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**Performance and Sensitivity Analysis of the VaR-Based Portfolio  
Insurance Strategy**

Empirical study of Sweden 1989-2011

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# **Performance and Sensitivity Analysis of the VaR-Based Portfolio Insurance Strategy**

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## **Abstract**

This paper evaluates the empirical performance of the VaR Based Portfolio Insurance (VBPI) relative to the Constant Proportion Portfolio Insurance (CPPI) based on Swedish data for 1989-2011. The evaluation emphasizes on the two strategies' ability to combine downside protection with upside potential, with the Omega measure as the main performance evaluator. Furthermore, the empirical implications of the inherent model risk of VBPI are evaluated with a sensitivity analysis focusing on the impacts of alternative estimates of the instantaneous growth rate and volatility of the risky asset. The conclusions of the paper are that the VBPI underperformed relative to CPPI on most performance measures, including Omega, in most scenarios during 1989-2011 and that the VBPI suffered severely from model risk.

*Key words:* VaR, Value at Risk, VBPI, CPPI, Portfolio Insurance, Omega, Black-Scholes, Geometric Brownian Motion, Gap risk, Portfolio Management, Expected Net Gain,

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## 1. Introduction

In this section we will introduce the concept of portfolio insurance (PI), mention a few related studies and outline the research questions. In Subsection 1.1, we will explain the idea behind portfolio insurance and discuss issues relating to measuring portfolio performance. Subsection 1.2 will discuss the research questions and the motivation behind the study. In Subsection 1.3, we will disclose a few relevant studies on the subject of PI.

### 1.1 Introduction and background

In this subsection, we will cover the background of our study. Looking at two centuries of data on returns of financial assets, Siegel (2008) among others has shown us that in the long run stocks are the preferred choice for achieving the highest asset value growth. Many times though, an investor will hold a portfolio not for the “long run” but for shorter, more time-specific periods. In that case, relying on the long run potential of risky assets can backfire severely as investors have experienced in several stock market falls during the last decades. One approach an investor or portfolio manager can use to prevent the portfolio from having these extreme drops in value is to implement some form of trading strategy to protect the value of assets in the portfolio. In academia investors are usually assumed to be risk averse to some degree and the degree of risk aversion will then also indicate the willingness to trade off potential growth to gain a more limited downside risk. The question is then how investors effectively can construct their portfolio in order to achieve desirable return potential and at the same time limit the downside risk for the investment period.

A structured way of setting a floor value, sometimes called protection level or insurance level, is called portfolio insurance (PI). The idea is to set a minimum amount that the portfolio is “guaranteed” to have at the end of the investment period, without cutting off all upside potential of the risky asset. The strategy is thus expected to reshape the return density function by cutting off the left tale of it while keeping the right tale as large as possible. Rubinstein and Leland (1981) were the first to introduce this idea with option based portfolio insurance (OBPI) which, in its simplest form, involves buying an out-of-the-money put option written on the portfolio, offsetting any portfolio loss under the guaranteed level at the maturity of the investment period. The set-up of OBPI depends on the assumption that a put option with strike and maturity matching the investor’s portfolio and investment period is available on the market, which often is not the case and certainly was not in the seventies. Using the reciprocal of the idea of Black and Scholes (1973), Rubinstein and Leland

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(1981) showed how an investor can replicate any option payoff structure by trading a combination of stock and cash, creating a synthetic option. In order to achieve this, the investor needs to make assumptions about the underlying dynamics of the risky and risk free asset. One of the most common assumptions in option-related finance theory is that the risky asset follows a geometric Brownian motion, leading to explicit formulas to calculate the weights to invest in each asset class in order to perform the option replication. A PI strategy based on a synthetic option approach will then by definition be model dependent.

The theory of constant proportion portfolio insurance, CPPI, was first developed by Black and Jones (1987) and Black and Perold (1992) as simple alternative to the previous more complex portfolios insurance theories based on option replication and specification of asset dynamics. It has its roots in expected utility maximization theory as pointed out by Zhu and Kavee (1988). Since the CPPI technique does not rely on assumptions on asset dynamics, it will be model independent. However, as pointed out by Pain and Rand (2008) the end value in a CPPI strategy is path dependent since it is a function of both the final price and the price history of the risky asset during the investment period.

Several PI strategies have been developed throughout the years. One particularly interesting PI strategy is the Value-at-Risk-based PI (VBPI) strategy suggested by Jiang et al (2009). Unlike the OBPI and CPPI strategies, the VBPI method targets the gap risk of the PI strategy, that is the risk that the portfolio falls under the pre-set floor at the end of the investment period. This is done in the VBPI method by applying the principles of value-at-risk (VaR) to the management of the portfolio.

The problem of evaluating the outcomes of different portfolio management strategies, such as PI strategies, is how to take into account all aspects of the distribution of the portfolio values at the maturity. Here the risk and return of the portfolio distribution are of course central aspects. Traditional performance measures focus on the first two moments of the return distribution and are therefore not enough to capture the whole idea and spirit of PI. The problem with these measures can be illustrated in Figure 1, showing two return distributions with the same mean and variance. If the two distributions were return distributions from different PI strategies it is clear that the mean and variance could not tell them apart. Looking at the distributions graphically, we would see the difference since a graph shows the whole distribution but deciding upon performance could still be difficult. A sufficient performance metric is needed that takes the whole distribution into account. At

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least two measures manage to do that, the Omega measure developed by Keating and Shadwick (2002) and the ENG measure (the expected net gain) proposed by Lee et al (2008).

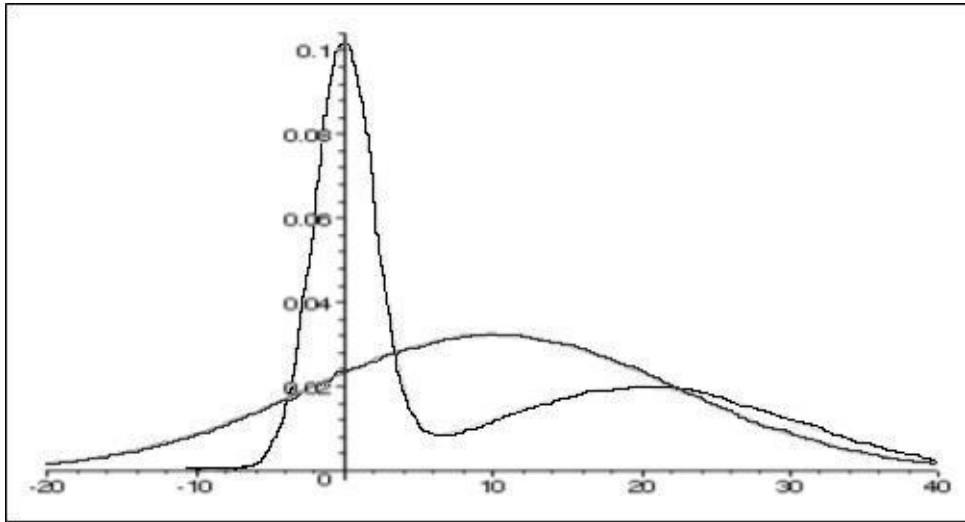


Figure 1 Density functions with the same mean and variance (Keating & Shadwick (2002)). Illustration of two return distributions, where standard statistics (mean & variance) cannot, from a PI perspective, evaluate performance.

## 1.2 Research questions

In this subsection we lay out the research questions and argue why they are relevant to the present discussion on portfolio management.

In this paper two PI strategies, VBPI and CPPI, will be compared to each other and to buy and hold (B&H) strategy. The methodology used will in many aspects replicate the setup of Jiang et al (2009) in terms of model and investment period in order to be able to make comparisons. The first question that we want to investigate in this thesis is:

*Has the VBPI outperformed the CPPI for 3-month investment periods during 1989-2011 in Sweden?*

Furthermore, the intention of this thesis is also to examine the sensitivity of the VBPI model to the accuracy of the estimates used in the model, which are the expected return on the risky and the risk-free asset and the volatility of the risky asset's return. The VBPI strategy is based on an assumption of the asset price dynamics that have been criticized in many articles (see e.g. B. Mandelbrot (1963)), although the criticism has not been directed specifically to the application of PI. Jiang et al (2009) discuss those issues briefly and leave it for further research. Hence, the second question that we try to answer is:

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*How sensitive is the performance of VBPI to assumptions about the underlying growth rate and volatility of the risky asset?*

The period of 1989 to 2011 includes several market plummets in Sweden, not to mention the financial crisis of 2008. These drops produce such volatility and downside in the assets that we find it to be suitable to test the performance of different PI strategies in this period. To the best of our knowledge, there has been no other study of the VBPI strategy covering the financial crisis of 2008, and especially no studies focusing on the sensitivity of the model dependency. Hence, in this thesis we investigate the sensitivity of the performance of VBPI to estimates of the underlying dynamics of the risky asset.

### **1.3 Previous relevant empirical studies on PI performance**

In this subsection, we discuss shortly three relevant studies on PI performance.

Several papers have done empirical studies of the performance of different PI strategies, either relative to each other or to a simple buy and hold (B&H) strategy. Loria et al (1991) use Australian data to demonstrate that even though the synthetic (futures based) OBPI strategy did provide an effective protection to downside risk, it also had several drawbacks. Firstly, the model had large exposure to the gap risk when experiencing large drops in the market. Secondly, they showed that the model assumptions had substantial influence on the performance of the strategy. As an example, reducing the expected growth of the risky asset increased the likelihood that the portfolio value at maturity would be above the pre-specified floor. Impacts from assumption on volatility also played a role both in upwards and downwards trending markets. Due to the model dependency of synthetic OBPI, investors utilizing this strategy always face the risk of making replication errors when performing the strategy. This problem could be avoided by buying a put option on the portfolio, but then instead the risk of illiquidity and/or counterparty risk can arise.

Annaert et al (2008) make use of block-bootstrap simulation which is based on a historical return distribution of US, UK, Japan, Austria and Canada equity markets to evaluate CPPI and OBPI relative to a B&H strategy. Like many others they verify the effectiveness of PI strategies to limit downside risk. However, in their study which is based on a stochastic dominance criterion, they cannot conclude that the benefits of a floor protection outweigh the decrease in expected return and are thus unable to determine whether portfolio

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insurance dominates a buy and hold strategy or vice versa. One of the performance measures they use is the above mentioned Omega measure. As explained in Subsections 2.3 and 4.5, the Omega measure is calculated for a certain threshold, for example the preferred minimum return. Since the choice of a threshold can affect the ranking of the portfolios, it is also possible to plot Omega as a function of different thresholds to provide a more thorough analysis of the portfolio performance. In the study of Annaert et al (2008) only one single threshold is used which limits the ability to make any deeper analysis of the results. However, they show how sensitive PI performance metrics are to changes in rebalancing frequency and the level of floor protection.

Jiang et al (2009) evaluate the relative performance of VBPI to CPPI and B&H, based on Chinese data from December 30, 1999 to December 28, 2007. Several performance measures, relevant to a risk-averse investor, are used in the evaluation but most focus is done on the Omega measure. Here, the Omega measure is presented as a function of different threshold the authors consider appropriate. The study concludes that the VBPI is to be preferred by risk-averse investors and that VBPI is generally more effective in sustaining the floor protection relative to CPPI. The biggest outperformance of the VBPI strategy against the CPPI method is when monthly rebalancing is used in the presence of transaction costs. The authors point out that VBPI underperforms when rebalancing is too frequent and that this fact might relate to model errors.

In Section 2 of this paper the VBPI and CPPI strategies will be presented in depth, along with the performance measures Omega and ENG. The dataset and the construction of portfolios are found in Section 3. Section 4 describes the methodology used in the study and results are analyzed in Section 5.



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## 2. Theoretical review

In this section we explain the two portfolio insurance strategies that are used in our study, as well as two performance measures that have been presented to particularly measure performance of portfolios by looking at the whole distribution of returns. In Subsection 2.1 we outline the CPPI strategy while Subsection 2.2 discusses the VBPI method. In Subsection 2.3 the Omega measure is described and in Subsection 2.4 the ENG measure is defined.

### 2.1 The CPPI strategy

The idea of constant proportion PI, CPPI, is the same as when buying a protective put; to set a pre-specified floor and prevent the portfolio value from falling below that floor. The investment mix is determined by a multiple ( $m$ ) and a cushion ( $C$ ), which equals the difference between the portfolio value minus the floor discounted by the risk-free rate:

$$C_t = P_t - F_T e^{-r(T-t)}$$

where  $P_t$  is the portfolio value at time  $t$ ,  $F_T$  is the floor at the end of the investment period and  $r$  is the risk-free rate. The multiple  $m$  reflects the investor's risk aversion with a lower multiple for a more risk-averse investor as will be specified below.

With the above quantities defined, the idea of CPPI is to invest the multiple  $m$  times the cushion in the risky asset, denoted by  $e_t$ , while holding the rest in a safe risk free asset, that is

$$e_t = mC_t = m(P_t - F_T e^{-r(T-t)}) \quad (1)$$

As time goes on and the value of risky asset varies heavily, the CPPI strategy requires the investor to dynamically rebalance the portfolio in order to hold the calculated cushion times the multiple in the risky asset. No more, no less. If the rebalancing is done continuously without transaction costs and with perfect liquidity in the market, the portfolio cannot fall below the floor. Therefore the CPPI method is in theory a perfect PI strategy. Black and Perold (1992) presented how the CPPI in this context is equivalent to a perpetual American call option. They also showed that following a CPPI strategy is the optimal investment strategy for certain assumed utility functions.

### 2.2 The VBPI strategy

In this subsection we explain the theory behind the VBPI strategy. In financial risk management Value at Risk (VaR) is an important concept with many areas of application. For

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most banks the use of VaR is mandatory in order to decide upon the regulatory capital provision needed (Bank for International Settlements (2004)). Recall that VaR is essentially a way to measure risk in a portfolio of assets and present it as a single number. The VaR measure provides the maximum amount of money that an asset portfolio is expected to lose at a specific confidence level in a fixed period of time. As an illustration, imagine that the ten day VaR of a certain portfolio is estimated to be 10 M SEK at the 99% confidence level. This means that the portfolio will not lose more than 10 M SEK in the next 10 business days with 99% probability. Furthermore, the probability of this event is computed using certain model assumptions.

In the context of portfolio insurance (PI), having a method to estimate this worst case loss, is equivalent to having a method that takes the gap risk of a PI strategy into consideration. In other words, with a PI strategy based on VaR the investor can choose not only the floor protection aligned with her risk aversion, but also the likelihood that the value of the portfolio is above the floor at maturity. As stated previously, with continuous trading, no transaction costs and perfect market liquidity no such additional feature would be necessary. In reality though, trading is discrete, transaction costs are present and markets are not perfectly liquid.

Since VaR in a VBPI should be equal to the expected maximum loss at maturity, the investor needs to specify the underlying dynamics of the risk free and risky asset. A common assumption in finance is that the risky asset follows the Black and Scholes (1973) framework. Although the assumptions it is based on have met critique in academia and in the finance industry (as mentioned in 1.2), the framework is well known in finance and also produces many convenient results. In the context of VBPI, the Black and Scholes framework implies closed formulas for the calculation of the weights to be invested in risky and risk free assets, respectively.

According to the derivation of Jiang et al (2009), which is based on the dynamic risk budgeting approach by Strassberger (2006), the risk-free asset is deterministic and follows the process  $dB_t = rB_t dt$  where the initial value of the risk-free asset  $B_0$  is positive and  $r$  is the risk-free rate. Hence,  $B_T = B_0 e^{rT}$ . The risky asset follows a geometric Brownian motion, that is

$$dS_t = S_t(\mu dt + \sigma dW_t) \quad (2)$$

where the initial value of the risky asset  $S_0$  is positive. Furthermore,  $\mu$  and  $\sigma$  are the instantaneous return and volatility of the risky asset, respectively. In this setup,  $\mu$  is assumed

to be greater than  $r$ . The process  $W_t$  is a standard Brownian motion which is, as explained in Hull (2009), a stochastic process, normal distributed with mean 0 and a variance rate of 1 per year.

The object is to find a formula for  $w$ , the weight to invest in the risk-free asset and  $(1-w)$  that will be invested in the risky asset. Assuming that the initial portfolio value is  $V_0$ , the number of units of the risk free asset  $\beta$  will be equal to  $\frac{wV_0}{B_0}$  and equivalently the number of units of the in the risky asset  $\eta$  will be equal to  $\frac{(1-w)V_0}{S_0}$ . Consequently, if no rebalancing is implemented until maturity, the value of the portfolio at maturity will be  $V_T = \beta B_T + \eta S_T$ . By using Itô's lemma, Equation (2) implies that the value of  $S_T$  at maturity is given by  $S_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W_T}$ . Furthermore, by plugging the definition of  $\beta$  and  $\eta$  into  $V_T$ , additional to the final values of  $B_T$  and  $S_T$ , the following expression of the portfolio value at the end of the investment period is derived:

$$V_T = wV_0 e^{rT} + (1-w)V_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W_T}$$

The expected value of the portfolio at the end of the investment period is

$$E(V_T) = wV_0 e^{rT} + (1-w)V_0 e^{\mu T}$$

where we have used the standard result  $E\left(e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W_T}\right) = e^{\mu T}$ . The VaR, for a given confidence level  $p$ , is in the context of VBPI equal to the difference of the expected value of the portfolio and the insured floor  $G$  at the terminal date. Hence,

$$VaR_p = (E(V_T) - G) = wV_0 e^{rT} + (1-w)V_0 e^{\mu T} - G.$$

Solving for  $w$ , the proportion invested in the risk-free asset, gives:

$$w = \frac{G + VaR_p - V_0 e^{\mu T}}{V_0 (e^{rT} - e^{\mu T})}.$$

Since the return of the risk-free asset is considered to be known and constant, the only source of risk in the portfolio is from the risky asset. The VaR of the portfolio is thus equal to the VaR of the risky asset, that is (see p. 187 in Jiang et al (2009))

$$VaR_p = (1-w)V_0 e^{\mu T} - (1-w)V_0 e^{(\mu - \frac{1}{2}\sigma^2)T - \sigma z_p \sqrt{T}}$$

where  $N(z_p) = p$  and  $N(x)$  is the standard normal cumulative distribution function and  $p$  is the confidence level. The VaR is the deviation of the value of the risky asset from its expected value, given a confidence level. Plugging the expression for VaR into the formula for  $w$  gives

$$w = \frac{G + (1-w)V_0 e^{\mu T} - (1-w)V_0 e^{(\mu - \frac{1}{2}\sigma^2)T - \sigma z_p \sqrt{T}} - V_0 e^{\mu T}}{V_0 (e^{rT} - e^{\mu T})}$$

so that

$$\begin{aligned}
& wV_0e^{rT} - wV_0e^{\mu T} + wV_0e^{\mu T} - wV_0e^{(\mu - \frac{1}{2}\sigma^2)T - \sigma z_p \sqrt{T}} \\
& = G + V_0e^{\mu T} - V_0e^{(\mu - \frac{1}{2}\sigma^2)T - \sigma z_p \sqrt{T}} - V_0e^{\mu T}
\end{aligned}$$

and hence,  $w$  is given by

$$w = \frac{G - V_0e^{(\mu - \frac{1}{2}\sigma^2)T - \sigma z_p \sqrt{T}}}{V_0e^{rT} - V_0e^{(\mu - \frac{1}{2}\sigma^2)T - \sigma z_p \sqrt{T}}} \quad (3)$$

where  $z_p = N^{-1}(p)$ .

The above closed formula indicates that two parameters must be chosen by the investor, namely the floor  $G$  and the confidence level  $p$ . The choice depends on the degree of risk aversion, with lower levels of both parameters resulting in a more aggressive investment. The closed formula also shows that the three parameters  $\mu$ ,  $r$  and  $\sigma$  need to be estimated. Estimation of the inputs will affect the amount invested in risky and risk free assets respectively. Overestimation of  $\mu$  and/or  $r$  will lead to overinvestment in the risky asset while overestimation of  $\sigma$  will lead to underinvestment in the risky asset, which implies that the VBPI strategy has an inherent model risk.

### 2.3 The Omega performance measure

In Keating and Shadwick (2002) the authors first developed the Omega measure as a universal performance measure, aimed at measuring performance of portfolios or investment strategies by taking the whole return distribution into consideration. The mathematical definition of Omega is the following:

$$\text{Omega}(V) = \frac{\int_V^b (1 - F(x)) dx}{\int_a^V F(x) dx}$$

where  $F(x)$  is the empirical cumulative distribution function of the risky asset's return, the interval between  $a$  and  $b$  is the return interval and  $V$  is the threshold to be specified by the investor. All returns below the chosen threshold  $V$  are considered to be losses while those above  $V$  are viewed as gains. For any investor, a portfolio with higher Omega for a chosen threshold  $V$  is always to be considered the preferred portfolio. The higher the threshold  $V$ , the lower Omega and thus Omega is a strictly decreasing function of the threshold  $V$ . Figure 2 shows the Omega function graphically.

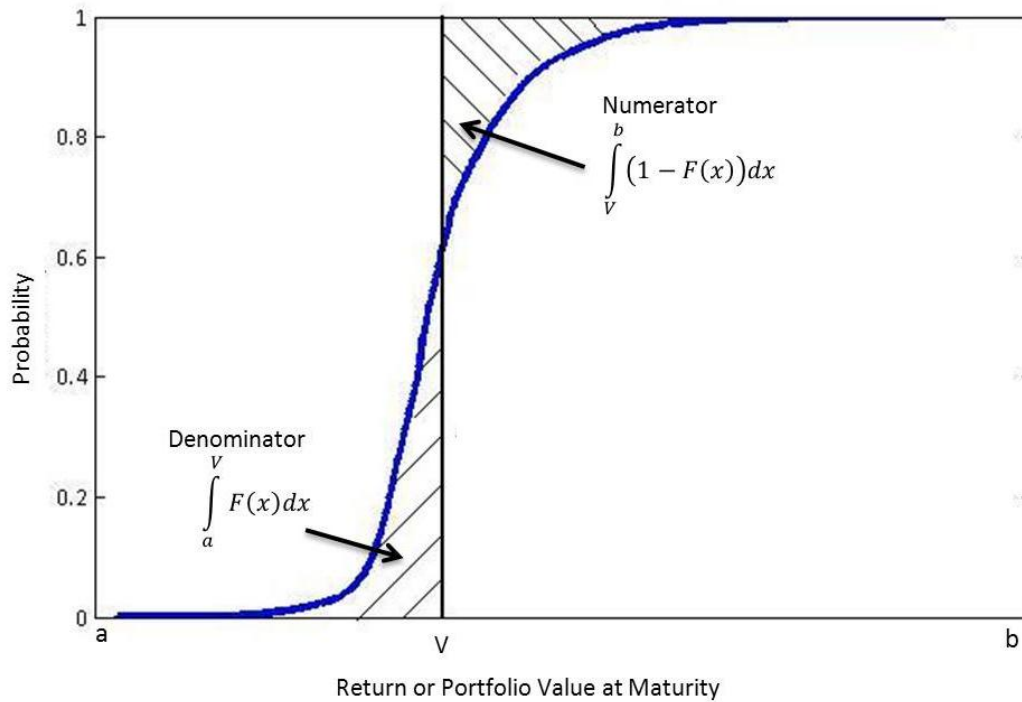


Figure 2 A graphical demonstration of the function for the Omega performance measure. The numerator is the area above the cumulative distribution function and the threshold  $V$  and the denominator the area under the cumulative distribution function and the threshold  $V$ . A strategy with a higher Omega at a certain threshold is preferred to another with a lower Omega.

## 2.4 The ENG measure (Expected net gain)

Lee et al (2008) propose an approach to evaluate PI strategies by calculating the expected net gain of the strategy, ENG. They define the loss as the sacrifice of the upside capture while the gain is the downside protection. Note that the gains and losses are defined in the opposite way compared to Omega. The net expected gain, ENG, is thus found by deducting the expected loss when stock price goes up from the expected gain of the strategy as stock price goes down. The mathematical definition of ENG is the following:

$$ENG = \theta_G * AG - \theta_L * AL$$

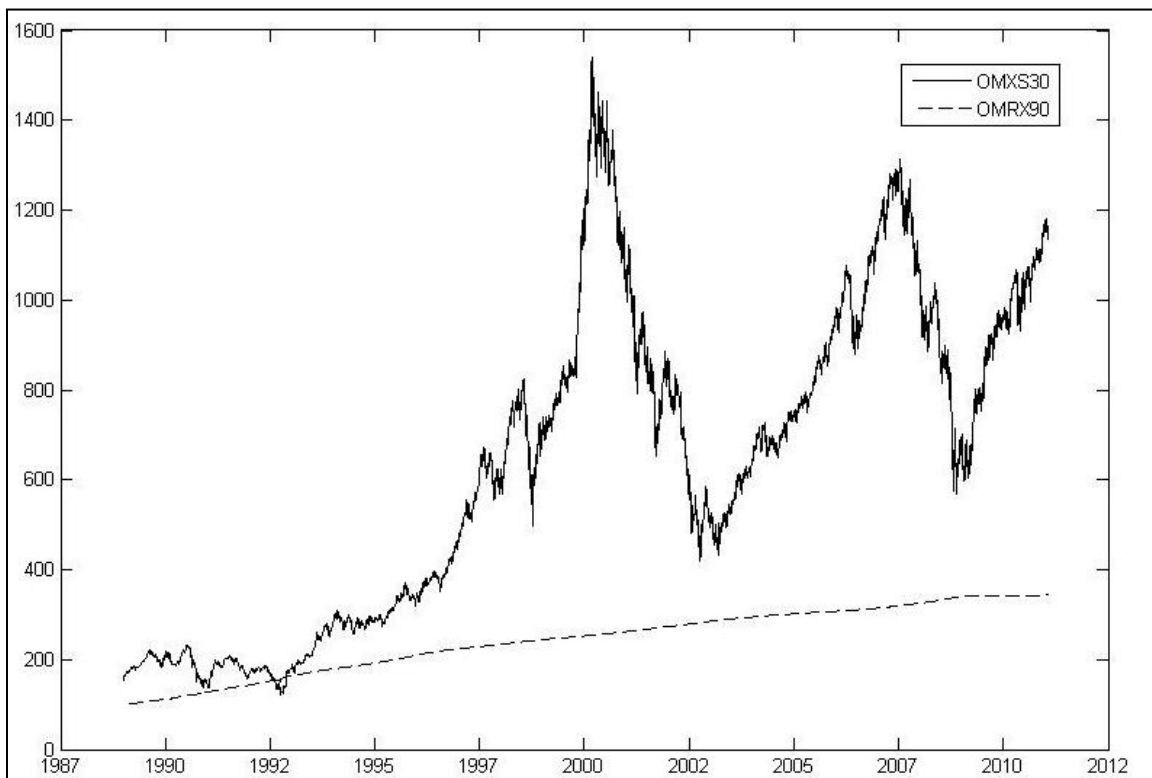
where AG is the average gain as stock price goes down, AL is the average loss as stock price goes up,  $\theta_G$  is the probability that stock price goes down and  $\theta_L$  is the probability that stock price goes up. This approach gives one single number for each strategy, using the buy-and-hold (B&H) strategy as a benchmark, where the portfolio with higher ENG is considered more effective.

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### 3. Data Description

In this section we discuss the data that will be used in our study. The data involves a risky asset represented by the OMXS30 index and a risk free asset represented by the OMRX90 index. The OMXS30 consists of the 30 most traded stocks on the Stockholm Stock Exchange in Sweden, while the OMRX90 is the total return index of Swedish treasury bills with 90 day maturity. The data was retrieved from DataStream on January 22, 2011 and database errors in the form of misplaced or missing decimals were cleaned by running a Matlab script.

The period covered in this study is from January 2, 1989 to January 21, 2011 and consists of daily observation of closing price adjusted for dividends. In total the dataset consists of 5755 observations. A graphical illustration of the evolution of the two indices for the relevant time period is shown in Figure 3.



**Figure 3** The time series of the Swedish stock index OMXS30 and the Swedish treasury bills index OMRX90 from January 2, 1989 to January 21, 2011, collected from Datastream on the 25<sup>th</sup> of January 2011. There are 5,755 observations for each series that are used to estimate parameters for and create 5,635 portfolios, one each day with an exception of the first 60 days that are used for parameter estimation. The portfolios, which have an investment period of 60 trading days, are in turn used to evaluate the performance of the PI strategies in this study.

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Two specific observations are important to keep in mind regarding the data. Firstly, in several time periods the trajectory of OMXS30 does not resemble the geometric Brownian motion. Large jumps in asset prices are very unlikely in a framework modeled by a Brownian motion and observing this many jumps for a period of 22 years is extremely unlikely. Secondly, the graph illustrates how the risky asset outperforms the risk free asset as expected when looking at the whole period, but clearly there are also many periods of heavy falls in the risky asset's price. By looking at a shorter investment period horizon, the dataset should be a good base to test how different PI strategies fare relative to a simple B&H, in addition to test the sensitivity of the model dependency of the VBPI. Based on the mismatch of assumed asset dynamics and historical outcome presented in Figure 3, we can expect that the VBPI strategy will be prone to model error.

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## 4. Methodology

In this section we describe the methodology that will be used in our study. Subsection 4.1 lays out the practical details while the implementation of the three different investment strategies, the CPPI method, the VBPI strategy and the B&H strategy is explained in Subsections 4.2, 4.3 and 4.4 respectively. In Subsection 4.5 we define the performance measures that are used and finally in Subsection 4.6 we discuss the sensitivity analysis of the VBPI strategy.

### 4.1 Summary of the research process

In this subsection we outline the practical details of our study. For every day in the observation period, a portfolio is constructed with an investment period of three months or 60 trading days and an investment of 10,000 monetary units. The parameters used in VaR-based portfolio insurance, VBPI, that have to be estimated are the expected return of the risky asset  $\mu$  and the expected return of the risk-free asset  $r$  and the volatility  $\sigma$  of the risky asset. In the study there are two approaches to this task, on one hand the parameters are estimated for the whole observation period and those estimates are used for all portfolios (fixed parameters). The expected return is calculated for a time-series running from the first to the last observation, and annualized. The volatility is measured as the standard deviation of daily returns for the same period (i.e. from the first to the last observation). Daily returns are calculated as

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

where  $S_t$  denotes the stock price at day  $t$ .

In the second estimation method, the parameters are estimated for each portfolio, based on the 60-day period preceding the start date of each portfolio (variable parameters). The expected return for the risk-free and the risky asset,  $r$  and  $\mu$ , are estimated by the annualized geometric mean of the return in the 60-day period. The volatility  $\sigma$  is the standard deviation of the annualized daily return of the risky asset in the 60-day period. This means that from the 5,755 observations there are  $5,755 - 2 * 60 = 5,635$  portfolios that will be used to evaluate CPPI and VBPI and there will be four scenarios in the basic analysis.

Case 1: using transaction costs and fixed parameters for all portfolios

Case 2: using transaction costs and different parameters for each portfolio

Case 3: no transaction costs and using fixed parameters for all portfolios

Case 4: no transaction costs and using different parameters for each portfolio



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Where transaction costs are involved the amount traded, and thus the amount for which transactions costs must be paid, is assumed to be the minimum amount of bonds and stocks that need to be traded in order to rebalance the portfolio. The whole portfolio is therefore not traded at each rebalancing date, which should be in line with how portfolio managers rebalance in reality. The transaction fee is assumed to be 18 basis points for stocks and 10 basis points for bonds, to align with the study of Jiang et al (2009) and allow for comparison of results. Transaction costs are withdrawn from the value of respective investment asset class after rebalancing. This approach is not the same as in the study of Jiang et al (2009) who only show how they calculate the division between asset classes taking transaction costs into account but not specifically how the costs are calculated and withdrawn. We also conducted the study where we calculated transaction costs of the total amount that should be invested in each asset class. That caused the effect of transaction costs to increase noticeably and as pointed out in Subsection 5.1 the effect on the performance measures was more in line with the study of Jiang et al (2009). This gives us a reason to believe that they assume that transaction costs are computed of the total amount at every rebalancing occasion.

The above four cases are run for B&H and daily, weekly and monthly rebalancing for CPPI and VBPI. Histograms illustrating the density of portfolio values at maturity and graphical illustration of the Omega function of VBPI and CPPI are retrieved together with several other performance measures illustrating different aspects of a PI.

For Case 1 and 3, with fixed parameters and with and without transaction costs, an additional sensitivity analysis is performed for VBPI. The analysis consists of running a loop of different inputs for  $\mu$ , the growth parameter of the risky asset and  $\sigma$ , the volatility of the risky asset. The outcome of this loop will be a graphical illustration of the different scenarios' impact on five performance measures. In addition, different Omega functions are plotted in the same graph, also illustrating the sensitivity of VBPI to parameter estimation.

## **4.2 Implementation of the CPPI strategy**

In this subsection we outline the implementation of the CPPI strategy in our empirical evaluation. The implementation follows the idea of Subsection 2.1 but the setup will be a little bit different in order to be able to make comparisons with VBPI, similar to Jiang et al (2009). The multiple  $m$  that reflects the investor's risk aversion in the CPPI model

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implementation will be set so that the initial weight of the risky asset is equal in CPPI and VBPI portfolios. At each rebalancing date, both models are run completely independent but they will have the same starting point. In this way, it is possible to perform relevant comparisons of the two strategies at maturity.

The floor in the CPPI approach is equal to the insured value of the VBPI so that at maturity both strategies' portfolio value will be compared to the same insured value/floor. The discount rate used in the floor calculation is the estimated risk free rate.

At the start of the investment period the multiple  $m$  is calculated based on the weight of risky assets in the VBPI set up, using the same notation as before,

$$m = \frac{(1 - w)V_0}{C_0}$$

where  $V_0$  is the initial portfolio value. The parameter  $w$  is the proportion that is invested in the risk free asset and hence  $1-w$  is the fraction invested in the risky asset. Recall the formula from Subsection 2.1

$$C_t = P_t - F_T e^{-r(T-t)}$$

which is used to calculate the cushion  $C_0$ . Knowing the multiple  $m$ , the initial number of stocks is then

$$\eta_0 = \frac{mC_0}{S_0}.$$

The number of bonds to hold will then be

$$\beta_0 = \frac{V_0 - mC_0}{B_0}.$$

At rebalancing dates, the value of the current portfolio is recorded given the number of bonds and stocks in the portfolio since the last rebalance date using

$$V_t = \beta B_t + \eta S_t$$

Given the portfolio value, the cushion is calculated in the same way as at day one, while the multiple  $m$  is fixed. The number of stock and bonds to trade comes from the same calculations as at day 1 but with the new cushion and asset prices as inputs to the formulas.

At each rebalancing date the above procedure is repeated until maturity and in this way, given no major jumps in asset prices in between the rebalancing dates, the portfolio should both be insured and offer upward potential. When transaction costs are present, transaction fees are withdrawn from the portfolio value at the day of rebalancing. In this setting neither

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borrowing at the risk free rate nor short selling stocks is allowed, hence the individual weight of stocks and bonds respectively are limited to the interval between 0 and 1 and the sum of the two weights are always equal to 1.

### **4.3 Implementation of the VBPI strategy**

In this subsection we discuss the implementation of the VBPI approach in our study. At day 1 the weights to invest in bonds and stocks are calculated via the closed formula (3) derived in Subsection 2.2. At each rebalancing date the current value of the portfolio value is recorded and used as input for the calculation of the new weight to hold in stocks and bonds, respectively. By this rebalancing the portfolio is constructed such that it once again is expected to be above the insured value maturity in accordance with the confidence level. In this way the portfolio is never guaranteed to be above the insured level at maturity, but the likelihood of that happening is controlled for in the strategy, and at the same time allowing for upward potential. Transaction costs are treated in the same way as for CPPI. The estimated parameters for VBPI, as explained in Subsection 4.1, are the same for all portfolios in case 1 and 3 but in case 2 and 4 each VBPI portfolio will be based on different parameter values. It is assumed that borrowing at the risk free rate or short selling stocks is not permitted when implementing the VBPI strategy, hence the individual weight of stocks and bonds respectively are limited to the interval 0 and 1 and the sum of the two weights are always equal to 1.

### **4.4 Implementation of the B&H strategy**

The buy-and-hold strategy (B&H) is defined in this study as the static portfolio of 100% invested in the risky asset which is in accordance with other studies that evaluate the performance of portfolio insurance (PI) strategies, e.g. Jiang et al (2009) and Lee et al (2008). This strategy is the benchmark to evaluate and illustrate the characteristics of PI strategies.

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#### 4.5 Specification of performance measures

In this subsection we explain the different performance measures that are used to evaluate the performance of PI strategies in our study. The basis for quantitatively analyze the results of the study will be seven performance measures. Five of the measures focus on specific characteristics of the distribution of portfolio values at maturity, relevant in the context of PI. The other two measures, the Omega and the ENG, take the whole distribution into account.

Since managing the downside risk is the key to an effective PI strategy, two specific measures are used to evaluate the performance of the portfolio on the left tail of the return density. The first measure records the value of the portfolio representing the 5<sup>th</sup> percentile (V5) of the return distribution, that is if all portfolios are ranked in a descending order it is the portfolio that has a higher value at maturity than 5% of the portfolios. The second measure is the average value of all portfolio values that are on or to the left of the above mentioned 5<sup>th</sup> percentile (AV5), i.e. the average of the 5% lowest portfolio values at maturity. Those two performance measures draw attention the worst performers in the set of portfolios tested in this study, and the higher the values of V5 and AV5 the better the strategy is in limiting extreme losses. In theory, the value of both V5 and AV5 in the CPPI method should be above the floor. In the case of VBPI the V5 should in expectation be equal to the floor and the AV5 below.

Gap risk is expected and controlled for in the VBPI strategy by setting a confidence interval. In the CPPI method, however, the value of the portfolio should in theory not go under the pre-set floor. In practice the strategies are facing a gap risk as a consequence of non-continuous perfectly liquid trading and transactions costs. It is thus valuable to ex post measure how often the strategies manage to keep the portfolio value above the insured level at maturity. This is measured by the protection ratio, defined as the percentage of portfolios with value above or equal to the insured level at maturity. The higher this measure the better the strategy has been to manage the gap risk for PI. In theory, the protection ratio for CPPI should be 100% and for VBPI should be expected to be equal to the confidence level, in this study 95%.

It is equally important to evaluate the PI strategy's ability to maintain as much of upside potential as possible. In the context of PI the extreme gains are not as important as the

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extreme losses. Therefore the measures of the upside potential do not focus on the far right end of the return density, the 5% highest portfolio values at maturity like V5 and AV5 focus on the lowest 5%. Instead the attention is directed to the 25% best-performing portfolios. The fourth performance measure is thus the 75<sup>th</sup> percentile of the portfolio value distribution at maturity (Q75), i.e. the value of the portfolio that is higher than 75% of the ranked portfolios. Furthermore, it is of outmost interest to calculate the average value of all portfolios that are greater or equal to Q75, referred to as AQ75. Both measures indicate the success of the PI to offer high expected returns in combination to limited downside risk.

The most comprehensive measure of the performance of a PI-strategy is the Omega measure. As explained in Subsection 2.3, Omega is a ratio of two integrals of the return distribution. The higher this ratio the better is the performance of the strategy, considering a given threshold. In the context of PI the most interesting thresholds to evaluate are those that lie in the lower region of the stock return. If a strategy gives higher Omega for all thresholds that are believed to be relevant, the strategy is considered to be dominating the other. Plotting the Omega measure as a function of different thresholds provides a graphical illustration of the performance of PI's and should provide more information than a single Omega measure that depends on the specific threshold chosen. In this study the thresholds are chosen to lie in the interval 9,800 and 10,300, when the initial investment is 10,000, to align with Jiang et al (2009) and enable comparisons. In further comparison of different strategies the focus will be on the thresholds of 9,800 and 10,000 as well as 9,900. Thus the focus will be on the thresholds that represent the floor, the initial invested amount and one in between those two. Azar (2004) has written a Matlab function for Omega which has been used for modeling in this paper.

In order to support the findings of Omega, the expected net gain (ENG) will also be utilized, which is another performance measure that was developed to evaluate PI strategies, as explained in Subsection 2.4. The B&H strategy serves as a benchmark and the initial investment 10,000 is the criterion for gain or loss. Following the procedure of Lee et al (2008) the process of finding ENG starts by calculating the profit of the portfolios in each strategy, including B&H strategy, by deducting the initial investment from the end portfolio values. Secondly, the profit of each portfolio in the different PI strategies is compared to the portfolios in the B&H strategy and the profit differences are calculated. Recall from Subsection 2.4 that gains and losses are defined in the opposite way compared to Omega. Thus in the third step, the profit differences for each strategy are separated in two groups,

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the gain group where the B&H portfolio end values are less than the initial portfolio value and the loss group where the B&H portfolios have a higher end value than 10,000. Finally, the average of each group is calculated and the ENG is found by subtracting the mean loss from the mean gain for each strategy.

#### **4.6 Specification of sensitivity analysis**

In this subsection we disclose the approach to the sensitivity analysis of the VBPI strategy. As mentioned in Subsection 1.2 the VBPI is model dependent and the estimated inputs to the model will influence the outcome of utilizing a VBPI strategy. In order to evaluate how sensitive the performance of the VBPI is under the setup present in this study a sensitivity analysis will be performed, highlighting the consequences of using different growth rates and volatility for the risky asset in the VBPI model. The analysis is based on case 1 and 3, presented in Subsection 4.1, using fixed parameters but with many different growth rates for the risky asset as an input. The procedure is then repeated with the volatility as the variable. The outcomes for all performance measures are saved and then plotted as a function of the growth rate of the risky asset on one hand and the volatility on the other hand. If the model is independent of those two estimates, the plots should show a straight horizontal line. The Omega measure that is already graphically presented as a function of thresholds will be demonstrated a little bit differently. In this case each Omega function that results from running the analysis with different inputs will be plotted on the same graph. If the Omega function is independent on the input of the growth rate or volatility, then all functions in the graph will lie together and appear as one. The different growth rates  $\mu$  used in this sensitivity analysis lie in the interval of 0.0575 and 0.2 and the volatility  $\sigma$  in the interval of 0.1 and 0.40.

One way to measure the sensitivity of the VBPI strategy would be to compare two periods with different volatility. We believe that such an analysis would fail to identify investment periods that have large drops in the stock price as volatility goes both ways. Instead we intend to investigate the VBPI performance in the presence of extreme negative returns. The dataset is divided into three subsets of portfolios, separated by the average of the 10 worst days' returns in the portfolio. The assessment will be based on the worst and the best subset. The idea is to evaluate how the VBPI is affected by extreme negative returns, given the model assumption about log normal daily returns.

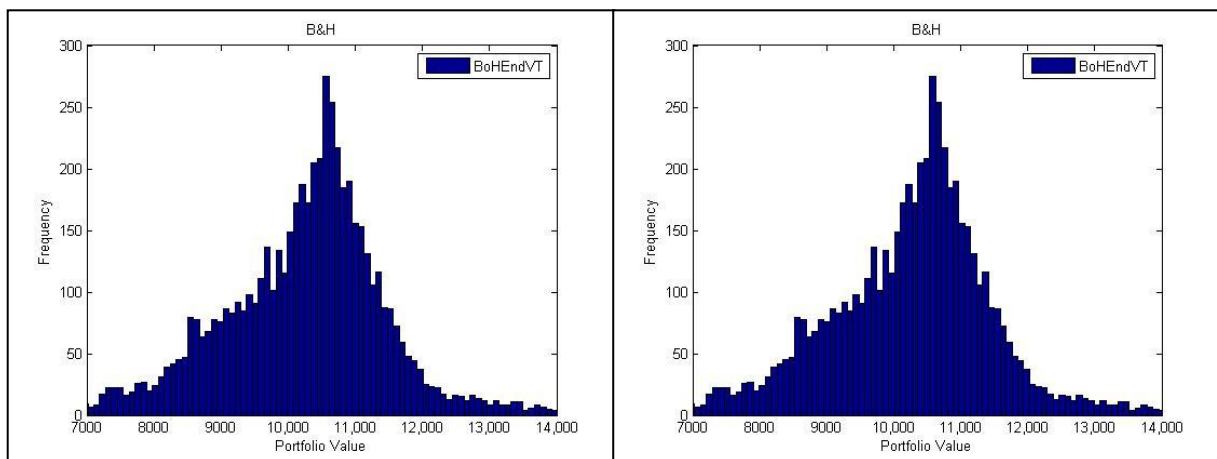
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## 5. Results

In this section we outline the results from our study. In Subsection 5.1 we compare the performance of the VBPI strategy and the CPPI method to each other and to the buy-and-hold (B&H) strategy. Subsection 5.2 analyses how sensitive the performance of the VBPI strategy is to a change in the estimated growth rate and volatility of the return of the risky asset. In Subsection 5.3 we compare the performance of the VBPI strategy to the CPPI method under two scenarios, one with large drops in asset prices and another with smaller drops in asset prices.

### 5.1 Performance comparison

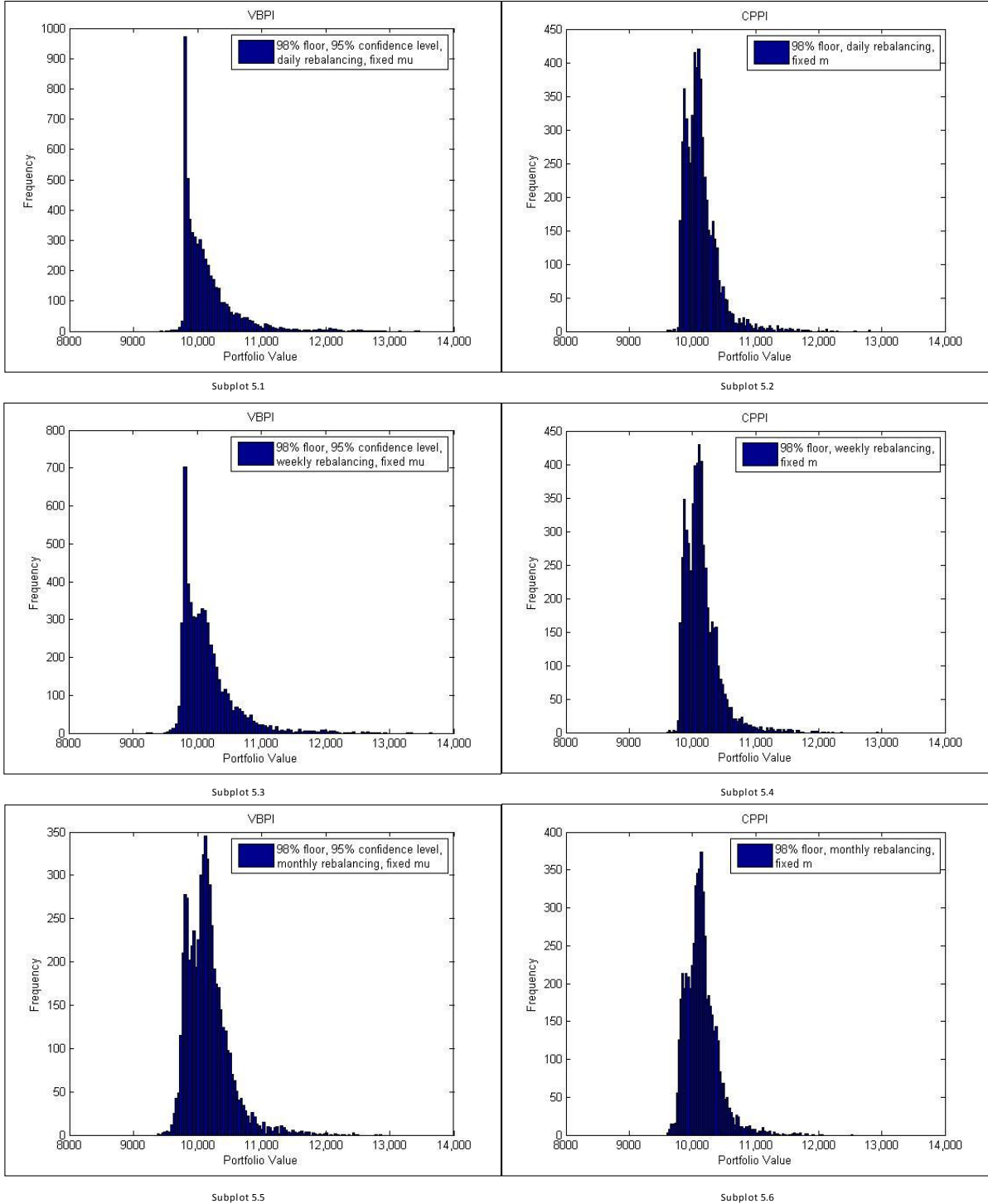
In this subsection we describe the basic results of the comparison of the performance of the VBPI strategy, the CPPI method and the B&H strategy. One of the purpose of using PI methods is to reshape the return distribution and preferably cut off the left tail while keeping as much of the right tail as possible. Figure 4 shows the distribution of the final value of 5,635 portfolios that only invested in stocks in the start of the investment period and kept them for 60 days without any rebalancing or bond investment (i.e. the B&H strategy), both with and without transaction costs. Although the final value of a large proportion of the portfolios lies between 10,000 and 11,000, the tails are quite fat, especially the left one.



**Figure 4** The empirical frequency distribution of portfolio value at maturity for buy-and-hold strategy. The 5,635 portfolios were constructed from the time series that are displayed in Figure 3 and all invested in stocks for the whole 60-day investment period. The histogram to left is with transaction costs and the one right without transaction costs.

This distribution of portfolio value at maturity in the B&H strategy is compared to the distribution for the PI strategies, VBPI and CPPI. Figure 5 shows the frequency of portfolio

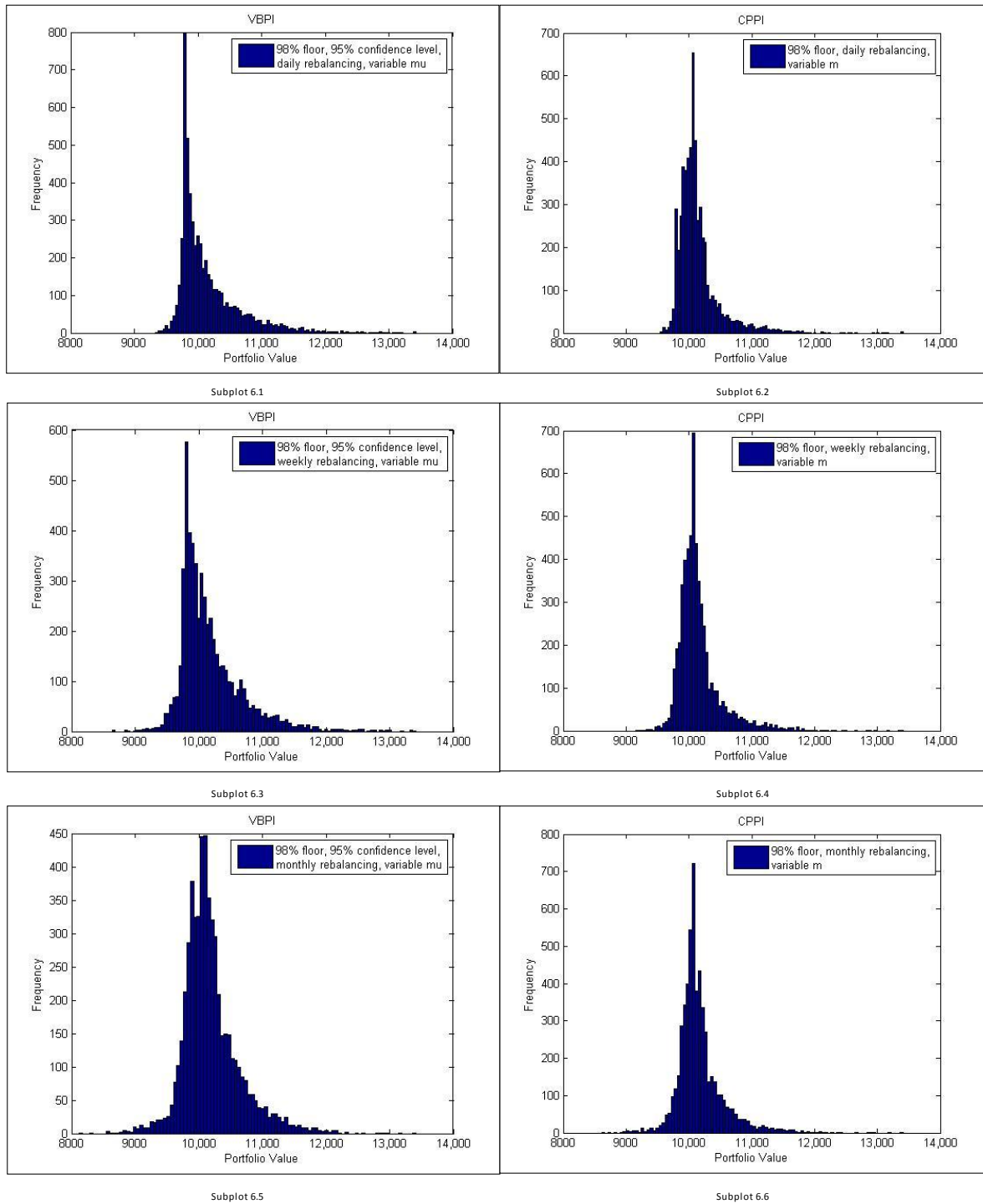
values at maturity for VBPI and CPPI with transaction costs and fixed parameters for all portfolios (case 1) while Figure 6 presents the frequency distribution with transaction costs and variable parameters (case 2).



**Figure 5** The empirical frequency distribution of portfolio value at maturity for case 1 with transaction costs and fixed parameters for all portfolios, with different rebalancing interval. The 5,635 portfolios were constructed from the time series that are displayed in Figure 3 and managed during a 60-day investment period using the VaR-based portfolio insurance strategy (VBPI) and the constant proportion portfolio insurance strategy (CPPI), respectively. Distribution for VBPI is to left and for CPPI to right.



The distributions for case 3 and 4, without transaction costs, can be found in Figures 12 and 13 in the Appendix and they are similar to those in Figures 5 and 6.



**Figure 6** The empirical frequency distribution of portfolio value at maturity for case 2 with transaction costs and variable parameters for all portfolios, with different rebalancing interval. The 5,635 portfolios were constructed from the time series that are displayed in Figure 3 and managed during a 60-day investment period using the VaR-based portfolio insurance strategy (VBPI) and the constant proportion portfolio insurance strategy (CPPI), respectively. The distribution for VBPI is to left and for CPPI to right.

The comparison reveals that both VBPI and CPPI are able to reshape the distribution, especially the left part of the distribution as it is supposed to, as only a small portion of portfolios have lower value than the pre-set floor of 98%. This, however, comes at a cost as the right tail of the distribution is also smaller which means that the higher return portfolios are fewer than in the B&H strategy. A more frequent rebalancing, e.g. daily rebalancing as seen in subplots 5.1, 5.2, 6.1 and 6.2 of Figures 5 and 6, increases the probability that the final portfolio value is above the floor of 98% but at the same time decreases the right tail.

Table 1 shows statistics of the empirical distributions for the three different strategies; B&H, VBPI and CPPI with various rebalancing intervals. The B&H strategy has considerably higher average return than the other strategies but its volatility is also quite much higher. Comparing VBPI and CPPI, VBPI has in general a higher return but similar to B&H the volatility is also high which makes the strategy riskier. Therefore it could be helpful to look at the Sharpe ratio which takes risk into account when ranking investments and is defined as

$$\text{Sharpe ratio} = \frac{\mu - r}{\sigma}$$

where  $\mu$  is the return on the risky asset,  $r$  is the risk free return and  $\sigma$  is the volatility of the risky asset.

		B&H	Daily Rebalancing		Weekly Rebalancing		Monthly Rebalancing	
			VBPI	CPPI	VBPI	CPPI	VBPI	CPPI
Case 1: Floor 98%, with transaction costs, variable inputs								
p=0.95	Return	15.71%	7.57%	6.51%	8.25%	7.09%	8.60%	7.60%
	Stdev	50.23%	23.24%	16.41%	22.83%	16.58%	20.03%	16.46%
	Skewness	0.83	2.58	3.65	2.25	3.21	1.73	2.43
	Sharpe ratio	0.21	0.09	0.07	0.12	0.10	0.16	0.13
Case 2: Floor 98%, with transaction costs, fixed inputs								
p=0.95	Return	15.71%	6.79%	5.94%	7.39%	6.23%	7.00%	6.34%
	Stdev	50.23%	20.03%	12.42%	19.56%	12.21%	15.34%	11.58%
	Skewness	0.83	3.11	3.24	2.98	2.93	2.47	2.22
	Sharpe ratio	0.21	0.08	0.06	0.11	0.08	0.12	0.10
Case 3: Floor 98%, without transaction costs, variable inputs								
p=0.95	Return	16.50%	10.20%	8.29%	9.93%	8.23%	9.63%	8.38%
	Stdev	50.49%	24.28%	17.48%	23.58%	17.24%	20.48%	16.79%
	Skewness	0.82	2.40	3.44	2.17	3.12	1.72	2.42
	Sharpe ratio	0.22	0.20	0.17	0.19	0.16	0.21	0.18
Case 4: Floor 98%, without transaction costs, fixed inputs								
p=0.95	Return	16.50%	9.08%	7.20%	8.79%	7.08%	7.80%	6.98%
	Stdev	50.49%	21.55%	13.51%	20.63%	12.85%	15.93%	11.91%
	Skewness	0.82	2.92	3.30	2.85	2.95	2.45	2.23
	Sharpe ratio	0.22	0.18	0.15	0.17	0.15	0.16	0.15

**Table 1** Statistics of the sample distributions for all strategies (buy and hold, VaR-based portfolio insurance and constant proportion portfolio insurance) with different rebalancing interval. The 5,635 portfolios that are used to evaluate the strategies were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The variable parameters are estimated using the 60-day period before the insured period starts and the fixed parameters,  $\mu=0.0918$ ,  $\sigma=0.2287$  and  $r=0.052$  were estimated for the entire set.

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According to Table 1, the Sharpe ratio is highest for B&H strategy and VBPI's Sharpe ratios are all higher than for CPPI. This would indicate that investors are compensated for the higher risk they take for the more volatile returns of B&H and VBPI. The Sharpe ratio only takes the first two moments of the distribution into account, assuming that it is sufficient to describe the return distribution. A quick look at the frequency distributions of the portfolio values in Figures 5 and 6 reveals that it is not the case as the distributions are both skewed and far from normal. It is therefore hard to draw conclusions about the investors' preferences from the Sharpe ratio.

In the absence of transaction costs, as for case 3 and 4 in the lower part of Table 1, the return for both VBPI and CPPI strategies becomes higher as the frequency of rebalancing is increased but the volatility is higher as well so the rebalancing frequency does not seem to have much effect on the Sharpe ratio. The picture changes though when transaction costs are taken into account, as in case 1 and 2 in the upper part of Table 1, since return increases and surprisingly the volatility decreases as the rebalancing frequency is lowered. In the presence of transaction costs and monthly rebalancing seems to be the most preferable rebalancing frequency.

Scott and Horvath (1980) show that investors prefer a positive skewness and although others like Brockett and Garven (1998) have managed to show that this is not always the case, we assume that risk-averse investors that are interested in PI prefer a positive skew since that means that the left tail is small and thus smaller probability of losses. In general, by looking at the skewness of the empirical distributions in Table 1, CPPI has a higher positive skew which increases with more frequent rebalancing.

Finally, there is a considerable difference of the return, volatility and skewness whether the parameters for VBPI are fixed or variable, which is the first indicator that the choice of parameters is an important factor of the performance of VBPI. The Sharpe ratio tends to be higher for variable inputs.

The different strategies can be evaluated by their ability to keep the pre-set floor, the protection ratio as defined in Subsection 4.5 and presented in Table 2. The B&H strategy performs worst, only managing to keep around 69% of the portfolios above the floor, which was expected after looking at the distribution in Figure 4. Here the CPPI performs much better than VBPI as it in all cases has a higher protection ratio, with the lowest difference in monthly rebalancing. Fixed inputs for VBPI parameters give higher protection ratios than variable ones, leading to an opposite conclusion compared to the above mentioned statistics from the sample distributions.

	B&H	Daily Rebalancing		Weekly Rebalancing		Monthly Rebalancing	
		VBPI	CPPI	VBPI	CPPI	VBPI	CPPI
Case 1: Floor 98%, with transaction costs, variable inputs p=0.95	68.78%	82.40%	94.87%	82.31%	93.79%	87.52%	92.51%
Case 2: Floor 98%, with transaction costs, fixed inputs p=0.95	68.78%	91.46%	99.73%	89.17%	99.13%	90.06%	95.95%
Case 3: Floor 98%, without transaction costs, variable inputs p=0.95	69.25%	86.25%	96.06%	84.99%	94.68%	88.29%	93.26%
Case 4: Floor 98%, without transaction costs, fixed inputs p=0.95	69.25%	93.06%	99.79%	90.45%	99.25%	90.86%	96.63%

**Table 2** Floor protection ratios for all strategies (buy and hold, VaR-based portfolio insurance and constant proportion portfolio insurance) with different rebalancing interval. The 5,635 portfolios that are used to evaluate the strategies were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The variable parameters are estimated using the 60-day period before the insured period starts and the fixed parameters,  $\mu=0.0918$ ,  $\sigma=0.2287$  and  $r=0.052$  were estimated for the entire set.

Not surprisingly, transaction costs cause the protection ratio to fall for all strategies and have of course most effect in daily rebalancing. The effect of transaction costs are however not as severe as in Jiang et al (2009) study since the method of this paper is to only calculate transaction costs on the lowest possible amount that is needed to rebalance the portfolio while Jiang et al (2009) seem to calculate transaction fees and taxes of the total amount on each rebalancing date. When implementing the latter method, the protection ratios fell considerably, especially with more frequent rebalancing. The effect was minimal for monthly rebalancing, protection ratios for weekly rebalancing fell approximately by 10 percentage points and protection ratios for daily rebalancing plummeted to 3-6%, which is similar to the effect of transaction costs and rebalancing in the paper by Jiang et al (2009). We believe that the former method is more realistic and we will therefore use this approach in the following analysis.

We also note that when variable parameter inputs were calculated using the investment period itself instead of the previous 60-day period as in the paper by Jiang et al (2009), the protection ratio was much higher for VBPI and more in line with protection ratios of CPPI. Instead of having a protection ratio of 86.25% as in case 3 and daily rebalancing as seen in Table 2 the protection ratio is 96.34% when same period parameters are used. The same trend is noticed for other rebalancing frequencies. In general it seems that the distribution of VBPI portfolio values at maturity is less dispersed when the same period parameters are used and the performance measure results are thus closer to CPPI. This difference in use of period to estimate the parameters could be one explanation to why our results are so different from the results of Jiang et al (2009).

	B&H	Daily Rebalancing		Weekly Rebalancing		Monthly Rebalancing	
		VBPI	CPPI	VBPI	CPPI	VBPI	CPPI
Case 1: Floor 98%, with transaction costs, variable inputs p=0.95	8,147	9,723	9,799	9,664	9,784	9,638	9,743
Case 2: Floor 98%, with transaction costs, fixed inputs p=0.95	8,147	9,796	9,843	9,780	9,840	9,759	9,810
Case 3: Floor 98%, without transaction costs, variable inputs p=0.95	8,162	9,747	9,810	9,677	9,797	9,655	9,758
Case 4: Floor 98%, without transaction costs, fixed inputs p=0.95	8,162	9,797	9,852	9,783	9,847	9,763	9,816

**Table 3 The 5<sup>th</sup> percentile (V5), the value of the 5% worst performing portfolio, with different rebalancing interval for all strategies (buy and hold, VaR-based portfolio insurance and constant proportion portfolio insurance). The 5,635 portfolios that are used to evaluate the strategies were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The variable parameters are estimated using the 60-day period before the insured period starts and the fixed parameters,  $\mu=0.0918$ ,  $\sigma=0.2287$  and  $r=0.052$  were estimated for the entire set.**

Table 3 presents the value of the 5% worst performing portfolio V5, the 5<sup>th</sup> percentile as explained before, and Table 4 the average of the 5% worst performing portfolios AV5, for each strategy and case as well as various rebalancing intervals. These two measures give an indication of how well the strategies are able to limit the downward tail of the frequency distribution. The results for both performance measures are in line with the protection ratio as CPPI portfolios are in every case higher than VBPI, the absence of transaction costs gives higher values and fixed inputs for VBPI give more favorable result than variable inputs. Here, more frequent rebalancing seems to give better results, with and without transaction costs.

	B&H	Daily Rebalancing		Weekly Rebalancing		Monthly Rebalancing	
		VBPI	CPPI	VBPI	CPPI	VBPI	CPPI
Case 1: Floor 98%, with transaction costs, variable inputs p=0.95	7,588	9,633	9,756	9,500	9,681	9,370	9,543
Case 2: Floor 98%, with transaction costs, fixed inputs p=0.95	7,588	9,774	9,819	9,730	9,812	9,692	9,767
Case 3: Floor 98%, without transaction costs, variable inputs p=0.95	7,602	9,664	9,766	9,525	9,697	9,397	9,563
Case 4: Floor 98%, without transaction costs, fixed inputs p=0.95	7,602	9,777	9,824	9,732	9,816	9,693	9,773

**Table 4 The average value of the 5% worst performing portfolios (AV5) with different rebalancing interval for all strategies (buy and hold, VaR-based portfolio insurance and constant proportion portfolio insurance). The 5,635 portfolios that are used to evaluate the strategies were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The variable parameters are estimated using the 60-day period before the insured period starts and the fixed parameters,  $\mu=0.0918$ ,  $\sigma=0.2287$  and  $r=0.052$  were estimated for the entire set.**

Table 5 shows the value of the 75<sup>th</sup> percentile Q75, the portfolio which has a higher value than 75% of the portfolios, while Table 6 shows the average value of the 25% best performing portfolios AQ75. Those measures focus on the upper tail of the frequency distribution and measure how effectively the strategies can keep higher returns while limiting the downside risk. As in line with the highest return in Table 1, the B&H strategy has the highest value of the 75<sup>th</sup> percentile and the highest average value of 25% best performing portfolios. When VBPI and CPPI are compared, it is obvious that VBPI is more able to retain higher returns in a bullish market since its portfolio values, according to these measures, are higher than for the CPPI portfolios for every case and rebalancing frequency.

	B&H	Daily Rebalancing		Weekly Rebalancing		Monthly Rebalancing	
		VBPI	CPPI	VBPI	CPPI	VBPI	CPPI
Case 1: Floor 98%, with transaction costs, variable inputs							
p=0.95	10,945	10,325	10,213	10,377	10,241	10,369	10,271
Case 2: Floor 98%, with transaction costs, fixed inputs							
p=0.95	10,945	10,270	10,231	10,294	10,247	10,303	10,266
Case 3: Floor 98%, without transaction costs, variable inputs							
p=0.95	10,965	10,431	10,261	10,434	10,264	10,403	10,293
Case 4: Floor 98%, without transaction costs, fixed inputs							
p=0.95	10,965	10,353	10,268	10,344	10,271	10,327	10,284

**Table 5 The 75<sup>th</sup> percentile (Q75), the value of the portfolio that is higher than 75% of all portfolios, for all strategies with different rebalancing interval. The 5,635 portfolios that are used to evaluate the strategies were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The variable parameters are estimated using the 60-day period before the insured period starts and the fixed parameters,  $\mu=0.0918$ ,  $\sigma=0.2287$  and  $r=0.052$  is estimated for the entire set.**

	B&H	Daily Rebalancing		Weekly Rebalancing		Monthly Rebalancing	
		VBPI	CPPI	VBPI	CPPI	VBPI	CPPI
Case 1: Floor 98%, with transaction costs, variable inputs							
p=0.95	11,654	10,872	10,597	10,877	10,628	10,797	10,650
Case 2: Floor 98%, with transaction costs, fixed inputs							
p=0.95	11,654	10,727	10,496	10,730	10,503	10,609	10,496
Case 3: Floor 98%, without transaction costs, variable inputs							
p=0.95	11,675	10,961	10,677	10,937	10,677	10,835	10,680
Case 4: Floor 98%, without transaction costs, fixed inputs							
p=0.95	11,675	10,822	10,550	10,794	10,538	10,643	10,519

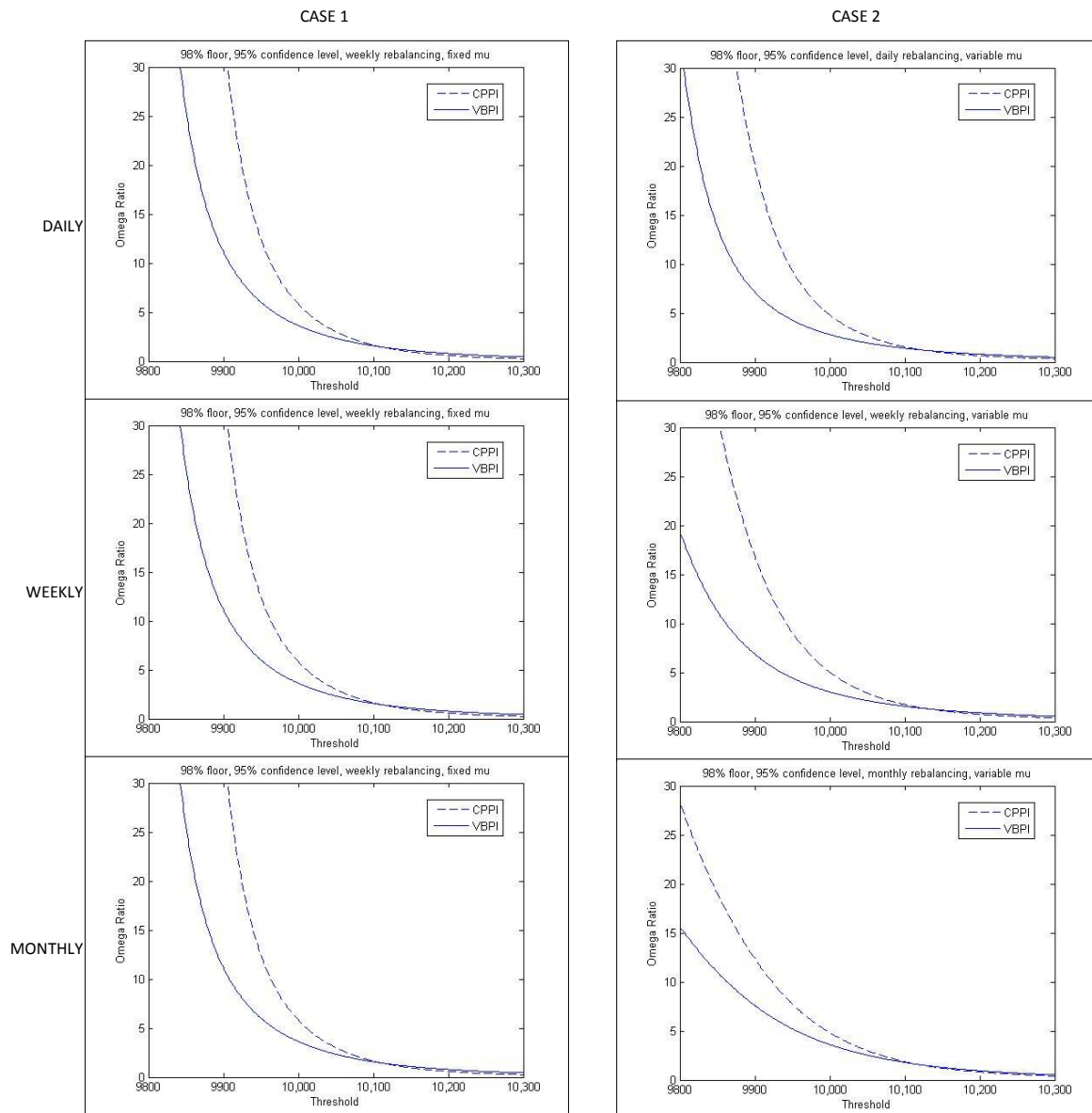
**Table 6 The average value of the 25% best performing portfolios for all strategies with different rebalancing interval. The 5,635 portfolios that are used to evaluate the strategies were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The variable parameters are estimated using the 60-day period before the insured period starts and the fixed parameters,  $\mu=0.0918$ ,  $\sigma=0.2287$  and  $r=0.052$  is estimated for the entire set.**

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The above measures that focus on the first two moments of the frequency distribution of end portfolio values give a contradictory conclusion to which strategy would be preferable for an investor. The CPPI strategy seems to be more effective in limiting the loss in case of a bearish market while at the same time the VBPI strategy's ability to retain higher returns is more advantageous. This is where the Omega measure enters the picture as it is able to include the effect of higher moments of a distribution. Recall that for a fixed threshold, a higher Omega is preferred by an investor. Figure 7 presents Omega for VBPI and CPPI for fixed and variable parameters, with transaction costs and different rebalancing interval. A similar figure without transaction costs can be found in the Appendix, but the conclusion is the same (see Figure 14 in the Appendix).

The Omega measure indicates that CPPI outperforms VBPI for all scenarios, especially when the focus is on the lower value part of the threshold. There is a break point around 10,100 above which the VBPI strategy outperforms CPPI but the difference in Omega seems to be minimal compared to the lower thresholds. Since the purpose of the study is to evaluate the strategies' insurance ability, this would indicate that CPPI outperforms VBPI as a tool for PI.

Although not shown here, the omega function for B&H strategy is slightly negatively sloped and on the left side it lies far below both VBPI and CPPI. The break point is approximately the same 10,100 above which the B&H outperforms both VBPI and CPPI but there it only lies just above the other Omega functions. Thus the outperformance on the upper side of the thresholds is small compared to the underperformance in the lower range.



**Figure 7** The Omega for VBPI and CPPI as a function of the threshold for fixed parameters to left and variable parameters to right, with transaction costs and different rebalancing interval. The 5,635 portfolios that are used to evaluate the strategies were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The variable parameters are estimated using the 60-day period before the insured period starts and the fixed parameters,  $\mu=0.0918$ ,  $\sigma=0.2287$  and  $r=0.052$  are estimated for the entire set.

Since Omega plays a crucial role in evaluating the PI strategies, the aim was to find an alternative measure of PI performance to support the findings of this paper. Table 7 shows the expected net gain (ENG) from using the different PI strategies compared to simple B&H strategy. The ENG measure confirms the conclusion drawn from the Omega measure when transaction costs are considered as ENG is higher for CCPI than VBPI for both variable and



fixed inputs and for all rebalancing frequencies, which indicates that CPPI outperforms VBPI when transaction costs are taken into account. Analyzing the individual parts that make up ENG, the gain and the loss, the same pattern appears as with Omega; CPPI gains more than VBPI, which means here that it is more able to minimize the loss when markets go down and VBPI loses less than CPPI, here manages to keep more of the upward potential. As an example, the average loss of VBPI with transaction costs, variable inputs and daily rebalancing, as in the upper left corner of Table 7, is 607 while the average loss of CPPI is 687.6. The gain of CPPI, however, is 938.4 compared to the gain of 834.5 for VBPI, resulting in a net gain of 227 for VBPI and 251 for CPPI.

In summary, the higher gain of CPPI is more than the lesser amount of loss of VBPI when transaction costs are considered, resulting in CPPI having higher ENG but without the transaction cost the smaller loss of VBPI dominates the higher gain of CPPI and VBPI's ENG becomes higher.

	Daily Rebalancing		Weekly Rebalancing		Monthly Rebalancing	
	VBPI	CPPI	VBPI	CPPI	VBPI	CPPI
Case 1: Floor 98%, with transaction costs, variable inputs p=0.95	227	251	252	271	278	279
Case 2: Floor 98%, with transaction costs, fixed inputs p=0.95	216	231	242	244	243	245
Case 3: Floor 98%, without transaction costs, variable inputs p=0.95	311	302	298	297	299	293
Case 4: Floor 98%, without transaction costs, fixed inputs p=0.95	284	263	276	259	255	253

**Table 7** The expected net gain, ENG, for both portfolio insurance strategies with different rebalancing interval. The 5,635 portfolios that are used to evaluate the strategies were constructed from the time series that are displayed in Figure 3. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The variable parameters are estimated using the 60-day period before the insured period starts and the fixed parameters,  $\mu=0.0918$ ,  $\sigma=0.2287$  and  $r=0.052$  are estimated for the entire set. The B&H strategy is used as a benchmark.

There are however some other interesting things that can be read from the Omega graphs and the ENG table. Firstly, the rebalancing frequency has less effect on Omega when the parameters are fixed than when they are variable which is evident by the fact that the graphs in the left part of Figure 7 are very much alike whether the rebalancing is daily, weekly or monthly. In the right part, with variable parameters, the Omega lines become steeper as the rebalancing frequency increases, indicating that daily rebalancing gives the best result when variable parameters are used, in spite of the transaction costs. This result is contrary to the conclusion of Jiang et al (2009) who found that VBPI performed better as the portfolios were rebalanced less frequent. The ENG measure for VBPI and CPPI indicates that more frequent

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rebalancing gives better results as long as there are no transaction costs and when transaction costs are considered, monthly rebalancing brings best performance. Hence, the ENG measure for CPPI is in line with the conclusion of Jiang et al (2009).

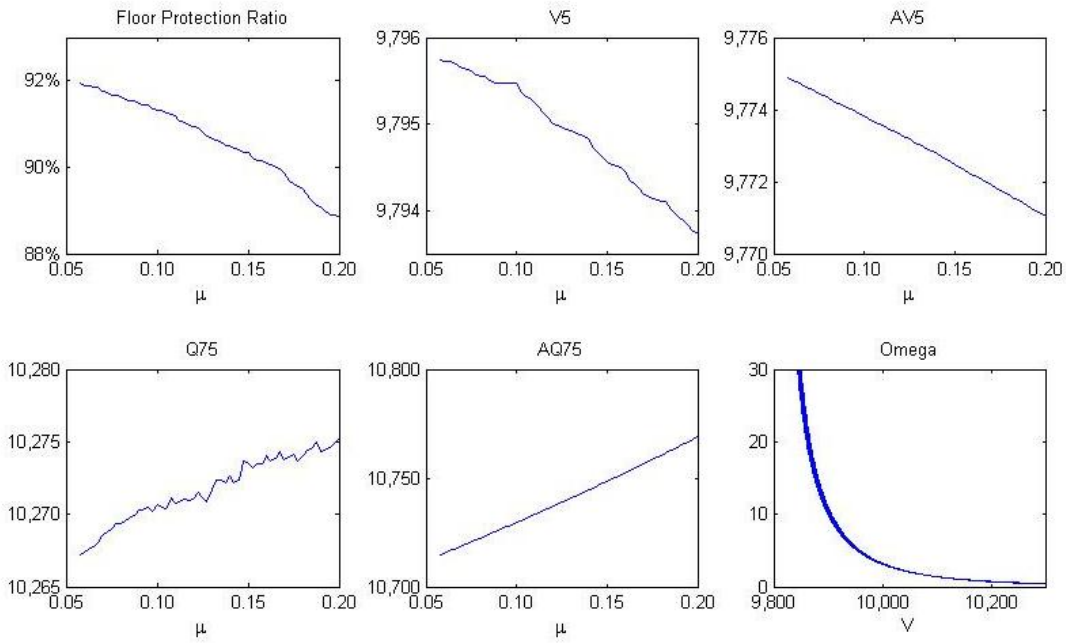
Secondly, the Omega is higher when the parameters are fixed than when the parameters are variable and that can be seen by the fact that the Omega lines in the left part lie higher than the Omega lines in the right part. Similarly, the ENG is higher for variable inputs than fixed, with transaction costs. When transaction costs are considered the opposite is observed.

This also suggest strongly that the choice of parameters in the VBPI strategy have an important effect on the strategy's efficiency, which is in line with Jiang et al (2009) who draw attention to the VBPI's model risk. It is therefore of interest to see the effect of different parameters in VBPI on the performance measures.

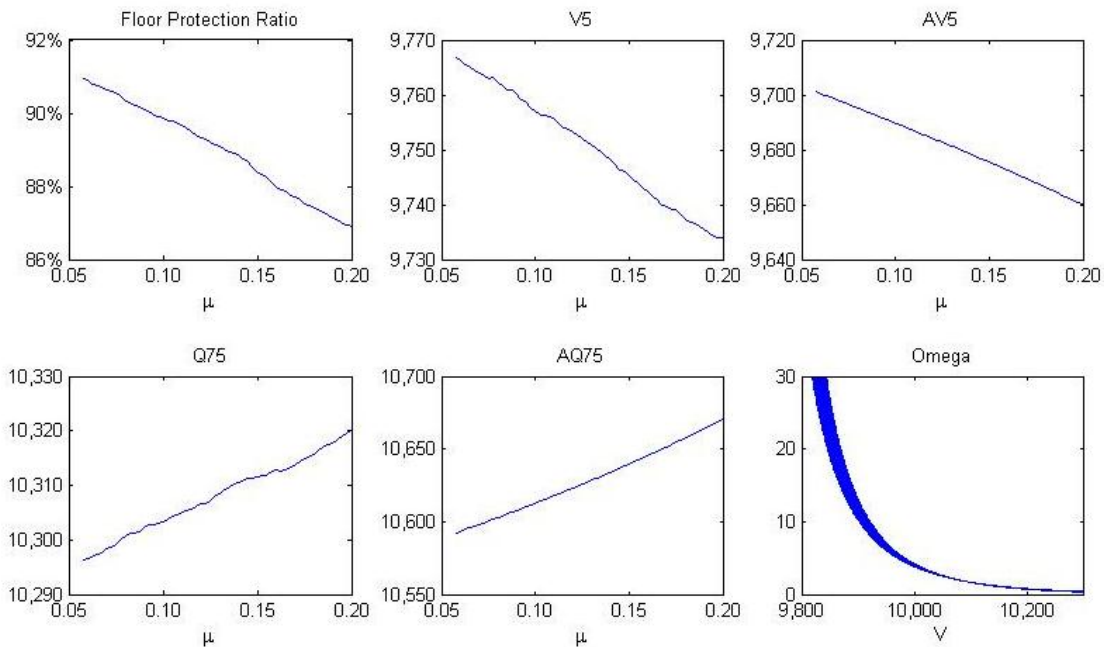
## **5.2 Sensitivity analysis**

In this subsection we present the results of the change in VBPI performance measures when the estimated growth rate and volatility of the risky asset are altered. Figures 8 and 9 present the sensitivity analysis for VBPI with transaction costs for daily and monthly rebalancing, respectively. The figures show that the higher the estimated expected return of the risky asset the more dispersed the return of VBPI's portfolios. In other words, the performance measures that focus on the left tail decrease as the expected return increases which can be seen by the negative slope of the lines in the graphs in the upper part of Figures 8 and 9. For daily rebalancing the floor protection ratio falls from 91.9% to 88.8% as the risky asset's growth rate goes from 5.75% to 20%.

At the same time the performance measures that focus on the right tail increase. For daily rebalancing the average value of the 25% best-performing portfolios goes from 10,715 to 10,769, or 0.51% as the estimated expected return of the risky asset increases. This is also apparent from the two left graphs of the lower part of Figures 8 and 9 as the slopes of the lines are positive.



**Figure 8** Performance measures for VBPI as a function of the parameter  $\mu$ , the expected return of the risky asset, and with transaction costs and daily rebalancing. The 5,635 portfolios that are used to evaluate the sensitivity of the VBPI strategy were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The other parameters that have to be estimated are fixed,  $\sigma=0.2287$  and  $r=0.052$  and were estimated using the entire time series period in Figure 3.



**Figure 9** Performance measures for VBPI as a function of the parameter  $\mu$ , the expected return of the risky asset, and with transaction costs and monthly rebalancing. The 5,635 portfolios that are used to evaluate the sensitivity of the VBPI strategy were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The other parameters that have to be estimated are fixed,  $\sigma=0.2287$  and  $r=0.052$ , and were estimated using the entire time series period in Figure 3.

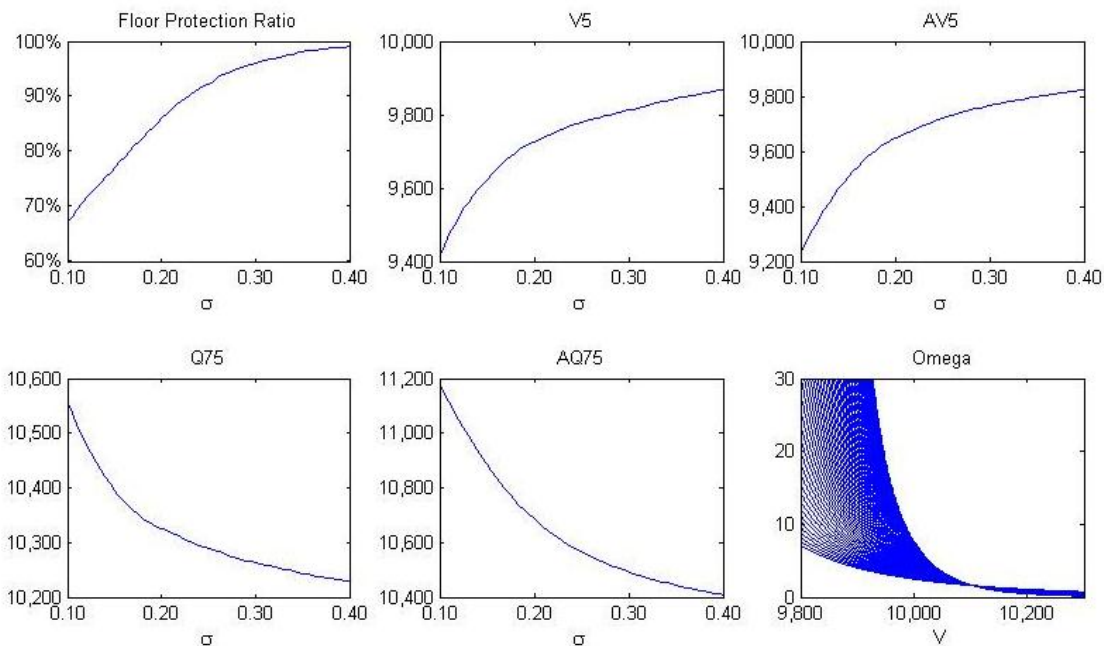
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In addition the effect is stronger for monthly rebalancing than it is for daily rebalancing. This observation is interesting, since monthly rebalancing is the frequency that has had the best (or least bad) performance in comparison to CPPI, both in this study and in the paper by Jiang et al (2009). By rebalancing monthly the floor protection ratio falls from 91% to 86.9% when the growth rate increases while the 25% best-performing average rises from 10,592 to 10,670, or by 0.74%.

Finally, the Omega measure in the right lower end of Figures 8 and 9 presents the fact that Omega is not independent of the expected growth rate that is used for the risky asset. This is obvious, as the Omega functions do not lie together and appear as one but instead form a band of all the different functions. In line with the observation before, this band is broader for monthly rebalancing than daily rebalancing. For the threshold of 10,000 the Omega ranges from 3.13 to 3.28 for daily rebalancing while the range is from 3.83 to 4.26 when the portfolios are rebalanced monthly. For the 9,900 threshold these ranges are 9.87 to 11.02 and 10.23 to 13.05, for daily and monthly rebalancing, respectively, showing that the range is larger for monthly rebalancing and the sensitivity to the parameters in the VBPI model is thus bigger. The same trend is seen for threshold 9,800 and also when transactions costs are not involved, which can be confirmed with Figures 15 and 16 in the Appendix.

It is also of interest to see what effect different  $\sigma$ , or the volatility of the return of the risky asset, has on the performance measures of VBPI, which is presented in Figure 10 for monthly rebalancing and with transaction costs. Here the effect of different parameter values is much more than for the expected return. As the volatility increases from 0.1 to 0.4 the floor protection ratio rises from 67.15% to 98.99% which is a huge difference. In contrast to the sensitivity to the expected return, the lower end performance measures are positively related to  $\sigma$ , the volatility, as seen by the positive slopes in the upper part graphs in Figure 10. Accordingly, the higher end performance measures are negatively correlated to the volatility. This indicates that the dispersion of end portfolio values in VBPI decreases as the  $\sigma$  parameter increases. This indicates that a higher volatility will induce a more conservative VBPI investment strategy.

As with the floor protection ratio, the effect of higher  $\sigma$  is strong on other performance measures. For example the average value of the 5% worst-performing portfolios increases by 6.4%, from 9,234 to 9,824 as  $\sigma$  increases while the average value of the 25% best-performing portfolios fall by 7.4%, from 11,178 to 10,409.



**Figure 10** Performance measures for VBPI as a function of the parameter  $\sigma$ , the expected volatility of return of the risky asset, with transaction costs and monthly rebalancing. The 5,635 portfolios that are used to evaluate the sensitivity of the VBPI strategy were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The other parameters that have to be estimated are fixed,  $\mu=0.0918$  and  $r=0.052$ , and were estimated using the entire time series period in Figure 3.

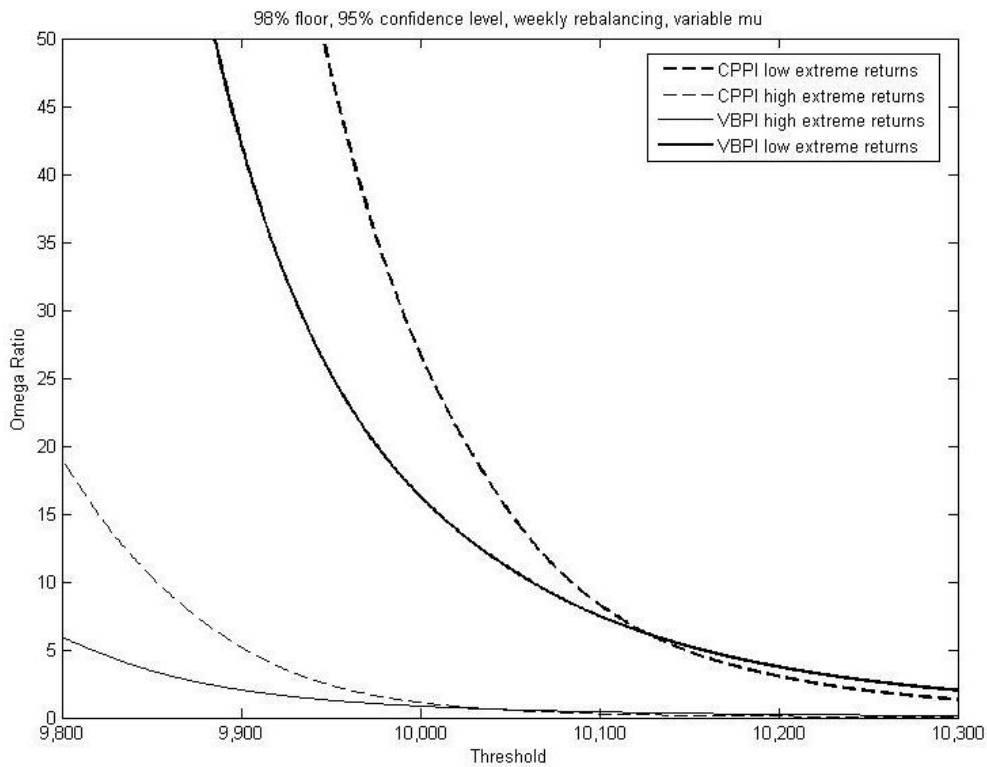
The band of Omega functions in the lower right corner of Figure 10 is much broader than before. For the threshold of 10,000 the omega ranges from 2.55 to 7.7 for monthly rebalancing with transaction costs, for the threshold of 9,900 the range is from 4.22 to 56.03 and the difference is huge for the threshold of 9,800 where the omega ranges from 7.1 to 867.

A sensitivity test of  $\sigma$  with daily rebalancing and transaction costs reveals the same trend as with monthly rebalancing although the effect of different  $\sigma$  is smaller. A graph of this can be found in Figure 17 the Appendix.

### 5.3 Two scenarios with different volatility

As shown in Subsection 5.2, the estimation of the parameters of the underlying dynamics of the risky asset in the VBPI calculations affects the eventual outcome of the strategy. Not only due to being dependent on estimating these inputs, the VBPI used in this paper, as well as in Jiang et al (2008), also explicitly assumes the Black & Scholes dynamics with log normal daily returns. In Figure 11, the Omega function is shown for both CPPI and VBPI under two

different scenarios. The scenario of high extreme negative returns consists of portfolios that have the average daily return of the 10 worst days lower than -2.14%. The scenario of low extreme negative returns consists of portfolios that have the average return of the 10 worst days higher than -1.40%. The sample of portfolios in each scenario is thus not from a specific period in Figure 3, but sampled based on the daily return in the investment period of each portfolio. Each of the two scenarios consists of one third of the 5,635 portfolios that were constructed to use in our study, while one third (with negative returns that lie in between) was not used for this analysis.



**Figure 11** The Omega for VBPI and CPPI as a function of the threshold for variable parameters, with transaction costs and weekly rebalancing interval. In each scenario there are 1/3 of the 5,635 portfolios, which were constructed from the time series displayed in Figure 3. One scenario consists of portfolios with large extreme negative daily returns and the other of portfolios with low extreme negative daily returns. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The variable parameters are estimated using the 60-day period before the insured period starts.

It is clear from Figure 11 that the two strategies perform better under investment periods that experience relatively low extreme drops in risky asset prices. Comparing the performance of CPPI and VBPI, it is apparent that when the investment period has low

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negative daily returns the difference between their average Omega is lower than when the investment period has with high negative daily returns, both in absolute terms (apparent in Figure 11) and in relative terms (1.34 for low and 1.45 for high). In a similar fashion, the protection ratio of CPPI under the high and low scenarios is 91.9% and 97.7% respectively. For VBPI the respective figures are 75.6% and 93.4%. Both the Omega and the protection ratios in this case indicate that the VBPI suffers from model risk in periods with high extreme negative returns.

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## 6. Conclusion

### 6.1 Discussion of results compared to the purpose of the paper

Based on standard statistics such as mean, variance and skewness the CPPI strategy seems to be more able to reshape the return distribution than VBPI on most scenarios evaluated although it comes at a cost as the VBPI has higher Sharpe ratios. Looking at specific aspects of a PI strategy, such as the gap risk, it is clear that the CPPI managed this risk much better than VBPI in all scenarios. Interesting to observe is that the VBPI did not manage to control the gap risk with the corresponding chosen confidence level in any scenario evaluated. The value  $V_5$  and  $AV_5$  measures also confirm this. We believe that these results are due to underestimation of the probability of extreme asset movements and discontinuous asset prices. The VBPI did however, return better results in the retained upside potential, based on  $Q_{75}$  and  $AQ_{75}$  for all scenarios evaluated. Turning to the Omega measure that incorporates the whole distribution of returns, the CPPI returns higher values for the threshold levels deemed as most interesting from a PI perspective. The VBPI is not dominated by the CPPI, but is underperforming for all thresholds below 10,100. These results are confirmed by the ENG measure. Given the evaluation from a PI perspective the first conclusion of this paper is that VBPI did not outperform CPPI for 3-month investment periods in Sweden during 1989-2011.

The results suggest that the VBPI is sensitive to the estimation of the parameters in the model. From a theoretical point of view it was expected that higher volatility would lead to less investment in stocks, and that higher expected growth rate of the risky asset would lead to more investment in stocks. The results of this paper confirm that, in showing the effects on the performance measures from different estimates. The second conclusion of this paper is that the parameter inputs have severe impact on the gap risk and the overall PI performance, as measured by Omega, and illustrates well the model risk VBPI faces. The VBPI is also found to be sensitive to periods that experience large daily drops in risky asset prices, compared to more stable periods. The gap risk of VBPI increased substantially, with lower protection ratio, as well as lower Omega in periods of large price drops. We believe that this result stems from the assumption that stock prices are driven by a geometric Brownian motion, that does not seem to fit the observed behavior of stocks in practice. The third conclusion of this paper is that the VBPI, as defined in this study, contains a model error that leads to suboptimal rebalancing as a PI strategy.



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## 6.2 Discussion of conclusions compared to other studies

The results of this study resemble other studies of option-based PI strategies (OBPI), such as Loria et al (1991), in the way that the estimation of the inputs to the model affects the ability of the strategy to meet the insurance target. Both this study, as well as OBPI studies, has chosen to use the Black & Scholes framework to set up the PI strategy, and the consequences are similar both in the VaR setting of this study as well as the option setting of OBPI studies. The risk of making erroneous choices in rebalancing, due to inherent model error or model risk, that drove the development of CPPI is still present in VBPI due to the same assumption of the underlying dynamics of the risky asset. In line with Jiang et al (2009), this paper has shown that the choice of threshold in the Omega measure can impact the conclusion of ranking two alternative PI's and showing Omega graphically as a function of the threshold appears to be the preferable way compared to the method used in Annaert et al (2008). The results of this paper are not in line with the results found in Jiang et al (2009), where the VBPI was found to be the superior PI strategy even though the methodology of this paper was set up in a way specifically to allow comparisons to their study. We believe that the reasons for the contradicting results are that Jiang et al (2009) applied the VBPI on a time series not well suited to test PI strategies, containing no sharp drops in the asset prices. The results of this thesis did however confirm the issue of model risk discussed briefly in the conclusion of the paper by Jiang et al (2009).

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## Appendix

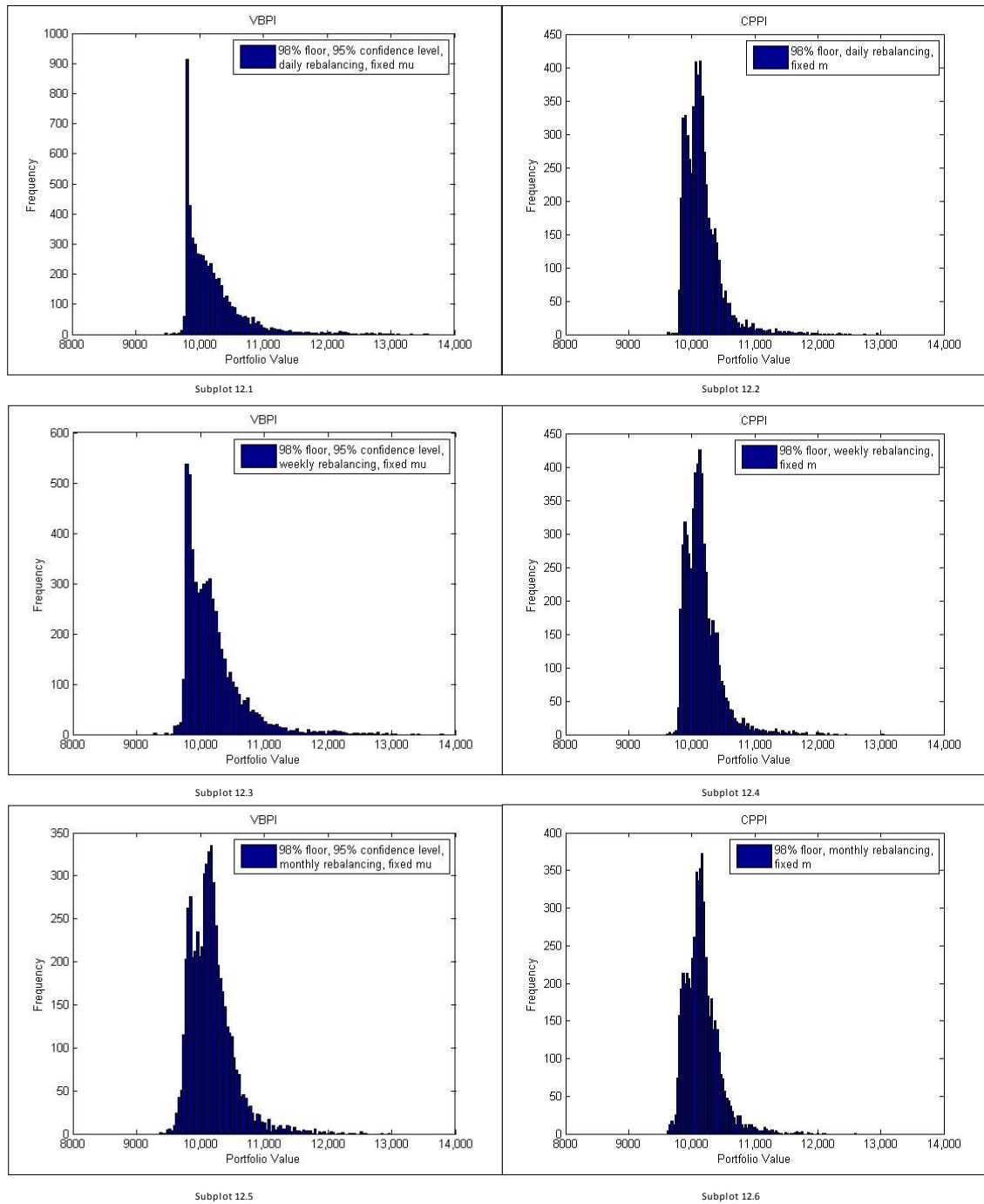


Figure 12 The empirical frequency distribution of portfolio value at maturity for case 3 without transaction costs and fixed parameters for all portfolios, with different rebalancing interval. The 5,635 portfolios were constructed from the time series that are displayed in Figure 3 and managed during a 60-day investment period using the VaR-based portfolio insurance strategy (VBPI) and the constant proportion portfolio insurance strategy (CPPI), respectively. Distribution for VBPI is to left and for CPPI to right.

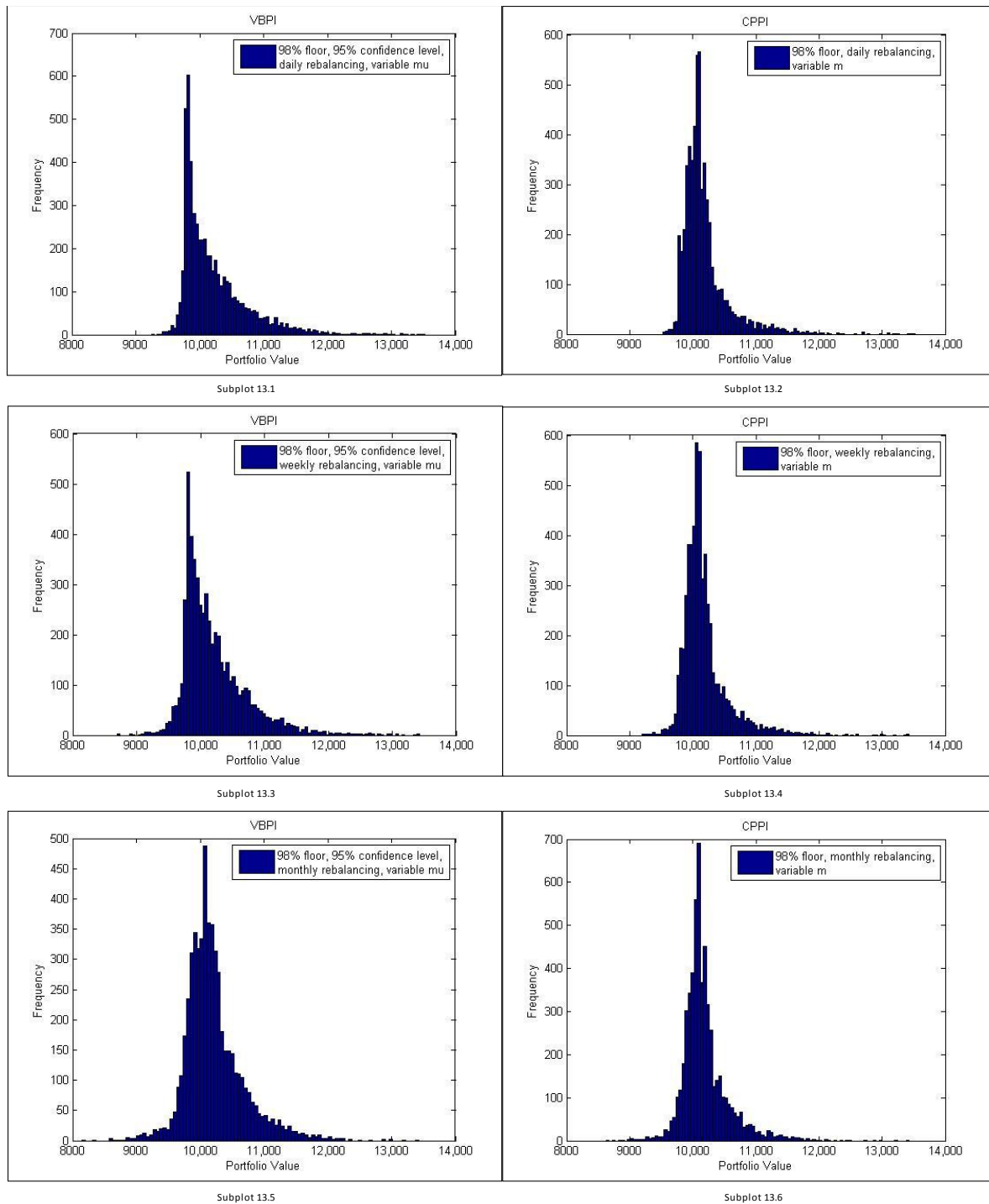


Figure 13 The empirical frequency distribution of portfolio value at maturity for case 4 without transaction costs and variable parameters for all portfolios, with different rebalancing interval. The 5,635 portfolios were constructed from the time series that are displayed in Figure 3 and managed during a 60-day investment period using the VaR-based portfolio insurance strategy (VBPI) and the constant proportion portfolio insurance strategy (CPPI), respectively. Distribution for VBPI is to left and for CPPI to right.

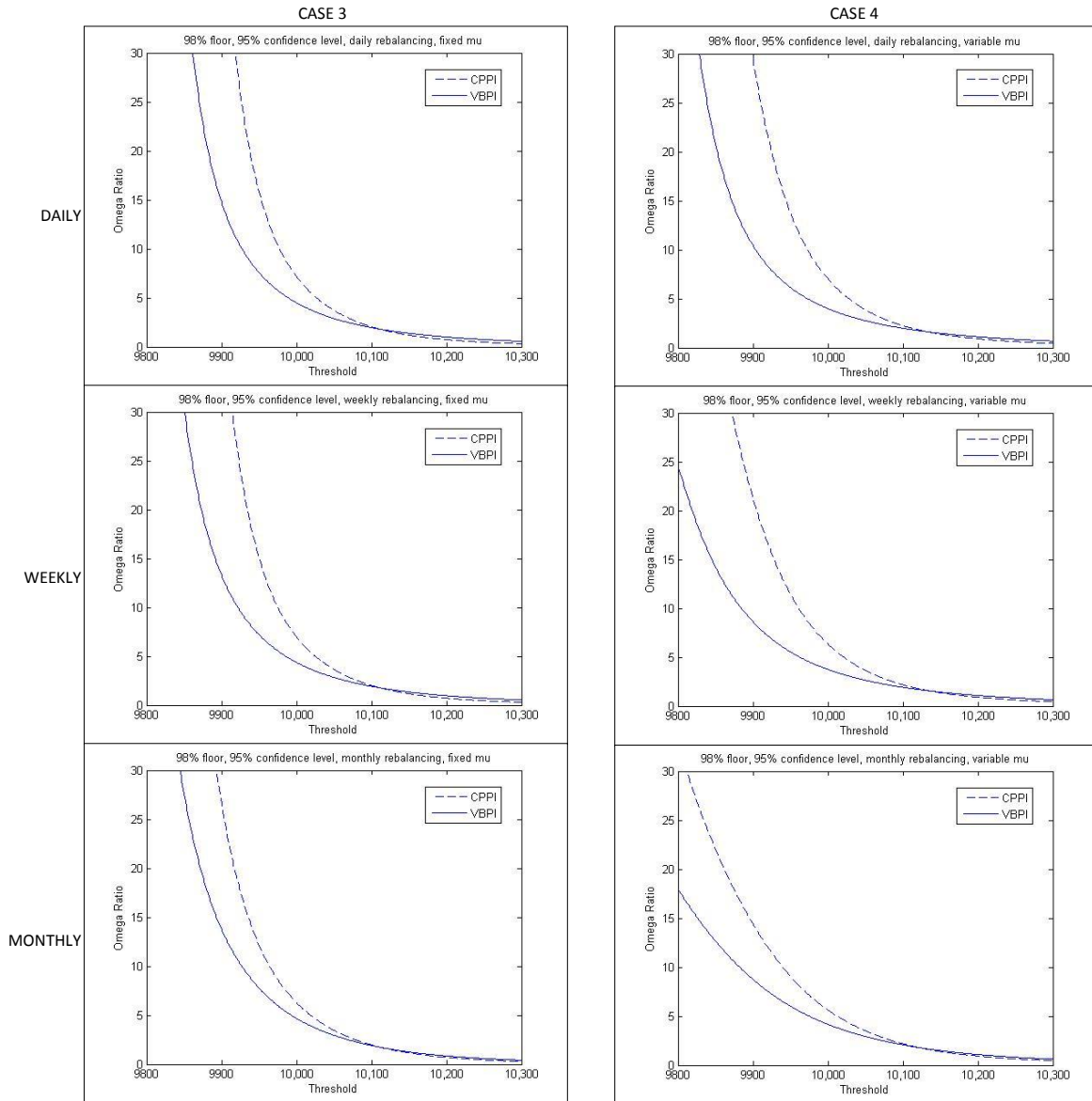


Figure 14 The Omega for VBPI and CPPI as a function of the threshold for fixed parameters to left and variable parameters to right, without transaction costs and different rebalancing interval. The 5,635 portfolios that are used to evaluate the strategies were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The variable parameters are estimated using the 60-day period before the insured period starts and the fixed parameters,  $\mu=0.0918$ ,  $\sigma=0.2287$  and  $r=0.052$  are estimated for the entire set.

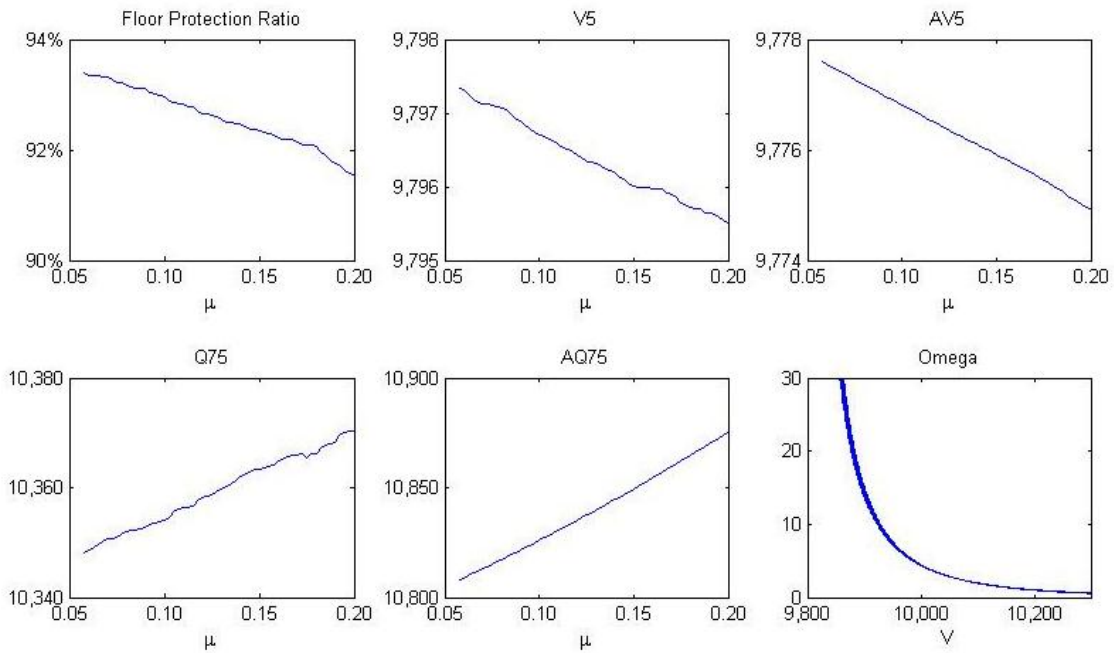


Figure 15 Performance measures for VBPI as a function of the parameter  $\mu$ , the expected return of the risky asset, and without transaction costs and daily rebalancing. The 5,635 portfolios that are used to evaluate the sensitivity of the VBPI strategy were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The other parameters that have to be estimated are fixed,  $\sigma=0.2287$  and  $r=0.052$ , and were estimated using the entire time series period in Figure 3.

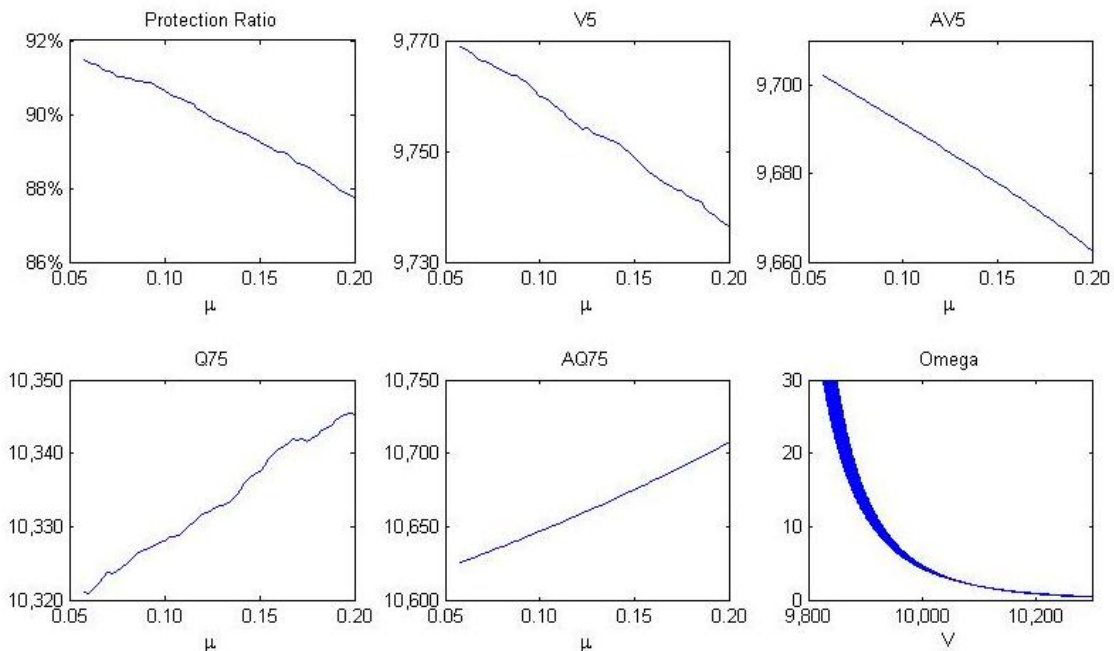


Figure 16 Performance measures for VBPI as a function of the parameter  $\mu$ , the expected return of the risky asset, and without transaction costs and monthly rebalancing. The 5,635 portfolios that are used to evaluate the sensitivity of the VBPI strategy were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The other parameters that have to be estimated are fixed,  $\sigma=0.2287$  and  $r=0.052$ , and were estimated using the entire time series period in Figure 3.

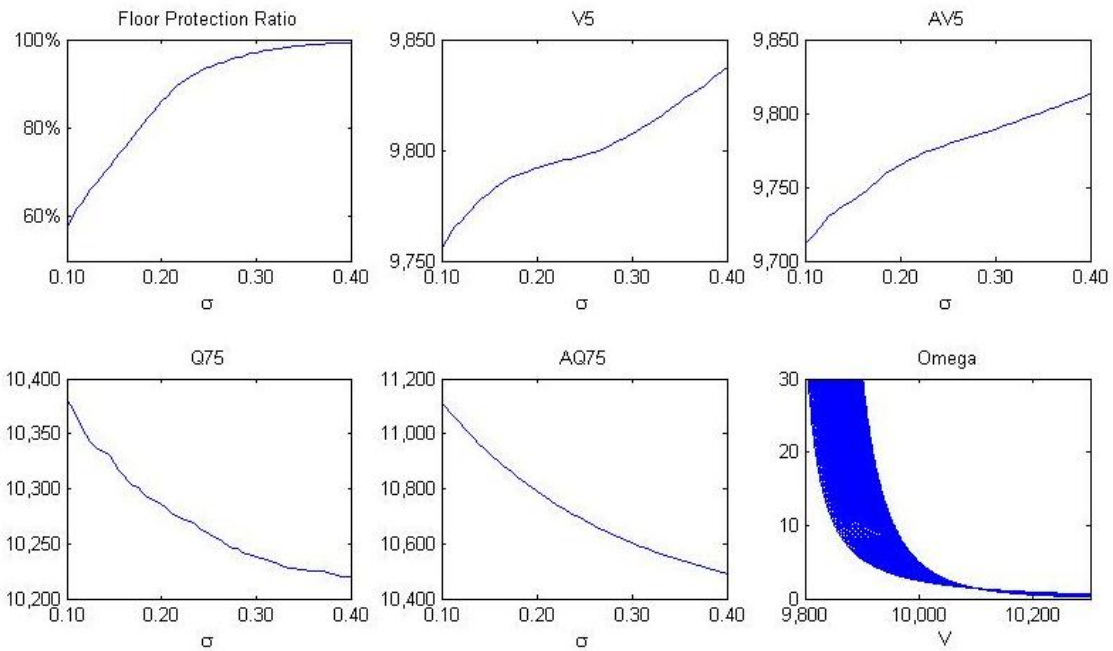


Figure 17 Performance measures for VBPI as a function of the parameter  $\sigma$ , the expected volatility of return of the risky asset, with transaction costs and daily rebalancing. The 5,635 portfolios that are used to evaluate the sensitivity of the VBPI strategy were constructed from the time series that are displayed in Figure 3. The investment period is 60 days. The pre-set floor is 98% of the initial investment and the confidence level is 95%. The other parameters that have to be estimated are fixed,  $\mu=0.0918$  and  $r=0.052$ , and were estimated using the entire time series period in Figure 3.



Matlab-script that provides results for case 1 and 2, with transaction costs.

```

% The first for-loop is only used for getting 100 observations for the
% sensitivity analysis, to see the effect of different mu and sigma on VBPI.
% When uncommenting the first loop, we also have to uncomment one line in
% the fixed parameters and comment the other PI methods+graphs, see note below.
% There are two loops, one for mu and one for sigma.
% To run the loop, a figure for omega has to be opened first and held
%for l=1:1:length(mu_fixed);
%for l=1:1:length(sigma_fixed);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
v0=10000; % Invested amount
G=9800; % Floor 98% of 10.000
p=0.95; % Confidence level
zp=norminv(p); % as in the formula in the article
fs=0.0018; % Trans.Cost Stock
fb=0.001; % Trans.Cost Bond
omega_low=9800; % lower limit of omega threshold
omega_high=10300; % upper limit of omega threshold
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% variable parameters
mu=ANN_LN_Rturn; % Risky growth rate, variable
sigma=ANN_SIGMA_OMXS30; % Volatility, variable
r=ANN_Return_T_bill; % Risk free rate, variable
mu_title='variable';

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% % fixed parameters
% mu=ones(1,length(ANN_LN_Rturn))*0.0918; % Risky growth rate, fixed
% sigma=ones(1,length(ANN_SIGMA_OMXS30))*0.2287; % Volatility, fixed
% r=ones(1,length(ANN_Return_T_bill))*0.052; % Risk free rate, fixed
% mu_title='fixed';

% used with first loop
%mu=ones(1,length(ANN_LN_Rturn))*mu_fixed(l); % Risky growth rate, fixed
%sigma=ones(1,length(ANN_SIGMA_OMXS30))*sigma_fixed(l); % Volatility, fixed
%
% This is for daily rebalancing
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Time length of insured period equal to 3 month (240 trading days per year)
T=60/240;
tau=[0 1/240:1/240:60/240]; % time for rebalancing
int=1; % The rebalancing interval
rebalancing='daily';
period=60; % The length of the investment period in trading days
bond=OMRX_T_Bill_90; % The bond data
stock=OMXS_30; % The stock data
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% % This is for weekly rebalancing
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% % Time length of insured period equal to 3 month (240 trading days per year)
% T=60/240;
% tau=[0 5/240:5/240:60/240]; % time for rebalancing
% int=1; % The rebalancing interval
% rebalancing='weekly';
% period=length(tau); % The length of the investment period in trading days
% bond=OMRX_T_Bill_90_weekly; % The weekly bond data
% stock=OMXS_30_weekly; % The weekly stock data
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

%
% %This is for monthly rebalancing
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% %Time length of insured period equal to 3 month (240 trading days per year)
% T=60/240;
% tau=[0 20/240:20/240:60/240]; % time for rebalancing
% int=1; % The rebalancing interval
% rebalancing='monthly';
% period=length(tau); % The length of the investment period in trading days
% bond=OMRX_T_Bill_90_monthly; % The monthly bond data
% stock=OMXS_30_monthly; % The monthly stock data
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 1.1 Startup portfolio and static insured portfolio (with transaction
% costs), used for starting portfolios in dynamic insured portfolio
% Initial weight in Risk free asset
W0=(G-v0*exp((mu-(0.5*(sigma.^2))*T-zp*sigma*sqrt(T)))/...
(v0*exp(r*T)-v0*exp((mu-(0.5*(sigma.^2))*T-zp*sigma*sqrt(T))));
for i=1:1:length(W0)
    if W0(1,i)<0
        W0(1,i)=0;
    end
    if W0(1,i)>1
        W0(1,i)=1;
    end
end

Beta=(W0*v0*(1-fb))./bond(1,:); % Units of the risk free asset at t=0
N=((1-W0)*v0*(1-fs))./stock(1,:); %Units of the risky asset at t=0

VT=Beta.*bond(end,:)+N.*stock(end,:);
OmegaVBPI_static=omega(VT,omega_low:omega_high); % added this for curiosity...

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 1.2 Dynamic insured portfolio (with transaction costs)
% Weight of riskfree asset for rebalancing every trading day
Wtau=zeros(period,length(stock));
Vt=zeros(period,length(stock));
Vts_beg(1,:)=(1-W0)*v0*(1-fs); % bond value in the beginning of the day
Vtb_beg(1,:)=(W0*v0)*(1-fb); % stock value in the beginning of the day
Vts(1,:)=Vts_beg(1,:); % bond value, after transaction & costs
Vtb(1,:)=Vtb_beg(1,:); % stock value, after transaction & costs
Vt(1,:)=Vtb(1,:)+Vts(1,:);
Wtau(1,:)=W0;
for j=1:1:length(bond)
    for i=int+1:int:period
        Vtb_beg(i,j)=Vtb(i-int,j)/bond(i-int,j)*bond(i,j);
        Vts_beg(i,j)=Vts(i-int,j)/stock(i-int,j)*stock(i,j);

        Wtau(i,j)=(G-(Vtb_beg(i,j)+Vts_beg(i,j))*exp((mu(j)-(0.5*...
            (sigma(j)^2))*(T-tau(i))-zp*sigma(j)*sqrt(T-tau(i)))/...
            ((Vtb_beg(i,j)+Vts_beg(i,j))*exp(r(j)*(T-tau(i)))-...
            (Vtb_beg(i,j)+Vts_beg(i,j))*exp((mu(j)-(0.5*(sigma(j)^2))*...
            (T-tau(i))-zp*sigma(j)*sqrt(T-tau(i)))));
        if Wtau(i,j)>1;
            Wtau(i,j)=1;
        end

        if Wtau(i,j)<0;

```

```

        Wtau(i,j)=0;
    end
    Vtb(i,j)=(Vtb_beg(i,j)+Vts_beg(i,j))*Wtau(i,j)-...
        abs((Vtb_beg(i,j)+Vts_beg(i,j))*Wtau(i,j)-Vtb_beg(i,j))*fb;
    Vts(i,j)=(Vtb_beg(i,j)+Vts_beg(i,j))*(1-Wtau(i,j))-...
        abs((Vtb_beg(i,j)+Vts_beg(i,j))*(1-Wtau(i,j))-Vts_beg(i,j))*fs;
    % to make sure that transaction costs will not cause negative
    % values, if all is sold then transaction costs are withdrawn from
    % other type of investment
    if Vtb(i,j)<0
        Vtb(i,j)=0;
        Vts(i,j)=(Vtb_beg(i,j)+Vts_beg(i,j))*(1-Wtau(i,j))-...
            abs((Vtb_beg(i,j)+Vts_beg(i,j))*(1-Wtau(i,j))-Vts_beg(i,j))*fs-...
            abs((Vtb_beg(i,j)+Vts_beg(i,j))*Wtau(i,j)-Vtb_beg(i,j))*fb;
    end
    if Vts(i,j)<0
        Vts(i,j)=0;
        Vtb(i,j)=(Vtb_beg(i,j)+Vts_beg(i,j))*Wtau(i,j)-...
            abs((Vtb_beg(i,j)+Vts_beg(i,j))*Wtau(i,j)-Vtb_beg(i,j))*fb-...
            abs((Vtb_beg(i,j)+Vts_beg(i,j))*(1-Wtau(i,j))-Vts_beg(i,j))*fs;
    end
    Vt(i,j)=Vts(i,j)+Vtb(i,j);
end
end

% The last row of Wtau collapses because tau=T, copy from above
Wtau(end,:)=Wtau(end-int,:);
% And there should not be any transaction the last day
Vt(end,:)=Vts_beg(end,:)+Vtb_beg(end,:);
EndVt=Vt(end,:);
EndWtau=Wtau(end,:);

% Performance Measures VBPI
IndicesV5VBPI=find(EndVt<=quantile(EndVt,0.05));
for i=1:length(IndicesV5VBPI);
    inputsAV5VBPI(i)=EndVt(1,IndicesV5VBPI(i));
end

IndicesQ75VBPI=find(EndVt>=quantile(EndVt,0.75));
for i=1:length(IndicesQ75VBPI);
    inputsAQ75VBPI(i)=EndVt(1,IndicesQ75VBPI(i));
end

returnVBPI=log(EndVt./v0); % return in the period
AVreturnVBPI=mean(returnVBPI);
returnVBPIann=(1+returnVBPI).^(240/60)-1; % use annualized return
AVreturnVBPIann=mean(returnVBPIann);
StandarddevVBPI=std(returnVBPIann);
SkewnessVBPI=skewness(returnVBPIann);

% Seven performance ratios
ProtectionRatioVBPI=length(find(EndVt>=G))/length(EndVt); % Protection Ratio
V5VBPI=quantile(EndVt,0.05); % 5% vigintile (V5)
% average value for the 5% poorest-performing portfolios (AV5)
AV5VBPI=mean(inputsAV5VBPI);
Q75VBPI=quantile(EndVt,0.75); % 75% quartile (Q75)
% average value for the 25% best-performing portfolios (AQ75)
AQ75VBPI=mean(inputsAQ75VBPI);
RiskReturnVBPI=AVreturnVBPI/std(returnVBPI);
RiskReturnVBPIann=AVreturnVBPIann/std(returnVBPIann);

```

```

SharpeVBPI=(AVreturnVBPIann-mean(r))/std(returnVBPIann);

OmegaVBPI=omega(EndVt,omega_low:omega_high);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Comment from here when running the first loop

% 2.1 Buy & Hold (100% risky asset, no rebalancing)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

BoHEndVT=((v0*(1-fs))./stock(1,:)).*stock(end,:); % End value of the position

% Performance Measures B&H
% Inputs
IndicesV5BoH=find(BoHEndVT<quantile(BoHEndVT,0.05));
for k=1:1:length(IndicesV5BoH);
    inputsAV5BoH(k)=BoHEndVT(1,IndicesV5BoH(k));
end

IndicesQ75BoH=find(BoHEndVT>quantile(BoHEndVT,0.75));
for k=1:1:length(IndicesQ75BoH);
    inputsAQ75BoH(k)=BoHEndVT(1,IndicesQ75BoH(k));
end

returnBoH=log(BoHEndVT./v0);
AVreturnBoH=mean(returnBoH);
returnBoHann=(1+returnBoH).^(240/60)-1; % annualized return
AVreturnBoHann=mean(returnBoHann);
StandarddevBoH=std(returnBoHann);
SkewnessBoH=skewness(returnBoHann);

% Seven performance ratios B&H
ProtectionRatioBoH=length(find(BoHEndVT>=G))/length(BoHEndVT);
V5BoH=quantile(BoHEndVT,0.05);
AV5BoH=mean(inputsAV5BoH);
Q75BoH=quantile(BoHEndVT,0.75);
AQ75BoH=mean(inputsAQ75BoH);
RiskBoH=AVreturnBoH/std(returnBoH);
RiskBoHann=AVreturnBoHann/std(returnBoHann);
SharpeBoH=(AVreturnBoHann-mean(r))/std(returnBoHann);

OmegaBoH=omega(BoHEndVT,omega_low:omega_high);

% 3.1 CPPI Static
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Starting floor is equal to present value of insured value in VBPI
F0=G*exp(-r*T); % The floor grows at risk free rate
Ft=repmat(F0,length(tau),1).*exp(tau'*r);
% Starting Value of the portfolio is equal in VBPI and CPPI
VCPPI0=v0;
C0=VCPPI0-F0; % Cushion at start
% The multiple that gives the same starting weights as in VBPI
m=((1-W0)*v0)./(C0);
%m=repmat(9,1,5635); % if we want to try a fixed m (comment next line above)
e0=m.*C0; % Starting value of risky assets in portfolio, without transaction costs
b0=(VCPPI0-e0); % Starting value of risk free asset, without transaction costs
BetaCPPI0=b0.*(1-fb)./bond(1,:); % Starting number of bonds, taking costs into account
NCPPI0=e0.*(1-fs)./stock(1,:); % Starting number of stocks, taking costs into account

```

```

VTCPPI=BetaCPPI0.*bond(end,:)+NCPPI0.*stock(end,:); % Value of portfolio at T

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 3.2 CPPI Dynamic (with transaction cost)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for j=1:1:length(bond);
    for i=int+1:int:period;
        betaCPPI(1,j)=BetaCPPI0(1,j);
        NCPPI(1,j)=NCPPI0(1,j);
        etCPPI(1,j)=e0(1,j);
        btCPPI(1,j)=b0(1,j);
        VtiCPPI(1,j)=VCPPI0;

        VtiCPPI(i,j)=betaCPPI(i-1,j)*bond(i,j)+NCPPI(i-1,j)*stock(i,j);

        etCPPI(i,j)=max(min(m(1,j)*(VtiCPPI(i,j)-Ft(i,j)),VtiCPPI(i,j)),0);
        btCPPI(i,j)=VtiCPPI(i,j)-etCPPI(i,j);

        NCPPI(i,j)=(etCPPI(i,j)-(abs(etCPPI(i,j)-NCPPI(i-
1,j)*stock(i,j))*fs))/stock(i,j);
        betaCPPI(i,j)=(btCPPI(i,j)-(abs(btCPPI(i,j)-betaCPPI(i-
1,j)*bond(i,j))*fb))/bond(i,j);
        if NCPPI(i,j)<0
            NCPPI(i,j)=0;
            betaCPPI(i,j)=(btCPPI(i,j)-(abs(btCPPI(i,j)-betaCPPI(i-1,j)*...
                bond(i,j))*fb)-(abs(etCPPI(i,j)-NCPPI(i-
1,j)*stock(i,j))*fs))/bond(i,j);
        end
        if betaCPPI(i,j)<0
            betaCPPI(i,j)=0;
            NCPPI(i,j)=(etCPPI(i,j)-(abs(etCPPI(i,j)-NCPPI(i-1,j)*...
                stock(i,j))*fs)-(abs(btCPPI(i,j)-betaCPPI(i-
1,j)*bond(i,j))*fb))/stock(i,j);
        end
    end
end
end

EndVTCPPI=VtiCPPI(end,:);

% Performance Measures CPPI
IndicesV5CPPI=find(EndVTCPPI<quantile(EndVTCPPI,0.05));
for j=1:1:length(IndicesV5CPPI);
    inputsAV5CPPI(j)=EndVTCPPI(1,IndicesV5CPPI(j));
end

IndicesQ75CPPI=find(EndVTCPPI>quantile(EndVTCPPI,0.75));
for j=1:1:length(IndicesQ75CPPI);
    inputsAQ75CPPI(j)=EndVTCPPI(1,IndicesQ75CPPI(j));
end

returnCPPI=log(EndVTCPPI./v0);
AVreturnCPPI=mean(returnCPPI);
returnCPPIann=(1+returnCPPI).^(240/60)-1; % annualized return
AVreturnCPPIann=mean(returnCPPIann);
StandarddevCPPI=std(returnCPPIann);
SkewnessCPPI=skewness(returnCPPIann);

```

```

% Performance ratios CPPI
ProtectionRatioCPPI=length(find(EndVTCPPPI>=G))/length(EndVTCPPPI);
V5CPPI=quantile(EndVTCPPPI,0.05);
AV5CPPI=mean(inputsAV5CPPI);
Q75CPPI=quantile(EndVTCPPPI,0.75);
AQ75CPPI=mean(inputsAQ75CPPI);
RiskCPPI=AVreturnCPPI/std(returnCPPI);
RiskCPPIann=AVreturnCPPIann/std(returnCPPIann);
SharpeCPPI=(AVreturnCPPIann-mean(r))/std(returnCPPIann);

OmegaCPPI=omega(EndVTCPPPI,omega_low:omega_high);

% create omega graph
threshold=OmegaVBPI.returnLevel;
%omega_values=[OmegaCPPI.omegaValues; OmegaVBPI.omegaValues];
omega_values=[OmegaCPPI.omegaValues; OmegaVBPI.omegaValues; OmegaBoH.omegaValues];
createfigure(threshold,omega_values,G/100,p*100,rebalancing,mu_title);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% ENG - Expected net gain performance measure
% using the starting value of the portfolios as criterion for gain or loss
Profit_diff_CPPI=(EndVTCPPPI-v0)-(BoHEndVT-v0);
Profit_diff_VBPI=(EndVt-v0)-(BoHEndVT-v0);
Indices_diff_less=find(BoHEndVT-v0<0);
Indices_diff_more=find(BoHEndVT-v0>=0);
for j=1:1:length(Indices_diff_less);
    inputs_diff__less_CPPI(j)=Profit_diff_CPPI(1,Indices_diff_less(j));
end
for j=1:1:length(Indices_diff_more);
    inputs_diff__more_CPPI(j)=Profit_diff_CPPI(1,Indices_diff_more(j));
end
average_gain_CPPI=mean(inputs_diff__less_CPPI);
average_loss_CPPI=mean(inputs_diff__more_CPPI);
ENG_CPPI=average_gain_CPPI+average_loss_CPPI;

for j=1:1:length(Indices_diff_less);
    inputs_diff__less_VBPI(j)=Profit_diff_VBPI(1,Indices_diff_less(j));
end
for j=1:1:length(Indices_diff_more);
    inputs_diff__more_VBPI(j)=Profit_diff_VBPI(1,Indices_diff_more(j));
end
average_gain_VBPI=mean(inputs_diff__less_VBPI);
average_loss_VBPI=mean(inputs_diff__more_VBPI);
ENG_VBPI=average_gain_VBPI+average_loss_VBPI;
ENG=[ENG_VBPI,ENG_CPPI];

% Stop commenting here when running the first loop
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% % This goes with the first loop
% Protection_Ratio_VBPI=[Protection_Ratio_VBPI ProtectionRatioVBPI];
% V5_VBPI=[V5_VBPI V5VBPI];
% AV5_VBPI=[AV5_VBPI AV5VBPI];
% Q75_VBPI=[Q75_VBPI Q75VBPI];
% AQ75_VBPI=[AQ75_VBPI AQ75VBPI];
% RiskReturn_VBPI=[RiskReturn_VBPI RiskReturnVBPI];
% subplot(3,3,6), plot(OmegaVBPI.returnLevel,OmegaVBPI.omegaValues,'DisplayName',...
% 'OmegaVBPI.returnLevel vs
OmegaVBPI.omegaValues','XDataSource','OmegaVBPI.returnLevel',...
% 'YDataSource','OmegaVBPI.omegaValues');figure(gcf);

```

```

% end

% % For the sensitivity analysis, uncomment one test below, depending on
% % which sensitivity analysis you are doing
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% % for mu sensitivity test
% title({'Omega'});
% xlim([9800 10300]);
% set(gca,'XTickLabel',{'9,800','10,000','10,200'},...
%   'XTick',[9800 10000 10200],'Position',[0.7 0.45332 0.235 0.195]);
% subplot(3,3,1), plot(mu_fixed,Protection_Ratio_VBPI(2:end));
% title({'Floor Protection Ratio'});
% xlim([0.05 0.2]);
% set(gca,'XTickLabel',{'0.05','0.10','0.15','0.20'},...
%   'XTick',[0.05 0.1 0.15 0.2],'Position',[0.09 0.75295 0.235 0.195]);
% subplot(3,3,2), plot(mu_fixed,V5_VBPI(2:end));
% title({'V5'});
% xlim([0.05 0.2]);
% set(gca,'XTickLabel',{'0.05','0.10','0.15','0.20'},...
%   'XTick',[0.05 0.1 0.15 0.2],'Position',[0.4 0.75295 0.235 0.195]);
% subplot(3,3,3), plot(mu_fixed,AV5_VBPI(2:end));
% title({'AV5'});
% xlim([0.05 0.2]);
% set(gca,'XTickLabel',{'0.05','0.10','0.15','0.20'},...
%   'XTick',[0.05 0.1 0.15 0.2],'Position',[0.7 0.75295 0.235 0.195]);
% subplot(3,3,4), plot(mu_fixed,Q75_VBPI(2:end));
% title({'Q75'});
% xlim([0.05 0.2]);
% set(gca,'XTickLabel',{'0.05','0.10','0.15','0.20'},...
%   'XTick',[0.05 0.1 0.15 0.2],'Position',[0.09 0.45332 0.235 0.195]);
% subplot(3,3,5), plot(mu_fixed,AQ75_VBPI(2:end));
% title({'AQ75'});
% xlim([0.05 0.2]);
% set(gca,'XTickLabel',{'0.05','0.10','0.15','0.20'},...
%   'XTick',[0.05 0.1 0.15 0.2],'Position',[0.4 0.45332 0.235 0.195]);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% % for sigma sensitivity test
% title({'Omega'});
% xlim([9800 10300]);
% set(gca,'XTickLabel',{'9,800','10,000','10,200'},...
%   'XTick',[9800 10000 10200],'Position',[0.7 0.45332 0.235 0.195]);
% subplot(3,3,1), plot(sigma_fixed,Protection_Ratio_VBPI(2:end));
% title({'Floor Protection Ratio'});
% xlim([0.1 0.4]);
% set(gca,'XTickLabel',{'0.10','0.20','0.30','0.40'},...
%   'XTick',[0.1 0.2 0.3 0.4],'Position',[0.09 0.75295 0.235 0.195]);
% subplot(3,3,2), plot(sigma_fixed,V5_VBPI(2:end));
% title({'V5'});
% xlim([0.1 0.4]);
% set(gca,'XTickLabel',{'0.10','0.20','0.30','0.40'},...
%   'XTick',[0.1 0.2 0.3 0.4],'Position',[0.4 0.75295 0.235 0.195]);
% subplot(3,3,3), plot(sigma_fixed,AV5_VBPI(2:end));
% title({'AV5'});
% xlim([0.1 0.4]);
% set(gca,'XTickLabel',{'0.10','0.20','0.30','0.40'},...
%   'XTick',[0.1 0.2 0.3 0.4],'Position',[0.7 0.75295 0.235 0.195]);
% subplot(3,3,4), plot(sigma_fixed,Q75_VBPI(2:end));
% title({'Q75'});
% xlim([0.1 0.4]);
% set(gca,'XTickLabel',{'0.10','0.20','0.30','0.40'},...

```

```

% 'XTick',[0.1 0.2 0.3 0.4],'Position',[0.09 0.45332 0.235 0.195]);
% subplot(3,3,5), plot(sigma_fixed,AQ75_VBPI(2:end));
% title({'AQ75'});
% xlim([0.1 0.4]);
% set(gca,'XTickLabel',{'0.10','0.20','0.30','0.40'},...
% 'XTick',[0.1 0.2 0.3 0.4],'Position',[0.4 0.45332 0.235 0.195]);

```

### The Matlab function that creates the omega figure

```

function createfigure(OmegaCPPI1, YMatrix1, title1,title2,title3,title4)
%CREATEFIGURE(OMEGACPPI1,YMATRIX1)
% OMEGACPPI1: vector of x data
% YMATRIX1: matrix of y data

% Auto-generated by MATLAB on 01-Mar-2011 16:01:48

% Create figure
figure1 = figure;

% Create axes
axes1 = axes('Parent',figure1,...
    'XTickLabel',{'9800','9900','10,000','10,100','10,200','10,300'},...
    'XTick',[9800 9900 10000 10100 10200 10300]);
xlim(axes1,[9800 10300]);
ylim(axes1,[0 30]); % the values on the y-axes
box(axes1,'on');
hold(axes1,'all');

% Create multiple lines using matrix input to plot
plot1 = plot(OmegaCPPI1,YMatrix1,'Parent',axes1);
set(plot1(1),'LineStyle','--','DisplayName','CPPI');
set(plot1(2),'Color',[0 0 1],'DisplayName','VBPI');
%set(plot1(3),'Color',[0 0 1],'LineStyle',':','DisplayName','B&H');

% Create xlabel
xlabel('Threshold');

% Create ylabel
ylabel({'Omega Ratio'});

% Create title
title([int2str(title1),'% floor, ',num2str(title2),'% confidence level,
'...
',title3,' rebalancing, ',title4,' mu']);

% Create legend
legend1 = legend(axes1,'show');
set(legend1,...
    'Position',[0.71 0.78 0.15 0.1])
    '%Location','NorthEast');

```