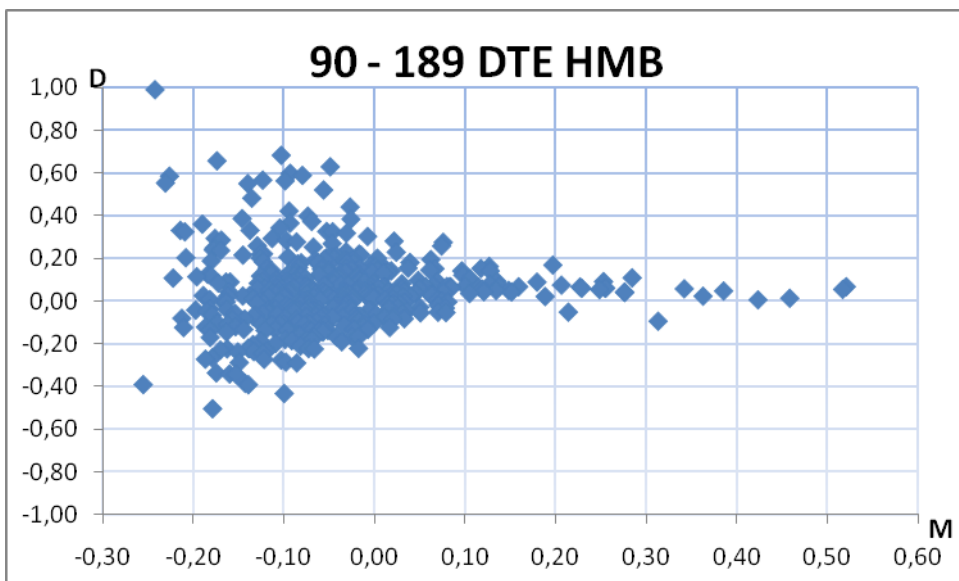


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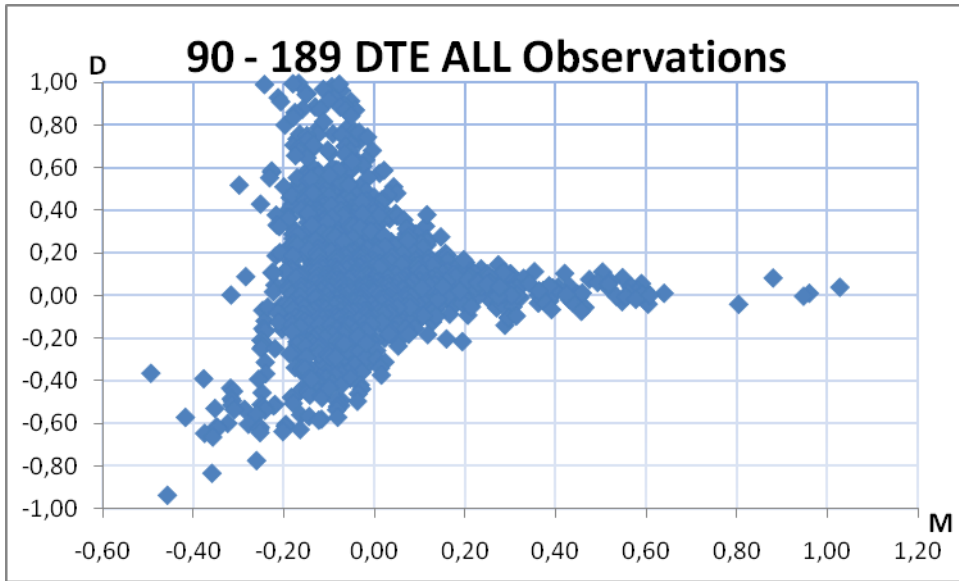


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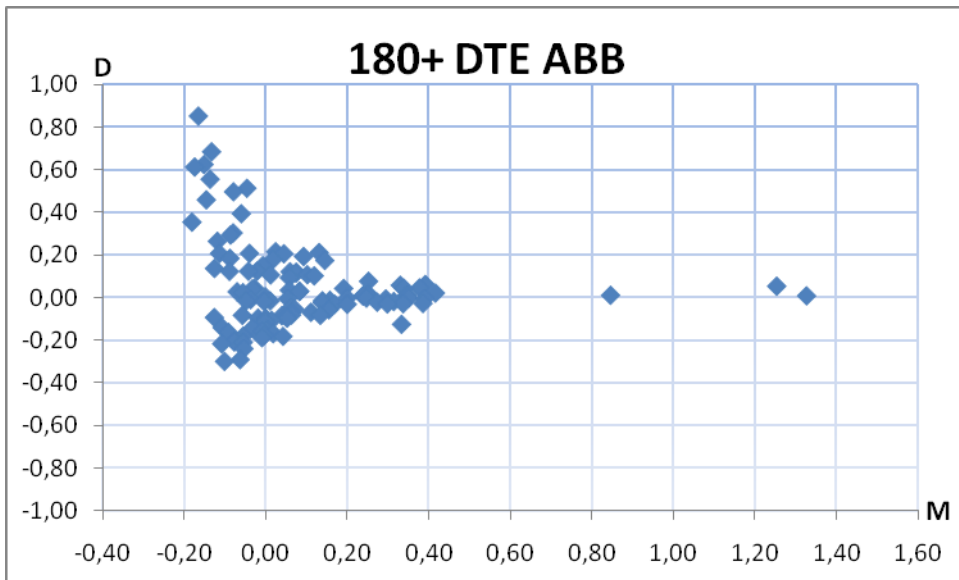


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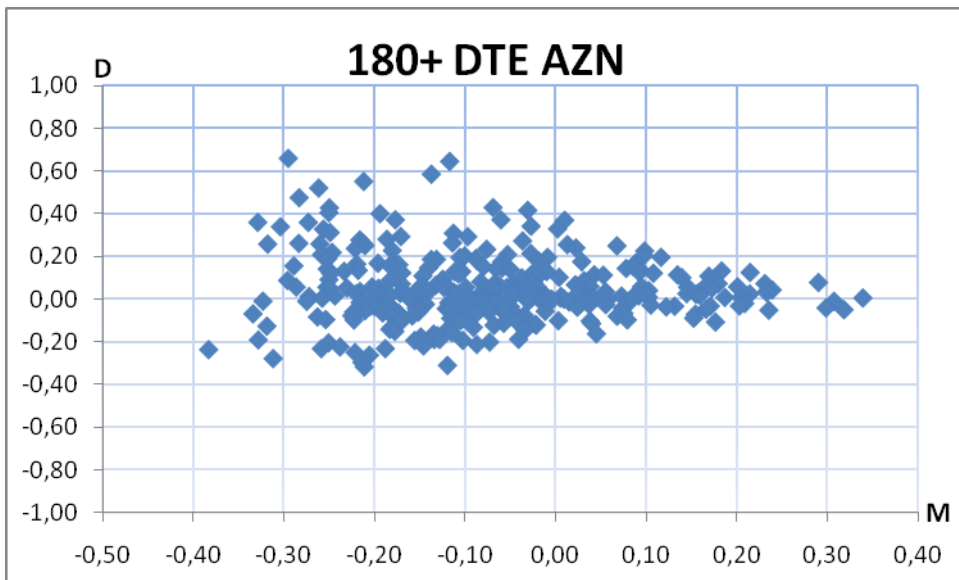


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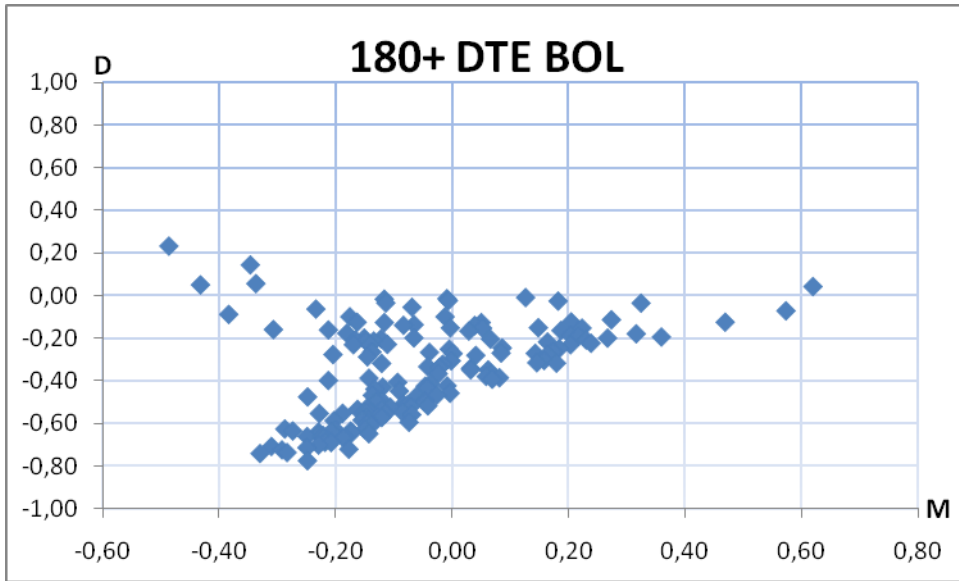


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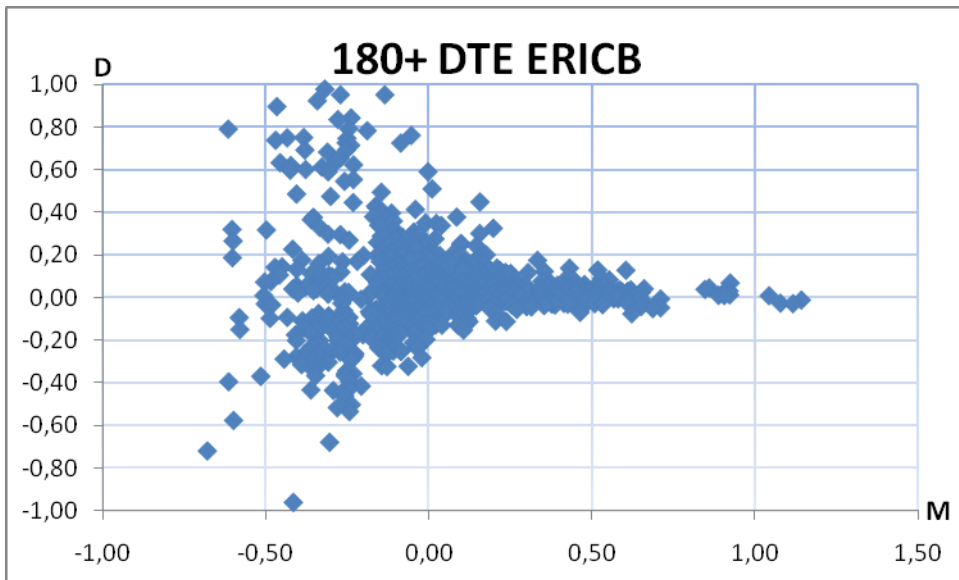


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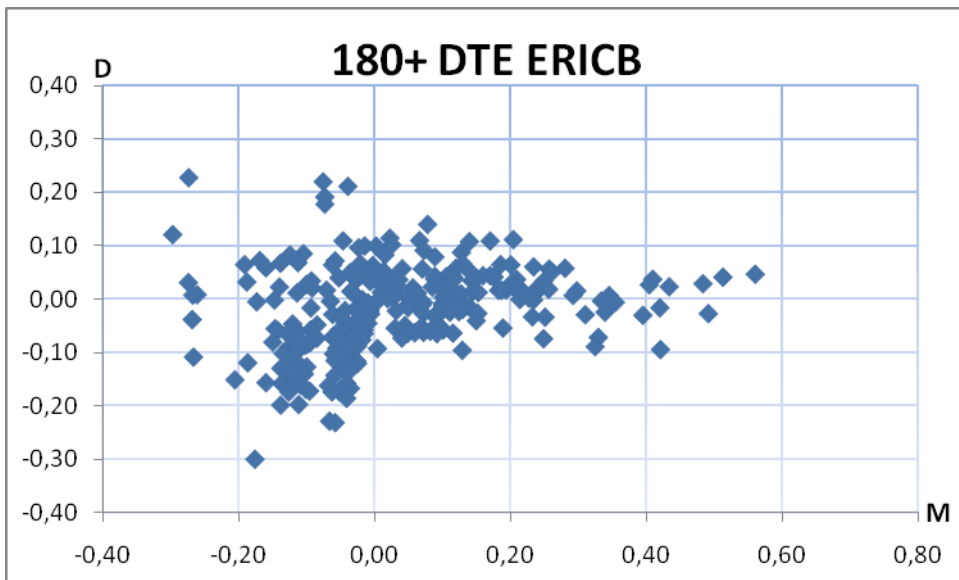


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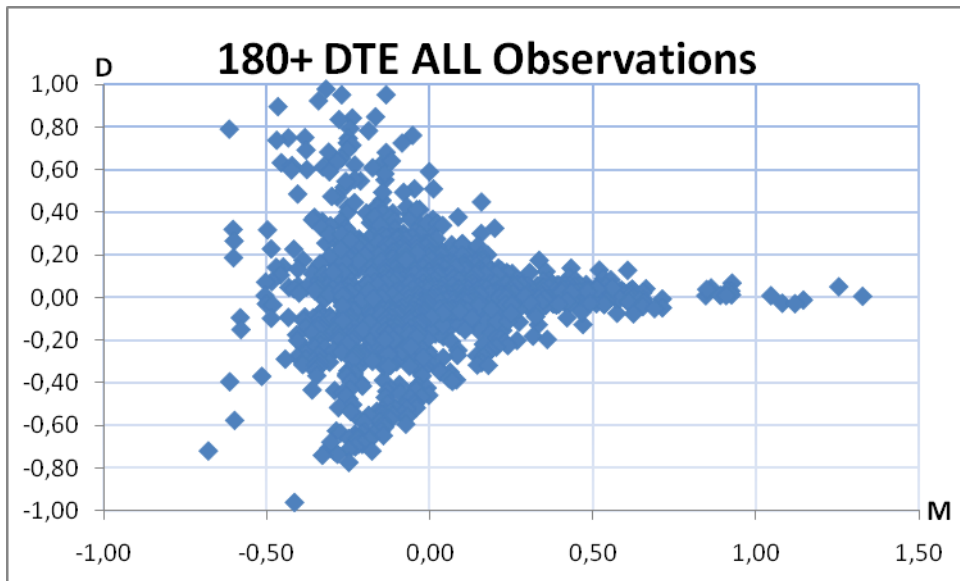


Figure 21

Appendix B

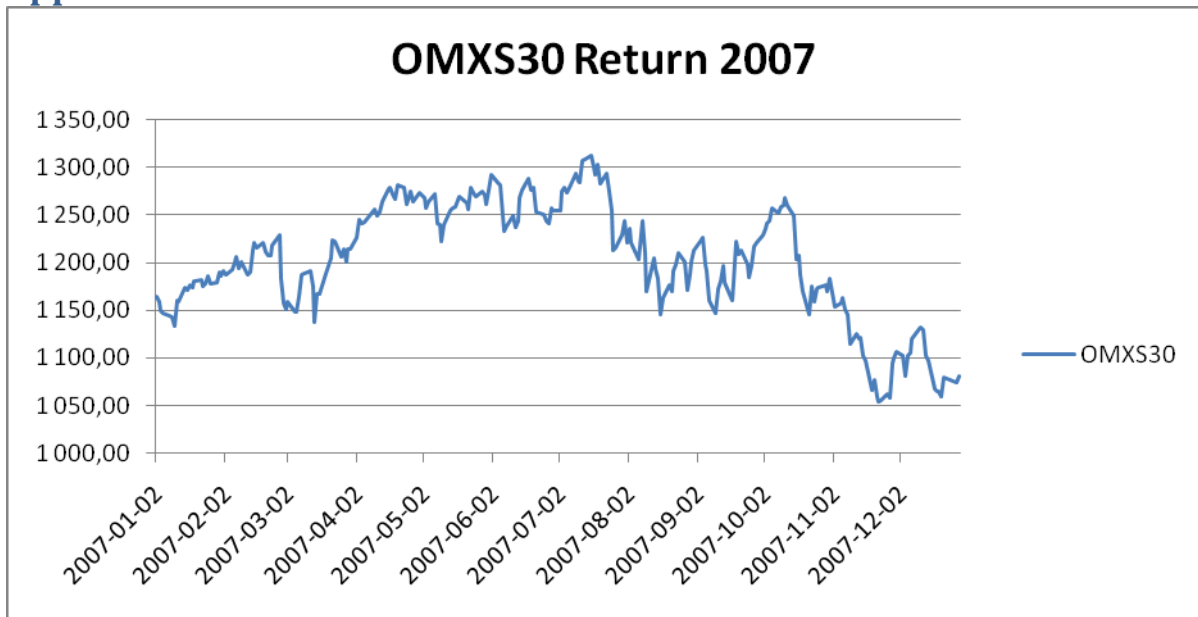


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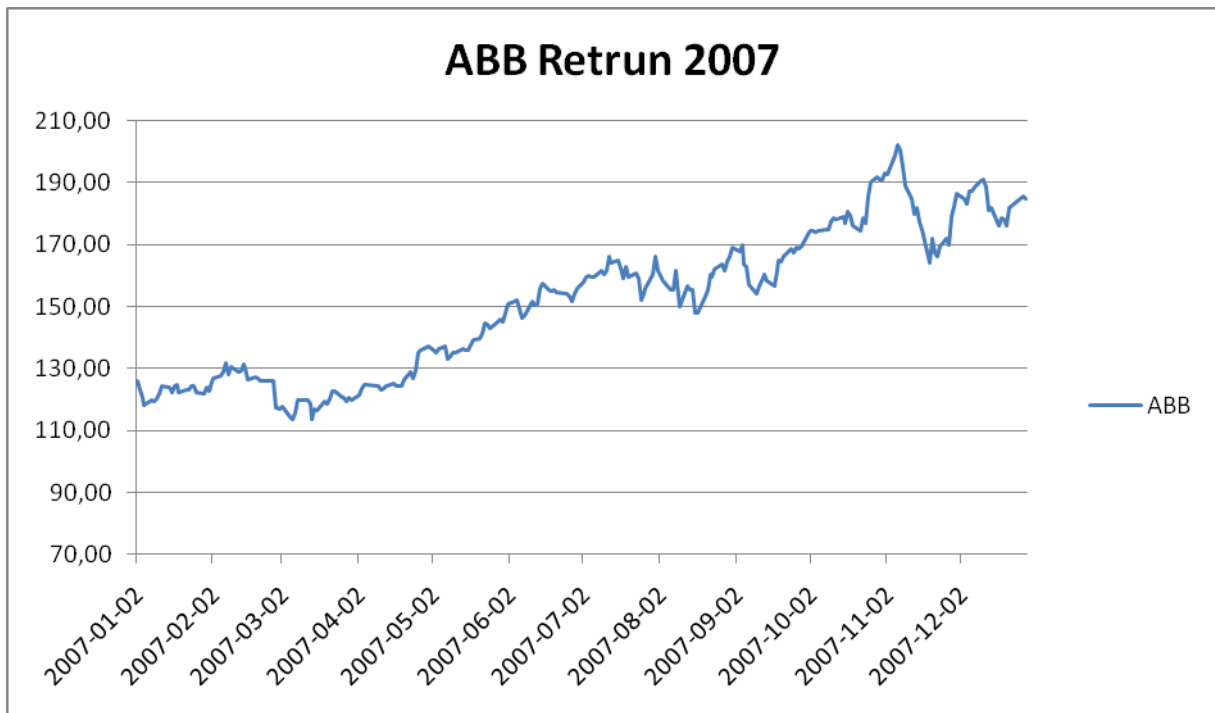


Figure 23



Figure 24

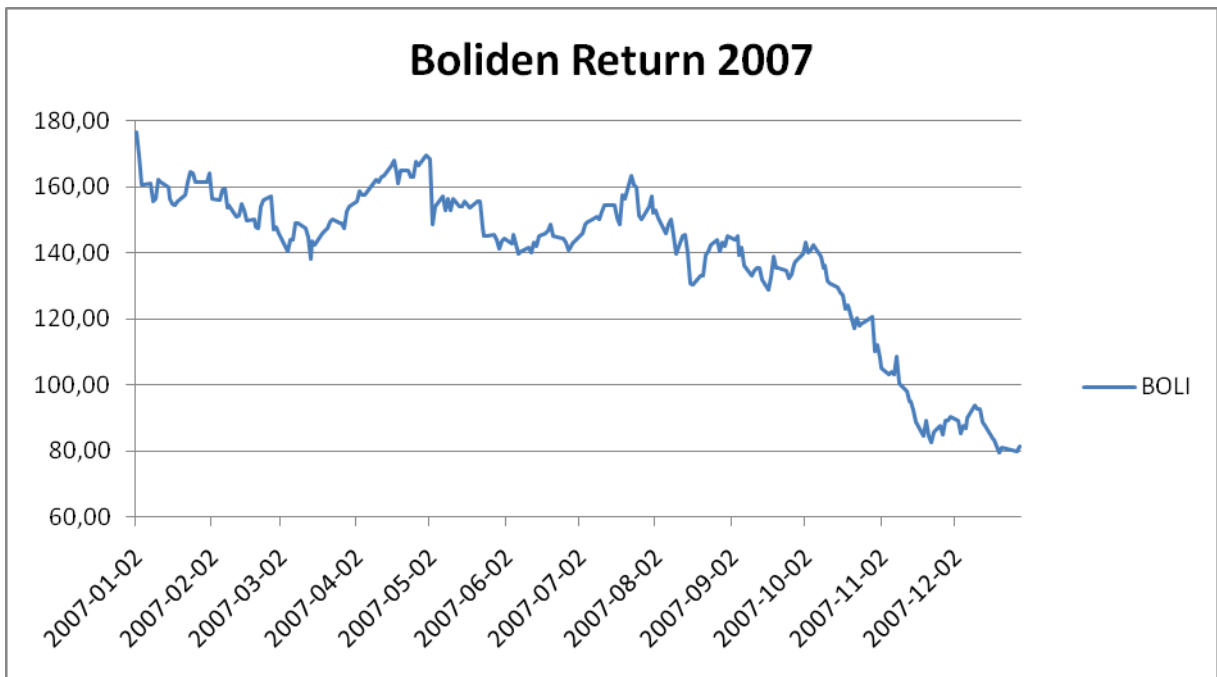


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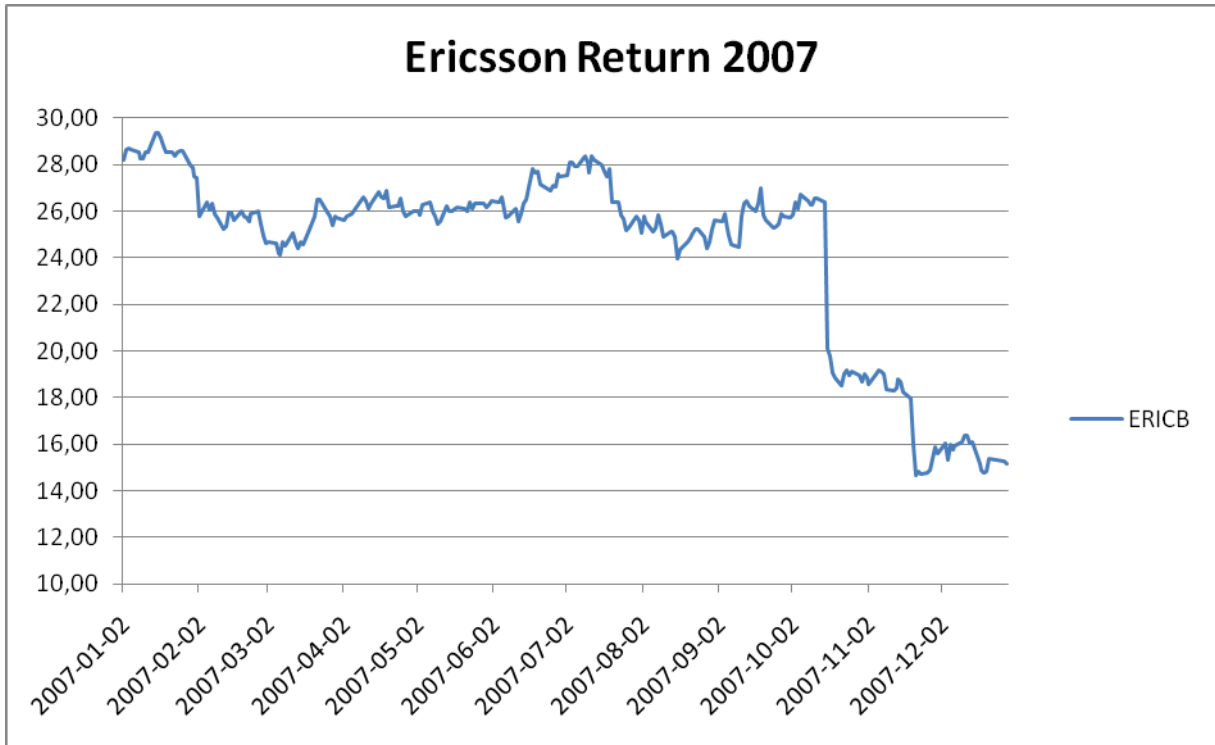


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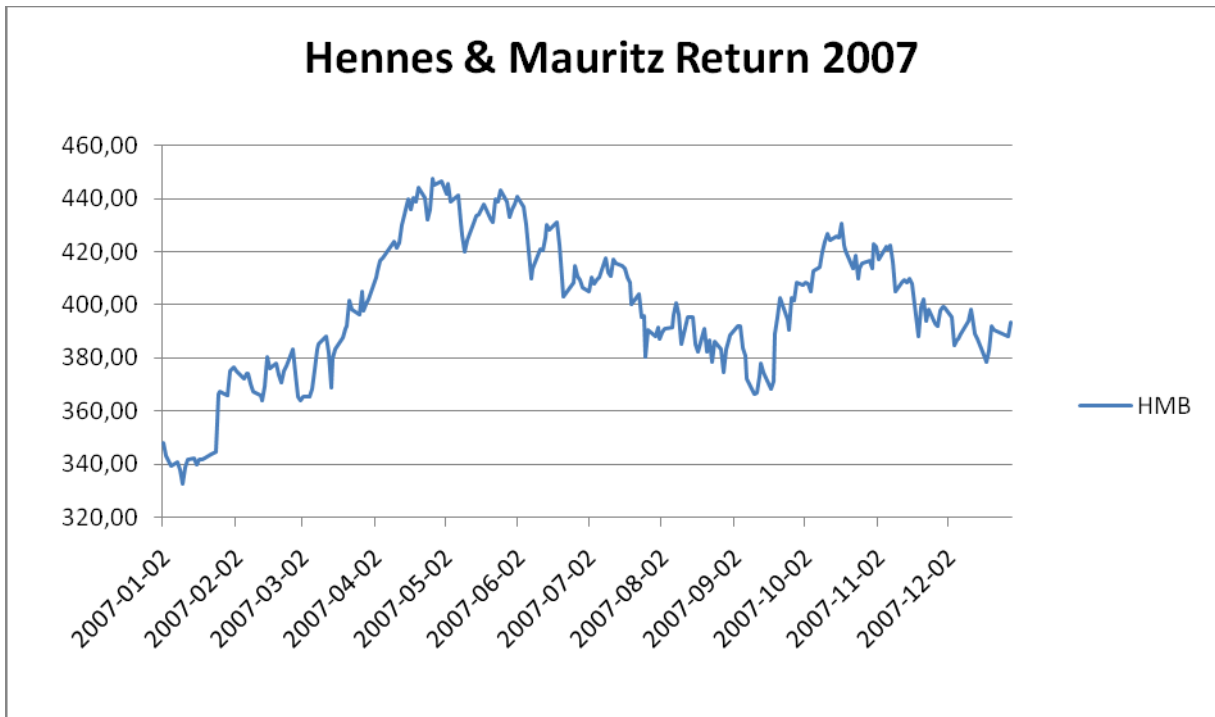


Figure 27

Appendix C

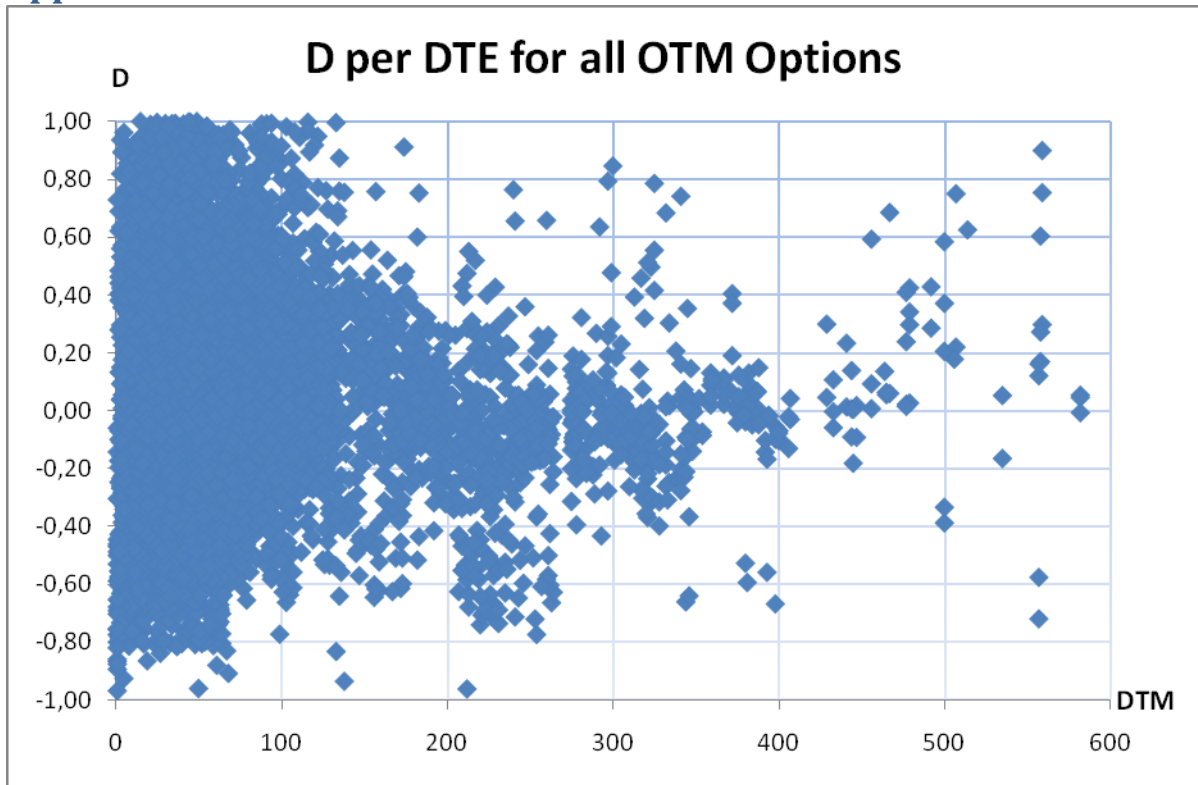


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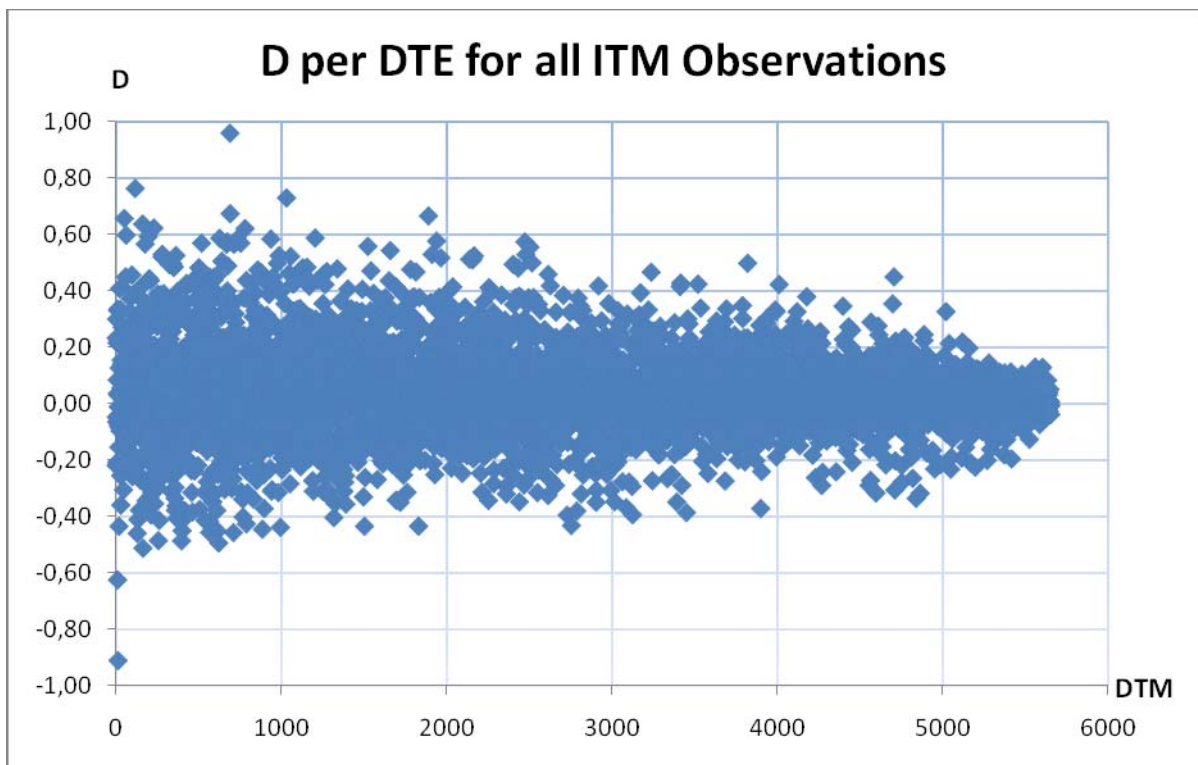


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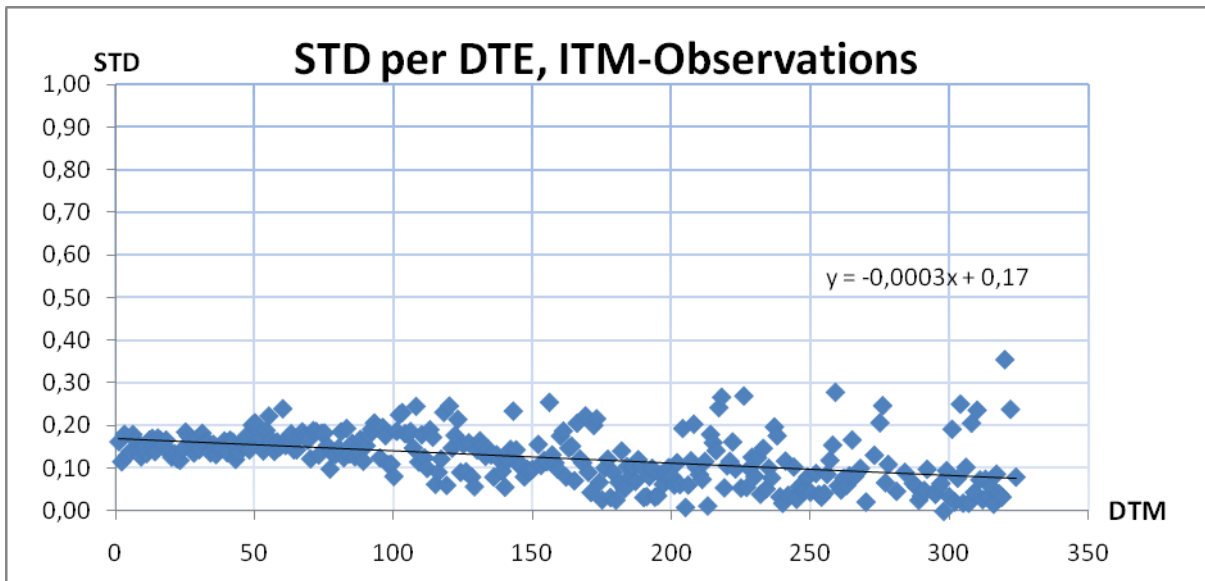


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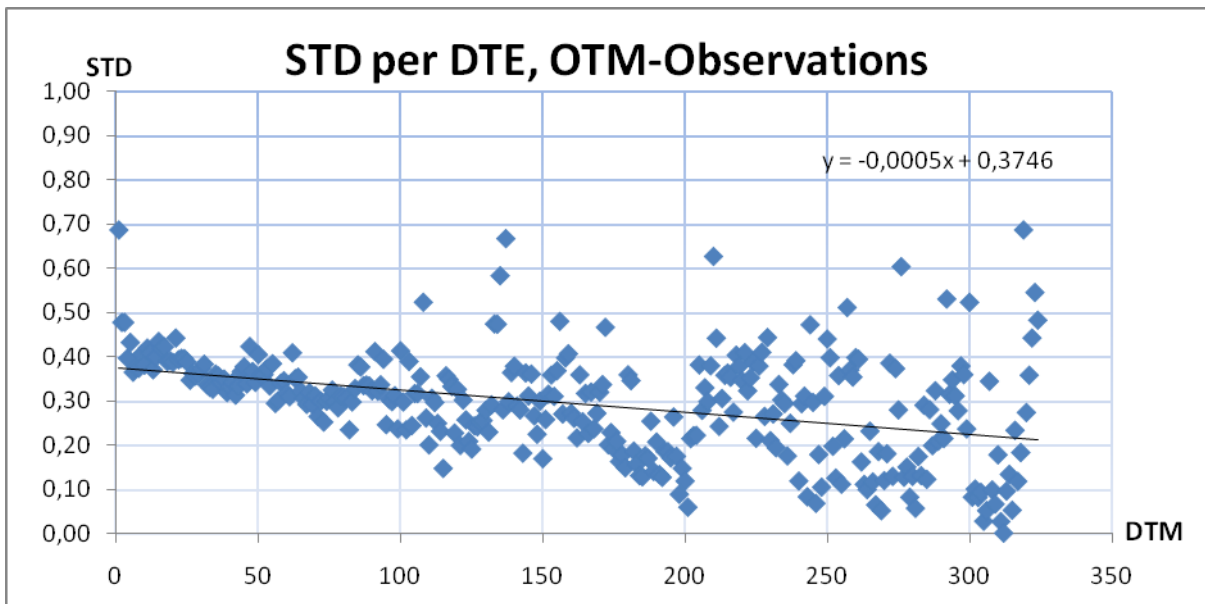


Figure 31

Appendix D

F-test, difference in standard deviations for options that are in-the-money resp. out-of-the-money.

	<i>In the money</i>	<i>Out of the money</i>
Average value	0,0352098573	0,0535556148
Variance	0,0205625745	0,1112313341
No. Observations	5823	8791
Degrees of freedom	5822	8790
F-value	0,1848631472	
P(F>F-obs)	0,0000000000	
Critical F-value	0,9613656705	

Table 10

F-test, difference in standard deviations from zero for options with more resp. less than 175 days to maturity for out-of-the-money options

	<i>ITM <175 DTM</i>	<i>ITM >175 DTM</i>
Average value	0,1500586502	0,0908760312
Variance	0,0016349588	0,0033707874
No. Observations	172	115
Degrees of freedom	171	114
F-value	0,4850376307	
P(F>F-obs)	0,0000088379	
Critical F-value	0,7575535987	

Table 11

F-test, difference in standard deviations from zero for options with more resp. less than 175 days to maturity for out-of-the-money options

	<i>OTM <175 DTM</i>	<i>OTM >175 DTM</i>
Average value	0,3339704236	0,2528644350
Variance	0,0062583728	0,0152184703
No. Observations	173	130
Degrees of freedom	172	129
F-value	0,4112353405	
P(F>F-obs)	0,0000000298	
Critical F-value	0,7643199334	

Table 12

