Discounting and relative prices

Discounting and relative prices in assessing future environmental damages

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Abstract

Environmentalists are often upset at the effect of discounting costs of future environmental damage, e.g. due to climate change. An often overlooked message is that we should discount costs but also take into account the increase in the relative price of the ecosystem service endangered. The effect of discounting would thus be counteracted, and if the rate of price rise of the item was fast enough it might even be reversed. The scarcity that leads to rising relative prices for the environmental good will also have direct effects on the discount rate itself. The magnitude of these effects depends on properties of the economy's technology and on social preferences. We develop a simple model of the economy that illustrates how changes in crucial technology and preference parameters may affect both the discount rate and the rate of change of values of environmental goods. The combined effect of discounting and the change of values of environmental goods is more likely to be low, or even negative, the lower is the growth rate of environmental quality (or the larger its decline rate) and the lower is the elasticity of substitution between environmental quality and produced goods.

Key Words: Discounting, future costs, scarcity, environment, climate change

JEL: H43 Q32 Q54

1. Introduction.

During the last 10-20 years, there has been a vast economic literature discussing various aspects of climate change and climate policies. Part of this literature gives cost-benefit analyses of various proposed measures to reduce greenhouse gases. In analyses of this type economists add up all benefits and costs of any proposed policy. This summation of costs and benefits includes both the current period and all future periods. In such cost-benefit analyses, future costs and benefits are discounted to present values through the use of a discount (or interest) rate. If some cost or benefit component at a future date t is of the magnitude V_t and the discount rate is r, the present value is $(1+r)^{-t}V_t$. Environmentalists are often upset at the effect of discounting on such benefits and costs. There have been numerous debates – related to the climate problem as well as other environmental problems - in which environmentalists are dismayed to find the cost of some expected environmental damage is discounted to a present value that is basically insignificant.

The logic of discounting is in fact dramatic – but correct as long as the assumptions hold. With 5% discounting per year a future cost of 100 dollars a century away is worth less than a dollar today. There are various arguments in favour of low or non-linear discount rates and a big professional literature on the subject[i], but in this article we want to focus on another, separate, factor namely that of changes in relative price. The cost V_t – for instance of future environmental damage should be valued in real future prices. However we only know the current prices today. Our best estimate of V_t is $V_t = V_0(1+p)^t$ where p is the expected rise per year in real price of the relevant good (i.e. the price relative to some general price level such as the consumer price index). For an environmental resource that will become scarce in the future, the relative price will typically rise – and this effect may again be exponential – and thus potentially just as powerful arithmetically as discounting. Indeed, combining the two formulae we see that the discounted, present, real cost of any given item of damage in the future will be $V_0(1+p)^t(1+r)^{-t}$. This general point is well known to economists but is still quite often overlooked.

The effect of future scarcity of environmental goods is hard to study empirically for two reasons. Firstly since we are speaking of a fairly recent phenomenon and secondly because many environmental goods are un-priced non-market goods and thus we are speaking more of a willingness to pay than of a regular price. However there are market goods (partly related to environmental factors) that can illustrate the effect of scarcity on prices. One clear example is offered by land and housing markets. We select this example because the stock of housing typically does exhibit scarcity. Houses can of course be built and modernised which no doubt explains a large part of the rise in price but the underlying scarcity of space for housing lots is likely an important explanatory factor why housing should appreciate faster in densely populated areas. We choose to illustrate this with the housing market in the UK where land scarcity certainly is important. Table 1 shows that the Greater London area where land is scarcest has a rate of price escalation of 17.8% compared to 11.5% in Scotland. The UK national average is 15.9%. The retail price index for the same period showed an inflation of 7,4%, far lower than even the lowest price increase in table 1. The rate of interest was also much lower, the repo rate of the Bank of England was for this period between 3.5 – 7.5%[ii].

Area	Annual % price increase 1993-2004	Area	Annual % price increase 1993-2004
Greater London	17,8	Wales	16,1
S.West	16,7	Yorkshire/Humbershire	15,7
E.Midlands	16,6	North	15,5
W.Midlands	16,6	N.West	15,2
S.East	16,3	N.Ireland	13,4
E.Anglia	16,2	Scotland	11,5

Table 1 The increase in the price of houses in the UK by region Source HBOS (2005)

The simple, but often overlooked, message is that, in calculating the costs of some future damage (say of excess carbon in the atmosphere), we should discount costs but also take into account the increase in the relative price of the ecosystem service endangered (biodiversity, clean air, water, arable land, coastal ecosystems etc) and thus the effect of discounting would be counteracted and if the rate of price rise of the item was fast enough it might even be reversed. This point is in its simplest form well established in the economics literature, and has been explicitly or implicitly used in discussions of climate policy.[iii] Matters are somewhat more complicated than they first appear however. The scarcity that leads to rising relative prices for the environmental good will also have direct effects on the discount rate itself. The magnitude of these effects depend on the growth rate of the economy (or more generally on properties of the economy's technology) and on properties of the social preference function. In this paper we develop a simple model of the economy that illustrates how changes in crucial parameters may affect both the discount rate and the rate of change of values of environmental goods.

2. Discounting and relative prices

In economic models of dynamic optimization and cost-benefit analyses a frequently used objective function is of the type

(1) $$W = \int_0^T e^{-\rho t} U(C(t)) dt$$

where $C(t)$ stands for consumption at time t and U can be interpreted as a measure of wellbeing or utility. At the individual (person or household) level the function given by (1) represents the person's preferences regarding alternative consumption profiles through life. In this case it is natural to interpret the time horizon T as the (maximal) remaining life time of the person. The tradeoffs between consumption at different points of time are given partly by the "utility discount rate" ρ, and partly by the utility function U. The larger is ρ, the more weight is given to the present relative to the future. Economists usually assume that $\rho > 0$, implying that if one initially is faced with a situation where consumption is constant over time and one is offered a particular increase in consumption in any year, one would prefer to have this consumption increase early rather than later. The function U is assumed to be strictly concave with $U(0)=0$, so that it increases less than proportionately with consumption. The more concave U is, the more weight is given to periods with "low" consumption relative to periods with "high" consumption. For the limiting case of $\rho = 0$ this means that if one is offered a particular increase in consumption in any year, one would prefer to have this consumption increase in a period when consumption is low rather than in a period when it is higher.

In the context of the climate problem, the time perspective is much longer than the life time for any particular generation. For such problems, the interpretation of equation (1) is therefore slightly different than what we gave above. For issues with time perspective of up to a century or even longer, it is natural to interpret the function (1) as a representation of society's preferences over distributions of consumption across generations. In this case we interpret $C(t)$ as average per capita consumption at time t.[iv] An assumption of $\rho > 0$ means that society gives the current generation more weight than future generations, and that future generations are given lower weight the more distant they are. In the economics literature, there has been an extensive discussion of whether a positive value of ρ can be given an ethical justification. For the points we make in the present article, it makes no difference whether $\rho > 0$ or $\rho = 0$.

The concavity of *U* measures "inequality aversion": The more concave *U* is, the more weight is given to generations with "low" consumption relative to generations with "high" consumption. In a situation with economic growth (rising $C(t)$), the future is thus given lower weight the more concave *U* is. In a situation with economic decline, however the value of future consumption might even be given a higher weight than current consumption (we would thus have a negative discount rate if the rate of economic decline was sufficient to outweigh the effect of utility discounting ρ). Finally, as it is natural to let *T* be very large for analyses of climate issues and several other environmental problems, economists often choose an infinite time horizon.

The appropriate interest rate *r* for discounting consumption when preferences are given by (1) is

(2) $$r = \rho + \frac{-\frac{d}{dt}U'(C(t))}{U'(C(t))}$$

The interpretation of this discount rate is (somewhat loosely) as follows. Imagine society makes some investment making present per capita consumption go down by one unit (e.g. one thousand dollars). What is the minimum increase in consumption one year ahead in order for *W* to not decline, provided there are no further changes in consumption after one year? The answer to this question is $1+r$, where *r* is given by (2). This discount rate has two parts: one pure time preference or "utility discounting" ρ, and one related to the fact that additional money is less valuable to those (in the future) who anyhow will be richer than today.

With a concave utility function, *U'* is declining over time when consumption is growing, so that both terms in this expression are positive for this case. It is often assumed that the utility function has the simple form

(3) $$U(C) = \frac{1}{1-\alpha}C^{1-\alpha} \text{ for } \{\alpha > 0, \alpha \neq 1\} \text{ and } U(C) = LnC \text{ for } \alpha = 1.$$

This specification has the advantage that the elasticity of utility with respect to consumption is constant[v]. In this case, the appropriate discount rate *r* is (4) often called the "Ramsey" rate:

(4) $$r(t) = \rho + \alpha g_C(t)$$

where $g_C(t)$ is the relative growth rate of consumption. If e.g. $\rho = 0.01$, $\alpha = 1.5$ and $g_C(t) = 0.025$, we find $r = 0.0475$, i.e. a discount rate of almost 5%. Notice that this discount rate will be constant over time only if the growth rate of consumption is constant over time. If we believe that consumption growth will be slower in the future than it is presently, future discount rates in our calculations should be set lower than present rates. For instance Azar & Sterner (1996) show that limits to future economic growth imply lower discount rates and therefore higher values of damage per ton of carbon emitted.

In the debate on growth and sustainability, the "pessimists" point to scarce resources as a reason for "limits to growth" while "optimists" point to technology and new sectors as sources of growth. Clearly communication and computing are two examples of phenomenal economic growth that use few scarce natural resources. But if future growth is concentrated to some sectors while others have constant levels then this growth implies a changing output composition and presumably rising prices in the sectors that do not grow. These could include solitary access to unspoilt nature, but conceivably also important goods such as clean water and other vital inputs. In studies of environmental issues it is therefore useful to explicitly distinguish between environmental goods and other consumption goods, which is not possible in the aggregate approach above. Let us use *E* to represent some aggregate measure of the environmental quality in society, while *C* is an aggregate measure of all other goods. Instead of the utility function given in (1), we assume *U*=*U*(*C,E*), so that the objective function is changed to (ignoring time references to simplify notation)

(5) $$W = \int_0^\infty e^{-\rho t} U(C,E)\,dt$$

With this change, the appropriate discount rate *r* is changed from (2) to

(6) $$r = \rho + \frac{-\frac{d}{dt}U_C(C,E)}{U_C(C,E)}$$

where subscripts represent partial derivatives. As argued earlier, to calculate the future value of a change in environmental quality we must consider both discounting and the change in the relative price (or valuation) of the environmental quality. The valuation of the environmental good is given by U_E/U_C . This fraction tells us by how much current consumption must increase to just offset a deterioration in current environmental quality of one unit (i.e. to make current utility or wellbeing the same before and after the change in environmental quality and consumption). The relative change in this price, previously denoted *p*, is thus

$$(7) \qquad p = \frac{\frac{d}{dt}\left(\frac{U_E}{U_C}\right)}{\left(\frac{U_E}{U_C}\right)}$$

This price change will depend on the development over time of both consumption and the environmental quality. If *C* increases over time while *E* is constant or declines, *p* will be positive for most reasonable specifications of the function *U*. The combined effect of discounting and the relative price increase of environmental goods is given by *r-p*. If both *r* and *p* are positive, the sign of the combined effect is ambiguous without further specification of the utility function *U*.

Writing this paper in Oslo in January, an example that comes to mind as an effect of climate change is the loss in skiing areas. This would be sad enough but the potential consequences of climate change are of course much more severe. For the roughly half of humanity that lives in Asia, assessments speak of rising temperature, decreasing water availability, drought, water and food shortage, spread of disease through several mechanisms (temperature directly as well as through various disease vectors), coastal inundation and population displacement[vi]. In this context we see that *E* is directly associated with major items such as water, shelter, land, homesteads and food. With this definition, a fall in *E* or a rise in the marginal cost of obtaining these ecosystem resources, obviously has very major welfare consequences.

3. The case of a constant elasticity of substitution

The properties of the utility function *U*(*C,E*) will of course depend on how we measure environmental quality. Nevertheless, it is useful to illustrate the points made above with a simple example. Consider the following constant elasticity of substitution (CES) utility function:

$$(8) \qquad U(C,E)=\frac{1}{1-\alpha}\left[(1-\gamma)C^{1-\frac{1}{\sigma}}+\gamma E^{1-\frac{1}{\sigma}}\right]^{\frac{(1-\alpha)\sigma}{\sigma-1}}$$

where σ is the elasticity of substitution, which is positive.[vii] This elasticity is easiest to interpret if we for a moment consider the hypothetical case where environmental quality is a good that consumers can purchase in the market. If the price of this environmental good increases by 1% relative to the price of other consumer goods, the purchase of the environmental good will decline by σ % relative to the purchase of other consumption goods. The lower is the elasticity of substitution, the less willing are consumers thus willing to substitute away from environmental quality as the price of environmental quality increases.

The parameter γ has no easy, direct interpretation. Consider however the following variable γ^* which will figure prominently in our derivation of the discount rate r based on (8):

$$(9) \qquad \gamma^*=\frac{\gamma E^{1-\frac{1}{\sigma}}}{(1-\gamma)C^{1-\frac{1}{\sigma}}+\gamma E^{1-\frac{1}{\sigma}}}$$

From (8) it follows that we alternatively could write this as

$$(9') \qquad \gamma^*=\frac{U_E E}{U_E E+U_C C}=\frac{\frac{U_E}{U_C}E}{\left(\frac{U_E}{U_C}E\right)+C}$$

Using the market analogy above, the variable γ^* may be interpreted as the "value share of environmental quality": This tells us what share of their total consumption expenditures consumers would use on environmental quality if environmental quality was a good that could be purchased in the same manner as other consumption goods. An alternative interpretation of γ^* is that $\gamma^*/(1-\gamma^*)$ tells us by how many percent the environmental quality must increase to just offset a reduction in the consumption level by one percent.

By a suitable choice of units, we may set $\gamma^* = \gamma$ at our initial time point (t=0). From the interpretation above, this initial value of γ^* tells us somewhat loosely how large the value of the environmental quality is relative to the total consumption value (i.e. of environmental quality and other consumption goods). As time passes, γ^* will change unless C and E are changing proportionately. Under the reasonable assumption that C is growing faster than E, γ^* will be rising (declining) over time if $\sigma < 1$ ($\sigma > 1$). For the CES utility function with $\sigma < 1$ we have the case that Gerlagh and van der Zwaan (2002) call "poor substitutability" between environmental quality and other goods. If C/E grows without bounds in this case, the value share γ^* will approach 1 as time passes[viii].

Notice that the utility function (8) includes a parameter α that has a similar interpretation as α had in (3). Moreover, (8) is identical to (3) for the special case of $\gamma = 0$. The same is true if E is proportional to C. For the general case of $0 < \gamma < 1$ and E and C growing at different rates the utility function (8) generally gives a different interest rate than (4). From (6) and (8) tedious but straightforward derivations give[ix]

$$(10) \qquad r = \rho + \left[(1-\gamma^*)\alpha + \gamma^* \frac{1}{\sigma} \right] g_C + \left[\gamma^* \left(\alpha - \frac{1}{\sigma} \right) \right] g_E$$

where g_E is the relative growth rate of environmental quality. Notice that (10) is identical to (4) if either $\gamma^*=0$, $g_C = g_E$, or $\alpha\sigma=1$. Generally, however, (10) may give a lower or a higher discount rate than (4). For the reasonable case of $g_C > g_E$ (10) gives a lower or higher interest rate than (4) depending on whether $\alpha\sigma > 1$ or $\alpha\sigma < 1$. Notice also that if $\sigma \neq 1$ and $\alpha\sigma \neq 1$ the discount rate will not be constant over time even if the growth rates for C and E are constant (since γ^* changes over time when $\sigma \neq 1$).

As for the relative price of environmental quality, it follows from (7) and (8) that

(11) $$p = \frac{\frac{d}{dt}\left(\frac{U_E}{U_C}\right)}{\left(\frac{U_E}{U_C}\right)} = \frac{1}{\sigma}(g_C - g_E)$$

The price change is thus positive provided consumption increases relative to environmental quality over time, and is larger the smaller is the elasticity of substitution. If e.g. the environmental quality is constant and consumption increases by 2.5% a year, and the elasticity of substitution is 0.5, this price will increase by a yearly rate of 5%.

Using *R* to denote the combined effect of discounting and the relative price increase of environmental goods, i.e. *R=r-p*, it follows from (10) and (11) that

(12) $$R = \rho + \left[(1-\gamma^*)\left(\alpha - \frac{1}{\sigma}\right)\right]g_C + \left[\gamma^*\alpha + (1-\gamma^*)\frac{1}{\sigma}\right]g_E$$

In Appendix B we derive derivatives of *r, p* and *R* with respect to the "technology variables" g_C and g_E and the preference parameters α and σ. The signs are summarized in Table 2.

	r	*p*	*R=r-p*
g_C	+	+	− if $\alpha\sigma < 1$ + if $\alpha\sigma > 1$
g_E	− if $\alpha\sigma < 1$ + if $\alpha\sigma > 1$	−	+
α	Depends on γ^*, g_C and g_E (+ if $g_C > 0$ and $g_E \geq 0$)	0	Depends on γ^*, g_C and g_E (+ if $g_C > 0$ and $g_E \geq 0$)
σ	− (if $g_C > g_E$)	− (if $g_C > g_E$)	+ (if $g_C > g_E$)

Table 2: Sign of derivatives of r, p and R with respect to g_C, g_E, α *and* σ

The table reveals several interesting results. First, we see that if $g_C > 0$ and $g_E \geq 0$, the discount rate is higher the higher is the value of the parameter α (which measures "the degree of inequality aversion"). This is the same as we had for the simple case of only one good in the utility function. Moreover, the combined effect of the discount rate and the change in relative prices (i.e. *R*) is also higher the higher is α .

A second result is that for the reasonable case of $g_C > g_E$, an increase in the elasticity of substitution between environmental quality and other consumption will reduce the discount rate, but nevertheless *increase* the combined effect of the discount rate and the change in relative prices.

A third result concerns the effects of changes in growth rates. Changes in the growth rates g_C and g_E change the discount rate and the combined effect of the discount rate and the change in relative prices in the same direction if $\alpha\sigma > 1$. The case of $\alpha\sigma < 1$ (low elasticity of substitution and limited inequality aversion which dose not seem unreasonable) is more interesting: In this case changes in the growth rates g_C and g_E change the discount rate and the combined effect of the discount rate and the change in relative prices in the *opposite* direction. In particular, the higher is the consumption growth rate, the lower is the combined effect of discounting and price changes.

Notice that *R* is more likely to be low, or even negative, the lower is the growth rate of environmental quality (or the larger its decline rate) and the lower is the elasticity of substitution between environmental quality and produced goods. Notice also that if this elasticity is below 1 (and $g_C > g_E$), the variable γ^* defined by (9) will be rising over time. As time passes, γ^* will approach 1, and it follows from (12) that *R* will approach $\rho + \alpha g_E$. In the numerical example above we assumed $\rho = 0.01$ and $\alpha = 1.5$. If *E* declines by 0.67% per year in the long run, the value of *R* approaches zero in the long run, implying that the present value of a specific future environmental damage should be approximately independent of how far into the future it occurs.[x]

It has sometimes been argued that conventional discounting may lead to too little mitigation today, and therefore to unacceptably large climate changes in the future (see e.g. Hasselmann et al. , 1997). Although there may be some truth in this, it is important to remember that the discount rate should be an endogenous variable that is not independent of the evolution of the economy. This is particularly clear when we consider the simple aggregate economy with the discount rate given by (4). If the future is "bad" in the sense that aggregate consumption declines in the long run, the second term in the expression will be negative. In such a situation the long-run discount rate will therefore be negative provided the utility discount rate ρ is sufficiently close to zero. In Appendix C we show that a similar result holds for the two-good economy we have considered in this paper. If the environment develops in such a bad way that utility or wellbeing is declining in the long run, then our combined discount and price change rate *R* will be negative provided the utility discount rate ρ is sufficiently close to zero.

4. Development of the discount schedule over time

In order to see the effects on discounting of changing shares in utility due to different growth rates we carried out a simulation assuming that the consumption good producing sector grows at 2.5% while the supply of the environmental good is constant. Initial values of the parameters E, C and γ are chosen so that γ^*, the "value share of the environment" is initially =0.1. (Had *E* been an ordinary good, then we would have spent 10% of our aggregate income on it). Initial values for *E* and *C* are normalised to 1 but *C* grows at 2.5% while *E* is constant. Thus in some sense *C* becomes dominant but with a low elasticity of substitution, (σ=0.5), *E* becomes more and more important for our utility the more its relative scarcity increases. This is measured by γ^*, which, as shown above, increases as long as σ<1. In our case γ^* grows from 0.1 to 0.5 in 90 years and to 0.9 in 180 years.

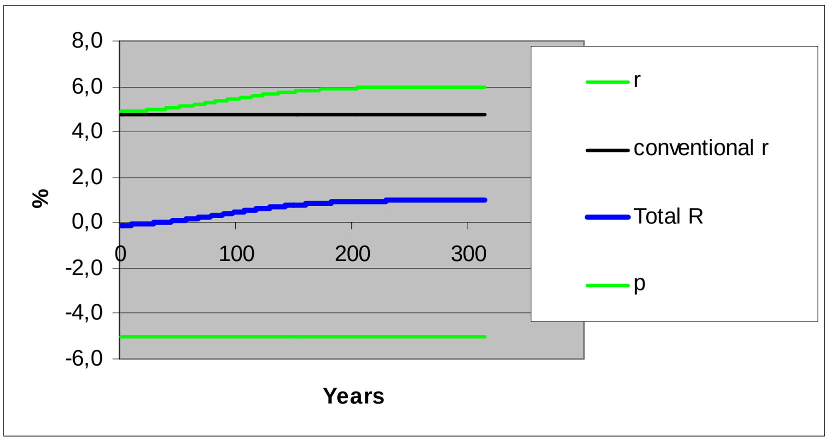

Figure 1 Components of discounting, ρ=0.01, σ=0.5, α=1,5, γ^*_0=0,1, g_C=0.025.

Figure 1 shows the components of discounting with a marginal elasticity of utility, α, of 1.5 and with an elasticity of substitution of 0.5. The conventional "Ramsey rule" discount rate would, as mentioned earlier, be 1.5*2.5 + 1 = 4.75%. Since $\alpha\sigma < 1$, our "corrected" discount rate is higher: it starts at 4.9% and rises slowly to 6% but it is counteracted by a relative price effect *p* of -5% so that the effective actual discounting starts at -0.1% and rises gradually to +1%. As mentioned above, the situation would be different if substitutability was easier. With an elasticity of substitution of 1 we have Cobb Douglas; $\alpha\sigma > 1$ and a constant utility share γ^* of 0.1. Consequently *r* is a constant 4.6% and lower than the conventional 4.75%. The price effect is in this scenario exactly mirrors the growth rate of *C* (i.e. -2.5%) giving a total combined discount and price change rate of 2.1%.

The fact that the discount rates change over time due to the change in the composition of our "utility basket" (when *C* rises and *E* is constant) is an important result of our method. However it makes it hard to readily compare overall results of changing the parameters. We have therefore calculated the constant rates that are equivalent to our varying rates for one century[xi].

α	*σ*	Convent *r*	*r*	*p*	TOT *R*
0.5	0.5	2.25	3.35	-5.00	-1.65
0.5	1	2.25	2.37	-2.50	-0.12
0.5	1.5	2.25	2.28	-1.67	0.61
1	0.5	3.5	4.24	-5.00	-0.76
1	1	3.5	3.50	-2.50	1.00
1	1.5	3.5	3.44	-1.67	1.77
1.5	0.5	4.75	5.12	-5.00	0.12
1.5	1	4.75	4.62	-2.50	2.13
1.5	1.5	4.75	4.60	-1.67	2.94

Table 3 A comparison of discount rates

We see from table 3 that low values of *σ* give values of our discount rate *r* that are high compared to the conventional rates (which only depend on α*)*. On the other hand low values of *σ* also give high price effects. (The value of *α* does not affect *p*). It is very hard to say what reasonable values are. However a value of *σ* of somewhere between 0.5 and 1 implies some reasonable degree of substitutability. Values above 1 would suggest substitutability is so high we need hardly worry about the environment.

Our total discount factor including the price effect is, for the values chosen, always lower than the conventional rate. It is possible however to find combinations of values that give a higher discount rate than the conventional rate. For the range of values where both *σ* and *α* are between 0.5 and 1.5, our total combined discount and price change rate is in the interval -1.6 to +2.9%, all considerably below the conventional discount rates.

5. Conclusion

We have in this paper taken as our starting point the concern felt by environmentalists that future environmental damage is given insufficient weight in economic calculations. We show that the broad intuition is correct that says that discounting should be complemented by a calculation of future prices that may well be rising if environmental goods are expected to become scarcer. By way of a simple example we illustrate the importance of scarcity by looking at the market for real estate in the UK where house prices rise much faster in areas of scarcity such as London than in all other areas. We built a model that is as simple as possible but still incorporates the following characteristics:

- Discounting is derived from a model of intertemporal utility maximisation
- Discounting rests on both pure time preference and the characteristics of the utility function.
- The utility function allows for different elasticities of utility
- There is consumption substitution (to varying degrees) between a consumption good *C* and one environmental aggregate *E*.

With the help of this model we show that the formula for the rate of discount itself is different in a two sector model with different growth rates than in an aggregate model of the economy. We can also directly relate price changes in the environmental good to scarcity and show that the same (substitution) parameter that is decisive for the rate of price appreciation also (together with other parameters) decides the discount rate. We show that future costs should be discounted at a rate that will not generally be constant – it will vary over time because the share of utility coming from consumption and environmental goods will vary over time. In order to our results with the traditional aggregate (Ramsey) formula we calculate one century equivalent average rates and show that for some likely parameter values the total combined discount and price change rate should generally be considerably lower than the conventional discount rate – and in some cases even negative. Particularly in the case when climate change causes "catastrophic change", the discount rate could be negative.

We believe that our results are sufficiently precise and detailed to be practicable at least at the level of numerical examples or sensitivity analyses conducted routinely as part of regular cost benefit analyses or similar calculations for instance in the framework of assessing investments and consequences in the area of climate economics.

Appendix A: Derivation of equation (10)

Using dots to denote derivatives with respect to time and El_x to denote an elasticity with respect to a variable *x,* differentiation of $U(C,E)$ gives

(A1) $$\frac{\dot{U}_C}{U_C}=\frac{U_{CC}\dot{C}}{U_C}+\frac{U_{CE}\dot{E}}{U_C}=\frac{U_{CC}C}{U_C}\frac{\dot{C}}{C}+\frac{U_{CE}E}{U_C}\frac{\dot{E}}{E}=\left(El_C U_C\right)g_C+\left(El_E U_C\right)g_E$$

From (8) it follows that

(A2) $$U_C=\frac{\sigma}{\sigma-1}\left[(1-\gamma)C^{1-\frac{1}{\sigma}}+\gamma E^{1-\frac{1}{\sigma}}\right]^{\frac{(1-\alpha)\sigma}{\sigma-1}-1}(1-\gamma)\left(1-\frac{1}{\sigma}\right)C^{-\frac{1}{\sigma}}$$

so

(A3) $$El_C U_C=\left(\frac{(1-\alpha)\sigma}{\sigma-1}-1\right)(1-\gamma^*)\left(1-\frac{1}{\sigma}\right)-\frac{1}{\sigma}=\left(\frac{1}{\sigma}-\alpha\right)(1-\gamma^*)-\frac{1}{\sigma}$$

Similarly, we find

(A4) $$El_E U_C=\left(\frac{1}{\sigma}-\alpha\right)\gamma^*$$

Inserting (A3) and (A4) into (A1) gives

(A5) $$\frac{-\dot{U}_C}{U_C}=\left[\left(\alpha-\frac{1}{\sigma}\right)(1-\gamma^*)+\frac{1}{\sigma}\right]g_C+\left(\alpha-\frac{1}{\sigma}\right)\gamma^* g_E$$

which may be rewritten as (10).

Appendix B: Derivation of results in Table 2

Derivatives of r

From (10) we find

$$\frac{\partial r}{\partial g_C}=(1-\gamma^*)\alpha+\gamma^*\frac{1}{\sigma}>0$$

$$\frac{\partial r}{\partial g_E}=\gamma^*\left(\alpha-\frac{1}{\sigma}\right)>0$$

which has the same sign as $\alpha\sigma - 1$.

$$\frac{\partial r}{\partial \alpha} = (1-\gamma^*)g_C + \gamma^* g_E$$

which is positive if both growth rates g_C and g_E are positive.

$$\frac{\partial r}{\partial \sigma} = -\frac{\gamma^*}{\sigma^2}(g_C - g_E)$$

which has the opposite sign of $g_C - g_E$.

Derivatives of R

From (12) we find

$$\frac{\partial R}{\partial g_C} = (1-\gamma^*)\left(\alpha - \frac{1}{\sigma}\right)$$

which has the same sign as $\alpha\sigma - 1$.

$$\frac{\partial R}{\partial g_E} = \gamma^* \alpha + (1-\gamma^*)\frac{1}{\sigma} > 0$$

$$\frac{\partial R}{\partial \alpha} = \frac{\partial r}{\partial \alpha}$$

which is positive if both growth rates g_C and g_E are positive (se above).

$$\frac{\partial R}{\partial \sigma} = \frac{(1-\gamma^*)^2}{\sigma^2}(g_C - g_E)$$

which has the same sign as $g_C - g_E$.

Derivatives of p

The derivatives of *p* with respect to g_C, g_E and α follow immediately from (11) and are not included in this Appendix.

Appendix C. Proof that long-run declining *U* implies $R-\rho<0$ in the long run

From (8) it follows that

$$\text{(C1)} \quad \frac{\frac{d}{dt}U(C,E)}{U(C,E)} = (1-\gamma^*)g_C + \gamma^* g_E$$

Clearly, if $g_C > 0$ and $g_E \geq 0$ utility will be increasing over time. The interesting case is where $g_C > 0$ and $g_E < 0$ i.e. the environmental quality is declining while consumption is growing. We are particularly interested in the possibility of *U* declining, and shall distinguish between the three cases of $\sigma > 1$, $\sigma < 1$ and $\sigma = 1$.

$\sigma > 1$

In this case γ^* approaches zero in the long run, so that (C1) implies that the utility level must be increasing in the long run (although *U* might decline in the short run if *E* is declining at a sufficiently high rate).

$\sigma < 1$

In this case γ^* approaches one in the long run, so that (C1) and $g_E < 0$ implies that the utility level must be declining in the long run. From (12) it follows that $R = \rho + \alpha g_E$ in the long run. For a sufficiently low utility discount rate ρ, the combined discount and price change rate must therefore be negative if the environmental quality is declining in the long run.

$\sigma = 1$

In this case γ^* is constant equal γ. For the utility level to be declining we see from (B1) that

$$\text{(C2)} \quad g_E < \frac{1-\gamma}{\gamma} g_C$$

So that (12) implies that

$$\text{(C3)} \quad R < \rho + \left[(1-\gamma)(\alpha-1)\right]g_C + \left[\gamma\alpha + (1-\gamma)\right]\left(-\frac{1-\gamma}{\gamma}\right)g_C$$

which may be rewritten as

(C4) $$R = \rho - \frac{1-\gamma}{\gamma} g_C < \rho$$

We thus have a similar result as for the case of $\sigma < 1$: For a sufficiently low utility discount rate ρ, the combined discount and price change rate must be negative if the long-run decline in the environmental quality is so strong that the utility level is declining.

Acknowledgments

Comments from Kjell-Arne Brekke, Reyer Gerlagh, Odd Godal, Richard Howarth, Åsa Löfgren and Snorre Kverndokk are highly appreciated. This article was written during our participation in the project "Environmental economics: policy instruments, technology development, and international cooperation" conducted at the Centre for Advanced Study (CAS) at the Norwegian Academy of Science and Letters in Oslo in 2005/2006. The financial, administrative and professional support of the Centre to this project is much appreciated. Thomas Sterner would also like to thank the Mistra CLIPORE program for support.

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[i] Important contributions include Arrow et al (1996), Cline (1992), Dasgupta (2001), Hasselmann (1999), Horowitz (1996), Heal (1997), Lind (1982), Nordhaus (1997), Portney and Weyant (1999), Schelling (1995), Shogren (2000), and Weitzman (1994, 1998).

[ii] http://www.bankofengland.co.uk/statistics/index.htm

[iii] See e.g. Nordhaus (1997), Hasselmann (1999), Vennemo (1997).

[iv] In this simple presentation we ignore all issues regarding distribution of consumption among different persons at any point of time.

[v] These utility functions are sometimes also referred to as constant relative risk aversion (CRRA) functions.

[vi] See, for instance, the impact assessments by IPCC Working Group II (the 2001 Assessment can be found at http://www.grida.no/climate/ipcc_tar/wg2/411.htm).

[vii] If σ =1 we get a Cobb Douglas function instead of (8): $U(C,E)=\frac{1}{1-\alpha}\left[C^{1-\gamma}E^{\gamma}\right]^{1-\alpha}$. If $\alpha=1$ we get $U(C,E)=\frac{\sigma}{\sigma-1}Ln\left[(1-\gamma)C^{1-\frac{1}{\sigma}}+\gamma E^{1-\frac{1}{\sigma}}\right]$ and if $\alpha=\sigma=1$ we get $U(C,E)=Ln\left[C^{1-\gamma}E^{\gamma}\right]$.

[viii] This could be illustrated if we consider for a moment *E* as a source of (or as equivalent to) "food and water". In an economy where all sectors grew but "food and water" declined, it is clear that the relative price of food and

water would increase so fast that the small amount of physical output in this sector soon assumed a share of close to 100% of total value in the economy.

[ix] See appendix A for the derivation.

[x] Notice that utility is declining in the long run in this case. The reason why we in spite of this get a non-negative value of R is that we have assumed that the utility rate of discount δ is positive.

[xi] The total result for our varying rates is compounded for 100 years and the corresponding fixed rate calculated.

These calculations were in this case carried out for a growth rate of 2.5% in C with E constant. $\rho=\gamma^*=0.1$.