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Monotonicity aspects on seasonal adjustment

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Monotonicity aspects on seasonal adjustment

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Abstract

Monotonicity is an important property in time series analysis. It is often of interest to know if the seasonal adjustment method used has altered the monotonicity or changed the time of turning points in a time series that exhibits cycles. The issue of whether the monotonicity of the trend cycle component of the original non-stationary time series is preserved after the series has been adjusted is treated in this report. The time of a turning point is defined as the time when the cycle changes from recession to expansion (or vice versa). In this report seasonal adjustment with moving average methods is analysed from monotonicity aspects. The time series is assumed to consist of three additive components: a trend cycle part, a seasonal part and a stochastic error part. No parametric model is assumed for the trend cycle. The behaviour of the adjusted series is analysed for two cases: a monotonically increasing trend cycle and a trend cycle with a peak. If the trend cycle is monotonic within the entire observed section the monotonicity is preserved. Unimodality is preserved but not always the time of the turning point.

Keywords: Seasonal adjustment, monotonicity, turning point, moving average

1. Introduction

When non-stationary time series are analysed, one important aspect is the monotonicity of the series. The time series examined in this report are non-stationary economic time series exhibiting cycles. The observation on the time series at time t is denoted $Y(t)$, where Y consists of three additive components, namely the component of long term change (hereafter trend cycle), the seasonal component and the error term. The cycles in the trend cycle component are not periodic. Monthly observations are made on Y and some of the variation is due to seasonality. The series is adjusted for seasonality in order to make the turning points of the trend cycle more easily identified. Moving average methods, used for seasonal adjustment, are analysed with regard to their monotonicity preserving properties. The question of whether the seasonally adjusted series preserves the time of the turning points in the cycles is treated. The timeliness of the estimates is also discussed. This is of importance in many cases, especially in order to obtain warnings.

In Section 2 the model for the time series is specified. Different suggestions for seasonal adjustment are discussed in Section 3. Two moving average estimators are analysed with regard to their ability to preserve the monotonicity in Section 4. The conclusions from this study are presented in Section 5.

2. Specifications

Seasonal variation in a time series is not often defined rigorously, but Wallis (1974) gives examples of some more explicit statements. A common description is that seasonal variation is fluctuations that are periodical with a period of one year. The series under observation, Y, contains a trend cycle and thus Y is not a stationary series. The seasonal adjustment is made in order to distinguish the trend cycle. It is sometimes proposed that a polynomial function should be used to model a deterministic trend. In this report the trend cycle is not restricted to any parametric function. The trend cycle function is unknown, apart from the important aspect of monotonicity and unimodality, which is not an assumption, but follows from the definition of a turning point. The errors are modelled as white noise. The case when the error term is modelled as an ARMA process will not be studied in this report. In this report the case when the accessible data is for a short or moderate time period. Only a part of a series that contains one turning point at most will be investigated.

The model used in this report for an observation of the time series at time t is

$$Y(t) = \mu(t) + S(t) + \varepsilon(t) \tag{2.1}$$

$\mu(t)$ is the trend cycle component,

$\mu(t) \in \wp$, $\wp$ is the family of all unimodal functions,

$S(t)$ is the seasonal component

and $\varepsilon(t)$ are iid N(0; σ^2).

The seasonal component in (2.1) is not modelled as a stochastic process. The case of a seasonal cycle of 12 time periods is studied. The seasonal component and $\mu(t)$ are defined by

$$\sum_{k=1}^{12} S(k+h) = 0, \qquad h = 0, 1, 2, \ldots \tag{2.2}$$

and by the restriction that there exists no decomposition of $\mu(t)$ such that

$\mu(t) = X(t) + Z(t)$,

with $Z(t) \neq 0$ for some t, where $\sum_{k=1}^{12} Z(k+h) = 0, \qquad h = 0, 1, 2, \ldots$

3. Different suggestions for adjustment of seasonality

3.1 The seasonal component is known

If the seasonal component, $S(t)$, is known, the estimation of the trend cycle component is reduced to a simple subtraction.

$$\hat{\mu}(t) = Y(t) - S(t) = \mu(t) + \varepsilon(t) \tag{3.1}$$

$$\mathrm{E}[\hat{\mu}(t)] = \mu(t) + \mathrm{E}[\varepsilon(t)] = \mu(t)$$
$$\mathrm{Var}[\hat{\mu}(t)] = \mathrm{Var}[\varepsilon(t)] = \sigma^2$$

3.2 The seasonal component is unknown

In Harvey (1993) some models for seasonal time series are discussed. One suggestion of how to model a deterministic seasonality is

$$Y(t) = \alpha + \beta * t + \sum_{j=1}^{s-1} \gamma_j z_j(t) + u(t),$$

where α is the mean, β is the slope, z_j are dummy variables, γ_j are seasonal coefficients that sum to zero and u is a stationary stochastic process. An alternative way of modelling a seasonal pattern is by a set of trigonometric terms at the seasonal frequencies, that is

$$Y(t) = \alpha + \beta * t + \sum_{j=1}^{s/2} (\gamma_j \cos \lambda_j t + \gamma_j^* \sin \lambda_j t) + u(t).$$

Regression methods can be used for estimating the components of the models above.

The observed variable Y can be assumed to comprise three unobserved component, namely trend cycle, seasonal and irregular components, that is

$$Y(t) = C(t) + S(t) + I(t)$$

If it can be assumed that these components each follows an ARMA process, an estimate of S can be obtained by applying a linear filer to the observations (Burridge and Wallis, 1990).

The time series can be modelled as consisting of a seasonal component and a non-seasonal component, i.e.

$$Y(t) = S(t) + N(t)$$

Several authors have suggested seasonal adjustment methods that involve fitting an ARIMA model to Y above and using this along with some assumptions to determine models for S and N. Some authors suggest using ARIMA models and deterministic terms to allow for both stochastic and deterministic components. These methods involve determining ARIMA models for the components and then using signal extraction theory to estimate them (Bell and Hillmer, 1984).

One method for seasonal adjustment is the X-11 method. The multiplicative model used for the X-11 method is assumed to be the following

$$Y(t) = TC(t) * S(t) * TD(t) * H(t) * I(t)$$

where $TC(t)$, $S(t)$, $TD(t)$, $H(t)$ and $I(t)$ are the unobserved trend-cycle component, the seasonal component, the trading-day component, the holiday component and the irregular component, respectively. The program for the X-11 method is divided into seven steps. The first version of this method, X-1, was a refinement of the ratio to moving average method (Hylleberg, 1992).

The aim of the seasonal adjustment in this report is to produce an estimate, $\hat{\mu}(t)$, that pertains the monotonicity of $\mu(t)$. Two moving average techniques are used, denoted M_1 and M_2 respectively. Data consists of monthly observations and a natural estimator is a 12-month moving average. A known property of the twelve point moving average is that it tends to cut corners at turning points (Leong, 1962). This report does not, however, deal with preservation of the level. An implication of this property is that a turning point does become less pronounced and thus more difficult to detect. Another aspect of using the moving average as a trend cycle estimator is the Slutsky-Yule effect, cited in Jorgenson (1964). This effect refers to the fact that if the random component of the original series is independently distributed over time, then the random component of a moving average of this series is not.

For the model in Section 2 other adjustment methods than a moving average might be more efficient. However, it is important to use a technique that is robust against slow changes in the seasonal component over years. It should also be considered that the number of available observations is not very large. Another reason for examining the technique of moving average is that this technique is frequently used.

Both a centred (M_1) and a non-centred (M_2) moving average are shown below. For the time series under study, Y, it is assumed that the seasonal component is constant over time or at least that any possible change is very slow.

The estimate of the trend cycle component by a centred moving average (M_1) is

$$\hat{\mu}^{M_1}(t) = \sum_{j=-6}^{6} w_j Y(t+j) \tag{3.2}$$

where $w_{-6}=w_6=1/24$ and $w_{-5}=w_{-4}=\ldots=w_5=1/12$.

$$E[\hat{\mu}^{M_1}(t)] = \sum_{j=-6}^{6} w_j \mu(t+j)$$

$$\mathrm{Var}[\hat{\mu}^{M_1}(t)] = \sigma^2\left(\frac{23}{288}\right).$$

The estimate of the trend cycle component by a non-centred moving average (M_2) is

$$\hat{\mu}^{M_2}(t) = \sum_{j=-11}^{0} w_j Y(t+j) \tag{3.3}$$

where $w_{-11}=w_{-10}=\ldots=w_0 = 1/12$.

$$E[\hat{\mu}^{M_2}(t)] = \frac{1}{12}\sum_{j=-11}^{0} \mu(t+j)$$

$$\text{Var}[\hat{\mu}^{M_2}(t)] = \sigma^2\left(\frac{24}{288}\right).$$

The centred estimator is based on the latest thirteen observations and the non-centred estimator is based on the latest twelve observations. A difference between these two estimators is that the centred moving average at time t, per definition, can not be calculated until six months later, thus producing an systematic delay.

4. Results

The investigation concerns whether the monotonicity of the trend cycle is preserved by the expected value of a moving average estimator. Two methods of estimation, M_1 and M_2, are investigated for two different cases, namely the case when the trend cycle is monotonic and the case of a peak in the trend cycle. The observations under study are denoted $y(1), y(2),..., y(n)$.

4.1 Moving average estimators for a monotonically non-decreasing trend cycle

The case when the μ-vector is monotonically non-decreasing within the entire observed section is studied in this section, that is

$$\mu(t-1) \leq \mu(t), \qquad \text{where } 2 \leq t \leq n.$$

The question of whether the monotonicity is preserved when two moving average techniques (centred and non-centred respectively) are used to estimate the monotonically non-decreasing μ-vector, is analysed in this section. That is, the correctness in the relation

$$E[\hat{\mu}^{M_i}(t-1)] \leq E[\hat{\mu}^{M_i}(t)], \qquad \text{where } i = \{1, 2\} \tag{4.1}$$

is investigated. The results are valid also for the opposite case (a monotonically non-increasing μ-vector).

4.1.1 Centred 12-month moving average

The trend cycle estimate is the centred 12-month moving average,

$$\hat{\mu}^{M_1}(t) = \frac{1}{24}(Y(t-6) + Y(t+6)) + \frac{1}{12}\sum_{j=-5}^{5} Y(t+j) = \tag{4.2}$$
$$= \frac{1}{24}(\mu(t-6) + \mu(t+6) + S(t-6) + S(t+6) + \varepsilon(t-6) + \varepsilon(t+6)) +$$
$$+ \frac{1}{12}\sum_{j=-5}^{5} \mu(t+j) + S(t+j) + \varepsilon(t+j).$$

The expected value, denoted $\kappa(t)$, is defined as

$$\kappa(t) = \mathrm{E}[\hat{\mu}^{M_1}(t)] = \frac{1}{24}(\mu(t-6) + \mu(t+6)) + \frac{1}{12}\sum_{j=-5}^{5} \mu(t+j). \tag{4.3}$$

Statement 1: *If the trend cycle is a monotonic function, the expected value of the centred 12-month moving average is also a monotonic function.*

The proof is given in the appendix.

4.1.2 Non-centred 12-month moving average

The trend cycle estimate is the non-centred 12-month moving average,

$$\hat{\mu}^{M_2}(\mathrm{t}) = \frac{1}{12}\sum_{j=-11}^{0} Y(t+j) = \tag{4.4}$$
$$= \frac{1}{12}\sum_{j=-11}^{0} \mu(t+j) + \frac{1}{12}\sum_{j=-11}^{0} S(t+j) + \frac{1}{12}\sum_{j=-11}^{0} \varepsilon(t+j)$$

The expected value, denoted $\eta(t)$, is

$$\eta(t) = \mathrm{E}[\hat{\mu}^{M_2}(t)] = \frac{1}{12}\sum_{j=-11}^{0} \mu(t+j). \tag{4.5}$$

Statement 2: *If the trend cycle is a monotonic function, the expected value of the non-centred 12-month moving average is also a monotonic function.*

The proof is given in the appendix.

4.2 Moving average estimators at a turning point in the trend cycle vector

The case when the μ-vector is inversely U-shaped with a peak at time t = p is studied in this section, that is

$$\mu(1) < \mu(2) < ... < \mu(p) \quad \text{and}$$
$$\mu(p) > \mu(p+1) > ... > \mu(n)$$

The question of whether the monotonicity and the time of the turning point is preserved when the two moving average techniques are used to estimate the inversely U-shaped μ-vector, is investigated in this section. That is, the correctness in the relation

$$\mathrm{E}[\hat{\mu}^{\mathrm{M_i}}(1)] < \mathrm{E}[\hat{\mu}^{\mathrm{M_i}}(2)] < ... < \mathrm{E}[\hat{\mu}^{\mathrm{M_i}}(p)] \text{ and}$$
$$\mathrm{E}[\hat{\mu}^{\mathrm{M_i}}(p)] > \mathrm{E}[\hat{\mu}^{\mathrm{M_i}}(p+1)] > ... > \mathrm{E}[\hat{\mu}^{\mathrm{M_i}}(n)] \qquad (4.6)$$

where i = {1, 2}

is investigated. The results are valid also for the opposite case (a trough).

4.2.1 Centred 12-month moving average

The observations under study are denoted $y(1), y(2),..., y(p), ..., y(n)$, where p is the time of the peak. The expected value of $\hat{\mu}^{\mathrm{M_1}}(t)$ is $\kappa(t)$. The values of κ for different time intervals are presented in Table 1. Note that the analysis at time t assumes that the observations $y(t+1), y(t+2), ..., y(t+6)$ are available.

Table 1
The expected value of the estimated trend cycle, $\kappa(t)$, at a peak at time p in the μ-vector

Decision time, t		$\mu(t)$		$\mathrm{E}[\hat{\mu}^1(t)] = \kappa(t)$
$2 \le t \le$ (p-6)		$\mu(t-1) < \mu(t)$ ↗		$\kappa(t-1) < \kappa(t)$ ↗
(p-5) $\le t \le$ p		$\mu(t-1) < \mu(t)$ ↗		Several possibilities
(p+1) $\le t \le$ (p+6)		$\mu(t-1) > \mu(t)$ ↘		Several possibilities
(p+7) $\le t \le$ n		$\mu(t-1) > \mu(t)$ ↘		$\kappa(t-1) > \kappa(t)$ ↘

In Table 1 the function $\kappa(t)$ is shown for different time intervals. The results assume that the observations $\{y(t+1), y(t+2),..., y(t+6)\}$ are available at time t. Nothing definite can be concluded about $\kappa(t)$ in the interval $\{p\text{-}5\text{: } p\text{+}6\}$. However, the time of the turning point of $\kappa(t)$ must occur in this interval. Thus the indication of a turning point can come before, at or after time t = p. The μ-vector is monotonically decreasing from time t = p and forward. From Table 1 we have that from time t = p+7 the value of κ is monotonically decreasing. This indicates that the delay of the expected value of this estimator is, at maximum, six time units. However the usual situation would be that the observations $\{y(t+1), y(t+2),..., y(t+6)\}$ are not available at time t. This situation does cause an systematic delay of six time units for the centred moving average estimator. This systematic delay must be added to the delay times presented in Table 1. Therefore the total delay of κ is, at maximum, twelve months. Because of this systematic delay, the centred moving average estimator will not be considered further.

4.2.2 Non-centred 12-month moving average

The observations under study are denoted $y(1), y(2),..., y(p), ..., y(n)$. The expected value of $\hat{\mu}^{M_2}(t)$ is $\eta(t)$. The values of η for different time intervals are presented in Table 2.

Table 2
The expected value of the estimated trend cycle, $\eta(t)$, at a peak at time p in the μ-vector

Decision time, t		$\mu(t)$		$\mathrm{E}[\hat{\mu}^2(t)] = \eta(t)$
$2 \le t \le$ p		$\mu(t-1) < \mu(t)$ ↗		$\eta(t-1) < \eta(t)$ ↗
(p+1) $\le t \le$ (p+12)		$\mu(t-1) > \mu(t)$ ↘		Several possibilities
(p+13) $\le t \le$ n		$\mu(t-1) > \mu(t)$ ↘		$\eta(t-1) > \eta(t)$ ↘

In Table 2 the function $\eta(t)$ is shown for different time intervals. Nothing definite can be concluded about $\eta(t)$ in the interval $\{p+1\text{: } p+12\}$. However, the time of the turning point of $\eta(t)$ must occur in this interval. The maximum delay of $\eta(t)$ is twelve months. Without further specifications of the kind of turning point nothing definite can be concluded.

The non-centred moving average estimator will now be further studied in three special cases of a peak at time t = p. The three cases considered are a symmetric turning point, an almost flat curve after the turning point and a steep slope after the turning point. For all cases we have that

$$\mu(1) < \mu(2) < ... < \mu(p) \quad \text{and} \qquad (4.7)$$
$$\mu(p) > \mu(p+1) > ... > \mu(n)$$

The observations under study for all three cases are denoted $y(1), y(2),..., y(p),..., y(n)$.

In the first case investigated, the μ -vector is symmetric around the peak at time t = p, i.e.

$$\mu(p-q)= \mu(p+q), \qquad q \geq 1. \tag{4.8}$$

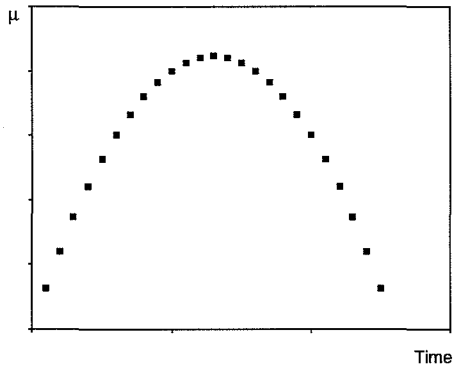

Fig. 1. Case 1, the μ -vector is symmetric around the peak

The table below shows the non-centred moving average estimator at a symmetric peak.

Table 3
The expected estimated trend cycle, $\eta(t)$, at a symmetric peak at time p in the μ -vector (case 1)

Decision time, t		$\mu(t)$		$E[\hat{\mu}^2(t)] = \eta(t)$
$2 \leq t \leq$ (p)		$\mu(t-1) < \mu(t)$ ↗		$\eta(t-1) < \eta(t)$ ↗
(p+1) $\leq t \leq$ (p+5)		$\mu(t-1) > \mu(t)$ ↘		$\eta(t-1) < \eta(t)$ ↗
t = (p+6)		$\mu(t-1) > \mu(t)$ ↘		$\eta(t-1) = \eta(t)$ →
(p+7) $\leq t \leq$ n		$\mu(t-1) > \mu(t)$ ↘		$\eta(t-1) > \eta(t)$ ↘

Table 3 shows the expected estimated trend cycle, $\eta(t)$, at a symmetric peak at time p. The function $\eta(t)$ is shown for different time intervals. The delay for the expected value of this estimator at a symmetric peak is six time units.

The second case considered is where the μ -vector forms an almost flat curve after the turning point at time t = p, that is

$$\mu(p+q) > \mu(p-1), \qquad q \in \{1, 2, \ldots, 11\}. \tag{4.9}$$

Since $\mu(1) < \ldots < \mu(p-2) < \mu(p-1)$, it follows that $\mu(p+m) > \mu(p-m)$, $m \in \{1, 2, \ldots, 11\}$.

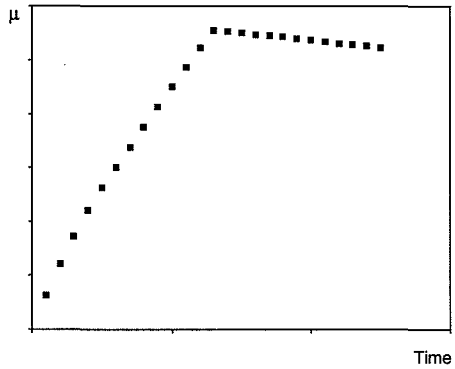

Fig. 2. Case 2, the μ-vector forms an almost flat curve after the peak.

The table below shows the non-centred moving average estimator at an unsymmetrical peak (case 2).

Table 4
The expected estimated trend cycle, $\eta(t)$, at an unsymmetrical peak at time p in the μ-vector (case 2)

Decision time, t		$\mu(t)$		$E[\hat{\mu}^2(t)] = \eta(t)$
$2 \leq t \leq (p)$		$\mu(t-1) < \mu(t)$ ↗		$\eta(t-1) < \eta(t)$ ↗
$(p+1) \leq t \leq (p+11)$		$\mu(t-1) > \mu(t)$ ↘		$\eta(t-1) < \eta(t)$ ↗
$(p+12) \leq t \leq n$		$\mu(t-1) > \mu(t)$ ↘		$\eta(t-1) > \eta(t)$ ↘

Table 4 shows the expected estimated trend cycle, $\eta(t)$, at an unsymmetrical peak at time p. The function $\eta(t)$ is shown for different time intervals. The delay for the expected value of this estimator at this kind of peak is eleven time units.

The third case considered is where the μ-vector forms a steep slope after the turning point at time $t = p$. It is assumed that

$$\mu(p+1) < \mu(p-11). \tag{4.10}$$

Since $\mu(p+1) > \mu(p+2)$ and $\mu(p-11) < \mu(p-10)$ the inequality $\mu(p+2) < \mu(p-10)$ must hold. From this result it is implicit that $\mu(p+m) < \mu(p-12+m)$, $m \in \{3, 4, \ldots, 11\}$.

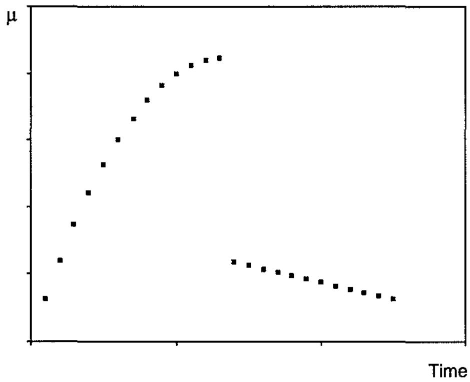

Fig. 3. Case 3, the μ-vector forms a steep slope after the peak.

The table below shows the non-centred moving average estimator at an unsymmetrical peak (case 3).

Table 5
The expected estimated trend cycle, $\eta(t)$, at a peak at time p in the μ-vector (case 3)

Decision time, t		$\mu(t)$		$E[\hat{\mu}^2(t)] = \eta(t)$
$2 \le t \le (p)$		$\mu(t-1) < \mu(t)$ ↗		$\eta(t-1) < \eta(t)$ ↗
$(p+1) \le t \le n$		$\mu(t-1) > \mu(t)$ ↘		$\eta(t-1) > \eta(t)$ ↘

Table 5 shows the expected estimated trend cycle, $\eta(t)$, at an unsymmetrical peak at time p. The function $\eta(t)$ is shown for different time intervals. There is no delay for the expected value of this estimator at this kind of peak.

The ability of $\eta(t)$ to preserve the time of the turning point depends on the shape of the μ-vector at the peak. For a symmetric peak, a delay of seven time units is to be expected. For a μ-vector that is almost flat after the turning point, the delay in $\eta(t)$ is twelve time units. For a μ-vector with a very steep slope after the turning point, there is no delay in $\eta(t)$.

Statement 3: *If the trend cycle is unimodal, the expected value of the non-centred 12-month moving average will preserve the unimodality (Frisén, 1986) but not always the time of the turning point in the trend cycle.*

5. Discussion

The centred and the non-centred moving average techniques have been investigated as possible methods of adjusting a time series for seasonality. The properties of these moving averages have been evaluated for a monotonic trend cycle and for a turning point in the trend cycle. The assumptions for the model used in this report might be too strong for many applications. However, some problems with the monotonicity evaluation of the seasonal adjustment were demonstrated even for a model with these assumptions.

Never the less, the following can be concluded from this monotonicity study: Using the simple moving average technique for seasonal adjustment does preserve the monotonicity of a monotonically non-decreasing trend cycle. It has been shown that if all observations of the moving average are within a monotonic section of the time series, both the centred and the non-centred moving average will preserve the monotonicity. For the case of an unspecified peak in the trend cycle, no definite conclusions can be made regarding the time of the turning point for either of the moving averages.

Because of a systematic delay in the timeliness, the centred moving average was not investigated further.

The non-centred moving average was investigated for three different kinds of peaks. For a unimodal section of the trend cycle, the non-centred moving average will preserve the unimodality. However, it has been shown in this investigation that the non-centred moving average does not always preserve the monotonicity of all parts in the unimodal case. Thus, the time of the turning point is not always preserved. In some cases the use of the moving average technique results in a delayed indication of a turning point.

The non-centred moving average as a method for seasonal adjustment is conservative in the sense that it does not give any false indications of a turning point. This moving average performs well at monotonic sections, but because of the possible delay at a turning point, it is important to try to use other methods. One possibility, if it agrees with the structure in the data on hand, is to use a large historical data set to estimate the seasonal components.

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Appendix. Proof of Statement 1 - 3

For **Statement 1** we have that the difference between two consecutive expected estimates of the trend cycle is

$$\kappa(t) - \kappa(t-1) =$$
$$= \frac{1}{24}\left[\mu(t+6) - \mu(t-1+6)\right] + \frac{1}{12}\left[\mu(t+5) - \mu(t-1-5)\right] + \frac{1}{24}\left[\mu(t-6) - \mu(t-1-6)\right]$$

The differences inside the three brackets are each greater than or equal to zero for all $t \geq 7$, according to the assumption of a non-decreasing function. Therefore $\kappa(t)$ - $\kappa(t-1) \geq 0$ and κ is a non-decreasing function for all $t \geq 7$.

For **Statement 2** we have that the difference between two consecutive expected estimates of the trend cycle is

$$\eta(t) - \eta(t-1) = \frac{1}{12}\left[\mu(t) - \mu(t-1-11)\right]$$

The difference inside the brackets is greater than or equal to zero, for all $t \geq 12$, according to the assumption of a non-decreasing function. Therefore $\eta(t)$ - $\eta(t-1) \geq 0$ and η is a non-decreasing function for all $t \geq 12$.

For **Statement 3** the proof is divided into the three cases that have been investigated. For each case investigated in Section 4.2.2 the proof for the different time intervals, denoted i), ii), iii) and iv), are given separately. The proof for the interval i) is the same as for Statement 2. For the rest of the intervals the proof for the first time point is showed, the rest follow easily.

Case 1

i) $\eta(p-a)$ - $\eta(p-1-a) > 0$, for $0 \leq a \leq$ (p-2). See proof of Statement 2.

ii) $\eta(p+1)$ - $\eta(p)$ =(1/12)*($\mu(p+1)$ - $\mu(p-11)$)=(1/12)*($\mu(p-1)$ - $\mu(p-11)$) >0
$\eta(p+b)$ - $\eta(p-1+b) > 0$, for $2 \leq b \leq 5$

iii) $\eta(p+6)$ - $\eta(p+5)$=(1/12)*($\mu(p+6)$ - $\mu(p-6)$)=(1/12)*($\mu(p-6)$ - $\mu(p-6)$) =0

iv) $\eta(p+7)$ - $\eta(p+6)$= (1/12)*($\mu(p+7)$ - $\mu(p-5)$)=(1/12)*($\mu(p-7)$ - $\mu(p-5)$)<0
$\eta(p+b)$ - $\eta(p-1+b) < 0$, for $b \geq 8$

Case 2

i) $\eta(p-a) - \eta(p-1-a) > 0,$ for $0 \le a \le$ p-2. See proof of statement 2.

ii) $\eta(p+1) - \eta(p) = (1/12)*(\mu(p+1) - \mu(p-11)) > 0$
$\eta(p+b) - \eta(p-1+b) > 0,$ for $2 \le b \le 11$

iii) $\eta(p+12) - \eta(p+11) = (1/12)*(\mu(p+12) - \mu(p)) < 0$
$\eta(p+b) - \eta(p-1+b) < 0,$ for $b \ge 13$

Case 3

i) $\eta(p-a) - \eta(p-1-a) > 0,$ for $0 \le a \le$ p-2. See proof of statement 2.

ii) $\eta(p+1) - \eta(p) = (1/12)*(\mu(p+1) - \mu(p-11)) < 0$
$\eta(p+b) - \eta(p-1+b) < 0,$ for $b \ge 2$

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