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Evaluation of some methods for statistical surveillance of an autoregressive process

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Evaluation of some methods for statistical surveillance of an autoregressive process

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Abstract

Statistical surveillance is used for fast and secure detection of a critical event in a monitored process. This paper studies the performance for AR(1) processes.

Two often suggested methods for detection of a shift in the mean, the modified Shewhart and the residual method, are compared and evaluated. Further, comparisons are made with direct Shewhart and a likelihood ratio method.

New evaluation measures, the probability for successful detection and the predictive value, are also applied together with the average run length and run length distributions.

We conclude that neither the modified nor the residual methods is uniformly optimal. The residual method is, however, optimal for immediate detection, but has inferior properties otherwise. For many parameter setups, the modified method will give the better performance.

1. Introduction

Statistical surveillance is used for systematic monitoring of a process with the purpose to detect an unwanted departure from a specified state. Methods for *Statistical Process Control* (SPC) have been widely used for industrial, medical, economical, environmental and many other applications. Several textbooks have been published, for example Box and Luceño (1997), Montgomery (1997) or Wetherill and Brown (1991). Note the difference between hypothesis testing for a change-point on a fix set of data and surveillance: In both cases we do not know if something has happened and when. But statistical surveillance is used for situations where new data arrives at each time step. The procedure is repeated and there is no fixed hypothesis.

One fundamental assumption required by standard methods is that the process is *iid* (*Independent and Identically Distributed*) - a requirement which is often not met in practise. Removing the assumption of independence will affect the performance of the surveillance procedures.

A survey by Alwan and Roberts (1995) of 235 quality control applications, where less than 50% of the studied applications were independent and less than 15% were *iid*, gives a good motivation for studying this problem. Further, Alwan and Roberts (1995) together with Caulcutt (1995) and the discussion following them, testified about the frustration they have met with engineers who tried to apply SPC methods to autocorrelated data since the resulting monitoring system does not have the wanted properties. Stone and Taylor (1995) also pointed out that sometimes not even the ARIMA model is sufficient for the description of the process.

The robustness of CUSUM and EWMA applied directly on the observed process have been discussed by for example Bagshaw and Johnson (1975), Harris and Ross (1991), Johnson and Bagshaw (1974), Montgomery and Mastrangelo (1991), Schmid and Schöne (1997), VanBrackle and Reynolds (1997) and Yashchin (1993).

Among others, two solutions for the non *iid* case have been proposed by several authors: We will call them the *modified Shewhart* method and the *residual* method, respectively. The methods will be described in detail below. The modified Shewhart method have been investigated by Vasilopoulos and Stamboulis (1978), for an AR(2) process. The residual method was suggested for ARIMA-processes by Berthoux *et al.* (1978) and Alwan and Roberts (1988). Since these methods are often suggested and used in practise it is interesting to compare them with each other. Furthermore, we will briefly examplify what will happen if the process parameters are estimated during run-in under an assumed *iid* situation. We will call this method the *direct Shewhart*.

Often comparisons between the methods are limited to average run length. We will extend the evaluation using the predictive value and the probability of successful detection suggested by Frisén (1992). We will in this paper also compare the modified Shewhart and the residual method with examples of the *likelihood ratio method* in order to further examine their properties.

In Section 2 a specification of the situation which is studied is given. In Section 3 the methods compared in this paper are defined in detail. Section 4 contains results on the evaluation measures considered. In Section 5 the results and conclusions are discussed.

2. Specifications

Consider a process that is observed at discrete time steps, $t = 1, 2, \ldots$. The data observed at time t is a continous stochastic variable denoted by $X(t)$. The cumulated data up to time t is denoted by $X_t = \begin{pmatrix} X(1) & \ldots & X(t) \end{pmatrix}$. Consistently, the current value of any variable is denoted by time within parentheses, *eg.* $X(t)$, $\mu(t)$, $\varepsilon(t)$ and $w(t)$, while the cumulated sets are denoted by time in index, *eg.* X_t, μ_t, ε_t and w_t. When the process behaves in the prescribed, wanted or expected way we say that it is "in control". Our general model for the in control part of the process is

$$X(t) = \mu(t) + w(t),$$

where

$$w(t) = \phi \cdot w(t-1) + \varepsilon(t). \tag{2.1}$$

and the correlation $|\phi| < 1$. The variable ε_t is normally distributed white noise with $Var[\varepsilon(t)] = \sigma^2$ and $\varepsilon(t)$ is independent of w_{t-1}. Note that we are defining σ^2 as the variance of the concealed error term, ε. We will in this paper assume that ϕ, μ and σ are known and we can therefore without loss of generality set $\mu(t) = 0$ and $\sigma = 1$.

At an unknown time, τ, the process is disturbed and goes "out of control". We study the case where a shift in μ to a known value, δ, occurs, *i.e.*

$$\mu(t) = \delta \cdot \mathbf{1}_{\{\tau \leq t\}}.$$

Hence the expected value of X is

$$E[X(t)] = \begin{cases} 0 \text{ when } t < \tau \\ \delta \text{ when } t \geq \tau \end{cases}.$$

At each time, s, we want to discriminate between two events, $D(s)$ and $C(s)$, where $D(s) = \{\tau > s\}$ is the event of the process being in control. $C(s) = \{\tau = s\}$ and $C(s) = \{\tau \leq s\}$ will be discussed.

Figure 1 shows an example of an AR(1) process with a shift $\delta = 10 \cdot \sigma$ with $\tau = 40$.

3. Methods for Monitoring an AR(1) Process

When the monitored process is not *iid* but autoregressive the properties of the standard methods are changed. In this paper we will study some methods that are often suggested in the literature for this case: "Direct Shewhart", where the time series structure is not taken into account; "Modified Shewhart", where the limits have been altered to give a specific average run length and "Residual Shewhart", where the forecast errors are used for monitoring. As a benchmark these methods will be compared with the likelihood ratio method. The name "modified Shewhart" was given by Schmid (1995) and exact limits for some processes have been given by Vasilopoulus and Stamboulis (1978). The residual method was suggested by Alwan and Roberts (1988) and Berthoux *et al.* (1978).

We will in this paper restrict attention to the AR(1) process (2.1) with $\phi > 0$.

3.1. Direct Shewhart

If time dependence is not taken into account a user might estimate the mean and the variance during run-in. In the case of an *iid* process, X, the Shewhart procedure, suggested by Shewhart (1931), prescribes that an alarm is called when

$$|X(t)| > k \cdot \sigma,$$

where the constant k is set to give a certain proabability of calling a false alarm. In traditional SPC litterature k is often 3 or 3.09. However, for a stationary AR(1) process the variance of X becomes

$$\sigma_x^2 = Var\left[X(t)\right] = \frac{\sigma^2}{1-\phi^2}.$$

Estimating the variance with a very large number of observations and using the same constant k an alarm will be called when

$$|X(t)| > k \cdot \sigma_x = k \cdot \sigma \cdot \frac{1}{\sqrt{1-\phi^2}}. \qquad (3.1)$$

Since $(1-\phi^2)^{-1/2} > 1$ these limits will become greater than the limits for an *iid* process with variance σ^2.

3.2. Modified Shewhart

The direct Shewhart will, as we will see in later Sections, have some undesirable properties, *eg.* an ARL_0 (Section 4.1) that is depending on ϕ. A straightforward solution to that problem could be to adjust the control limits of the Shewhart chart to give the wanted ARL_0.

Define $c(\phi)$ as the factor adjusting the limits of the *iid* Shewhart so that an alarm is called when

$$|X(t)| > k \cdot \sigma \cdot c(\phi).$$

Since $\phi > 0 \Longrightarrow Var[X] > \sigma^2$ it follows that $c(\phi) > 1$. In Table 3.1, the adjusting factors have been estimated by computer simulation to yield $ARL_0 = 11$, the limits are also plotted in Figure 3 together with the limits obtained by using the direct Shewhart (3.1) with $ARL_0 = 11$ for $\phi = 0$.

ϕ	**Modified** $c(\phi)$	**Direct** $(1-\phi^2)^{-1/2}$	ARL_0
0.0	1.000	1.000	11.00
0.2	1.014	1.020	11.26
0.4	1.060	1.091	12.17
0.6	1.155	1.250	14.36
0.8	1.363	1.667	20.99

Table 3.1: *Comparison between the adjusting factors of the modifed and direct Shewhart.*

We see that $c(\phi) < (1-\phi^2)^{-1/2}$, *i.e.* the direct Shewhart is having higher alarm limits than the modified. Therefore it follows that that the ARL_0 is higher for the direct than for the modified Shewhart.

3.3. Residual Method

The idea of the residual method is that the current value, $X(s)$, and its expectation given the past value are compared and the difference is used for monitoring. A similar approach is used by the *Food and Drug Administration* (FDA) as a guideline in postmarketing surveillance of adverse effects of drugs, where consequtive quarters are compared (Sveréus, 1995). Also the *National Institute for Radiation Protection* (SSI) uses differences in mean between consequtive 24 hour-period means to detect suddenly increasing background radiation levels (Kjelle, 1987). Other examples of applications of the residual method can be found in Harris and Ross (1991), Montgomery (1997), Notohardjono and Ermer (1986) and Pettersson (1998).

Based on the second last observation, $X(s-1)$, a forecast of $X(s)$ is

$$\widehat{X}(t) = \phi \cdot x(t-1).$$

The residual is here defined as the difference between the observed value and its forecast, *i.e.*

$$R(t) = X(t) - \widehat{X}(t) = X(t) - \phi \cdot X(t-1).$$

When $t < \tau$ the residual $R(t) = \varepsilon(t)$. But generally the residual becomes

$$R(t) = \varepsilon(t) + \Delta(t),$$

where

$$\Delta(t) = E[R(t)] = \begin{cases} 0 & \text{when } t < \tau \\ \delta & \text{when } t = \tau \\ (1-\phi)\delta & \text{when } t > \tau \end{cases}. \tag{3.2}$$

For a fixed value of τ

$$Var[R(t)] = Var[\varepsilon(t)] = \sigma^2.$$

and a Shewhart test used for R would call an alarm when

$$|R(t)| > k \cdot \sigma,$$

where k is a constant.

When $\phi > 0$, the expected value will decrease after τ and

$$E[R(t)] < E[R(\tau)], \text{ for } t = \tau+1, \tau+2, \ldots$$

In Figure 1 we see an example of an simulated AR(1) process, with a shift at $t = 40$ of the size 10σ. Figure 2 shows the residuals, *i.e.* forecast errors, of the process in Figure 1, where $E[R(40)] = 10$ and $E[R(t)] = 5$ for $t > 40$. That have earlier been observed by among others Harris and Ross (1991), Ryan (1991), Superville and Adams (1994) and Wardell *et al.* (1994) and for time series analysis by among others Enders (1995), Fox (1972) and Wei (1990).

3.4. Likelihood Ratio Method

It is possible to derive a method which have certain optimality properties. For a fixed false alarm rate and a fixed time, an alarm set based on the *likelihood ratio statistic* (*lr*) have the highest probability of calling an alarm when the process have gone out of control (Frisén and de Maré, 1991). Sequential procedures with minimal expected delay are based on this statistic. This approach will not be studied in detail in this paper, except for some illustrative examples intended to give insight in the properties of the methods studied.

The likelihood ratio statistic, $lr(X_s)$, is defined as

$$lr(X_s) = \frac{f(X_s \mid C(s))}{f(X_s \mid D(s))},$$

where $f(X_s \mid D(s))$ and $f(X_s \mid C(s))$ is the probability density function of X_s under the in- and out-of control states, respectively. Since $X(t)$ given X_{t-1} is normally distributed with

$$E[X(t) \mid X_{t-1}] = \phi \cdot X(t-1) + \Delta(t)$$

and $Var[X(t)] = \sigma^2$ the probability distribution function becomes

$$f_{X(t)|X_{t-1}}(x(t), x(t-1)) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2\sigma^2}(x - \phi \cdot x(t-1) + \Delta(t))^2\right\},$$

where $\Delta(t) = 0$ for $t < \tau$ (3.2). Further, using that

$$f(X_s) = f(X(s) \mid X_{s-1}) \cdot f(X(s-1) \mid X_{s-2}) \cdot \ldots \cdot f(X(1))$$

the lr statistic for $D(s) = \{\tau > s\}$ and $C(s) = \{\tau = k \leq s\}$ reduces to

$$lr(X_s) = \prod_{i=k}^{s} \frac{f(X(i) \mid X_{i-1}, \tau = k)}{f(X(i) \mid X_{i-1}, \tau > s)}.$$

Cancelling constants and using the properties of the exponential function we find that the lr statistic depends on the data only through

$$\begin{aligned}
&\sum_{i=k}^{s} (X(i) - \phi \cdot X(i-1) + \Delta(i))^2 - (X(i) - \phi \cdot X(i-1))^2 \\
&= \sum_{i=k}^{s} [2 \cdot X(i) \cdot \Delta(i) - 2\phi \cdot X(i-1) \cdot \Delta(i)] \\
&= 2\sum_{i=k}^{s} [X(i) - \phi \cdot X(i-1)] \cdot \Delta(i) = 2\sum_{i=k}^{s} R(i) \cdot \Delta(i).
\end{aligned}$$

Now, using the specification (3.2) for Δ we find that the lr statistic depends on the data only through

$$R(k) + (1-\phi) \sum_{i=k+1}^{s} R(i),$$

for $C\{\tau = k\}$ when $k < s$ and $R(s)$ for $C\{\tau = s\}$. Hence the likelihood ratio statistic for immediate detection, $C(s) = \{\tau = s\}$, depends on the data only through $R(s)$. However, for other specifications of $C(s)$ this is no longer the case. The likelihood ratio statistic for $C(s) = \{\tau = k\}$ becomes a function of $R(k), \ldots, R(s)$.

4. Results

In this section numerical results comparing the methods are presented. To compare different methods, several evaluation measures have been suggested, see Frisén (1992) and Frisén and Wessman (1998) for overviews. The choise of which measure should be used as guidance has to be decided by using knowledge of the specific application.

We will study an AR(1) process with parameter $0 < \phi < 1$ and without loss of generality we set $\mu = 0$ and $\sigma = 1$. We will use a two-sided Shewhart test, with the limits set to give $ARL_0 = 11$. For many applications this might be too small but it will anyway show the impact of the autocorrelation on the surveillance procedures.

The critical event is a shift in mean from 0 to $\delta \cdot \sigma$ occuring at time τ. To calculate the predictive value and probability of successful detection we need knowledge of the run length given any value of τ, which is an extension from earlier papers on this matter, where only the cases $\tau = 1$ or $\tau = \infty$ have been considered. At calculation of the predictive value, we will a priori assume that τ is geometrically distributed,

$$f_\tau(t) = \nu \cdot (1-\nu)^{t-1}, t = 1, 2, \ldots,$$

where ν is the failure rate or incidence, *i.e.* $\nu = P\{\tau = t \mid \tau \geq t\}$, for $t = 1, 2, \ldots$.

4.1. The Run Length Distribution

The time to the first alarm, that is the run length, t_A, is of special interest. When $t_A < \tau$ the alarm is false and otherwise it is true. The stochastic variable t_A is a stopping time with outcomes in $\{1, 2, \ldots\}$. Figure 4 shows the the probability density function for the run length, f_{t_A}, for the modified and residual method when $\mu(t) \equiv 0$ which is denoted by $\tau = \infty$.

An often used summarizing value is the *Average Run Length* (*ARL*). More specifically, we define

$$ARL_0 = E[t_A \mid \tau = \infty],$$

the average run length when the process is in control. In quality control literature, the ARL_0 is often compared with

$$ARL_1 = E[t_A \mid \tau = 1].$$

For the residual method, the probability of calling a false alarm at a specific time is

$$p_0 = P(|R(t)| > k\sigma) = 2(1 - \Phi(k)),$$

where Φ denotes the cumulative probability density function for the standard normal distribution. The expectation $E[R(t)]$ is depending on the time since the shift (3.2). The probability of calling an alarm for $t \geq \tau$ becomes

$$p_{A0} = P(t_A = \tau) = P(|R(t)| > k \mid t = \tau) = 1 - \Phi(k - \delta) - \Phi(-k - \delta)$$

and

$$p_{A1} = P(t_A = t \mid t > \tau) = 1 - \Phi(k - \delta \cdot (1 - \phi)) - \Phi(-k - \delta \cdot (1 - \phi)).$$

The average run lengths ARL_0 and ARL_1 becomes

$$\begin{aligned} ARL_0 &= \sum_{i=1}^{\infty} i \cdot P(t_A = i \mid \tau = \infty) \\ &= \sum_{i=1}^{\infty} i \cdot p_0 \cdot (1 - p_0)^{i-1} = \frac{1}{p_0} \end{aligned}$$

and

$$\begin{aligned} ARL_1 &= 1 \cdot P(t_A = 1 \mid \tau = 1) + E[t_A \mid t_A > 1] \cdot P(t_A > 1 \mid \tau = 1) \\ &= p_{A0} + \left(\frac{1}{p_{A1}} + 1\right) \cdot (1 - p_{A0}) \\ &= \frac{1 - p_{A0} + p_{A1}}{p_{A1}}. \end{aligned}$$

For the direct Shewhart the ARL_0 depends on ϕ (Figure 5). Therefore it is not directly comparable with the other methods. It will be excluded from analyses with measurements of detection power.

Figure 6 presents the ARL_1 for the residual and modified Shewhart where the values for the latter have been obtained using computer simulations. Comparing them, we find that they both have an ARL_1 that increases with ϕ, but ARL_1 for the residual method is higher than the ARL_1 for the modified method. Using the run lengths would therefore favour the modified Shewhart method. When $\phi \approx 0.6$ there is a substantial difference.

These ARL functions have earlier been described by Schmid (1995), Wardell *et al.* (1994) and Zhang (1997). They found that the modified method has a smaller ARL_1 than the residual method, given a fixed ARL_0. Also Schmid and Schöne (1997) and Superville and Adams (1994) have found the same.

4.2. Probability of Successful Detection

For some applications it is crucial that a change is detected within a certain time, say d time steps. If an alarm is called within d time steps, actions can be taken to prevent the negative effects of the change. A relevant measure for such applications is the *Probability of Successful Detection* (PSD). We define

$$PSD(s, d, \phi) = P\{t_A < s + d \mid t_A \geq s, \tau = s\} = \frac{P\{s \leq t_A < s + d \mid \tau = s\}}{P\{t_A \geq s \mid \tau = s\}}$$

(Frisén, 1992). The PSD is generally a function of the time of the change, τ. The properties of the Shewhart test implies that the PSD for the residual method is constant over time:

$$PSD_{res}(d, \phi) = 1 - (1 - p_{A0}) \cdot (1 - p_{A1})^{d-1}.$$

Also the PSD for the modified Shewhart is constant over time and have been estimated by computer simulations.

In the special case where $d = 1$, *i.e.* the probability of immediate detection, the residual is better than the modified (Figure 7). When $\phi = 0$ the PSD for both the methods are equal. From Figure 8, where some values of the PSD for $d > 1$ are plotted, we see that the performance of both the residual and modified methods get worse when ϕ grows. Further, we can see that for values $\phi \approx 0.7$ or smaller the modified method will have a higher probability of calling an alarm here. When ϕ is close to one, the PSD becomes higher for the residual method than for the modified depending on that it still has a high probability of calling an alarm at $t = \tau$.

4.3. Predictive Value

As an alarm is called we want to know how certain we can be that a change has occured. A measure for this is the *Predictive Value* (PV), defined as

$$PV(s) = P\{\tau \leq s \mid t_A = s\},$$

(Frisén, 1992). It can be rewritten as the proportion of motivated alarms of all alarms at time s, *i.e.*

$$PV(s) = \frac{P\{t_A = s \wedge \tau \leq s\}}{P\{t_A = s\}} = \frac{PMA(s)}{PMA(s) + PFA(s)},$$

when $P\{t_A = s\} > 0$. When PV is close to 1 the alarm is highly motivated.

We define the *Probability of a False Alarm* (PFA) occuring at time s as

$$PFA(s) = P\{t_A = s \mid \tau > s\} \cdot P\{\tau > s\}. \quad (4.1)$$

The *Probability of a Motivated Alarm* (*PMA*) is not only depending on the time of the alarm, s, but also on the actual time of the change, τ, and the event to be detected. The PMA is calculated by conditioning on τ and using the distribution of τ

$$PMA(s,\delta) = \sum_{t=1}^{s} P\{t_A = s \mid \tau = t\} \cdot P\{\tau = t\}. \tag{4.2}$$

To derive the PV for the residual method we find the PFA using (4.1)

$$PFA(t) = p_0 \cdot (1-p_0)^{t-1} \cdot (1-\nu)^t,$$

which is independent of ϕ. Secondly, we use (4.2) to find PMA

$$PMA(s) = \sum_{t=1}^{s} \nu (1-\nu)^{t-1} \cdot l(s,t,\phi),$$

where $l(s,t,\phi) = P\{t_A = s \mid \tau = t\}$ for $t \leq s$. For the residual method $l(s,t,\phi)$ can be calculated exactly and

$$\begin{aligned} PMA_{res}(s) = {} & \nu \cdot (1-\nu)^{s-1} \cdot (1-p_0)^{s-1} \cdot p_A(0) \\ & + \sum_{t=1}^{s-1} \nu \cdot (1-\nu)^{t-1} \cdot (1-p_0)^{t-1} (1-p_A(0))^{s-t-1} \cdot p_A(1). \end{aligned}$$

For the modified Shewhart the l-function, p_0 and p_A have been estimated by computer simulations. In Figure 9 PV for $\phi = 0$, $\phi = 0.2$ and $\phi = 0.9$ of the residual and modified method are plotted. Often it is reasonable to choose an ARL_0 high enough to ensure that the monitoring stops before the $t = ARL_0$ when $\tau = 1$. When $t < ARL_0$

$$\phi' < \phi'' \Rightarrow PV(t,\phi') > PV(t,\phi''),$$

for the cases presented in the figure. Further, when $t = 3, 4, \ldots$ the predictive value for the modified Shewhart is higher than for the residual method. Initially the modified Shewhart is having a very poor PV, which is depending on the high flase alarm probability at $t = 1$ (Figure 4). At $t = 2$ the methods are almost equal, but the modified method is better when $\phi = 0.9$ and the residual when $\phi = 0.2$.

5. Discussion

The direct Shewhart method will have an ARL_0 which is increasing with ϕ. In order to obtain a constant ARL_0 the limits would have to be adjusted and set equivalent with the modified Shewhart. Hence direct Shewhart is not fully comparable with the others.

The likelihood ratio method is optimal in the Neyman-Pearson sense, *i.e.* have the highest probability of calling a true alarm given a specific false alarm probability (Frisén and de Maré, 1991). When the method is optimized to detect a change immediately, *i.e.* when $C(s) = \{\tau = s\}$, the lr method and the residual method become equivalent, and in the special case $\phi = 0$ also the modified and direct Shewhart methods become equvivalent with the likelihood ratio. For other specifications of $C(s)$, *eg.* an event occuring at a specified time, $t < \tau$, the lr statistic is not a function of $R(s)$ or $X(s)$ only.

Apart from the case $C(s) = \{\tau = s\}$, both the residual and the modified Shewhart are suboptimal. As was seen above, the residual method is not monitoring the *level* of the mean, but instead the *change* in level of the mean. This effect have been observed and discussed by among others Harris and Ross (1991), Ryan (1991), Superville and Adams (1994) and Wardell *et al.* (1992, 1994). Wardell *et al.* (1994) showed that the run length distribution after a shift has occured is almost equal to the in control run length distribution, one time step after the shift.

Clearly, if we can identify the process under study and the requirements we have on the surveillance procedure, it might be possible to construct an optimal lr procedure. However, reducing the data with the tranformations presented above will, except in a few special cases, lead to a loss of information and suboptimal procedures. Applying EWMA or CUSUM on the reduced data can not get that information back.

Summarizing the findings about the ARL_1, PSD and PV (for $t < ARL_0$) we see that for most of the cases studied both the modified and the residual methods have worse performance for larger values of ϕ. For $\phi \approx 0.7$ and smaller, the modified method will be better, except for immediate detection. The low $PV(1)$ for the modified method is due to the high probability of a false alarm at $t = 1$. Zhang (1997) pointed out, as a rule of thumb, that the residual method is to prefer when $\delta > 2$ and $\phi > 0.8$.

As have been noted by many authors before, the residual chart does not give a full picture of the process and is only sufficient for some specifications of the considered process. To overcome the disadvantages of the residual and modified methods Adams *et al.* (1994) suggested that both the observed and residual processes should be used simultanously. Alwan (1992) compared the alarms given by either of the observed and residual process. This approach will lead to a multivariate monitoring problem, discussed by *eg.* Jones *et al.* (1970), Kramer and Schmid (1997) and Wessman (1998).

From the results in the earlier Sections we find that one method is not always uniformly better than the other. Neither the residual nor the modified method are optimal except in a few special settings, for example the residual method for the situation of immediate detection of a shift.

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Legend to Figures

1. Simulated example of an AR(1) process ($\phi = 0.5$) with a shift in mean, $\mu(t) = 10 \cdot \mathbf{1}\{t = 40, 41, \ldots\}$.

2. The residuals, $R(t)$, from the process in Figure 1.

3. The alarm limits for the direct (solid) and modified (dotted) shewhart for different values of ϕ.

4. The *pdf* of the in control run length, $f_{t_A}^{0}(t)$, for the modified Shewhart (cross) and the residual method (ring) for a process with high autocorrelation ($\phi = 0.9$).

5. The ARL_0 for direct shewhart for different values of ϕ. The limits were set to make $ARL_0 = 11$ for $\phi = 0$.

6. The ARL_1 for the modified Shewhart (dotted) and the residual method (solid) for different values of ϕ, where $ARL_0 = 11$.

7. The $PSD(1)$, *i.e.* the probability of immediate detection after a change, for the modified (dotted) and residual method (solid) for different values of ϕ, where $ARL_0 = 11$.

8. The $PSD(d)$, *i.e.* the probability of detection before d timesteps after the change, for the modified (dotted) and residual (solid) method for different values of ϕ, where $ARL_0 = 11$. The upper and lower pairs have $d = 7$ and $d = 3$, respectively.

9. The $PV(t)$, *i.e.* the predictive value of an alarm at time t, for the modified and residual method, where $\phi = 0.2$ and $\phi = 0.9$. The solid lines for the residual method and dotted for the modified Shewhart. (X) marks for the situation where $\phi = 0.2$ and (o) for $\phi = 0.9$. The dotted line without marks is for $\phi = 0$. The incidence $\upsilon = 0.1$.

Fig 1

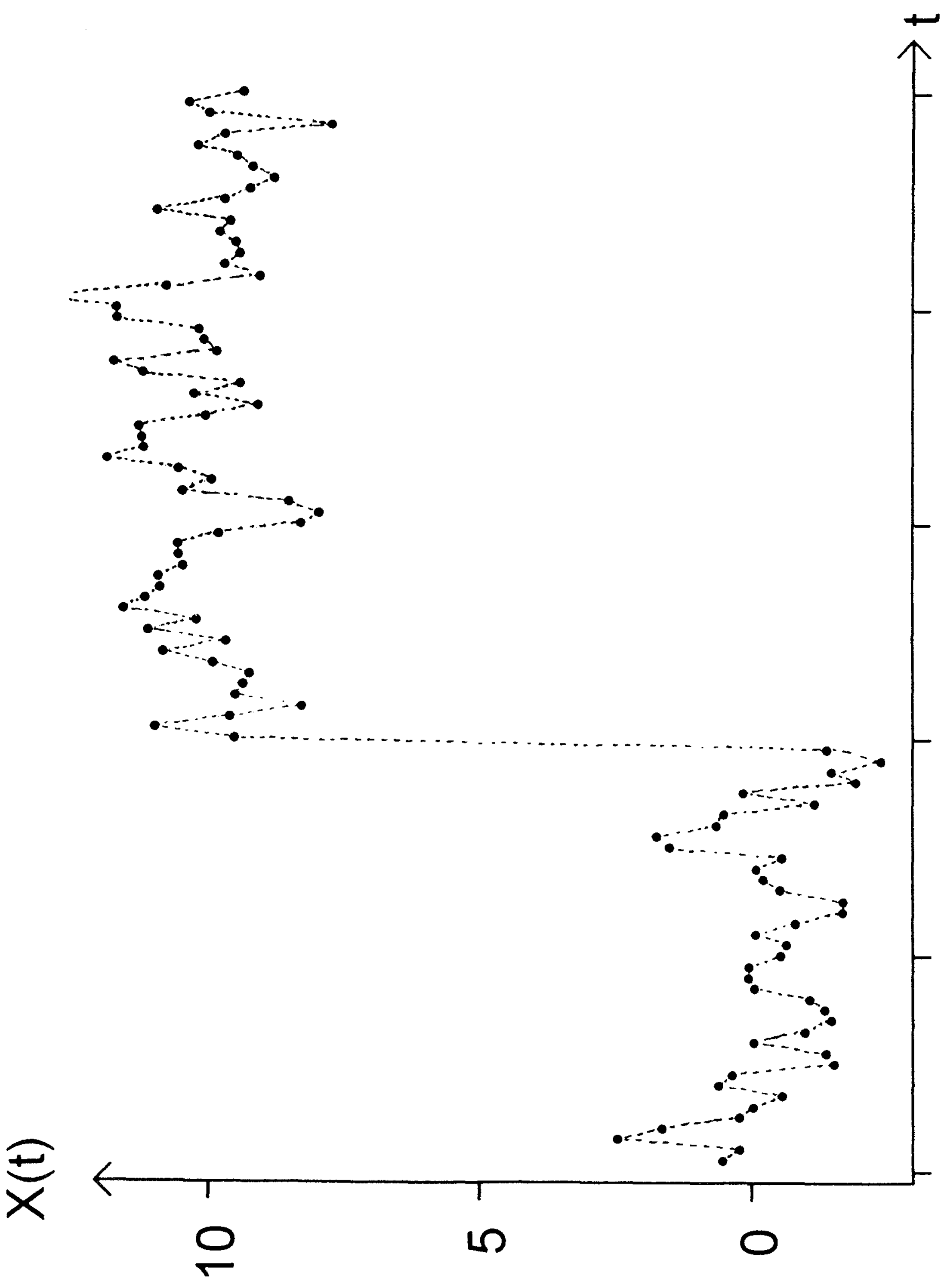


Fig 2

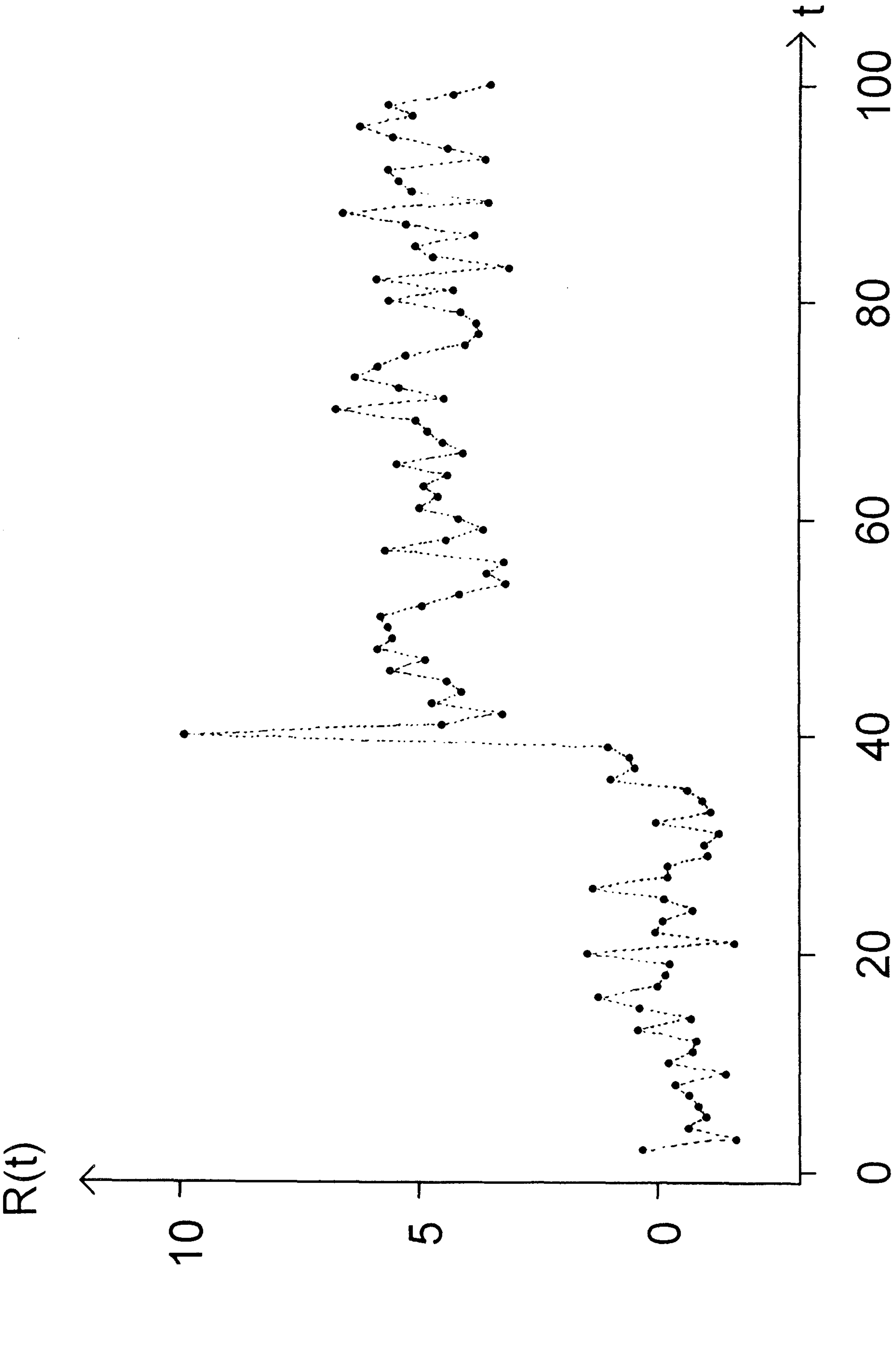


Fig 3

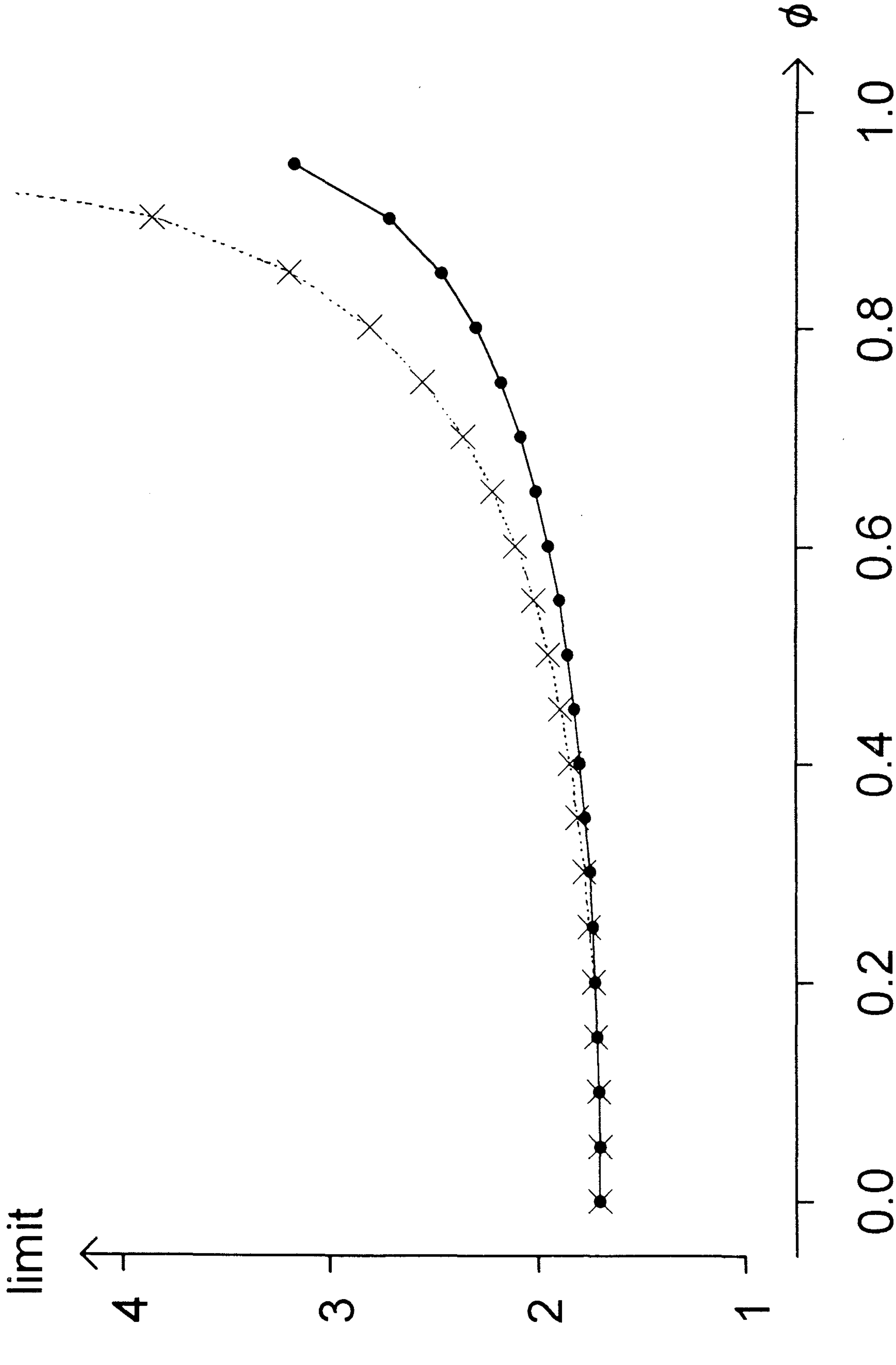
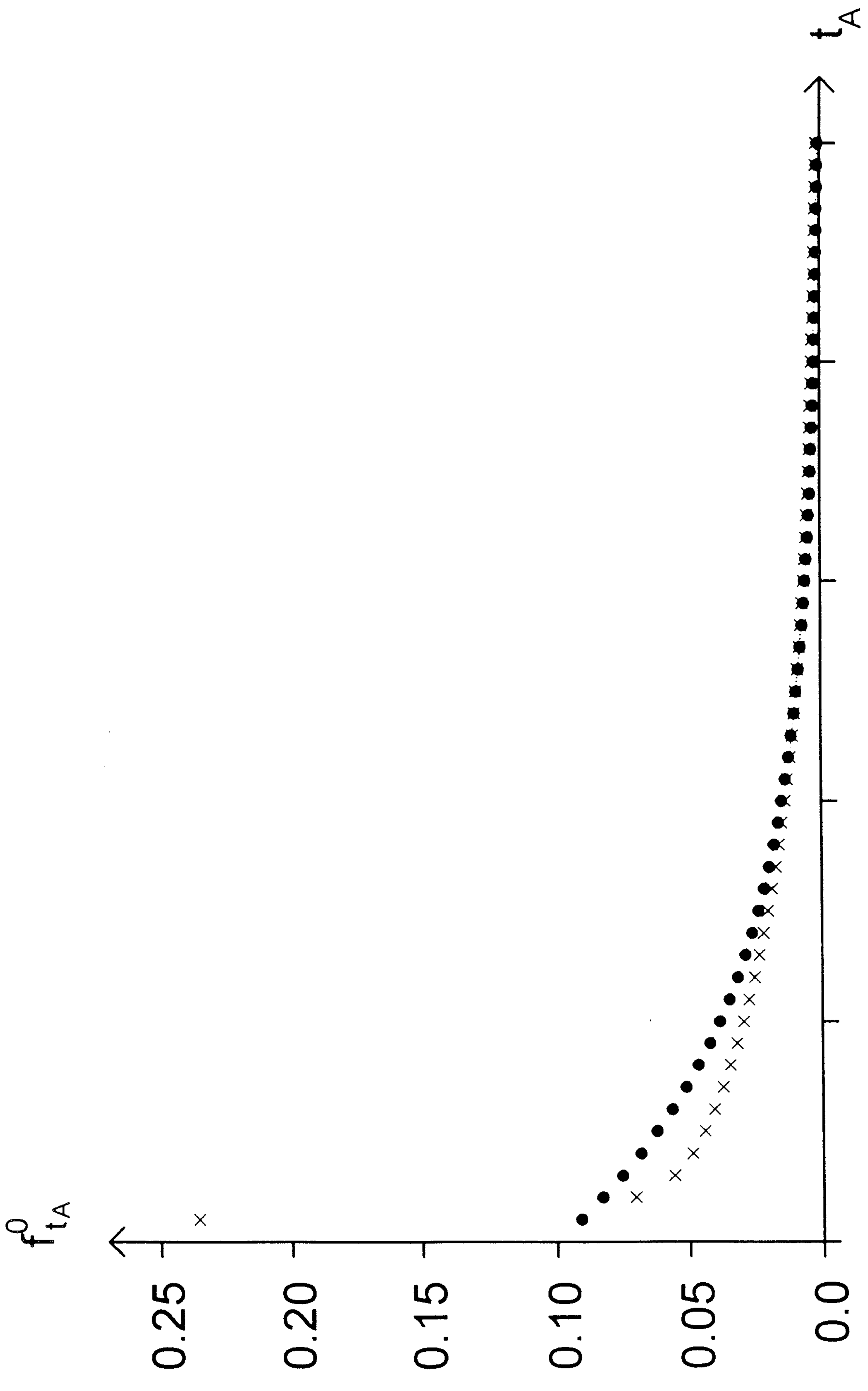


Fig 4



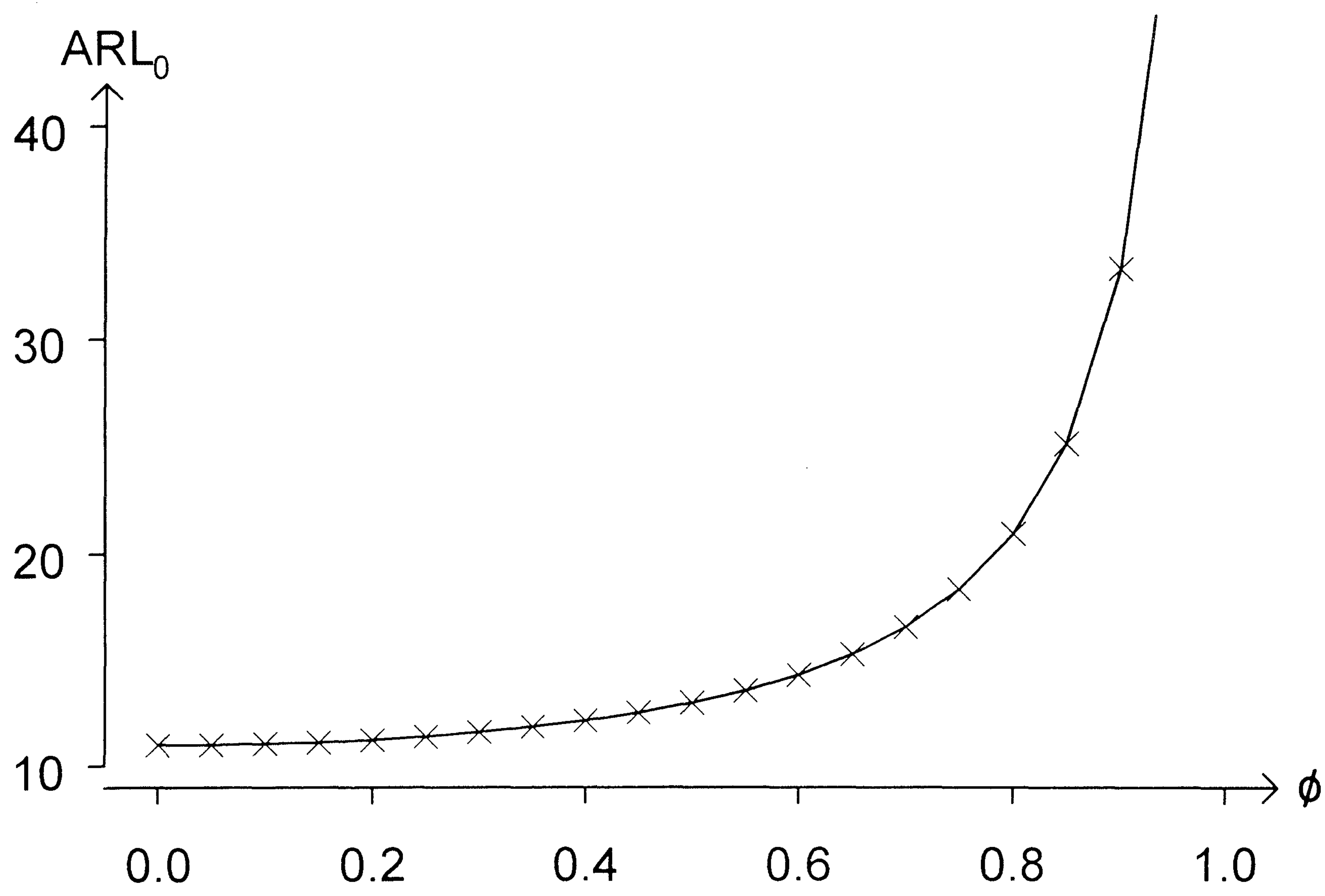


Fig 5

Fig 6

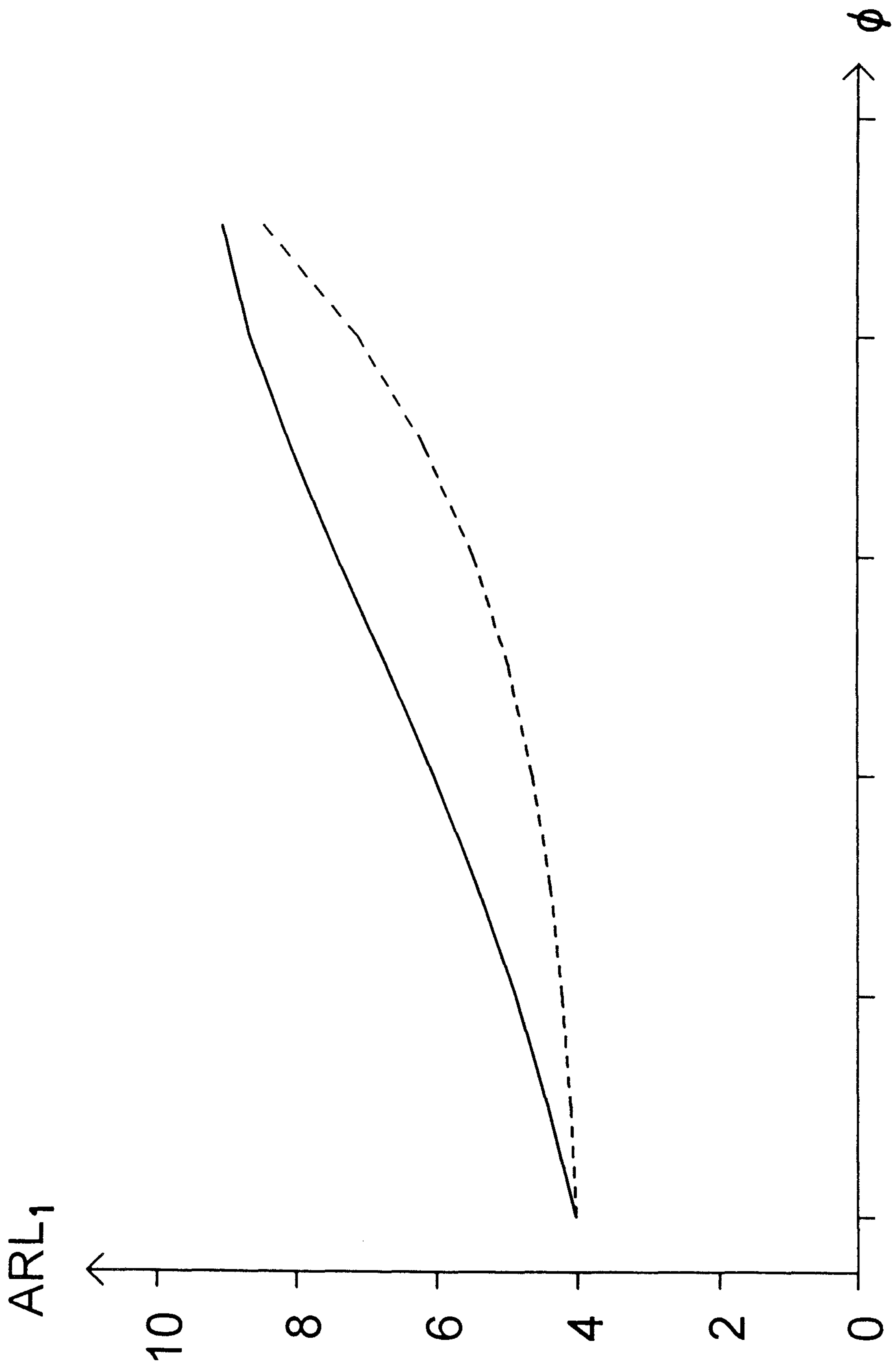


Fig 7

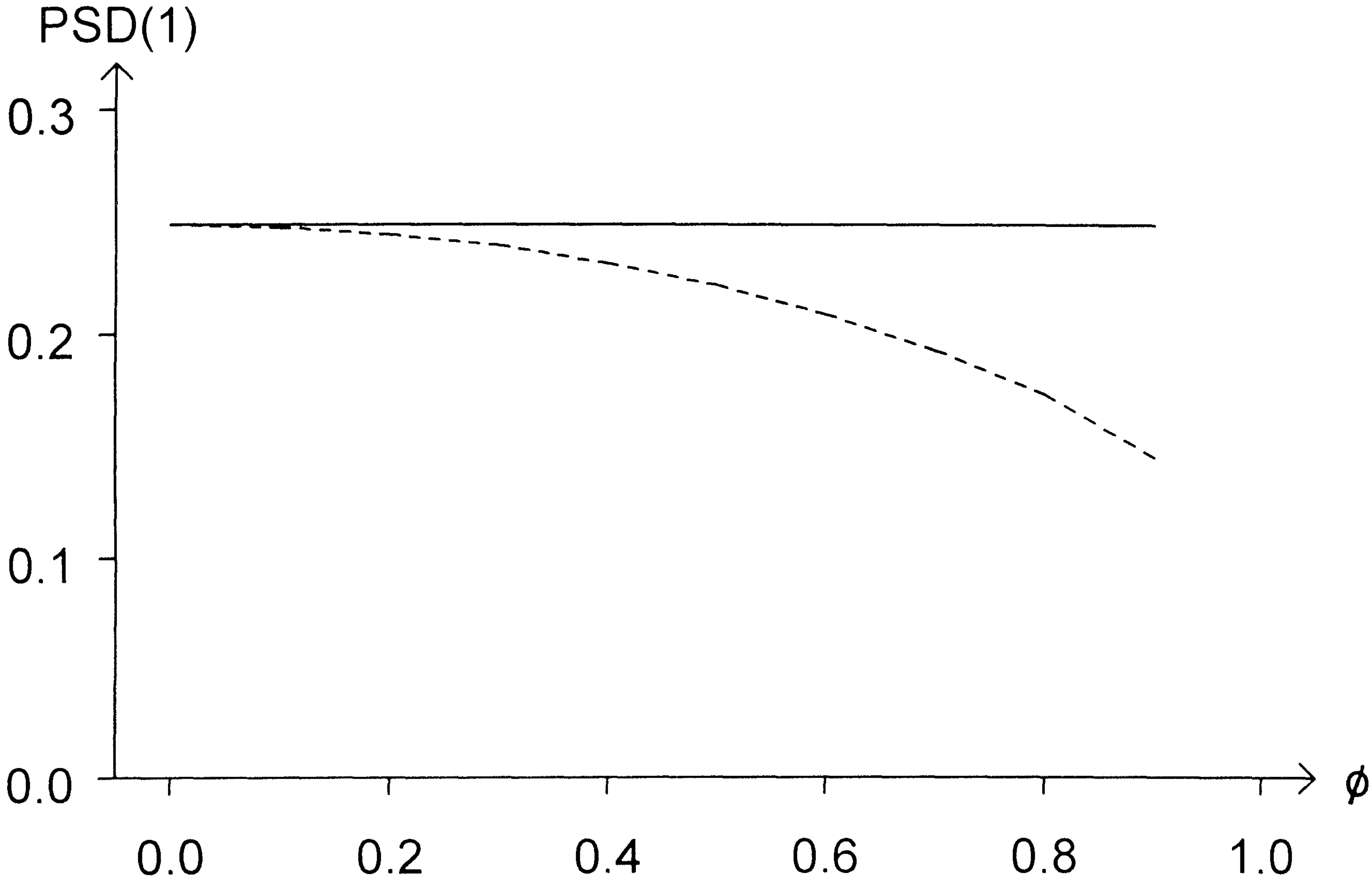


Fig 8

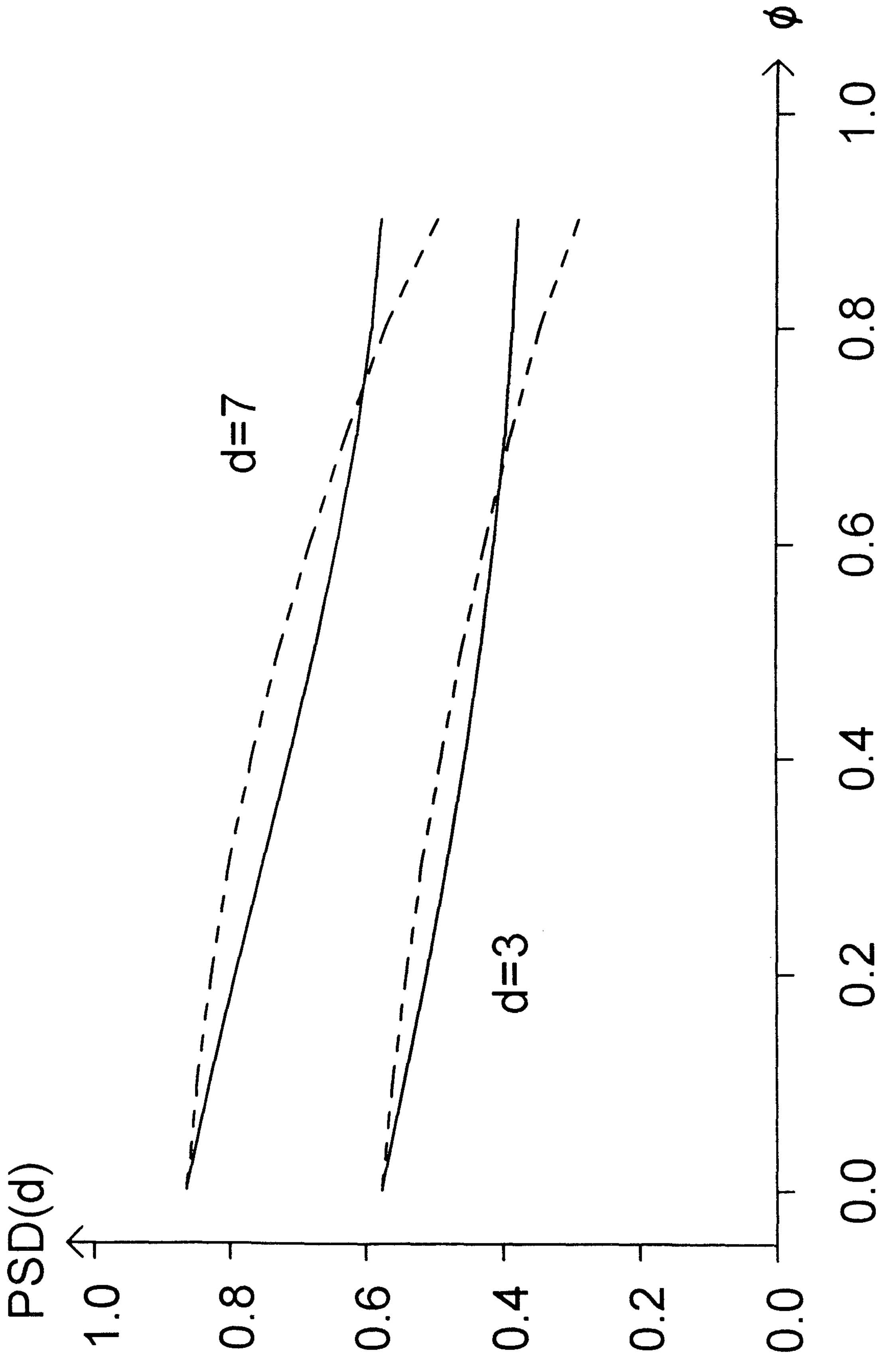
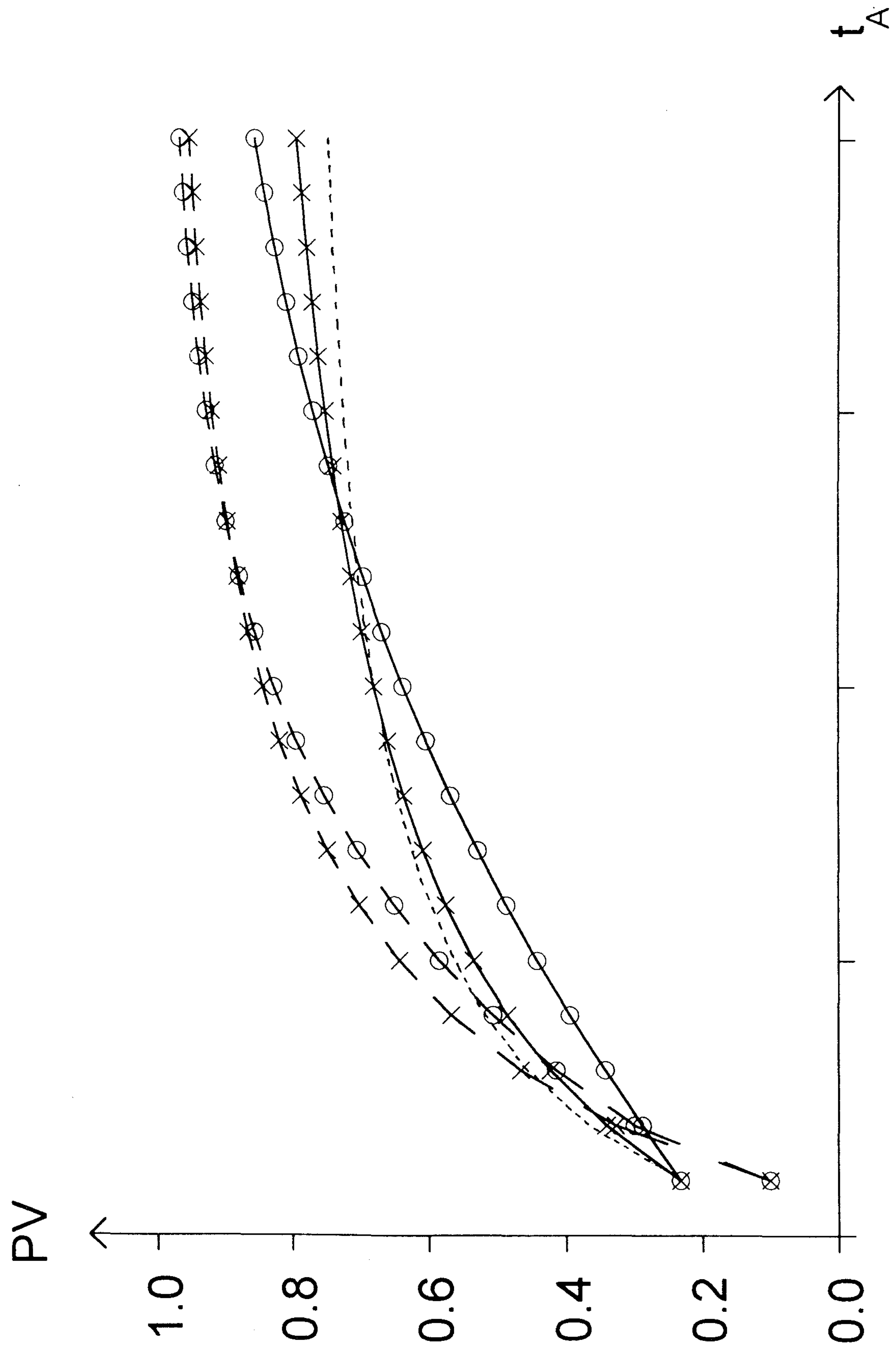


Fig 9



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1996:2	Wessman, P	Some principles for surveillance adopted for multivariate processes with a common change point.
1996:3	Frisén, M. & Wessman, P	Evaluations of likelihood ratio methods for surveillance.
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