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Trend-free repeated measurement designs

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TREND-FREE REPEATED MEASUREMENT DESIGNS

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ABSTRACT

The existence and non-existence of trend-free repeated measurement designs are investigated. Two families of efficient / optimal trend-free repeated measurement designs have been identified and methods of construction of these designs are presented.

1. INTRODUCTION

In many industrial, medical and agricultural experiments, treatments are applied to each experimental unit sequentially in time or space. There is a possibility that a systematic effect, or trend, influences the observations in addition to the experimental unit, treatment and residual effects. This type of effects should be taken into account both when the experiment is planned and when the results are analysed. One way to account for the presence of trends is to use the analysis of covariance, treating trend values as covariates. However, one may use suitable designs in the presence of trends to avoid the complications of analysis of covariance and increase design efficiencies.

Some studies in this direction have been made by Atkinson and Donev (1996), Box (1952), Box and Hay (1953), Bailey, Cheng and Kipnis (1992), Bradley and Yeh (1985), Bradley and Odeh (1988), Chai and Majumdar (1993), Cheng (1985), Cheng and Jacroux (1988), Coster (1993), Coster and Cheng (1988), Cox (1951,1952,1958), Daniel (1976, chapter 15), Daniel and wilcoxon (1966), Githinji and Jacroux (1998), Hill (1960), Jaroux (1993, 1994), Jacroux and Ray (1990), Joiner and Campbell (1976), Lin and Dean (1991), Mukerjee and Sengupta (1994), Ogilvie(1963), Philips (1964,1968a,1968b), Prescott (1981), Stufken (1988), Taylor (1967), Yeh and Bradley (1983), Yeh, Bradley and Notz (1985), Whittinghill (1989) and Wilkie (1987).

In the present paper, we consider two families of repeated measurement designs (RMDs) and show that efficient/optimal trend-free RMDs exist.

In Section 2 we give the notation and basic definitions for RMDs. The model and conditions for trend-free RMDs are discussed in Section3. Finally, Section 4 contains the main results.

2. NOTATION AND DEFINITIONS

In an RMD there are t treatments, n experimental units and p periods. Each experimental unit receives one treatment during each period. Thus a RMD denoted by
RMD (t,n,p) is a $p \times n$ array with treatments as entries, rows representing periods and columns as experimental units. The set of all such arbitrary RMDs is designated by $\Omega_{t,n,p}$. Let us formally define some useful concepts.

Definition 2.1. A design $d \in \Omega_{t,n,p}$ is said to be uniform on periods if each treatment is administered λ_1 times during each period.

Definition 2.2. A design $d \in \Omega_{t,n,p}$ is said to be uniform on experimental units if each treatment is administered to each experimental unit λ_2 times.

Definition 2.3. A design $d \in \Omega_{t,n,p}$ is said to be uniform if it is uniform on periods and experimental units simultaneously.

Definition 2.4. A design $d \in \Omega_{t,n,p}$ is said to be balanced for residuals if in the pxn array d each treatment is preceded by each other treatment (not including itself) λ times.

Definition 2.5. A design $d \in \Omega_{t,n,p}$ is said to be strongly balanced for residuals if each treatment is preceded by all the other treatments (including itself) λ times.

The necessary conditions for the respective RMDs listed above are:
$n = \lambda_1 t$, $p = \lambda_2 t$, $n(p-1) = \lambda t(t-1)$ and $n = \lambda_1 t$, $p = \lambda_2 t$, $n(p-1) = \lambda t^2$, respectively.

Definition 2.6. In case p<t we call a design $d \in \Omega_{t,n,p}$ balanced if it is uniform on periods, balanced for residuals and each experimental unit is administered distinct treatments.

Definition 2.7. In case p<t we call a design $d \in \Omega_{t,n,p}$ strongly balanced if it is uniform on periods and strongly balanced for residuals.

The necessary conditions for the respective RMDs listed above are:
$n = \lambda_1 t$, $n(p-1) = \lambda t(t-1)$ and $n = \lambda_1 t$, $n(p-1) = \lambda t^2$ respectively.

Definition 2.8. A design $d \in \Omega_{t,n,p}$ is called circular balanced if the collection of ordered pair (d(i, j), d(i+1, j)), $1 \leq i \leq p$, $1 \leq j \leq n$ contains each pair of distinct treatments λ times.

Definition 2.9. A design $d \in \Omega_{t,n,p}$ is called circular strongly balanced if the collection of ordered pairs (d(i, j), d(i+1, j)), $1 \leq i \leq p$, $1 \leq j \leq n$ contains each ordered pair of treatments (distinct or not) λ times.

In the last two definitions, when i = p, i+1 = 1, since we count the pairs (d(p, j), d(1, j)) as well.

The necessary conditions for the respective designs listed above are:
$np = \lambda t(t-1)$ and $np = \lambda t^2$ respectively.

3. Model and conditions for trend-free RMDS

We assume that, within each experimental unit there is a common polynomial trend of order k on the p periods, which can be expressed by the orthogonal polynomials $\phi_\alpha(l)$, $1 \leq \alpha \leq k$, on $l = 1, \ldots, p$, where ϕ_α is a polynomial of degree α. The polynomials $\phi_1(l)$, $\ldots$, $\phi_k(l)$ satisfy

$$\sum_{l=1}^{p} \phi_\alpha(l) = 0, \qquad \sum_{l=1}^{p} \phi_\alpha(l)\phi_{\alpha'}(l) = \delta_{\alpha\alpha'},$$

Where $\delta_{\alpha\alpha'}$ denotes the Kronecker delta, $\alpha, \alpha' = 1, ..., k$.

Let $\mathbf{X}_\beta = [\mathbf{I}_n \otimes \mathbf{1}_p]$, $\mathbf{X}_\theta = [\mathbf{1}_n \otimes \Phi_k]$, where $\mathbf{1}_n$ is a $n \times 1$ vector of unit elements, $\mathbf{I}_n$ is a $n \times n$ identity matrix, $\otimes$ denotes a Kronecker product and Φ_k is a $p \times k$ matrix with elements $\phi_\alpha(l)$ in row l and column α, $l = 1,..., p$, $\alpha = 1,..., k$. Then listing the response variables in the vector Y in order of period position within successive experimental units, the standard model for a RMD with trend is:

(3.1) $E(Y) = X_\mu \mu + X_\tau \tau + X_\rho \rho + X_\beta \beta + X_\theta \theta$,

Where μ is a constant, $X_\mu = 1_{np}$, τ is the t $\times$1 vector of treatment parameters, $X'_\tau = [\Delta_1 \Delta_2,...,\Delta_n]$ where for i $=1,...,$ n, Δ_n is a pxt matrix whose (k,j)th element is unity if treatment j is applied in period k, and zero otherwise, ρ is the tx1 vector of residuals, $X'_\rho = [\Pi_1, \Pi_2,...,\Pi_n]$ where for i= 1,...,n, Π_i is a pxt matrix whose (k,j) element is unity if treatment k is applied to the period preceding the j-th period, and zero otherwise.

Now we are in a position to define trend-free RMDs.

Definition 3.1 An RMD is trend-free with respect to treatments if $X'_\tau X_\theta = 0$.

Definition 3.2 An RMD is trend-free with respect to residuals if $X'_\rho X_\theta = 0$.

Definition 3.3. An RMD is trend-free if it is trend-free with respect to both treatments and residuals.

For design d let u_{dil} and v_{djl} denote the number of times treatment i appears in period l and the number of times residual j appears in period l, respectively. A design is linear trend-free with respect to treatments if and only if

(3.2) $\sum_{i=1}^{p} u_{dil}\phi_1(l) = 0, \qquad i = 1,...,t.$

A design is linear trend-free with respect to residuals if and only if

(3.3) $\sum_{j=1}^{p} v_{djl}\phi_1(l) = 0, \qquad j = 1,...,t.$

The polynomial, $\phi_1(l)$ satisfies $\phi_1(l) = -\phi_1(p-l+1)$. For k odd $\phi_1((k+1)/2) = 0$. Clearly (3.2) and (3.3) are true whenever

(3.4) $u_{dil} = u_{di(p-l+1)}$ and $v_{djl} = v_{dj(p-l+1)}$ $l = 1,..., [(p+1)/2]$, i, j = 1,...,t,

where [ ] denotes the largest integer less than or equal to (p+1)/2.

Conditions (3.2) and (3.3) do not imply condition (3.4). Condition (3.4) is necessary and sufficient for a design to be odd-degree trend-free [Lin and Dean (1991), corollary 2.1.2].

The following theorem is a modification to Corollary (2.1.1) of Lin and Dean (1991).

Theorem 3.1. A design d $\in$ RMD(t,n,p) which is trend-free with respect to treatments (residuals) for a (p-1)th order polynomial trend exists if and only if it is possible to arrange the treatments (residuals) so that each treatment (residual) occurs $p^{-1}r$ times in each periods.

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4. Main results.

In this section we are going to obtain some classes of trend-free RMDs and RMDs which are trend-free with respect to treatments.

First we shall consider RMDs which are trend-free with respect to treatments.

Theorem 4.1. All the balanced or strongly balanced RMDs which are uniform on periods are also trend-free with respect to treatments.

Proof . Each treatment appears the same number of times in each period so by Theorem (3.1) they are trend-free with respect to treatments.

There are two families of balanced RMDs.

Family 1. p = t. Balanced RMD(t, t, t) exists for all even t. A method of construction is presented below.

Construction 4.1. Construct a txt table in which columns represents experimental units and rows represent periods. Number the t experimental units successively from 1 to t. Assign integer 1 to t to the t cells in the first column by entering successive numbers in every other cell from top to bottom beginning with the first. Reverse the direction, going from bottom to top, once the end of a column has been reached, making sure that the return starts from the t-th cell in the column. Finally, complete each row in a cyclic manner.

Example 4.1. Let t = 4. Then the above procedure produces the following design

$$
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
4 & 1 & 2 & 3 \\
2 & 3 & 4 & 1 \\
3 & 4 & 1 & 2
\end{array}
$$

If t is odd then balanced RMD(t, t, t) does not exist for t = 3, 5, 7. Balanced RMD(t, t, t) exists for t = 9, 15, 21, 27, 39, 55 and 57. The existence and non-existence of balanced RMD(t, t, t) is not known. One can trivially construct balanced RMD(t, 2t, t) for all odd t. An easily remembered method is given next.

Construction 4.2. Construct a design for t experimental units analogous to method 3.1 outlined for t even. Similarly, construct for the remaining t experimental units a design by letting the first column be the reverse of the first column of the design constructed for the initial t units. The following example illustrates the above method.

Example 4.2. Let t = 5. Then the above procedure produces the following design.

$$
\begin{array}{cccccccccc}
1 & 2 & 3 & 4 & 5 & 3 & 4 & 5 & 1 & 2 \\
5 & 1 & 2 & 3 & 4 & 4 & 5 & 1 & 2 & 3 \\
2 & 3 & 4 & 5 & 1 & 2 & 3 & 4 & 5 & 1 \\
4 & 5 & 1 & 2 & 3 & 5 & 1 & 2 & 3 & 4 \\
3 & 4 & 5 & 1 & 2 & 1 & 2 & 3 & 4 & 5
\end{array}
$$

Unfortunately these designs are not trend-free with respect to residuals. To get around this problem one can consider the class of circular balanced RMDs. In this class the structure of residuals and treatments is the same and therefore designs in this class are trend-free.

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One can introduce a preperiod (period 0) giving all the periods residual effects. The period 0 would not be used in analysis. Circular RMDs exist whenever t > 2. Sonnemann, quoted in Kunert (1985), gave the following construction method for a circular balanced uniform RMDs with a minimum number of experimental units whenever t > 2 is an even integer.

Construction 4.3. Let t = 2k and obtain the first set of t columns by developing the column $(2k\ 1\ 2k\text{-}1\ 2\ 2k\text{-}2\ 3\ ...\ k\text{+}1\ k)'$ and call it D_0. Let π = $(\ 1\ 2...k\text{-}1\ 2k\text{-}1...k)$ and let D_i = $\pi^i D_0$, i = 1, 2, ..., t-2. Then D_0, D_1, ..., D_{t-2} can be juxtaposed to obtain the required RMD.

Example 4.3. Let k = 3, t = 6; then π = (12543) and

$$
D_0 = \begin{matrix} 6\ 1\ 2\ 3\ 4\ 5 \\ 1\ 2\ 3\ 4\ 5\ 6 \\ 5\ 6\ 1\ 2\ 3\ 4 \\ 2\ 3\ 4\ 5\ 6\ 1 \\ 4\ 5\ 6\ 1\ 2\ 3 \\ 3\ 4\ 5\ 6\ 1\ 2 \end{matrix}
\qquad
D_1 = \begin{matrix} 6\ 2\ 5\ 1\ 3\ 4 \\ 2\ 5\ 1\ 3\ 4\ 6 \\ 4\ 6\ 2\ 5\ 1\ 3 \\ 5\ 1\ 3\ 4\ 6\ 2 \\ 3\ 4\ 6\ 2\ 5\ 1 \\ 1\ 3\ 4\ 6\ 2\ 5 \end{matrix}
$$

$$
D_2 = \begin{matrix} 6\ 5\ 4\ 2\ 1\ 3 \\ 5\ 4\ 2\ 1\ 3\ 6 \\ 3\ 6\ 5\ 4\ 2\ 1 \\ 4\ 2\ 1\ 3\ 6\ 5 \\ 1\ 3\ 6\ 5\ 4\ 2 \\ 2\ 1\ 3\ 6\ 5\ 4 \end{matrix}
\qquad
D_3 = \begin{matrix} 6\ 4\ 3\ 5\ 2\ 1 \\ 4\ 3\ 5\ 2\ 1\ 6 \\ 1\ 6\ 4\ 3\ 5\ 2 \\ 3\ 5\ 2\ 1\ 6\ 4 \\ 2\ 1\ 6\ 4\ 3\ 5 \\ 5\ 2\ 1\ 6\ 4\ 3 \end{matrix}
$$

$$
D_4 = \begin{matrix} 6\ 3\ 1\ 4\ 5\ 2 \\ 3\ 1\ 4\ 5\ 2\ 6 \\ 2\ 6\ 3\ 1\ 4\ 5 \\ 1\ 4\ 5\ 2\ 6\ 3 \\ 5\ 2\ 6\ 3\ 1\ 4 \\ 4\ 5\ 2\ 6\ 3\ 1 \end{matrix}
$$

Afsarinejad (1990), using disjoint directed Hamiltonian cycles, constructed circular balanced uniform RMDs with minimum number of experimental units whenever t is an odd number.

Construction 4.4. Let t = 2k+1 and label the t treatments by0, 1, 2, ..., 2k. Then construct 2k columns as follows:

$$C_i^+ = (\ 0\ i\ i\text{+}1\ i\text{-}1\ i\text{+}2\ i\text{-}2\ ...\ i\text{+}(k\text{-}1)\ i\text{-}(k\text{-}1)\ i\text{+}k\ )'$$

$$C_i^- = (\ 0\ i\text{+}k\ i\text{-}(k\text{-}1)\ i\text{+}(k\text{+}1)\ ...\ i\text{-}2\ i\text{+}2\ i\text{-}1\ i\text{+}1\ i\ )'$$

for $1 \le i \le k$. All the elements except 0 are taken as the positive integers 1,2,...,2k (mod 2k).

These 2k columns are initial columns of 2k squares, which can be developed in turn to obtain 2k squares. Juxtaposing these squares gives a circular balanced uniform RMD with a minimum number of experimental units.

Example 4.5. Let t = 5. Then the four columns are:

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$$C_1^+ = (0\ 1\ 2\ 4\ 3)'$$
$$C_2^+ = (0\ 2\ 3\ 1\ 4)'$$
$$C_1^- = (0\ 3\ 4\ 2\ 1)'$$
$$C_2^- = (0\ 4\ 1\ 3\ 2)'$$

The required design is:

```
0 1 2 4 3 0 2 3 1 4 0 3 4 2 1 0 4 1 3 2
1 2 4 3 0 2 3 1 4 0 3 4 2 1 0 4 1 3 2 0
2 4 3 0 1 3 1 4 0 2 4 2 1 0 3 1 3 2 0 4
4 3 0 1 2 1 4 0 2 3 2 1 0 3 4 3 2 0 4 1
3 0 1 2 4 4 0 2 3 1 1 0 3 4 2 2 0 4 1 3
```

Family 2. p < t.
Afsarinejad (1983) constructed all the possible balanced minimal RMDs and strongly balanced minimal RMDs. These designs are all trend-free with respect to treatments.
Construction 4.5. A balanced minimal RMD can be obtained by developing, in turn, mod t, each of the following λ_1 columns.

$$c_1\ c_p \ldots c_{(i-1)p-(i-2)} \ldots c_{(\lambda_1-1)p-(\lambda_1-2)}$$
$$c_2\ c_{p+1} \ldots c_{(i-1)p-(i-3)} \cdots c_{(\lambda_1-1)p-(\lambda_1-3)}$$

$$\begin{matrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{matrix}$$

$$c_p\ c_{2p+1} \ldots c_{ip-(i-1)} \quad \ldots \quad c_t$$

where $(c_1, c_2, \ldots, c_t) =$

$(1, t, 2, t\text{-}1, 3, \ldots, t/2, (t+2)/2)$	if t is even
$(1, t, 3, t\text{-}2, 5, \ldots, (t+1)/2, (t+5)/2, \ldots, t,1)$	if $t = 4\gamma+1$
$(1, t, 3, t\text{-}2, 5, \ldots, (t-1)/2, (t+3)/2, \ldots, t, 1)$	if $t = 4\gamma + 3$.

The set $U_{i=2}^t (c_i - c_{i-1})$ contains each non-zero number mod t and so each pair of distinct treatments will appear once in the final design.
Example 4.6. Let t = 10 and p = 4. Then the above procedure produces the following column: (1, 10, 2, 9, 3, 8, 4, 7, 5, 6) and the design is presented below.

```
1  2  3  4  5  6  7  8  9  10 9  10 1  2  3  4  5  6  7  8  4  5  6  7  8  9  10 1  2  3
10 1  2  3  4  5  6  7  8  9  3  4  5  6  7  8  9  10 1  2  7  8  9  10 1  2  3  4  5  6
2  3  4  5  6  7  8  9  10 1  8  9  10 1  2  3  4  5  6  7  5  6  7  8  9  10 1  2  3  4
9  10 1  2  3  4  5  6  7  8  4  5  6  7  8  9  10 1  2  3  6  7  8  9  10 1  2  3  4  5
```

Now, let t = 9 and p = 5. Then the above procedure produces the following column:
(1, 9, 3, 7, 5, 7, 3, 9, 1) and the design is given below.

```
1 2 3 4 5 6 7 8 9 5 6 7 8 9 1 2 3 4
9 1 2 3 4 5 6 7 8 7 8 9 1 2 3 4 5 6
3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2
7 8 9 1 2 3 4 5 6 9 1 2 3 4 5 6 7 8
5 6 7 8 9 1 2 3 4 1 2 3 4 5 6 7 8 9
```

The following construction method due to Afsarinejad (1983) establishes the existence of strongly balanced minimal RMDs whenever $p < t$.

Construction 4.6. The construction method is the same as Construction (3.5) with the exception that the column is:

$(c_1, c_2, ..., c_t, c_{t+1}) =$

$(1, t, 2, t-1, ..., t/2, (t+2)/2, (t+2)/2)$	if t is even;
$(1, t, 3, t-2, ..., (t+5)/2, (t+1)/2, (t+1)/2, ..., t, 1)$	if $t = 4\gamma+1$;
$(1, t, 3, t-2, ..., (t-1)/2, (t+3)/2, (t+3)/2, ..., t, 1)$	if $t = 4\gamma+ 3$.

The set U_2^{t+1} $(c_i - c_{i-1})$ contains each number mod t and so each ordered pair of treatments will appear precisely once in the final design.

Example 4.7. Let t = 10 and p= 6. Then, $(c_1, c_2, ..., c_{11}) = (1, 10, 2, 9, 3, 8, 4, 7,5, 6, 6)$ and the deign obtained is presented below.

```
1 2 3 4 5 6 7 8 9 10 8 9 10 1 2 3 4 5 6 7
10 1 2 3 4 5 6 7 8 9 4 5 6 7 8 9 10 1 2 3
2 3 4 5 6 7 8 9 10 1 7 8 9 10 1 2 3 4 5 6
9 10 1 2 3 4 5 6 7 8 5 6 7 8 9 10 1 2 3 4
3 4 5 6 7 8 9 10 1 2 6 7 8 9 10 1 2 3 4 5
8 9 10 1 2 3 4 5 6 7 6 7 8 9 10 1 2 3 4 5
```

Now , let t = 9 and p = 4. Then
$(c_1, c_2, ..., c_{10}) = (1, 9, 3, 7, 5, 5, 7, 3, 9,1)$ and the design is given below.

```
1 2 3 4 5 6 7 8 9 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6
9 1 2 3 4 5 6 7 8 5 6 7 8 9 1 2 3 4 3 4 5 6 7 8 9 1 2
3 4 5 6 7 8 9 1 2 5 6 7 8 9 1 2 3 4 9 1 2 3 4 5 6 7 8
7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 1 2 3 4 5 6 7 8 9
```

Again, these designs are not trend-free with respect to residual effects. Designs, which are trend-free with respect to both treatment effects and residual effects, exist and they are called circular balanced RMDs.

Fortunately, circular balanced RMDs can be constructed using nearest neighbour designs. The construction of these designs was given by Rees (1967) and has been considered by several authors, for example, Hwang (1973), Hwang and Lin (1974, 1976, 1977, 1978), Lawless (1971) and Street (1982), Street and Street (1987, chapter 14). We

shall present a construction method and for the rest of methods we refer the interested readers to the above mentioned authors.

All it is needed is to construct an initial block to satisfy the following conditions, (I) all the differences, forward and backward, must be distinct; (II) the sum of the forward differences must be zero (mod t). Then use this initial block and develop a design cyclically. Now find the mirror image of this initial block and develop another design cyclically. These two designs together constitute a circular balanced RMD.

Construction 4.7. Given $t = 4\gamma + 3$ and $p = 2\gamma + 1$ circular RMDs exists whenever t is a prime power. Let x be a primitive root then the initial block for the first part of deign would be : $(x^0, x^2, x^4, \dots, x^{4\gamma})$ and the initial block for the second part of design would be $(x^{4\gamma}, \dots, x^4, x^2, x^0)$.

Example 4.7. Let t = 7 and p = 3. Then initial block of the first part is. (1, 2 , 4) and the initial block of second part is (4 , 2 , 1). The design is:

```
1  2  3  4  5  6  7  4  5  6  7  1  2  3
2  3  4  5  6  7  1  2  3  4  5  6  7  1
4  5  6  7  1  2  3  1  2  3  4  5  6  7
```

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