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Asymptotics and dynamics in first-passage and continuum percolation

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Abstract

This thesis combines the study of asymptotic properties of percolation processes with various dynamical concepts. First-passage percolation is a model for the spatial propagation of a fluid on a discrete structure; the Shape Theorem describes its almost sure convergence towards an asymptotic shape, when considered on the square (or cubic) lattice. Asking how percolation structures are affected by simple dynamics or small perturbations presents a dynamical aspect. Such questions were previously studied for discrete processes; here, sensitivity to noise is studied in continuum percolation.

Paper I studies first-passage percolation on certain 1-dimensional graphs. It is found that when identifying a suitable renewal sequence, its asymptotic behaviour is much better understood compared to higher dimensional cases. Several analogues of classical 1-dimensional limit theorems are derived.

Paper II is dedicated to the Shape Theorem itself. It is shown that the convergence, apart from holding almost surely and in L^1, also holds completely. In addition, inspired by dynamical percolation and dynamical versions of classical limit theorems, the almost sure convergence is proved to be dynamically stable. Finally, a third generalization of the Shape Theorem shows that the above conclusions also hold for first-passage percolation on certain cone-like subgraphs of the lattice.

Paper III proves that percolation crossings in the Poisson Boolean model, also known as the Gilbert disc model, are noise sensitive. The approach taken generalizes a method introduced by Benjamini, Kalai and Schramm. A key ingredient in the argument is an extremal result on arbitrary hypergraphs, which is used to show that almost no information about the critical process is obtained when conditioning on a denser Poisson process.

Keywords: First-passage percolation, noise sensitivity, continuum percolation, Gilbert model, limit theorems, shape theorem, stopped random walks, large deviations, dynamical percolation.