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*Olof Johansson-Stenman*

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SCHOOL OF BUSINESS, ECONOMICS AND LAW, UNIVERSITY OF GOTHENBURG

*Department of Economics*

*Visiting address* Vasagatan 1,

*Postal address* P.O.Box 640, SE 405 30 Göteborg, Sweden

*Phone* + 46 (0)31 786 0000

# HEALTH INVESTMENTS UNDER RISK AND AMBIGUITY

*Olof Johansson-Stenman*

Department of Economics, School of Business, Economics and Law,  
University of Gothenburg, SE – 405 30 Gothenburg, Sweden  
E-mail: [Olof.Johansson@economics.gu.se](mailto:Olof.Johansson@economics.gu.se)

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## ABSTRACT

This paper discusses how a decision maker should deal with uncertainty, both in the sense of a well-known probability distribution of different outcomes and as a situation where also the probability distribution is unknown. A simple baseline model is used throughout the paper, where the decision maker can invest in order to decrease the health risk. Since the investment is risky, the question concerns how much to invest. We derive and compare the optimal investment level for a number of different decision rules: a best guess rule, a maximin rule, an expected value rule, an expected utility rule, and three different rules that beyond risk aversion also reflect ambiguity aversion. Finally, these decision rules are evaluated more broadly.

Keywords: Investment under uncertainty, risk aversion, ambiguity aversion

JEL: D81, H51, I18

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## 1. INTRODUCTION

It is obvious that food-related public health investments, and regulations more generally, have to deal with uncertainty. For example, how should we deal with genetically engineered food and various chemical food additives? On the one hand, these new technologies offer potentially very large productivity improvements, with corresponding potential welfare improvements. This is not least important in developing countries, where about one billion of the world's population live on less than one dollar per day (Collier 2007) and about the same number of people are malnourished (FAO 2008); see also Chapter 16 in this Handbook (Abdulai and Kuhlitz 2010) on issues related to food security in developing countries. On the other hand, there are of course various risks associated with these technologies. Somehow we must deal with both the potential benefits and the risks. The question of the present paper is how, in principle, this should be done. In other words, we are intrinsically concerned with the normative *ought*-question concerning how a public decision maker should behave rather than the descriptive *is*-question corresponding to how such a decision maker behaves, or is expected to behave, under uncertainty.

While uncertainty has been incorporated into mainstream economic theory for a long time (e.g. Arrow 1971; Drèze and Modigliani 1972; Dreze 1987), there are many problems with applying the conventional approach in practice. In particular, there is a fair amount of evidence that people often deviate systematically from von Neumann-Morgenstern's (1944) expected utility (EU) theory. Indeed, by now there are a large number of competing non-EU models, of which prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992; Schmidt et al. 2008) constitutes the most prominent example.<sup>1</sup> However, whether there are any direct implications of these alternative theories for normative conclusions, in the sense of how a social decision maker ought to act, is less clear. It appears reasonable to view much of the behaviour reflecting deviations from expected utility as indications of what Kahneman et al. (1997) and Kahneman and Thaler (2006) denote *decision* utility, simply reflecting choice, as opposed to *experienced* utility, reflecting well-being. Consequently, one

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<sup>1</sup> See Starmer (2000) for an overview of non-expected utility theory, Chapter 37 in this Handbook (Fox 2010) for an overview of Risk Preferences and Food Consumption, and Chapter 10 in this Handbook (Just 2010) for a more general discussion of behavioral economics and the food consumer.

can argue that many of the observed deviations from EU theory have no direct implications for how a social decision maker should act.

However, the conventional von Neumann-Morgenstern (1944) approach to EU theory assumes that the probability distribution is known, whereas this is rarely the case in reality, where there are instead often largely diverging views even among the experts. One can argue, and it is indeed often argued, that this fact makes the conventional EU approach unsuitable for social decision making under uncertainty.

Still, according to *subjective* expected utility (SEU) theory, as famously expressed and axiomatised already by Savage (1954),<sup>2</sup> rational decision makers should form their own subjective probability distributions and behave *as if* these probabilities are the objective ones. For example, suppose your decision regarding what kind of margarine to buy depends in part on how healthy (and unhealthy) the different kinds are. In making this judgment you will obviously have to rely on external experts, and typically also on secondary sources of these opinions as expressed e.g. by media and friends. If you read another article claiming that type A margarine is better for you than type B, you would perhaps update your judgment somewhat in favour of type A margarine, etc.

Note that SEU theory doesn't say much about how these subjective probabilities are formed. Indeed, one individual may generally trust medical experts with respect to food recommendations, another may agree with a particular type of alternative medicine school, while a third may be largely guided by religious beliefs. Obviously, these three individuals may arrive at very different subjective probabilities regarding the health consequences of different kinds of food. SEU theory doesn't say that one individual's subjective probabilities are 'better' than others, nor does it say that they are equally good. SEU theory is simply silent on these issues.

However, SEU theory does imply restrictions on the structure of these expected utilities for each individual. Notably, it implies that compound lotteries should be evaluated at their resulting net probabilities. For example, suppose that 100 experts are judging whether or not

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<sup>2</sup> See Ramsey (1931) and de Finetti (1937) for earlier contributions to SEU theory that Savage (1954) incorporated into the von Neumann-Morgenstern (1944) framework.

a certain food is unhealthy or not. For analytical simplicity, assume that you know that precisely one of them is right and that you consider them equally likely to be right, i.e. you believe that each has a 1 % probability of being right. One of them believes that the food is unhealthy with a probability of 90%, while all others believe that the food is unhealthy with a probability of 1%. The net probability that the food is unhealthy in this compound lottery is then equal to  $0.01 \cdot 90\% + 0.99 \cdot 1\% = 0.9\% + 0.99\% = 1.89\%$ .<sup>3</sup>

However, while this kind of reasoning may seem plausible, much experimental and empirical evidence suggests otherwise. In fact, it seems that people typically have a particular aversion to unknown risks and hence place more weight on the judgments of more pessimistic experts. If we denote the uncertainty with respect to the true probability *ambiguity*, it seems, in other words, that people often tend to be *ambiguity averse* (Camerer and Weber 1992). An ambiguity averse individual would then behave as if, in the above example, the resulting probability that the food is unhealthy is higher than 1.89%.

Ambiguity aversion has been shown to be economically relevant and to persist in many different experimental settings and samples (Sarin and Weber 1993; Gilboa 2004) including business owners and managers who are supposedly familiar with decisions under uncertainty (Chesson and Viscusi 2003). Additionally, it is often found that people are willing to spend substantial amounts of money to avoid ambiguous processes in favour of processes that are equivalent in terms of SEU theory (Becker and Brownson 1964; Chow and Sarin 2001). There is also evidence in terms of conventional empirical studies, in particular from the financial sector, that the observed pattern cannot be explained by conventional theory, but which is consistent with theories incorporating ambiguity aversion; see Camerer and Weber (1992), Mukerji and Tallon (2001), Chen and Epstein (2002) and Gilboa (2004).

With respect to food safety, Shogren (2005) compared the monetary equivalents for risk elimination under non-ambiguous and ambiguous probability scenarios, respectively, in a survey about the food-borne pathogen Salmonella. He found a higher mean willingness-to-

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<sup>3</sup> Moreover, SEU theory implies that the individual would update the subjective probabilities in a Bayesian way when new information becomes available. For example, if the 99 more optimistic experts change their minds so that they now believe that the food is unhealthy with a probability of 2%, then the new resulting probability that the food is unhealthy is equal to  $0.01 \cdot 90\% + 0.99 \cdot 2\% = 0.9\% + 1.98\% = 2.88\%$ .

pay for a given probability reduction under the ambiguous scenario, although the difference is not large enough to be statistically significant. Other health-related studies include Ritov and Baron (1990), who, based on a hypothetical experiment, found reluctance to vaccination under missing information about side effects of the vaccine, and Riddle and Show (2006), who, based on survey evidence from Nevada residents, found a large effect of ambiguity on attitudes towards risks related to nuclear-waste transport. Theoretically, Treich (2010) shows that ambiguity aversion tends to increase the value of a statistical life.

The present paper deals with the question of how a public decision maker should think about issues of known and unknown risks. In doing this, a simple baseline model is used throughout the paper, where a public decision maker can invest in order to decrease the health risk. Since the investment is risky, the question concerns how much to invest. While we will model a simple investment that decreases the health damage and that can be bought at a given per unit price, one can interpret the investment much more broadly as any public measure that has positive expected health consequences and that is associated with some social costs; cf. e.g. Lichtenberg and Zilberman (1988), Lichtenberg et al. (1989) and Cropper (1992). For example, the food industry faces a large number of detailed regulations including labelling and food safety standards motivated ultimately by health reasons. Strengthening these regulations is in most cases costly; see e.g. Chapters 11 (Marette and Roosen 2010) and 17 (Hoffman 2010) in this Handbook. This is so whether the costs eventually fall on the food company owners as lower profits or on the consumers as higher prices.

The optimal investment levels are then derived and compared for a number of different decision rules, starting with the simplest ones and then gradually adding more complexities. Section 2 discusses three decision rules: the best guess, the maximin and the expected value decision rules. The best guess decision rule simply implies maximisation of the relevant decision variable, here consumption, for the most likely outcome of the risky variable. The maximin decision rule implies that we are making the outcome as good as possible for the worst case scenario, whereas with the expected value decision rule we maximise the expected value of consumption, implying that we take all possible outcomes and their associated probabilities into account.

Section 3 presents the St Petersburg paradox, which clearly shows that the expected value decision rule cannot be universally applied. Section 4 introduces a non-linear utility function

to the model, meaning that we can handle risk aversion and also resolve the St Petersburg paradox. The optimal investment rules are then derived for different utility specifications. Section 5 presents the optimal investment rules for a special case of a state-dependent expected utility model, namely when consumption and the absence of damage are imperfect substitutes.

Whereas Sections 2-5 handle the probabilities as exogenously given, Sections 6-8 in contrast deal with the problem when the decision maker does not know the objective probabilities. Section 6 presents yet another paradox, the Ellsberg paradox, which illustrates that most people do not seem to apply SEU theory as their universal decision rule when the probabilities are not known. Section 7 deals with the problem of unknown probabilities by adding probability distributions of the probabilities. Decision rules for three different models that allow for ambiguity aversion (in addition to risk aversion), i.e. that put a larger weight on the more pessimistic probability distributions, are then derived and discussed. Section 8 returns to the more fundamental question regarding whether models of ambiguity aversion can be justified for normative analysis or whether we should after all stick to SEU models. Section 9 concludes the paper.

## 2. BEST GUESS, MAXIMIN AND EXPECTED VALUE DECISION RULES

In order to be able to focus clearly on how to deal with uncertainty, the basic model will throughout the paper be kept very simple and deal with the choice of a single health investment level,  $I$ . For the same simplicity reason, the model will deal with a representative individual in a static framework, implying that distributional, discounting and timing issues are ignored and that no meaningful distinction can be made between income and wealth. Strategic interaction between agents will also be ignored, such that all decision rules are conducted in games against nature.

### 2.1 *The Basic Model*

Consider a representative individual who faces the budget

$$C = Y - I - D, \tag{1}$$

where  $C$  is consumption,  $Y$  is gross income,  $I$  is health investments and  $D$  is damage costs related to imperfect food safety. This formulation implies that consumption and the absence of health damage are perfect substitutes, which is not a central assumption here but will be central when introducing risk aversion. The damage costs, in turn, are written as

$$D = D_0 f(I), \quad (2)$$

where  $f'(I) < 0$ ,  $f''(I) > 0$  and  $f(0) = 1$ ,  $f(\infty) = 0$ .  $D_0$  is a stochastic variable with  $n$  possible outcomes,  $D_0^1, \dots, D_0^n$  occurring with (objective) probabilities  $p^1, \dots, p^n$ , respectively. Thus, the damage cost equals  $D_0$  if no investment is made and the larger the investment, the lower the cost; yet the damage cost will always be positive irrespective of the investment. This pattern appears fairly realistic for most potential food safety investments in practice. The following exponential function constitutes an example of a functional form of  $f$  that is consistent with this pattern:

$$f(I) = e^{-\alpha I}. \quad (3)$$

The problem of the decision maker is to choose the investment level  $I$  before knowing which value of  $D_0$  that will materialise. What should the decision maker then do?

## 2.2 The Best Guess Decision Rule

Perhaps the most straightforward alternative for a decision maker is to go for the most likely outcome and then invest optimally given that this outcome will occur. Suppose that the most likely outcome is given by  $D_0^L$ . This clearly implies that

$$C = Y - I - D_0^L f(I), \quad (4)$$

which is maximised for

$$\frac{\partial C}{\partial I} = -1 - D_0^L f'(I) = 0, \quad (5)$$

so that

$$f'(I) = -1 / D_0^L. \quad (6)$$

We can then, in principle, solve for  $I$  by using the inverse function of  $f'$ . Since we will do this repeatedly for different cases in this paper, we will go through this procedure in detail. Consider first the general case where

$$f'(I) = A, \quad (7)$$

where  $A$  is a constant. Then we can, implicitly, solve for  $I$  such that



$$I = g(A), \quad (8)$$

where  $g$  is the inverse function of  $f'$ . Since  $f'(I) < 0$ , we know that  $g'(A) > 0$ . Moreover, since  $A$  has a monotonic relation with  $-1/A$ , we can alternatively write

$$I = h(-1/A) = h(-1/f'(I)), \quad (9)$$

where  $h'(-1/A) > 0$ . In the case here, where  $A = -1/D_0^L$ , we then have

$$I = h(D_0^L), \quad (10)$$

where  $h' > 0$ . Thus, we have, not surprisingly, found that the larger the most likely damage costs (in absence of any investments), the larger the optimal investments.

In the special case where  $f$  has the specific exponential function form mentioned above in (3), we have

$$I = \frac{1}{\alpha} \ln(\alpha D_0^L). \quad (11)$$

An obvious advantage with this approach is that it is cognitively straightforward and computationally undemanding, which is presumably the reason it is often used, including in scientific contexts. For example, in the literature dealing with how much to invest in order to decrease the negative effects of global warming, the optimisation is in most cases made based on the assumption of a known temperature increase for a given emission trajectory and known costs for a given temperature increase, whereas both relations are highly uncertain; see e.g. Stern (2007).

Yet, an equally obvious drawback with this decision rule is that it completely ignores the outcomes of the (perhaps only slightly) less likely alternatives. For example, suppose that there are two possible outcomes,  $I$  and  $II$ , where the initial damage in  $I$  is zero while it is very large in  $II$ , and where we assume that the probability that  $I$  occurs is 55% and that  $II$  occurs is 45%. Then it does hardly seem reasonable to optimise based on the assumption that  $I$  will occur.

Thus, although the above decision rule might be a way in which we often solve problems in practice, since it is cognitively quite straightforward and computationally undemanding, it is difficult to justify as a general principle.

### 2.3 The Maximin Decision Rule

An alternative decision rule, which is equally straightforward as the one above, is the maximin decision rule, meaning that we make the outcome as good as possible for the worst case scenario. This means that we maximise consumption for the case where the initial damage  $D_0$  is greatest among the possible alternatives, i.e. irrespective of the probabilities. Thus, the decision maker chooses an optimal investment for the case where high damage occurs. We would then maximise

$$C = Y - I - D_0^{Max} f(I) , \quad (12)$$

implying that

$$f'(I) = -1 / D_0^{Max} . \quad (13)$$

Using (9) where  $A = -1 / D_0^{Max}$ , we then have

$$I = h(D_0^{Max}) . \quad (14)$$

By comparing (10) and (14), it clearly follows that the optimal investment is larger by using the maximin decision rule than when using the best guess decision rule. This is of course also true if we use the specific functional form according to (3), in which case we obtain that

$$I = \frac{1}{\alpha} \ln(\alpha D_0^{Max}) . \quad (15)$$

This alternative can be seen as the application of some precautionary principle, interpreted loosely.

However, while it may make perfect sense to apply some kind of precautionary measures (e.g. Gollier et al. 2000; Eeckhoudt et al. 2005), it is difficult to defend the maximin criterion as a general principle. For example, according to Bostrom (2002), the probability that an asteroid with a diameter larger than 1 km in diameter will hit Earth in a single year is approximately 1/500,000, and the probability that it will affect a single country or part of a country is of course correspondingly smaller. Suppose that the worst outcome for a particular food-related prospect is that the area will be hit by a large asteroid. Clearly, it does not make sense to make the optimisation regarding which investments to make based on the assumption that the area will be hit by an asteroid next year.

More generally, it appears difficult to base a general decision rule on only a subset of the possible outcomes. We therefore next turn to a decision rule that takes all possible outcomes into account.

### 2.4 The Expected Value Decision Rule

An alternative to the above decision rules is to instead maximise the expected consumption, which is equivalent to minimising the expected costs in terms of  $I$  and  $D$  together, meaning that we would use the information about all possible outcomes. Then we would maximise

$$E(C) = \sum_{i=1}^n p^i (Y - I - D_0^i f(I)), \quad (16)$$

implying that

$$f'(I) = -\frac{1}{\sum_{i=1}^n p^i D_0^i} = -\frac{1}{E(D_0)} \quad (17)$$

so that the optimal investment, using (9), is given by

$$I = h\left(\sum_{i=1}^n p^i D_0^i\right) = h(E(D_0)). \quad (18)$$

Thus, the optimal investment is larger than in the best guess scenario but lower than when using the maximin decision rule. This is of course again true if we use the specific functional form according to (3), in which case we obtain that

$$I = \frac{1}{\alpha} \ln\left(\alpha \sum_{i=1}^n p^i D_0^i\right) = \frac{1}{\alpha} \ln(\alpha E(D_0)). \quad (19)$$

Note in particular that the optimum conditions are independent of the initial income  $Y$  and hence also of uncertainty regarding the income level.

## 3. THE ST PETERSBURG PARADOX

So far, the principle of maximising the expected value appears easier to defend than the alternative ones. However, also this principle is difficult to defend generally, which has been known for some hundred years. The most well-known example that clearly shows the limitations of simply maximising the expected value, or expected consumption in our case, is obtained from the so-called St Petersburg paradox:

Consider a lottery where a fair coin is flipped repeatedly until it comes up tails. The total number of flips,  $n$ , determines the prize, which equals  $\$2^n$ . For example, if the coin comes up tails the first time, the prize is  $\$2^1 = \$2$ , and then the lottery ends. If it instead comes up

heads the first three times and then comes up tails, the prize is  $\$2^4 = \$16$ , and then the lottery ends, etc. Now, what is the value of this lottery? The expected dollar value is simply given by

$$\sum_{i=1}^{\infty} p(\text{total number of flips} = i) \cdot 2^i = 0.5 \cdot 2 + 0.5^2 \cdot 2^2 + 0.5^3 \cdot 2^3 \dots = 1 + 1 + 1 \dots, \quad (20)$$

and is thus clearly infinite. Yet, most people are not willing to pay very much for participating in such a lottery, and one can moreover not credibly argue that rational people *should*. This shows clearly that the maximising expected value decision rule does not constitute a reasonable universal decision rule either. Alternatively expressed, risk neutrality, which is implicitly assumed in the expected value decision rule, is generally not a valid assumption.

A solution to the St Petersburg paradox was proposed already by Daniel Bernoulli in 1738, who assumed that people maximise *utility* rather than *money*, and that utility is concave in money, in turn implying risk aversion. Bernoulli assumed a logarithmic utility function, but the essential assumption is that the utility function is concave in income (or wealth). How much then would a utility-maximising individual be willing to pay for participating in such a lottery? Consider an individual with (a cardinal) utility function  $U = \ln Y$ , where  $Y$  is income. An individual who maximises expected utility would then at most be willing to pay  $CV$  for the lottery, such that

$$\ln(Y) = \sum_{i=1}^{\infty} 0.5^i \cdot \ln(Y - CV + 2^i). \quad (21)$$

While this maximum willingness to pay is not possible to find analytically,<sup>4</sup> it is straightforward to obtain it numerically. It is moreover easy to show that the maximum willingness to pay increases monotonically with the initial income  $Y$ . For example, when  $Y$  is 10 million USD, the maximum willingness to pay is still less than 40 USD. Thus, simply introducing a logarithmic utility function can explain the St Petersburg paradox. It is also

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<sup>4</sup> Yet, it is easy to solve analytically for the case where utility is a function of the payoff only, i.e. for the case where there is no asset integration with other sources of income. Then we simply have that

$$\ln(X) = \sum_{i=1}^{\infty} 0.5^i \cdot \ln(2^i - X) = \sum_{i=1}^{\infty} 0.5^i \cdot \ln 2^i / \ln X \quad \text{so} \quad \text{that} \quad \ln(X) = \sqrt{\sum_{i=1}^{\infty} 0.5^i \cdot \ln 2^i} \quad \text{and}$$

$X = e^{\sqrt{\sum_{i=1}^{\infty} 0.5^i \cdot \ln 2^i}} \approx 2.2$ . Hence, the individual would not be willing to pay much more than \$2 for participating in the lottery. However, it should be pointed out that such a model is inconsistent with the conventional model where different sources of income are dealt with in the same way; see e.g. Rabin (2000), Rabin and Thaler (2001) and Johansson-Stenman (2010). Yet, as these authors also point out, there is ample empirical evidence that people do not perfectly integrate gamble gains with other sources of income or wealth.

worth mentioning that the degree of concavity implicitly assumed by using a logarithmic utility function is not at all extreme, but rather, if anything, on the low side.<sup>5</sup>

The example with the St Petersburg paradox thus shows that introducing a concave utility function, i.e. introducing risk aversion, can have a very large impact on optimal behaviour. Risk aversion is also the standard explanation behind why it can be fully rational for consumers to buy insurances despite the fact that they know that the insurance companies are making profits, and hence that their own expected value must be negative on average. Hence, it appears worthwhile to explore the implications of risk aversion for the optimal investment decision in the basic model considered in the present paper, which is the task of the next two sections.

#### 4. EXPECTED UTILITY

Let us make the same assumptions as above, but introduce a strictly concave utility function such that utility  $U = u(C)$ , where  $u'(C) > 0$  and  $u''(C) < 0$ . Before dealing with the risky case, consider the benchmark case with certainty. In this case we obtain

$$U = u(Y - I - D_0 f(I)). \quad (22)$$

Thus, as before, we assume (for analytical simplicity) that consumption and absence of health damage are perfect substitutes, which of course is a strong assumption and which will be relaxed in Section 5. The first order condition corresponding to (22) is given by:

$$u'(Y - I - D_0 f(I))(D_0 f'(I) + 1) = 0, \quad (23)$$

so that

$$f'(I) = -1/D_0. \quad (24)$$

Using again the inverse function technique based on (9), we obtain

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<sup>5</sup>A logarithmic utility function implies a constant relative risk aversion parameter, defined by  $-C u''(C)/u'(C)$ , equal to unity. Many studies estimate this parameter. For example, Blundell et al. (1994) and Attanasio and Browning (1995) estimate the relative risk aversion parameter based on consumption decisions over the life-cycle and find in most of their estimates the relative risk aversion parameter to be in the order of magnitude of 1 or slightly above. Vissing-Jørgensen (2002) estimates this parameter based on observed behavior in risky decisions and finds that the relative risk aversion parameter differs between stockholders (approx. 2.5 to 3) and bond holders (approx. 1 to 1.2).

$$I = h(D_0). \quad (25)$$

Intuitively, when there is no uncertainty involved, maximisation of  $U = u(Y - I - D_0 f(I))$  is equivalent to the maximising of net consumption  $Y - I - D_0 f(I)$ .

#### 4.1 Optimal safety investment under risk aversion

Consider now again the case where  $D_0$  is stochastic, as in the previous section, so that expected utility is given by

$$EU = \sum_{i=1}^n p^i u(Y - I - D_0^i f(I)). \quad (26)$$

It can then be shown (see Appendix) that we can write the optimal investment level as

$$I = h \left( E(D_0) \left( 1 + \text{cov} \left[ \frac{D_0}{E(D_0)}, \frac{u'(C)}{E(u'(C))} \right] \right) \right). \quad (27)$$

Thus, the optimal investment level exceeds the level implied by the expected value maximisation if and only if the normalised covariance between the damage in the absence of any investments,  $D_0$ , and the marginal utility of consumption,  $u'(c)$ , is positive.<sup>6</sup> And from (22), it is easy to see that it is. This result may seem surprising. Indeed, taking risk aversion into account typically tends to decrease the size of a given risky investment; see e.g. Rothschild and Stiglitz (1970, 1971).

Why, then, does risk aversion here increase, and not decrease, the optimal investment? The reason is that risk aversion, as the name suggests, implies a willingness to pay for reducing the risk, i.e. the variation in terms of the outcome (here consumption). And a higher investment here implies a lower expected ex-post variation of consumption (in addition to the expected damage), which is contrary to the typical investment decision where an increase in a risky investment tends to increase the overall risk.<sup>7</sup>

In the special case where  $f$  is given by (3), we similarly obtain

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<sup>6</sup> Note that the covariance expression is of course in itself a function of  $I$ , but the comparison of the optimal investment with the previous cases is still equally valid.

<sup>7</sup> See also Howarth (2003) and Brekke and Johansson-Stenman (2008) for discussions of similar mechanisms when discussing optimal social discount rates regarding investments in order to combat global warming.

$$I = \frac{1}{\alpha} \ln \left( \alpha E(D_0) \left( 1 + \text{cov} \left[ \frac{D_0}{E(D_0)}, \frac{u'(C)}{E(u'(C))} \right] \right) \right). \quad (28)$$

#### 4.2 Optimal safety investment when also income is uncertain

In reality, both health damage and income are uncertain. Let us for simplicity start with the case with no uncertainty about health costs. From (25), we then have that  $I = h(D_0)$ , implying that the optimal investment level is independent of the income *level*. But what about *variation* in income? Let  $Y$  be a stochastic variable with  $m$  different values,  $Y^1, \dots, Y^m$ , so that expected utility is given by

$$EU = \sum_{i=1}^m p^i u(Y^i - I - D_0 f(I)). \quad (29)$$

It can then be shown (see Appendix) that we here too can write the optimal investment level as

$$I = h(D_0). \quad (30)$$

Intuitively, the maximisation of  $u$  is always independent of  $Y$ , and hence also independent of variations of  $Y$ .

Consider next the case where both  $D_0$  and  $Y$  are stochastic, so that expected utility is given by

$$EU = \sum_{i=1}^n \sum_{j=1}^m p^{ij} u(Y^j - I - D_0^i f(I)) = \sum_{i=1}^n \sum_{j=1}^m p^{ij} u(C^{ij}), \quad (31)$$

where  $p^{ij}$  is the probability that the damage in the absence of investments is equal to  $D_0^i$  and that the income is equal to  $Y^j$ , and hence that the resulting consumption is given by  $C^{ij}$ . It can then be shown (see Appendix) that we can write the optimal investment level as in (27), i.e.

$$I = h \left( E(D_0) \left( 1 + \text{cov} \left[ \frac{D_0}{E(D_0)}, \frac{u'(C)}{E(u'(C))} \right] \right) \right). \quad (32)$$

However, here we cannot a priori determine whether the normalised covariance expression is positive or negative. Clearly, if the distributions of  $Y$  and  $D_0$  are sufficiently positively correlated, then the overall tendency may be that utility is greater when the damage is greater, in turn implying that the covariance between damage and the marginal utility of consumption

becomes negative. One may for example think of cases where the expected damage is proportional to the consumption of a certain good, which in turn is highly income elastic. Yet, in the benchmark case where damage and marginal utility of consumption are independently distributed, and hence uncorrelated, which perhaps is a reasonable starting point for many food-related health risks, we know that the covariance is larger than zero, and hence that the riskiness (in health damage) tends to increase the investment compared to in the expected value case.

## 5. STATE-DEPENDENT EXPECTED UTILITY: WHEN CONSUMPTION AND ABSENCE OF DAMAGE ARE IMPERFECT SUBSTITUTES

So far, we have assumed that private consumption and absence of damage are perfect substitutes, implying for example that the marginal willingness to pay for reducing the damage further is independent of the income level. This is clearly a very restrictive assumption. In order to analyse the optimal investment level more generally, we will here consider the case where private consumption and absence of damage are imperfect substitutes. Let us make the same assumptions as above, but relax the assumption of perfect substitutability, such that utility  $U = u(C, -D)$ , where  $\frac{\partial u}{\partial(-D)} > 0$  and  $\frac{\partial^2 u}{\partial(-D)^2} < 0$  and where as before  $\frac{\partial u}{\partial C} > 0$  and  $\frac{\partial^2 u}{\partial C^2} < 0$ ; also assume that  $u$  is strictly quasi concave. This formulation implies that under risk, we have a case of a state-dependent EU model, since the value of the damage will generally depend on the consumption levels; see e.g. Karni (1985, 2009a) for overviews of state-dependent EU theory.

Before dealing with the risky case, however, consider the benchmark case with certainty, in which we obtain  $U = u(Y - I, -D_0 f(I))$ , implying the first order condition, with respect to  $I$ ,

$$\frac{\partial u}{\partial C} + D_0 \frac{\partial u}{\partial(-D)} f'(I) = 0, \text{ which can be rewritten as } f'(I) = -\frac{\partial u}{\partial C} / \left( \frac{\partial u}{\partial(-D)} D_0 \right), \text{ implying}$$

from (9) that

$$I = h\left(D_0 \frac{\partial u}{\partial(-D)} / \frac{\partial u}{\partial C}\right) = h(D_0 MRS_{-D,C}), \quad (33)$$



where  $MRS_{-D,C} = \frac{\partial u}{\partial(-D)} / \frac{\partial u}{\partial C}$  is the marginal willingness to pay, in terms of private consumption, for reducing the damage. In the special case where  $f$  is given by (3), we similarly obtain

$$I = \frac{1}{\alpha} \ln(\alpha D_0 MRS_{-D,C}). \quad (34)$$

### 5.1 Optimal safety investment under risk aversion

When we introduce uncertainty in damage,  $D$ , expected utility is given by

$$EU = \sum_{i=1}^n p^i u(Y - I, -D_0^i f(I)). \quad (35)$$

The optimal investment level can then be written as (see Appendix):

$$I = h \left( E(D_0) \frac{E\left(\frac{\partial u(C, -D)}{\partial(-D)}\right)}{E\left(\frac{\partial u(C, -D)}{\partial C}\right)} \left( 1 + \text{cov} \left[ \frac{D_0}{E(D_0)}, \frac{\frac{\partial u(C, -D)}{\partial(-D)}}{E\left(\frac{\partial u(C, -D)}{\partial(-D)}\right)} \right] \right) \right). \quad (36)$$

In order to interpret this result, let us compare (36) with (33). The factor  $MRS_{-D,C} = -\frac{\partial u}{\partial D} / \frac{\partial u}{\partial C}$  in (33) here corresponds to the factor

$E\left(\frac{\partial u(C, -D)}{\partial(-D)}\right) / E\left(\frac{\partial u(C, -D)}{\partial C}\right)$ . However, we also have the covariance expression

associated with the insurance value of the investment. Note that the normalised covariance is not between  $D_0$  and the marginal utility of consumption, but between  $D_0$  and the marginal

utility of reduced damage. Still, since  $\frac{\partial u}{\partial(-D)} > 0$  and  $\frac{\partial^2 u}{\partial(-D)^2} < 0$ , we have that the

normalised covariance expression is positive and hence contributes to a larger investment level. When  $f$  is given by (3), we correspondingly obtain

$$I = \frac{1}{\alpha} \ln \left( \alpha E(D_0) \frac{E\left(\frac{\partial u(C, -D)}{\partial(-D)}\right)}{E\left(\frac{\partial u(C, -D)}{\partial C}\right)} \left( 1 + \text{cov} \left[ \frac{D_0}{E(D_0)}, \frac{\frac{\partial u(C, -D)}{\partial(-D)}}{E\left(\frac{\partial u(C, -D)}{\partial(-D)}\right)} \right] \right) \right). \quad (37)$$

### 5.2 When both Damage and Income are Stochastic

Consider finally the case where both  $D_0$  and  $Y$  are stochastic, so that expected utility is given by

$$EU = \sum_{i=1}^n \sum_{j=1}^m p^{ij} u(Y^j - I, -D_0^i f(I)). \quad (38)$$

The optimal investment level can then be written as (see Appendix):

$$I = h \left( E(D_0) \frac{E\left(\frac{\partial u(C, -D)}{\partial(-D)}\right)}{E\left(\frac{\partial u(C, -D)}{\partial C}\right)} \left( 1 + \text{cov} \left[ \frac{D_0}{E(D_0)}, \frac{\frac{\partial u(C, -D)}{\partial(-D)}}{E\left(\frac{\partial u(C, -D)}{\partial(-D)}\right)} \right] \right) \right). \quad (39)$$

Hence, here too we obtain an identical algebraic expression when we also allow for income variations, i.e. (39) is identical to (36), and (37) will also continue to hold for the special case where  $f$  is given by (3). Yet, the value is of course likely to differ, depending in particular on how the health damage covaries with income.

## 6 THE ELLSBERG PARADOX

The above analysis based on the EU decision rule has introduced considerable sophistication beyond the simple EU decision rule, and this increased complexity has made it possible to explain phenomena such as the St Petersburg Paradox. Yet, as mentioned in the introduction, there is nevertheless considerable evidence that people's choices under uncertainty tend to be inconsistent with the implications of EU theory, including SEU utility theory. A well-known example is given by the so-called Ellsberg (1961) paradox, as follows: Suppose you have an urn containing 30 red balls and 60 balls that are either black or yellow; the balls are well mixed. You do not know (but you may of course have a subjective guess) the relative shares of black and of yellow balls. Consider now the choice between Gamble A and Gamble B:

Gamble A	Gamble B
You receive \$100 if you draw a red ball	You receive \$100 if you draw a black ball

Consider next the choice between Gamble C and Gamble D:

Gamble C	Gamble D
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You receive \$100 if you draw a red or yellow ball	You receive \$100 if you draw a black or yellow ball
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It turns out in surveys as well as real-money experiments that most people prefer A to B and D to C (e.g. Becker and Brownson 1964; Slovic and Tversky 1974; Einhorn and Hogarth 1986; Curley and Yates 1989). However, this violates SEU theory. To see this, note that if you prefer A to B, your subjective probability that the ball is red must be larger than that the ball is black. But if this is true, then the probability that the ball is either red or yellow must be larger than the probability that the ball is either black or yellow. Therefore, preferring A to B and D to C implies a contradiction.

Why then do most people seem to prefer A to B and D to C? A plausible explanation goes as follows: In Gamble A, the individual knows that the probability that the ball is red is  $20/60 = 1/3$ . In Gamble B, the individual does not know the objective probability that the ball is black; it can be either lower or higher than  $1/3$  and take any value from 0 to  $2/3$ . If the individual is a bit 'pessimistic', he/she might conjecture that it is lower than  $1/3$ , and hence go for A.

In Gamble C, the individual does not know the probability that the ball is either red or yellow; it can be anything from  $1/3$  to 1. In Gamble D, in contrast, the probability that the ball is either black or yellow is known and equals  $40/60 = 2/3$ . In this case, an individual who is a bit pessimistic regarding the probabilities in Gamble C will go for Gamble D.

Note that choosing A over B and D over C, *if taken separately*, is not inconsistent with SEU theory. Rather, both choices may seem perfectly reasonable in an SEU perspective. Indeed, if a firm (or an individual) offers you a gamble, it is reasonable to suspect that the firm does so for a reason, which is presumably that the expected profit for the firm, which knows the objective probabilities, is positive if you accept the offer. Thus, if a firm offers you to sell Gamble A and instead obtain Gamble B, it would make perfect sense to believe that the objective probability that the ball is black is lower than  $1/3$ , and hence you should turn down the offer. Similarly, if a firm offers you to sell Gamble D and instead obtain Gamble C, it would be reasonable to expect that the objective probability that the ball is yellow is lower than  $1/3$  and therefore that the objective probability that the ball is either red or yellow is

lower than  $2/3$ . Hence, you should turn down this offer too. The violation of SEU theory is thus related to *both* choosing A over B and D over C, and not to each of these choices separately.

In the next section, we will consider alternative theoretical formalisations that are consistent with the behaviour in the above example, i.e. that have the power to explain the Ellsberg Paradox. These formalisations have in common that they share some kind of ambiguity aversion, meaning, somewhat loosely, an attitude or preference for known risks over unknown risks.

## 7. DECISION MODELS BASED ON AMBIGUITY AVERSION

This section describes three different ways of formalising ambiguity aversion. In order to make comparisons with the previously described decision rules based e.g. on expected value and on expected utility maximisation, we will stick to the same basic assumptions as before in our highly stylised model. As before, there are  $n$  possible outcomes,  $D_0^1, \dots, D_0^n$ . However, now we do not know the ‘true’ probability distributions. Instead, there are  $k$  possible probability distributions that the decision maker has to consider. Without loss of generality, we can order the probability distributions, such that the implied expected utility derived from them is in an increasing order  $P^1, \dots, P^k$ , where  $P^j = \{p^{j1}, \dots, p^{jn}\}$  and where hence

$$\sum_{i=1}^n p^{i1} u(Y - I - D_0^i f(I)) < \dots < \sum_{i=1}^n p^{ki} u(Y - I - D_0^i f(I)). \quad (40)$$

Moreover, the decision maker does not necessarily consider all probability distributions to be equally likely. The decision maker’s subjective probability, obtained with the help of experts and other information, that probability distribution  $P^j$  is the correct one is given by  $q^j$ , etc. Then, how should the decision maker proceed?

### 7.1 The Gilboa and Schmeidler’s Maximin Expected Utility Approach

The maximin EU approach by Gilboa and Schmeidler (1989)<sup>8</sup> simply implies that the decision maker should only take into account the beliefs of the most pessimistic probability

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<sup>8</sup> See also Schmeidler (1989) for a model that under some conditions is very similar to the one considered here.

distribution,  $P^1$ , in the sense that this distribution is associated with the lowest expected utility of the  $k$  different probability distributions.

Thus, the objective function of the decision maker,  $W$ , which we without loss of generality can denote *welfare*, is given by the maximisation of the expected utility as reflected by the most pessimistic beliefs regarding the probability distributions

$$W = EU(P^1) = \sum_{i=1}^n p^i u(Y - I - D_0^i f(I)). \quad (41)$$

This expression of course looks exactly the same as (31), with the only difference that the previously ‘objective’ probabilities are here replaced by the most pessimistic one of the alternatives. Let us use the short notation  $SE^1(D_0)$  for the expected value of  $D_0$  associated with the most pessimistic probability distribution, and

$$SE^1(u'(C)) = \sum_i p^i u'(C^i) \quad (42)$$

for the expected marginal utility of consumption based on the most pessimistic probability distribution. We can then write the optimal investment as

$$I = h \left( SE^1(D_0) \left( 1 + \text{cov}^1 \left[ \frac{D_0}{SE^1(D_0)}, \frac{u'(C)}{SE^1(u'(C))} \right] \right) \right), \quad (43)$$

where  $\text{cov}^1 \left[ \frac{D_0}{SE^1(D_0)}, \frac{u'(C)}{SE^1(u'(C))} \right]$  is the normalised covariance based on the most pessimistic probability distribution, i.e. probability distribution no. 1, between the marginal utility of consumption and  $D_0$ . We can of course again use the functional form according to (3) and obtain

$$I = \frac{1}{\alpha} \ln \left( \alpha SE^1(D_0) \left( 1 + \text{cov}^1 \left[ \frac{D_0}{SE^1(D_0)}, \frac{u'(C)}{SE^1(u'(C))} \right] \right) \right). \quad (44)$$

Thus, this approach implies a maximin decision rule with respect to the expected utilities of different experts, or to probability distributions more generally. As such, it is clearly less extreme than the maximin decision rule in terms of outcomes that were presented in Section 2.3. The two decision rules will coincide in the case where the most pessimistic expert perceives that the most pessimistic outcome will occur with probability one. On the other hand, it tends to imply a higher optimal investment level than one based on SEU

maximisation.<sup>9</sup> Still, it may be questioned on the grounds that it only takes into account the most pessimistic probability distribution and that it hence ignores all other probability distributions.

To see that this kind of ambiguity aversion can indeed explain the Ellsberg paradox outlined in the previous section, assume that an individual who does not know the distributions of the black and the yellow balls quite reasonably considers all combinations possible. Then it cannot be ruled out that 60 balls are black and that 0 are yellow, that 10 are black and 50 yellow, that 0 are black and 60 yellow, etc.

Consider now the choice between Gamble A and Gamble B above. In Gamble A, the objective probability that the ball is red is  $20/60 = 1/3$ , whereas in Gamble B, the objective probability that the ball is black is unknown and can be anything from zero to  $2/3$ . Applying the decision rule by Gilboa and Schmeidler, the individual will then consider the most pessimistic of the possible probabilities that the ball is black, which is zero. Hence, the individual will go for A, since  $1/3$  is clearly larger than zero.

Consider similarly the choice between Gamble C and D. In Gamble C, the probability that the ball is either red or yellow is not known and can be anything from  $1/3$  to 1. Applying the Gilboa and Schmeidler decision rule then again implies that the action is based on the most pessimistic probability, which is that the probability that the ball is either red or yellow is  $1/3$ . In Gamble D, the probability that the ball is either black or yellow is known and equal to  $2/3$ , which is clearly higher than  $1/3$ . Hence, the individual would choose D. Taken together, an individual who uses the decision rule by Gilboa and Schmeidler would act consistent with the

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<sup>9</sup> I write ‘tends to’ since this has not been shown formally. Indeed, in the related problem of determining the optimal investment in a risky asset (which is not linked to safety improvement as here), Gollier (2009) shows that one can generally not say that the optimal investment is lower under ambiguity aversion compared to in the standard expected utility case (for the same underlying utility function). Yet, he derives sufficient conditions for when this is the case. My conjecture in the present case is that it is presumably possible to construct an example where ambiguity aversion decreases the optimal investment level (although this problem is harder than in the case considered by Gollier, since the investment here affects the safety level as well). Nevertheless, I do not consider this possibility to be economically important, and my conjecture is that for all reasonable functional forms of the utility functions one may think of, the introduction of ambiguity aversion of the Gilboa and Schmeidler type increases the optimal investment level.

choice of most people, as discussed in the previous section, and hence choose A over B and D over C.

This example also illustrates that the decision rule by Gilboa and Schmeidler implies a rather extreme ambiguity aversion, and it appears that also less extreme ambiguity aversion may be able to explain the Ellsberg paradox. In the following two sub-sections, we will therefore consider decision rules with potentially less extreme ambiguity aversion.

### 7.2 Klibanoff, Marinacci and Mukerji's Smooth Ambiguity Approach

The 'smooth ambiguity' approach presented by Klibanoff, Marinacci and Mukerji (2005) implies a generalisation of the approach by Gilboa and Schmeidler (1989).<sup>10</sup> It is 'smooth' in the sense that it introduces *degrees* of ambiguity aversion, in contrast to the maximin approach by Gilboa and Schmeidler (1989), and as such it implies smooth indifference curves. Instead of only focusing on the most pessimistic probability distribution, the smooth ambiguity approach can be seen as a weighted aggregation of all probability distribution. The objective function based on smooth ambiguity aversion can be written

$$W = \sum_j \psi(EU(P^j)) q^j = \sum_j \psi\left(\sum_{i=1}^n p^{ji} u(Y - I - D_0^i f(I))\right) q^j, \quad (45)$$

where  $q^j$  reflects the probabilistic weight attached to the probabilistic scenario  $j$  (sometimes denoted second order probabilities) and the function  $\psi$  reflects ambiguity aversion. The larger the degree of ambiguity aversion, as reflected by the curvature of  $\psi$ , the larger the differences in weights attached to pessimistic and optimistic probability distributions. This means that in the most ambiguity averse case, the smooth ambiguity approach converges to the maximin EU approach by Gilboa and Schmeidler (1989), whereas in the case of no ambiguity aversion, it converges to the conventional subjective EU approach.

Before deriving the optimal investment for this case, we will derive the optimal investment for the benchmark case of no ambiguity aversion where then  $\psi'(EU(P^j))$  is a constant for

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<sup>10</sup> For simplicity, we consider a discrete version of their model, whereas they use continuous probability distributions. See also Klibanoff, Marinacci and Mukerji (2009) for an extension of this model to an intertemporal context.

all probability distributions. In order to do this, we will proceed as before by differentiating the objective functions with respect to  $I$ , setting this expression to zero and solving for  $I$ .

Let us use the short notation

$$SE^G(u'(C)) = \sum_j \sum_i q^j p^{ji} u'(C^i) \quad (46)$$

for the decision maker's subjective expected marginal utility of consumption when taking all information into account, i.e. both the uncertainty with respect to the probability distributions and the uncertainty within each probability distribution, but without any different weighting through the  $\psi$ -function. The optimal investment can then be written (see Appendix):

$$I = h \left( SE^G(D_0) \left( 1 + \text{cov}^G \left[ \frac{D_0}{SE^G(D_0)}, \frac{u'(C)}{SE^G(u'(C))} \right] \right) \right). \quad (47)$$

Note that this expression is almost identical to (32). The only difference is that (47) is based on subjective probabilities, where the overall problem can be seen as a compound lottery (i.e. involving probabilities of probabilities), whereas (32) is based on objective probabilities and a simple lottery.

With this benchmark case at hand, let us now return to the more general derivation of the optimal investment level. Using the short notations

$$SE^G(\psi'(EU)u'(C)) = \sum_j \sum_i q^j p^{ji} \psi'(EU(P^j))u'(C^i) \quad (48)$$

for the decision maker's subjective expected marginal welfare of consumption (i.e. how a unit of consumption contributes to welfare,  $W$ ), we can write the optimal investment as (see Appendix)

$$I = h \left( SE^G(D_0) \left( 1 + \text{cov}^G \left[ \frac{D_0}{SE^G(D_0)}, \frac{\psi'(EU)u'(C)}{SE^G(\psi'(EU)u'(C))} \right] \right) \right). \quad (49)$$

By comparing (49) to (47), it can be observed that the only difference is that the normalised covariance expression is here between the initial damage and the marginal *welfare* of consumption, instead of between the initial damage and the marginal *utility* of consumption. We can alternatively rewrite (49) as



$$I = h \left\{ SE^G(D_0) \left( 1 + \text{cov}^G \left[ \frac{D_0}{SE^G(D_0)}, \frac{u'(C)}{SE^G(u'(C))} \right] \right) \right. \\ \left. \left( \text{cov}^G \left[ \frac{D_0}{SE^G(D_0)}, \frac{\psi'(EU)u'(C)}{SE^G(\psi'(EU)u'(C))} - \frac{u'(C)}{SE^G(u'(C))} \right] \right) \right\}. \quad (50)$$

Hence, the optimal investment level is higher than that based on the subjective EU approach, corresponding to the expression on the first line of (50), if  $\psi'(EU)u'(c)$  covaries more positively with  $D_0$  than does  $u'(c)$ . Since  $\psi$  is a concave transformation, this tends of course to be the case (although not strictly shown, and the caveat in Footnote 9 applies here too). Using again the functional form of  $f$  according to (3), we obtain

$$I = \frac{1}{\alpha} \ln \left\{ \alpha SE^G(D_0) \left( 1 + \text{cov}^G \left[ \frac{D_0}{SE^G(D_0)}, \frac{u'(C)}{SE^G(u'(C))} \right] \right) \right. \\ \left. \left( \text{cov}^G \left[ \frac{D_0}{SE^G(D_0)}, \frac{\psi'(EU)u'(C)}{SE^G(\psi'(EU)u'(C))} - \frac{u'(C)}{SE^G(u'(C))} \right] \right) \right\}. \quad (51)$$

### 7.3 Gajdos, Hayashi, Tallon and Vergnaud's Ambiguity Approach

An even more recent approach to how one may make ambiguity aversion instrumental is the approach of Gajdos et al. (2008). They provide an axiomatic analysis suggesting a functional form where welfare (in the sense of objective function) consists of a weighted average of on one hand the lowest expected utility of the different probability distributions, i.e. the objective function in the model by Gilboa and Schmeidler (1989), and on the other hand the subjective expected utility taking all probability distributions into account, as follows:

$$W = \phi EU(P^1) + (1-\phi) \sum_j q^j EU(P^j) \\ = \phi \sum_{i=1}^n p^{i1} u(Y - I - D_0^i f(I)) + (1-\phi) \sum_j q^j \sum_{i=1}^n p^{ji} u(Y - I - D_0^i f(I)). \quad (52)$$

Thus, (52) corresponds to the Gilboa and Schmeidler's Maximin Approach when  $\phi = 1$  and to the SEU model when  $\phi = 0$ . By using (42) and (46), we can then write the optimal investment as (see Appendix):

$$I = h \left\{ \frac{\phi SE^1(u'(C))\Omega^{1G} + (1-\phi)SE^G(u'(C))}{\phi SE^1(u'(C)) + (1-\phi)SE^G(u'(C))} \right. \\ \left. SE^G(D_0) \left( 1 + \text{cov}^G \left[ \frac{D_0}{SE^G(D_0)}, \frac{u'(C)}{SE^G(u'(C))} \right] \right) \right\}, \quad (53)$$

where

$$\Omega^{1G} = \frac{SE^1(D_0) \left( 1 + \text{cov}^1 \left[ \frac{D_0}{SE^1(D_0)}, \frac{u'(C)}{SE^1(u'(C))} \right] \right)}{SE^G(D_0) \left( 1 + \text{cov}^G \left[ \frac{D_0}{SE^G(D_0)}, \frac{u'(C)}{SE^G(u'(C))} \right] \right)}.$$

Hence, the expression inside the  $h$ -function consists of two factors. The second one, on the second line, constitutes in itself the expression that would result if the public decision maker would be subjective expected utility maximisers. The first factor, on the first line, is clearly larger than one if and only if  $\Omega^{1G}$  exceeds one. And since, again, superscript 1 denotes the most pessimistic probability distribution, in expected utility terms, it is reasonable to believe that  $SE^1(D_0) > SE^G(D_0)$ . There is moreover no reason to believe that the corresponding covariance expression is larger based on the decision maker's subjective probability distributions (based on all existing feasible probability distributions) than based on the most pessimistic one. (Yet, the same qualification as before applies here too; see Footnote 9.) How much larger the optimal investment is compared to the baseline SEU-maximising case depends not only on  $\Omega^{1G}$ , and hence on the factors that it consists of, but also on  $\phi$ . This is logical, since the larger the  $\phi$ , the larger the weight put on the most pessimistic probability distribution. As for the approach of Klibanoff, Marinacci and Mukerji (2005), the optimal investment level here tends to be smaller than that based on the maximin approach by Gilboa and Schmeidler (1989). There is no reason to expect that either of the investment levels implied by the approaches of Klibanoff, Marinacci and Mukerji (2005) and Gajdos et al. (2008) would exceed the other. Let us for completeness again use the functional form of  $f$  according to (3) and obtain

$$I = \frac{1}{\alpha} \ln \left\{ \alpha \frac{\phi SE^1(u'(C))\Omega^{1G} + (1-\phi)SE^G(u'(C))}{\phi SE^1(u'(C)) + (1-\phi)SE^G(u'(C))} \right. \\ \left. SE^G(D_0) \left( 1 + \text{cov}^G \left[ \frac{D_0}{SE^G(D_0)}, \frac{u'(C)}{SE^G(u'(C))} \right] \right) \right\}. \quad (54)$$

Finally, by comparing (49) and (53), we obtain that the investment level implied by Klibanoff et al. exceeds the one by Gajdos et al. if

$$\text{cov}^G \left[ \frac{D_0}{SE^G(D_0)}, \frac{\psi'(EU)u'(C)}{SE^G(\psi'(EU)u'(C))} - \frac{u'(C)}{SE^G(u'(C))} \right] > \frac{\phi SE^1(u'(C))\Omega^{1G} + (1-\phi)SE^G(u'(C))}{\phi SE^1(u'(C)) + (1-\phi)SE^G(u'(C))}, \quad (55)$$

and vice versa. Thus, this tends to be the case the larger the degree of curvature imposed through the  $\psi$ -function and the smaller the  $\phi$ , which follows intuition.

## 8. REFLECTIONS: SHOULD POLICY MAKERS REALLY BE AMBIGUITY AVERSE?

As mentioned above, there is evidence that people, at least sometimes, tend to be ambiguity averse. A related issue is whether public policy tends to reflect ambiguity aversion as well. There are some indications that it does. For example, Viscusi (1998) argues that policymakers are too stringent when they face ambiguous risks; he exemplifies with the higher regulation of synthetic risks compared to more familiar but often more severe carcinogens. Viscusi and Hamilton (1999) argue that '(t)hese biases, in effect, institutionalise ambiguity aversion biases' (Viscusi and Hamilton 1999: 1013). Similarly, Sunstein (2000) argues that, in the presence of divergent risk scenarios, policymakers focus too much on the worst-case scenario and do not sufficiently account for the low probabilities involved.

Thus, that people tend to be ambiguity averse is well documented, and as indicated above, there is also some evidence that actual policy tends to reflect some ambiguity aversion. Moreover, as illustrated in the previous section, there are also a couple of recent models that operationalise ambiguity aversion and hence make it possible to incorporate such aspects into the decision rules; see e.g. Chambers and Melkonyan (2009) and Karni (2009b) for recent papers on regulation under ambiguity aversion.

Yet, to conclude from this that the decision maker's regulation policy ought to reflect ambiguity aversion would be to derive an *ought* from an *is*. Still, one may perhaps argue that the principle of consumer sovereignty implies that if people are ambiguity averse, then it

should be reflected by a corresponding ambiguity aversion in policy making. While this kind of reasoning has some appeal, it is not difficult to come up with counter arguments.

The most obvious one draws on the fact that consumer sovereignty may not be the ultimate social goal in itself. Indeed, one may, following e.g. Broome (1999), Ng (1999), O'Donoghue and Rabin (2006) and Johansson-Stenman (2008), assume that what matters intrinsically is well-being rather than choice. Or, using the terminology of Kahneman, Wakker and Sarin (1997), we are intrinsically interested in *experienced* utility rather than *decision* utility.

Hence, if the ultimate goal is to maximise social well-being, and it is believed that respecting people's preferences, as revealed by their choices, is an effective way of obtaining this goal, then it follows that it is indeed a good idea for policy makers to respect the principle of consumer sovereignty. However, this is then contingent on the assumption that people do know, and act in accordance to, what is best for them (in terms of their well-being). This assumption, as a general reliable rule, has been questioned in recent behavioural economics literature. In particular, it has been argued that people tend to have self-control problems that imply time inconsistency, and that they make short-sighted decisions that they end up regretting, and hence fail to act in accordance with their own will. As a result, policy measures based on different kinds of paternalism have been proposed. For example, Gruber and Köszegi (2002) argue in favour of cigarette taxation, not in order to internalise *externalities* (which they argue are rather limited anyway), but in order to internalise what they denote *internalities*, i.e. in order to help them act in accordance with their own ultimate will and interest. Similarly, O'Donoghue and Rabin (2006) argue in favour of 'fat taxes,' and other 'sin taxes'. For good overviews of such arguments more generally, see Camerer et al. (2003) and Thaler and Sunstein (2008). See also Sugden (2004) and Bernheim and Rangel (2007, 2009) for different arguments and alternative choice-based approaches when people make mistakes. Regardless of how one feels about such paternalistic policies, it is not easy to argue in favour of time-inconsistent public policy in order to mimic the time inconsistencies of citizens.

The question here is whether ambiguity aversion should be seen as a genuine preference that the decision maker ought to reflect just as much as it should reflect other values of its citizens, or whether it should be seen as an internally inconsistent decision rule, similar in nature to time inconsistency or loss aversion, and as such be seen as a kind of irrationality

that the decision maker has no reason to mimic. My own view, following e.g. Savage (1972), Drezé (1987) and Al-Najjar and Weinstein (2009a, b), is basically in line with the latter. I believe it is difficult to find good arguments in normative analysis against the axioms underlying subjective expected utility theory, including Savage's (1954) *Sure-Thing Principle*,<sup>11</sup> which is typically sacrificed in alternative axiomatically motivated models of ambiguity aversion (e.g. Gilboa and Schmeidler 1989; Gajdos et al. 2008). Likewise, I find it difficult to argue that compound lotteries should be evaluated fundamentally differently than the resulting simple lotteries. As expressed by Al-Najjar and Weinstein (2009b: 364), 'The formal models in the ambiguity aversion literature rely on taste to fit observed behaviour, offering no substantive insights into *why* decision makers cannot form probability judgments.'

Yet, the literature on ambiguity aversion is rapidly increasing, and there are certainly several highly intelligent and prominent authors who disagree with my view. For example, Gilboa, Postlewaite and Schmeidler (2009: 285) argue that it is sometimes 'more rational not to behave in accordance with a Bayesian prior than to do so',<sup>12</sup> and that this is in particular the case when there is no or very limited information available to form a prior in an SEU assessment. They then ask the natural question (287), 'What would then be the rational thing to do, in the absence of additional information?' to which they answer (287), 'Our main point is that there may not be any decision that is perfectly rational'. However, such an answer is not very helpful when contemplating how a public decision maker ought to act. Gilboa and Schmeidler (2001: 17-18) provide an alternative definition of rationality: 'an action, or

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<sup>11</sup> Suppose that a decision maker knows that two mutually exclusive events  $A$  and  $B$  will occur with probabilities  $p_A$  and  $1 - p_A$ , respectively, where  $p_A$  may be unknown. The Sure-Thing Principle then says that if the decision maker would take a certain action if he/she knew that  $A$  would occur, and also if he/she knew that  $B$  would occur, then he/she would take the action also in an uncertain case when  $p_A$  is completely unknown. In the words of Savage (1954): 'A businessman contemplates buying a certain piece of property. He considers the outcome of the next presidential election relevant. So, to clarify the matter to himself, he asks whether he would buy if he knew that the Democratic candidate were going to win, and decides that he would. Similarly, he considers whether he would buy if he knew that the Republican candidate were going to win, and again finds that he would. Seeing that he would buy in either event, he decides that he should buy, even though he does not know which event obtains, or will obtain, as we would ordinarily say.' Yet, one may argue that the Sure-Thing Principle has less intuitive appeal when lotteries constitute the events. □

<sup>12</sup> 'Bayesian' refers here to a person who behaves according to SEU theory, and not as the term is typically used in statistics where it simply reflects a person who updates the probability judgments according to Bayes rule.

sequence of actions, is rational for a decision maker if, when the decision maker is confronted with an analysis of the decisions involved, but with no additional information, she does not regret her choices.’ Whether ambiguity aversion is consistent with such a definition of rationality is debated. For example, Gilboa and Schmeidler (2001) argue that it is while Al-Najjar and Weinstein (2009) argue that it is not. Personally, I find the definition in itself to be somewhat problematic since it implies, or at least seems to imply, that the same action may be considered rational for an individual with low cognitive capacity and irrational for an individual with high cognitive capacity.

Nevertheless, I do have some caveats. First, choice situations under ambiguity may induce fear to a larger extent, and it appears just as reasonable to deal with this kind of fear as with other kinds of negative welfare effects, as recently argued also by Treich (2010). More generally, people may experience feelings (e.g. feelings of regret; cf. Loomes and Sugden 1982) through the decision processes per se. In principle, though, one can describe the different states of the world to which the SEU theory applies in a comprehensive way that includes such feelings.<sup>13</sup> Second, such effects of fear and other feelings may induce indirect welfare effects through consumer adaptations, and such effects should presumably be considered too (cf. Johansson-Stenman 2008).

Third, a decision maker may use a decision rule with ambiguity aversion in order to trade off other unavoidable shortcomings. For example, suppose that a decision maker is aware of seemingly unavoidable time inconsistency in the decision making process. Then, conditional on such time inconsistency, ambiguity aversion may under some conditions work as a commitment and help combat the negative welfare implications of the time inconsistency (cf. Siniscalchi 2009a, b). This is similar in nature to the finding by Benabou and Tirole (2002) that it can be ‘rational’ for a time inconsistent individual to be over-optimistic with respect to own abilities.

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<sup>13</sup> One might perhaps object that it appears unnatural to write (cardinal) utility as a function of feelings, since utility per se is often seen to reflect subjective well-being as reflected by emotions. Yet, utility reflects everything that is in the person’s interest and not only the emotions associated with the process. Moreover, it is common to write utility functions in terms of some sub-utility function, e.g. in terms of private consumption, and such sub-utility functions are presumably also related to subjective well-being.

Fourth, and perhaps most important, real decisions about risk at a social level will always have to simplify reality. Such simplifications are not always innocuous. More specifically, in situations where there are several risks involved, at different levels, formal analysis will almost always (have to) ignore some of the risks. This means that for actually applied decision rules, it is implicitly assumed for many sub-problems that the most likely outcome will occur, and the most likely outcome tends to be where nothing bad happens. Now, if this is a systematic pattern, the net effect tends to be that the overall social risk will be biased downwards. One could therefore argue that ambiguity aversion is a way to correct for neglect of some risks involved in more complex risky problems.

## 9. CONCLUSION

This paper has analysed the basic question of how a public decision maker should think when faced with issues of known and unknown risks by means of a simple baseline model where the decision maker can invest in order to decrease the health risk. Since the investment is risky, the question concerns how much to invest. Optimal investment levels have been derived and compared for a number of decision rules, namely the best guess, the maximin, the expected value and the expected utility rules. Three different rules that incorporate ambiguity aversion into the expected utility model were also analysed. Overall, taking risk aversion into account through the expected utility approach tends to increase the optimal investment compared to when using the simple expected value approach. Similarly, ambiguity aversion tends to increase the optimal investment beyond what corresponds to subjective utility maximisation.

Finally, it was discussed whether it makes sense to incorporate ambiguity aversion into public policy decision rules. It was concluded that this is doubtful, since it may be argued that the empirical evidence that people tend to be ambiguity averse is a reflection of inconsistencies and irrationality rather than of their true preferences that are linked to their well-being. However, it is worth pointing out again some examples of what SEU theory does not say. It does not say that we should trust expert judgments (or for that matter that we should not trust them). Moreover, it does not say that policy should not be largely motivated by very unlikely catastrophic outcomes. Indeed, my personal view is that this part of the probability distribution is actually the most important one when it comes to actions related to global

warming, and it can certainly not be ruled out that this may be the case also for some food risk related issues.

Yet, some caveats were also presented. In particular it was argued that when dealing with complex social which includes several sub-problems, one will for practical reasons have to ignore some of the risks involved. This is an important problem that deserves more attention, and one practical way of doing this is to incorporate some kind of ambiguity aversion as a practical way of adjusting for this kind of ignorance of some risks. Whether ambiguity aversion in terms of the models presented above, and similar ones, is a good way of correcting for such risk neglects is an open question that deserves more research.

## APPENDIX

### Derivation of Equation (27)

The first order condition associated with (26) is given by

$$\sum_{i=1}^n p^i u'(Y - I - D_0^i f(I)) (-1 - D_0^i f'(I)) = 0. \quad (\text{A1})$$

Solving for  $f'(I)$  implies that

$$f'(I) = - \frac{\sum_{i=1}^n p^i u'(C^i)}{\sum_{i=1}^n p^i u'(C^i) D_0^i}, \quad (\text{A2})$$

where  $u'(C^i) = u'(Y - I - D_0^i f(I))$ . Then we have

$$I = h \left( \frac{\sum_{i=1}^n p^i u'(C^i) D_0^i}{\sum_{i=1}^n p^i u'(C^i)} \right) = h \left( \frac{E(D_0 u'(C))}{E(u'(C))} \right). \quad (\text{A3})$$

Since we, by definition, have that  $E(D_0 u'(C)) = E(D_0) E(u'(C)) + \text{cov}[D_0, u'(C)]$ , it follows that

$$\frac{E(D_0 u'(C))}{E(u'(C))} = E(D_0) + \frac{\text{cov}[D_0, u'(C)]}{E(u'(C))} = E(D_0) \left( 1 + \text{cov} \left[ \frac{D_0}{E(D_0)}, \frac{u'(C)}{E(u'(C))} \right] \right). \quad (\text{A4})$$

Substituting (A4) into (A3) gives (27).

### Derivation of Equation (30)



The first order condition associated with (29) is given by

$$\sum_{i=1}^n p^i u'(Y^i - I - D_0 f(I))(-1 - D_0 f'(I)) = 0, \quad (\text{A5})$$

in turn implying that

$$f'(I) = -\frac{\sum_{i=1}^n p^i u'(C^i)}{\sum_{i=1}^n p^i u'(C^i) D_0} = -\frac{1}{D_0}. \quad (\text{A6})$$

Using (9) finally implies (30).

### Derivation of Equation (32)

The first order condition associated with (31) is given by

$$\sum_{i=1}^n \sum_{j=1}^m p^{ij} u'(Y^j - I - D_0^i f(I))(-1 - D_0^i f'(I)) = 0. \quad (\text{A7})$$

Solving for  $f'(I)$  implies that

$$f'(I) = -\frac{\sum_{i=1}^n \sum_{j=1}^m p^{ij} u'(C^{ij})}{\sum_{i=1}^n \sum_{j=1}^m p^{ij} u'(C^{ij}) D_0^i}, \quad (\text{A8})$$

where  $u'(C^{ij}) = u'(Y^j - I - D_0^i f(I))$ . Then, using (9) we have that

$$I = h \left( \frac{\sum_{i=1}^n \sum_{j=1}^m p^{ij} u'(C^{ij}) D_0^i}{\sum_{i=1}^n \sum_{j=1}^m p^{ij} u'(C^{ij})} \right) = h \left( \frac{E(D_0 u'(C))}{E(u'(C))} \right). \quad (\text{A9})$$

However, the second way of writing this expression is identical to (A3). Hence, we can here too write the optimal investment as in (27).

### Derivation of Equation (36)

The first order condition associated with (35) is given by

$$\sum_{i=1}^n p^i \left( \frac{\partial u(C, -D^i)}{\partial C} + \frac{\partial u(C, -D^i)}{\partial (-D^i)} D_0^i f'(I) \right) = 0, \quad (\text{A10})$$

in turn implying that

$$f'(I) = -\frac{\sum_{i=1}^n p^i \frac{\partial u(C, -D^i)}{\partial C}}{\sum_{i=1}^n p^i \frac{\partial u(C, -D^i)}{\partial (-D^i)} D_0^i}, \quad (\text{A11})$$

so that, using (9), we have

$$I = h \left( \frac{\sum_{i=1}^n p^i \frac{\partial u(c, -D^i)}{\partial(-D^i)} D_0^i}{\sum_{i=1}^n p^i \frac{\partial u(c, -D^i)}{\partial c}} \right). \quad (\text{A12})$$

Multiplying and dividing by the same expression, we can rewrite this as

$$I = h \left( \frac{\sum_{i=1}^n p^i \frac{\partial u(C, -D^i)}{\partial(-D^i)} D_0^i \sum_{i=1}^n p^i \frac{\partial u(C, -D^i)}{\partial(-D^i)}}{\sum_{i=1}^n p^i \frac{\partial u(C, -D^i)}{\partial(-D^i)} \sum_{i=1}^n p^i \frac{\partial u(C, -D^i)}{\partial C}} \right). \quad (\text{A13})$$

By using the definitions of expected value, we can rewrite (A13) as

$$I = h \left( \frac{E \left( \frac{\partial u(C, -D)}{\partial(-D^i)} D_0 \right) E \left( \frac{\partial u(C, -D)}{\partial(-D^i)} \right)}{E \left( \frac{\partial u(C, -D)}{\partial(-D^i)} \right) E \left( \frac{\partial u(C, -D)}{\partial C} \right)} \right). \quad (\text{A14})$$

We can then rewrite the first ratio in the parentheses in (A14) by a covariance expression as follows:

$$\frac{E \left( \frac{\partial u(C, -D)}{\partial(-D)} D_0 \right)}{E \left( \frac{\partial u(C, -D)}{\partial(-D)} \right)} = E(D_0) \left( 1 + \text{cov} \left[ \frac{D_0}{E(D_0)}, \frac{\frac{\partial u(C, -D)}{\partial(-D)}}{E \left( \frac{\partial u(C, -D)}{\partial(-D)} \right)} \right] \right). \quad (\text{A15})$$

Substituting (A15) into (A14) implies (36).

### Derivation of Equation (39)

The first order condition associated with (38) is given by

$$\sum_{i=1}^n \sum_{j=1}^m p^{ij} \left( \frac{\partial u(C^j, -D^i)}{\partial C^j} + \frac{\partial u(C^j, -D^i)}{\partial(-D^i)} D_0^i f'(I) \right) = 0. \quad (\text{A16})$$

Solving for  $f'(I)$  implies that

$$f'(I) = - \frac{\sum_{i=1}^n \sum_{j=1}^m p^{ij} \frac{\partial u(C^j, -D^i)}{\partial C^j}}{\sum_{i=1}^n \sum_{j=1}^m p^{ij} \frac{\partial u(C^j, -D^i)}{\partial(-D^i)} D_0^i}. \quad (\text{A17})$$

Then, using (9), we have

$$I = h \left( \frac{\sum_{i=1}^n \sum_{j=1}^m p^{ij} \frac{\partial u(C^j, -D^i)}{\partial (-D^i)} D_0^i}{\sum_{i=1}^n \sum_{j=1}^m p^{ij} \frac{\partial u(C^j, -D^i)}{\partial C^j}} \right) = h \left( \frac{E \left( \frac{\partial u(C, -D)}{\partial (-D)} D_0 \right) E \left( \frac{\partial u(C, -D)}{\partial (-D)} \right)}{E \left( \frac{\partial u(C, -D)}{\partial (-D)} \right) E \left( \frac{\partial u(C, -D)}{\partial C} \right)} \right). \quad (\text{A18})$$

By then using (A15), we obtain (39).

#### Derivation of Equation (47)

By differentiating (45) with respect to  $I$ , we obtain

$$\sum_j \psi' (EU(P^j)) \left( \sum_i p^{ji} u'(C^i) (1 + D_0^i f'(I)) \right) q^j = 0, \quad (\text{A19})$$

so

$$f'(I) = - \frac{\sum_j \sum_i q^j p^{ji} \psi' (EU(P^j)) u'(C^i)}{\sum_j \sum_i q^j p^{ji} \psi' (EU(P^j)) u'(C^i) D_0^i}.$$

(A20)

Since  $\psi'$  is constant, we can rewrite (A20) as

$$f'(I) = - \frac{\sum_j \sum_i q^j p^{ji} u'(C^i)}{\sum_j \sum_i q^j p^{ji} u'(C^i) D_0^i}. \quad (\text{A21})$$

Let us use the short notation

$$SE^G(u'(C)D_0) = \sum_j \sum_i q^j p^{ji} u'(C^i) D_0^i \quad (\text{A22})$$

for the decision maker's subjective expected value of the product of the marginal utility of consumption and  $D_0$ . In other words,  $SE^G(u'(C)D_0)$  reflects a weighted mean value of the initial damage, where the weights are given by the marginal utility of consumption. Substituting (46) and (A22) into (A21) then implies

$$f'(I) = - \frac{SE^G(u'(C))}{SE^G(u'(C)D_0)}. \quad (\text{A23})$$

Using again (9), we obtain

$$I = h \left( \frac{SE^G(u'(C)D_0)}{SE^G(u'(C))} \right), \quad (\text{A24})$$

where we can substitute

$$SE^G(u'(C)D_0) = SE^G(D_0) \left( 1 + \text{cov}^G \left[ \frac{D_0}{SE^G(D_0)}, \frac{u'(C)}{SE^G(u'(C))} \right] \right), \quad (\text{A25})$$

and obtain (47).

#### Derivation of Equation (49)

Let us use the short notation

$$SE^G(\psi'(EU)u'(C)D_0) = \sum_j \sum_i q^j p^{ji} \psi'(EU(P^j))u'(C^i)D_0^i \quad (\text{A26})$$

for the decision maker's subjective expected value of the product of the marginal welfare of consumption and initial damage  $D_0$ . Thus,  $SE^G(\psi'(EU)u'(C)D_0)$  reflects a weighted mean value of the initial damage, where the weights are given by the marginal welfare of consumption. Substituting (48) and (A26) into the first order condition (A21), we obtain

$$f'(I) = - \frac{SE^G(\psi'(EU)u'(C))}{SE^G(\psi'(EU)u'(C)D_0)}. \quad (\text{A27})$$

Then, using (9), we obtain

$$I = h \left( \frac{SE^G(\psi'(EU)u'(C)D_0)}{SE^G(\psi'(EU)u'(C))} \right). \quad (\text{A28})$$

Using finally that

$$SE^G(\psi'(EU)u'(C)D_0) = SE^G(D_0) \left( 1 + \text{cov}^G \left[ \frac{D_0}{SE^G(D_0)}, \frac{\psi'(EU)u'(C)}{SE^G(\psi'(EU)u'(C))} \right] \right), \quad (\text{A29})$$

and substituting (A29) together with (46) into (A28), we obtain (49).

#### Derivation of Equation (53)

The first order condition for an optimal investment level corresponding to (52) is given by

$$\phi \sum_{i=1}^n p^{li} u'(C^i) (1 + D_0^i f'(I)) + (1 - \phi) \sum_j \sum_{i=1}^n q^j p^{ji} u'(C^i) (1 + D_0^i f'(I)) = 0, \quad (\text{A30})$$

implying

$$f'(I) = - \frac{\phi \sum_{i=1}^n p^{li} u'(C^i) + (1 - \phi) \sum_j \sum_{i=1}^n q^j p^{ji} u'(C^i)}{\phi \sum_{i=1}^n p^{li} u'(C^i) D_0^i + (1 - \phi) \sum_j \sum_{i=1}^n q^j p^{ji} u'(C^i) D_0^i}, \quad (\text{A31})$$

and, using (9),

$$I = h \left( \frac{\phi \sum_{i=1}^n p^i u'(C^i) D_0^i + (1-\phi) \sum_j \sum_{i=1}^n q^j p^j u'(C^i) D_0^i}{\phi \sum_{i=1}^n p^i u'(C^i) + (1-\phi) \sum_j \sum_{i=1}^n q^j p^j u'(C^i)} \right). \quad (\text{A32})$$

Let us use the short notation

$$SE^1(u'(C)D_0) = \sum_i p^i u'(C^i) D_0 \quad (\text{A33})$$

for the expected value of the product of the marginal utility of consumption and initial damage associated with the most pessimistic probability distribution. Substituting (42), (46), (A25) and (A33) into (A32) then implies

$$I = h \left( \frac{\phi SE^1(u'(C)D_0) + (1-\phi) SE^G(u'(C)D_0)}{\phi SE^1(u'(C)) + (1-\phi) SE^G(u'(C))} \right). \quad (\text{A34})$$

By next using that

$$SE^1(u'(C)D_0) = SE^1(D_0) \left( 1 + \text{cov}^1 \left[ \frac{D_0}{SE^1(D_0)}, \frac{u'(C)}{SE^1(u'(C))} \right] \right) \quad (\text{A35})$$

and substituting (A22) into (A34), we obtain (53).

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