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EMPIRICAL TEST OF MARKET EFFICIENCY OF OMX OPTIONS

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Abstract

This thesis examines the market efficiency of the Swedish index option (OMX) market. The empirical tests are carried out on an ex-ante basis using the index future contracts to hedge the index options. Two efficiency tests have been performed, explicitly lower boundary and put call parity conditions test and dynamic hedging strategy. The first test shows that the discovered deviations from the lower boundary conditions and put call parity condition do not generate abnormal returns, particularly after all transaction costs have been accounted for. The second test, delta neutral dynamic hedging strategy, is simulated by comparing option market prices with the Black Scholes prices calculated using two volatility estimators, namely historical volatility (HSD) and weighted implied standard deviation (WISD). The strategy evidences that no systematic abnormal returns can be found in the OMX option market, therefore supporting the hypothesis of no arbitrage opportunity and market efficiency.

Keywords: Low boundary conditions; Put call parity; Delta neutral hedging; OMX options; Transaction costs; Implied volatility; Historical volatility; Market efficiency

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LIST OF ABBREVIATIONS

BS	Black Scholes Model
PCP	Put Call Parity
ATM	At the money
ITM	In the money
OTM	Out of the money
TTM	Time to maturity
WISD	Weighted Implied Standard Deviation
HSD	Historical Standard Deviation
IV	Implied Volatility
OMX	Swedish Index
S&P 500	U.S. Standard and Poor´s Index (500 most traded stocks)
S&P 100	U.S. Standard and Poor´s Index (100 most traded stocks)
FTSE-100	London Index
MIB30	Italian Index
MIBO30	Italian Index Options
DAX	German Index
CAC40	French Index
CBOE	Chicago Board Option Exchange

Part one: Background

The background introduces the general framework of this study, which includes Chapter 1 and Chapter 2.

In the introductory Chapter 1, we start with a literature and historical review of the options market and in particular the index options market both from a global and a regional (refers to Swedish market in particular) point of view. The importance of option market efficiency is argued thereafter. Finally, the purpose and motivation as well as the outline and delimitations of the paper are presented. Chapter 2 covers an overview of the structure of the Swedish option market, including an introduction of the financial instruments that will be used throughout this thesis.

Chapter 1 . Introduction

1.1 Background

One of the exciting developments in finance over the last 25 years has been the growth of derivatives markets (Hull, 2003). Since the first traded options were introduced by the Chicago Board Options Exchange in April 1973,(www.cboe.com, 2003), futures and options are now traded actively on many exchanges throughout the world. Forward contracts, SWAPs, and many different types of options are regularly traded outside exchanges by financial institutions, fund managers, and corporate treasurers in what is termed the over-the-counter market. Nowadays, many financial institutions hold nontrivial amount of derivative securities in their portfolio (Nandi & Waggoner, 2000).

Index options markets have had a significant role in financial markets since the Chicago Board Options Exchange introduced the first index option contract in 1983, and it has been one of the most successful of the many innovative financial instrument introduced over the last few decades, as their high trading volume indicates (Ackert and Tian, 2000). The introduction of new financial instruments, especially the Exchange Funds on U.S. market in the early nineties as well as the European markets a decade later, offers the possibility of index options to grow continuously (Deville, 2004).A majority of European countries introduced index options from the late 1980's.[1]

Like other European countries, in September 1986, with the purpose to serve as an underlying asset for trading in standardized European style options and future contracts the index options were introduced in Sweden. Since then, the OMX option market has grown tremendously.[2]

Well-functioning financial markets are vital to a thriving economy because these markets facilitate price discovery, risk hedging, and allocating capital to

[1] For example: options on the Italian index MIB30 were first introduced in November 1995 (Cavallo and Mammola, 2000), options on the German index DAX were introduced in August 1991 (Mittnik and Rieken 2000), options on the Swiss index SMI were introduced on 7th December 1988 (Chesney, Gibson and Loubergé 1995), options on the French index CAC 40 were introduced in 1991 (Capelle-Blancard and Chaudbury, 2001), options on the Finish index FOX were first traded 2nd May 1988 (Martikainen and Puttonen, 1996) options on the Spanish index IBEX 35 were first traded in January 1992 (Corredor and Santamaría, 2002).

[2] http://www.omxgroup.com

its most productive uses (Chaudhury and Capelle-Blancard, 2001). The importance of option efficiency could also be seen from the political and economic points of view, as Andersson (1995) argued that options could decrease the economic burdens of the taxpayers. Hence, we can say that efficient options markets provide well-functioning financial and economic system, same for the index option markets. In particular, index options give market participants the ability to participate in anticipated market movements, without having to buy or sell a large number of securities, and they permit portfolio managers to limit downside risk (Ackert & Tian, 1999). Therefore, the pricing efficiency of the option markets is of great importance to *academics, practitioners, and arbitrageurs, even for politicians*.

1.2 Index Option Market Efficiency Test

As in a majority of other studies, we define efficiency[3] as "A market is efficient with respect to information set θ_t if it is impossible to make economic profits by trading on the basis of information set θ_t" (Jensen, 1978), or the efficiency test is based on a principle called *no arbitrage opportunity*, which is critical for ensuring market efficiency because it forces asset prices to return to their implied, no arbitrage values (Ackert and Tian, 2000). However, in some situations, if arbitrageurs are subject to capital constraints and they can not raise the capital necessary to form risk-less hedging, they will be unable to undertake trades that would move the market toward an efficient state (Shleifer and Vishny, 1997).

Based on the interpretation of the market efficiency, there are many studies that have tested the efficiency of the options market. They have followed two main methods: 1) Lower boundaries and put call parity conditions test 2) Dynamic hedging strategy[4]. The first method is based on the basic upper and lower boundaries as well as put call parity conditions. And the second method is performed based on a Delta neutral portfolio created by comparing the market and model prices. Although dynamic hedging is a model dependent test; it is very popular in recent studies.

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[3] The concept of market efficiency will be discussed more in Chapter 5.

[4] We employ both of these two methods to test market efficiency of OMX index option market, details see our discussion in Chapters 5, 6 and 7.

Using econometric models[5] to estimate options prices was very popular before 1973. However, after Black Scholes (1973)[6] published their seminal paper on option-pricing, Black-Scholes' arbitrage-free option pricing formula has dominated finance academics for market efficiency tests. However, according to Hull (2003), a number of problems which relate to the assumptions of the Black Scholes Model arise in carrying out empirical research based on BS Models.[7] The first problem is that any statistical hypothesis about Market efficiency (EMH) has to be a joint hypothesis to the effect that (1) the option pricing formula is valid and (2) market is efficient. To distinguish between the two hypotheses, market efficiency and model validity, one of the two has to be taken as an assumption. The second problem concerns the choice of the best estimate of the stock price volatility. The third problem is to ensure that data on the stock price and option price are synchronous. This is also a crucial requirement for Put Call Parity, which was first introduced by Stoll (1969)[8] and is another method used by many empirical studies to test market efficiency. In order to avoid the problems of the Black Scholes Model, other efficiency tests are based on the Cox and Ross (1976) and Cox et al.´s (1979) binomial tree model and other models.[9]

However, in the earlier studies, most of the efficiency tests focus on stock options in the U.S. market.[10] Later on, the index option markets efficiency tests become a popular topic.[11] Meanwhile the introduction and fast growing index options in Europe have also called for the attention of empirical research to these markets. Since the mid 90´s, a few papers have investigated relatively

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[5] Econometric models assume that past relation between an option's price and its determinants is stationary, the econometrically fitted functional forms to past data can be interpreted as the empirical pricing equations for the option (Kassouf, 1965).

[6] Black and Scholes (1973) demonstrated that a risk-less hedge portfolio can be formed and the fair price of an option can be derived from the hedge portfolio.

[7] This study will use the Black Scholes Model for option pricing and efficient test, how to avoid problems for applying Black Scholes Model will be presented in Chapter 5, Methodology and Data.

[8] More studies based on Put Call Parity will be reviewed in Chapter 5 Methodology and data.

[9] For example: constant-elasticity-of-variance model (Cox and Ross, 1976), Jump-diffusion model (Merton, 1976 and Naik and Lee, 1990), Variance-gamma option pricing model (Madan and Milne, 1991), Bivariate Diffusion model (Hull and White, 1987). Other stochastic models include: Scott (1987), Stein and Stein (1991), Wiggins (1987), and Hetson (1993).

[10] E.g. Gould and Galai (1974) on OTC options, Klemkoski and Resnick (1980) on stock options traded on the CBOE.

[11] The empirical studies on index option market include Evnine and Rudd (1985), Chance (1986), Finucane (1991) and Wagner, Ellis and Dubofsky (1996) tested on S&P 100, and Kamara and Miller (1995), Ackert and Tian (2001) and Bharadwaj and Wiggins (2001) tested on S&P 500, these last two studies test other arbitrage relationships, such as the box-spread, also tested by Billingsley and Chance (1985), Chance (1987) and Ackert and Tian (1998).

new European index option markets, specifically: Puttonen (1993) on the Finnish option index market; Chesney et al. (1995) on the Swiss index option market; Capelle-Blancard and Chaudhury (2001) and Deville (2004) on the French (CAC 40) index option market; Mittnik and Rieken (2000) on German DAX index option market; Cavallo and Mammola (2000) and Brunetti and Torricelli (2003) on Italian MIB 30 index option market. What makes these tests of particular interest is that besides Cavallo and Mammola (2000), put Call Parity is the dominated method for rest of these existing studies. [12] Although Cavallo and Mammola (2000) have performed dynamic hedging, they have not deeply examined the predictability of volatility estimators.

Hence, compared with the empirical studies in U.S. market, studies on European markets are still limited, in particular, few papers have investigated the Swedish option and Index OMX option market, besides Andersson (1995)[13] who tested Swedish call option market with dynamic hedging strategy and put call parity, and Byström (2000)[14] who tested pricing bias in the OMX option market with both stochastic and Black Scholes Model implied volatility. However, the option market and Stockholm stock exchange during the 1990's has gone through a power change and the turnover of OMX has grown tremendously since it was first introduced in September 1986 (http://www.omxgroup.com). Although OMX market still cannot be compared with index option markets in U.S., such as S&P 500 or other European countries, such as FTSE -100, the trading volume is really large enough to obtain arbitrage free profit. Efficiency in small markets is equally important as in the large ones, in fact it is the new and smaller markets that are more likely to lack efficiency (Capelle-Blancard and Chaudhury, 2001), and thus it is possible to affirm that efficiency tests in small markets are very necessary.

[12] The reason may be that Put Call Parity makes it possible to assess the efficiency of options market without resorting to any pricing model.
[13] Andersson (1995) first estimates extreme volatility estimator in the case of non-trading and drift in the log-price process, and evaluated the predictive abilities of alternative volatility predictors and finally the efficiency of the Swedish call options market is tested by applying general test form—put call parity, and specific test form—Dynamic hedging strategy by applying Black Scholes model. Efficiency is evidenced from no arbitrage profit and no abnormal return.
[14] The paper investigates OMX Index call option market efficiency, through stochastic volatility option prices that are calculated through Fourier-Inversion and a dynamic hedging scheme. The author demonstrated lager ex ante profits, excluding transaction costs, however, the inefficiency according to author was due to the Black Scholes Model invalidity.

1.3 Purpose and Motivations of the Thesis

Being aware of the importance of option market efficiency, we decided to test OMX index option market efficiency. Byström tested the OMX index option market, but the method employed was dynamic hedging strategy only; the Put-call parity, which is model independent, was not applied. Moreover, the data used in his research covers too short of a period and it is out of date (from October 1993 to July 1994). Andersson (1995) employs both Put Call Parity and Dynamic hedging strategy. However, he only tests the market of stock options rather than the OMX index market, therefore, the need of studies on OMX index option market with relative wide coverage of the topic and relatively new data is obvious.

Hence, **the purpose of this study is to test whether OMX –Swedish index option market is efficient using data on OMX index options from June 1, 1994 to June 30, 2004.** The efficiency test will be fulfilled by several subtasks. Following Andersson (1995) and Cavallo and Mammola (2000), we will first test Put Call Parity conditions which are model independent. The efficiency identification will be concluded from two lower boundary tests, short and long hedge strategy (Put-Call Parity), violations from these tests signal an inefficient market. However, since put-call Parity is only a general form of test for market efficiency, we will further investigate the possibility to generate profitable positions through the simulation of ex-ante dynamic hedging strategies.[15] An abnormal return indicates an inefficient market. Additionally, we will also carefully examine both the historical volatility and implied volatility estimators and find out which is the better method for volatility forecasting.

Consequently, three Efficient Market Hypotheses[16] (EMH) will be designed and tested to draw the efficiency conclusion. These EMHs include:

EMH1: there are not significant violations from lower boundary condition for both put and call index options (market is efficient)

EMH2: there are not significant violations from PCP conditions for both long and short hedge strategies (market is efficient)

[15] Explanations of Ex-ante tests see chapter 5 Methodologies and data.

[16] For further discussions of EMH see chapter 5 Methodologies and data.

EMH3 : there are not significant abnormal returns from the dynamic hedging strategy for both OMX put and call index options (OMX market is efficient)

These three hypotheses will be tested in Chapters 6 and 7 respectively in this study. The conclusion of market efficiency will be drawn from the results of these tests. Failing to reject the hypotheses signals market efficiency, and vice versa.

Compared to other studies, our tests of the above three hypotheses are handled in several different ways.

First, in order to reduce the data synchronous problem, instead of duplicating the entire index basket, we hedge the index options with the index futures. It is very common that market makers use futures to hedge their positions (Draper & Fung, 2002). Indeed, for European options and futures that share the same underlying asset and common expiration day, the option can be priced as if it is an option on the futures contract (Black, 1976). However, this is the first paper which hedges OMX index options by OMX futures

Second, volatility will be forecasted from both historical standard deviation and implied volatility methods, which will be used to calculate the model option prices, and a paired T-test will be performed in order to get a clearer picture of the deviations between model prices and market prices as well as the predictability of historical volatility and implied volatility. The main motivation behind this paper is that no in-depth study of OMX Index options market has been done by using both Put Call Parity and Dynamic Hedging with recent data, specially. Even in European countries, only a few studies have applied dynamic hedging strategy for index option market efficiency testing is very limited.

1.4 *Outline of This Thesis*

The main goal of this study is to perform an efficiency test of the Swedish Index Option (OMX) market. To accomplish this, we divide the study into five sections in terms of background, theoretical framework, methodology and data, empirical research, and conclusions and recommendations.

Background: Chapters 1 & 2

In Chapter 1, an introductory background of the option and index option market is given, subsequently, the main purpose, methodologies as well as motivations and contributions of the paper are presented after a brief description of the problem situation - option market efficiency test. This chapter ends with the presentation of the outline and delimitations of this thesis. Chapter 2 covers a short Index Option (OMX) history, a definition of an option and future contract, and a description of the market structure of OMX options in Sweden.

Theoretical Framework: Chapters 3 & 4

In Chapter 3, the option pricing model - Black Scholes Model - to be used in this study will be presented, together with the empirical evidence for pros and cons for the Black Scholes Model. Finally, the chapter will be ended by the conclusion of the reasons why we use the Black Scholes Model. In Chapter 4, different measures of volatility that will be used to simulate dynamic hedging strategy are derived. First, we define and present two major ways for volatility estimation and forecasting: Historical volatility and Implied volatility. Thereafter, the evaluation from other empirical studies of different methods for historical and implied volatility will be reviewed, followed by the presentation of chosen volatility methods in this study.

Methodology and Data: Chapter 5

In Chapter 5, we describe the methodologies used in this study, namely, lower boundary and put call parity condition tests and dynamic hedging strategy (includes paired T-test). Thereafter, an empirical literature review of these two methods will be presented with the purpose to explain the reasons why we choose them in this study. Meanwhile, we will show the ways we try to avoid these common problems in option market efficiency tests. These are, briefly, by using index future to hedge index options to avoid non-synchronous data and dividend problem, by testing put call parity to avoid model-dependent problems, by applying both historical and implied volatility to avoid controversial ways to choose between them. Finally, the data used in this study will be presented.

Empirical Research: Chapters 6 & 7

Chapter 6 executes lower boundary and put call parity conditions test. The lower boundary and put call parity conditions in the presence of transaction costs are derived and subjected to empirical test on the Swedish index options market using daily options and index prices. The results from these tests will be presented thereafter. Chapter 7 conducts dynamic hedging strategy which includes two paired T-tests between Black Scholes Model prices and market prices. First, historical (HSD) and implied (WISD) volatility will be caudated and both of them will be used to calculate option prices. Consequently, option prices from the Black Scholes Model will be paired to market prices to perform paired T-tests and results from these Paired T-tests will be presented thereafter. Finally, Delta neutral dynamic hedging strategy will be performed with both historical and implied volatility to verify the possibility to realize systematic abnormal returns on the Swedish OMX market. Results from dynamic hedging are presented at the end of this chapter.

Conclusions and Recommendations: Chapter 8

Some concluding remarks are offered in this section. The conclusion from the empirical tests will be given, followed by the further research suggestions which can be of interest to examine by other researchers.

1.5 Delimitations

There are several problems encountered when we carry out the OMX efficiency test, which were believed to affect the efficiency results, and of which the majority have been overcome in a satisfactory way. However, there are still several delimitations we would like to point out.

1.5.1 The level of data synchronization

In order to overcome the data synchronous problem, we use index futures to hedge index options, as the index future and index option markets are more synchronized than stock and option market (Granath & Krisell, 2004), especially because the OMX index options and futures expire on the same date. However, there is still a data synchronization problem, such as option prices and futures prices are reported at different times within one day. In this case, the intraday data (e.g. every ten minutes) would perform much better than daily data. But this problem can not be solved due to time and economic constraints. Fortunately, the efficiency result from this study seems quite reasonable compared with other studies, however, intraday data is highly recommended for efficiency tests.

1.5.2 Transaction costs

The second delimitation in this study concerns the transaction costs, particularly those regarding the brokerage fee. For the efficiency test in this study, three different types of costs are considered, bid-ask spread, trading and clearing fee, and commission fee (e.g. brokerage fee). An accurate estimation of the actual transaction costs, especially brokerage fees, are very difficult, not only do the transaction costs tend to vary over time but they also depend on the particular strategy and the size of the transaction. Although the transaction costs could be collected by performing interviews or questionnaires for the related organization, this could not be conducted in this thesis due to time constraints.

In order to overcome the delimitation of transaction costs, for the lower boundaries and put call parity tests, we have performed sensitivity analysis with respect to the different level of transaction costs assumptions. For the dynamic hedging test we have considered the trading and clearing fee and bid-ask spread, and so leaving out the brokerage fee as even when excluding the brokerage fee the results from dynamic hedging are mainly negative.

Therefore, the results from efficiency tests in this study have been carefully interpreted and analyzed and we believe they already reflect the influence of the transaction costs.

However the results from these tests may be affected by the assumed transaction cost.

Chapter 2 . Swedish OMX Market

2.1 Introduction

This chapter provides an overview of the characteristics of the financial instruments that will be employed throughout the thesis. It starts with a brief description of the OMX index, followed by an overview of the OMX options and OMX futures. Finally, we discuss the Swedish exchange market structure and system.

2.2 History

The trading of options began in Sweden on June 12, 1985. At that time, only call options on six stocks could be traded (Andersson, 1995). Almost a year later the index option “OMX” was introduced on September 30, 1986, with the purpose of serving as the underlying asset for standardized European options and futures.

In the 80’s, many changes occurred in the Stockholm Stock Exchange, meaning that many companies were created to trade in the stocks and over-the-counter markets and many new financial instruments were introduced.

In 1987, the so called “puppet tax” was introduced on Sweden (Andersson, 1995), which charged the stock brokers. According to some researchers, this had a negative impact, reverting the trading activity and deteriorating the stock markets. Besides this, the stock and derivatives markets have grown tremendously both in number of traded contracts and the creation of new financial instruments.

Another successful instrument, the OMX futures, was introduced in Sweden April 3, 1987 (Sutcliffe, 1997). The futures began to be traded with a more modest number (1.7 millions) of contracts than the options (6.73 million contracts in 1987). But today, the OMX futures are highly liquid and traded.

Now, the Swedish Stock Exchange is placed 3rd in the rank of leading European Derivative Exchanges (according to the number of contracts) in 2003 (www.omxgroup.com).[17] Additionally, it is well-known for its highly secure and computerized trading system managed by OMX Technology.

2.3 OMX Index

The OMX Index[18] was listed at an initial level of 500.00 Kr. On April 27, 1998, the OMX index was divided by four and thus the current base value equals 125 Kr. It is constructed with the 30 most traded Swedish stocks[19] on the Stockholm Stock Exchange and calculated as an arithmetic market value weighted index. Therefore, it should closely reflect the development of the stocks listed in the Exchange (Sutcliffe, 1997).[20]

The OMX index is a price index, it only automatically adjusts dividends which are characterized as special dividends and for the part representing the excess 10% per share of the Index share's last paid price (excess dividend amount) (http://omxgroup.com).The index is recalculated twice a year –January 1 and June 1. The control periods for the stocks that shall be included in the OMX index are six months beginning seven months prior to the recalculation dates.

Figure 2.1 shows the development of the OMX options for the period 1987 until September 2004. As we can observe, the volume of OMX option contracts traded fluctuates between 4 and 7 million annually. The OMX options were introduced at the end of 1986, at that time the volume traded in 1987 was already considerably high. However, a noticeable drop in the volume traded in 1988 was probably due to the incorporation of the "puppet tax". From 2000, the traded volume of OMX options continually increased and this upward trend denotes a great potential growth perspective.

[17] See Appendix 1. Annual Activity in Stockholm Derivatives Market.
[18] It is continuously computed by AB Nyhetsbyrån Direkt.
[19] See Appendix 3. OMX Index Composition (2003)
[20] Market value weighted system is very common, and it provides the advantage that each share price is weighted accordingly to importance in the portfolio of shares, this means that changes in the index measure the changes in the market as a whole.

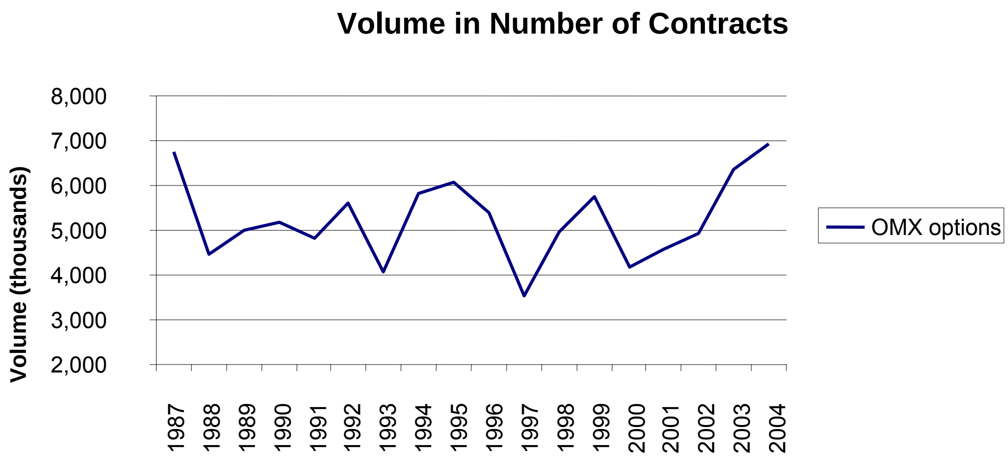

Figure 2.1 OMX Evolution from 1987 until 09/2004

(Source: www.omxgroup.com)

2.4 Characteristics of Index Options Contracts (OMX-options)

An option is a financial security that gives the right but not the obligation to buy (call option) or sell (put option) a specific amount of a given property, at a specified price (the strike price) during a specified period of time (time to maturity).

For each option, there is a buyer that holds the option contract[21] and a seller that writes the option.[22] If the option is exercised, the writer must deliver the underlying asset. When the underlying security is an index, the contract is settled in cash. For the holder, the potential loss is limited to the price (premium) paid to purchase the option. On the other hand, the potential loss of the writer of a call option is unlimited, (unless the seller already owns the underlying security), because the price can go up infinitely, whereas the writer of a put option can lose the value of the underlying security, as the price can go down infinitely as well.

There are two types of option contracts in terms of American and European style. When options can be exercised before maturity they are called American option, while European options can only be exercised at maturity date.

[21] The buyer of the option is said to have a long position.
[22] The seller of the options is said to have a short position.

In this study, we deal with European options whose underlying asset is the Swedish index, OMX. OMX options[23] are standardized option contracts with cash settlement. The expiration day of the options is the fourth Friday of the expiration month of the expiration year, and if the expire date is not a bank day it will be the preceding bank day. The times to maturity are available in three, six or twenty four months.[24] Index option contracts are generally traded on a 100 index point basis, as is the OMX Index.

An important attribute of OMX options is that at the expiration day the options become Asian, that is, at expiration day the payoff of the options depends on the average price of the underlying asset for that day (Granath & Krisell, 2004). In order to avoid possible complications due to this special feature, options with very short time to maturity will be screened out.[25]

2.5 Characteristics of the Index Future Contracts (OMX-Futures)

A future contract is a standardized, transferable, exchange-traded contract to deliver the underlying asset at a specified price, on a specified future date, futures are distinguished from generic forward contracts in that they contain standardized terms, are traded on a formal exchange, i.e. are regulated by the exchange agencies, and are guaranteed by clearinghouses. Hence, in order to insure that payment will occur, futures normally have a margin requirement that must be settled daily.

Investors do not pay a premium to acquire futures contracts unless the transaction takes place after the issuing of the futures contract. The premium will be positive or negative depending on the underlying asset movements (Granath & Krisell, 2004). Most of the futures contracts do not end up in delivery since the traders choose to close out their positions before delivery time by taking an opposite position from the original one (Hull, 2003).

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[23] See Appendix 2. OMX Options Specifications and Appendix 4. Leading European Derivatives Markets (2003).
[24] Before 2004 there was also options for 1 year time to maturity (www.omxgroup.com)
[25] Refer to Chapter 5, 6, 7 for further details.

In the same manner as the OMX options, the expiration day of the futures is the fourth Friday of the expiration month of the expiration year. Additionally, the maturity times available are three, six or twenty four months. However, OMX futures do not fulfill the general definition of futures contract in two ways; they do not involve payment of margins and they are not settled on a daily basis (Granath & Krisell, 2004).[26]

2.6 Market Structure and Option trading at OMX

Trading of options in Sweden takes place at OMX.[27] It performs both as clearing centre and market place. The Exchange takes care of all the information regarding the trade, which is continuously available, namely the bid and ask prices, paid prices, maturities, and volumes. All the derivatives trading is done within a computerized system. The trading system comprehends an electronic limit order book managed by OMX. When it is possible, the new orders are matched automatically to orders that are already in the limit order book. When it is not possible to match an order this is added to the book. Thus, the trading system is both an electronic matching system and a market making system (Berchtold and Nordén, 2004).

The clearing centre supervises that all the contracts are fulfilled through the margins system. The Clearing House has a number of clearing members.[28] Customers are represented in the clearing operations by a Clearing Member who is liable for the customer's obligations. Nevertheless, the customer is fully responsible to the Clearing House for the performance of the obligations. The Clearing House becomes a seller in relation to the buyer and a buyer in relation to the seller, and thereby assumes all of the rights and obligations in accordance with the relevant contract specifications. Additionally the Clearing House keeps

[26] Usually, the broker asks the investor to deposit an initial margin in a margin account and at the end of each trading day, the margin account is adjusted to reflect the investor's gain or loss. This practice is known as marking to market the account. This is the main characteristic that distinguishes future contracts from forward contracts. Hence, in our case, there is no difference between OMX futures and forwards.

[27] OMX Exchanges is the name of the company that comprises different divisions, OM Technology division (develops transaction technology as well as processing and outsourcing services) and HEX Integrated Markets division formed through the combination of Stockholmsbörsen, Helsinki Exchanges, HEX Tallinn and HEX Riga and the Vilnius Stock Exchange. In the present year OMHEX adopted the new name of OMX.

[28] In the same manner as the investor has a margin account with the broker, a clearing member is required to maintain a margin account with the Clearing House.

track of the daily transactions calculating the net positions of each of the clearing members.[29]

This overall system assures the non existence of losses due to default. This becomes true since the number of defaults have been insignificant at least in the major exchanges (Hull, 2003).

2.7 Conclusion

The Stock Market in Sweden is managed by OMX and is very well known for its highly computerized system and high degree of transparency. It is placed among the top three European Stock Markets with respect to the volume traded. OMX offers a wide range of financial instruments besides OMX options and futures. OMX options are considered to be sophisticated and sufficiently liquid instruments as well as their futures counterparts.

In the chapter, we have reviewed the history and characteristics of the Swedish OMX market as well as a brief presentation of the basic features of the OMX index, options and futures. Finally, a description of the market structure and the role of OMX have also been discussed.

[29] http://www.omxgroup.com

Part Two: Theoretical Framework

Before continuing with the empirical research, two key determinants to carry out the efficiency test are presented, that is the option pricing model in Chapter 3 and the volatility estimator in Chapter 4. The well known Black Scholes option pricing model will be introduced as well as the reasons to choose the model. Additionally, a literature review of alternative option pricing methodologies will be presented. The volatility concept and its importance for option valuation will be argued, and volatility estimating and forecasting methods used in this thesis will be discussed.

Chapter 3 . The Black Scholes Model

3.1 Introduction

Before we proceed with the empirical research, we present the widely known Black Scholes model for valuing Put and Call European options. Despite many option pricing models that have been proposed for calculating the option values and the market volatility expectations, the Black Scholes model is still the most commonly used model.

The Black Scholes model[30] was introduced in 1973 by Fisher Black, Myron Scholes and Robert Merton. They developed a pioneering formula for the valuation of stock options that smoothed the way for economic valuations in many areas. Additionally it helped to generate new types of financial instruments and eased risk management.

However, some authors criticize the model due to its simplifying assumptions. Nevertheless, as will be argued later in the chapter, it is the most commonly used model by the market participants and it will also be used in this thesis.

In this chapter we will first present the Black and Scholes option pricing model in Section 3.2, while the future pricing formula will be introduced in Section 3.3, the reasons to utilize the Black Scholes model will be argued in Section 3.4, and finally a conclusion will be given in Section 3.5.

3.2 The Black Scholes Pricing Model for European Options

First, we will introduce the underling assumptions of the Black Scholes model:[31]

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[30] Black and Scholes (1973) demonstrate that a risk less hedge portfolio can be formed and the fair price of a call option can be derived from the hedge portfolio.

[31] See Appendix 5. Black-Scholes Model Derivation

1. The stock price follows the Ito's lemma process (dS =μsdt + σsdz) with μ and σ constant.
2. The variance of the rate of return on the stock is constant over the lifetime of the option and is known by the market participants.
3. The short selling of securities with full use of proceeds is permitted.
4. There are no transaction costs or taxes.
5. There are no dividends during the lifetime of the derivatives.
6. There are no risk-less arbitrage opportunities.
7. Securities trading are continuous.
8. The risk free rate of interest, r, is constant and the same for all maturities.

The Black-Scholes model is used to calculate a theoretical option price and it employs the five key determinants of an option's price: stock price, strike price, volatility, time to expiration, and short-term (risk free) interest rate.

The formula for calculating the theoretical option prices are as follows:

European Call Option:

$$c = S_0 \, N(d_1) - K \, e^{-rT} N(d_2) \qquad \text{Eq. 3.1}$$

European Put Option:

$$p = K \, e^{-rT} \, N(-d_2) - S_0 \, N(-d_1) \qquad \text{Eq. 3.2}$$

Where:

$$d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \qquad \text{Eq. 3.3}$$

$$d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \qquad \text{Eq. 3.4}$$

The variables are:

S_0 = stock price today

K = strike price

T = time remaining until expiration, expressed as a percent of a year

r = current continuously compounded risk-free interest rate

σ = annual volatility of stock price (the standard deviation of the short-term returns over one year)

The statistical terms are:

ln = natural logarithm

N(x) = standard normal cumulative distribution function[32]

e = the exponential function

3.3 The Pricing of Futures

In this section we present the formula[33] used to calculate the futures over the index, which relates the futures price and the spot price:

$$F_0 = S_0\, e^{(r-q)T} \qquad \text{Eq. 3.5}$$

Where:
q = Dividend yield on the portfolio represented by the index
S_0 = Spot price today
F_0 = Futures or forward price today
T = Time until delivery date
r = Risk-free interest rate for maturity T

For the formula to be true it is important that the index represents an investment asset. In other words, changes in the index must correspond to changes in the value of a tradable portfolio (Hull, 2003).

3.4 Is the Black Scholes Model the Best Available Option Pricing Model?

Most academics and practitioners agree that the BS model is the best existing option pricing model. There are several reasons that make this model the best choice. First of all, for it simplicity the option prices are easily calculated and the only unknown parameter is the volatility.[34] Moreover, a call can be replicated creating a position synthetically, by borrowing money[35] and buying

[32] The probability that a variable with a Standard normal distribution is less than or equal to x.
[33] In section 5.4.1 we show that when using futures to hedge the options on a dividend paying stock the dividend effect drops.
[34] Refer to chapter 4 for volatility estimation.
[35] $K\, e^{-rT} N(d_2)$

the stock.[36] Additionally, the only factor that affects the option price is the variation in the underlying stock, and the volatility is considered to be constant (Andersson, 1995). Black and Scholes (1973) also suggested that a perfect hedge position can be created for a short period of time by taking long position in the stock and a short position in the call option and yielding a risk-less rate (Jarrow, 1999). Therefore, the value of the option that would prevent the establishment of a profitable hedge is the Black Scholes option value.

However, the assumptions of the BS model have been criticized for being simplistic; for instance, 1) The model does not take dividends and early exercise of the option into account, 2) The volatility fails to be constant and the distribution of the stock prices deviates from log normality and 3) The risk free interest rates change over time.

Besides these, recent studies have proved the deviations in option pricing due to possible variations of the underlying assumptions are on average negligible (Granath and Krisell, 2004), thus the Black Scholes option pricing formula continues to be the most widely used model.

Next, we discuss empirical studies whose purpose is to test the superior (inferior) predictive ability of other option pricing models with respect to BS price.

3.4.1 Model evaluation from empirical result of other studies

A number of papers have been conducted to analyze alternative option pricing models. Following Andersson (1995), these studies can be grouped by four different approaches.

The first group of studies tries to empirically asses the performance of the model under variations in the assumptions of the Black Scholes model.[37] The

[36] $S_0\,N(d_1)$

[37] Such as Merton (1976) who simulated the effects of specification errors in the BS option prices, he found that the effects of the misspecification were rather small. Boyle and Ananthabarayanan (1977) used an estimate of the variance in option pricing models, finding that the bias in the average option value was small. Bhattacharaya (1980) created neutral hedge positions using model prices for options and the actual distribution of stock prices and concluded that the BS prices show no major mispricings. Bookstaber (1981) calculated the impact of the synchronicity problem of options and stock quotations and concluded that the problem is significant when there are less than twenty options traded in the day.

results show that the deviations of the model under violation of the assumptions are irrelevant. These deviations are inversely related to the time to maturity and the closeness to in-the-money.

The second group of studies compares the actual prices with the model prices.[38] The disadvantage of this approach is that either the model prices or the deviations between them and the actual prices are non-stationary. The main conclusions that can be drawn from these studies are:

- The Black Scholes model is not a good predictor for out-of-the-money close to maturity options.
- The volatility of the stock returns is non-stationary, i.e., not constant or not predictable.
- Alternative models do not prove to be better estimators than the Black Scholes model, since the non-stationary problem applies for all of them.

The third approach consists of creating risk-neutral hedge positions and tests the behavior of the results of the investments as suggested by Black and Scholes (1973). The idea is to buy (sell) one option and sell (buy) a fraction of the underlying stock. The yield of the hedged portfolio minus the costs carried in the investment should be equal to zero, otherwise they imply market inefficiency.[39] Most of the studies carried agree that abnormal returns disappeared when transaction costs are accounted for.

[38] For example, Trippi (1977) performed an ex-post test buying all the options undervalued by 15% and short selling all the options overvalued by more than 15%, he observed a weekly average return of 11.4% concluding market in CBOE was inefficient. Macbeth and Merville (1979) performed two tests, one in (1979) comparing the actual and model prices for call options, were they found that mispricing decrease as time to maturity decrease and another in (1980) where they compared the model prices and the Cox (Constant Elasticity of Valuation model) prices concluding that the stochastic processes generating stock prices should be reconsidered. Blomeyer and Klemkosky (1983) compared the Black Scholes model vs. the market prices and the Roll model prices against the market prices; they found no difference in the pricing bias of both models. Rubinstein (1985) compared the BS model prices versus five different models (pure jump model, mixed diffusion-jump model, constant elasticity of variance diffusion model, compound diffusion model, displaced diffusion model) which relaxed the assumptions that the stock prices follows a continuous path over time and that the volatility is non stochastic, he found no model consistently better than other.

[39] Black and Scholes (1972) found out that profits disappeared when transaction costs were considered. Galai (1977) performed both ex-ante and ex-post tests, the former showed that profits vanished when the bid-ask spread cost was considered and the ex-post tests showed no abnormal returns when including transaction costs but he found synchronicity problem when he added the dividend adjustment to the BS model. Chiras and Manaster (1978) using spread positions for options that deviated by 10% or more from market prices, dividend effect and WISD found market inefficiency. Phillips and Smith (1980) introduced the bid-ask spread and other transaction costs over Chiras and Manaster (1978) data and found that the profits were eliminated.

The fourth group of the studies is based on the assumptions that the only unknown parameter is the volatility, which is calculated based on the actual option prices and under the assumption that the markets are efficient and synchronous. These studies are carried out on different volatility estimators.[40] There are two common ways to estimate volatility, one based on historical information (HSD) and another based on the actual market prices (IV). However, it is arguable which the best estimator is.

Based on empirical research the supports for the Black Scholes Model are:

- There is no model that conclusively shows better predictions than the BS.
- The Black Scholes model is a good tool when valuing at-the-money options.[41]
- Although the Black Scholes model assumes non-stationarity of the volatility, it shows good predictions for near maturity options.
- The data synchronization problem biases the results showing the importance of high quality data.

3.5 Conclusion

In this chapter, we have presented the theory of the Black Scholes option pricing model. The introduction of the model implies a major breakthrough for financial markets, allowing for a faster expansion of derivative markets in equities, foreign currencies, interest rates, and commodities. Following, we introduced the futures over stock indices pricing formula. Later, we discussed the reasons why the Black Scholes model is a good model for option pricing still today. Even though the Black Scholes model presents some unrealistic assumptions it has not been proved yet to be inferior to other option pricing models. The discussion is followed by an exposition of different empirical researches carried on alternative models. The chapter concludes evidencing the validity of the model. Therefore, we will use the Black Scholes price for dynamic hedging test.

[40] Latané and Rendleman (1976) and Chiras and Manaster (1978) found that WISD as better estimator than HSD, on the other hand Beckers (1980) and Canina and Figlewski (1991) found no predictive superiority of ISD over HSD.

[41] Deviations are mostly found for deep in and out-of-the money options.

Chapter 4 . Volatility Forecasting

4.1 Introduction

Empirical analysis of option pricing has focused on two related but logically distinct concerns, model choosing and volatility forecasting (Edey & Elliott, 1992). Black Scholes Model will be used for test. Now, we have to choose the suitable volatility.

The volatility plays a central role in the valuation of financial derivatives such as options and futures, and can, in fact, have a greater influence on the value of derivative securities than price movements in the underlying assets (Claessen & Mittnik, 2002). Especially in this study, to assess the fair value of OMX option and to perform dynamic hedging strategy, we need to choose a good ex-ante measure of volatility return regarding the future volatility.

The purpose of this chapter is to choose volatility forecasting method for option pricing application. A definition of volatility will be presented in Section 4.2, followed by the volatility measuring and forecasting methods in Section 4.3. Thereafter, in Sections 4.4 and 4.5, we describe the forecasting methods used in this study (both ISD and HSD methods). The reasons for WISD will be explored in Section 4.6. Finally, conclusions from this chapter will be drawn in Section 4.7

4.2 What is Volatility?

Hull (2003) defines volatility as "a measure of the uncertainty of the return realized on an asset". However, the volatility has no exact definition, although depending on the investment, the volatility can be described in several different ways[42]. Volatility is commonly referred to as the standard deviation of the

[42] Volatility can be defined as: 1. The long run price variability over the option life. 2. Average standard deviation in the immediate future 3. An indication of how far the price of the underlying asset might be over our investment horizon 4. The implied volatility. 5. "Risk" (Andersson, 1995)

returns of the underlying asset from today until the expiration of the option contract. The underlying asset can be the stocks, index or futures, etc.

4.3 Volatility Measuring and Forecasting Methods

There are basically two approaches to generate volatility forecasting. One is based on the Historical Standard Deviation (HSD) or actual volatility (Taleb, 1997). To extract information about the variance of future returns from their history, estimating historical volatility and projecting it forward is a very common approach for volatility forecasting in practice. The simple variance method is most popular.

An alternative method of obtaining a volatility forecast is called implied volatility, which was first proposed by Latane and Renleman (1976) and applied and developed by others. This method elicits market expectations about the future volatility from observed options prices, namely, volatility obtained by equating the option prices from option formula with market option prices. Thus, implied volatility measures the expected volatility of an underlying security as reflected in the price of an option on that particular security (Fontanills, 2003).

However, the superiority in the estimate and predictive ability of these two groups of volatility methods are quite controversial in academics and practices. A number of empirical investigations support the idea of using implied volatilities as a predictor for future volatility.[43] They argue that as a market contains all of the information, using historical estimates of standard deviation is not appropriate, and some traders tend to regard implied volatilities as a measure of how the market is currently pricing a given option relative to its underlying asset, without worrying too much about whether it is an accurate forecast of the actual volatility that will be realized over the options futures lifetime (Figlewski, 2004). However, the null hypothesis that historical returns add no information to that already contained in the implied volatilities was empirically rejected by a number of studies.[44] Although there are several

[43] For example: Latane and Rendleman (1976), Chiras and Manaster (1978), Gemmill, (1986) , Shastri and Tandon (1986), Scott and Tucker (1989).

[44] For example: Canina and Figlewski (1991), Day and Lewis (1992), Lamoureux and Lastrapes (1993), Xu and Taylor (1995). Specially, Day and Lewis (1992) in the case of the S&P 100 index and by Lamoureux and

problems associated with the historical standard deviation method[45], option traders and many academic researchers typically ignore them and calculate historical volatility estimates by the variance method.

As of today, there is not a definitive set of prescription on how to get the best volatility forecast other than that further research needs to be done. Volatility forecasting is vital for derivatives trading, but it remains very much an art rather than a science, particularly among derivatives traders (Figlewski, 2004). Hence, we use both of these two approaches for volatility forecasting in this study. A predictive ability of these two methods will be explored by the Paired T-test.[46] In the next section, we present the volatility of these two methods in detail.

4.4 Historical Standard Volatility (HSD)

The most common used method to estimate historical volatility is the basic standard historical volatility estimator.[47] The formula is in the following (Hull, 2003):

Define:

N+1 = number of observations

S_i= Stock price at end of i^{th} (=0, 1, 2…n) interval.

T = Length of time interval in years

And let

$u_i = \text{Ln}(S_i/S_{i-1})$ for i =1,2,..,n

The standard deviation, $(\sigma)\ HSD = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2}$ the annualized volatility is $\sigma * \sqrt{252}$

Lastrapes (1993) found support for this hypothesis using currency options. Xu and Taylor (1995) find support for this hypothesis using currency options traded on the Philadelphia stock exchange.

45 E.g. Serial autocorrelation in stock return, non-stationary variance and infrequent trading (Figlewski,2004).

46 Paired T-test will be discussed in detail in chapter 7 of this study.

47 Another historical volatility method is from ARCH family (includes GARCH). However, ARCH or GARCH are more suitable for short time horizon volatility forecasting. (Figlewski, 2004). As our forecast period is more than 10 years, according to Andersson (1995),GARCH (1,1) and Moving average 20 days volatility predictor are all doing a good job in predicting the future volatility, we use the basic historical volatility estimate method and use 20 days moving average for volatility forecasting.

The length of time to be used in estimating the volatility with historical estimator is unclear, it can be seen that HSD is an unbiased estimator of σ^2 (the variance). However, $\sqrt{HSD}$ is biased estimator of σ (Rubinstein, 1985). Based on the simple variance estimation, moving average methods are widely used in practice (Hwang and Satchell, 1998). Therefore the moving average HSD20 days will be used for the volatility forecasting.

4.5 Implied Volatility

Implied volatility is the value of a stock's standard deviation of returns which, when employed in the option pricing formula, will equate an observed option price with the prices calculated from the price formula. Hence, an option pricing equation like the Black Scholes model[48] giving the fair value for an option as a function of the price of the underlying asset, the options strike price and time to maturity, the risk less interest rate, and the volatility parameter is necessary. Of all these variables, only volatility is not directly observable from different markets (Hull, 2003). Unfortunately, it is not possible to invert in the pricing formula so that σ is expressed as a function of S_0, K, r, T and option price.

However, looking at the option pricing formula, it is possible to solve the model backwards from the observed price to determine what the market volatility input must be.

If we denote the market price and model value for a given option as C_{market}[49] and C_{model} respectively, and implied volatility as σ_{IV}, we can write:

$$C_{market}(\sigma_{IV}) = C_{model}$$

According to this basic principle, so it is easy to find the implied volatility by numerical methods.[50] Perhaps the easiest method is the Visual Calculus - Method of Bisection (Figlewski, 2004).[51]

[48] Black Scholes Model is the most common used option pricing model, there are also other models like Binomial tree or diffusion model, etc (we presented in the chapter one), but none of these have been as widely used and accepted as the Black Scholes Model.
[49] We calculate the market option price by using the midpoint of high and low daily prices.
[50] Besides numerical approximations, as an alternative is to run the regression. Interested can read Beckers (1981) and Whaley (1982).
[51] One begins with the options market price C_{market} and two trial volatilities that will bracket the true value, $\sigma_{high} < \sigma_{IV} < \sigma_{low}$, so we have $C(\sigma_{high}) < C_{market} < C(\sigma_{low})$. Now we define the bisect range: define the midpoint of

This is the method used in this study.[52] Naturally, these calculations can not be done manually. All implied volatilities will be computed by using Microsoft Access with a computer program written with a macro (Visual Basic) function. The basic principle of these macros follows a Bisection method.

However, one very crucial problem for implied volatility is that for the same underlying asset, we can obtain different implied volatilities.[53] This is so-called implied volatility smile.

4.5.1 Implied volatility smile

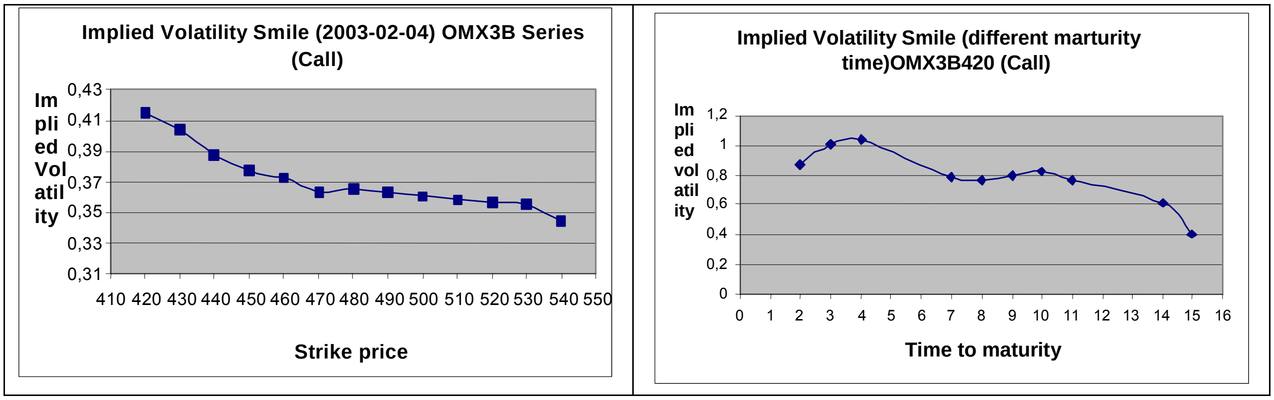
According to the Black Scholes Model, the volatility of an asset is constant over the life time of an option, consequently, all options with the same underlying asset should provide the same implied volatility (Canina and Figlewski, 1993). However, in practice the Black Scholes implied volatility tends to differ across strike price and time to maturity on the same underlying asset. This is the so-called "volatility smile", for equity and index options, a "sneer" appears—the implied volatility decreases monotonically as the strike prices rises relative to the index level, with the rate of decrease increasing for options with shorter time to expiration (Dumas et al, 1998).

the volatility is $\sigma_{mid} = (\sigma_{high} + \sigma_{low})/2$ and compute $C(\sigma_{mid})$. If $C_{market} < C(\sigma_{mid})$, then σ_{IV} is less than σ_{mid}, so we replace σ_{high} by σ_{mid}, and bisect the new narrower range. Otherwise if $C(\sigma_{mid})$ is below C_{market}, σ_{IV} lies in the upper half of the range, so we keep the same σhigh but replace σ_{low} by σ_{mid}, and continue the bisection. Each iteration cuts possible range for σ_{IV} in half, and the procedure continues until the desired accuracy is obtained.

[52] Implied volatility can be computed by another method, Newton-Raphson Search. This is another numerical linear approximation procedure which can achieve converge in a few steps. But it requires calculation of both of the options price and its vega or partial derivative with respect to volatility, at each iteration Let σ_{TRIAL} denote some initial trial value for IV and $v = \partial C/\partial\sigma$ be the Vega evaluated at σ_{TRIAL}. We want to know how much to change σ_{TRIAL} to get σIV. A linear approximation, Δ, can be calculated as $\Delta = (C_{MKT} - C(\sigma_{TRIAL})) / V$. Set the new value of σ_{TRIAL} equal to the old $\sigma_{TRIAL} + \Delta$, and repeat the process, until convergence to the desired accuracy is attained (typically only a couple of iterations). Newton-Raphson search converges much faster than bisection, but requires enough extra computation at each stage that it may not be faster in practice, especially if numerical derivatives are needed to obtain V.

[53] Figlewski (2004) stated that the problem of different (noise) Implied Volatility is due to synchronized data, bid ask spread and also infrequently traded options.

Figure 4.1 Volatility smile for OMX3B, OMX3B420



This relationship can be seen from the Figure 4.1, the implied volatility is plotted against strike price (moneyness) and time to maturity. As we can see from the figure, implied volatility changes significantly with the changes in strike prices and maturity time. Options with longer time to maturity and in- and out-of-the-money call options are more sensitive to the change with respect to volatility changes.

Researchers frequently try to deal with noise in implied volatilities by averaging across a number of options on the same underlying asset. In some cases, a simple average is used (often known by the acronym AISD for Average Implied Standard Volatility). However, for several reasons, taking a weighted average (WISD, for Weighted Implied Standard Volatility) is more common.

4.6 Weighted Implied Standard Deviations

One reason for weighted average volatility is that some options are traded more often than others, so their reported prices are expected to contain more reliable information. Another reason for weighting is to adjust for differing sensitivities of option values to the volatility parameter. There are considerable differences in the sensitivity of options prices changes with respect to stock return variance. This can be seen from Figure 4.1, for options that are deep-in or out-of-the money, a small price change has a big impact on the implied volatility. Hence, simply average implied volatility will not be sound (Latane and Rendleman, 1976). It is reasonable that options displaying high sensitivity to changes in standard deviation should be given more weight than options displaying low sensitivity (Chiras and Manaster, 1978).

Therefore, we follow Chiras and Manaster's method, the price elasticity of each option with respect to its implied volatility is considered when we construct the WISD from a set of options ISDs, the elasticity is the measure of the percentage change in the option value with respect to the percentage change in the ISD, this is the so-called VEGA. After obtaining the WISD for each day, we use it to forecast future volatility by advancing one day.[54]

Hence, the key to calculate WISD is finding the correct value of VEGA (V). We follow the calculation procedure mentioned by Cahill (1998):[55]

$$V_j = \frac{\delta W_j}{\delta \sigma_j} \approx F\sqrt{t\left(\frac{1}{\sqrt{2\pi}}\right)e^{\frac{-(dl_j)^2}{2}}}$$

Where,

F = Current futures price

T = Time to maturity

W_j =Price of option

$$d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

Then we revise the WISD with the following formula (Chiras and Manaster, 1978):[56]

$$WISD = \sum_{j=1}^{N} \sigma_j \left(\frac{V_j\left(\frac{\sigma_j}{W_j}\right)}{\sum_{j=1}^{N} V_j\left(\frac{\sigma_j}{W_j}\right)} \right)$$

Where,

N = Number of options recorded for a particular asset on a particular date.

σ_j = Implied standard deviation for option j of the asset.

[54] This is the most used methods by other studies.

[55] Computer programming for Vega see Appendix 8.

[56] Computer programming for WISD see Appendix 9

4.7 Conclusion

In this chapter, we have discussed volatility estimating and forecasting. We started the definition of volatility and continued with a discussion of commonly used volatility forecasting methods. There are two main groups for volatility estimating and forecasting, namely, historical and implied volatility methods, we have different choices among each group. Although there are several problems regarding forecasting, the sample variance method is still the most popular one, both in academics and in practice. Consequently, in the implied volatility group, WISD is the most common one. As there is not a definitive set of prescriptions regarding how to get the best volatility forecast, we decide to use both HSD and IV for forecasting. However, the predictive ability of them will be examined in Chapter 7.

Part Three: Methodology and Data

In this section the concept of market efficiency and the methodological issues are presented. How we tackle the problems of efficiency tests are carefully examined as well as a description of the data transformation in this study.

Chapter 5 . Methodologies and Data

5.1 Introduction

This chapter concerns general methodological issues and data used to perform the efficiency tests in this study. In tests of the EMH, the information subset of interest is the sequence of past security price movements. Tests using this kind of information are called Weak Form tests of EMH.

There are two main methodologies to test the efficiency of the option market. One is boundary conditions test including upper and lower boundary, which was first introduced by Merton (1973), and Put Call parity conditions, which was first developed by Stoll (1969) and then extended by others such as Courtadon and Bodurtha (1986). Another approach is a specific form of test (Andersson, 1995), namely, dynamic hedging scheme. This form of test usually uses a specific option pricing model which requires more restrictive assumptions about the behaviors of individuals and markets. The idea behind the latter one is very simple: Find an option which is mis-priced by comparing its model price to the market prices and create (theoretically) risk-less positions (Delta or Gamma neutral) for the option on the stock, then liquidate the position after a time period or rebalance Delta or Gamma frequently[57] and liquidate it on the expiration date,[58] and finally, the abnormal rate of return (Or NPV) is calculated. If the return is higher than the risk-free interest rate (or the positive NPV) ,[59] then the market is said to be inefficient.

Applying both put call parity and dynamic hedging methods to perform efficiency tests are common,[60] and some studies use only one of them.[61]

[57] According to Delta, the rebalancing will be made by purchasing options "undervalued" by the market (under the assumption that the theoretical price established by the model is the "fair value") and selling overvalued ones.

[58] Rebalance Delta frequently for European options and liquidate after a time period see Galai (1975).

[59] Black and Scholes and Merton model (1973) indicate that if a certain portfolio is formed consist of a risky asset, such as a stock, and a call option on that asset, then the return of the resulting portfolio will be approximately equal to the return on a risky-free asset, at least over short period of time.

[60] For example: Tucker (1985), Andersson (1995), Cavallo and Mammola (2000).

[61] For dynamic hedging strategy see Shastri and Tandon, (1986), Byström (2000), Bhargava and Clark, 2003), Chau and Allen (2002), etc. For Put Call Parity see Stoll (1969), Evnine and Rudd (1985), Kamara and Miller (1995), Ackert and Tian (2001).

Among the choices between the two methods, we decided to follow Andersson (1995) and Carvallo and Mammolla (2000) to perform both lower boundary and the put call parity condition test and dynamic hedging strategy.

This chapter is organized in the following way:
First, we present the concept of market efficiency in Section 5.2, and in Section 5.3 we describe the reasons to perform both boundary and PCP conditions test and dynamic hedging strategy. In Section 5.4 we present the general issues regarding efficiency tests problems and how we tackle them, as well as the reasons for utilizing ex-ante tests in this study.[62] The data used in this thesis are presented in Section 5.5. Finally, a summary of this chapter will be made in Section 5.6.

5.2 The Efficiency Concept

Efficiency can be interpreted in an economic and financial way. From the economists' point of view, efficiency is in terms of Pareto optimality, where it is asked if it is possible to improve the welfare of at least one economic agent without worsening the welfare of any other agent. In the context of the options market, the relevant question is whether the creation of a new market place for trading options increases the welfare for the whole society. The second meaning of efficiency (in financial terms) examines the possibility of earning profits that are higher than normal given the risk taken (Andersson, 1995). The second meaning, referred to as "the efficient market hypothesis," is the topic of study that prices of the securities are examined for the possibility of earning higher than normal return.

5.2.1 Three categories of efficiency

In an efficient market, we assume that the market responds immediately to all available information. The new information is rapidly reflected in the price of a traded security. According to the differential response rates, researchers divided efficiency into three categories, each dealing with different kinds of information content. Weak form efficiency (information on past prices) says that the market fully incorporates the information in past stock prices, thus, the

[62] In this chapter we briefly present these methodologies, the detail testing procedures for each method will be presented in different chapters in Part four: Empirical research.

strategy based on past information can not generate profits if weak-form efficiency holds. In the second category, the semi-strong form efficiency, the concern is the speed of price adjustment to publicly available information, in addition to the sequence of past prices. Finally, the third category, strong form of efficiency, asks whether any investor or group of investors (managers or mutual funds) have monopolistic access to any information relevant for the information of prices (Ross et al, 2002).

In our study, the efficiency tests based on the past information belong to the weak form of test.[63]

5.2.2 The efficiency market hypothesis

According to Jensen (1978), the Market Efficiency Hypothesis (MEH) is an extension of the zero profit competitive condition from the certainty world of classic price theory to the dynamic behavior of prices in speculative markets under conditions of uncertainty. Our Market Efficiency Hypothesis is centrally around the definition of market efficiency:

> *"A market is efficient with respect to information set θ_t if it is impossible to make economic profits by trading on the basis of information set θ_t" (Jensen, 1978, p3).*

By economic profits, we mean the risk adjusted returns[64] net of all costs. Therefore, our efficiency definition is no arbitrage profit or the inability of any trader to consistently generate above normal returns after transaction costs have been take into account for either Boundary tests and Put-call Parity or dynamic hedging strategy. However, in some situations, if arbitrageurs are subject to capital constraints and they can not raise the capital necessary to form the risk-less hedge, they will be unable to undertake trades that would move the market toward an efficient state (Shleifer and Vishny, 1997).[65]

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[63] However, Cavallo and Mammola (2000) argue that dynamic hedging is a little "stronger" test compared to Boundary and put call parity test.

[64] Adjusting for the changing risk of a position in an option can be accomplished by creating a neutral hedge position (Delta Neutral portfolio), if this hedge return will be greater than zero it is considered supernormal (Black and Scholes, 1973).

[65] E.g., in Swiss market, the short selling is prohibited for investors; therefore, we can not conclude the market is inefficient even there are significant arbitrage free profits, as no one can exploit them.

5.3 Why Do We Perform Both Dynamic Hedging and Boundary Tests?

As a specific form of testing, dynamic hedging offers us a clearer picture of pricing deviations between market price and model price, and efficiency identified by the existence of abnormal returns. Furthermore, the deviation between market prices and model prices are compared by running Paired T-tests, which show us an even clearer figure about the performance of HSD (Standard Historical Deviation) and IMD (Implied Volatility) as well as the factors influencing the deviation between market and model prices. However, recall our discussion in Chapter 1, the disadvantages of this method include: difficulty of choosing the best volatility forecast method, joint efficiency test,[66] and the data synchronization problem. Hence, to avoid choosing the controversial volatility methods, we use both HSD and IMV for dynamic hedging. Furthermore, for the joint-efficiency hypothesis problem, we also perform lower boundary and Put Call Parity condition tests, which do not depend on any option pricing model, and avoid the confounding nature of hypothesis of market efficiency and model specification prevalent in many efficiency studies. However, the data synchronous problem is even more critical for lower boundary and Put Call Parity condition tests. It is more popular to tackle this problem by using intraday data to investigate option markets, which can minimize data synchronous problem, but this is not possible for us to obtain intraday data, so we avoid this problem by using futures to hedge against index options, reasons and advantages of which will be presented in next section.

5.4 General Methodological Issues for Efficiency Tests

5.4.1 Future contracts against index options

Data synchronization is one of the most crucial requirements for boundary and put call parity as well as dynamic hedging tests. In order to avoid this problem, many studies use the intra-day data (every few minutes) to test the market efficiency. However, to use futures to hedge index options is another good choice when intra-day data is not available. It is very common that market makers use futures to hedge their positions (Darper & Fung, 2002). Indeed, for

[66] Like Black Scholes Model, every pricing model has different assumptions. To test efficiency of market one has to assume the model is valid, namely, market efficiency and model validity is a joint hypotheses.

European options and futures that share the same underlying asset and common expiration day, the option can be priced as if it is an option on the futures contract (Black, 1976). In reality, futures and futures options are traded side by side in the same exchange, and they close at the same maturity day, this minimizes data synchronization problems. Besides avoiding data synchronization problems, there are also other advantages for using futures against options.

Avoiding cost and difficulty of duplicating the underlying index basket

If we do not use futures to hedge index options, we have to replicate the whole index basket, which is not possible for us since we are not able to collect the data regarding the index basket duplication. Hence, it is cheaper and more convenient to hedge the options with futures. A number of authors[67] have examined the pricing efficiency of index options and index future contracts with varying degrees of concern for the numerous frictions that impede the arbitrage process, resulting from cost and difficulty of trading the underlying index basket.

It is more convenient to use futures against options

It is natural that people choose to trade options on futures rather than options on the underlying asset. The main reason appears to be that a futures contract is, in many circumstances, more liquid and easier to trade than an underlying asset. Furthermore, a futures price is known immediately from trading on the futures exchange, whereas the spot price of the underlying asset may be not so readily available. *An important point is that exercising it does not usually lead to delivery of the underlying assets. This facilitates hedging, arbitrage, and speculation (Hull, 2003).*

Avoiding dividend effect

As we discussed in Chapter 2, the OMX index does not adjust the dividend payment automatically, we have to take into account dividend effect[68] which is

[67] E.g. Kavussanos and Nomikos, (2000), Draper and Fung (2002).
[68] The options prices will be affected by dividend payment if the dividend is paid during the options' maturity time. In order to get unbiased option prices, we have to deduct the present value of dividend.

very complicated as we have to get either the dividend payment or dividend yield rate for all of the 30 weighted stocks comprising the OMX. However, we are using the future to hedge, therefore, this problem can be ignored as a future contract does not really lead to delivery of the underlying asset, since the position will be closed on the cash settlement prior to delivery (Granath, and Krisell, 2004).[69] As we explained in Chapter 1, futures options are analogous to options on stocks paying a dividend yield. This results in that a futures price behaves in the same way as a stock paying a dividend yield at the domestic risk-free interest rate, *r*. PCP relationships for future option price is the same as that for options on a stock paying a dividend yield at rate *q* when the stock is replaced by the futures price and $q = r$ (Hull, 2003).

5.4.2 Ex-post and Ex-ante efficiency test

The test conducted for efficiency can be divided into two types. The first is ex-post test, which says that based on information at time t, a trading strategy is devised and a position is established based on the same prices; the position is liquidated at the time t+1 at prices that are theoretically unknown at time t. The second type is an ex-ante test, which says that based on information at time t, a trading strategy is devised, but the position is established at time t+1 at prices that are unknown at time t, and the position is liquidated at time t+2.

In general, supernormal profits for the ex post tests may indicate either market inefficiency or that the markets are not synchronized given the assumption that the option pricing model is correct (Andersson, 1995). Galai (1978) argues that the market efficiency tests should be based on ex-ante tests implying that a trading strategy is devised at time t, whereas the hedge position is established with prices that are realized at time t+h. In practice, ex-post tests assume ability

[69] Numerical proof of avoiding dividend effect: The put call parity formulas for an option on a dividend paying stock (*q*= yield rate) are $C = S_0 e^{-qT} N(d_1) - Ke^{-rT} N(d_2)$ and $P = Ke^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1)$. The future pricing formula is $F_0 = S_0 e^{(r-q)T} or S_0 = F_0 e^{-(r-q)T} = F_0 e^{(q-r)T}$. Since we are using the future to hedge, we substitute the future price in the put call parity formulas then, $C = F_0 e^{(q-r)T-qT} N(d_1) - Ke^{-rT} N(d_2) = F_0 e^{-rT} N(d_1) - Ke^{-rT} N(d_2)$ and in the same way $P = Ke^{-rT} N(-d_2) - F_0 e^{-rT} N(-d_1)$. As can be seen from the derivation, the dividend yield drops from the formulas. Therefore, when we use the OMX futures to hedge the OMX options, the dividend effect can be ignored.

to simultaneously execute all legs of the arbitrage at the prices that, to start with, indicate potential arbitrage opportunities this seems unrealistic, especially for multi-market arbitrage and for smaller traders. Therefore, in this study, ex-ante tests that simulate the trading opportunities of an actual trader will be performed. The realistic time delays between identification of possible profitable investment and the actual execution of the position must be incorporated, after the comparison between market and model prices, the Paired T-test will investigate the significantly mispriced options (over 15% of the model prices) that will be filtered and used for dynamic hedging.

5.5 The Data

Both primary and secondary data will be used in this study. The secondary data includes market data in terms of options, futures and Index price, strike price, volume from June 1, 1994 to June 30, 2004, and risk free interest rate (STIBOR). The futures and options data is provided by Stockholmsbörsen. The STIBOR interest rate is collected from Riksbanken after 1994, and before 1994 is extracted from REUTERS. Primary data include Black Scholes Model prices, historical volatilities and implied volatilities will be derived by our own calculations.

5.5.1 The OMX Index, options and futures

The historical data of OMX options and futures is obtained from Stockholmsbörsen. They are tab delimited text files, which contain trading date, series, last ask price, last bid price, close price, highest price, lowest price and volume. The historical data of the OMX index is downloaded directly from the website of Stockholmsbörsen, which contains the trading date and the closing price.

The data is nearly perfect for analysis. The only problem is that they are the latest values for each trading day. As all our tests employ hedging strategies, which involve different positions in different types of options and underlying futures, the value we used for trade options or futures should be synchronized for each hedge. However, last ask and bid transacted prices of options and futures are very likely to happen at very different times, especially for those less liquid series. In addition, the last bid and ask price as well as the close price are sometimes not really tradable. The market may already be closed when the arbitrage opportunity is spotted, and the action has to be taken at the beginning of the following day, whereas the data available for the following day is taken at the closing time. Clearly, these problems introduce noise to the data, which the trading simulation is based on and eventually may distort the results.

Ideally, as we presented in the methodology part, we should use the spot value captured for every relative short time interval. For example, Cavallo and Mammola (2000) use intraday prices captured every 15 minutes throughout all trading hours to test put-call parity, and monitor the price minute by minute for deriving "the Black Scholes Model option price." However, this data is usually difficult to obtain, and in reality, a substantial number of studies in this area are still using the daily price.[70] From a statistical point of view, given the data covering a relative long period, the daily data could very well represent the movement of the market, and any significant pattern spotted in the daily data should also be quite valid in reality.

In order to minimize the noise due to the unavailability of intraday data, we tackled the data from various angles.

First, unlike many researches that only use a few years' data; we use 10 years' data. We use a daily trading history of all options, which traded and expired between June 1, 1994 and June 30, 2004. This gives us 303,824 options trading records and 12,201 futures trading records.

Second, in order to reduce the noise of the closing price, we use the middle point of ask and bid price to supply the simulation model with both option price and future price.[71]

Third, the liquidity, moneyness and time to maturity are considered when filtering the data.[72] For example, the record with 0 trading volume are unlikely to be used, as the price listed may not be tradable in reality. For different

[70] Evnine and Rudd (1985) and Kamara and Miller (1995) find very similar results using intraday data and closing price data for S&P 100 and S&P 500, respectively.
[71] The transaction price is the mid point of bid and ask price, this is the most popular way to deal with transaction prices in derivatives market, as Phillip and Smith (1980) point out that any tests based on bid and ask prices would be biased in favor of the market efficiency hypothesis.
[72] See chapters 6 and 7.

trading strategies, the level of consideration is different for each item, and therefore the filters are different as well.

Finally, some data that is obviously unusable or incorrect is also filtered out. Although it is a very small amount, these are mainly series which are very illiquid. The criteria is that any records with ask price or bid price 0 or 0.01 (it simply means that the price is not available rather than price is 0) or with a difference between ask and bid price of more than 10 points (the average difference is 3.1 points, while 95% of the records have a difference of less than 6 points, therefore, a difference of more than 10 points is a gap too wide) will be excluded.

Of course, the noise is just moderately reduced, and we still need to be aware of it when analysis and interpreting the research result.

5.5.2 Risk Free Rate

For risk free rate, there are basically two options to choose. One is the REPO rate and the other is the STIBOR rate.

REPO is the rate that banks receive or pay when depositing or borrowing funds at the central bank for a period of seven days. The STIBOR rate is the interest rate banks pay when borrowing from other banks at maturities other than overnight. This rate is available for different borrowing periods. Therefore, being an institution engaged in options trading, the interest rate they can get should be more close to the STIBOR rate rather than the REPO rate. Of course the actual rate must be higher than STIBOR, and the difference could vary a lot. Therefore, the test result must be interpreted in the consideration of this varied difference.

As each trading strategy involves the transaction when the hedge is established and liquidated, and for all the models, we assume a constant interest rate during the life of the hedge. We consider the interest rate when the hedge is established, and the "time to maturity" is taken into consideration. If the life of the hedge is less than 30 days, we use the weekly rate, if it is between 30 to 89 days, we use the monthly rate, 3-month rate is used for 90-182 days, while 6-month rate for 183-272 days.

According to the above principles, we pre-calculated the discount rate e^{-rT} for each option record. This simplified later calculations.

5.5.3 Cost of hedge

The cost of hedge is a very wide scope. It is not only comprised of the commission paid for each transaction, but also the administrative cost. As the administrative cost is mainly in the form of overheads, it is not properly, and also not possible, to breakdown into cost per hedge and to apply them into any of our simulation models. Therefore, we exclude the administrative cost. If there is a pattern of existing arbitrage profit, given a significant trading volume, the overhead can likely be covered.

The broker's commission is usually a variable cost. However, the cost per transaction varies a lot from broker to broker. In addition, the members of the stock exchange centre (broker dealers) do not pay this commission. Therefore, we decided not apply any broker's commission for dynamic hedging from an institutional investor point of view. However, the sensitivity analysis for the test result in respect to various levels of commission will be discussed in lower boundary and put call parity condition tests. The average commission cost in FTSE-100 and MIB30 is available in several studies,[73] and they are used as a proxy to handle the sensitivity analysis.

The stock exchange centre (Stockholmsbörsen) charges a fee to all the members. This cost, though rather small, is published, and is accounted for all transactions. It is SEK3.5 per contract for options and SEK4.5 per contract for futures. This cost is considered in all our hedge simulations.

The other cost we considered is the bid ask spread. This is because we are unlikely to buy exactly at bid price, and sell exactly at ask price. This difference is called ask bid spread. The actual price you can trade is assumed at the middle point of ask bid price. As we have replaced closing price with the average of ask bid price, this spread is already moderately taken into account[74], and should be further applied.

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[73] Cavallo and Mammolla, (2000); Deville (2003); Dumas, et al. (1998).

[74] Fully consider the bid ask spread, see 6.2.2, assuming buy at ask prices and selling at bid prices.

5.5.4 Transformation of the data

We first write a VB procedure to load all data files (one file for each month), and then filter out all series which are not OMX related. Next we extract all option records and all future records in two separate tables. Thereafter, based on series name, the option type, strike price, expiration year and month are also computed in individual fields of the tables. The trading date of each sample is also converted to date format from its original format.

The expiration date of each month is derived from the date of the latest available option, which will expire in that month.

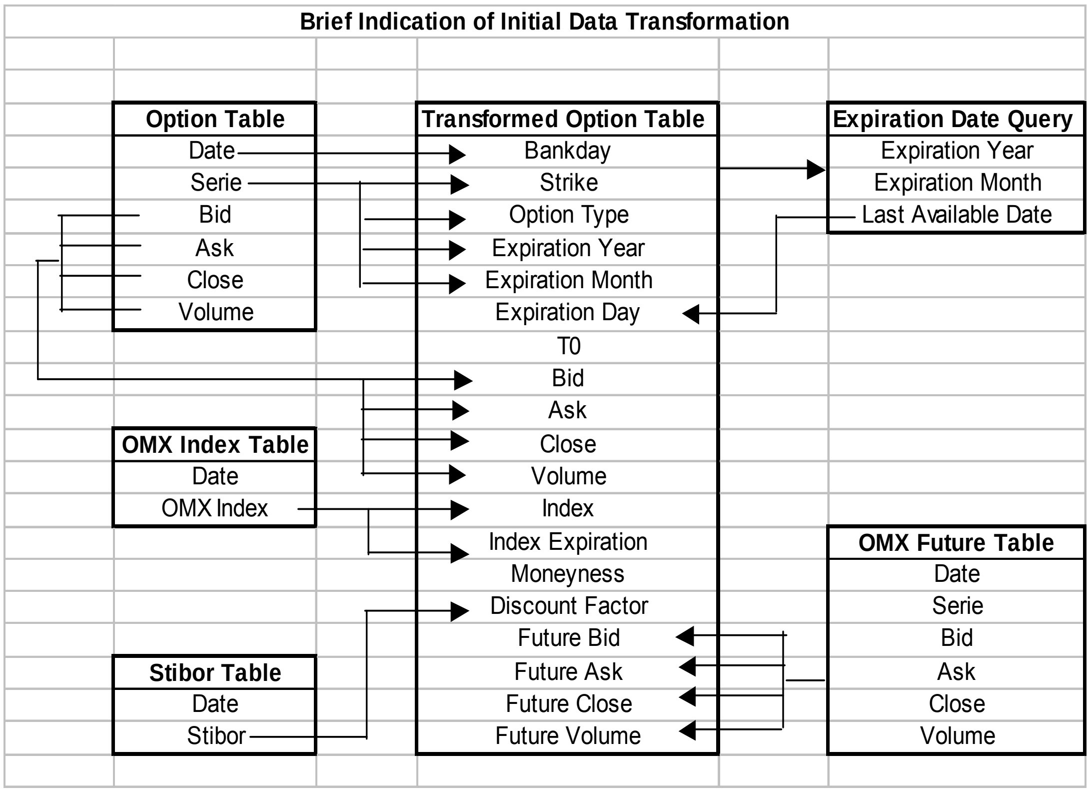
By matching the date, the current index and the index on the expiration date are set for each option. Similarly, by matching the date and the expiration date, the trading data of the underlying future is also set for each option.

With the above information, we could easily derive a few additional fields, such as T_0 (time to maturity), moneyness, etc.

Finally, based on "T_0" and the "bank day" field, we select the right interest rate from the STIBOR table for each option, and, thereafter, calculate the time discount factor respectively.

In addition, as there is a contract split in the market on August 27, 1998, a query has to be devised to recalculate the price and volume of the options traded before that day.

Figure 5.1 Data Transformation



Now this option table is ready for implementing a number of tests.

When implementing the put call parity test, there is another major transformation job. This is to match the puts with the calls. We run two queries to extract all calls and all puts. Then we create a new table –named "PutCallTable" – by matching the "bankday", "strike price" and "expiration date" of both queries. Long and short hedges will be performed by using data from the "PutCallTable."

5.6 Conclusion

In this chapter we have presented the methodologies and data used in this study in order to test the efficiency of OMX options market. First, we show a brief literature review of the methodologies used in this thesis for market test, namely lower boundary test and put call parity tests, and dynamic hedging strategy. Then, we discussed the concept of market efficiency, which depend on the price response rates to different levels of information content.

The market efficiency hypothesis of this study assumes that there are no arbitrage profits or that the arbitrageurs are unable to consistently generate abnormal returns after transaction costs have been taken into account for both boundary and put call parity tests, and dynamic hedging.

There are three major concerns when carrying out the efficiency test. First, we use both volatility estimators and historical and implied volatility to calculate the Black Scholes prices. Second, in order to avoid the joint efficiency hypothesis problem, we perform lower boundary and the put call parity condition tests, which are model independent. Third, the data synchronization problem is solved by using OMX futures to hedge index options instead of replicating the whole index basket.

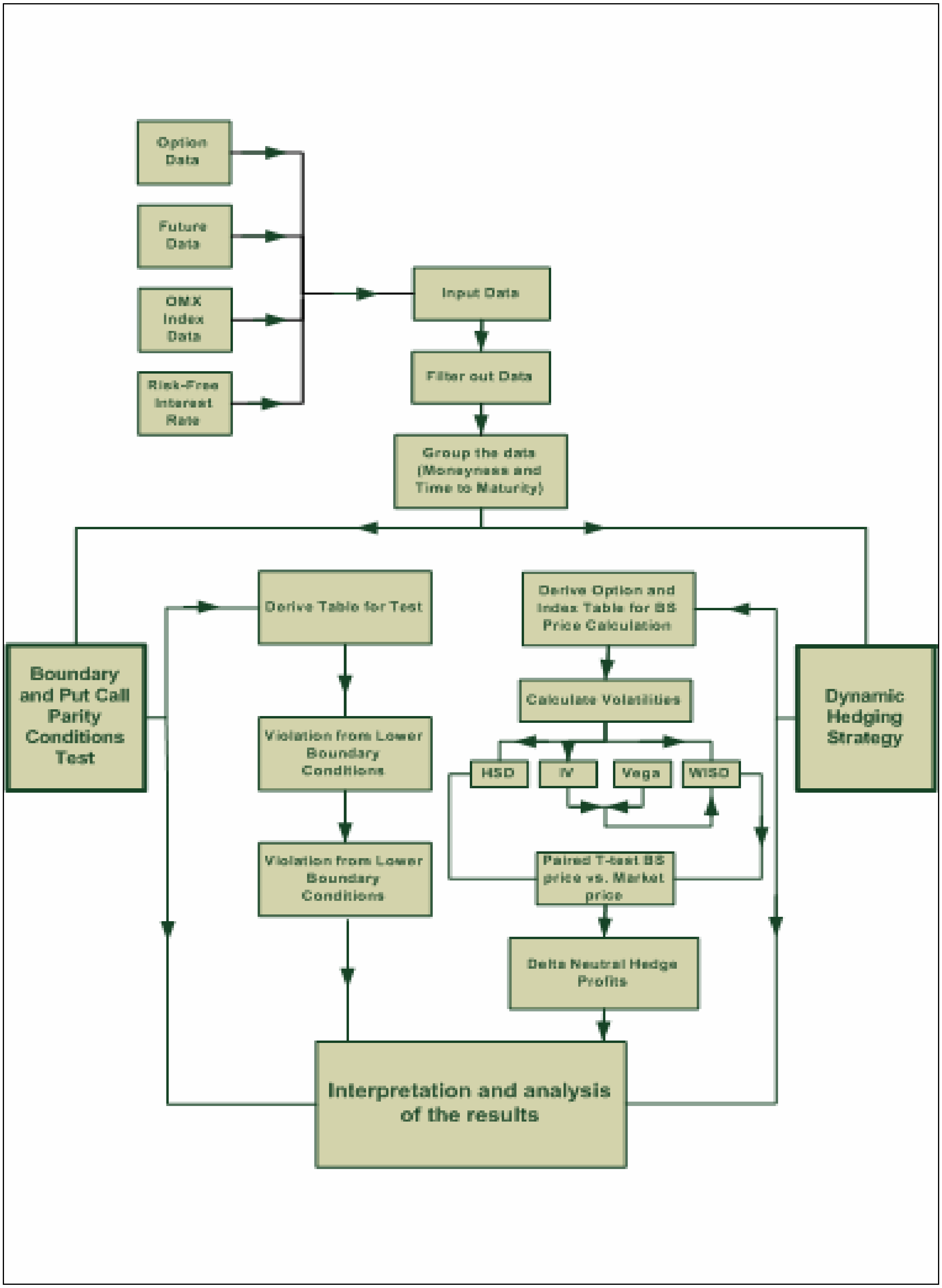
Finally, in this chapter we described how we transformed the data to be ready for efficiency tests.

Part four: Empirical Research

In this part, the two tests for market efficiency are performed. They are lower boundary and put call parity condition tests, and dynamic hedging strategy.

In Chapter 6, lower boundary and put call parity condition tests are implemented. The results from these condition tests are carefully presented and analyzed, and additionally a sensitivity analysis is performed. In Chapter 7, the dynamic hedging simulation is performed; consequently, the results are interpreted, analyzed, and validated in different ways.

Figure 0.1 Test procedure for Lower Boundary and PCP conditions and Dynamic Hedging Strategy



Chapter 6 Boundary and put call parity condition tests

6.1 Introduction

In this chapter, we test several no arbitrage conditions that include the lower boundary condition and put call parity condition tests, taking into account transaction costs. These conditions concern arbitrage across the index and options market, and are intended to explore the relative efficiency of the index options market. Overall, our results support efficiency of the OMX index option market, as the frequency of arbitrage condition violation is low compared with other efficient markets. These tests join the body of research on the index options market in US[75], continental Europe and elsewhere.[76]

The remainder of this chapter is organized as follows. First, we derive lower boundary and put call parity conditions tests with transaction cost and bid ask spread, which are used in this study. Thereafter, we illustrate how we have constructed the data to be ready for efficiency tests. Consequently, the results from these tests will be presented. Besides, we also test the same boundaries and put call parity conditions under different scenarios with respect to the changes in the transaction cost, the results from this sensitivity analysis will be presented and analyzed.

6.2 Derivation of Boundary and Put-Call Parity Conditions

Lower and upper boundary conditions and put call parity tests are based on the given stock and options market and aimed at checking the violations of no-arbitrage relationships among these prices. Rational upper and lower boundary conditions and put call parity conditions must be fulfilled for any put and call

[75] Evnine and Rudd (1985, US, S&P 100), Chance (1987, US, S&P100), Kamara and Miller (1995, US, S&P500), Ackert and Tian (1999,2000,US, S&P 500).

[76] Ackert and Tian (1998, Canada, TIP 35) Puttonen (1993, Finnish Index), Chesney et al. (1995, Swiss index), Capelle-Blancard and Chaudhury (2001 French CAC 40), Deville (2004 French CAC 40), Mittnik and Rieken (2000 German DAX), Cavallo and Mammola (2000, Italian MIB 30) and Brunetti and Torricelli (2003 Italian MIB 30).

options market, namely, *if an option price is above the upper boundary or below the lower boundary, there are profitable opportunities for arbitrageurs, so the market is not efficient* (Hull, 2003). Meanwhile, Put Call Parity condition shows that the value of a European call with a certain exercise date can be deduced from the value of a European put with the same exercise price and maturity date, and vice versa. Therefore, *violation of the put call parity implies, arbitrage profit, the market is not efficient.*

The hypotheses for lower boundary and PCP conditions tests are:

EMH1: there is not significant violation from lower boundary condition for both put and call index options (market is efficient)

EMH2: there is not significant violation from PCP conditions for both long and short hedge strategies (market is efficient)

6.2.1 Derivation of the Lower boundary test for puts and calls[77]

If we define,

S_0: Current index price,

F_0: Current futures price

K: Strike price /Exercise price

R: Continuously compounded risk-free interest rate for an investment maturing in time T

T: Time to maturity of option

C: Value of European call option to buy one share

P: Value of European put option to buy one share

A) Without bid ask spread, trading at transaction price[78]

The lower boundary for call option[79] is: $C \geq S_0 - K e^{-rT}$ or

[77] Upper boundary for call is $C \leq S_0$, for put is $P \leq K$. If call upper boundary is not satisfied, arbitrager profit can be made by selling the call and buying the stock, if put upper boundary is violated, arbitrager profit can be obtained by sell the put option and save it in the bank at risk free interest rate.

[78] The transaction price is the mid point of bid and asks price, this is the most popular way to dealing with transaction price for market price of derivatives, as Phillip and Smith (1980) point out that any tests based on bid and ask prices would be biased in favor of the market efficiency hypothesis.

[79] If this boundary is violated, arbitrage profit can be obtained by buying the call option and borrowing and shorting the stock at (So). When the option expires on T, he needs to return stock. if St>k, call option expires, he can buy at K, after return this stock, profit for arbitrager is $So\text{-}Ke^{-rT}\text{-}C$. If St<K, call is worthless, arbitrageur can earn even more than this amount. Detailed derivation for this condition see Appendix 6. Derivation of Lower Boundary for Call and Put Options.

$C-(F_0-K)e^{-rT} \geq 0$ **Eq. 6.1**

The lower boundary for call option[80] is: $P \geq Ke^{-rT}-S_0$ or

$P+(F_0-K)e^{-rT} \geq 0$ **Eq. 6.2**

B) With bid ask spread[81]

$C_{ask}-(F_{0bid}-K)e^{-rT} \geq 0$ **Eq. 6.3**

$P_{ask}+(F_{0ask}-K)e^{-rT} \geq 0$ **Eq. 6.4**

As we are using futures contracts to hedge the index option,[82] $S_0 = F_0 e^{-rT}$, taking into account the transaction cost like Courtadon and Bodurtha (1986), we use the formula derived by Hull (2003) for put and call future boundary, to derive the call and put lower boundary tests as follows:

A) Without bid ask spread:

$C-(F_0-K)e^{-rT}+TC_{BC}+TC_{SF}+Tk \geq 0$ **Eq. 6.5**

$P+(F_0-K)e^{-rT}+TC_{BP}+TC_{BF}+Tk \geq 0$ **Eq. 6.6**

B) With bid ask spread:

$C_{ask}-(F_{0bid}-K)e^{-rT}+TC_{BC}+TC_{SF}+Tk \geq 0$ **Eq. 6.7**

$P_{ask}+(F_{0ask}-K)e^{-rT}+TC_{BP}+TC_{BF}+Tk \geq 0$ **Eq. 6.8**

Where, TC_{BC} and TC_{BP} are transaction costs for buying call and put options. TC_{SF} and TC_{BF} are transaction cost for short future contract and buying future contracts. Tk includes brokerage fees.

[80] If this boundary is violated, arbitrage profit can be made through borrowing money to buy both stock and put option. On the expiration date, if St<K, the risk free arbitrager is Ke^{-rT}-P-So, if St>k, the arbitrager profit is even larger. Detail derivation, see Appendix 6. Derivation of Lower Boundary for Call and Put Options.

[81] If we buy options, we would like to buy at the lowest price (bid price), or to sell it at highest price, however, it is impossible in the market, we usually buy it at higher price or sell it at lower prices beyond our expectation, which is the effect of bid ask spread. Bid ask spread is different for different investors and there is no matured theoretical way to estimate it, however, if we assume that an investor buys at ask price and sells it at bid price, then the bid ask spread will be maximized imposed, therefore, in this study, following Cavallo and Mamolla (2000), we fully impose the bid ask spread by assuming that investors buy at ask prices and sell at bid prices.

[82] As we are using future to hedge the index, the dividend effect has dropped out, hence, we use the formula without considering the dividend.

6.2.2 Put Call Parity

The well-known basic put-call parity relationship for options and underlying assets (no dividends) is the following:

$$C + K e^{-rT} \equiv P + S_0 \qquad \textbf{Eq. 6.9}$$

Therefore, if the market is efficient the put call parity condition must be fulfilled, otherwise there are arbitrage opportunities. The Put Call Parity condition for European future options is derived from the above formula by Hull (2003), as:

$$C + K e^{-rT} \equiv P + F_0 e^{-rT} \qquad \textbf{Eq. 6.10}$$

The non-arbitrage conditions can be derived from the above equation by establishing two portfolios. The first represents a long-hedge position, because it involves taking a long position in the underlying asset. The second portfolio represents a short-hedge position, as it takes a short position in the underlying share. In the absence of transaction costs, these conditions for put call future parity are[83]:

A) Without bid ask spread:

$C - P - (F_0 - K)e^{-rT} \leq 0$ **Eq. 6.11**

$P - C + (F_0 - K)e^{-rT} \leq 0$ **Eq. 6.12**

B) With bid ask spread:

$C_{bid} - P_{ask} - (F_{0ask} - K)e^{-rT} \leq 0$ **Eq. 6.13**

$P_{bid} - C_{ask} + (F_{0bid} - K)e^{-rT} \leq 0$ **Eq. 6.14**

If the expression representing a long hedge (Eq.6.11) is not satisfied, a profit can be made by buying the put option, purchasing the future contract and shorting call option. The investor can borrow Initial investment from a bank at a risk-free rate. If the equation 5.12 is violated, the arbitrage profit can be obtained through shorting the put option, and shorting the future contract, and buying the call option. We follow Cavallo and Mamolla's (2002) method,

[83] Appendix 7. Derivation of Put Call Parity Conditions.

options and futures will be purchased at the ask price and sell at bid price. The PCP conditions are derived as follows:

A) Without bid ask spread

$$C - P - F_0 e^{-rT} + Ke^{-rt} \leq TC_{SC} + TC_{bp} + TC_{bf} + Tk \qquad \textbf{Eq.6.15}$$

$$P - C + F_0 e^{-rT} - Ke^{-rt} \leq TC_{sp} + TC_{bc} + TC_{sf} + Tk \qquad \textbf{Eq.6.16}$$

B) With bid ask spread:

$$C_{bid} - P_{ask} - F_{0ask} e^{-rT} + Ke^{-rt} \leq TC_{SC} + TC_{bp} + TC_{bf} + Tk \qquad \textbf{Eq.6.17}$$

$$P_{bid} - C_{ask} + F_{0bid} e^{-rT} - Ke^{-rt} \leq TC_{sp} + TC_{bc} + TC_{sf} + Tk \qquad \textbf{Eq.6.18}$$

Where, TCsc, TCsp, and TCsf are, respectively, the transaction cost for writing call, put and futures contracts, TCbp, TCbc, and TCbf are the transaction costs for purchasing put, call and future contracts.

6.3 Test Procedure

In this section, we present how we implement the lower boundary and PCP condition tests, following the test procedure in Figure 0.1.

6.3.1 Constructed data

Recalling our discussion in Chapter 5, we use daily data for options, futures, and index prices, daily data for interest rates[84] from June 1, 1994 to June 30, 2004. In order to avoid problems with infrequent trading and non-synchronous data problems, the data selection criteria mentioned in Chapter 5 is fully applied:

1. Most studies eliminate options with less than several days and more than one year to expiration[85]. The options with a maturity of fewer than seven days usually have relatively small time premiums, and contain little information about the volatility function, hence the estimation of volatility is extremely

[84] As Capelle-Blancard and Chaudhury (2001) and Cavallo and Mammola (2000) we ignore the borrow and lending spread.

[85] Example see Mittik and Rieken (2000) use data 2 days –1 year to marturity; Chaudhury and Capelle-Blancard (2001) use data from 2 days—180 days to maturity; Draper and Fung (2002) use data within 90 days to maturity, Wang (2002), Tandon and Shastri, (1986), Robert and Resnick, (1979).

sensitive to non-synchronous option prices and other possible measurement errors. Options with more than six months or longer time to maturity days are not actively traded. However, we only exclude options traded on the expiration date and have more than one year to maturity. As we find the result is not significantly different by excluding seven days or one day for OMX options. We exclude data with longer than one year to maturity not only for the infrequently traded option, but also the longest interest rate is available only for no more than one year.

2. Furthermore, like Andersson (1995), we only include those options and futures whose prices were more than 0 or 0.01 kr, and the volume for both options and index and futures are larger than zero. Thus we have 59,025 calls and 63,607 puts.

3. For put call parity, we have to pair put and call options, futures having the same maturity time, banking day and strike price. We have 32,742 samples for long hedge and 32,740 samples for short hedge.

6.3.2 Transaction costs

It is important to take transaction costs into account. Recalling our data presentation in Chapter 5, the transaction costs[86] include commissions fee, trading and clearing fee, cost derived from bid ask spread, short selling, etc. However, besides the trading and clearing fee for futures = 4.50 SEK/contract, trading and clearing fee for options = 3.50 SEK/contract, an accurate estimation of the actual transaction cost are very difficult, not only the transaction costs tend to vary over times, they may also depend on a particular strategy and the size of transactions. *A common approach is to assume the bid ask spread is constant, then it can be estimated based on a sample of bid ask quotation* (Phillip and Smith, 1980) or derived from the movements of the transaction prices (Roll, 1984, Stoll, 1989, Smith and Whaley, 1994). However, none of these methods can fully contain all information for bid ask spread besides sensitivity analysis. Therefore, we follow Cavallo and Mammola´s (2000) method, carry out our empirical study of boundary and PCP condition

[86] Fee lists see Appendix 2. OMX Options Specifications.

tests assuming with and without bid ask spread scenarios, which will be divided into three sub-scenario with respect to different transaction levels.[87]

1. TC = 0, the transaction costs are equal to zero.

2. TC is at a lower level. The trading and clearing fee for futures =4.50 SEK/contract, trading and clearing fee for options =3.50 SEK/contract, will be taken into account, but without broker fees. Investors in this category can be members and brokers of OM.

3. TC is at the highest level, which represents the individual investor. They have to pay the trading cost and brokerage commissions, we follow the Eq. 6.3 and 6.4. However, they have to pay the broker fee (Tk). Following Capelle-Blancard and Chaudhury (2001) and Cavallo and Mammola (2000), we assume a brokerage commission of 0.05% of the contract value for traded options and futures for individuals. This transaction level represents individual investors.

6.3.3 Moneyness and time to maturity

In order to control the influence of time to maturity and the moneyness of the contracts on the efficiency of market, several sensitivity analyses will be conducted. For <u>the time to maturity</u>, we sort the contracts for any particular day into three different field as (1) short term (less than 30days), (2) medium term (30-60 days) and (3) long term (60-1 year). For the <u>moneyness</u>, which is defined in this study by the S/K.[88] Define S-K as the time $_t$ --intrinsic value of a call, a call option is then said to be at the money (ATM) if 0,98≤ S/K ≤1.02, out of the money (OTM) if S/K< 0,98, and in the money (ITM) if S/K >1.02.

[87] As buying and selling futures only involve cash settlement in maturity, also futures contracts do not need to pay the margin settlement, we assume zero short selling cost for futures. As we will liquidate position the day after, the cost for borrowing money to buy options is really small, so we ignore it.

[88] Fung, (2002), Dumas et al., (1998) and Clewlow et al. (1998) have sorted data by moneyness in the same way. The definition of moneyness indicates the moneyness of call options, our in the money options group, for example, comprises of in the money call and out of the money put options.

6.4 Empirical results

In this section, we discuss the empirical results regarding the efficiency of the OMX index options market that are reported in Tables 1 through 3. In order to verify our result, we also compare our result from the OMX market to the U.S. and other EU market evidence. The main statistic in our discussion is the frequency (or percentage) of violations of the arbitrage conditions defined as:

Frequency (Percentage) of Violation = Number of Violations identified /Number of Observations Examined.

6.4.1 Lower boundary conditions

The result from lower boundary conditions for put and call reported in Table 1 corresponds to using the transaction prices (mid-point bid ask spread) in panel I and maximized bid ask spread by assuming investors buying at ask prices and selling at bid prices in panel II. For both panel I and II, the table reports three columns with the results obtained under three different assumptions on the level of transaction costs. In the first column, to make the result comparable we ignore transaction cost, so TC is zero. In the second and third columns, we consider the level of transaction costs incurred, respectively, by an arbitrageur or member of OMX (TC = trading cost), and an occasional investor (TC = brokerage fee + trading cost).

In panel 1, of the 59,025 observations that remain after screening, the lower boundary conditions for call options appear to have been violated in only 446 cases (0.756%), without taking any transaction costs into account (Panel I, TC=0). After taking into account trading costs and brokerage fees they are respectively reduced to 0.647% and 0.378%. Comparatively, the violation of lower boundary condition for put is 454 (0.71%) without transaction costs, and after taking into account transaction cost it reduced to 398 (0.626%) and 259 (0.41%) with brokerage fees.

In panel II, with the bid ask spread cost, both the number of the violation of boundary condition for call and put are reduced to nearly zero. These incidences compare quite favourably with those in the U.S. and elsewhere. For example, Ackert and Tian (1999) report frequency of violations for the S&P 500 index options at more than 5% for call options and at more than 2% for put options from February 1992 to January 1994. In the much smaller Finish index option market, from May 2, 1988, to December 21, 1990, Puttonen (1993) finds 7% violation for at-the-money call options with in-the-money options exhibiting a significantly greater incidence.[89] The lower boundary condition from Capelle-Blancard and Chaudbury (2001) for French CAC 40 index option market appears to have been violated in 0.88% for call options and 0.51% for put options.

Hence, from the above analysis, we do not reject the null hypothesis of EMH test 1: (H_0) there are not significant violations from lower boundary condition for both put and call options (market is efficient), and the hypothesis is true. Therefore, we conclude that at least on the basis of the most fundamental arbitrage relationship for the stock index and options prices, the OMX market appears quite efficient, especially considering its smaller size (in terms of traded value of the underlying index) and age.

6.4.2 Long hedge and short hedge from put call parity condition

Table I summaries the result of the put call parity tests under different assumptions about transaction costs. The results from both long hedge and short hedge of put call parity conditions for the Swedish OMX market indicate some inefficiency due to the large number of violations. However, after taking into account the bid ask spread and other transaction costs, the arbitrage profits are nearly wiped away. This consistently matches our expectations, the number of hedges, which would have provided a profit opportunity decrease substantially when transaction cost and bid-ask spread are included.

As can be seen in panel 1 in Table 1, the result from the PCP conditions for long hedge violations (long future and put, short call) are 45.978%, 33.92%, 13.032% and short hedge (short put and future, long call) violations are 54%, 41.72% and 18.47%, respectively. While the size of deviation from PCP conditions exhibit a clear tendency to decline substantially as the transaction cost increases, it can be observed that after fully imposing the bid ask spread in panel II, the PCP violations frequency drops dramatically to 1.216% (2.8%),

[89] Based on the CBOE equity transaction data for 196 trading days ending in June 2, 1977, Bhattacharya (1983) finds the frequency of the ex-post violation at 8%. This compares with Galai (1978) frequency of 2.95% for equity options during the first six months of the CBOE operation in 1973. Using transactions data for the Canadian equity options market, Halpern and Turnbull (1985) report an ex-post violation incidence of around 10% during 1978 and 1979.

0.928% (2.19%) and 0.611% (1.25%) for long (short) hedge under scenarios in which the transaction cost is zero, transaction cost includes only trading cost and transaction costs include both trading cost and brokerage fees.

Although non-negligible violations (1.25%) for short hedge still exist in arbitrage opportunities, compared with the efficiency conclusion from other studies in other index option markets the violations for long and short hedge in OMX index option market are very small. For the S&P Index (American) options, Evnine and Rudd (1985) report 52% (short arbitrage) and 22% (long hedge) violation.[90] For the S&P 500 Index (European) options, Kamara and Miller (1995) report he PCP violation frequencies at 23% (short hedge) and 10% (long hedge). When taking into account the cost and constraints, the violation frequencies drop to 3% and 5% respectively. For the smaller and newer Italian Index option market, Cavallo and Mammola (2000) report a 49% violation frequency for the short as well as the long hedge arbitrage. Considering the bid ask spread and other transaction costs, they find the violation frequency drops to 2% for both short and long hedge.[91]

What is interesting to note in the results is that the result from this study is perfectly consistent with Cavallo and Mammola (2000). We have not observed a larger number of profitable short hedges than profitable long hedges, and in contrast with the findings of Klekosky and Resnick (1980) on CBOE and Nisbet (1992) on the London Traded Option Market (LTOM). This is also consistent with our expectation, as we are using futures contracts instead of duplicating the index to hedge the index options, which can avoid the influence from short selling market if it is inefficient. There are several possible reasons to interpret the result: (i) Investors in Sweden prefer to use index futures to hedge index options, rather than to duplicate the whole index basket. (ii) The futures hedge is not particularly difficult or costly compared with replicating

[90] In this paper, authors use intra day data to examine the pricing of the options on the S&P Index options. Although the result of significant violations of the PCP conditions suggests that the market is some degree, authors believe the result is highly influenced by the data non-synchronization. However, without taking into account the transaction cost can be another reason for the higher violations.

[91] For the Swiss equity options (American) option market, Lefoll (1994) undertakes put call parity conditions for thirteen stocks from October 1989 to June 1990. Considering only the bid-ask spread cost, he reports 1.1% (0.8%) and 0.5% (0.3%) ex-post and (ex ante) violation of the short and long hedge PCP conditions respectively. Berg, Brevik and Saettem (1996) study Oslo Stock Exchange equity options (American style) on four stocks from May to July 1991. Taking into account transaction costs but using daily closing data, they report a 54.5% frequency of violation of the 1,209 observations. Of the 659 violations, there are 503 (156) violations of the short (long) hedge strategies for an average profit of NOK 0.95 (0.97).

the index, and (iii) We mentioned previous studies having higher short hedge profit opportunities, because they have not allowed for fully imposing the bid ask spread.

This conclusion can be verified by the result from the average arbitrage profit value from short and long hedge. From Table 6-1, after taking into account the transaction cost, it can be seen that the average arbitrage profit for short hedge (2.38 index points) is smaller than the long hedge (2.54 index points),[92] which can explain that the short hedge is not affected by the short selling market, this shows that the options markets of Italy and Sweden have similar PCP experiences.

Although the violations for PCP of OMX is not zero, according to Nandi and Waggoner (2000), a quick look at the daily closing prices as reported in any daily newspaper reveals a few violations of arbitrage conditions; the question is whether or not the violations are large enough for us to reject the market efficiency hypothesis and whether the deviations are a result of having non-synchronous data. Even if the tests do indeed reveal violations of the boundary conditions, it is not possible to say that the market is inefficient. Therefore, we argue that when compared with other efficient tests, the violation from this study is too small to reject the null hypothesis of OMX market efficiency. Hence, from the above analysis, we fail to reject the null hypothesis of EMH test 2: (H_0) there are not significant violations from PCP conditions for both put and call options (market is efficient), and the null hypothesis is true. Therefore, we accept and conclude that from the analysis of PCP condition tests, the OMX market has similar lower boundary and PCP condition results as the Italy index option market, and the market appears quite efficient.

[92] Many studies such as Klekosky and Resnick (1980) on CBOE and Nisbet (1992) London Traded Option Market (LTOM) get higher short hedge arbitrage.

Table 6-1 Summary Statistics of the Profitable Hedges for Panel I and II for cases 1, 2 and 3 for different levels of transaction costs

	Profitable Hedges	Panel I ----Transaction Prices			Panel II----Bid-Ask Spread		
		TC=0	TC=TC1	TC=TC2	TC=0	TC=TC1	TC=TC2
Lower Boundary Call	*Total sample: 59025*						
	Number of violations	446	382	223	31	29	24
	% of the violations	0.76%	0.65%	0.38%	0.05%	0.05%	0.04%
	Average Value	-1.38	-1.52	-2.15	-2.44	-2.62	-2.79
Lower Boundary Put	*Total sample:63607*						
	Number of violations	454	398	259	35	34	28
	% of the violations	0.71%	0.63%	0.41%	0.06%	0.05%	0.04%
	Average Value	-2.114	-2.325	-3.126	-3.10	-3.11	-3.36
Long Hedge	*Total sample:32742*						
	Number of violations	15054	11106	4267	398	304	200
	% of the violations	45.98%	33.92%	13.03%	1.22%	0.93%	0.61%
	Average Value	0.90	1.04	1.42	1.87	2.27	2.54
Short Hedge	*Total sample:32740*						
	Number of violations	17678	13658	6048	918	716	408
	% of the violations	54.0%	41.7%	18.5%	2.8%	2.2%	1.2%
	Average Value	1.12	1.28	1.70	1.66	1.95	2.38

Panel 1: Lower boundary call test: $C-(F_0-K)e^{-Rt}+TC_{BC}+TC_{SF}+Tk \geq 0$;
Lower boundary put test: $P+(F0-K)e^{-rt}+TC_{BP}+TC_{BF}+TK\geq 0$
Put Call Parity Long Hedge: $C- P- (F_0-K)\, e^{-rT} \leq TC_{sc}+TC_{bc}+TC_{sf}+ Tk$
Put Call Parity Short Hedge: $P-C + (F_0-K)\, e^{-rT}\leq TC_{sp}+TC_{bc}+TC_{sf}+Tk$
Panel II: Lower boundary call test: $C_{ask} – (F_{0ask} – K)e^{-rT} +TC_{BP}+TC_{BF}+Tk \geq 0$
Lower Boundary put test: $P_{ask} + (F_{0ask} - Ke^{-rT}) + TC_{BP}+TC_{BF}+Tk \geq 0$
Put Call Parity Long Hedge: $C_{bid}- P_{ask}- (F_{0ask}-K)\, e^{-rT} \leq TC_{sc}+TC_{bc}+TC_{sf}+ Tk$
Put Call Parity Short Hedge: $P_{bid}-C_{ask} + (F_{0bid}-K)\, e^{-rT}\leq TC_{sp}+TC_{bc}+TC_{sf}+Tk$

Where, TC is the total transaction cost, in the first column of the table, TC = 0, in the second column, TC1 represents the transaction cost for the brokers of OMX, only trading cost is paid. In the third column, TC2 is the transaction cost incurred by an individual investor.

* Average value is expressed in index points.

Table 6-2 Sensitivity Analysis with Respect to Moneyness and Time to Maturity

CASE 1: With zero transaction costs										
		At the money			In the money			Out the money		
	Overall	0< t ≤ 30	30 < t < 60	60 ≤ t < 360	0< t ≤ 30	30 < t < 60	60 ≤ t < 360	0< t ≤ 30	30 < t < 60	60 ≤ t < 360
Lower boundary Call										
Sample	**31**	2	0	0	17	2	10	0	0	0
Mean	-2.44354	-1.5958646	0	0	-1.3640667	-4.1727369	-4.102347425	0	0	0
SD	3.689824	2.205315	0	0	1.6925718	2.84988006	5.724010565	0	0	0
Maximum	0.071131	-0.0364714	0	0	0.0711314	-2.1575673	0.04844104	0	0	0
Median	-1.10173	-1.5958646	0	0	-0.8832821	-4.1727369	-1.246002887	0	0	0
Minimum	-16.765	-3.1552578	0	0	-7.146675	-6.1879064	-16.76499517	0	0	0
Lower boundary Put										
Sample	**35**	5	0	0	26	1	2	0	1	0
Mean	-3.10231	-1.4859536	0	0	-8.3665367	-3.216493	-2.289393979	0	-0.2413375	0
SD	3.219363	1.1818005	0	0	4.2018906		0.303305264	0		0
Maximum	-0.04044	-0.2489416	0	0	-3.5861885	-3.216493	-2.07492477	0	-0.2413375	0
Median	-2.15544	-0.972427	0	0	-6.5606747	-3.216493	-2.289393979	0	-0.2413375	0
Minimum	-11.9837	-2.9721954	0	0	-17.403671	-3.216493	-2.503863188	0	-0.2413375	0
Long Hedge										
Sample	**398**	81	19	7	91	43	13	93	38	13
Mean	1.871	1.004	1.843	1.900	1.652	1.596	2.715	2.147	2.686	4.544
SD	2.697	1.545	1.813	1.773	2.999	2.012	2.745	2.558	2.783	6.171
Maximum	19.259	6.983	6.199	5.581	19.259	9.503	8.343	11.579	15.758	17.261
Median	0.926	0.397	1.743	1.246	0.283	1.037	1.399	0.990	2.114	1.562
Minimum	0.000	0.000	0.084	0.354	0.001	0.007	0.535	0.000	0.024	0.011
Short Hedge										
Sample	**918**	146	34	3	219	138	22	243	85	28
Mean	1.655	1.337	1.461	3.815	1.093	2.682	2.967	1.184	2.391	3.471
SD	2.290	2.039	1.370	5.676	1.324	2.983	3.026	1.499	2.820	4.708
Maximum	16.625	12.904	5.603	10.363	5.754	11.837	10.039	8.411	11.948	16.625
Median	0.777	0.529	1.080	0.786	0.532	1.341	1.351	0.522	1.185	2.116
Minimum	0.000	0.000	0.002	0.296	0.001	0.005	0.022	0.001	0.029	0.025

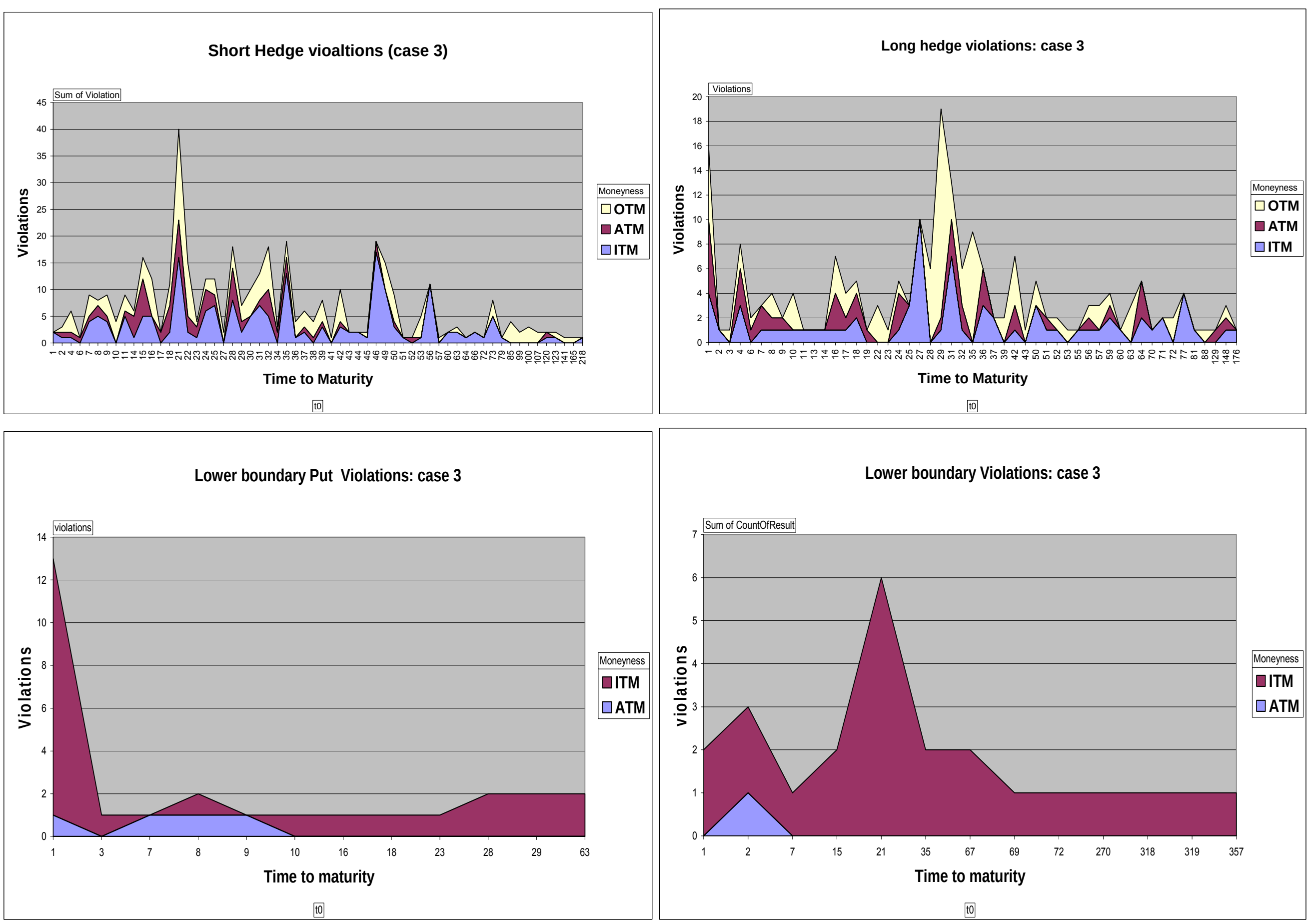
CASE 2: With Trading Costs										
		At the money			In the money			Out the money		
	Overall	0< t ≤ 30	30 < t < 60	60 ≤ t < 360	0< t ≤ 30	30 < t < 60	60 ≤ t < 360	0< t ≤ 30	30 < t < 60	60 ≤ t < 360
Lower boundary Call										
Sample	**29**	2	0	0	16	2	9	0	0	0
Mean	-2.61619	-1.5958646	0	0	-1.4537665	-4.1727369	-4.563546143	0	0	0
SD	3.756166	2.205315	0	0	1.7058382	2.84988006	-4.563546143	0	0	0
Maximum	-0.03647	-0.0364714	0	0	-0.1041124	-2.1575673	-0.158529165	0	0	0
Median	-1.2093	-1.5958646	0	0	-0.8990934	-4.1727369	-1.282707531	0	0	0
Minimum	-16.765	-3.1552578	0	0	-7.146675	-6.1879064	-1.282707531	0	0	0
Lower boundary Put										
Sample	**34**	5	0	0	25	1	2	0	1	0
Mean	-3.11236	-1.4059536	0	0	-3.7596763	-0.2184593	-2.209393979	0	-0.1613375	0
SD	3.222719	1.1818005	0	0	3.4958799		0.303305264	0		0
Maximum	-0.01813	-0.1689416	0	0	-0.0181253	-0.2184593	-1.99492477	0	-0.1613375	0
Median	-2.12148	-0.892427	0	0	-2.8854138	-0.2184593	-2.209393979	0	-0.1613375	0
Minimum	-11.9037	-2.8921954	0	0	-11.90373	-0.2184593	-2.423863188	0	-0.1613375	0
Long Hedge										
Sample	**304**	50	16	7	57	41	13	75	35	10
Mean	2.272	1.435	2.010	1.740	2.440	1.512	2.556	2.489	2.750	5.740
SD	2.863	1.722	1.795	1.773	3.465	2.031	2.745	2.611	2.789	3.631
Maximum	19.099	6.823	6.039	5.422	19.099	9.343	8.183	11.419	15.599	17.101
Median	1.188	0.833	1.668	1.087	1.192	0.920	1.239	1.332	2.131	6.476
Minimum	0.004	0.020	0.004	0.194	0.044	0.007	0.375	0.065	0.107	0.096
Short Hedge										
Sample	**716**	93	30	3	162	120	21	181	81	25
Mean	1.945	1.905	1.494	3.655	1.299	2.913	2.948	1.409	2.346	3.721
SD	2.409	2.253	1.345	5.676	1.362	3.010	3.026	1.561	2.841	4.828
Maximum	16.466	12.744	5.443	10.203	5.594	11.678	9.879	8.251	11.788	16.466
Median	1.046	0.893	1.103	0.626	0.823	1.504	1.347	0.753	1.116	2.062
Minimum	0.000	0.001	0.000	0.136	0.004	0.003	0.086	0.010	0.007	0.001

CASE 3: With both transaction Costs										
		At the money			In the money			Out the money		
	Overall	0< t ≤ 30	30 < t < 60	60 ≤ t < 360	0< t ≤ 30	30 < t < 60	60 ≤ t >360	0< t ≤ 30	30 < t < 60	60 ≤ t < 360
Lower boundary Call										
Sample	**24**	1	0	0	13	2	8	0	0	0
Mean	-2.79479	-2.4286328	0	0	-1.4138383	-3.6959869	-4.859317796	0	0	0
SD	3.930589	0	0	0	1.7401697	2.85482981	5.978566886	0	0	0
Maximum	-0.04871	-2.4286328	0	0	-0.0487128	-1.6773173	-0.597762955	0	0	0
Median	-1.35926	-2.4286328	0	0	-0.6596178	-3.6959869	-1.359259159	0	0	0
Minimum	-16.4427	-2.4286328	0	0	-6.76305	-5.7146564	-16.44274517	0	0	0
Lower boundary Put										
Sample	**28**	4	0	0	22	0	2	0	0	0
Mean	-3.35598	-1.3229878	0	0	-3.8846566	0	-1.606518979	0	0	0
SD	3.239618	1.1021985	0	0	3.4559552	0	0.324518468	0	0	0
Maximum	-0.11362	-0.312927	0	0	-0.1136156	0	-1.37704977	0	0	0
Median	-2.26189	-1.2651644	0	0	-3.050911	0	-1.606518979	0	0	0
Minimum	-11.5447	-2.4486954	0	0	-11.54473	0	-1.835988188	0	0	0
Long Hedge										
				1.682			2.276			4.078
Sample	**200**	25	13	5	34	24	13	49	30	7
Mean	2.536	1.716	1.821	1.511	3.037	1.815	1.977	2.881	2.348	7.006
SD	2.989	1.773	1.722	1.765	3.764	2.241	2.557	2.553	2.731	6.627
Maximum	17.833	5.754	5.422	4.481	17.833	8.807	7.083	10.042	14.207	16.203
Median	1.460	1.269	1.235	0.486	2.402	0.925	0.576	2.233	1.635	0.308
Minimum	0.009	0.013	0.165	0.372	0.043	0.009	0.009	0.013	0.086	5.317
Short Hedge										
				4.475			2.527			2.831
Sample	**408**	52	19	1	82	82	17	84	53	18
Mean	2.377	2.349	1.400	9.677	1.488	3.281	2.812	1.921	2.649	3.924
SD	2.548	2.320	1.275		1.373	2.995	2.908	1.534	2.838	5.077
Maximum	15.255	11.356	4.777	9.677	4.931	11.053	9.240	7.642	11.105	15.255
Median	1.491	1.878	0.876	9.677	0.936	2.720	1.492	1.553	1.462	1.676
Minimum	0.002	0.086	0.099	9.677	0.027	0.043	0.126	0.033	0.002	0.126

6.4.3 Sensitivity analysis of condition violations with respect to time to maturity and moneyness

Although the efficiency conclusion has been drawn from the results of PCP and lower boundary tests, it is worthwhile to examine the factors influencing the arbitrage violations, which can confirm our efficiency result and help us to understand the difference between the OMX index options market with others. Like most other studies, we investigate the two most reported factors in terms of moneyness and time to maturity. The main focus is violations with respect to three different transaction levels under the Panel II scenario when we fully impose the bid as k spread. The results from the sensitivity analysis are presented in Table 6-2 and Figure 6.1.

Figure 6.1 From Long (a) and Short Hedge (b), Lower Boundaries Put (c) and Call (d)



Time to maturity

Consistent with our expectation, the percentage of violations from this study is higher for options with the short term maturities than options with a long time to maturity. This can easily be observed from Table 6-2 and Figure 6.1. For example, for the short hedge in case 3 (the highest level of transaction costs), the frequency of violations is 218 (52+82+84) for options with time to maturity less than one ($t \leq 30$), the frequency of violations is 154 (19+82+53) for options with time to maturity less than 60 and larger than 30 ($30 < t < 60$), and the frequency of violations is 36 (1+17+18) for options with time to maturity less than 360 and larger than or equal to 60 ($60 \leq t < 360$). The frequency of violations seems to increase as the time to expiration decreases in both long and short hedge.

However, the mean values of the arbitrage profits seem to increase as the time to maturity increases for short hedge. Analyzing the short hedge in case 3, the mean of arbitrage profits is 2.35 for the short term and 9.68 for the long term for ATM options. For ITM options, the mean is 1.49 for the short term options and 2.81 for long term options. And for OTM options, the mean values are 1.92 for the short term and 3.92 for the long tem options. This is not the case for long hedge, where we find the opposite pattern; the arbitrage profits for long hedge seem to decrease as the time to expiration increases.

This contrasts with the result from Kamara and Miller (1995), for the post-1987 crash period when the longer maturity options volume declined, Kamara and Miller (1995) found that PCP violations increase with the time to maturity. However, the result from our study is consistent with Ackert and Tian (1999), who found that for the violations from short (long) hedge to decrease (increase) with time to expiration. When we look at the OMX index option market, we find that our result is quite reasonable; there are no more than 3% of options with maturity time longer than 60 days for both long and short hedge. Options that have long maturity time usually do not attract as much trading activity (Capelle-Blancard and Chaudhury, 2001). This is also the case in the OMX index option market. Most of the options are actively traded in short and medium term time to maturity, the more traded options, the more the violations will derive from PCP conditions, which is consistent with our result.

Moneyness

Examining violations from the moneyness point of view, we observe that the frequency of violations is generally higher for those options that are away from the money (out-of and at-the-money) for both long and short hedge, which can be seen in Table 6-2, case 3. There are less violations, 43 (72), for at the money options for long (short hedge), than 71 (181) for in the money and 86 (155) for out of the money for long (short) hedge.

This is consistent with both Ackert and Tian (2001) and Kamara and Miller (1995) on S&P 500 index options that the PCP violations are greater when options are away from money as the market will be more volatile. This can also be explained by looking at OMX index options samples, the away from money options are more liquidated (in and out of the money options) than in the money options. For both short and long hedge, we have 20.64%, 46.03% and 33.33% at-, in- and out-of-the money options respectively. The violations from put and call lower boundary tests for out of the money options are zero. However, even for in and at the money, the violation frequency for put and call lower boundary tests are nearly zero, and we can regard that the violations for put and call lower boundary tests are zero.

For the size of the arbitrage profit, away from the money options also have a larger size than at the money options for long hedge. For instance, the mean values of the arbitrage profits are 1.682 for ATM, 2.276 for ITM and 4.078 for OTM options (See Table 6-2). However, for short hedge, at-the-money options have larger arbitrage profit than away from the money options. For instance, the mean values of the arbitrage benefits are 4.475 for ATM, 2.527 for ITM and 2.831 for OTM options.

In contrast, the higher short hedge size of violations for at the money is consistent with Capelle-Blancard and Chaudhury (2001) from the CAC 40 index options market. They report the same result, that the size of deviations is higher for at-the-money options and tend to go down for in-the-money and out-of -the-money options.

As a conclusion, from the sensitivity analysis we find that the number of PCP condition violations increase when the time to maturity decreases, and moneyness moving from "at the money" to "away from the money", this is consistent with result from Ackert and Tian (1999) for the S&P 500 options market. The highest violations are short term away from money options as they are more liquidated in the OMX market,[93] this is consistent with both Ackert and Tian (2001) and Kamara and Miller (1995).

However, the mean values of the arbitrage profits increase as the time to maturity increases for short hedge, which confirms our efficiency result that investors use index futures to hedge index options instead of duplicating the whole index basket. In terms of the size of violations, there is strong evidence that the deviation is higher for at the money options and tends to go down for in the money and out of the money options, which is consistent with Capelle-Blancard and Chaudhury (2001).

Therefore, the sensitivity analysis with respect to time to maturity and moneyness verifies our efficiency OMX market result, and indicates that OMX market has the same systematic patterns of PCP violations with the U.S. and other European index options markets such as the French CAC 40 market.

6.5 Conclusion

In this chapter, by taking the transaction cost into account, we have examined the efficiency of the OMX options market by performing the lower boundary and put call parity conditions tests, using daily data for options, futures, and index prices, daily data for interest rate from June 1, 1994 to July 31, 2004. These lower boundary and put call parity conditions tests have been done centrally around two efficiency markets null hypotheses:

EMH 1: there are not significant violations from lower boundary condition for both put and call index options (market is efficient)

EMH 2: there are not significant violations from PCP conditions for both long and short hedge strategies (market is efficient)

[93] In OMX index options market, 67% of total options are in and at the money and no more than 3% of options with maturity time longer than 60 days.

However, there are two key considerations for preparation for efficiency conditions tests regarding data filtering and transaction costs.

First, in order to avoid problems with infrequent trading and non-synchronous data problems, we construct imported and transformed data in several ways: 1) We exclude options that expire on the trading day and with longer than 360 days time to maturity, as maturity fewer than seven days usually have a relatively small time premium and contain little information about the volatility function, hence the estimation of volatility is extremely sensitive to non-synchronous option prices and other possible measurement errors. On the other hand, options with longer time to maturity are not actively traded. 2) We exclude these options and futures whose prices were more than 0 or 0.01 kr., and the volume for both options and index and futures are larger than zero.

Second, as it is very difficult to collect all the necessary transaction cost data, we decided to do the sensitivity analysis with respect to different levels of transaction cost. Tests have been implemented under three scenarios: 1) With no transaction cost. 2) With only trading and clearing cost. 3) With trading and clearing cost, and an assumed brokerage fee.

Overall, our result support efficiency of the OMX index option market, we fail to reject both EMH1 and EMH2, and there are not significant violations from lower boundary and PCP conditions. The frequencies of the violation of arbitrage condition violations from both lower boundary and Put Call Parity conditions are low.

For the most fundamental of all the arbitrage relations, lower boundary conditions for options, we find below one percent incidence of lower boundary violations in the OMX index option market. After fully imposing the bid ask spread, these violations are nearly close to zero. And these results compare favourably with those tests performed in the S&P 500 index option market (Ackert and Tian, 2001) and in the CAC 40 index option market (Capelle-Blancard and Chaudbury, 2001), and other European markets such as the Finnish index option market (Puttonen,1993).

With respect to the put call parity, the PCP efficiency in the OMX index options market seems quite similar to that in the Italian MIB30 index option market. Cavallo and Mammola (2000) report a 49% violation frequency for the short as well as the long hedge arbitrage. Considering the bid ask spread and other transaction costs, they find the violation frequency to drop to 2% for both short and long hedge. We report 46% violation frequency for long hedge and 54% violation frequency for short hedge. After incorporating various market transaction cost, especially bid ask spread, the violation frequency reduces to 0.61 % for long hedge and 1.25% for short hedge. The mean value of arbitrage profit for short hedge (2.38) is smaller than long hedge (2.54), therefore, the result from this study contrasts with the finding of Klemkosky and Resnick (1980) on the CBOE and Nisbet (1992) on the LTOM. We do not observe a larger number of profitable short hedge than profitable long hedge. This may be explained in that most investors and arbitrageurs in the Swedish OMX market hedge index options by Index futures, which is not difficult and is less costly compared with duplicating the OMX Index.

The sensitivity analysis with respect to time to maturity and moneyness verifies our efficiency OMX market result, and indicates that the OMX market has the same systematic patterns of PCP violations as U.S and other European index options markets. First, we find that the number of PCP condition violations increases when the time to maturity decreases, and moneyness moving from "at the money" to "away from the money", this is consistent with results from Ackert and Tian (1999) for the S&P 500 options market. Most traded options are these with short term maturity time and those away from money (in- and out-of -the money) in the OMX market. Secondly, we find the arbitrage profit for short hedge increases with the longer time to maturity, evidence that futures contracts are the most commonly used instrument for index options hedge in the OMX market. Finally, we observe the higher short hedge size of violations for at the money, which is consistent with Capelle-Blancard and Chaudhury (2001) from the CAC 40 index options market.

However, the main insight we can see from our presented result is that the tests of market efficiency critically depends upon the treatment of transaction costs. This is a clear illustration that a study of market efficiency based on the test of rational pricing boundaries must be done with transaction costs, especially the bid ask spread cost. The results from other studies also express a clear illustration that a study of market efficiency based on the test of rational pricing boundaries must be done with transaction costs.

Chapter 7 . Dynamic Hedging Test

7.1 Introduction

In this chapter, we test the market efficiency with the well-known Black Scholes Model (BS Model) and dynamic hedging simulation strategy. The Black Scholes model indicates which is the theoretically reasonable price in an efficient market, and the dynamic hedge strategy verifies whether the deviation between this "reasonable price" and actual price is allowed in terms of market efficiency.

Recalling our discussion in previous chapter for option pricing, the most popular valuation model indicates that if a certain portfolio is consisting of risky assets, (such as stocks, index, etc.) and a call option on that asset, then the return of the portfolio will be approximately equal to the return on a risk-free asset, at least over short periods of time. This type of portfolio is often called a *hedged replicating portfolio*. By properly rebalancing the positions in the underlying asset and the option, the return on the hedge portfolio can be made to approximate the return of the risk-free asset over longer periods of time. This approach is referred to as dynamic hedging strategy (Nandi and Waggoner, 2000).

Dynamic hedging strategy test will be implemented by pursuing a dynamic simulation strategy attempting to exploit deviations between option prices and theoretical prices. It is also called volatility trading strategy, more precisely, a volatility trading consists of formulating trading strategies on the basis of one's own anticipation of the future volatility of the security underlying the option contract (Cavallo and Mammola, 2000). Expecting a future volatility higher (lower) than that currently registered on the markets corresponds to foreseeing a rise (lowering) in the price of the options. The related strategy consists of purchasing (selling) the options.

Therefore, there are two key processes in the dynamic hedging strategy: First, volatility forecasting, and second, Delta neutral portfolio creation, which involves two of the most important Greek letters -Delta and Vega.

Volatility forecasting

Before we simulate the dynamic hedging scheme, Vega will be calculated in order to perform the volatility forecasting. We will follow the procedure and formula presented in Chapter 4 to calculate Vega and WISD.

Delta neutral portfolio creation

Most traders use Delta, Gamma and Vega to create risk neutral portfolios, however, Delta neutral is the most commonly used (Andersson, 1995). The Delta of an option, Δ, is defined as the rate of change of the option price with respect to the price of the underlying asset.[94] Delta formula (Δ) for call is:

$$\Delta = \frac{\partial C}{\partial S} \equiv N(d_1),$$

and for put is:

$$\Delta = \frac{\partial C}{\partial S} \equiv N(d_1) - 1 \qquad \text{(Hull, 2003)}$$

Delta will be calculated in order to decide the portion of futures we need to buy or sell to create the Delta neutral portfolio. And as the delta hedged position only remains for a relatively short time, the hedge has to be adjusted periodically, this is called rebalancing.

Suppose that an investor has sold or bought call options for g shares, the investor's position could be hedged by buying or selling $\frac{\partial C}{\partial S}\text{g} = N(d_1)\text{g}$ shares of the underlying asset. This is the so-called hedging ratio. This ratio will be rebalanced to maintain the Delta neutral.

The remaining section in this chapter will perform dynamic hedging strategy, the detailed implementation and the results from different steps will be presented in each sub-section (see Figure 0.1 for basic procedure).

It starts in section 7.2, by applying a number of selection criteria to the sample data, apart from the generic selection criteria, which we discussed in Chapter 5.

[94] Suppose that the delta of a call option on stock is 0,6, this means that when stock price changes by a small amount, the option price changes by about 60% of that amount.

These criteria ensure that the samples are valid for carrying out dynamic hedge and the noise of the data reduced to an acceptable level.

The next step is to forecast daily volatility of the OMX index future by using two different methodologies. One is Historical Volatility (HSD) and the other is Weighted Implied Volatility (WISD). Therefore, in Section 7.3 we present the result from HSD and WISD calculation as well as the critical analysis from HSD and WISD. As WISD is artificially derived from the market, we validate it theoretically and statistically.

Thereafter, in Section 7.4, the forecasted volatility is applied to the Black Scholes Model, which eventually generates the "theoretical price" of each sample. Naturally, to compare theoretical price (hereafter, we call it Model Price) and the actual market price (hereafter, we call it Market Price), we run a paired t-test. The P value may likely indicate that there is a significant difference between Market and Model Prices; however, given the cost, trading limitation and the accuracy of forecasted volatility issues, the difference is inevitable. Therefore, the descriptive statistic values of the paired T-test, which illustrate a more detail picture of the relationship between the Model Price and the Market Price, are also reviewed.

Finally, in Section 7.5, in order to evaluate whether the difference between the model price and the market price generates abnormal profits in reality, we run dynamic hedging simulation under two scenarios. One does not consider spread cost, while the other does.

We first present more specific designations of this simulation, and start to identify the mis-priced options based on the result from Section 7.4. At the same time, we neutralize the risk by buying or selling a certain number of OMX index futures. This hedge is maintained until the maturity of the option. The number of futures contracts holding is determined by the delta hedge ratio which is calculated with VB function in Access, and it is rebalanced on the daily basis with the newest delta value. Finally, we add up all cash flows, and calculate the Net Present Value (NPV) for each hedging. The results from the dynamic hedging will be presented with figures and tables to draw the efficiency conclusions.

Additionally, the supplementary result from the dynamic hedging, the identification of the superior volatility estimator with superior volatility predictability will be concluded in Section 6, which is followed by the summary and conclusion from this chapter in Section 7.7

However, before we start dynamic hedging, we would like to emphasize that according to the efficiency definition in this study, the efficiency will be concluded if we do not find significant abnormal returns or significant positive NPV from the dynamic hedging result. The hypothesis for dynamic hedging simulation is:

EMH3: there is no abnormal return from the dynamic hedging strategy for both OMX put and call index options (OMX market is efficient)

7.2 Data Selection

As we presented in Chapter 5, dynamic hedging strategy, liquidity and sensitivity to the non-synchronous problems are the main concerns for data filtration in order to reduce the noise.

As the BS Model is based on holding different positions and number of options and underlying assets, the price we used must be tradable. Following the general data filter procedure that we discussed in Chapter 5, we set the following preliminary data filter criteria:

a) Any index options and futures with ask price, bid price and trading volume of 0 or 0,01 will be excluded
b) Any index options and futures with a difference between the bid and ask price of more than 10 points will be excluded

To further ensure the liquidity, a common approach is to select the options with strike close to latest index (central strike) or the underlying future price, as they are usually more liquidated. Here we choose to use the price close to the underlying future price, as both options and futures contain the market´s expectation for the day when they expire. Many researchers, such as Nandi and Waggoner (2000), filter out all options with strike price over or below the underlying future price more than 10%. We evaluated our data and found 93%

of the volume is traded within this range, and its average traded volume is 461, which is about 3 times higher than those out of the range. Therefore, it is reasonable to adopt this selection criterion.

Another reason for the above selection criteria is that deep-in or deep-out-of money options have small time premiums and hence contain little information about the volatility function (Dumas, Fleming and Whales 1998).

Like deep-in and deep-out-of money options, the short term options also have very small time premium. The estimation of volatility becomes more sensitive towards the synchronous problem and other errors when the options are close to expiration. Therefore, as with the PCP condition test, most researchers in this area exclude the very short term options. As for the definition of “very short term”, H. Byström (2000) chose 15 days for the OMX Index market test, while the majority of researchers, such as Draper and Fung (2003) or Deville (2003), chose six days. We disagree with H Byström, as nearly 70% of the trading volumes are traded within 15 days. If we exclude all options shorter than 15 days, the applicability of the result is rather questionable. Instead, we chose six days, which only excludes 20% of the total trading volume of the market.

Finally, we also exclude long term options, which not only have liquidity problems, but are also less applicable in the Black Scholes Model. This is because Black Scholes Model assumes a constant volatility throughout the hedge, while it actually changes, and the more time it spans, the more it changes. Given a long period, the Black Scholes Model will produce a substantial error due to the volatility change. We decided to use “90 days” as the lower boundary of long term option. It excludes about 1.5% of the total market trading volume.

With all above filters, 81,745 option records were finally selected. They represent 69% of the total trading volume of the market.

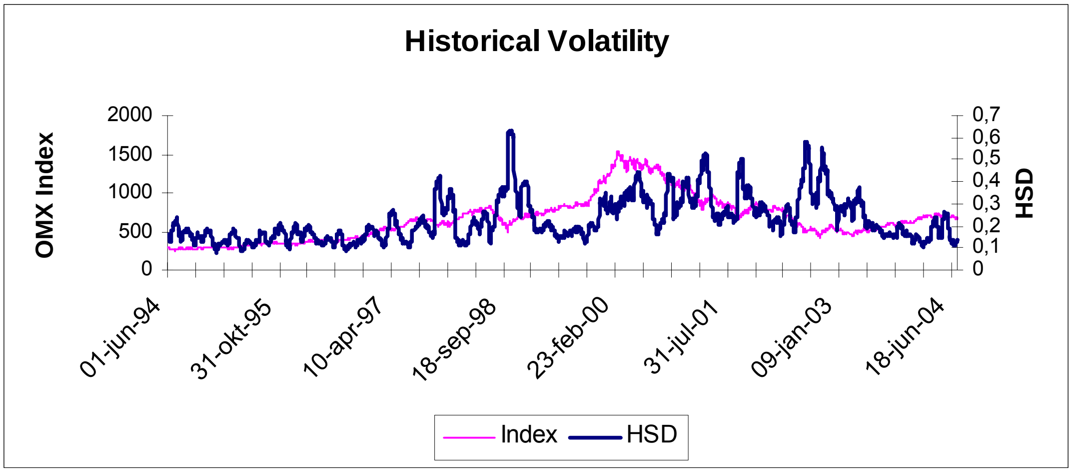
7.3 Historical Volatility (HSD) and Weighted Implied Volatility (WISD)

Before applying the sample data to the Black Scholes Model, we need to forecast the volatility. As discussed in Chapter 4, we chose to forecast both HSD and WISD.

7.3.1 HSD

As most of the options expired within one month, we decided to follow Andersson (1995) and Figlewski (2004) to calculate the Historical Volatility (HSD) on a 20 consecutive trading day basis. Based on the daily OMX index, we calculated the expected return for each day ($ln(S_t/S_{t-1})$). Next, we constructed a query to calculate the Standard Deviation of the 20-trading day moving average for each trading day (see Appendix 8 and Appendix 9 for query detail). Finally, we annualize the result with a multiplier of 15.81139, which is the square root of 250 (we assume the total trading day of every year is 250). Figure 7.1 presents the HSD during the entire tested period. From the figure we can see that HSD changes consistently with the change in index. During the years when index is very volatile, the HSD not only becomes higher, but also becomes more volatile.

Figure 7.1 HSD



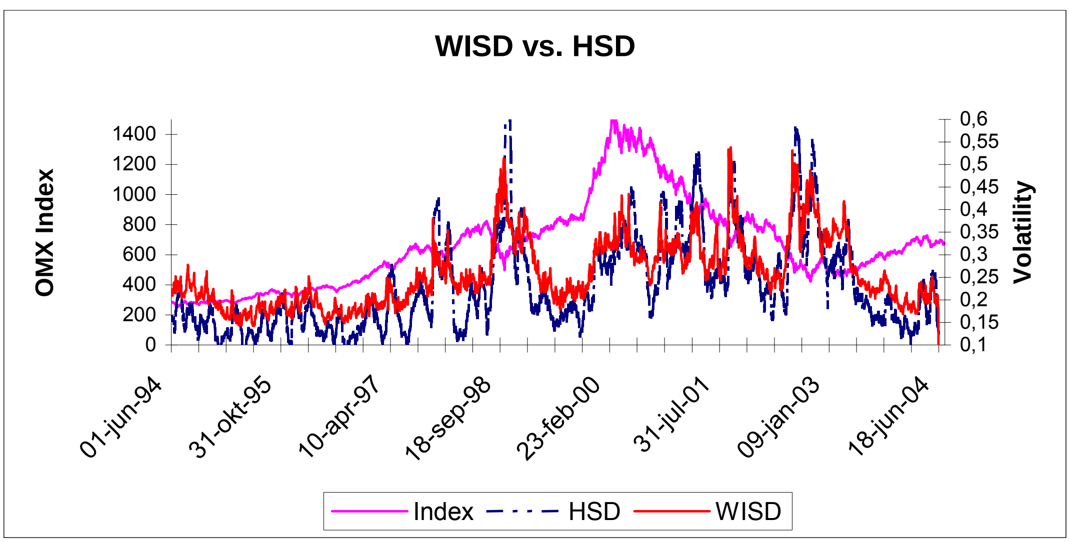
7.3.2 WISD

Implied volatility is calculated from the observed option prices through the BS Model. It is the market's opinion of the volatility. We calculate this volatility by using the VB function (Appendix 8) which follows the Bi-section method, presented in Chapter 4. This function tries different volatilities with the BS Model. It starts with the middle point of the upper boundary of 2 and the lower boundary of 0. If the price is higher than actual, the upper boundary will be reduced to this point, while if lower, the lower boundary will be increased to this point. This process is iterated until the distance of both boundaries reduces to 0.000001.

With the guidance of the formula in Chapter 4, to obtain a single WISD for each day we first wrote a VB function to calculate the Vega (Appendix 9) for each option, and then it is used in two queries to calculate weighted average value for each trading day.

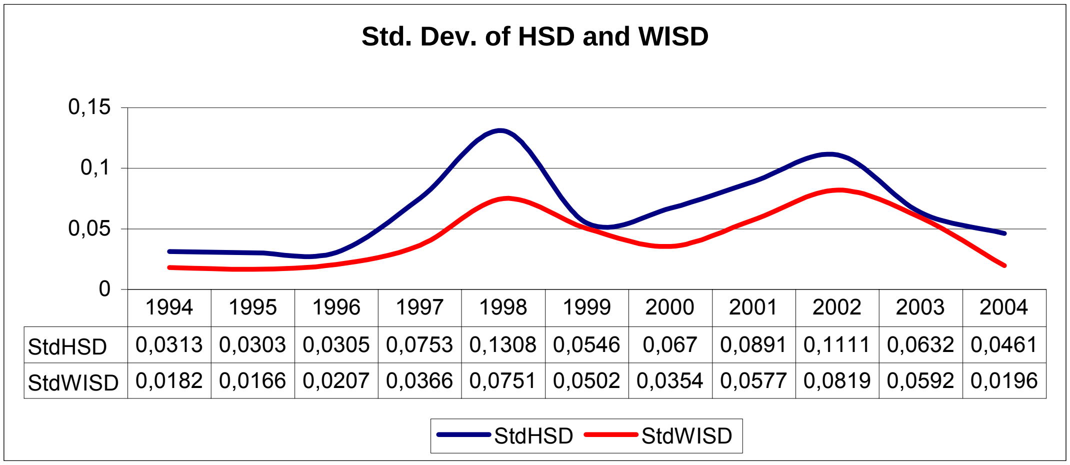
The WISD is presented in Figure 7.2, with reference to HSD. It is quite obvious that the WISD is in general higher than HSD. The explanation could be that as HSD is calculated from index, while WISD is implied directed from the option market, the WISD compounds not only the pure volatility of the asset, but also some cost issue related to setting up the hedge in the market. This makes WISD more volatile. However, it is also interesting to discover that when the market at a very high volatile status, the WISD becomes lower than HSD.

Figure 7.2 WISD vs. HSD



This indicates that when the asset market is getting extremely volatile; the derivative market tends to moderate it. This suspicion is further confirmed in the following Figure 7.3, which shows that WISD is more stable than HSD.

Figure 7.3 Standard deviation of HSD and WISD



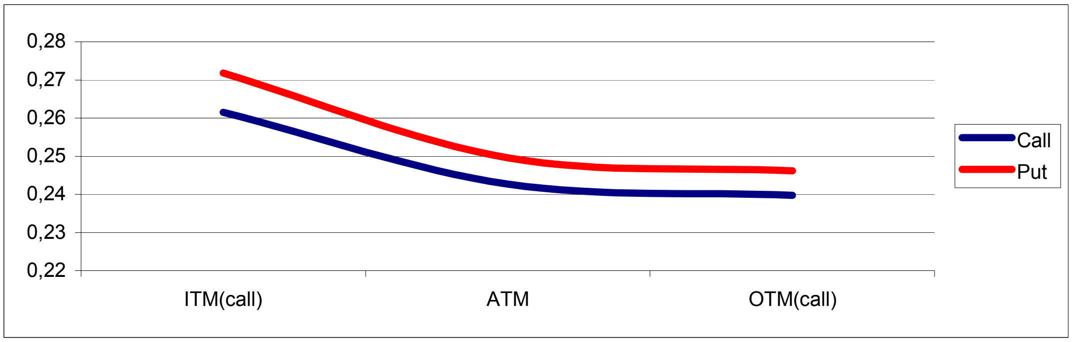
	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
StdHSD	0,0313	0,0303	0,0305	0,0753	0,1308	0,0546	0,067	0,0891	0,1111	0,0632	0,0461
StdWISD	0,0182	0,0166	0,0207	0,0366	0,0751	0,0502	0,0354	0,0577	0,0819	0,0592	0,0196

As WISD is artificially derived from the market, validation of the result is very important to build up the confidence in our research. We therefore valid it from a number of aspects:

a) Volatility Smile

When we average the implied volatility by moneyness, and plot it against it, it presents a left skew pattern (Figure 7.4). This means that when strike price increases, the implied volatility drops. The rate of dropping slows down when it becomes out of the money. In addition, put options gives higher volatility than call. This result is consistent with our expectation, and is already confirmed by many studies such as MacBeth and Merville (1979), Lauterbach and Schultz (1990), Rubinstein (1994) and Jackwerth and Rubinstein (1996). They all claim that options with lower strike prices have higher implied volatilities than those with higher strike prices. According to Hull (2003), the reason may be that the investors fear a repetition of the crash of 1987. Prior to October 1987, implied volatilities were much less dependent on strike prices.

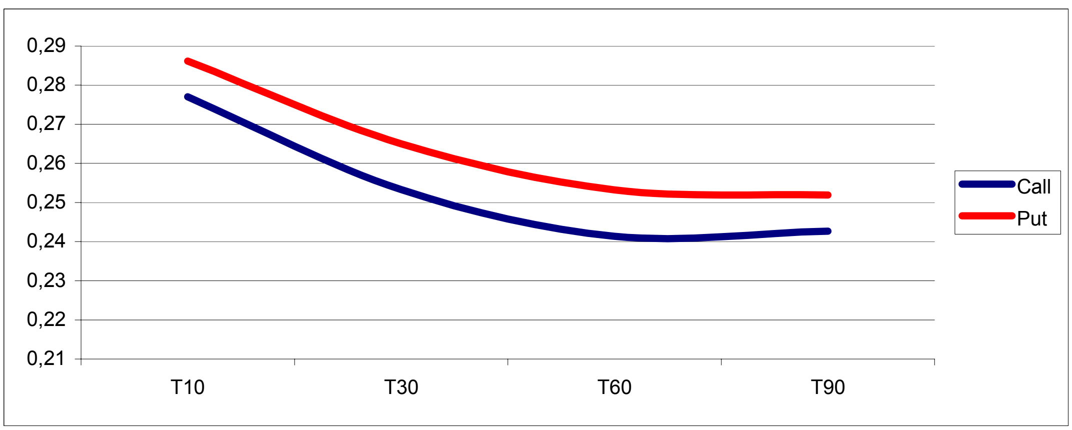
Figure 7.4 Volatility Smile



b) Term Structure

The term structure of the volatility shows a downward slope (Figure 7.5). This is contrary to Cavallo and Mammola's (2000) finding of the MIB30, but is consistent with Goldman Saches's research on the Japan market. Empirical evidence indicates that upward sloping term structure is characteristic of markets with lower volatility (as in the United States), while high volatility markets (notably Japan) usually present downward-sloping term structure, which means that the volatility tends to be an increasing function of maturity when short-dated volatilities are historically high (Hull, 2003).

Figure 7.5 Term Structure



c) Stationary---Unit Root Tests

In order to examine the stationarity of the time series of implied volatility, we perform the Unit Root tests following Hill et al. (2000). The results from these tests are represented in Table 7-1 (the auto-regression result of implied volatility).

We first examine the stationarity of the time series by test directly with a unit root test. The AR(1) model for the time series variable y_t is:

$$y_t = \rho y_{t-1} + v_t \qquad \text{E.q. 7.1}$$

In this model, if ρ=1, then y_t is the non-stationary random walk, $y_t = \rho y_{t-1} + v_t$, and it is said to have a unit root, because the coefficient ρ=1. From the Table 7-1, we observe that all ρ<1, then implied volatilities from our results are stationary time series. This is consistent with Cavolla and Mammola (2000)'s argument, the decline of autocorrelation at longer lags is an indication of stationarity.

In order to verify the stationarity, we look at the difference series. The test is put into a convenient form by subtracting y_{t-1} from both side of (E.q 7.1), to obtain:

$$y_t - y_{t-1} = \rho y_{t-1} - y_{t-1} + v_t$$

$$\Delta y_t = (\rho - 1) y_{t-1} + v_t = \gamma \cdot y_{t-1} + v_t \qquad \text{E.q. 7.2}$$

Where $\Delta y_t = y_t - y_{t-1}$, and $\gamma = \rho - 1$, then,

$H_0 : \rho = 1 \leftrightarrow H_0 : \gamma = 0$

$H_0 : \rho < 1 \leftrightarrow H_0 : \gamma < 0$

As can be seen in Table 7-1, the estimated coefficients present negative serial correlation at Lag 1 for all implied volatilities estimates. This result is consistent with the finding of French et al (1987) on the S&P 500, and Harvey and Whaley (1992) on the S&P 100 Index, and may indicate the predictability of implied volatility. Since y_t can be made stationary by taking the first difference, are said to be integrated of order 1.

Meanwhile in order to support our conclusion of stationarity, we perform an additional test, the Dickey-Fuller test. After applying the first difference to the series, we collected the t-statistics, known for this particular test as tau-statistics (τ)[95]. As we can observe in Table 7-1, there are large negative values of the tau-statistic at lag 1 for all the series. We fail to reject the stationarity of the time series, and therefore, we conclude that they are Stationary Integrated series.

As a conclusion, the above analysis indicates that the implied volatilities we get from the OMX index options market display all typical patterns of other derivative markets such as in the U.S. and other European markets. Moreover, the WISD is a stationary time series and has a relatively high predictability. Therefore, we should have high confidence to apply the WISD in the BS Model.

[95] The critical value for Dickey Fuller tests for 5% degrees of confidence is -2.86. The tau must be smaller than -2.86 in order to reject the null hypothesis of non-stationarity.

Table 7-1 Summary Statistics and Autocorrelations for OMX Index Options Annualized Implied Volatilities

		Mean	SD	Min	Max	Skew	Autocorrelations p1	p2	p3	p4	p5	p6	p10
Call options IV													
At the money	level	0.2539572	0.083482	0.053070068	0.633362	0.842652	0.951408	0.938161	0.924818	0.912108	0.904041	0.891062	0.857586
	1st Difference	7.219E-05	0.030768	-0.341705322	0.219727	-0.442069	-0.361999	0.000268	-0.006979	-0.04669	0.050469	-0.041967	0.004444
	tau						-18.15099	-3.082855	0.976812	0.478695	-0.423371	-1.055677	-0.875937
In the money	level	0.2573475	0.084024	0.067202409	0.556797	0.909961	0.934484	0.924121	0.90845	0.898347	0.890692	0.879684	0.851823
	1st Difference	-2.39E-05	0.030465	-0.224367142	0.209525	0.066429	-0.419872	0.040075	-0.041365	-0.018826	0.025369	-0.02803	-0.018811
	tau						-20.67472	1.791996	-1.851098	-0.841845	1.134323	-1.253145	-0.839849
Out of the money	level	0.247248	0.071644	0.120895386	0.545057	0.747285	0.964795	0.946214	0.93361	0.919741	0.906914	0.893336	0.854132
	1st Difference	-1.29E-05	0.019047	-0.180198669	0.176703	-0.314623	-0.234102	-0.086279	0.017453	-0.01481	0.011171	-0.05521	-0.031556
	tau						-11.05088	-3.974421	0.800896	-0.67943	0.512312	-2.535142	-1.446702
Put options IV													
At the money	level	0.2539549	0.083462	0.053070068	0.633362	0.842933	0.933046	0.915746	0.907616	0.896628	0.884156	0.872921	0.832358
	1st Difference	-8.33E-05	0.030764	-0.219726563	0.341705	0.442875	-0.37041	-0.067649	0.021353	0.010487	-0.009168	-0.023239	-0.019301
	tau						-18.15348	-3.086531	0.971889	0.476947	-0.416788	-1.056508	-0.876715
In the money	level	0.2605667	0.084671	3.05176E-05	0.666992	0.457457	0.8456	0.833972	0.821919	0.802335	0.793226	0.776714	0.724316
	1st Difference	-1.97E-05	0.047077	-0.380015055	0.330353	-0.08741	-0.461485	0.001362	0.023414	-0.033725	0.023229	-0.012515	0.040706
	tau						-21.48428	0.056239	0.96657	-1.392198	0.958374	-0.516138	1.678152
Out of the money	level	0.268193	0.08055	0.142049154	0.585667	0.942857	0.973912	0.959471	0.946568	0.935681	0.924989	0.915742	0.877013
	1st Difference	-2.38E-05	0.018471	-0.120188395	0.228923	1.411731	-0.222446	-0.030941	-0.03862	-0.002452	-0.02773	0.043825	-0.018316
	tau						-10.76208	-1.459354	-1.82164	-0.115512	-1.306428	2.065603	-0.862375
Weigthed Implied Volatilities													
WISDc	level	0.2434504	0.075825	0.12672758	0.54881	0.860312	0.974705	0.961808	0.95177	0.940582	0.929949	0.918715	0.880604
	1st Difference	-1.43E-05	0.017099	-0.122019283	0.161132	0.371226	-0.24322	-0.057559	0.021695	-0.010794	0.012596	-0.017292	-0.061674
	tau						-12.46657	-2.865958	1.078433	-0.536352	0.625746	-0.858908	-3.070186
WISDpc	level	0.2569126	0.074762	0.14201981	0.538232	0.922747	0.981318	0.967754	0.956686	0.945154	0.933525	0.922007	0.881217
	1st Difference	-1.51E-05	0.01451	-0.096278947	0.163216	0.693834	-0.136173	-0.069158	0.011788	0.004328	-0.002709	-0.012326	-0.053298
	tau						-6.834151	-3.444971	0.585695	0.214897	-0.134511	-0.611862	-2.650216
WISDp	level	0.2718005	0.076982	0.146183954	0.589391	0.941441	0.977387	0.962965	0.949962	0.938259	0.926864	0.915855	0.87284
	1st Difference	-1.69E-05	0.016431	-0.105808722	0.16716	0.490767	-0.180366	-0.033407	-0.028676	-0.005209	-0.008815	0.01037	-0.024035
	tau						-9.117428	-1.661099	-1.425273	-0.258581	-0.437462	0.514591	-1.192584

7.4 BS Model Price vs. Market Price

Now we are ready to calculate BS Price. As the volatility is calculated from the current transaction price, and it is not available before the transaction takes place. Therefore, we can not apply current day's volatility in the BS Model. Instead, as we discussed in Chapter 4, we forecast volatility by projecting volatility one more day. This means that we use the previous day's volatility to forecast the current day. After obtaining the volatility, the Black Scholes Model option price is computed in Microsoft Access.[96]

7.4.1 Paired T-test

After calculating the price from the model, we run two paired t-tests for the two model prices and market price. The results of the tests reject the hypothesis of Model Price = Market Price for all type and all moneyness options regardless of using WISD or HSD,[97] as all probability values are smaller than 0,001. The size effect[98] is generally small to medium, except the OTM call of WISD price and OTM put of HSD price. This further confirms the significance between the market price and the model prices.

However, this does not mean the model is wrong or the market is inefficient. With the trading cost and other trading limitations, the efficient price is a range rather than a single value. Moreover, as the volatility applied is merely a forecast, the accuracy of the volatility also affects the deviation of the model price and market price. Therefore, it is not surprising that both prices are different. However, the other descriptive values of the test could be used to see how closely both prices fit to each other. These values are summarized in Table 7-2.

[96] Queries and Functions for calculating model price is in Appendix 8 and Appendix 9

[97] According to Pallant (2001), a paired T-test will tell if there is a significant difference between in the mean scores for Time 1 and Time 2. If the probability value is less than 0,05, then we can conclude that there is a significant difference between our two scores.

[98] Size effect indicates the relative magnitude of the difference between the means. Eta squared is one of the most commonly used effect size statistics, the formula is: Eta Squared = $t^2/(t^2+N-1)$, to interpret the eta squared values following the guidelines can be used from Cohen (1988): 0,01=small effect, 0,06=moderate effect, 0,14 = large effect.

Table 7-2 Paired T-test Results

Paired T Test Result Between Model Price and Market Price (P is 0 for all)														
	Call							**Put**						
	Cor. R.	**Mean**	**SD**	**SD Err**	**t**	**Df**	**Eta**	**Cor. R.**	**Mean**	**SD**	**SD Err**	**T**	**Df**	**Eta**
WISD														
ITM	0.995	0.363	1.982	0.014	25.44	19317	0.032 S*	0.997	0.207	2.053	0.019	10.76	11223	0.0102 S
ATM	0.996	0.575	1.677	0.019	30.65	7995	0.105 M*	0.995	0.130	1.722	0.021	6.237	6864	0.0056 S
OTM	0.994	0.758	1.779	0.013	57.35	18088	0.154 L*	0.992	-0.34	1.766	0.012	-28.1	21287	0.0358 S
HSD														
ITM	0.972	0.952	4.777	0.039	24.69	15366	0.038 S	0.978	0.910	5.379	0.051	17.93	11223	0.0278 S
ATM	0.974	1.011	4.062	0.045	22.26	7995	0.058 M	0.971	1.323	4.068	0.049	26.95	6864	0.0957 S
OTM	0.954	0.157	5.042	0.038	4.183	18088	0.001 S	0.951	1.625	4.263	0.029	55.63	21287	0.1269 L

***S: Small Size Effect, M: Medium Size Effect, L: Large Effect**

a) Correlation factor

Model prices derived from both WISD and HSD are highly positive correlated to the market price, while the rate is higher for WISD prices in all option types and through all moneyness. This gives high confidence that the option market is guided by BS model, and it seems WISD plays a more important role than HSD.

b) Mean value

The mean value of the difference (Market Price –Model Price) is between 0.34 and 0.7583 index point for WISD Price, which is about 0.5 index point lower than HSD Prices except the OTM Call, it is only 0.5 index point higher than HSD (but it has much lower standard deviation value).

c) Standard deviation

By looking at the standard deviation, it is roughly in the range of 1.5 to 2 for WISD price, with standard error of about 0.015, while it is about 4 to 4.5 for HSD with standard error of about 0.04. This means the market price is much more diverted from the HSD price than the WISD price.

d) Distribution of the difference between market price and model price

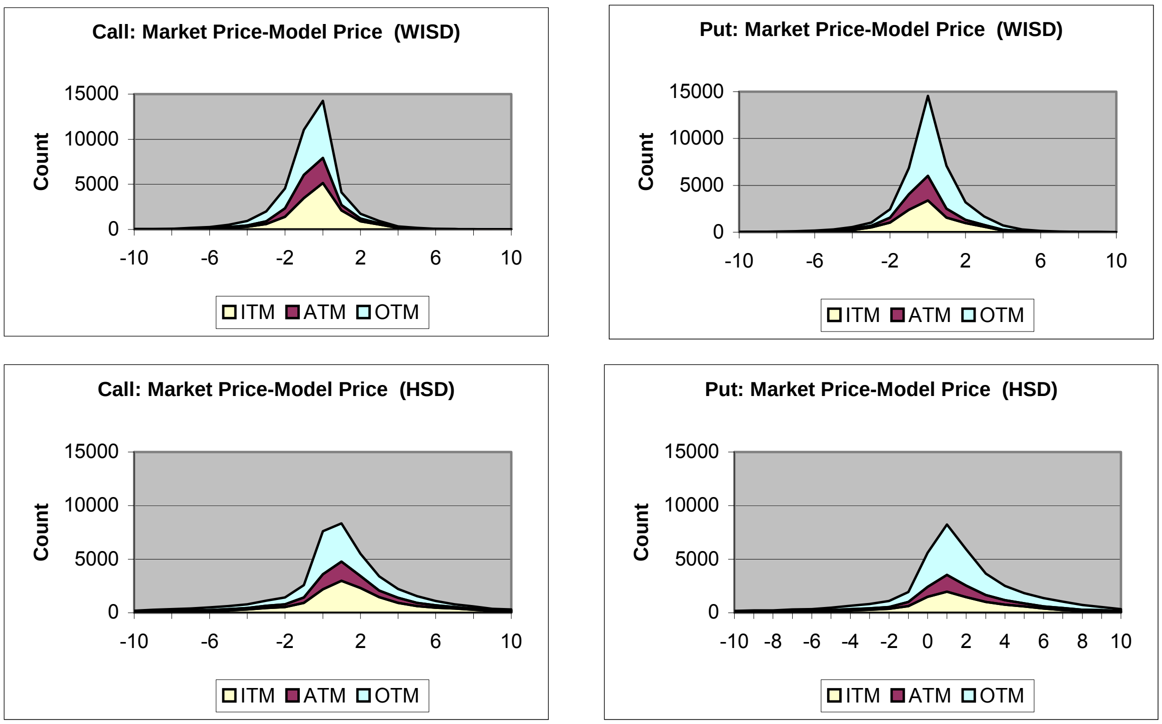
Figure 7.6 illustrates the distribution of the difference. The difference is expressed in index points. Looking at the WISD prices, the difference is very much normally distributed with mean value around 0 for all moneyness types and all option types. As for the HSD prices, the difference is also quite normally distributed but with a mean value around 1. Compare with WISD price, the "bell" is, obviously, much less kurtosis.

For WISD prices, the frequency reduced to a very low level when the difference beyond ± 2 index points, and fades to zero when it reaches ± 4. For HSD prices, this range is at least doubled.

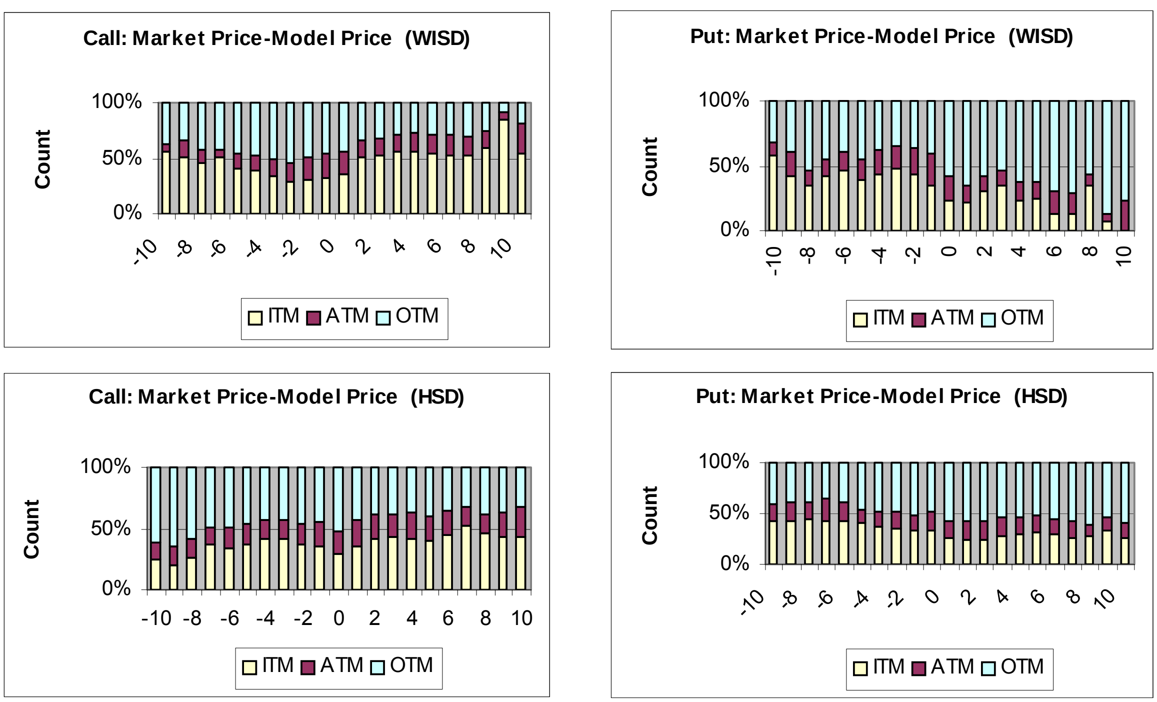
When looking at the composition of different moneyness type, we spot a pattern that for call options, there are more OTM options which are theoretically under priced (Market Price lower than Model Price), while there are more ITM options which are theoretically over priced. Whereas for put options, we see more ITM options under priced and more OTM options over priced. This result is consistent with Cavallo and Mammola (2000).

Figure 7.6 Difference Between Model Price and Market Price for Put and Call Options

Frequency Chart of Price Difference



Composition of Moneyness Frequency Against Price Difference



7.5 Dynamic Hedge Simulation

7.5.1 Cost issues related to dynamic hedge

Although the statistic analysis of both prices gives us some indications of the price efficiency, we are still not clear whether the price difference between the Market Price and Model Price is reasonable. This means that we need to know whether the price difference could be covered by the cost related to setting up dynamic hedge strategy. Besides the hedge costs we presented in Chapter 5; there are other costs for dynamic hedging that come from the following aspects:

- The commission cost for every transaction
  Apart from the fees charged by the stock exchange center to the members, which is a small amount, the traders are usually liable to pay commission to the brokers. The cost varies for different traders. Mostly, the cost is transaction based, but could also be structured in different ways. Therefore, it is very difficult to make a simple assumption.

- The liquidity of the market at a certain time spot
  The amount of options or futures may not be able to be fully traded with the same listed price and the same time spot.

- The complexity of the capital market
  The interest rate we are using is STIBOR. However, the real interest rate we get for setting up hedges could be very different. Moreover, there are many practical limitations for borrowing and depositing funds, and also the non-interest cost involved for the transactions in capital market.

- The accuracy of the volatility forecasting
  The volatility is a fact, but it only can be estimated. The inaccuracy of the volatility forecast also results that the market price deviates from the model price

- The limitation of market practice
  The BS model considers very little about the limitation of the actual market. One obvious example is that the model assumes that we can buy or sell underlying assets to neutralize the risk

- Other costs
  The cost of resources needed to set up dynamic hedging, such as human resource, IT resource, etc.

7.5.2 The design and implementation of dynamic hedging simulation

To cope with the cost issues, we designed this dynamic hedge simulation to explore the market efficiency more precisely.

This simulation will take into account the issues of

- Fees charged by the stock exchange center
- Spread cost
- The limitation of the market
- The inaccuracy of the volatility forecast

If the result shows abnormal profit, we will run sensitive analysis to take into account the other cost issues.

7.5.2.1 Spot mispricing

The simulation starts from spotting occurrences of possible mispricing. Model Price and Market price are compared on a the daily basis. Once we spot a mispricing occurrence, a hedge is set up accordingly on the following day.

Following Cavolla and Mammola (2000), we assume more than 15% difference as over price, and beyond –15% as under price. We therefore spotted 12,456 mispriced calls and 11,193 mispriced puts, which represents 30% of all calls and 28% of all puts. When benchmarking the HSD price, we got 58% mispriced calls and 68% mispriced puts. Among them, the majority are the under priced calls and overpriced puts.

Table 7-3 Number of Mispriced Options Spotted by WISD and HSD

Number of Mis-priced options spotted by WISD and HSD								
	Call				Put			
Moneyness	Under	Over	Fare	Total	Under	Over	Fare	Total
WISD								
ITM	2060	261	13046	15367	210	788	10226	11224
	13,4%	1,7%	84,9%	_	1,9%	7,0%	91,1%	_
ATM	2466	247	5283	7996	265	985	5615	6865
	30,8%	3,1%	66,1%	_	3,9%	14,3%	81,8%	_
OTM	6148	1274	10667	18089	861	8084	12343	21288
	34,0%	7,0%	59,0%	_	4,0%	38,0%	58,0%	_
Total	10674	1782	28996	41452	1336	9857	28184	39377
	25,8%	4,3%	70,0%	_	3,4%	25,0%	71,6%	_
HSD								
ITM	1786	4710	8871	15367	958	3426	6840	11224
	11,6%	30,7%	57,7%	_	8,5%	30,5%	60,9%	_
ATM	1075	4016	2905	7996	495	3342	3028	6865
	13,4%	50,2%	36,3%	_	7,2%	48,7%	44,1%	_
OTM	3527	9221	5341	18089	1895	14748	4645	21288
	19,5%	51,0%	29,5%	_	8,9%	69,3%	21,8%	_
Total	6388	17947	17117	41452	3348	21516	14513	39377
	15,4%	43,3%	41,3%	_	8,5%	54,6%	36,9%	_

The fact of having more under priced calls and overpriced puts may lead us to draw the conclusion that the cost of short selling underlying asset is relatively higher than holding long position in the underlying asset, as both scenarios require shorting underlying asset. This conclusion had been drawn by a number of researchers, such as Klemkosky and Resnick (1980) on the CBOE and Nisbet (1992) on the London Traded Option Market (LTOM).

However, we found that the under priced calls and over priced puts are relatively of lower value, which makes them easier to be spotted as mis-pricing with the same value of price difference. Therefore, if we look at Figure 7.6, which expresses the price difference in index point, this pattern does not exist.

Therefore, if the trading cost is not correlated to the price of the option, the above conclusion is not valid.

Now, all mis-pricing occurrences are stored in two MS Access tables, one for WISD, the other for HSD.

7.5.2.2 Simulate dynamic hedging

Once we spot an over valued call, on the following day (as the price we are using is the last price of the day, we are unable to make new transaction within the day when a mis-pricing is spotted), a hedge will be setup by selling 10 contracts of calls, and buying "D" contracts of related index futures. In theory, in order to keep the hedge with zero risk, the D number should be the absolute value of the delta value for each option each day, which is multiplied by ten. However, as we are unlikely to buy a fraction of the future contract, D number has to be integer.

In addition, as we do not know the current delta value when we establish the hedge, the D value has to be based on the delta value of the previous day.

Likewise, if we spot an under valued call, 10 contracts of call will be bought, and D numbers of future contracts will be sold on the next day.

For puts, overvalue signal will result in selling both 10 contracts of put and D contracts of futures, while undervalue signal requires buying both 10 contracts of put and D contracts of futures.

Once the hedge is established, we remain it until expiration. During that period, the D value is revised on the daily basis, and the number of futures contracts holding is rebalanced accordingly. This means, on a daily basis, we will need to buy or sell futures contracts to neutralize the risk according to the indication of the delta.

We devised another two MS Access tables to record the futures contract volume/value which is traded for each hedging for each day.

On the expiration day, all options and futures contracts are cleared. We create two more MS Access tables to calculate all cash flow involved for each hedge. The fee charged by the stock exchange centre, which is SEK 3.5 for each options contract, SEK 4.5 for each futures contract, is considered throughout the life of each hedge.

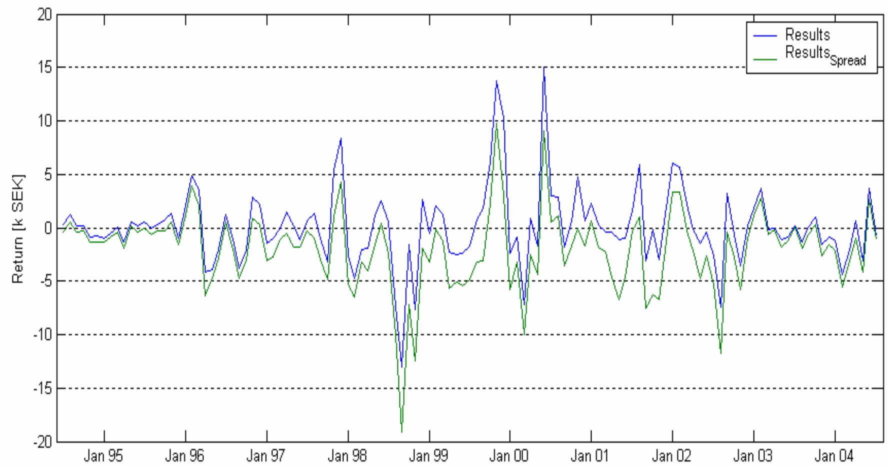
Finally, we add up the Net Present Value (NPV) of all cash flow of every hedge, to see whether the NPV is systemically positive.

We do this simulation with two scenarios. One is under the condition that all transactions are done at the middle point of the last ask, bid price. In the second scenario, we can only buy at the ask price, and sell at the bid price.

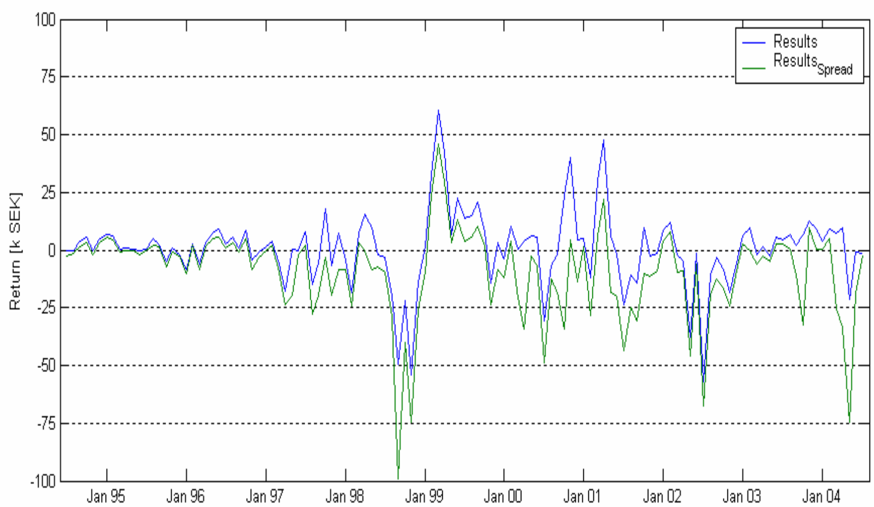
7.5.3 Empirical result of dynamic hedging simulation

Figure 7.7 NPV of Dynamic Hedging with WISD and HSD (monthly basis)

NPV of Dynamic Hedging (WISD)



NPV of Dynamic Hedging (HSD)



Without spread cost, the NPV for hedges based on both WISD and HSD are somewhat around zero. While looking at the spread cost scenario, the NPV reduced significantly, especially for HSD based hedges. This result preliminary indicates that there is no obvious arbitrage profit that exists, even the BS model has spotted some occurrences of having the opportunity to gain arbitrage profit. This means that the difference we have spotted between the model generated price and the market price could be more or less covered by the cost occurred during the life of the hedge even without considering the commission charges from the broker.

When ask bid spread cost steps into the picture, we are generally losing money. This clearly indicates that it is not worth to invest in risk neutral dynamic hedge for OMX index options.

We can get more comprehensive pictures when looking at the NPV for each year and each moneyness type from Table 7-4 and Table 7-5 (the average value in the total rows are not the mean value of each year's average, as the number of hedges are different from year to year. Therefore, this value is averaged from all hedges). As can be seen from the tables, there is a pattern that the hedges of ITM options are more unprofitable than that of the OTM options. This means that the profits of the hedge are more generated from the speculation of the underlying assets rather than the speculation of the options.

We also observe that without considering spread cost, the average profits are slightly positive. However, they are highly negative when we consider the spread cost. This confirms the above discovery numerically.

When looking at the average profit for each year and the moneyness type, someone may conclude that if we devise the hedge only for OTM options and could also somewhat reduce the spread cost, we might be able to achieve arbitrage profit. However, as the moneyness is unclear when we spot the mispricing, this strategy will require the speculation of moneyness, which is actually the speculation of the price of the underlying asset. Therefore, to achieve this profit, we have to bare the risk which is similar to the market risk of the asset itself.

Therefore, the dynamic hedging simulation result suggests that there is no arbitrage profit existing in the market, although there are many options with market value that are significantly different from the value generated by the BS Model. The OMX Index option market is efficient.

Table 7-4 Dynamic Hedging Arbitrage Result (WISD)

Dynamic Hedging Arbitrage Result (Forecast volatility with WISD)											
Moneyness	Year	Sum		Mean		Std. Dev.		Min		Max	
		NS*	S*	NS	S	NS	S	NS	S	NS	S
ITM	1994	49	-214	1,09	-4,76	25,72	24,22	-49	-54	59	44
	1995	-675	-2219	-2,10	-6,91	17,64	17,30	-42	-46	46	38
	1996	4866	-644	7,36	-0,97	33,85	35,16	-71	-94	100	97
	1997	-3345	-9633	-6,40	-18,42	42,77	41,91	-147	-155	203	179
	1998	-31025	-41092	-74,40	-98,54	78,86	80,79	-275	-291	255	226
	1999	15068	4042	31,66	8,49	72,27	70,80	-200	-236	326	272
	2000	-4415	-10035	-29,83	-67,80	232,20	231,68	-1468	-1510	293	245
	2001	-8885	-19763	-25,83	-57,45	99,54	100,54	-376	-417	374	307
	2002	-21933	-30507	-53,11	-73,87	103,88	103,45	-304	-334	305	276
	2003	-995	-1812	-11,71	-21,32	39,22	41,08	-125	-157	102	81
	2004	-638	-990	-16,79	-26,05	35,07	35,21	-96	-107	88	76
ITM Total		**-51928**	**-112867**	**-14,96**	**-32,52**	**87,34**	**89,26**	**-1468**	**-1510**	**374**	**307**
ATM	1994	-2090	-3049	-8,86	-12,92	19,91	19,28	-57	-63	77	57
	1995	-2174	-3872	-4,51	-8,03	18,34	17,67	-58	-62	64	59
	1996	-11083	-14688	-15,68	-20,78	28,77	30,05	-85	-101	58	56
	1997	-3016	-8293	-4,86	-13,38	38,63	37,38	-139	-154	157	120
	1998	-11509	-16564	-31,62	-45,51	96,75	96,24	-794	-811	738	709
	1999	-17121	-23844	-33,50	-46,66	68,51	71,93	-298	-365	189	158
	2000	16772	9774	68,46	39,89	289,44	288,15	-1718	-1766	567	530
	2001	4221	-3838	13,57	-12,34	102,79	101,90	-244	-297	404	311
	2002	601	-2896	2,58	-12,43	76,35	81,07	-255	-462	194	186
	2003	-4538	-5830	-26,54	-34,09	59,24	60,49	-144	-174	124	112
	2004	-9447	-11133	-43,33	-51,07	168,98	169,41	-600	-604	606	601
ATM Total		**-39384**	**-84233**	**-9,61**	**-20,55**	**101,16**	**100,59**	**-1718**	**-1766**	**738**	**709**
OTM	1994	1378	-1324	1,16	-1,11	11,33	11,63	-44	-51	45	36
	1995	5765	1396	3,05	0,74	12,69	12,30	-61	-64	55	52
	1996	2338	-3781	1,14	-1,84	12,59	13,70	-94	-115	98	92
	1997	14897	758	6,91	0,35	31,80	30,89	-144	-162	150	112
	1998	10331	-13539	6,09	-7,99	113,20	114,98	-730	-769	753	739
	1999	26406	415	12,08	0,19	42,62	41,74	-226	-268	282	240
	2000	6231	-15457	6,00	-14,89	144,84	148,21	-1551	-1583	1496	1483
	2001	11079	-14103	8,19	-10,42	58,54	61,57	-513	-595	334	287
	2002	17944	823	14,17	0,65	54,56	53,72	-223	-260	252	201
	2003	3223	-2154	2,84	-1,90	21,62	22,15	-82	-105	96	81
	2004	4128	-91	5,68	-0,13	38,16	37,98	-128	-162	667	661
OTM Total		**103720**	**-47057**	**6,21**	**-2,82**	**60,29**	**61,25**	**-1551**	**-1583**	**1496**	**1483**
Grand Total		**12408**	**-244157**	**0,51**	**-10,06**	**73,44**	**74,54**	**-1718**	**-1766**	**1496**	**1483**

*NS: Without Spread Cost S: With Spread Cost

Table 7-5 Dynamic Hedging Arbitrage Result (HSD)

Dynamic Hedging Arbitrage Result (Forecast volatility with HSD)											
Moneyness	Year	Sum		Average		Std		Min		Max	
		NS	S	NS	S	NS	S	NS	S	NS	S
ITM	1994	109	-1902	0,31	-5,40	18,02	17,18	-57	-61	52	36
	1995	-6550	-10651	-9,19	-14,94	20,98	20,82	-68	-78	48	37
	1996	-1573	-10069	-1,44	-9,19	31,30	31,74	-116	-142	111	107
	1997	-20441	-40473	-13,01	-25,76	46,64	47,82	-211	-244	172	131
	1998	-95683	-141471	-59,06	-87,33	85,19	96,31	-764	-839	687	637
	1999	19487	-15544	15,60	-12,45	81,90	86,66	-286	-729	288	214
	2000	-122	-117670	-0,14	-136,98	130,12	381,94	-531	-2912	1384	1339
	2001	9410	-69670	7,52	-55,69	114,36	186,76	-307	-2171	440	385
	2002	-77003	-105688	-62,05	-85,16	88,47	91,03	-357	-445	165	147
	2003	7409	-36733	12,47	-61,84	32,53	235,81	-141	-2341	84	69
	2004	-9950	-91996	-42,34	-391,47	183,44	731,17	-650	-3322	736	726
ITM Total		**-174907**	**-641867**	**-16,22**	**-59,54**	**87,71**	**194,91**	**-764**	**-3322**	**1384**	**1339**
ATM	1994	8456	4484	11,49	6,09	24,21	24,04	-56	-61	61	54
	1995	4279	-217	4,37	-0,23	24,05	24,19	-94	-98	65	57
	1996	8125	3556	9,56	4,21	23,92	23,38	-69	-75	59	48
	1997	-11992	-59547	-9,02	-45,56	45,34	141,07	-141	-1589	171	138
	1998	1102	-18285	1,24	-20,80	58,77	61,24	-162	-240	164	137
	1999	62464	40283	53,76	34,91	77,00	72,09	-203	-246	295	246
	2000	39585	16913	69,45	29,67	288,28	289,59	-442	-1171	1609	1580
	2001	-9013	-40629	-12,35	-55,73	100,21	131,75	-290	-935	336	277
	2002	-5049	-14615	-9,54	-27,68	80,20	80,28	-188	-214	252	181
	2003	15954	-7068	21,16	-9,39	38,03	116,17	-117	-1149	143	135
	2004	-7313	-40992	-13,77	-77,34	241,45	391,82	-1248	-2638	1245	1194
ATM Total		**106598**	**-116115**	**11,76**	**-12,92**	**110,14**	**149,64**	**-1248**	**-2638**	**1609**	**1580**
OTM	1994	11569	5163	6,45	3,59	12,15	13,83	-40	-44	52	37
	1995	1714	-6820	0,68	-3,68	16,02	19,19	-95	-100	63	56
	1996	23020	8275	7,15	3,43	14,84	17,30	-114	-120	46	39
	1997	14315	-37544	3,67	-11,29	35,26	72,58	-197	-1054	160	129
	1998	-53908	-164534	-18,25	-60,87	89,21	249,58	-764	-4217	753	727
	1999	125077	67032	34,86	21,27	51,56	52,51	-201	-287	270	227
	2000	19313	-81512	8,21	-36,34	116,13	240,03	-583	-3855	1549	1520
	2001	34709	-55834	13,27	-22,00	76,33	118,43	-301	-1716	348	297
	2002	-49174	-88149	-21,14	-40,31	64,45	71,36	-321	-766	252	205
	2003	34619	3555	13,05	1,52	32,81	72,99	-150	-1392	683	661
	2004	19933	-17376	17,17	-18,20	146,31	230,93	-752	-1413	778	769
OTM Total		**181187**	**-367743**	**6,23**	**-14,62**	**67,05**	**133,77**	**-764**	**-4217**	**1549**	**1520**
Grand Total		**112878**	**-1125724**	**2,31**	**-25,06**	**81,95**	**154,89**	**-1248**	**-4217**	**1609**	**1580**

7.6 WISD is Outperforming HSD as Volatility Indicator

Now the market efficiency is proven. This implies that the price deviation between Model Price and Market Price is only due to the cost involved in the dynamic trading.

Given the fact that the price deviation is far wider when using HSD to forecast volatility than using WISD, we can conclude that, compared with HSD, WISD is a superior volatility estimator and forecaster. This result is identical with a number of researches, such as Cavallo and Mammola (2000) for the Italian MIBO30, and Shastri and Tandon (1986) for currency options in Philadelphia Stock Exchange.

By looking at Figure 7.6, we can see that it is quite practical to use the BS Model price (with WISD as volatility) to assess the market price. A simple conclusion may be that the reasonable market price of OMX options should most likely be the Model Price plus minus two index points. If it is beyond 4 points, we should start to question its reasonability.

7.7 Conclusion

This chapter tries to test the market efficiency with the hypothesis that there is no abnormal return from the dynamic hedging strategy for both OMX put and call index options.

Considering the liquidity and data non-synchronous problem, the data is carefully selected to reduce the noise of the samples.

The BS Model is used to preliminary assess the market. The result shows that the market price is very close to the "reasonable price" which is generated by the BS Model. However, there is still a significant difference between Model and Market Price.

To assess whether this difference is reasonable, we run dynamic hedging simulation directly with an ex-ante approach. The result shows slight arbitrage profit when we only consider some fee cost and market limitation of setting up dynamic hedge. However, the profit is totally wiped out, even showing negative profit, when spread cost is considered. Therefore, we are unable to make profit by investing in risk neutral dynamic hedging. Thus, the market is efficient.

Furthermore, volatility forecast is also discussed in this chapter. All empirical results suggest that WISD is more outperformed than the HSD as a method of volatility forecasting.

Part Five: Conclusions and Recommendations

In the last part of the study we depict the final conclusions of the study as well as recommendations for further research.

Chapter 8 . Conclusions

Market efficiency of the OMX index option is of central importance to this research. Three Hypotheses are tested with over 10 years of complete historical data. To minimize the cost issue, liquidity issue and the dividend issue, the OMX index future is treated as its underlying asset, instead of duplicating the whole basket of stocks. As the options and the futures have the same trading time and same expiration date, the data synchronous is minimized.

The historical data is on a daily basis. Compare with the intra-day data, it contains significant noise for implementing the research. However, this noise is moderated through the research design by being carefully filtered with the consideration of liquidity and various cost issues, and this noise is also considered when interpreting the research results.

There are three efficiency market hypotheses around no arbitrage profit in this study. The first hypothesis is that there are no significant violations from the lower boundary conditions. This means any participant in the OMX market can not make risk-free profit by investing in hedges, which comprise of fully mutual covered option contracts and future contracts. The result shows no more than 1% of violations exist in the market when no transaction cost is taken into account. This figure drops close to zero when spread cost is considered. Given the noise of the data non-synchronization, a small amount of violation is inevitable, therefore the hypothesis is failed to be rejected, and we conclude that OMX market is efficient from lower boundary conditions.

The second hypothesis is that there are no significant violations from the Put Call Parity Condition. This condition is based on the long and short hedges, which includes different position of the puts and calls, as well as buying and selling the related futures. The puts and calls must have the same expiration date and the same strike price, and have to be traded at the same time. The options shall also be paired with futures with the same trading time, strike price and maturity time. The test result shows great sensitivity towards spread cost. When no cost is considered, the violation is around 54% (45%) for long and short hedge, while it falls to 1.25% (0.61%) when spread cost is considered.

This result is consistent with a number of researches in other markets. The total number of violations is only 600 out of total sample 65,482 and the average profit is about 2 index points, which could be easily interpreted as the effect of noise of the data and the additional cost of the hedge. Therefore, again, this second hypothesis is failed to be rejected.

The third hypothesis is that there is no arbitrage profit by investing in risk neutral (delta neutral, in practice) dynamic hedges. The well-known Black-Scholes Model (BS Model) is a model based on the above hypothesis to generate a "reasonable" options price. We compare the market price with the model generated price, although they are highly correlated with very small mean value of the difference, the difference is still significant. However, as the model neglects a number of important market limitations and cost issues, the difference is inevitable. Therefore, to further explore the hypothesis, we simulate the dynamic hedging directly with ex-ante approach. The simulation uses the BS Model to detect possible over priced or under priced candidate options. Once a mis-priced is detected, options are sold or bought on the following trading day. Meanwhile, futures are traded to neutralize the risk with the guidance of "delta". This hedge is held until expiration, and futures contracts are rebalanced on a daily basis with the delta hedge ratio. The result shows that over the last ten years, small arbitrage profit exist when little cost is considered, and all hedges are systematically losing money when spread cost is considered. Therefore, the research indicates that the cost of dynamic hedging is high enough to prevent any market participant to consider making profit by investing in risk neutral dynamic hedges. This result also positively supports our hypothesis that we do not reject null hypothesis, the market is efficient from dynamic hedging strategy result.

During the dynamic hedging test, the issue of volatility forecasting is also extensively discussed, as it is the key factor to determine "what is a risk-free hedge." This is a subject with enormous studies which suggest dozens of methodologies. Therefore, to narrow the scope of the research, we only select two mainstream methodologies, namely Historical Volatility (HSD) and Weighted Implied Volatility (WISD). The result shows that both HSD and WISD well illustrate the risk and the return of the OMX index market.

However, the WISD is clearly outperforming the HSD as a volatility indicator. This conclusion is also confirmed by a number of researches in other markets.

Recommendation of Further Research

There are a few issues we would like to address for further research of this topic.

First, although we had used some techniques to moderate the noise of data non-synchronization, the problem still obviously exists. The only way to clear this problem is to use intra-day data, as Cavallo and Mammola did for MIB30 in 2000. Therefore, in future research, we should try to obtain intra-day data, and run all our test procedures again. The time span may not be as long as 10 years, but one year should be the minimum.

Second, in the family of volatility forecasting, another important school, GARCH (Generalised Autoregressive Conditional Heteroskedastic), has not been discussed in the research. This has been recommended by a number of researches as a preferred volatility forecasting method. Therefore, we also would like to apply GARCH into the dynamic hedging test. We will not only see the NPV of the hedging return, but also compare its predictability with that of HSD and WISD.

Finally, although the commission cost plays a big role in all hedges, we are unable to clearly quantify it in this research. Despite the difficulty of knowing the commission cost of the market and the actual variation of the commission cost we believe it is still possible to gain a rough overview about the commission cost structure. This overview will guide us to run the test with more meaningful sensitive analysis, and also increase the credibility of the test result.

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Appendixes

Appendix 1. Annual Activity in Stockholm Derivatives Market

Annual Activity in Stockholm Derivatives Markets (1994-2003)
Source: www.omxgroup.com

Volume in number of contracts	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
Stock options	10,055,373	12,807,778	12,920,145	19,485,816	20,589,273	26,824,117	30,691,587	34,729,075	35,795,942	43,098,768
Stock futures	208,386	92,389	272,514	288,841	533,508	1,129,453	2,144,767	1,468,018	1,290,181	1,424,890
Stock loans	29,912	10,226	16,151	14,549	24,066	3,228				
OMX options	5,812,435	6,067,268	5,399,227	3,545,967	4,947,486	5,733,106	4,167,448	4,587,544	4,916,726	6,371,381
OMX futures	1,706,984	1,593,408	1,625,391	2,163,560	9,265,510	11,931,352	11,477,162	14,906,505	13,331,795	14,567,900
OMSX options			274							
OMSX futures			130							
VOLX futures			10							
FTSE 250 options	2,140									
FTSE 250 futures	9,565									
NOX options						1,554	40			
NOX futures						234	60			
Total Swedish Stock related products	**17,824,795**	**20,571,069**	**20,233,842**	**25,498,733**	**35,359,843**	**45,623,044**	**48,481,064**	**55,691,142**	**55,334,644**	**65,462,939**
Number of trading days	**253**	**251**	**251**	**249**	**250**	**252**	**251**	**250**	**250**	**249**
Average daily volume	**70,454**	**81,956**	**80,613**	**102,405**	**141,439**	**181,044**	**193,152**	**222,765**	**221,339**	**262,903**

Appendix 2. OMX Options Specifications

OMX-Options (Options in the Swedish OMX share index)
Source: www.omxgroup.com

Type of Contract	Standardized Options Contracts with Cash Settlement.
Style of Options	European option.
Contract Base	The OMX share index.
Indexer	Stockholmsbörsen AB (the Exchange).
Index Ombudsman	An independent person or organization appointed by the Exchange, presently KPMG.
Exercise Price	The index value (exercise index) contained in the series designation multiplied by one-hundred. Re-calculation of the Exercise Price may occur in certain cases in accordance with the section on Re-calculation.
Index Calculator	An independent person or organization appointed by the Exchange, presently SIX AB. The OMX share index is calculated continuously during the day at the Index Calculator through automated data retrieval from the Exchange and is provided to the Exchange by the Index Calculator. In the event of computer failure or lack of information from the Exchange or the Index Calculator, alternative data retrieval may be used. As a result of this, the frequency of calculations and reports may be altered. In "Addendum OMX" under the section "Terms for OMX index" are contained, in addition to definitions, the bases for calculation of the OMX share index, i.e. regulations regarding how adjustments shall be made in an issue, the payment of dividends, etc., under what circumstances shares can be excluded upon a calculation of the index as well as what documentation of the index shall exist.
The Index Ombudsman	The Index Calculator shall report corrections of the index value and the reasons for such to the Index Ombudsman. Current index values are definitive where the Index Calculator has not reported corrections or where objections have not been made as set forth below.

Objections to the index shall be made in writing to the Index Ombudsman no later than at 15.00 hours the second Bank Day following the day the index in question is provided to the Exchange by the Index Calculator. The grounds for the objection shall be contained in the application. A copy of the application shall be filed simultaneously with the Exchange and the Index Calculator.

Decisions of the Index Ombudsman concerning objections shall be taken no later than the second Bank Day following the day the objection is received by the Index Ombudsman. Exchange Members, Clearing Members and Customers shall accept the decisions of the Index Ombudsman and shall waive any right to file proceedings in lieu thereof. The Exchange and the Index Calculator shall be immediately informed of any decisions. To the extent that decisions of the Index Ombudsman state that another index value than that employed should have been employed, correction of the value in question shall take place immediately. The corrected value shall thereafter apply as the correct basis for the index for the time in question. The Index Ombudsman shall inform Exchange Members and Clearing Members for their own benefit and for the benefit of Customers regarding decisions and whether the Indexer has made a material correction of an index value or calculation basis.

Fix

Fix is comprised of a weighted index (average index) regarding the Expiration Day calculated in accordance with "The Terms for the OMX index". The decision in such regard shall be taken by the Index Calculator and shall be available at the latest at 15.00 hours on the Bank Day following the Expiration Day. The Counterparty shall accept decisions of the Index Calculator and shall waive any right to file proceedings in lieu thereof. The Exchange shall inform Exchange Members and Clearing Members for their own benefit and for the benefit of Customers regarding Fix.

Re-calculation

In the event of a planned index change other than that which is set forth in the terms for the index (deflation of the index or other similar event), the Index Ombudsman shall approve that re-calculation of the Exercise Index proposed by the Exchange or other change which may occur with regards to the planned index change. The Exchange shall, in a timely manner, inform Exchange Members and Clearing Members for their own benefit and for the benefit of Customers regarding the change.

Expiration Day	The fourth Friday of the Expiration Month of the Expiration Year, or where such day is not a Bank Day, the preceding Bank Day.
Expiration Month	The month listed in the series designation.
Expiration Year	The year listed in the series designation.
Premium	Agreed to by the parties. The premium shall be expressed in Swedish kronor and cover the price for one one-hundredth of an Options Contract.
Premium Settlement Day	The third Bank Day following Registration.
Tick size	The tick size is 0.01 where the Premium is less than 0.1; 0.05 where the Premium is greater than, or equal to, 0.1 but less than 4.0; and 0.25 where the Premium is greater than, or equal to, 4.0.
Final time for trading	The time for EMP's closing on the Expiration Day.
Final time for Registration	Application for Registration must be received by the Exchange not later than 120 minutes after EMP's normal closing on the Expiration Day.
Automatic Exercise	Cash Settlement shall occur for the option holder on the Expiration Day provided that the settled amount following the Exchange's fees in accordance with the Fee List is positive in favor of the Counterparty. Cash Settlement shall occur for the option issuer provided that the Exchange carries out Cash Settlement for the option holder in the same Series. Amounts payable by the Counterparty of such posts following the Exchange's fees in accordance with the Fee list shall be paid as Settlement.
Settlement	Payment of Settlement shall occur on the Expiration Settlement Day in accordance with the Exchange's instructions.
Final Settlement Day	The third Bank Day following the Expiration Day.
Setting-Off of Contracts	Setting-Off of Contracts may occur during the Term.
Listing	Exchange Listing as well as Clearing Listing.
Series Term	Three, six or twenty-four months.

Series Designation Each series shall be designated by the designation for the Contract Base, Expiration Year, exercise index, Expiration Month and Option Type.

Listing of Series Unless otherwise expressly stated, Series are listed in accordance with the provisions set forth in section 4.2.13.1. In such circumstances, the Exchange initially sets the exercise index for one Call Option Series and one Put Option Series at the value of the OMX stock index at the closing of trading on the immediately preceding Bank Day rounded off in accordance with the provisions adopted by the Exchange. Additional Series with the same Expiration Month are normally listed during the Term.

For Series with Terms of more than six months, at least two Put Option Series and two Call Option Series shall be listed on the First Listing Day. Normally, no new Series with the same Expiration Month are listed during the Term.

Fees Stock Index Options: Transaction and Closing = 3.50 SEK/contract.

Stock Index Futures: Transaction and Closing = 4.50 SEK/contract.

OMX Index: Stocks, Weighting and Business Sectors (as of 2nd of June 2003 fo the period July - December 2003)			
Source: www.omxgroup.com			
Company	**Market Cap. (MSEK)**	**Weight %**	**Business Sector**
Autoliv	11.458	0.84	
TOTAL	11.458	0.84	**Automobiles&Components**
Nordea	116.420	8.49	
SHB A	87.093	6.35	
FörenSparbank A	57.795	4.21	
SEB A	54.577	3.98	
TOTAL	315.885	23.02	**Banks**
Sandvik	51.742	3.77	
Volvo B	51.193	3.73	
Assa Abloy B	27.219	1.98	
Atlas Copco A	26.371	1.92	
SKF B	18.611	1.36	
Skanska B	16.081	1.17	
Atlas Copco B	12.337	0.90	
Alfa Laval	8.990	0.66	
ABB	8.474	0.62	
TOTAL	221.018	16.11	**Capital Goods**
Securitas B	27.659	2.02	
TOTAL	27.659	2.02	**Commercial Serv&Supplies**
Electrolux B	47.900	3.49	
TOTAL	47.900	3.49	**Consumer&Durables&Apparel**
Investor B	24.141	1.76	
TOTAL	24.141	1.76	**Diversified Financials**
Swedish Match	21.799	1.59	
TOTAL	21.799	1.59	**Food Beverage & Tobacco**
Skandia	22.825	1.66	
TOTAL	22.825	1.66	**Insurance**
SCA B	51.240	3.73	
Holmen B	12.129	0.88	
Stora Enso R	11.537	0.84	
TOTAL	74.906	5.46	**Materials**
Eniro	11.132	0.81	
TOTAL	11.132	0.81	**Media**
AstraZeneca	124.716	9.09	
TOTAL	124.716	9.09	**Pharmaceuticals&Biotechnol.**
Drott B	7.904	0.58	
TOTAL	7.904	0.58	**Real Estate**
Hennes & M. B	126.713	9.24	
TOTAL	126.713	9.24	**Retailing**
Ericsson B	126.904	9.25	
Nokia	21.849	1.59	
TOTAL	148.753	10.84	**Technology Hardware&Equip.**
TeliaSonera	148.205	10.80	
Tele2 B	36.976	2.70	
TOTAL	185.181	13.50	**Telecommunication Services**
TOTAL	1,371.989	100.00	

Appendix 3. OMX Index Composition (2003)

Appendix 4. Leading European Derivatives Markets (2003)

Source: www.omxgroup.com

LEADING EUROPEAN DERIVATIVE EXCHANGES

Stock Related Products, options and futures

Ranking	Derivative Exchange [1)]	Total No. of contracts 2,003
1	Eurex	452,732,788
2	Euronext	*412,571,153*
3	Stockholmsbörsen	65,462,939
4	Spanish Exchanges (BME)	30,399,095
5	Borsa Italiana	17,731,994
6	Athens Derivatives Exchange	4,767,299
7	Warszawa	4,378,910
8	Oslo Børs	3,386,871
9	HEX Helsinki	2,007,309
10	Wiener Börse	1,392,601
11	Budapest Stock Exchange	983,512
12	Copenhagen Stock Exchange	768,893

Interest Rate Products, options and futures

Ranking	Derivative Exchange [1)]	Total No. of contracts 2,003
1	Eurex	562,199,527
2	Euronext	*275,824,644*
3	Stockholmsbörsen	6,674,408
4	Athens Derivatives Exchange	66,260
5	Spanish Exchanges (BME)	1,382
6	Budapest Stock Exchange	100

1) Members of World Federation of Exchanges

Source: World Federation of Exchanges

Appendix 5. Black-Scholes Model Derivation

BLACK-SCHOLES PARTIAL DIFFERENTIAL EQUATION (Hull, 2003)

The Black-Scholes model assumes that stock prices move according to a stochastic process known as a geometric Brownian motion, which is a particular type of a Markov Process. This means that our predictions for the future should be unaffected by the price movements in the past. This property is consistent with the weak form of Market Efficiency, which means that the present price of a stock impounds all the information contained in a record of past prices.

1. **Behavior of Stock prices**

Wiener Process

A Wiener process has a mean change of zero and a variance rate of 1 per year. It is also known as Brownian motion.

A variable z that follows a Wiener process has two properties:

The change δz during a small period of time δt is

$$\delta z = \varepsilon\sqrt{\delta t} \qquad 1.$$

where ε is a random drawing from a standardized normal distribution, φ(0,1)

This property implies that δz has a normal distribution with

Mean = 0
Standard Deviation = $\sqrt{\delta t}$
Variation = δt

The values of δz for any two different short intervals of time δt are independent.

The second property implies that z follows a Markov process.

Generalized Wiener Process

The basic Wiener process defined so far has a drift rate of zero (this means that the expected value of z at any future time is equal to its current value) and a variance rate of 1 (which means that the variance of the change in z in a time interval of length T equals T).

A generalized Wiener process for a variable *x* can be defined in terms of *dz* as follows:

$$dx = a \cdot dt + b \cdot dz \qquad 2.$$

where *a* and *b* are constants.

The term $a \cdot dt$ means that the variable x has an expected drift rate of *a,* per unit of time.
The term $b \cdot dt$ can be regarded as adding noise or variability to the path followed by the variable *x*. The noise is *b* times a Wiener process. Therefore, *b* times a Wiener process has a standard deviation of *b*. In a small time interval *δt,* the change *δx* in the value of *x* is given by equations 1 and 2 as:

$$\delta x = a\delta t + b\varepsilon\sqrt{\delta t} \qquad 3.$$

Thus, the generalized Wiener process given in equation 3, has an expected drift rate of *a* and a variance rate of b^2 .

Ito Process

Another stochastic process, known as the Ito process, follows a generalized Wiener process in which the parameters *a* and *b* are functions of the value of the underlying variable *x* and time *t*. It can be written as:

$$dx = a(x,t)dt + b(x,t)dz \qquad 4.$$

Both the expected drift rate and variance rate of an Ito process are liable to change over time. In a small time interval between t and $t + \delta t$, the time changes from x to $x + \delta x$, where

$$\delta x = a(x,t)\delta t + b(x,t)\varepsilon\sqrt{\delta t}$$

It assumes that the drift and variance of x remain constant, equal to $a(x,t)$ and $b(x,t)^2$, respectively, during the time interval between t and $t + \delta t$.

2. Process of Stock Prices

The generalized Wiener process has a constant expected drift rate and a constant variance rate. This model would fail to represent the stock prices. That is, the expected percentage return required by investors from a stock is independent of the stock's price.

Therefore, the constant expected drift-rate assumption is inappropriate and needs to be replaced by the assumption that the expected return (i.e. expected drift divided by the stock price) is constant.

If S is the stock price at time t, the expected drift rate in S should be assumed to be μS for some constant parameter μ. This means that in a short interval of time, δt, the expected increase in S is $\mu S\delta t$. The parameter μ is the expected rate of return on the stock, expressed in decimal form.

If the volatility of the stock price is always cero, the model is:

$$\delta S = \mu S\delta t$$

In the limit as $\delta t \rightarrow 0$

$$dS = \mu S dt$$

Or

$$\frac{dS}{S} = \mu dt$$

Integrating between time cero and T, we get

$$S_T = S_0 e^{\mu T} \qquad 5.$$

Where, S_0 and S_T are the stock price at time zero and time T. The equation shows that when the variance rate is zero, the stock price grows at a continuous compounded rate of μ per unit of time.

In practice, a stock price shows volatility, assuming that the variability of the percentage return in a short period of time, δt, is the same regardless of the stock price. Therefore the standard deviation of the change in a short period of time δt should be proportional to the stock price, the following model is obtained:

$$dS = \mu S dt + \sigma S dz \qquad 6.$$

Or

$$\frac{dS}{S} = \mu dt + \sigma dz \qquad 7.$$

The variable σ is the volatility of the stock price and μ is its expected return.

3. Ito´s Lemma

The price of any derivative is a function of the stochastic variables underlying the derivative and time.
Suppose that the value of a variable x follows an Ito process

$$dx = a(x,t)dt + b(x,t)dz \qquad 8.$$

Where dz is a Wiener process and a and b are functions of x and t. The variable x has a drift rate of a and a variance of b^2. Ito´s Lemma shows that a function G of x and t follows the process:

$$dG = \left(\frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} b^2 \right) dt + \frac{\partial G}{\partial x} b \cdot dz \qquad 9.$$

where the dz is the same Wiener process as in equation 7. Thus G also follows an Ito process. It has a drift rate of

$$\frac{\partial G}{\partial x}a+\frac{\partial G}{\partial t}+\frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2$$

and a variance rate of

$$\left(\frac{\partial G}{\partial x}\right)^2 b^2$$

From Ito's Lemma follows that the process followed by a function G of S and t is

$$dG=\left(\frac{\partial G}{\partial S}\mu S+\frac{\partial G}{\partial t}+\frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2\right)dt+\frac{\partial G}{\partial S}\sigma S dz \qquad 10.$$

Both S and G are affected by the same underlying source of uncertainty, dz

4. Derivation of Black-Scholes-Merton Differential Equation

The stock price process assumed is the one developed in the formula $dS=\mu S dt+\sigma S dz$

Supposing that f is the price of a call option or other derivative contingent on S. The variable f must be some function of S and t. And using equation 10.

$$df=\left(\frac{\partial f}{\partial S}\mu S+\frac{\partial f}{\partial t}+\frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt+\frac{\partial f}{\partial S}\sigma S dz \qquad 11.$$

The discrete versions of the formulae 6 and 10

$$\delta S=\mu S\delta t+\sigma S\delta z \qquad 12.$$

$$\delta f=\left(\frac{\partial f}{\partial S}\mu S+\frac{\partial f}{\partial t}+\frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\delta t+\frac{\partial f}{\partial S}\sigma S\delta z \qquad 13.$$

where δS and δf are the changes in S and f in a small time interval δt . The Wiener process underlying S and f is the same, (in equations 12 and 13 the δz $(= \varepsilon\sqrt{\delta t}\,),$ are the same). It follows that, by choosing a portfolio of the stock and the derivative, the Wiener process can be eliminated.

The portfolio contains:

-1: Derivative

$+\frac{\partial f}{\partial S}$: Shares

The holder of the portfolio is short one derivative and long an amount $\frac{\partial f}{\partial S}$ shares.

Define Π as the value of the portfolio. By definition,

$$\Pi = -f + \frac{\partial f}{\partial S} S \qquad 14.$$

The change $\delta\Pi$ in the value of the portfolio in the time interval δt is given by

$$\delta\Pi = -\delta f + \frac{\partial f}{\partial S}\delta S \qquad 15.$$

Substituting equations 12 and 13 into 15 yields

$$\delta\Pi = \left(-\frac{\partial f}{\partial t} - \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2 \right)\delta t \qquad 16.$$

Because, this equation does not involve δz, we have eliminated the Wiener process and the portfolio must be riskless during the time δt. This implies that the portfolio should earn the same return as any other risk-free security.

It follows that

$$\delta\Pi = r\Pi\delta t$$

where r is the risk-free interest rate.

Substituting from equations 14 and 15 we obtain

$$\left(\frac{\partial f}{\partial t}+\frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\delta t = r\left(f-\frac{\partial f}{\partial S}S\right)\delta t$$

The Black-Scholes-Merton differential equation is the obtained:

$$\frac{\partial f}{\partial t}+rS\frac{\partial f}{\partial S}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2}=rf \qquad 17.$$

This equations have many solutions and can be used for any derivative that has S as the underlying variable. Depending on the boundary conditions imposed at the values of S and t.

For European call option, the key boundary condition is,

$f = \max(S - K, 0)$ when t = T

For European put option,

$f = \max(K - S, 0)$ when t = T

Appendix 6. Derivation of Lower Boundary for Call and Put Options

1. Derivation of call options price boundary:

The basic conditions for call options is $C \geq S_0 - Ke^{-rt}$ If this condition is violated, arbitrage profit can be made by buying call option and short selling the stocks.

Table 1. Derivation of Call Option Price Boundary

Stock												
Begin			St>K					St<K				
						Total					Total	
	Stock Trade	Cash Flow		Stock Trade	Cash Flow	Stock Hold	NPV		Stock Hold	Cash Flow	Stock Hold	NPV
Buy Call		-C-TCbc	Excise Call		(St-k)			Give up Call	0	0		
Short Stock	1	S0-TCss	Buy Stock	-1	-St-TCbs			Buy Stock	-1	-St		
SubTotal	**1**	-C+S0-TCbc-TCss		-1	-k-TCbs	0	S0-c-K*e-rt-TCbc-TCss-TCbs*e-rt			K	0	**S0-c-K*e-rt-TCbc-TCss-TCbs*e-rt**
Future												
Begin			St>K					St<K				
						Total					Total	
	Future	Cash Flow		Future	Cash Flow	Futrue	NPV		Future	Cash Flow	Future	NPV
Buy Call		-c-TCbc	Excise Call		(St-k)			Give up Call	0	0		
Short Future	1	0-TCsf	Clear Future	-1	-(St-F)	0		Clear Future	-1	-(St-F)		
SubTotal	**1**	-c-TCbc-TCsf		-1	F-K	0	-C+Fe-rt -Ke-rt-TCbc-TCsf		-1	F-K	0	-C+Fe-rt -Ke-rt-TCbc-TCsf

Therefore, the lower boundary for EU Call options condition is:

$$-C - Ke^{-rt} + F_0 e^{-rt} \leq TC_{bc} + TC_{sf} + T_k \qquad \textbf{E.q. 6.5}$$

2. Derivation of put options price boundary

Table 2. Derivation of Put Options Price Boundary

Stock												
Begin			St>K					St<K				
						Total					Total	
	Stock Trade	Cash Flow		Stock Trade	Cash Flow	Stock Hold	NPV		Stock Hold	Cash Flow	Stock Hold	NPV
Buy put		-p-TCbp	Give up Put		0			Exercise Put	0	K-St		
buy Stock	1	-S0-TCbs	Sell Stock	-1	St			Sell Stock	-1	St-TCss		
SubTotal	**1**	**-p-S0-TCbp-TCbs**		-1	**k-TCss**	**0**	**-P-TCbp-TCbs +Ke-rt -TCss*e-rt**			**k-TCss**	**0**	**-P-TCbp-TCbs +Ke-rt -TCss*e-rt**

Future												
Begin			St>K					St<K				
						Total					Total	
	Future	Cash Flow		Future	Cash Flow	Futrue	NPV		Future	Cash Flow	Future	NPV
Buy Put		-P-TCbp	Give up put		0			Exercise put	0	K-St		
Buy Future	1	0-TCbf	Clear Future	-1	(St-F)	0		Clear Future	-1	(St-F)		
SubTotal	**1**	**-P-TCbp-TCsf**		**-1**	**K-F**	**0**	**-P-TCbp-TCsf + Ke-rt -Fe-rt**		**-1**	**K-F**	**0**	**-C+Fe-rt -Ke-rt-TCbp-TCsf**

Therefore, the lower boundary for EU Call options condition is:

$$-P + K_0 e^{-rt} - F_0 e^{-rt} \le TC_{bp} + TC_{bf} + T_k$$ **E.q 6.6**

Appendix 7. Derivation of Put Call Parity Conditions

The basic principle of put call parity indicates that EU call with a certain exercise date and price can be deduced from the value of a European put with the same exercise price and exercise date, therefore,

$$C + Ke^{-rt} = P + S_0$$

If this condition can not be satisfied, there are two ways to make arbitrage profit, namely, through long hedge and short hedge.

1. Long hedge

when $C + Ke^{-rt} > P + S_{0,}$ arbitrage profit can be obtained by selling call options, buying put and stocks or futures

Table 1. Derivation for Long Hedge Condition

Stock												
Begin			St>K					St<K				
						Total					Total	
	Stock Trade	Cash Flow		Stock Trade	Cash Flow	Stock Hold	NPV		Stock Hold	Cash Flow	Stock Hold	NPV
Sell Call		c-TCsc	Excise Call		-(St-k)			Give up Call	0	0		
Buy Put		-p-TCbp	Give up Put		0			Execise Put		K-St		
Buy Stock	1	-S0-TCbs	Sell Stock	-1	St-TCss			Sell Stock	-1	St		
SubTotal	**1**	**-S0+c-p-TCsc-TCbp-TCbs**		**-1**	**k-TCss**	**0**	**-S0+c-p+Ke-rt-TCsc-TCbp-TCbs-TCss*e-rt**			**k-TCss**	**0**	**-S0+c-p+Ke-rt-TCsc-TCbp-TCbs-TCss*e-rt**
Future												
Begin			St>K					St<K				
						Total					Total	
	Future	Cash Flow		Future	Cash Flow	Futrue	NPV		Future	Cash Flow	Future	NPV
Sell Call		c-TCsc	Excise Call		-(St-k)			Give up Call	0	0		
Buy Put		-p-TCbp	Give up Put		0			Execise Put		K-St		
Buy Future	1	-TCbf	Clear Future	-1	St-F	0		Clear Future	-1	St-F		
SubTotal	**1**	**c-p-TCsc-TCbp-TCbf**		**-1**	**K-F**	**0**	**c-p+(K-F0)e-rt-TCsc-TCbp-TCbf**		**-1**	**K-F**	**0**	**c-p+(K-F0)e-rt-TCsc-TCbp-TCbf**

Therefore, the long hedge condition for put call parity EMH is:

$$C - P - F_0 e^{-rt} + Ke^{-rt} \leq TC_{SC} + TC_{bp} + TC_{bf} + Tk \qquad \textbf{E.q 6.15}$$

Table 2. Derivation for Short Hedge Condition

Stock												
Begin			St>K					St<K				
						Total					Total	
	Stock Trade	Cash Flow		Stock Trade	Cash Flow	Stock Hold	NPV		Stock Hold	Cash Flow	Stock Hold	NPV
Buy Call		-c-TCbc	Excise Call		St-k			Give up Call	0	0		
Sell Put		+p-TCsp	Give up Put		0			Execise Put		-(K-St)		
Sell Stock	1	S0-TCss	Buy Stock	-1	-St-Tcbs			Buy Stock	-1	-St-Tcbs		
SubTotal	1	S0-c+p-TCbc-TCsp-TCss		-1	-k-TCbs	0	**S0-c+p--Ke-rt-TCbc-TCsp-TCss-TCbs*e-rt**			-k-TCbs	0	**S0-c+p--Ke-rt-TCbc-TCsp-TCss-TCbs*e-rt**
Future												
Begin			St>K					St<K				
						Total					Total	
	Future	Cash Flow		Future	Cash Flow	Futrue	NPV		Future	Cash Flow	Future	NPV
Buy Call		-c-TCbc	Excise Call		St-k			Give up Call	0	0		
Sell Put		+p-TCsp	Give up Put		0			Execise Put		-(K-St)		
Sell Future	1	0-TCsf	Clear Future	-1	-(St-F)	0		Clear Future	-1	-(St-F)		
SubTotal	1	-c+p-TCbc-TCsp-TCsf		-1	F-K	0	**F0e-rt-c+p-Ke-rt-TCbc-TCsp-TCsf**		-1	F-K	0	**F0e-rt-c+p-Ke-rt-TCbc-TCsp-TCsf**

Therefore, the short hedge condition for put call parity EMH is:

$$P - C + F_0 e^{-rt} - Ke^{-rt} \leq TC_{sp} + TC_{bc} + TC_{sf} + Tk \qquad \textbf{E.q 6.16}$$

Appendix 8. Queries for Volatility and BS Price Calculation

qryHSD_01MakeHSD: Generate HSD

```
SELECT      StDev(OMXIndex_1.Rj)*15.81139 AS StDevOfRj
FROM   OMXIndex AS OMXIndex_1 INNER JOIN OMXIndex
ON          ([OMXIndex].[Day]-OMXIndex_1.Day>=0)
            AND
            ([OMXIndex].[Day]-OMXIndex_1.Day<=19)
GROUP BY    [omxindex].[bankday]
```

qryBS_09WISD_SP1_index: first step of generating WISD

```
SELECT      Cl_Alloptions.BankDay,
            Sum([F_vega]*([f_iv]/(([ask]+[bid])/2))) AS WISD_sp1
FROM   Cl_Alloptions
WHERE       (((Cl_Alloptions.F_Vega) Is Not Null))
GROUP BY    Cl_Alloptions.BankDay
```

qryBS_10WISD_index: second step of generating WISD

```
SELECT      qryBS_09WISD_SP1_index.BankDay,
            Sum((([f_vega]*([f_iv]/(([ask]+[bid])/2)))/[wisd_sp1])*[f_iv]) AS WISD
FROM   Cl_Alloptions INNER JOIN qryBS_09WISD_SP1_index
ON          Cl_Alloptions.BankDay = qryBS_09WISD_SP1_index.BankDay
WHERE       (((Cl_Alloptions.F_Vega) Is Not Null And (Cl_Alloptions.F_Vega)<>0))
GROUP BY    qryBS_09WISD_SP1_index.BankDay
```

qryBS_12SetBSPriceCall: Generate BS Call Price with WISD

```
UPDATE      Cl_Alloptions
INNER JOIN  OMXIndex
ON          Cl_Alloptions.BankDay = OMXIndex.Predict_day
SET         Cl_Alloptions.F_BS_Price
            =CallOption([f_AB]*[ert],[strike],[t0]/365,[cl_alloptions].[interestrate]/100,[omxindex].[F
            _wisd])
WHERE       (((OMXIndex.F_wisd) Is Not Null) AND ((Cl_Alloptions.CP)="call"));
```

qryBS_12SetBSPriceCall: Generate BS Put Price with WISD

```
UPDATE      Cl_Alloptions
INNER JOIN  OMXIndex
ON          Cl_Alloptions.BankDay = OMXIndex.Predict_day
SET         Cl_Alloptions.F_BS_Price
             =
       putOption([f_AB]*[ert],[strike],[t0]/365,[cl_alloptions].[interestrate]/100,[omxindex].[F_wisd])
WHERE       (((OMXIndex.F_wisd) Is Not Null) AND ((Cl_Alloptions.CP)="put"));
```

Appendix 9. VB Functions for Dynamic Hedging Test

```
Public Function dOne(stock, exercise, time, interest, sigma) 'by Maja Sliwinski
   dOne = (Log(stock / exercise) + interest * time) / (sigma * Sqr(time)) _
      + 0.5 * sigma * Sqr(time)
End Function

Public Function CallOption(stock, exercise, time, interest, sigma) 'by Maja Sliwinski
   CallOption = stock * Excel.Application.NormSDist(dOne(stock, exercise, _
      time, interest, sigma)) - exercise * Exp(-time * interest) * _
    Excel.Application.NormSDist(dOne(stock, exercise, time, interest, sigma) _
    - sigma * Sqr(time))
End Function

Public Function PutOption(stock, exercise, time, interest, sigma) 'by Maja Sliwinski
   PutOption = CallOption(stock, exercise, time, interest, sigma) _
       + exercise * Exp(-interest * time) - stock
End Function

Public Function CallVolatility(stock, exercise, time, interest, target) 'by Maja Sliwinski
   High = 2
   Low = 0
   Do While (High - Low) > 0.000001
   If CallOption(stock, exercise, time, interest, (High + Low) / 2) > _
      target Then
          High = (High + Low) / 2
          Else: Low = (High + Low) / 2
   End If
   Loop
   CallVolatility = (High + Low) / 2
End Function

 Public Function PutVolatility(stock, exercise, time, interest, target) 'by Maja Sliwinski
   High = 2
   Low = 0
   Do While (High - Low) > 0.0001
   If PutOption(stock, exercise, time, interest, (High + Low) / 2) > _
      target Then
          High = (High + Low) / 2
          Else: Low = (High + Low) / 2
   End If
   Loop
   PutVolatility = (High + Low) / 2
End Function

Public Function vega(stock, exercise, time, interest, target, sigma)
           vega = stock * (time ^ 0.5) * (Exp(-dOne(stock, exercise, time, interest, sigma) ^_
           2 / 2)) / ((2 * 3.1415926535) ^ 0.5)
End Function

Public Function Delta(cp, stock, exercise, time, interest, sigma)
           Delta = Excel.Application.NormSDist(dOne(stock, exercise, time, interest, sigma))
           If cp = "put" Then Delta = Delta - 1
End Function
```