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## Surveillance of spatio-temporal patterns

Change of interaction in an Ising dynamic model

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# Surveillance of Spatio-Temporal Patterns: Change of Interaction in an Ising Dynamic Model

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## Abstract

Surveillance to detect changes of spatial patterns is of interest in many areas such as environmental control and regional analysis. A model which possesses both spatial and time dependence is the Markov chain Markov field. Here a special case of this, called the Ising dynamic model with zero external field, and change in its spatial interaction parameter is considered. A method for simulation exactly according to this Ising dynamic model, is proposed. Surveillance methods corresponding to common methods for the time independent case, are derived.

*Key words:* Markov chain Markov field, Heterogeneous external field, Zero external field perfect simulation, on-line change detection.

*AMS 2000 Mathematics Subject Classification:* 82B20, 60K35, 62L15.

## Introduction

Due to several environmental issues, such as forestry disease surveillance, voter proportion change detection, earthquake warning system and others, results about methods for judging whether a change in the behaviour of a spatio-temporal process has occurred or not, are strongly demanded.

For e.g. monitoring a suspected breakout of a disease in a forest one could suppose that indications of the disease are present without any spatial dependence when the forest is in control (i.e. not an epidemic). A sudden breakout of the disease could then be detected as a change in the appearance of clustered diseased trees (i.e. epidemic) which in turn could be recognised as a suddenly increased level of attraction.

Guyon [8] and Bayomog [1] introduced a spatio-temporal lattice process which they called Markov chain Markov field. This is a sequence of lattice variables forming a Markov chain indexed by time  $t$ . At each fixed time  $t$  given the previous state at time  $t-1$ , the lattice variable at  $t$  is a Markov field. They investigated properties of pseudo-likelihood estimators of and tests for pair-wise interaction and temporal interaction parameters in the distribution of this process. Several random lattice models were also considered by Liggett [17].

At large this work is the proceedings of Järpe [12] and [13]. While we in that paper considered the Ising model with zero external field assuming time independence given the time of change, we allow for both temporal and spatial dependence in this one. In those papers the problem was also the monitoring of Ising patterns for detecting a change in the interaction parameter. This problem was dealt with by reducing the observation process to a statistic which was minimal sufficient for the interaction parameter and, using the statistic's asymptotic distribution as an approximation of the statistic's distribution in a finite lattice, methods for univariate surveillance could be applied.

In this paper the approach is different. In Section 1 we introduce the Markov chain Markov field (Guyon [8] and Bayomog [1]). The model considered here is called Ising dynamic model with zero external field. It is a spatio-temporal model and a special case of a Markov chain Markov field. In short it is a sequence of random fields  $X(1), X(2), \dots$  and the sequence is a Markov chain. Each field,  $X(t)$ , given the previous field,  $X(t-1)$ , is a Markov field according to the Ising model with heterogeneous external field. This observation is useful for the suggested technique, explained in Section 2, to simulate exactly from the Ising dynamic model with zero external field.

In Section 3 we consider some spatio-temporal surveillance methods to detect a change in the interaction parameter of the space-time model. The aim is to establish appropriate methods for fast and accurate detection of a change. Some common surveillance methods are derived for the Markov chain case. From simulations of the Ising dynamic model it is possible to evaluate the performance of these surveillance methods. Finally in Section 4, the results are discussed.

## 1 Space-time Model

We briefly introduce the space-time model Markov chain Markov field and a special case of it called the Ising dynamic model which is based on the Ising model. A more thorough presentation of Markov field Markov chain is in

Guyon [8] and Baymogh [1] and of the Ising model in e.g. Kindermann and Snell [15].

## 1.1 Markov Chain Markov Field

Let  $A_N$  be a set consisting of  $N$  positions, called sites, symbolically denoted by  $i \in \{1, 2, \dots, N\}$ , forming a lattice. Let  $(E, \mathcal{E})$  be a measurable set and  $\nu$  a positive  $\sigma$ -finite measure on  $E$ . Let the random process  $X(t) = \{X_i(t) : i \in A_N\}$  be a random field defined on  $(E^N, \mathcal{E}^{\otimes N}, \nu^{\otimes N})$ ,  $t \in \mathbb{Z}_+$  with some reflexive neighbourhood relation where site  $i$  being a neighbour of site  $j$  is denoted  $j \in \partial i$  and the variable at time  $t$  and sites  $i \in B \subseteq A_N$  is denoted by  $X_B(t)$ . Given the previous field,  $X(t-1)$ , let  $X(t)$  be a random field,  $t \in \mathbb{Z}_+$ . Also let  $X = \{X(t) : t \in \mathbb{Z}_+\}$  be a stochastic process with state space  $E^N$ .

**Definition 1 (Guyon [8])** *The stochastic process  $X$  is called a Markov chain Markov field if*

$$X_i(t) | X_{A_N \setminus \{i\}}(t), \{X(s) : s = 1, 2, \dots, t-1\} \stackrel{\mathcal{D}}{=} X_i(t) | X_{\partial i}(t), X(t-1)$$

for each  $i \in A_N$  and  $t \in \mathbb{Z}_+$ .

This means that  $\{X(t) : t \in \mathbb{Z}_+\}$  is a Markov chain with transition probabilities

$$P_t(B, x) = P[X(t) \in B | X(t-1) = x], \quad x \in E^N.$$

In this paper we consider time homogeneous Markov chains, for which there exists a transition probability  $P$  on  $E^N$  such that  $P_t = P$  for all  $t \in \mathbb{Z}_+$ .

## 1.2 Ising Dynamic Model with Zero External Field

The Ising dynamic model with zero external field is a special case of the Markov chain Markov field. Let  $A_n$  be a finite set consisting of  $n^2$  positions, called sites, symbolically denoted by  $i \in \{1, 2, \dots, n^2\}$ , forming a square lattice in  $\mathbb{Z}^2$ .

Let  $F = \{-1, 1\}$  and  $\mathcal{F} = \sigma(F)$ , the smallest  $\sigma$ -algebra of  $F$ . Then the space  $(F, \mathcal{F})$  is measurable with respect to the counting measure,  $\mu$ , on  $F$ . Furthermore, let  $X(t)$  be a random field defined on  $(F^{n^2}, \mathcal{F}^{\otimes n^2}, \mu^{\otimes n^2})$ ,  $t \in \mathbb{Z}_+$  with a neighbourhood relation according to Definition 2. If  $B \subset A_n$ , we denote  $\{X_i(t) : i \in B\}$  by  $X_B(t)$ .  $X$  is assumed to have a distribution with frequency function  $p(x)$ .

**Definition 2** *The sites  $i$  and  $j$  are called (first order) neighbours, denoted by  $i \sim j$ , if the Euclidean distance between  $i$  and  $j$  is exactly 1. The neighbourhood of a site  $i$  is the set  $\partial i = \{j \in A_n : i \sim j\}$ .*

To avoid edge problems, we map our square study region onto a torus.

We consider the special case of a Markov chain Markov field where given the state of the neighbourhood at time  $t$ ,  $x_{\partial i}(t)$ , and the state at  $i$  at time  $t-1$ ,  $x_i(t-1)$ , the random variable at site  $i$  and time  $t$ ,  $X_i(t)$  is conditionally

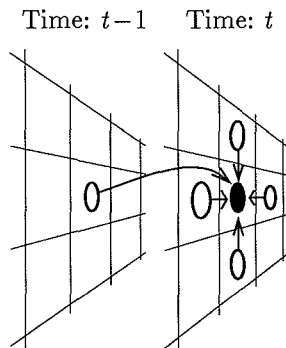


Figure 1: *The state  $X_i(t)$  at a site,  $i$ , is conditionally independent of states at other sites given the states of its spatial neighbours,  $X_{\partial i}(t)$ , and its temporal neighbour,  $X_i(t-1)$ .*

independent of all other sites. More precisely, the process  $X = \{X_i(t) : i \in A_n, t \in \mathbb{Z}_+\}$  considered, satisfies

$$X_i(t) | X_{A \setminus i}(t), \{X(s) : s=1, 2, \dots, t-1\} \stackrel{\mathcal{D}}{=} X_i(t) | X_{\partial i}(t), X_i(t-1)$$

for each  $i \in A_n$  and  $t \in \mathbb{Z}_+$  (see Figure 1). We assume that, for each  $t \in \mathbb{Z}_+$ ,  $X(t)$  fulfils a positivity condition (Hammersley and Clifford [9]):  $\mathbb{P}[X_i(t) = x_i(t)] > 0$  for each  $i \in A_n$  (which implies that  $\mathbb{P}[X_1(t) = x_1(t), \dots, X_{n^2}(t) = x_{n^2}(t)] > 0$ ). For the sake of simplicity we denote  $\{X_i(u) : i \in A_n, 1 \leq u \leq t\}$  by  $X_{<t}$  and  $x(t)$  by  $x$ ,  $x(t-1)$  by  $x'$ ,  $x(t-2)$  by  $x''$  and so on, whenever this is convenient.

**Definition 3** *The global distribution function, denoted  $p(x|x')$ , of the Ising dynamic model with zero external field is the transition probability in the Markov chain  $X(1), X(2), \dots$*

$$\mathbb{P}[X(t)=x|X(t-1)=x'] = Z^{-1} \exp(\phi Q(t) + \psi R(t))$$

where  $Z = Z(x', \phi, \psi) = \sum_x \exp(\phi Q(t) + \psi R(t))$  is a normalising constant and  $Q(t) = \sum_{i \sim j} x_i x_j$ ,  $R(t) = \sum_i x_i x'_i$  are energies where summation with index  $i \sim j$  means summing over all  $i \in A_n$ ,  $j \in \partial i : j < i$  and summation index  $i$  means summing over all sites  $i \in A_n$ .  $\phi$  and  $\psi$  are spatial interaction

and temporal interaction parameters respectively on some parameter space  $\Phi \times \Psi \subseteq \mathbb{R}^2$ .

One way of simulating from the Ising dynamic model, is via the Ising model with heterogeneous external field. A model without time in it is the Ising model. In the **Ising model with heterogeneous external field**, the lattice process  $X = \{X_i : i \in A_{n^2}\}$  is defined by the distribution

$$P[X=x] = Z^{-1} \exp\left(\phi \sum_{i \sim j} x_i x_j + \lambda_1 \sum_{i \in {}_1A_{n^2}} x_i + \lambda_2 \sum_{i \in {}_2A_{n^2}} x_i\right)$$

where  $({}_1A_{n^2}, {}_2A_{n^2})$  is a partition of  $A_{n^2}$ , and  $(\sum_{i \in {}_1A_{n^2}} x_i, \sum_{i \in {}_2A_{n^2}} x_i)$  is called **heterogeneous external field**. If  $\lambda_1 = \lambda_2$ , the external field is **homogeneous**, and  $\lambda_1 = \lambda_2 = 0$  makes a **zero external field**. When not otherwise stated, we consider the zero external field version of the Ising and Ising dynamic models, and therefore this term (“with zero external field”) will be suppressed subsequently.

When  $\phi$  is positive the neighbours tend to have the same values thus rendering clustered patterns and when  $\phi$  is negative they tend to have different values which renders regular patterns. In the same way positive  $\psi$  makes each site more likely to remain the same for each time step while negative  $\psi$  gives each site the tendency to be the opposite of the previous state.

Denoting  $\sum_{j \in \partial_i} x_j$  by  $v_i$ , the local distribution function,  $p(x_i | x_{\partial_i}, x'_i)$ , of the random variable at site  $i$ , given the states of its spatial and temporal neighbourhood, is

$$P[X_i(t) = x_i | X_{\partial_i}(t) = x'_{\partial_i}, X_i(t-1) = x'_i] = Z_i^{-1} \exp\left(x_i(\phi v_i + \psi x'_i)\right)$$

where  $Z_i = 2 \cosh(\phi v_i + \psi x'_i)$  is the local normalizing constant. Due to the positivity condition, this conditional probability takes its values on  $(0,1)$  for all values of the neighbourhood. The Ising dynamic model may also be expressed a logistic linear model by

$$\log\left(\frac{p(1|x_{\partial_i}, x'_i)}{1 - p(1|x_{\partial_i}, x'_i)}\right) = 2(\phi v_i + \psi x'_i).$$

### 1.2.1 Stationarity

In the zero external field Ising model, Onsager [19] found that with  $\phi_- = \log(\sqrt{2}-1) = -\phi_+$  the existence of a unique stationary distribution of  $X$  is guaranteed for  $\phi \in \Phi$  compact on  $(\phi_-, \phi_+)$ .

Pickard [21] investigated the homogeneous external field Ising model and found that for the non-zero external field ( $\lambda \neq 0$ ), it is sufficient that  $\phi > \phi'_-$ , where  $\phi'_-$  is a function of the external field parameter  $\lambda$ , and bounded by  $\log(\sqrt{2}-1)$ .

Guyon [8] and Bayomog [1] studied the Markov Chain Markov Field. Bayomog concluded that for the auto-Poisson model, the temporal interaction part of the energy function,  $\alpha_i(x|\theta)$ , uniformly bounded in  $x, \theta$  for all  $i \in A_n$  implies the existence of a unique stationary distribution of  $X$ .

For the Ising dynamic model, we will assume that for  $\phi$  and  $\psi$  in the parameter interval  $\Phi \times \Psi$ , there is a stationary distribution,  $\pi$ , of  $X(t)$  and we only consider parameters within that interval.

## 2 Simulation

Perfect simulation is a technique to simulate exactly to a stationary distribution (introduced by Propp and Wilson [22]). It is used in this paper for the monotone case and, with the modification by Häggström and Nelander [10], for the anti-monotone case.

The method of perfect simulation of the Ising model is applicable in the case with Ising dynamic model. Apart from being nice for illustrations (see Figure 2) this way of getting samples simulated exactly according to the Ising dynamic model is the base for the simulated moment results in the end of Section 1.

**Observation 1** *The global distribution of the Ising dynamic model is a global distribution of an Ising model with heterogeneous external field. Spatial interaction parameter  $\phi$  and temporal parameter  $\psi$  in the Ising dynamic model corresponds to interaction parameter  $\phi$  and external field parameter  $\psi x'$ .*

Since there is no problem to simulate Ising heterogeneous external field patterns (using e.g. the Gibbs sampler), Observation 1 says that we may simulate exactly according to the Ising dynamic model.

### Gibbs Sampler

The Gibbs sampler was invented by Suomela [24] but usually the credit goes to Geman and Geman [6]. It is a stepwise procedure where one, at each time-step  $t = 1, 2, 3, \dots$ , cruises along the sites  $i \in A_n$  so that each site is almost surely visited infinitely often. This cruise could be made in a number of ways e.g.  $i = 1, 2, \dots, n^2$ . At each visit the state possessed by that site, is

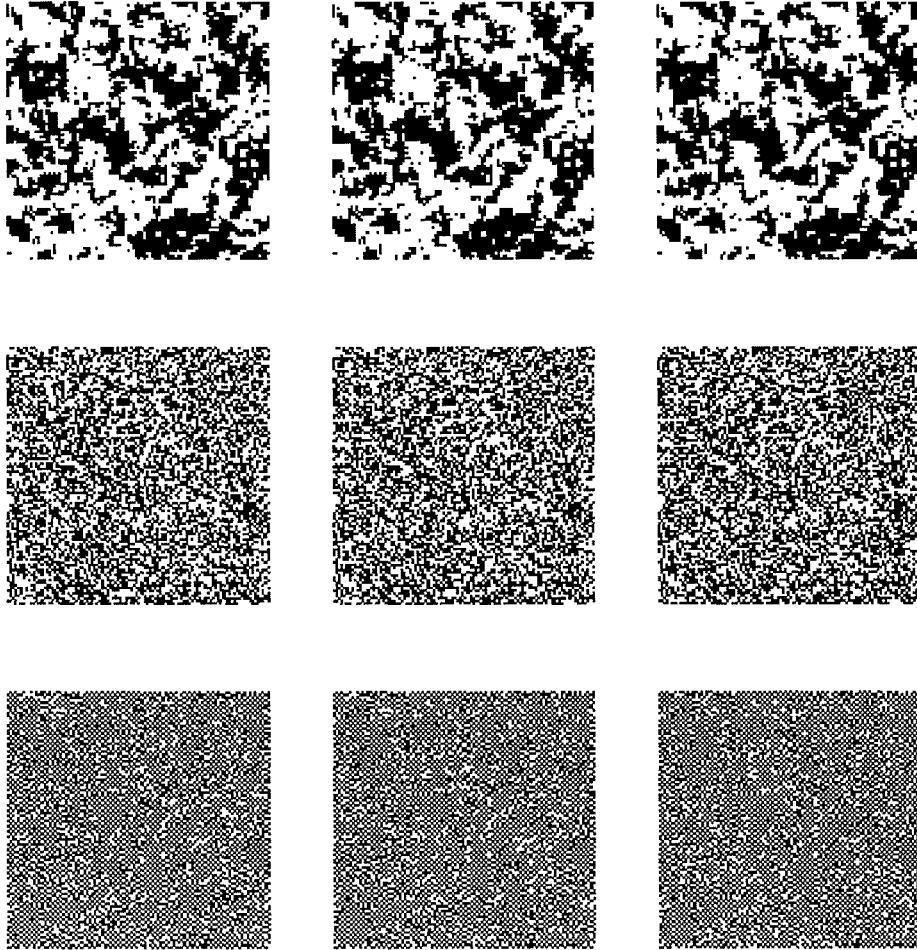


Figure 2: *Three  $100 \times 100$  square lattice patterns at times 1, 2, 3 simulated exactly according to an Ising dynamic model with  $\psi = 3$ . In the first row  $\phi = 0.7$  (attraction), in the second (spatial independence) and in the third  $\phi = -0.7$  (repulsion).*



updated according to the map  $g : F^{n^2} \times F^{n^2} \times A_n \times [0, 1] \rightarrow F^{n^2}$  as

$$g(x(t), x(t-1), i, u_i(t)) = \begin{cases} x_{A_n \setminus i}(t) \cup \{1\} & \text{if } p(1 | x_{\partial i}(t), x_i(t-1)) > u_i(t) \\ x_{A_n \setminus i}(t) \cup \{0\} & \text{otherwise} \end{cases}$$

where  $\{u_i(t) : i = 1, \dots, n^2, t = 0, 1, 2, \dots\}$  is a sequence of independent observations of the uniform distribution on  $[0, 1]$ . For some fixed  $t = t_0$ , the states are  $\{x_1(t_0), \dots, x_{n^2}(t_0)\}$ . As all sites are visited and updated again the clock snaps one tick to  $t = t_0 + 1$  and the states are  $\{x_1(t_0 + 1), \dots, x_{n^2}(t_0 + 1)\}$ , and so on. Starting with arbitrary states  $\{x_1(0), \dots, x_{n^2}(0)\}$  and updating according to this rule, the sequence  $\{x(0), x(1), \dots\}$  of configurations achieved at each "full round", forms a Markov chain. So we may approximately simulate an Ising configuration according to the desired global distribution.

### Perfect Simulation

Let  $\{u_i(t) : i = 1, \dots, n^2, t = -M, \dots, 0\}$  be a sequence of independent observations of a uniform random variable on  $[0, 1]$ . The main idea is the following: impose the partial ordering relation denoted by  $\preceq$  meaning that  $x \preceq y$  if  $x_i \leq y_i$  for each  $i \in A_n$ . Then generate two monotone Markov chains  $\{x(t)\}_{t=-M}^0$  and  $\{y(t)\}_{t=-M}^0$  according to the "Coupling-from-the-past-protocol" starting with  $x(-M) = \hat{0}$  being the minimal state and  $y(-M) = \hat{1}$  the maximal state and terminating with  $x(0) = y(0)$  which is the simulated Ising configuration. The time  $-M$  is unknown stochastic and it is determined during the evaluation of the algorithm. The algorithm is the pseudocode

```

T ← 1
repeat
  upper(-T-1) ←  $\hat{1}$ 
  lower(-T-1) ←  $\hat{0}$ 
  for t = -T to 0
    for i = 1 to  $n^2$ 
      upper(t) ← g(upper, upper(t-1), i, u_i(t))
      lower(t) ← g(lower, lower(t-1), i, u_i(t))
  T ← 2T
until upper = lower
return upper

```

where the map  $g$  may be chosen as the previous sampling algorithm Gibbs sampler or other Metropolis-Hastings algorithms (see Kendall and Møller [14]) or some other that results in a Markov chain and which preserves the partial ordering.

However, the distribution  $p$ , that we want to simulate samples from, must satisfy a monotonicity condition (Propp and Wilson [22]) in order for the method to be valid. When  $\phi$  is negative (attractive case),  $p$  is monotone but otherwise not. On the other hand, when  $\phi$  is positive (repulsive case),  $p$  satisfies an anti-monotonicity condition and with a slight modification of the updating map  $g$ , realisations can be simulated in this case as well (Häggröm and Nelander [10]). In short this is to say, provided that we have chosen the Gibbs sampler as the map  $g$  in the case of positive  $\phi$ , then the modification is just to update each site of say the chain referred to in the pseudocode as *upper*, not according to its neighbours but rather according to the corresponding neighbourhood of the partner chain *lower*. The same goes for updating each site of the *lower* chain according to the corresponding neighbourhood in the *upper* chain. Häggröm and Nelander showed that this change makes the resulting Ising pattern distributed exactly according to the stationary distribution in the anti-monotone case. Thus we are able to simulate Ising configurations regardless of the value of  $\phi$ .

### 3 Surveillance

There are many different names (surveillance, change detection, monitoring etc.) for the task of detecting a shift. In this paper we consider the case where the data accumulates in time and it is decided "on-line" whether or not a change has occurred. For further reading see e.g. Friséen [2],[3], Friséen and de Maré [4], Friséen and Wessman [5], Järpe and Wessman [11], Lai [16] or Wessman [25].

We make consecutive observations  $x(1)$ , then  $x(2)$ , then  $x(3)$  and so on, of  $X = \{X_i(t) : t \in \mathbb{Z}_+, i \in A_n\}$ , a process according to the Ising dynamic model where, conditional on  $X(t-1) = x(t-1)$ ,

$$X(t) \stackrel{\mathcal{D}}{=} \begin{cases} p(x(t)|x(t-1); \phi = \phi_0) & \text{if } t < \tau \\ p(x(t)|x(t-1); \phi = \phi_1) & \text{if } t \geq \tau \end{cases}$$

where the distribution  $p(\cdot; \phi)$  is as given in Section 1,  $\phi_1 \in (\phi_-, \phi_+)$  and  $\tau$  is an unknown random time-point. We consider the problem of detecting a change of  $\phi$  from  $\phi_0$  to  $\phi_1$ .

#### 3.1 Markov Chain Surveillance Methods

There are several general surveillance methods suggested in the literature. We recall the definition of the Cusum, Shiryaev-Roberts (SR) and likelihood ratio (LR) methods for for the i.i.d. (independent and identically distributed)

variables before and i.i.d. variables after change case (henceforth this will be referred to simply as “the i.i.d. case”) and then we derive the same methods for the Markov chain case. Let

$$\text{lr}(x(t)|x(t-1)) = \frac{p(x(t)|x(t-1), \tau \leq t)}{p(x(t)|x(t-1), \tau > t)}.$$

In the i.i.d. case,  $\text{lr}(x(t)|x(t-1)) = \text{lr}(x(t)) = p(x(t)|\tau \leq t)/p(x(t)|\tau > t)$ . All three methods are stopping rules on the form

$$T = \inf\{s : a(X_{\leq s}) > c\}$$

where  $a(\cdot)$  is an alarm function and  $c$  is a threshold. The Cusum method for the i.i.d. case  $\{y(t) : t \in \mathbb{Z}_+\}$ , suggested by Page [20] and studied by Lorden [18], is defined with alarm function

$$a(y_{\leq s}) = \max_{1 \leq t \leq s} \log(f_{Y_{\leq s}}(y_{\leq s}|\tau=t)/f_{Y_{\leq s}}(y_{\leq s}|\tau > s)).$$

The Cusum method for the Markov chain,  $X$ , has alarm function, conditional on the initial state  $X(0) = x_0$ ,

$$\begin{aligned} a(x_{\leq s}) &= \max_{1 \leq t \leq s} \log \frac{p(x_{\leq s}|x_0, \tau=t)}{p(x_{\leq s}|x_0, \tau > s)} \\ &= \begin{cases} (\log \text{lr}(x(1)|x_0))^+ & s = 1 \\ (\log \text{lr}(x(t)|x(t-1))) + a(x_{\leq s-1})^+ & s = 2, 3, \dots \end{cases} \end{aligned}$$

The SR method for the i.i.d. case  $\{y(t) : t \in \mathbb{Z}_+\}$ , derived by Shiryaev [23], has alarm function

$$a(y_s) = \sum_{t=1}^s f_{Y_{\leq s}}(y_{\leq s}|\tau=t)/f_{Y_{\leq s}}(y_{\leq s}|\tau > s).$$

The SR method for the Markov chain,  $X$ , has alarm function

$$\begin{aligned} a(x_{\leq s}) &= \sum_{t=1}^s \frac{p(x_{\leq s}|x_0, \tau=t)}{p(x_{\leq s}|x_0, \tau > s)} \\ &= \begin{cases} \text{lr}(x(1)|x_0) & s = 1 \\ \text{lr}(x(1)|x_0) \prod_{r=2}^s \text{lr}(x(r)|x(r-1)) \\ \quad + \sum_{t=2}^s \prod_{r=t}^s \text{lr}(x(r)|x(r-1)) & s = 2, 3, \dots \end{cases} \\ &= \begin{cases} \text{lr}(x(1)|x_0) & s = 1 \\ \text{lr}(x(s)|x(s-1))(a(x_{s-1}) + 1) & s = 2, 3, \dots \end{cases} \end{aligned}$$

The LR method for the i.i.d. case  $\{y(t) : t \in \mathbb{Z}_+\}$ , presented by Friséen and de Maré [4], has alarm function

$$a(y_{\leq s}) = \sum_{t=1}^s p_\tau(t) / (1 - \sum_{w=1}^s p_\tau(w)) \cdot (f_{Y_{\leq s}}(y_{\leq s} | \tau = t) / f_{Y_{\leq s}}(y_{\leq s} | \tau > s))$$

where  $p_\tau(t) = P[\tau = t]$ . The LR method for the Markov chain,  $X$ , has alarm function

$$\begin{aligned} a(x_{\leq s}) &= (1 - \sum_{t=1}^s p_\tau(t))^{-1} \sum_{t=1}^s \frac{f(x_{\leq s} | \tau = t)}{f(x_{\leq s} | \tau > t)} \\ &= (1 - \sum_{t=1}^s p_\tau(t))^{-1} \left( p_\tau(1) \text{lr}(x(1) | x_0) \prod_{r=2}^s \text{lr}(x(r) | x(r-1)) \right. \\ &\quad \left. + \sum_{t=2}^s p_\tau(t) \prod_{r=t}^s \text{lr}(x(r) | x(r-1)) \right). \end{aligned}$$

By using simulations of the Ising dynamic model, we may evaluate the performance of these surveillance methods.

## 4 Discussion

This study shows that, monitoring a change in interaction in a sequence of finite square lattice Ising dynamic patterns with zero external field, can be performed using methods for surveillance of a Markov chain. It is suggested that simulation exactly according the Ising dynamic model with zero external field, is used for the evaluation of the surveillance methods presented.

Sometimes, methods for treating a simultaneous change in both the chemical potential and the interaction parameters, are necessary. The reason for choosing the Ising model was that it is a simple model which nevertheless possesses a non-trivial property. An extension to e.g. a Potts model is one possibility of how to proceed. Also surveillance in spatial point process models (see e.g. Grabarnik and Särkkä [7]) could be of interest.

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