



Research Report
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Research Report 2001:4
ISSN 0349-8034

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ON SEASONAL FILTERS AND MONOTONICITY

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ABSTRACT

Seasonal adjustment is important in for example economic time series where the variation can be due to both seasonal and cyclical movements. In a situation where we want to detect a turning point of a cyclical process exhibiting seasonal variation, it is very important that the seasonal adjustment does not adversely affect the ability to detect the turning points. Thus, it is important that the seasonal adjustment does not alter the monotonicity. In this report, seasonal adjustment using differentiation and moving average methods is analyzed with respect to the effect on turning points.

Key words: Seasonal adjustment; Moving average; Differentiation; Monotonicity; Unimodality; Turning point.

1 INTRODUCTION

Timely prediction of a turn in the business cycle is important for government as well as industry. The turn is a change from a phase of recession to one of expansion (or vice versa). By applying a system for detection of the turning points of a leading indicator, we can receive early signals about the future behavior of the business cycle. Since quick detection is important, monthly or quarterly observations are used rather than yearly data. However, monthly and quarterly data often have seasonal variations. Thus data must be adjusted for seasonality in order to distinguish the cycles. The problem of seasonality in economic time series and the evaluation of methods for its adjustment has for a long time been of great interest, see Grether and Nerlove (1970), Granger (1978), Bell and Hillmer (1984) and Ghysel and Perron (1996). The choice of method depends on the users' loss function. Grether and Nerlove (1970) use minimum mean square error as a criterion of optimality. When it is important that the times of the turning points in the cycle are preserved also other criteria such as minimum delay are relevant. In research on business cycles, there are many suggested methods for on-line turning-point detection, where the aim is to detect the next turning point as soon as possible, see Neftci (1982), Hamilton (1989), Frisé (1994), Koskinen and Öller (1998), Birchenhall et al. (1999) and Andersson et al. (2001). At each new time point (e.g. each month) a new observation is made and a new decision has to be made as to whether the process has reached a turning point or not. Since this involves repeated decisions, the methodology of statistical surveillance is appropriate, see Shiryaev (1963), Frisé and de Maré (1991) and Srivastava and Wu (1993). In this paper the

effect of seasonal adjustment is examined with emphasis on a surveillance situation. In a turning point detection context, the problem of whether seasonal adjustment alters the turning point times of the cycles is of special interest. Therefore, an important aspect of seasonal adjustment is the monotonicity of the adjusted series. The altering of the turning point may be due to certain smoothing and/or lagging properties of the seasonal adjustment procedure.

The paper is organized as follows. Section 2 contains a review of different methods for seasonal adjustment. Section 3 contains a review on earlier investigations concerning the effect of seasonal adjustment on change point analyses. Further in Section 3, the effects of seasonal adjustment by moving average methods and differentiation are analyzed with regard to their effects on change point detection. The monotonicity preserving properties of moving average methods are analyzed. The question of whether the seasonally adjusted series preserves the time of the turning points in the cycles is treated. The effect of using seasonally adjusted data in a surveillance system is investigated, with special emphasis on the delay of a change point indication. Section 4 contains a summary and discussion.

2 MODELS AND METHODS IN SEASONAL ADJUSTMENT

Seasonal variation in a time series is not often defined rigorously, but Wallis (1974) gives examples of some more explicit statements. A common description is that seasonal variation is fluctuations that are periodical with a period of one year. Monthly or quarterly data often contain seasonal variation, which can be considerable, as can be seen from Figure 1. If seasonality is neglected in the modeling and in a surveillance situation it could lead to serious wrong conclusions, therefore seasonal adjustment must be made. However, when choosing the adjustment method it is important to consider the effect on the structure of the original series. Grether and Nerlove (1970) express this by requiring that the method should remove the peaks which appear at the seasonal frequencies in the original series, but should affect the remainder of the spectral densities as little as possible. Granger (1978) expresses this by stating that a desirable property of the adjustment procedure is that it should leave non-seasonal time series unaffected.

In Nerlove (1964), economic time series are described in the frequency domain. It is shown that a slowly changing seasonal pattern or a stochastic seasonal pattern will reveal itself in the spectrum. Several investigated methods for seasonal adjustment eliminate more than the seasonality and produce several phase shifts at lower frequencies.

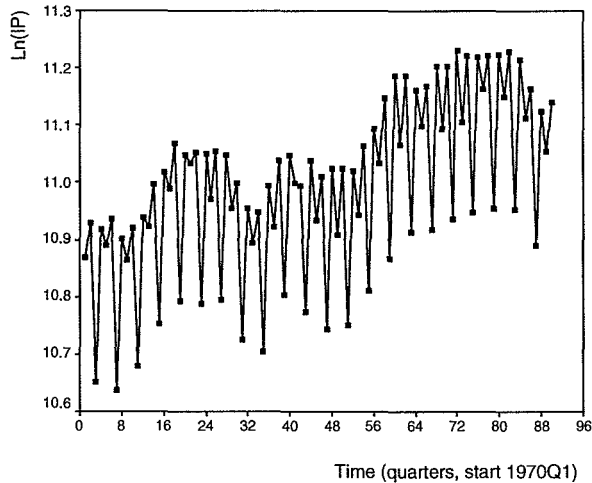


Figure 1: *Industrial production, quarterly data (1970Q1: 1992Q2).*

Akaike (1980) categorized approaches to seasonal adjustment of time series into three classes: methods based on moving averages, methods based on multiple regression and methods based on time series models. Sometimes a categorization is made between empirical and model-based approaches, where e.g. procedures based on moving averages or differentiation are considered empirical as the seasonal components are eliminated directly from the data at hand without an explicit model. The two latter classes are considered model-based in the sense that they are based on a clearly defined model of the time series.

In seasonal adjustment methods based on multiple regression parametric functions are used to model the trend and seasonal components at time t , see Harvey (1993).

The components of a time series can be modeled as additive, multiplicative or a mixture of additive and multiplicative components. When using a completely additive (multiplicative) model, one common approach is to assume that the time series consists of a sum (product) of components (trend-cycle, season and an irregular term). The task is then to separate these three components and to eliminate the seasonal by subtraction (division). With an additive model, the components can be estimated using for example ordinary regression technique. With a completely multiplicative model one common estimation procedure is to make the model additive by a logarithmic transformation and then proceed as for the additive model. For a model with both multiplicative and additive components, a simple iterative procedure was suggested by Frisén (1979).

2.1 Constant seasonal pattern

The most commonly used model is

$$Y(t) = \mu(t) + S(t) + \varepsilon(t) \quad (1)$$

where ε is a stationary stochastic process, often assumed to be iid $N(0; \sigma^2)$.

For the interpretation of S as a constant seasonal component with s seasons, we need

$$\sum_{k=1}^s S(t+k) = 0, \text{ for any integer } t,$$

while there exists no further decomposition

$$\mu(t) = X(t) + Z(t),$$

with $Z(t) \neq 0$ for some t

$$\text{and } \sum_{k=1}^s Z(t+k) = 0, \text{ for all integers } t.$$

Different models can be assumed for $S(t)$ and $\mu(t)$ in (1). If a constant seasonal pattern is assumed along with parametric functions for $\mu(t)$ and $S(t)$, then linear regression can be used to estimate the components. One example of a parametric model for S is the trigonometric one

$$S(t) = \sum_{i=1}^{s/2} (a_i \cos(\lambda_i \cdot t) + b_i \sin(\lambda_i \cdot t)),$$

$$t = \{1, 2, \dots\},$$

$$s = \# \text{ seasons},$$

$$a_i, \lambda_i, b_i \text{ are constants.}$$

This model is sometimes called harmonic regression model.

Seasonal adjustment of model (1) can be made by

$$Y(t) - \hat{S}(t), \text{ where } \hat{S}(t) \text{ is an estimator.}$$

Another approach for seasonal adjustment is seasonal differentiating, considered by Yule (1926). For recent use of seasonal differentiating, see Öller (1986) and Öller and Tallbom (1996).

Seasonal adjustment can also be made by estimating the μ -component by applying moving averages. One method based on moving averages is the well-known X-11 method, see Shiskin et al. (1967). A moving average can be both centered and non-centered and the use of moving average techniques are suited for time series where it is assumed that the seasonal component is constant over time or at least that any possible change is very slow.

2.2 Stochastic seasonal pattern

A large class of models where the components are stochastic are structural models, see Harvey (1993). Structural time series models start from an additive model where all components are stochastic. Engle (1992) gives an example of a structural model

$$\mu(t) = \beta_1 \mu(t-1) + \beta_2 \mu(t-2) + \omega(t), \omega \sim \text{iid } N(0; \sigma^2_1),$$

$$S(t) = \alpha S(t-4) + \xi(t), \xi \sim \text{iid } N(0; \sigma^2_2),$$

$$\varepsilon \sim \text{iid } N(0; \sigma^2_3),$$

$$\omega, \xi, \varepsilon \text{ are independent of each other,}$$

where a Kalman filter procedure was used to estimate the regression parameters (α, β_1, β_2) and the variances. A maximization algorithm was used, which provides a set of recursive formulas for calculating the mean and variance of the unobserved components at each time, conditional on a particular set of information.

Young et al. (1999) described time series analysis using dynamic harmonic regression, where the phase of the harmonic components can vary:

$$\mu(t) = \phi(t) + \omega(t), \omega \text{ is iid, } E[\omega]=0, \text{Var}[\omega]=\sigma^2,$$

$$S(t) = \sum_{i=1}^{R_s} \{a_{i,t} \cos(\lambda_i \cdot t) + b_{i,t} \sin(\lambda_i \cdot t)\},$$

where $a_{i,t}$ and $b_{i,t}$ are stochastic variables

and $\lambda_i, i = 1, \dots, R_s$, are the frequencies associated with the seasonality.

Filtering equations are used in the estimation procedure.

Time series models where each of the three components, $\{\mu(t), S(t), \varepsilon(t)\}$, are assumed to follow an ARIMA process have been suggested. When the stochastic structure of the components are known and by imposing certain restrictions on the ARIMA models for μ and S , Cleveland and Tiao (1976) demonstrated that weight functions, W , can be determined and that the components can be estimated as

$$\hat{S}(t) = W_S \cdot Y(t),$$

$$\hat{\mu}(t) = W_\mu \cdot Y(t).$$

If the stochastic structure is unknown a decomposition that uniquely determines S and μ can be made by putting certain restrictions on the components (Hillmer and Tiao (1982)).

3 THE EFFECT OF ADJUSTMENT METHODS ON TURNING POINTS

3.1 Seasonal adjustment and change point problems

The problem of altered change points by seasonal adjustment has been briefly discussed, by e.g. Ghysel and Perron (1996) and Franses and Paap (1999). However, often the effects of seasonal adjustment have been studied first after additional transformations have been applied (Öller (1986) and Öller and Tallbom (1996)).

A linear approximation of the Census X-11 method (see Young (1968)) based on moving averages, was investigated by Ghysel and Perron (1996) in relation to a test for structural change in a fixed sample. In the case of an abrupt level shift in a non-seasonal time series it was found that the magnitude of the discrete jump was reduced and a saw-toothed pattern appeared before and after the shift. The effect of the linear approximation to the Census X-11 method on size and power of the test for structural change (MacNeill (1978)) was investigated. The result was that the seasonal adjustment filter will give a test that is oversized and where the relative efficiency is lower for an early shift. The incorrect size is a result of the so called Slutsky-Yule effect (cited in Jorgenson (1964)), where the random component of the original series is independently distributed over time, but the random component of a moving average of this series is not. If the dependency structure is ignored, one result is an incorrect size. When testing for a shift, the sample is divided into one pre-shift sample and one post-shift sample and the parameters of pre-shift and post-shift are estimated. The small power for early shifts is a result of the small sample from which to estimate the pre-shift level.

In Franses and Paap (1999), a Hidden Markov model (HMM) with two states (1, 2) was used to analyze seasonally adjusted data. Prior to the HMM, the linear approximation of the Census X-11 was applied. The result was a positive bias of the estimator of the probability of staying in a particular regime. The dates of the shift

between the two states were estimated using both seasonally adjusted data and non-adjusted data on the same time series. The result was that the estimated dates of the break points differed between adjusted and non-adjusted data.

Ghysels (1997) investigated whether business cycle turning points are clustered around certain times of the year, i.e. if the probabilities of transition between phases of the business cycle were varying with seasonality.

Leong (1962) points out that a known property of the moving average is that it tends to smooth the turning points. The result is that the turning points are more difficult to detect.

In a surveillance situation, it is relevant that the time of the turning point is preserved after the adjustment.

3.2 Model specifications

The investigation starts from model (1), with the additional restriction

$$\mu(t) \in \wp, \text{ where } \wp \text{ is the family of all unimodal functions.}$$

Since Y contains the component μ , Y is not a trend stationary series.

In the following the results and examples are given for the situation when the turning point is a peak, but the results are valid also for the opposite case (a trough).

By the definition of a turning point, the regression μ is monotonic within each regime. That is, for a peak we have the following monotonicity and unimodality restrictions

$$\mu_t = \begin{cases} \mu(1) \leq \mu(2) \leq \dots \leq \mu(t), & t \leq p \\ \mu(1) \leq \dots \leq \mu(p) \text{ and } \mu(p) \geq \dots \geq \mu(t), & t > p \end{cases} \quad (2)$$

where p is the time of the peak.

Observe that the dependency of $E(Y(t) | t, p) = \mu(t)$ on p makes $\mu(t)$ a stochastic variable.

The seasonal adjustment is made in order to distinguish μ from the other components. The part of Y that is investigated contains one turning point at most. No parametric assumptions are made about $\mu(t)$ as a function of time, the only information is that $\mu(t)$ is unimodal according to (2). In this report the case when the accessible data are for a short or moderate time period is studied. Since no parametric model is assumed for $\mu(t)$, none of the parametric based estimation methods discussed in Section 2 is appropriate. Instead, we adjust for seasonality by applying differentiation or moving average. Other adjustment methods than a moving average might be more efficient. However, it is important to use a technique that is robust against slow changes in the seasonal component over years. It should also be noted that the number of available observations might, in many situations, not be very large. Another reason for examining the technique of moving average is that this technique is frequently used.

The aim of the seasonal adjustment is to adjust for seasonality without altering the turning point time of $\mu(t)$. The properties of seasonal differentiating as well as moving average technique are investigated. Since the data consist of monthly observations, the twelfth difference and twelve-month moving average are used.

The twelfth difference is

$$D(t)=Y(t)-Y(t-12)$$

and the expected twelfth difference at time t , under the assumptions of model (1), is

$$\delta(t) = E[D(t)] = \mu(t) - \mu(t-12) . \quad (3)$$

A centered moving average will not be suitable in a monitoring situation, since it results in an automatic lag of m time points (for monthly data $m = 6$). In the turning point detection situation, the aim is to detect the turning point in μ as soon as possible and thus a non-centered filter should be considered instead. A monthly centered moving average is based on the latest thirteen observations and a consequence of the centering is that the centered moving average at time t , per definition, can not be calculated until six months later, thus producing a systematic delay. Therefore a non-centered moving average is considered for the surveillance situation.

The non-centered moving average is

$$\tilde{Y}(t) = \frac{1}{12} \sum_{j=-11}^0 Y(t+j) .$$

and the expected value of $\tilde{Y}(t)$ under the assumptions of model (1) is

$$\eta(t) = E[\tilde{Y}(t)] = \frac{1}{12} \sum_{j=-11}^0 \mu(t+j) . \quad (4)$$

3.3 Change-point preserving properties

The investigation concerns the question whether indicators of turning points are timely after seasonal adjustment by differentiating and by moving average. Two different cases are investigated namely the case when the trend cycle is monotonic and the case of a peak in the trend cycle.

3.3.1 μ is monotonically increasing

The case when the μ -vector is monotonically non-decreasing within the entire observed section is studied in this section, that is

$$\mu(t-1) \leq \mu(t), t \geq 2.$$

The question of whether the sign of the slope of a monotonically increasing μ is preserved after differentiation is analyzed. That is, the correctness in the relation

$$\delta(t) \geq 0,$$

is investigated.

Statement 1a: *If μ is monotonically increasing, the expected seasonal difference is positive. Correspondingly, if μ is monotonically decreasing, the expected seasonal difference is negative.*

The question of whether the monotonicity is preserved when a non-centered moving average is used to estimate a monotonically increasing μ is analyzed. That is, the correctness of the relation

$$\eta(t-1) \leq \eta(t),$$

is investigated. The results are valid also for the opposite case (a monotonically non-increasing μ -vector).

Statement 1b: *If μ is a monotonic function, the expected value of the non-centered 12-point moving average is also a monotonic function.*

Details of Statement 1b are given in Appendix A.

3.3.2 μ has a turn

The case when the μ -vector is inversely U-shaped with a peak at time p is studied in this section, that is

$$\mu(1) < \mu(2) < \dots < \mu(p) \text{ and } \mu(p) > \mu(p+1) > \dots$$

The questions investigated in this section are i) whether the sign of the differentiated series can be used for indication of turning points and ii) whether the monotonicity and the time of the turning point is preserved when a moving average technique is used. That is, the correctness of the relations

$$\delta(t) > 0, t \leq p, \text{ and } \delta(t) < 0, t > p$$

and

$$\eta(1) < \eta(2) < \dots < \eta(p) \text{ and } \eta(p) > \eta(p+1) > \dots$$

is investigated

The seasonal differentiating and the non-centered moving average estimator will be studied in three special cases of a peak at time $t = p$. We study one symmetric turning point and two extreme turning points (a peak with a slowly decreasing post-peak slope and a peak with a steep post-peak slope). To a large extent economic theory and applied work rely on the assumption of symmetric cycles, see Falk (1986). However, it was early pointed out by Keynes (1936) that the business cycle appears to have abrupt transitions at troughs and smooth transitions at peaks. Neftci (1984) finds evidence of different absolute slopes in expansions and recessions for the unemployment rate. For research on asymmetry of macroeconomic time series, see also McQueen and Thorley (1993).

In the first case investigated, μ is symmetric around the peak at time $t = p$, i.e.

$$\mu(p-q) = \mu(p+q), q \geq 1.$$

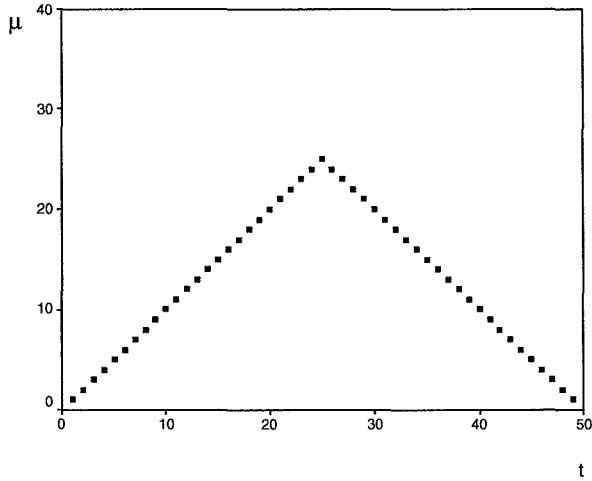


Figure 2: Case 1: μ at a symmetric peak.

The table below shows the result for the two seasonal adjustment methods at a symmetric peak.

Table 1: Case 1: Expected twelfth difference, $\delta(t)$ and expected value of non-centered moving average, $\eta(t)$.

Decision time, t	$\mu(t)$	$\delta(t)$	$\eta(t)$
$t \leq p$	$\mu(t-1) < \mu(t)$	$\delta(t) > 0$	$\eta(t-1) < \eta(t)$
$p+1 \leq t \leq p+5$	$\mu(t-1) > \mu(t)$	$\delta(t) > 0$	$\eta(t-1) < \eta(t)$
$t = p+6$	$\mu(t-1) > \mu(t)$	$\delta(t) = 0$	$\eta(t-1) = \eta(t)$
$t \geq p+7$	$\mu(t-1) > \mu(t)$	$\delta(t) < 0$	$\eta(t-1) > \eta(t)$

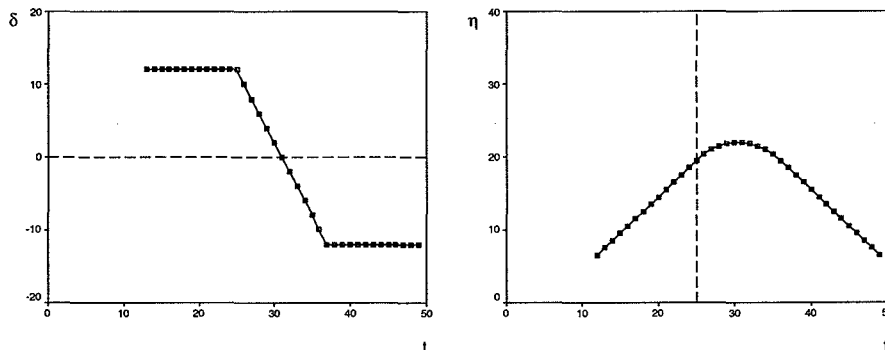


Figure 3: Case 1: The expected twelfth difference and the expected value of a non-centered 12 point moving average for μ at a symmetric turning point.

Table 1 shows that both the expected difference and the expected estimated trend cycle have a delay of six time units for a symmetric peak.

The second case considered is where the peak is non-symmetrical with a slowly decreasing post-peak slope, that is

$$\mu(p+q) > \mu(p-1), q \in \{1, 2, \dots, 11\}.$$

Since $\mu(1) < \dots < \mu(p-2) < \mu(p-1)$, it follows that $\mu(p+m) > \mu(p-m)$, $m \in \{1, 2, \dots, 11\}$.

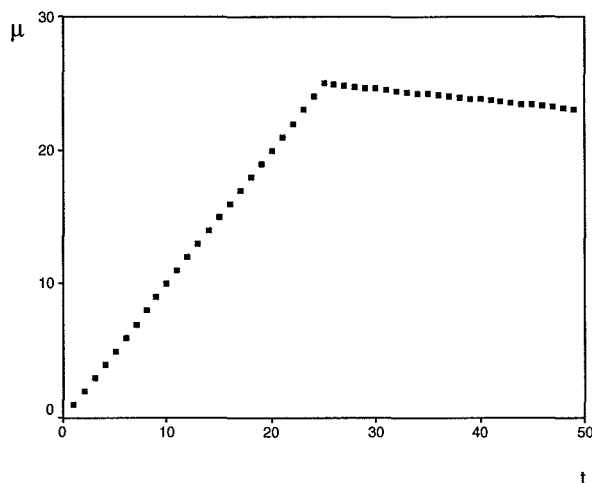


Figure 4: Case 2: μ at a non-symmetric peak with a slowly decreasing post-peak slope.

The table below shows the result for the two seasonal adjustment methods at a non-symmetric peak with slowly decreasing post-peak slope.

Table 2: Case 2: Expected twelfth difference, $\delta(t)$ and expected value of non-centered moving average, $\eta(t)$.

Decision time, t	$\mu(t)$	$\delta(t)$	$\eta(t)$
$t \leq p$	$\mu(t-1) < \mu(t)$	$\delta(t) > 0$	$\eta(t-1) < \eta(t)$
$p+1 \leq t \leq p+11$	$\mu(t-1) > \mu(t)$	$\delta(t) > 0$	$\eta(t-1) < \eta(t)$
$t \geq p+12$	$\mu(t-1) > \mu(t)$	$\delta(t) < 0$	$\eta(t-1) > \eta(t)$

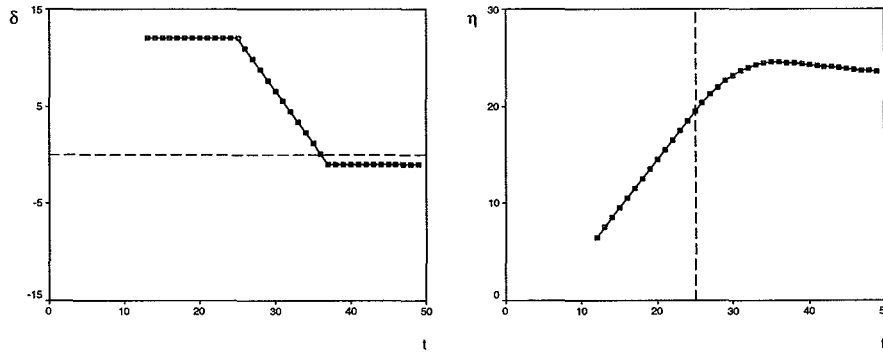


Figure 5: Case 2: The expected twelfth difference and the expected value of a non-centered 12 point moving average for μ at a non-symmetric peak with a slowly decreasing post-peak slope.

Table 2 shows that the delay is eleven time units for the expected twelfth difference and the expected value of the non-centered moving average at a peak with a slowly decreasing post-peak slope.

The third case considered is where μ has a non-symmetrical peak with a steep post-peak slope. It is assumed that

$$\mu(p+1) < \mu(p-11).$$

Since $\mu(p+1) > \mu(p+2)$ and $\mu(p-11) < \mu(p-10)$ the inequality $\mu(p+2) < \mu(p-10)$ must hold. From this result it is implicit that $\mu(p+m) < \mu(p-12+m)$, $m \in \{3, 4, \dots, 11\}$.

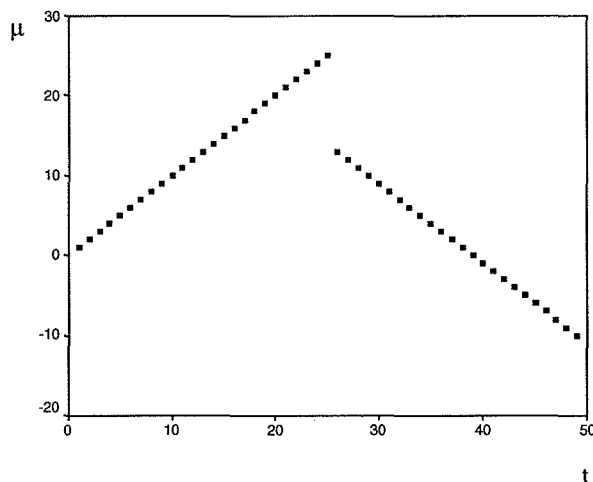


Figure 6: Case 3: μ at a non-symmetric peak with a steep post-peak slope.

The table below shows the result for the two seasonal adjustment methods at a non-symmetric peak with a steep post-peak slope.

Table 3: Case 3: expected twelfth difference, $\delta(t)$, and expected value of non-centered moving average, $\eta(t)$.

Decision time, t	$\mu(t)$	$\delta(t)$	$\eta(t)$
$t \leq p$	$\mu(t-1) < \mu(t)$	$\delta(t) > 0$	$\eta(t-1) < \eta(t)$
$t \geq p+1$	$\mu(t-1) > \mu(t)$	$\delta(t) < 0$	$\eta(t-1) > \eta(t)$

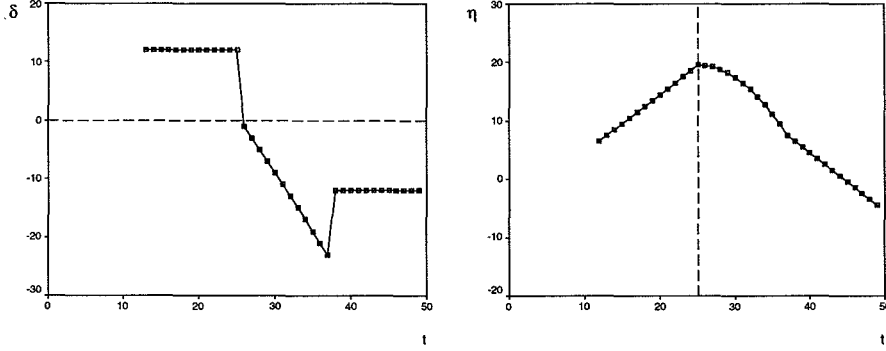


Figure 7: Case 3: The expected twelfth difference and the expected value of a non-centered 12 point moving average for μ at a non-symmetric peak with a steep post-peak slope.

Table 3 shows that there is no delay for the expected difference, nor for the expected value of the non-centered moving average at a non-symmetrical peak with a steep post-peak slope.

Statement 2a: *If μ is unimodal, the sign of the expected twelfth difference will not always indicate the slope of μ .*

Frisén (1986) proved that if μ is a unimodal function, the expected value of the moving average will preserve the unimodality. This means that the turning point itself is preserved, but that is not the same as if the time of the turning point is preserved.

Statement 2b: *If μ is unimodal, the expected value of the non-centered 12-point moving average will preserve the unimodality of μ (Frisén (1986)) but not always the time of the turning point in μ .*

Details of Statement 2b are given in Appendix A.

3.4 Using moving average and differentiating in turning point detection

3.4.1 Turning point indication

In the surveillance situation the aim is to detect a turning point in the cyclical process as soon as possible after it has happened. When the non-centered moving average is used for seasonal adjustment, a turn in the moving average is used as an indication of a turning point.

The properties of the sign of $\delta(t)$ as indicator for a turning point will be further illustrated. If

$$\delta(t) = \begin{cases} c, & \text{if } t \leq p \\ -c, & \text{if } t > p \end{cases} \quad (5)$$

where c is a constant

then Y , conditional on t , has the following expected value:

$$\mu(t) = \begin{cases} c \cdot t/12, & \text{if } t \leq p \\ -2c + c \cdot t/12, & \text{if } p < t \leq p+12 \\ -4c + c \cdot t/12, & \text{if } p+12 < t \leq p+24 \\ \dots & \end{cases}$$

(See Appendix B.)

To illustrate the effect, the difference to the previous value is given in Table 4.

Table 4: *The first difference, $\mu(t) - \mu(t-1)$, in relation to the seasonal difference, $\delta(t) = \mu(t) - \mu(t-12)$.*

Decision time, t	$\delta(t)$	$\mu(t) - \mu(t-1)$
$t \leq p$	c	$c/12$
$t = p+1$	$-c$	$-2c+c/12$
$p+2 \leq t \leq p+12$	$-c$	$c/12$
$t = p+13$	$-c$	$-2c+c/12$
$p+14 \leq t \leq p+24$	$-c$	$c/12$

An illustration is given in Figure 8.

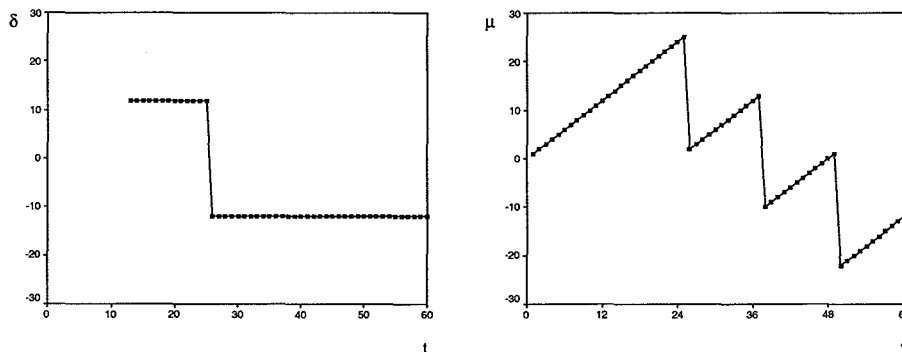


Figure 8: *The expected value of a differentiated series is δ in expansion and $-\delta$ in recession (left panel). Then the expected value of the undifferentiated series, $\mu(t)$, is illustrated in the right panel.*

The implication of defining the turning point as a shift in the series of seasonally differentiated data is that the original data is not at a turning point. Instead this definition implies that the original post-peak slope has a saw-toothed pattern. This problem occurs when seasonal differentiating is used, but not for the first order difference.

3.4.2 Differentiating and moving average in a system of surveillance

We investigate the consequence of using either the twelfth difference or the moving average for seasonal adjustment in a surveillance situation when the peak occurs at time p . We want to detect the peak as soon as possible after it has occurred. For that purpose we construct an alarm system, with an alarm statistic and alarm limit. We compare the probability of an alarm at different time points after the turning point for three processes, namely data that are free from seasonality, a moving average series and a series of differentiated data.

For an observation on $Y(t)$ at time t we have

$$Y(t) \sim \begin{cases} F^0, & t \leq p \\ F^1, & t > p \end{cases}$$

where F^0 and F^1 are the distribution functions, conditional on expansion (0) and recession (1), respectively.

The turning point is modeled using linear functions

$$Y(t) \sim \begin{cases} \alpha + \beta_1 \cdot t + S_j(t) + \varepsilon(t), & t \leq p \\ \alpha + \beta_1 \cdot p - \beta_2 \cdot (t - p) + S_j(t) + \varepsilon(t), & t > p \end{cases}$$

where $\varepsilon \sim \text{iid } N(0; \sigma^2)$

and $S_j(t)$ is the seasonal component at time t , season j , as given in 1).

The series that is free from seasonality is modeled as

$$X(t) \sim \begin{cases} \alpha + \beta_1 \cdot t + \varepsilon(t), & t \leq p \\ \alpha + \beta_1 \cdot p - \beta_2 \cdot (t - p) + \varepsilon(t), & t > p \end{cases}$$

We want to detect the change from F^0 to F^1 as soon as possible without too many false alarms. We give examples of two criteria for determining the alarm limits for a stochastic variable $U(t)$ which here could be either of the standardized versions of $X(t)$, $D(t)$ or $\tilde{Y}(t)$, where $D(t)$ and $\tilde{Y}(t)$ are the twelfth order differentiated series and the moving average series of $Y(t)$, respectively. The standardization is made so that $E(U(t))=0$ and $\text{Var}(U(t))=1$. Usually, surveillance systems for different processes are made comparable by adjusting the alarm limits to have the same average run length, conditional on F^0 , i.e. the same ARL^0 . Sometimes the median run length (MRL^0) is used instead of the arithmetic average.

The Shewhart approach (see e.g. Frisén (1992)) for constructing the alarm limit is used. Thus an alarm is triggered as soon as $U > k$. For the Shewhart method of surveillance we have the following relation between ARL^0 and the probability of an alarm at any time point, p_0 , given that no turn has happened:

$1/\text{ARL}^0 = p_0 = P(U > k \mid U \in F^0) = 1 - \Phi(k)$,
 where Φ is the normal probability function,
 see Frisén (1992).

Let $\text{ARL}^0=20$ and thus $p_0=0.05$. When $\Phi(k) = 0.95$ we get the value $k = 1.64$. We start with determining the alarm limit k_X for data free from seasonality, $X(t)$.

$$U(t) = (X(t) - (\alpha + \beta_1 \cdot t)) / \sigma \text{ implies}$$

$$k_X(t) = \alpha + \beta_1 \cdot t - 1.64 \cdot \sigma.$$

For the differentiated data, $D(t)$, we have $U(t) = (D(t) - 12\beta_1) / (\sqrt{2\sigma^2})$, which implies

$$k_D = 12\beta_1 - 1.64\sqrt{2\sigma^2}$$

and for the moving average series, $\tilde{Y}(t)$, we have

$$U(t) = (\tilde{Y}(t) - (\alpha + \beta_1 \cdot t - 5.5)) / (\sqrt{\sigma^2 / 12}) \text{, which implies}$$

$$k_{\tilde{Y}}(t) = \alpha + \beta_1 \cdot t - 5.5 - 1.64 \cdot \sigma / \sqrt{12}.$$

We investigate the case where the alarm limits are adjusted to yield $\text{ARL}^0=20$ ($p_0=0.05$) for all three processes ($X(t)$, $D(t)$ and $\tilde{Y}(t)$). At time $t = p$ the process is at a peak, which means that the earliest possible indication of a turning point comes at time $t = p+1$. Three peaks are investigated, namely a symmetric peak $\{\beta_1=0.0069, \beta_2=0.0069\}$, a peak with flat recession $\{\beta_1=0.0069, \beta_2=0.0051\}$ and a peak with steep recession $\{\beta_1=0.0069, \beta_2=0.0087\}$. The variance is set to $\text{Var}[\varepsilon] = 0.00026$. These parameter values are estimated from Swedish data on logarithmized industrial production.

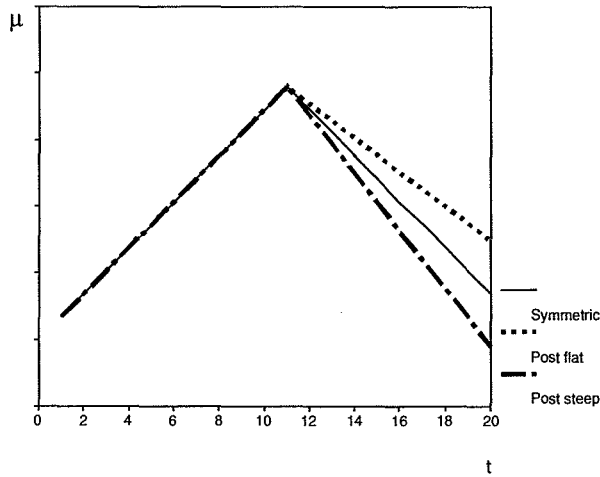


Figure 9: The expected value of the process, exemplified for $p=11$. The pre-peak slope and the post-peak slope are modeled using linear functions: $\{\beta_1=0.0069, \beta_2=0.0069\}$, $\{\beta_1=0.0069, \beta_2=0.0051\}$, $\{\beta_1=0.0069, \beta_2=0.0087\}$.

We calculate the measure probability of successful detection

$$\text{PSD}(\tau, d) = P(t_A - \tau < d | t_A \geq \tau = \tau_0)$$

where τ_0 = time of change (here $p+1$)

and $t_A = \min[t: U(t) > k]$

for the time points $\{p+1, p+2, \dots, p+12\}$, that is $d = \{1, 2, \dots, 12\}$.

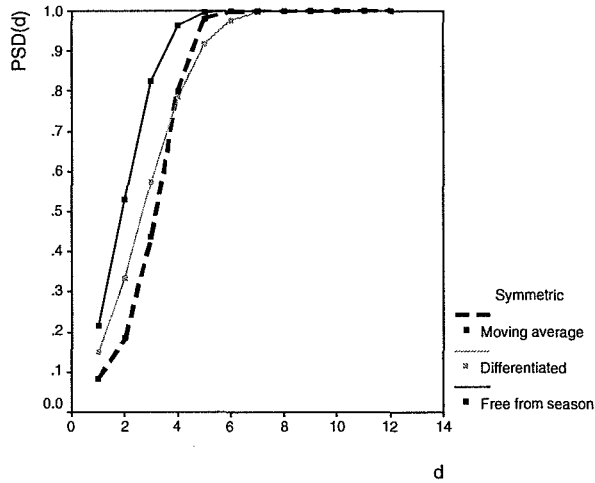


Figure 10: The probability of successful detection, using Shewhart limits. $ARL^0 = 20$, $\beta_1 = 0.0069$, $\beta_2 = 0.0069$.

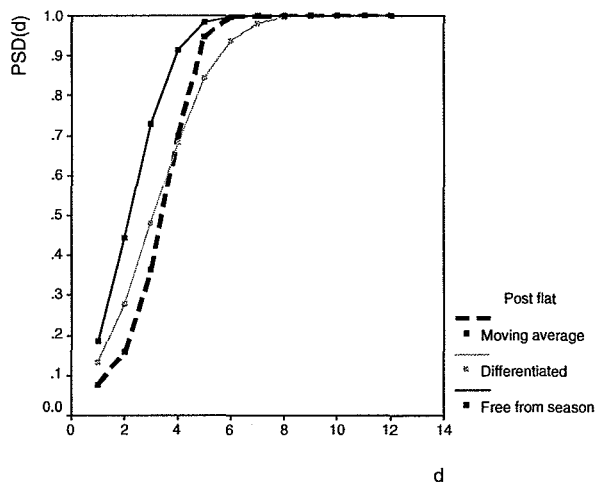


Figure 11: The probability of successful detection, using Shewhart limits. $ARL^0 = 20$, $\beta_1 = 0.0069$, $\beta_2 = 0.0051$.

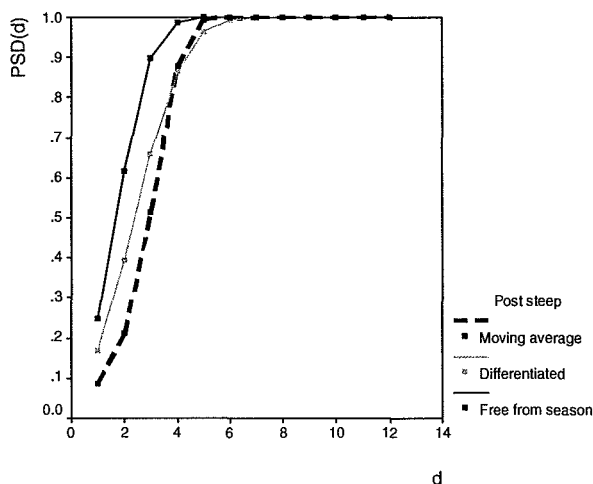


Figure 12: The probability of successful detection, using Shewhart limits. $ARL^0 = 20$, $\beta_1 = 0.0069$, $\beta_2 = 0.0087$.

We have demonstrated that if transformed data are used, then the probability of detecting a change from expansion to recession is reduced. We also see that the reduction is the greatest for requirements of very quick detection ($d < 4$) by the moving average, whereas for cases where more time is acceptable ($d > 4$) the reduction is the greatest for the differentiated series. The difference is based on two observations, whereas the moving average is based on twelve observations. For example, at time $t = p+1$ we have, for $D(p+1)$,

$$Y(p-11) \in F^0$$

$$Y(p+1) \in F^1$$

and for $\tilde{Y}(t)$ we have

$$\{Y(p-10), Y(p-9), \dots, Y(p)\} \in F^0$$

$$Y(p+1) \in F^1.$$

Thus, a majority of the observations in $\tilde{Y}(t)$ is in the in-control-state, F^0 . However, for detection at later time points, the variable $D(t)$ still consists of only one observation from the out-of-control state, whereas in $\tilde{Y}(t)$ more and more information from the out-of-control state is included. Therefore the PSD at later time points is larger for the moving average. As we can see when comparing Figure 10-12, the symmetry of the peak has no major influence on the relation between the PSD-functions for $X(t)$, $D(t)$ and $\tilde{Y}(t)$.

4 CONCLUDING REMARKS

Moving average techniques and differentiating have been investigated as possible methods of adjusting a time series for seasonality. A centered moving average does produce a systematic delay in the timeliness and therefore a non-centered moving average is considered. The properties of the non-centered moving average and the seasonal differentiating have been evaluated for a monotonic trend cycle and for a turning point in the trend cycle.

When using differentiating for seasonal adjustment, the issue is whether the sign of the slope is an indicator of a turning point, i.e. if the expected difference is positive for an increasing trend cycle (and negative for a decreasing trend cycle). The study showed that if both observations, $y(t)$ and $y(t-12)$, are within a monotonic section, then the expected difference will be positive for a monotonically increasing trend cycle and negative for a monotonically decreasing trend cycle.

Seasonal differentiating was investigated for three different peaks. It was shown that the sign of the expected difference will not always indicate the slope of the trend cycle at a turning point. The result is a delay in indication of a turning point.

If differentiating is used for seasonal adjustment, the peak (i.e. change in monotonicity) is sometimes defined as a change in level, from a positive level to a negative one. This study shows that defining the turning point from differentiated data implies that the undifferentiated data display a saw-toothed pattern (i.e. several turning points).

It has been shown that if all observations of the moving average are within a monotonic section of the time series, the non-centered moving average will preserve the monotonicity.

The non-centered moving average was investigated for three different kinds of peaks. For a unimodal section of the trend cycle, the non-centered moving average will preserve the unimodality. However, it has been shown in this investigation that the non-centered moving average does not always preserve the monotonicity of all parts in the unimodal case. Thus, the time of the turning point is not always preserved. In some cases the use of the moving average technique results in a delayed indication of a turning point.

The non-centered moving average as a method for seasonal adjustment is conservative in the sense that it does not give any false indications of a turning point. This moving average performs well at monotonic sections, but because of the possible delay at a turning point, it is important to try to use other methods. One possibility, if it agrees with the structure of the data on hand, is to use a large historical data set to estimate the seasonal components.

The assumptions of a constant (or slowly changing) seasonality used in this report might be too strong for many applications. However, some problems with the monotonicity evaluation of the seasonal adjustment were demonstrated even for a model with these assumptions.

Most data-driven transformations can have serious effects on the possibility to detect change points in a time series. Canova (1999) evaluates different methods for trend adjustment (including first order differentiating) ability to identify turning points of the business cycle. One conclusion is that the change point times of the trend-adjusted series may differ substantially compared to the official dating of the change points. Thus, information from historical data or other prior knowledge on the seasonal pattern is very valuable.

ACKNOWLEDGEMENTS

This paper is a part of a research project on statistical surveillance at Göteborg University, supported by the Swedish Council for Research in the Humanities and Social Sciences. The work on this paper has also been supported by the Oscar Ekman Stipendiefond. The authors wish to thank our supervisor, Professor Marianne Frisén, for her guidance and inspiration throughout this work. Also many thanks to Associate

Professor Ghazi Shukur for his valuable comments on earlier versions of the manuscripts.

REFERENCES

- Akaike, H. (1980) Seasonal adjustment by Bayesian modelling. *Journal of Time Series Analysis*, **1**, 1-13.
- Andersson, E., Bock, D. and Frisé, M. (2001) Likelihood based methods for detection of turning points in business cycles. A comparative study. *Research report 2001:5*, Department of Statistics, Göteborg University, Sweden.
- Bell, W. R. and Hillmer, S. C. (1984) Issues involved with the seasonal adjustment of economic time series. *Journal of Business and Economic Statistics*, **2**, 291-320.
- Birchenhall, C. R., Jessen, H., Osborn, D. R. and Simpson, P. (1999) Predicting US business-cycle regimes. *Journal of Business & Economic Statistics*, **17**, 313-323.
- Canova, F. (1999) Does detrending matter for the determination of the reference cycle and the selection of turning points. *Economic journal*, **109**, 126-151.
- Cleveland, W. P. and Tiao, G. C. (1976) Decomposition of seasonal time series: A model for the Census X-11 Program. *Journal of the American Statistical Association*, **71**, 581-587.
- Engle, R. F. (1992) Estimating structural models of seasonality. In *Modelling seasonality*(Ed, Hylleberg, S.) Oxford University Press, X, pp. 291-318.
- Falk, B. (1986) Further Evidence on the Asymmetric Behavior of Economic Time Series over the Business Cycle. *Journal of Political Economy*, **94**, 1097-1109.
- Franses, H. P. and Paap, R. (1999) Does Seasonality Influence the Dating of Business Cycle Turning Points? *Journal of Macroeconomics*, **21**, 79-92.
- Frisé, M. (1979) A note on Alternating estimation in non-linear regression. *Journal of Statistical Computation and Simulation*, **9**, 19-23.
- Frisé, M. (1986) Unimodal regression. *The Statistician*, **35**, 479-485.
- Frisé, M. (1992) Evaluations of Methods for Statistical Surveillance. *Statistics in Medicine*, **11**, 1489-1502.
- Frisé, M. (1994) Statistical Surveillance of Business Cycles. *Research report 1994:1* (Revised 2000), Department of Statistics, Göteborg University, Sweden.
- Frisé, M. and de Maré, J. (1991) Optimal Surveillance. *Biometrika*, **78**, 271-80.
- Ghysel, E. and Perron, P. (1996) The Effect of Linear Filters on Dynamic Time Series With Structural Change. *Journal of Econometrics*, **70**, 69-97.
- Ghysels, E. (1997) On seasonality and business cycle durations: A nonparametric investigation. *Journal of Econometrics*, **79**, 269-290.
- Granger, C. W. J. (1978) Seasonality: Causation, Interpretation and Implications. In *Seasonal Analysis of Economic Time Series*(Ed, Zellner, A.) Department of Commerce, Washington DC.
- Grether, D. M. and Nerlove, M. (1970) Some Properties of "Optimal" Seasonal Adjustment. *Econometrica*, **38**, 682-703.
- Hamilton, J. D. (1989) A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, **57**, 357-384.
- Harvey, A. C. (1993) *Time Series Models*, Harvester Wheatsheaf, New York.
- Hillmer, S. C. and Tiao, G. C. (1982) An ARIMA-based approach to seasonal adjustment. *Journal of the American Statistical Association*, **77**, 63-70.

- Jorgenson, D. W. (1964) Minimum variance, linear unbiased seasonal adjustment of economic time series. *Journal of the American Statistical Association*, **59**, 681-724.
- Keynes, J. (1936) *The General Theory of Unemployment, Interest and Money*, Harcourt, Brace and Company, New York.
- Koskinen, L. and Öller, L.-E. (1998) A Classifying Procedure for Signalling Turning Points. *Working Paper*, National institute of economic research, Stockholm, Sweden.
- Leong, Y. S. (1962) The use of an iterated moving average in measuring seasonal variations. *Journal of the American Statistical Association*, **57**, 149-171.
- MacNeill, I. B. (1978) Properties of sequences of partial sums of polynomial regression residuals with applications to tests for change of regression at unknown time. *Annals of Statistics*, **6**, 422-433.
- McQueen, G. and Thorley, S. (1993) Asymmetric business cycle turning points. *Journal of Monetary Economics*, **31**, 341-362.
- Neftci, S. (1982) Optimal prediction of cyclical downturns. *Journal of Economic Dynamics and Control*, **4**, 225-41.
- Neftci, S. (1984) Are Economic Time Series Asymmetric over the Business Cycle? *Journal of Political Economy*, **92**, 307-328.
- Nerlove, M. (1964) Spectral Analysis of Seasonal Adjustment Procedures. *Econometrica*, **32**, 241-285.
- Shiryayev, A. N. (1963) On optimum methods in quickest detection problems. *Theory of Probability and its Applications*, **8**, 22-46.
- Shiskin, J., Young, A. H. and Musgrave, J. C. (1967) The X-11 Variant of the Census Method II Seasonal Adjustment. *Technical Paper*, Bureau of the Census U.S Department of Commerce.
- Srivastava, M. S. and Wu, Y. (1993) Comparison of EWMA, CUSUM and Shiryayev-Roberts Procedures for Detecting a Shift in the Mean. *Annals of Statistics*, **21**, 645-670.
- Wallis, K. F. (1974) Seasonal adjustment and relations between variables. *Journal of the American Statistical Association*, **69**, 18-31.
- Young, A. H. (1968) Linear approximations to the Census and BLS seasonal adjustment methods. *Journal of the American Statistical Association*, **63**, 445-471.
- Young, P. C., Pedregal, D. J. and Tych, W. (1999) Dynamic harmonic regression. *Journal of Forecasting*, **18**, 369-394.
- Yule, G. U. (1926) Why Do We Sometimes Get Nonsense-Correlations Between Time-Series? *Journal of the Royal Statistical Society*, **89**, 1-64.
- Öller, L.-E. (1986) A note on exponentially smoothed seasonal differences. *Journal of Business and Economic Statistics*, **4**, 485-489.
- Öller, L.-E. and Tallbom, C. (1996) Smooth and timely business cycle indicators for noisy Swedish data. *International Journal of Forecasting*, **12**, 389-402.

Appendix A. Details regarding Statement 1b, 2b

Statement 1b: If μ is a monotonic function, the expected value of the non-centered 12-point moving average is also a monotonic function.

Detailed description:

We have that the difference between two consecutive expected estimates of the trend cycle is

$$\eta(t) - \eta(t-1) = \frac{1}{12} [\mu(t) - \mu(t-1-11)].$$

For the difference $\mu(t) - \mu(t-1-11)$ we have

$\mu(t) - \mu(t-12) \geq 0$, all $t \geq 12$,
according to the assumption of a non-decreasing μ .

Therefore $\eta(t) - \eta(t-1) \geq 0$ and η is a non-decreasing function for all $t \geq 12$.

Statement 2b: If μ is unimodal, the expected value of the non-centered 12-point moving average will preserve the unimodality of μ (Frisén, 1986) but not always the time of the turning point in μ .

Detailed description:

For Statement 2b the description is divided into the three cases that have been investigated. For each case the description for the different time intervals, denoted i), ii), iii) and iv), are given separately.

Case 1 (a symmetric peak)

i) $\eta(p-i) - \eta(p-1-i) > 0$, for $0 \leq i \leq (p-2)$. See proof of Statement 1b.

ii) $\eta(p+i) - \eta(p+i-1) = (1/12) * ((\mu(p-i) - \mu(p-i-12)))$.

Since $(\mu(p-i) - \mu(p-i-12)) > 0$ for $i = \{1, 2, \dots, 5\}$, it follows that
 $\eta(p+i) - \eta(p+i-1) > 0$.

iii) $\eta(p+6) - \eta(p+5) = (1/12) * (\mu(p+6) - \mu(p-6)) = (1/12) * (\mu(p-6) - \mu(p-6)) = 0$

iv) $\eta(p+i) - \eta(p+i-1) = (1/12) * (\mu(p+i) - \mu(p-(i-2))) = (1/12) * (\mu(p+i) - \mu(p+i-2))$.

Since $(\mu(p+i) - \mu(p+i-2)) < 0$ for $i \geq 7$, it follows that
 $\eta(p+i) - \eta(p+i-1) < 0$.

Case 2 (non-symmetric peak with a slowly decreasing post-peak slope)

i) $\eta(p-i) - \eta(p-1-i) > 0$, for $0 \leq i$. See proof of Statement 1b.

ii) $\eta(p+i) - \eta(p+i-1) = (1/12) * ((\mu(p+i) - \mu(p+i-12)))$.

Since $(\dots < \mu(p-11) < \dots < \mu(p-1))$ and $\mu(p+i) > \mu(p-1)$, $i = \{1, 2, \dots, 11\}$, it follows that $\eta(p+i) - \eta(p+i-1) > 0$.

iii) $\eta(p+i) - \eta(p+i-1) = (1/12) * (\mu(p+i) - \mu(p+i-12))$.

Since $(\mu(p+i) - \mu(p+i-12)) < 0$, $i \geq 12$, it follows that $\eta(p+i) - \eta(p+i-1) < 0$.

Case 3 (non-symmetric peak with a steep post-peak slope)

i) $\eta(p-i) - \eta(p-1-i) > 0$, for $0 \leq i$. See proof of Statement 1b.

ii) $\eta(p+i) - \eta(p+i-1) = (1/12) * ((\mu(p+i) - \mu(p+i-12)))$.

Since $(\dots < \mu(p-11) < \dots < \mu(p-2) < \mu(p-1))$ and $\mu(p+1) < \mu(p-11)$, it follows that $\eta(p+i) - \eta(p+i-1) < 0$, for $i \geq 1$.

Appendix B. Expected value of Y(t).

Since $E[Y(t) - Y(t-12)] = c$, $t \leq p$, it follows that $E[Y(t)] = \frac{c}{12} \cdot t$, $t \leq p$.

Therefore $E[Y(p-11)] = \frac{c}{12} \cdot (p-11)$.

We have that

$$E[Y(p+1) - Y(p-11)] = -c,$$

i.e. $E[Y(p+1)] = -c + E[Y(p-11)],$

i.e. $E[Y(p+1)] = -c + \frac{c}{12} \cdot (p-11),$

i.e. $E[Y(p+1)] = -2c + \frac{c}{12} \cdot (p+1).$

Analogical results holds for $E[Y(p+2)], \dots, E[Y(p+12)],$ so that

$$E[Y(p+j)] = -2c + \frac{c}{12} \cdot (p+j), j = \{2, 3, \dots, 12\}.$$

We have that

$$E[Y(p+13) - Y(p+1)] = -c,$$

$$\text{i.e. } E[Y(p+13)] = -c + E[Y(p+1)],$$

$$\text{i.e. } E[Y(p+13)] = -c + (-2c) + \frac{c}{12} \cdot (p+1),$$

$$\text{i.e. } E[Y(p+13)] = -4c + \frac{c}{12} \cdot (p+13).$$

Analogical results holds for $E[Y(p+14)]$, ..., $E[Y(p+24)]$, so that

$$E[Y(p+j)] = -4c + \frac{c}{12} \cdot (p+j), j = \{14, 15, \dots, 24\}.$$

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