Research Report
Department of Statistics
Göteborg University
Sweden

Testing for cointegrating relations -
A bootstrap approach

Panagiotis Mantalos
Ghazi Shukur

Testing for Cointegrating Relations-

A Bootstrap Approach

Panagiotis Mantalos

Department of Statistics

Lund University

Ghazi Shukur

Department of Statistics

Göteborg University

ABSTRACT

Using Monte Carlo methods together with the Bootstrap critical values, we have studied the properties of two tests (Trace and L-max), derived by Johansen (1988) for testing for cointegration in VAR systems. Regarding the size of the tests, the results show that both of the test methods perform satisfactorily when there are mixed stationary and nonstationary components in the model. The analyses of the power functions indicate that both of the test methods can effectively detect the present of cointegration vector(s). Finally, when considering the size and power properties, we could not find any noticeable differences between the two test methods.

Keywords: Testing for cointegration in VAR systems, Bootstrap, Monte Carlo methods.

1. Introduction

To test for cointegration among a set of economic time series, various test procedures have been proposed. The most frequently used methods for testing for cointegration in empirical investigations are the Johansen (1988, 1991), and Johansen and Juselius (1990a). These tests are derived from the likelihood- based method for testing for cointegration in a Vector Autoregressive (VAR) Model.

1

Recently, a number of authors have proposed the Bootstrap method to improve the critical values of the cointegration test in order that the size of the test approaches its nominal value. Mantalos and Shukur (1998) improved the critical values in the Error Correction Model (ECM) cointegration test by employing Bootstrap technique. Given the "Bootstrap critical values", they studied the size and power of the cointegration test based on ECM. Van Giersbergen (1996), using the stationary Bootstrap, studied the Johansen trace statistic in VAR models. Mantalos (1998) examined the VECM cointegration test by employing Bootstrap technique, and found that the Bootstrap method has the right size properties when testing the null hypothesis of no cointegrating relations.

While it might be supposed that a Bootstrap test should do at least as well as the corresponding asymptotic test, Harris and Judge's (1998) found, by Bootstrapping the Johansen test, that the Bootstrap test statistic has poor size properties when there are mixed stationary and nonstationary components in the model, i.e. when there is at least one cointegration vector present. Note that Harris and Judge (1998) results from Bootstrapping the Johansen's test, for 50 observations and 0-cointegration vectors, are the same as Mantalos (1998) results, i.e. 0.05 actual size for 5 % nominal size and 0.10 for 10% (see Harris and Judge (1998) table1, Mantalos (1998) table 1).

In general, however, the results of the above - mentioned studies is the confirmation that the Bootstrap test approach appears to do very well when we test if there is cointegration or not. But there remain some questions that should be answered; for example,

a) Do the bootstrap test works well when there are mixed stationary and nonstationary components in the model?

b) How the Bootstrap test behaves in the case of overspecifying the possible number of variables in the cointegrating vector(s)?

In this paper, we address these remaining problems by Bootstrapping the Johansen test. We demonstrate a way to apply the VAR cointegration test by using the Bootstrap critical values. And we show the efficiency of the Bootstrap method to improve the critical values of the

VAR cointegration test for testing more than one cointegrating relation, in such a way that the size of the test approaches its nominal value.

The paper is arranged as follows. In the next section we present the model we analyse. In Section 3 we introduce the "Bootstrap test". Section 4 presents the design of our Monte Carlo experiment. In Section 5 we describe the results concerning the size of the test, while power is analysed in Section 6. Finally, a brief summary and conclusions are presented in Section 7.

2. Model Specification

Let $\mathbf{Y}_t$ denote the entire vector of the integrated of order one I(1) time series under study, of dimension k x 1, from the following data generating process:

$$\Delta \mathbf{Y}_t = u_t, \tag{1}$$

$$u_t = C(L)\mathbf{e}_t, \tag{2}$$

where $C(L)$ denotes an infinite order polynomial in the lag operator[1] that is,

$$C(L) = \sum_{j=0}^{\infty} C_j L^j, \tag{3}$$

$$\mathbf{e}_t \sim \text{i.i.d.}(0, \Sigma_e), \tag{4}$$

where $\mathbf{Y}_t = \left(y_1 y_2 \cdots y_k\right)'$, $\Delta = y_t - y_{t-1}$, and t= 1, ..., T. Then from the Granger Representation Theorem (Engle and Granger (1987)) we know that, if $\mathbf{Y}_t$ is cointegrated with cointegrating rank r, it has the error correction representation :

$$\Delta \mathbf{Y}_t = \alpha \beta' \mathbf{Y}_{t-1} + \mathbf{e}_t, \tag{5}$$

---

[1] The lag operator is indicated here by "L" and for any integer k , $L^k x_t = x_{t-k}$

where β is a (k x r) matrix of cointegrating vectors, and α is (k x r) matrix of adjustment coefficients. In the regression model (5), the null hypothesis of correct specification implies that the error terms e_t will be independently and identically distributed, and this is the case when the $u_t \sim \text{i.i.d.}(0, \Sigma_u)$. However, because this assumption might be too optimistic we extend our analysis to the following p-th order VECM:

$$\Delta \mathbf{Y}_t = \sum_{i=1}^{p-1} A_i \Delta \mathbf{Y}_{t-i} + \alpha \beta' \mathbf{Y}_{t-p} + \mathbf{e}_t, \quad t = 1, \ldots, T, \tag{5a}$$

where p is enough large so that $\mathbf{e}_t$ is, or is near to a zero mean independent white noise process with non-singular covariance matrix Σ_e.

The maximum likelihood estimation of (5a) by Reduced Rank Regression (RRR) for $\mathbf{e}_t \sim \text{i.i.d.}(0, \Sigma_e)$ and the asymptotic properties of this estimator is discussed in Johansen (1988) and here we give a brief overview of the RRR.

To introduce the RRR estimators and further the two versions of the LR test statistics, the trace and λ-max statistic we first define by regressing $\Delta \mathbf{Y}_t$ on $\Delta \mathbf{Y}_{t-1}, \cdots, \Delta \mathbf{Y}_{t-p}$, and $\mathbf{Y}_{t-p}$ on $\Delta \mathbf{Y}_{t-1}, \cdots, \Delta \mathbf{Y}_{t-p}$, the residuals $\hat{\mathbf{u}}_t$, $\hat{\mathbf{w}}_t$ respectively. And then, given the residuals $\hat{\mathbf{u}}_t$, $\hat{\mathbf{w}}_t$ we require the cross product matrices:

$$S_{uu} = T^{-1} \sum_{t=1}^{T} \hat{\mathbf{u}}_t \hat{\mathbf{u}}_t', \tag{6}$$

$$S_{yu} = T^{-1} \sum_{t=1}^{T} \hat{\mathbf{w}}_{t-1} \hat{\mathbf{u}}_t', \tag{7}$$

$$S_{yy} = T^{-1} \sum_{t=1}^{T} \hat{\mathbf{w}}_{t-1} \hat{\mathbf{w}}_{t-1}', \tag{8}$$

and

$$S_{uy} = S_{yu}'. \tag{9}$$

Define now:

$$S(\lambda) = \lambda S_{yy} - S_{yu} S_{uu}^{-1} S_{uy}, \tag{10}$$

then the RRR estimator of the cointegrating vectors, $\hat{\beta}$, is the (k x r) matrix of the eigenvectors corresponding to the r largest eigenvalues satisfying:

$|S(\lambda)| = 0$.

Then given the k ordered eigenvalues $\lambda_1 > \cdots > \lambda_k > 0$, and to test that there are at most r cointegrating vectors, Johansen (1988) derives two likelihood ratio (LR) tests, the *trace* statistic:

$$LR_\lambda = -T \sum_{i=r+1}^{k} \ln(1 - \hat{\lambda}_i),$$ (11)

and the maximum-eigenvalue, λ-max statistic:

$$LR_{\lambda-\max} = -T \ln(1 - \hat{\lambda}_{i+1}), \quad i = 0, 1, \cdots, k-2, k-1.$$ (12)

3. Bootstrap Hypothesis Testing, Critical Values

In the case of the Johansen's test, the distributions of the used test statistics are known only asymptotically. Johansen (1988) tabulated simulated values for selected percentiles of those asymptotic distributions. As a result, when using those critical values, the tests may not have the correct size in small samples, and inferential comparisons and judgements based on them might be misleading. However, by using Bootstrap technique we try to improve the critical values so that the true size of the test approaches its nominal value.

According to the basic principle of Bootstrap hypothesis testing, the Bootstrap data should be generated from the model under the null hypothesis. In our study, with k variables we have k different null hypotheses:

$H_{01} = 0$ CI, $H_{02} = 1$ CI, $\cdots H_{0k} = r = k-1$ CI.

Then the following hypothesis: $H_0 : \alpha\beta' = $ rank $r = 0$ is used to test whether the variables are cointegrated or not. And the Bootstrap data should be generated from model (5a) with the restriction that $\hat{\beta}$ be equal to zero, where $\hat{\beta}$ is the ML estimator of the β in (5a).

When the variables are cointegrated the Bootstrap data should be generated from model (5a) with the restriction that the $\hat{\beta}$ will only contain the i-largest eigenvectors, $i = 0, 1, \cdots, k-2, k-1$, for testing that there are at the most i cointegration vectors. A direct residual resampling gives:

$$\Delta \mathbf{Y}_t^* = \sum_{i=1}^{p-1} \hat{A}_i \Delta \mathbf{Y}_{t-i}^* + \mathbf{e}_t^*, \tag{5b}$$

to test if the variables are cointegrated, and

$$\Delta \mathbf{Y}_t^* = \sum_{i=1}^{p-1} \hat{A}_i \Delta \mathbf{Y}_{t-i}^* + \hat{\alpha}\breve{\beta}' \mathbf{Y}_{t-1}^* + \mathbf{e}_t^*, \tag{5c}$$

to test that there are at the most i cointegration vectors. In (5c), $\mathbf{e}_t^*$ are i.i.d observations $\mathbf{e}_1^*,...,\mathbf{e}_T^*$, drawn from the empirical distribution ($\hat{F}_e$) putting mass 1/T at the adjusted ML residuals ($\hat{\mathbf{e}}_t - \overline{\mathbf{e}}_t$) from models (5b), (5c) by using the RRR procedure. $\breve{\beta}$ is the restricted $\hat{\beta}$ that contains only the i largest eigenvectors, t = 1, . . ., T. This method is called the *Bootstrap based on residuals*, abbreviated RB, proposed by Efron (1979). Note that, in what follows, all Bootstrap statistics will marked by an asterisk $^{(*)}$.

Let us denote by T_s the LR test statistics (11) and (12) as given in chapter 2, and by T_s^* the "Bootstrap LR test statistics". Then the basic principle of Bootstrap hypothesis testing is, then, to draw a number of "Bootstrap samples" from the model under the null hypothesis, calculate the "Bootstrap test statistics" (T_s^*), and compare it with the observed test statistic.

The "Bootstrap test statistics" (T_s^*) can then be calculated by repeating this step N_b number of times. We then take the $(1-\alpha):th$ quintile of the Bootstrap distribution of T_s^* and get the α - level "Bootstrap critical values" ($c_{t\alpha}^*$). And, finally, we then reject the null hypothesis if $T_s \geq c_{t\alpha}^*$. This is our Bootstrap test approach to investigate the size by using the conventional way to report the results, i.e. in form of tables. A Bootstrap estimate of the P-value for testing is P*$\{ T_s^* \geq T_s \}$ and this approach we use to examine the size of the Bootstrap test with the help of the "P-value *plot*". The number of the Bootstrap sample used to estimate Bootstrap critical values and P-values in our study is $N_b = 200$.

4. The Monte Carlo Experiment

In this section we illustrate in a Monte Carlo study the cointegration test in VECM systems with I(1) variables. We calculate the *estimated size* by simply observing how many times the null is rejected in repeated samples under conditions where the null is true. To judge the reasonability of the results we use an approximated 95% confidence interval for the actual size (π):

$$\hat{\pi} \pm 2\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{N}} , \tag{13}$$

where $\hat{\pi}$ is the estimated size and N is the number of replications. It should also be noted that, in our Monte Carlo experiment, we use 10.000 replications per model for the estimating of the size. Accordingly, the confidence interval when using the 5% significance level, will lie between 4,56% and 5,44%. Results that lie outside the above interval will be considered as an indication of bad performances.

Following the Monte Carlo analysis of Podivinsky (1998), we have generate the following process:

$$\Delta_1 \mathbf{Y} = \begin{bmatrix} \Delta_1 y_1 \\ \Delta_1 y_2 \\ \Delta_1 y_3 \end{bmatrix}_t = \mathbf{e}_t, \tag{14}$$

for analysing the size of the test of no cointegration, and

$$\Delta_1 \mathbf{Y} = \begin{bmatrix} \Delta_1 y_1 \\ \Delta_1 y_2 \\ \Delta_1 y_3 \end{bmatrix}_t = \begin{bmatrix} -0.4 & 0.1 \\ 0.1 & 0.2 \\ 0.1 & 0.3 \end{bmatrix} \begin{bmatrix} 1 & -2 & \theta \\ \gamma & -\gamma/2 & -\gamma\theta/2 \end{bmatrix} \mathbf{Y}_{t-1} + \mathbf{e}_t, \tag{15}$$

to study the size and the power of the tests when there is one and two cointegration vectors, and where $\mathbf{e}_t \sim \text{I.I.}N(0,100 I_3)$, $\mathbf{Y}_{t-1} = (y_1 y_2 y_3)'_{t-1}$, $\Delta_1 = y_t - y_{t-1}$. The process has cointegration rank r = 1 if $\gamma = 0$ and rank r = 2 if $\gamma = 1$. Therefor we use $\gamma = 0$ to study the size and power of the tests when there is only one cointegration vector ($y_{1t} = 2y_{2t} - \theta y_{3t}$), and we use the value $\gamma = 1$ to study the size and power of the tests when there are two cointegration vectors (i.e. the $y_{1t} = 2y_{2t} - \theta y_{3t}$ and $y_{1t} = (y_{2t} + \theta y_{3t})/2$). For $\theta = 0$, only two variables are in cointegration vectors and when $\theta = 1$ all three variables are in cointegration vectors.

The order of p of the process (5a) is assumed to be known, and we fitted the (5a) model with p=1. For that reason, the RRR estimators require the cross product matrices:

$$S_{uu} = T^{-1} \sum_{t=1}^{T} \Delta \mathbf{Y}_t \Delta \mathbf{Y}_t', \qquad (6a)$$

$$S_{yu} = T^{-1} \sum_{t=1}^{T} \mathbf{Y}_{t-1} \Delta \mathbf{Y}_t', \qquad (7a)$$

$$S_{yy} = T^{-1} \sum_{t=1}^{T} \mathbf{Y}_{t-1} \mathbf{Y}_{t-1}'. \qquad (8a)$$

However, since the assumption, that the order of p of the process (5a) is known might be too optimistic, we also fitted the (5a) model with p=3.

For each time series, 25 presample values are generated with zero initial conditions, taking net sample sizes of T = 50, 75 and 100. Finally, the " Bootstrap test statistic " (T_s^*) is calculated. Since in our study, we are primarily interested in the behaviour of the tails of the distributions, only results using the conventional 5% nominal significance level have been analysed in Table 1.

5. Analysis of the Size of the Tests

In this section we present the results of our Monte Carlo experiment concerning the sizes of the Bootstrap tests statistics using VECM models. A conventional way to report the results of a Monte Carlo experiment is to tabulate the proportion of how many times the null hypothesis is rejected in repeated samples under conditions where the null is true. In Table 1, we present the estimated size for both versions of the Johansen (1988) cointegration test at the 5% nominal size. The table shows the properties of the tests under conditions when two or three variables are included in the model, and when there are none, one or two cointegration vector(s). Moreover, the test properties have been investigated for different number of observations in the samples. In the case of 50 observation, we study the combination when two or three variables are included in the model, and when, there are none, one or two cointegration vector(s). For the sample sizes of 75 and 100 observations, results are shown only in the case when three variables are used in the model. Results from the other case of two

8

variables have shown to be identical to those of three variables and will not be referred to in what follows.

Now, looking at the first column with 50 observations, both the Trace and L-max tests exhibit the correct size when we have two variables and one or two cointegration vector(s). In the case of three variables, the tests perform satisfactorily when there are none or one cointegration vector. In the case of two cointegration vectors, both tests under-reject very slightly. The same conclusions hold for the case with 75 observations, but in the case of two cointegration vectors, both tests lay almost on the lower bound (4.56 –5.44) of the estimated confidence interval. Finally, when we have 100 observations, the L-max test performs well in all situations, while the Trace test under-reject slightly in the absence of cointegration vector.

Table 1. Estimated Size for the Cointegration Test at 5% Nominal Size

	50 OBS				75 OBS		100 OBS	
5%	(2V)		(3V)		(3V)		(3V)	
	TRACE	L-MAX	TRACE	L-MAX	TRACE	L-MAX	TRACE	L-MAX
0 CI	–	–	0.0467	0.0484	0.0496	0.0491	0.0446	0.0468
1 CI	0.0468	0.0469	0.0468	0.0476	0.0496	0.0498	0.0511	0.0515
2 CI	0.0499	0.0499	0.0445	0.0445	0.0454	0.0454	0.0482	0.0482

The (2V) indicates two while (3V) indicates three variables in the cointegration vector(s).

Davidson and MacKinnon (1997) argue that graphical methods provide more information, about the size of the tests, and in our case we use the P-value plots that are easy to implement and interpret.

The graphs we use in our study, the P-value plots and Size-Power plots, are based on the empirical distribution function (EDF) of the P-values of (T_s). More precisely, we use only the Wald-Bootstrap test, and then, the P-value associated with (T_s) is P*$\{ T_s^* \geq T_s \}$, where

s = 1, ... , N, and N is the number of replications in our Monte Carlo experiment.

The (EDF) of the P_s is simply an estimate of the cumulated density function (c.d.f.) of $p(T_s)$ which at any point x_j in the (0,1) interval, it is defined by

$$\hat{F}(x_j) \equiv \frac{1}{N} \sum_{s=1}^{N} I(P_s \le x_j),$$

(16)

where $I(P_s \le x_j)$ is an indicator function that takes the value 1 if its argument is true and 0 otherwise. The EDF (16) is either evaluated at every data point, or if the N is large, as often it is, only at m points j = 1, ..., m. Davidson and MacKinnon (1997) advocate the use of m =107 corresponding to intervals 0,01 with 0,002 extra points in the tails to do not miss any unusual behaviour in tails, or m = 217, with intervals 0,005 and 0,001 extra points in the tails. However, in our cse, we evaluate at m = 217 points. Finally, we mention that the P-value plot we use is a simple plot of $\hat{F}(x_j)$ against x_j.

Figure 1: P-value plots for Bootstrap tests -Trace, L-max for no cointegration.

Figure 1a: 50 obs, 0 CI

Figure 1a: 75 obs, 0 CI
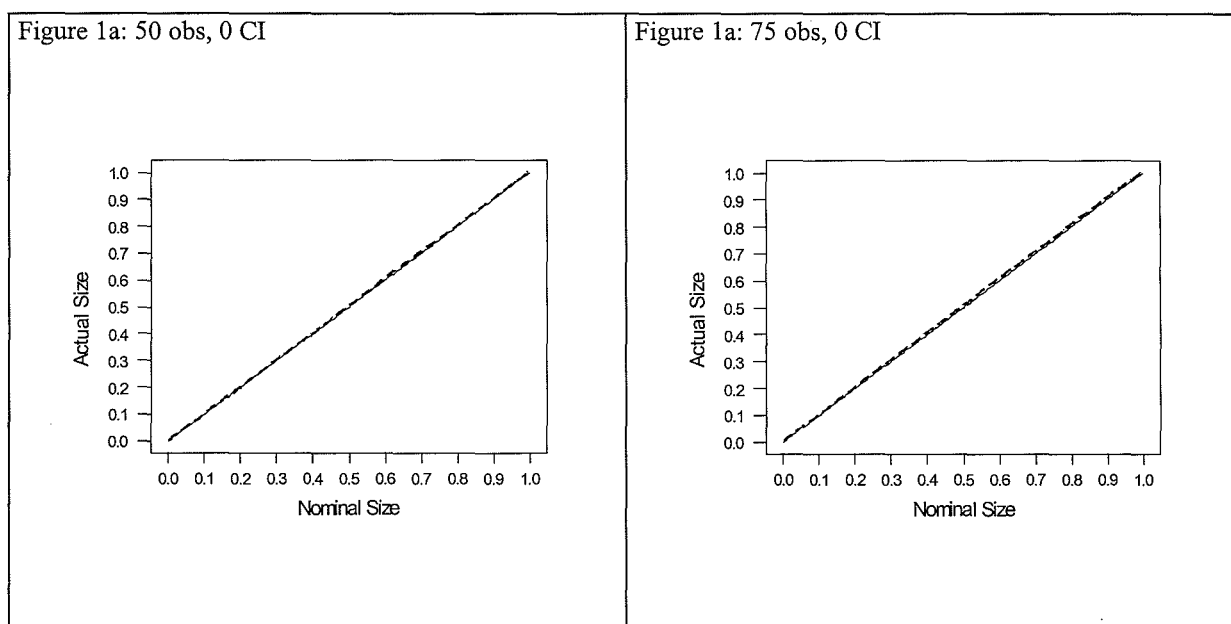
Now as for the P-value plots, we have that if the distribution used to compute the p_s is correct, each of the p_s should be distributed as uniform (0,1). Therefore, when $\hat{F}(x_j)$ is plotted against x_j, the resulting graph should be close to the 45° line. Figure 1 shows some of the advantages of the P-value plots, since they make it easy to distinguish between tests that work badly, and test that work well. The P-value plots should also make it possible and easy to distinguish between tests that systematically over-reject or under-reject, and tests that reject the null hypothesis about the right proportion of the time. In our case, the Bootstrap test rejects the null hypothesis about the right proportion of the time.

Figure 2: Truncate P-value plots for Bootstrap Trace and L-max tests, no cointegration vector.
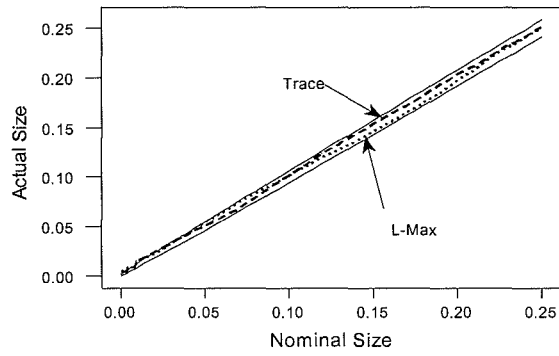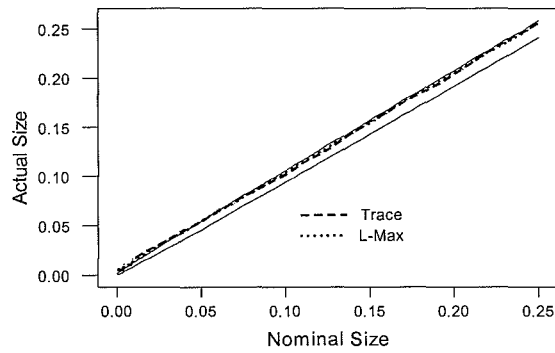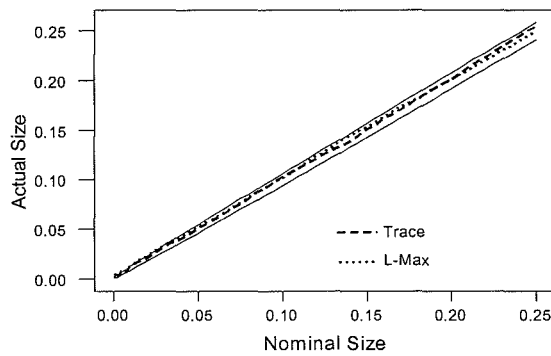
Figure 2a: 50 obs.

Figure 2b: 75 obs.

Figure 2c: 100 obs.

Note that the whole lines for the figures are the approximate 95% confidence interval for actual size.

Figure 3: P value plots for Bootstrap tests-Trace, L-max for 1-2 cointegration vector(s).
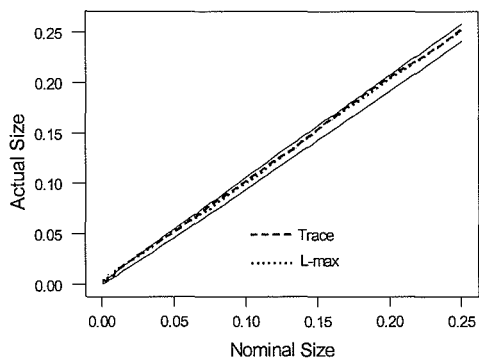
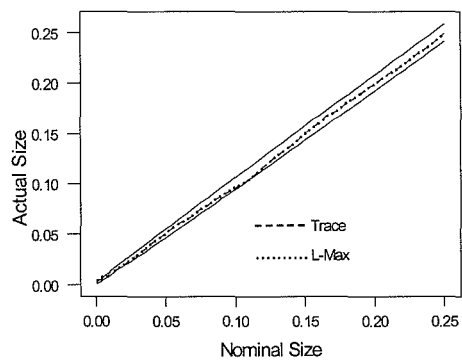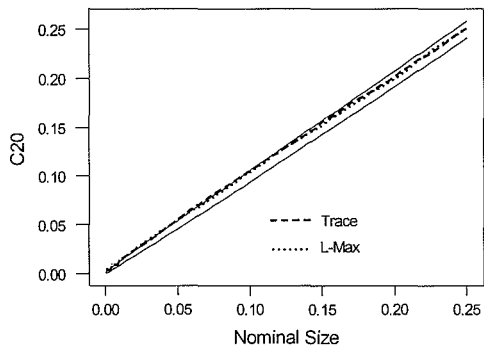| Figure 3a: 50 obs, CI	Figure 3b: 50 obs, CI

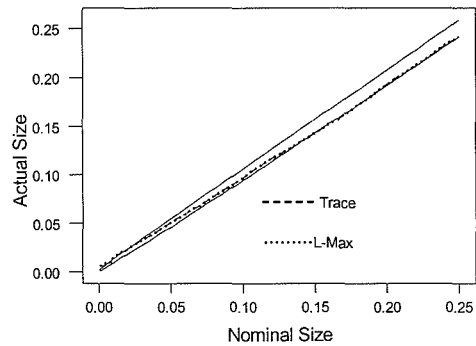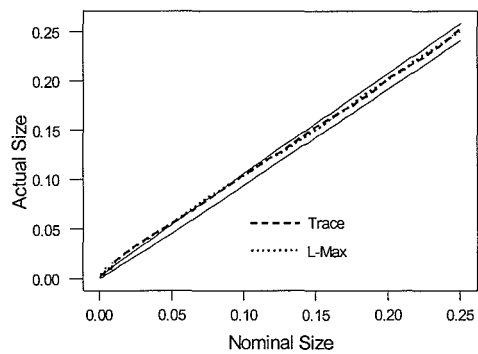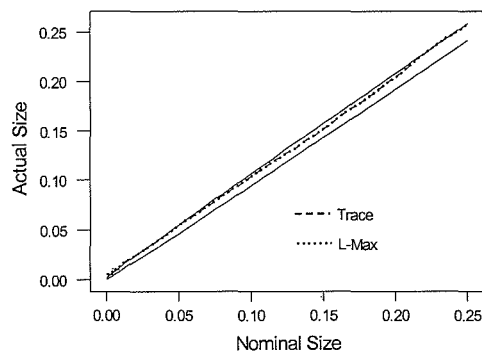| Figure 3c: 75 obs, 1CI	Figure 3d: 75 obs, CI

Figure 3e: 100 Obs 1CI	Figure 3f: 100 Obs 2CI

Note that the whole lines for the figures are the approximate 95% confidence interval for actual size.

The disadvantage of these P-value plots is that we cannot see patterns in the behaviour of the tests when all perform well (see Figure 1). To reduce this problem, we truncate hereafter the P-value plots at x = 0.25.

Figure 2, shows the truncated P-value plots for the Bootstrap tests for 50, 75, and 100 observations and in the case when there is no cointegration vector in the VAR(1) model. While in Figure 3, we present results when there are one or two cointegration vector(s). Looking at these figures, it is not difficult to make the inference that, using the Bootstrap critical values, both tests perform adequately especially in large samples. Only occasionally, the results can lie either on the confidence bounds or strictly slightly outside the bounds. Generally speaking, in this study, the Bootstrap test rejects sufficiently under the null hypothesis and it is difficult to se any noticeable differences between the performances of the Trace and L-max tests. In the case when there is no cointegration, the Bootstrap test rejects reject about the right proportion of the time for all samples (50, 75 and 100). Finally, there are not noticeable effects on the size of the tests from including two extra lags in model (5a), i.e. by fitting a VAR(3) model, so it is not shown here.

The conclusion of our size investigation is that Bootstrap works well even when there mixed stationary and nonstationary components in the model. That is, the Bootstrap test behaves approximately the way that we expected, and the plots look roughly like 45° lines, for all samples

6. Analysis of the Power of the Tests

In this section we discuss the most interesting results of our Monte Carlo experiment, designed to gather evidence concerning the power of the various test versions. We analyse the power of the Bootstrap test using sample sizes 50, 75, and 100 observations. The power functions are estimated by calculating the rejection frequencies in 1000 replications using values of $\gamma = 0$ in (15), when there is only one cointegration vector ($y_{1t} = 2y_{2t} - \theta y_{3t}$), and $\gamma =$ 1 when there are two cointegration vectors (i.e. $y_{1t} = 2y_{2t} - \theta y_{3t}$) and $y_{1t} = \left(y_{2t} + \theta y_{3t} \right) / 2$).

For $\theta = 1$ all three variables are in cointegration vectors. For $\theta = 0$, only two variables are in cointegration vectors, and in this case the power functions are estimated by calculating the rejection frequencies in 1000 replications using a sample size of 50 observations.

In this section we present *the estimated power functions* of the tests in various graphical forms. We use the Size-Power Curves to compare the power functions of alternative test statistics. The main point of interest is to compare the results for the different tests in different sets of conditions. We follow the same process to evaluate the EDF's denoted as $\hat{F}^{\oplus}(x_j)$ by using the same sequence of random numbers as for the size of the tests (see section 5). We then plot the estimated power functions against the estimated size, that is, $\hat{F}^{\oplus}(x_j)$ against $\hat{F}(x_j)$, to obtain the Size-Power curves.

When we apply model (15), all the estimated power functions quickly exhibit the maximal power value of 100%, which make it impossible to visually detect any differences in the power functions of the tests in these situations. The reason behind this is mainly the magnitude of the parameters used in model (15), which, of course, signify a rapidly increasing power function. Hence, to allow a realistic picture of the power functions, and to make it easier to see if there are any differences in the properties between the models, we chose to use half of the coefficients in (15) and apply the following model instead:

$$\Delta_1 Y = \begin{bmatrix} \Delta_1 y_1 \\ \Delta_1 y_2 \\ \Delta_1 y_3 \end{bmatrix}_t = \begin{bmatrix} -0.2 & 0.05 \\ 0.05 & 0.1 \\ 0.05 & 0.15 \end{bmatrix} \begin{bmatrix} 1 & -2 & \theta \\ \gamma & -\gamma/2 & -\gamma\theta/2 \end{bmatrix} \mathbf{Y}_{t-1} + \mathbf{e}_t \tag{15a}$$

In Figure 4, we present results for the Bootstrap test for no cointegration relations when the true model has one cointegration vector or two cointegration vectors. Looking at this figure, we can see that the Trace test and the L-max test have almost the same power. We can see that even when we use model (15a), the tests exhibit sufficient power.

Figure 4: Power-Size plots for Trace and L-max statistics, Ho: No cointegration relations.

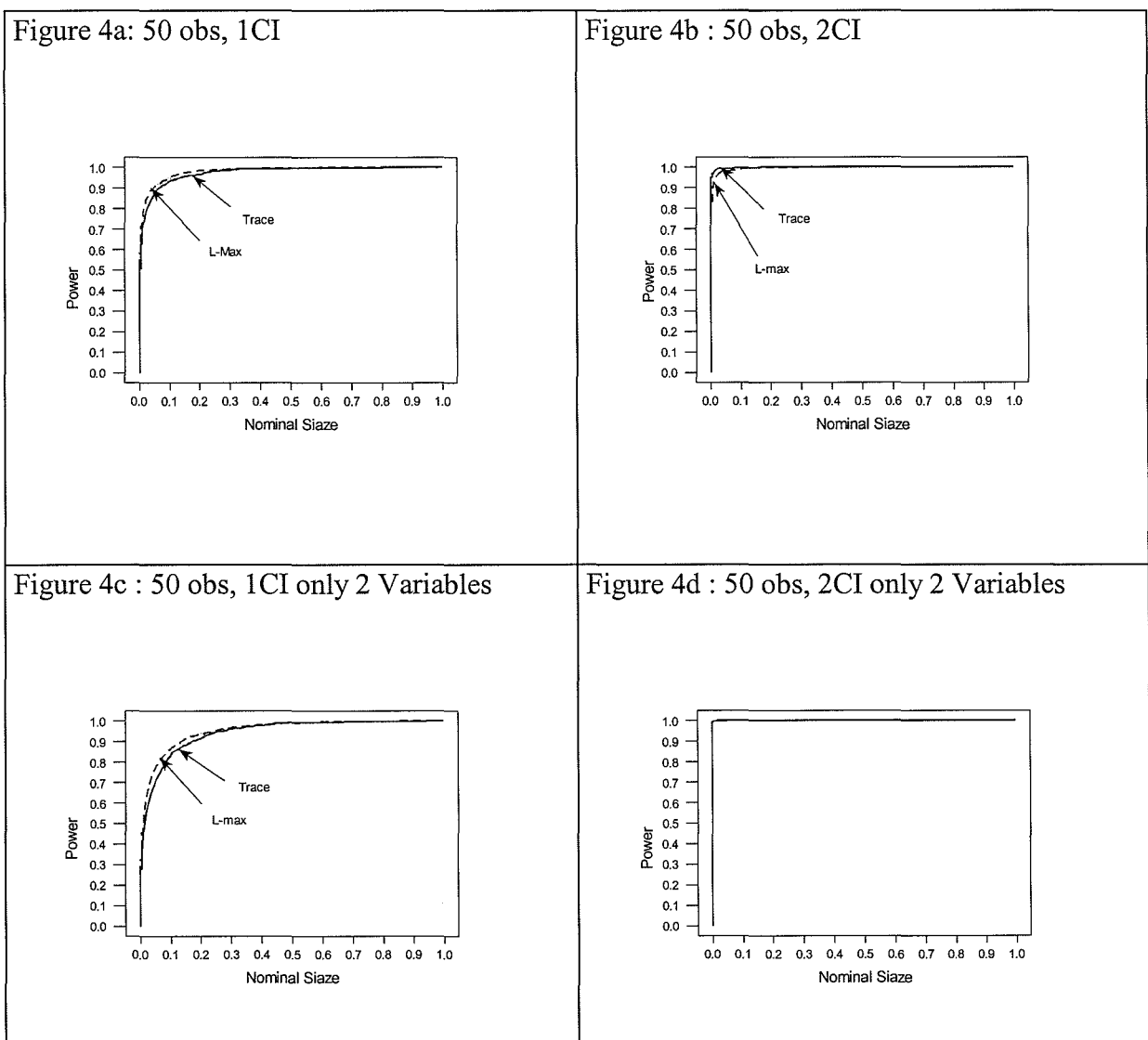| Figure 4a: 50 obs, 1CI	Figure 4b : 50 obs, 2CI
Figure 4c : 50 obs, 1CI only 2 Variables	Figure 4d : 50 obs, 2CI only 2 Variables

Figure 5 presents the Size-Power Curves for the tests, when there are two or three variables in the cointegration vectors. As we see, there is no effects on the power of the tests if there are 2 or 3 variables in the cointegration vector.

Figure 6a shows that the Bootstrap tests have almost the same power in both cases, when we test if there is cointegration or not and when we test if there is one cointegration vector and all three variables are in cointegration vector. On the other hand, Figure 6b shows that there is a slight different between these two cases.

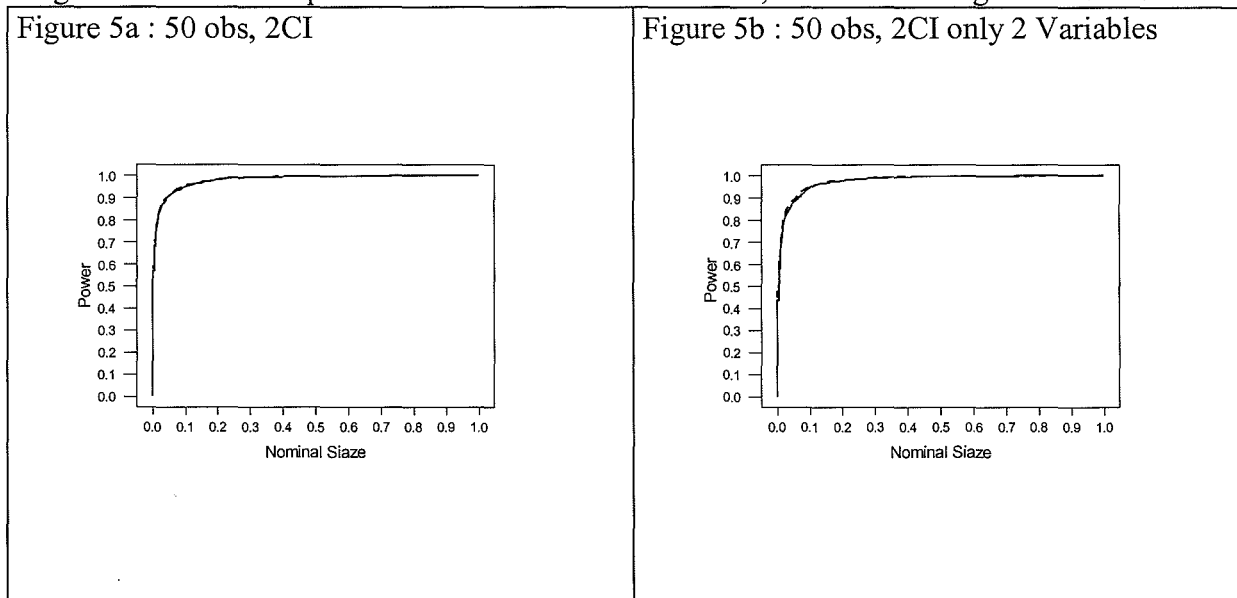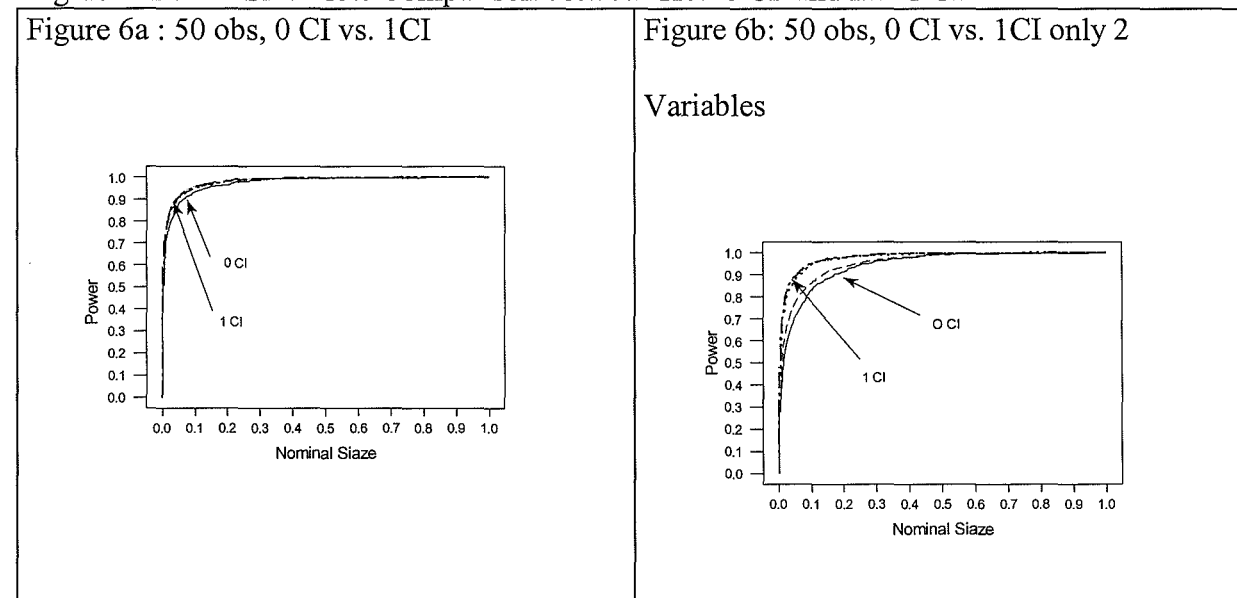Figure 5: Power-Size plots for Trace and L-max statistics, Ho: One cointegration relations.

Figure 5a : 50 obs, 2CI	Figure 5b : 50 obs, 2CI only 2 Variables

Figure 6: Power-Size Plots Comparison between Ho: 0 CI and Ho: 1 CI

Figure 6a : 50 obs, 0 CI vs. 1CI	Figure 6b: 50 obs, 0 CI vs. 1CI only 2 Variables

Finally, Figure 7, shows as expected the lost in the power functions for the 0CI and 1CI tests, by including two extra lags in model (5a), i.e. by fitting a VAR (3) model.

Figure 7: Power plot for model (5) and (5a) with p = 3.

Figure 7a: 50 obs, 0 -Cointagration Vector	Figure 7b: 50 obs, 1-Cointagration Vector
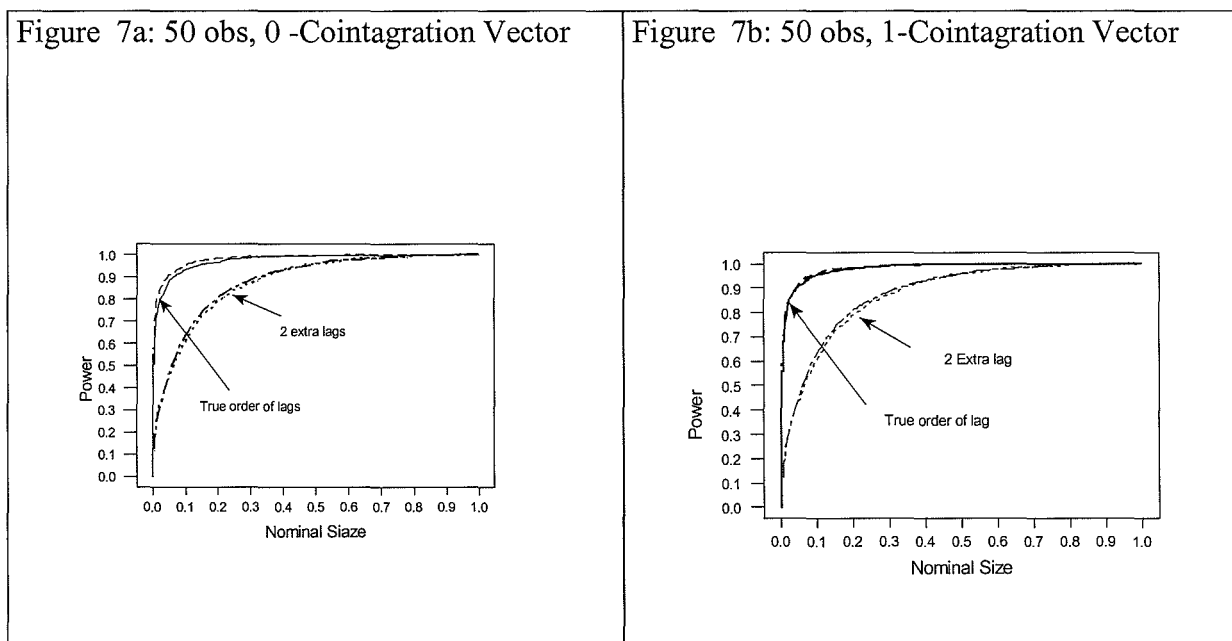
7. A Brief Summary and Conclusions

When we investigate the properties of a test procedure, two aspects are of prime importance. First, we wish to see if the *actual size* of the test is close to the *nominal size*. Second given that actual size is a reasonable approximation to the nominal size, we then wish to investigate the *actual power* of the test for an alternative hypothesis. When comparing different tests, we will therefore prefer those in which (a) actual size lies close to the nominal size and, given that (a) holds, (b) have greatest power. The argument that the Bootstrap test has actual size that lies close to the nominal size make us choose the Bootstrap test in our VAR model given that it has adequate power. The tests has shown to be adequate, even in small samples, in testing both the null hypothesis both in the case to test the null hypothesis that there are no cointegration relations and in the case when there is cointegration.

In this paper we have studied the properties of two tests (Trace and L-max), derived by Johansen (1988) for testing for cointegration in VAR systems.

The investigation has been carried out using Monte Carlo simulations together with Bootstrap statistics. Different combinations were investigated regarding the size of the tests, where the

17

number of observations, the number of the cointegration vectors in the system and the number of the variables in the cointegration vectors have been varied. For each model we have performed 10,000 replications and studied three different nominal sizes. The power properties have been investigated in the same manner using 1000 replication per model and different alternative hypotheses.

The results regarding the size of the tests have been presented both in form of table and P-value plots. Our analysis has revealed that, using the Bootstrap test statistics, the Trace and L-max have been showed to perform adequately when there are mixed stationary and nonstationary components in the model. The difference between the two tests is really quite negligible, and it is difficult to see any pattern that cannot be a result of random error in the experiment. In other words, the Bootstrap test have exhibit well performances in all samples (50, 75 and 100).

As regarding the power of the tests, the power functions have been presented only graphically. The analyses of the power indicate that both of the tests can sharply detect the presence of CI vector(s), and that the differences between the power functions of them are very trivial. The power functions, as expected, become lower when including extra lags in the VAR model.

ACKNOWLEDGMENTS

References

Davidson, R. and J. G. MacKinnon (1997), "Graphical Methods for Investigating the Size and Power Of Hypothesis Tests," Working Paper, Department of Economics, University of Queen's, Canada.

Efron, B. (1979), "Bootstrap methods: Another look at the jackknife," *Annals of Statistics*: 7, 1-26.

Engle, R. F. and Granger, C. W. J. (1987), "Co-integration and Error Correction: Representation, Estimation and Testing", *Econometrica,* 55: 251-276.

Harris, R. I. D. and Judge, G (1998), "Small sample testing for cointegration using the Bootstrap approach," *Economics Letters,* 58 (1998).

Johansen, S. (1988), "Statistical Analysis of Cointegration Vectors," *Journal of Economic Dynamics and Control,* 12: 231-254.

Johansen, S. (1991), "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models," *Econometrica,* 59:1551-1580.

Mantalos, P. and G. Shukur (1998), "Size and Power of the Error Correction Model (ECM) Cointegration Test- A Bootstrap Approach," *Oxford Bulletin of Economics and Statistics,* 60, 2 (1998).

Mantalos, P. (1998), "Bootstraping the VECM test for Cointegrating relations," mimeograph, Department of Statistics, University of Lund, Sweden.

Podivinsky, J. M. (1990), "Testing misspecified cointegration relationships," Working Paper 19-1990. Department of Econometrics, Monash University.

Shukur, G. and P. Mantalos (1997), "Size and Power of the RESET Test as Applied to Systems of Equations: A Bootstrap Approach," Working paper 1997:3. Department of Statistics, University of Lund, Sweden.

Van Giersbergen (1996), " Bootstrapping The trace Statistic in VAR Models: Monte Carlo Results and Applications," *Oxford Bulletin of Economics and Statistics,* 58, 2 (1996).

Research Report

1998:6	Dahlbom, U.:	Least squares estimates of regression functions with certain monotonicity and concavity/convexity restrictions.
1998:7	Dahlbom, U.:	Variance estimates based on knowledge of monotonicity and concavity properties.
1998:8	Grabarnik, P. & Särkkä, A.	Some interaction models for clustered point patterns.
1998:9	Afsarinejad, K. & Hedayat, S.:	Repeated measurement designs for models with self and mixed carryover effects.
1998:10	Hatemi-J, A. & Shukur, G.:	The causal nexus of government spending and revenue in Finland: A bootstrap approach.
1998:11	Shukur, G.:	The robustness of the systemwise Breauch--Godfrey autocorrelation test for non-normal distributed error terms.
1999:1	Andersson, E.:	On monotonicity and early warnings with applications in economics.
1999:2	Wessman, P.:	The surveillance of several processes with different change points.
1999:3	Andersson, E.:	Monotonicity aspects on seasonal adjustment.
1999:4	Andersson, E.:	Monotonicity restrictions used in a system of early warnings applied to monthly economic data.