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Index theory in geometry and physics

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Abstract

This thesis contains three papers in the area of index theory and its applications in geometry and mathematical physics. These papers deal with the problems of calculating the charge deficiency on the Landau levels and that of finding explicit analytic formulas for mapping degrees of Hölder continuous mappings.

The first paper deals with charge deficiencies on the Landau levels for non-interacting particles in $\mathbb{R}^2$ under a constant magnetic field, or equivalently, one particle moving in a constant magnetic field in even-dimensional Euclidian space. The K-homology class that the charge of a Landau level defines is calculated in two steps. The first step is to show that the charge deficiencies are the same on every particular Landau level. The second step is to show that the lowest Landau level, which is equivalent to the Fock space, defines the same class as the K-homology class on the sphere defined by the Toeplitz operators in the Bergman space of the unit ball.

The second and third paper uses regularization of index formulas in cyclic cohomology to produce analytic formulas for the degree of Hölder continuous mappings. In the second paper Toeplitz operators and Henkin-Ramirez kernels are used to find analytic formulas for the degree of a function $f : \partial\Omega \to Y$, where Ω is a relatively compact strictly pseudo-convex domain in a Stein manifold and Y is a compact connected oriented manifold. In the third paper analytic formulas for Hölder continuous mappings between general even-dimensional manifolds are produced using a pseudo-differential operator associated with the signature operator.

Keywords: Index theory, cyclic cohomology, regularized index formulas, Toeplitz operators, pseudo-differential operators, quantum Hall effect.