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Habit Formation in the Environmental Quality: Dynamic Optimal Environmental Taxation

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Abstract

In this article we propose a model in which individuals experience habit formation in environmental quality. Further, a consumption good causes a negative external effect on the environment. The intertemporal utility of a benevolent social planner depends on consumption of two goods and the environment. Hence, the choice of the social planner is to maximize utility given the negative effect of the consumption good on the environment, taking into account that there is habit formation in environmental quality. Given a simple model we show that the level and time path of the optimal tax is affected by the assumption of habit formation; more specifically the stronger the habit, the higher the optimal level of environmental quality in steady state, and the faster the transition towards the steady state tax level. Furthermore the initial value of the habit stock is of crucial importance for the time path of the tax. Also, we solve for, and analyze, dynamics and the existence and characteristics of equilibria in the model and discuss how the characteristics of the habit formation affect steady state.

JEL classification: C61, D90

Keywords: Habit formation; environment; intertemporal choice; transitional dynamics.

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1 Introduction

It is not a controversial statement that the state of the environment (environmental quality) affects utility, but how and through which mechanisms, are subject to more discussion. Could it be that the utility derived from environmental quality is not only dependent on the current state of environmental quality, but rather on the difference between the current state of the environment and some experience of past states of environmental quality? If this is true, the question arises about how we should model the "experience of past states of environmental quality." We propose that one way to model this is to treat the "experience of past states of environmental quality" as a habit stock. Hence, the link between the utility derived from the current state of the environment and past states is habit formation. This implies that environment is a habitual "good." The definition of a habitual good, which we use throughout the paper, is summarized succinctly by Pollak (1970). The author defines a habit such that *(i)* past consumption influences current preferences and hence, current demand and *(ii)* a higher level of past consumption of a good implies, *ceteris paribus*, a higher level of present consumption of that good. The incorporation of habit formation in an environmental framework has not been well investigated, and in this article we aim at shedding some light on whether or not it is of importance to account for habit formation when dealing with environmental problems.

Our contribution to the literature is firstly that we model habit formation in a good that is of a public good character (environmental quality), and secondly that we analyze how this assumption affects an optimal environmental tax, the time-path of such a tax, and environmental quality in steady state. As is shown, the level and time path of the tax are affected by the assumption of habit formation: the stronger the habit, the higher the optimal quality of environment in steady state, and the faster the transition towards the steady state tax level. Furthermore, the initial value of the habit stock is of crucial importance for the time path of the tax. An initially low habit stock corresponds to a decreasing tax over time, and an initially high habit stock corresponds to an increase in tax over time.

Given that we have habit formation in environmental quality, we have intertemporally dependent preferences by assumption, and this is nothing new *per se*. A significant share of the literature on growth, for example, has over time dealt with optimal growth paths under different forms of utility functionals. Kurz (1968) develops a model of economic growth incorporating wealth effects. This is analogous with many of the growth models incorporating environment or pollution as an argument in the utility function (see for example Beltratti, 1996; Gradus and Smulders, 1993; and Smulders, 1995). The formal structure of such models is similar to the structure of the model proposed in this paper. The main difference between these models and the one presented in this article is the explicit modeling of habit formation, i.e. we include two parameters that reflects the degree of habit formation and focus on their effect on optimal environmental taxation. Also, there is a difference in the motivation behind the models. In this paper we argue that the utility derived from environmental quality is dependent not primarily on the current state of environmental quality, but rather on the differences of the current state of environmental quality compared to past states or levels of environmental quality. An individual builds up a habit stock that is positively dependent on past levels of environmental quality, and then the individual compares the current state of environmental quality to this habit stock. A deterioration of environmental quality gives a negative effect on the utility, while an increase in level of environmental quality gives a positive effect on the utility. It seems to be a reasonable assumption that people are not solely concerned about the current level of environmental quality; but rather compare it to historical levels. Think about a grandfather who tells his grandson what it was like when he was a child and was able to swim in the lake. The knowledge of the state of the environment deteriorates over time, since when the grandson becomes a grandfather he will have no knowledge of how it was to swim in the lake. We model this as deterioration of the habit stock, which is thoroughly explained in the Model section below.

In traditional economic modeling, preferences are assumed to be separable over time. Psychological findings continuously indicate that this is not true, and have been incorporated in several articles over the years (see for example Loewenstein and Elster, 1992; and Rabin, 1998, for extensive overviews). The literature on non-separable utility over time can be divided into two parts (Chaloupka, 1991). The first part is referred to as endogenous tastes or habit formation (see e.g. Gorman, 1967; Pollak, 1970, 1976; and Boyer, 1983). The other part consists of the research on "rational addiction" (Stigler and Becker, 1977; Becker and Murphy, 1988; Becker et al., 1994), which gives an explanation for the existence of addiction in an economic framework. In Phlips (1983), this distinction of research on changing preferences is referred to as "purely semantic." Phlips shows that rational addiction can be seen as a special case of a more general habit formation model.

The model in this paper is based on both the rational addiction model developed by Stigler and Becker (1977) and Becker and Murphy (1988), and the model developed by Pollak (1970) in which utility is not dependent on the consumption level *per se*, but rather on the change between past consumption and current consumption levels. The main difference between this paper and Becker and Murphy (1988) and Pollak (1970) is that we include a public good that displays habit formation (environmental quality), and not a private consumption good.[1] Furthermore, the mentioned public good is affected by a negative external effect.

The paper is organized in the following way. First we develop the model. Second, we solve for the problem of the social planner, where we characterize the equilibrium, discuss dynamics and solve for the steady state, and subsequently we solve for the decentralized problem. We graphically illustrate and analytically derive the optimal tax path and discuss the effect of habit formation before ending with concluding remarks.

2 The Model

We assume that the intertemporal utility of an individual depends on consumption of two different goods and environmental quality. The two consumption goods are substitutes, but one of the goods gives rise to a negative external effect on environmental quality (from now on we refer to this good as *the environmental bad*). The environmental quality is of a public good character, and so individuals do not take into account that their consumption of the environmental bad has a negative effect on the environment. We assume that a social planner implements a tax to correct for this behavior. Hence, the choice of the social planner is to maximize utility given the negative effect of the environmental bad, taking into account that there is habit formation in environmental

[1] Several studies take into account that different private goods (most studied is probably the consumption of cigarettes) display habit formation (see e.g. Chaloupka, 1991; and Becker et al., 1994).

quality.

To formalize the discussion above, the total discounted utility is given by a quadratic utility function:

$$\int_{t=0}^{\infty}(aN(t)+bX(t)+cZ(t)+dS(t)+AN(t)^2+ BX(t)^2+CZ(t)^2+DS(t)^2+2EX(t)N(t)+2GN(t)S(t)+ 2FX(t)S(t)+2HZ(t)X(t)+2IN(t)Z(t)+2JZ(t)S(t))e^{-\rho t}dt. \quad (1)$$

a, b, c, d, A, B, C, D, E, F, G, H, I, and J are constant parameters, and $N(t)$, $X(t)$, $Z(t)$, and $S(t)$ are variables that change over time. The variables are defined as follows:

$N(t)$ = Environment which displays habit formation.

$X(t)$ = The "dirty" consumption good (the environmental bad).

$Z(t)$ = The "clean" consumption good.

$S(t)$ = The habit stock related to the environment.

Without loosing the key features of the model, the following assumptions are made regarding the variables:

$$N(t)=n-\gamma X(t) \quad (2)$$

$$Z(t)=y-X(t) \quad (3)$$

$$\dot{S}(t) = \beta N(t) - \delta S(t). \tag{4}$$

Environmental quality (2) is defined as a flow variable, i.e. we do not consider environmental quality as a stock. This is because we want to be able to focus on the effect of habit formation, and therefore want to use the simplest possible model. The environment is affected negatively by consumption of the good X, with a parameter $0 < \gamma < 1$, that illustrates how bad the good is for the environment. We have a goods market equilibrium condition: (3). We assume an exogenously given income equal to y in each period and normalized prices to one, and assume that it is neither possible to save nor borrow. $\dot{S}$ is the change in S over time, and the habit stock increases with βN, $(0 < \beta < 1)$, where β is a "habit formation coefficient" and depreciates over time with a depreciation rate equal to δ $(0 < \delta < 1)$. This way of modelling the habit stock follows Becker and Murphy (1988), but is also a variation of the discrete model of Pollak (1970). Due to tractability of the analysis we leave out time when referring to a variable, hence $X = X(t)$, except when we want to make a specific point where the time dimension is of importance.

It should be noted that the choice of a quadratic utility function is made mainly for analytical convenience, and could be comparable to a linearization around a steady state (since taking derivatives of a quadratic function yields linear relationships between the variables).

We assume further that individuals do not take into account the negative effect of consuming X on environmental quality, i.e. the environment has a public good character. Subsequently the individual treats environmental quality as given $(\bar{N}(t))$ (and consequently take the habit stock as given). The corresponding utility function for an individual (a representative agent) is given by:

$$\int_{t=0}^{\infty}(a\bar{N}(t)+bX(t)+cZ(t)+dS(t)+A\bar{N}(t)^2+ BX(t)^2+CZ(t)^2+DS(t)^2+2EX(t)\bar{N}(t)+2G\bar{N}(t)S(t)+ 2FX(t)S(t)+2HZ(t)X(t)+2I\bar{N}(t)Z(t)+2JZ(t)S(t))e^{-\rho t}dt \quad (5)$$

The goods market equilibrium condition for the individual is given by:

$$Z(t)=y+v-\tau X(t) \quad (6)$$

Where $\tau = 1 + r$, is a tax on good X. Further, we assume that the tax revenue is returned to the individual via a lump sum tax, v.

Given the definition of the utility function the following must hold: $A = \frac{1}{2}u_{NN}$, $B = \frac{1}{2}u_{XX}$, $C = \frac{1}{2}u_{ZZ}$, $D = \frac{1}{2}u_{SS}$, $E = \frac{1}{2}u_{NX}$, $F = \frac{1}{2}u_{NS}$, $G = \frac{1}{2}u_{NX}$, $H = \frac{1}{2}u_{ZX}$, $I = \frac{1}{2}u_{NZ}$, and $J = \frac{1}{2}u_{ZS}$. To make the model as stringent as possible, we make some assumptions on the utility function and parameters, which hold throughout the paper: $u_X > 0$, $u_N > 0$, $u_Z > 0$, $u_S > 0$, $d = -a$, $A = \frac{1}{2}u_{NN} < 0$, $B = \frac{1}{2}u_{XX} < 0$, $C = \frac{1}{2}u_{ZZ} < 0$, $D = \frac{1}{2}u_{SS} < 0$, $E = \frac{1}{2}u_{NX} = 0$, $F = \frac{1}{2}u_{NS} = 0, H = \frac{1}{2}u_{ZX} = 0$, $I = \frac{1}{2}u_{NZ} = 0$, $J = \frac{1}{2}u_{ZS} = 0$, $G = \frac{1}{2}u_{NS} > 0$.

Special attention should be given to the assumptions $u_S > 0$, $d = -a$, and $G = \frac{1}{2}u_{NS} > 0$. The first of these three assumptions is an assumption that indicates that we have a *beneficial* habit.[2] This term is

[2] As can be shown, the choice of a benefical good puts a restriction on the relative size of A in relation to D (A must be smaller in magnitude than D), but since this doesn't alter the results or intuition of the results, we do not develop the restriction further in the paper. The calculations can be obtained from the author upon request.

used in the rational addiction literature, and refers to an addiction/habit that does not have any negative effects on the individual through the stock effect (not to be mixed up with the negative effect on environmental quality assumed in this paper). The opposite of a beneficial addiction is a *harmful* addiction, where the most cited example is smoking. The instant effect of smoking is positive, but the habit stock effect is negative. The second assumption indicates that individuals get utility not from environmental quality per se, but rather from a reference level which is dependent on earlier consumption. This follows the approach by Pollak (1970). The third assumption is necessary (but not sufficient, as is shown below) for a higher level of past environmental quality to imply, *ceteris paribus*, a higher level of present environmental quality, i.e. an increase in past levels of environmental quality increases the marginal utility of present level of environmental quality. Furthermore we assume that $N, Z, X > 0$.

We can now restate the utility functions given the assumptions of the parameters:

$$\int_{t=0}^{\infty} (a(N-S)+bX+cZ+AN^2+BX^2+CZ^2+DS^2+2GNS)e^{-\rho t}dt \quad (7)$$

$$\int_{t=0}^{\infty} (a(\bar{N}-S)+bX+cZ+A\bar{N}^2+BX^2+CZ^2+DS^2+2G\bar{N}S)e^{-\rho t}dt. \quad (8)$$

Now the basic set up of the model is finished, and what we are going to do next is solve the optimization problem of the social planner. This exercise aims at finding and characterizing the optimal paths of the model. We then turn to the question of optimal environmental taxation (the decentralized problem).

3 The Problem of the Social Planner

The problem for the social planner is to maximize (1) subject to (2), (3), and (4).

Given one state (S) and one control variable (X), we are clearly able to study transitional dynamics. To identify a possible optimal path we use the Maximum Principle of Pontryagin (introduced by Pontryagin, Boltyanskii, Gamkrelidze and Mishchenko (1964)). The subsequent presentation of and solution to the maximization problem follows Sierstad and Sydsaeter (1987), and Sydsaeter, Strom and Berck (2000) closely. The current value Hamiltonian for our problem is stated as follows (where we have substituted in for restrictions [2] and [3]):

$$H^c = a((n-\gamma X)-S)+bX+c(y-X)+A(n-\gamma X)^2+BX^2+ \\ C(y-X)^2+DS^2+2G(n-\gamma X)S+\lambda(\beta(n-\gamma X)-\delta S), \quad (9)$$

where λ is the current value shadow price. The analysis of optimality hinges upon the following conditions:

Mangasarian's Sufficient Conditions. Infinite Horizon.

Suppose that an admissible pair $(S^*(t), X^*(t))$ satisfies the following conditions for all $t \geq t_0$:

(1) $X^*(t)$ maximizes $H^c(t, S^*(t), X(t), \lambda(t))$

(2) $\dot{\lambda}(t) - \rho\lambda(t) = -\partial H^c(t, S^*(t), X^*(t), \lambda(t))/\partial S.$

(3) $H^c(t, S(t), X(t), \lambda(t))$ is concave w.r.t $(S(t), X(t))$.

(4) $\lim_{t\to\infty} [\lambda(t)e^{-\rho t}(S(t) - S^*(t))] \geq 0$ for all admissible $S(t)$.

Then $(S^*(t), X^*(t))$ is optimal.

Conditions (1) and (2) represent the Maximum Principle of Pontryagin, and are *necessary conditions* for optimality to hold. Conditions (3) and (4) are *sufficient conditions* for optimality, but we acknowledge that we could still find optimal solutions even if (3) and (4) do not hold (for a further discussion see Sierstad and Sydsaeter, 1977). We also impose the transversality condition $\lim_{t\to\infty} S(t) > 0$ (following from $N > 0$). It can be shown that the current value Hamiltonian is concave w.r.t habit stock and the consumption good X (Condition [3]) iff $D(B + C + A\gamma^2) > G^2\gamma^2$. Given Condition (1), an interior maximum of H^c requires that $\frac{\partial H^c}{\partial X} = 0$. Hence,

$$\frac{\partial H^c}{\partial X} = b - c + 2BX - 2C(y - X) - a\gamma - 2GS\gamma - 2A\gamma(n - \gamma X) - \beta\gamma\lambda = 0, \tag{10}$$

and solving for λ and X we find that

$$\lambda = \frac{b - c + 2BX - 2C(y - X) - a\gamma - 2GS\gamma - 2A\gamma(n - \gamma X)}{\beta\gamma} \tag{11}$$

$$X = \frac{-b + c + 2Cy + a\gamma + 2A\gamma n + 2GS\gamma + \beta\gamma\lambda}{2(B + C + A\gamma^2)}. \tag{12}$$

Hence λ is a decreasing function in X (note that $A, B, C < 0$). Using Condition (2), we can solve for $\dot{\lambda}$:

$$\frac{\partial H^c}{\partial S} = -a + 2DS + 2G(n - \gamma X) - \delta\lambda = \rho\lambda - \dot{\lambda} \Longrightarrow$$

$$\dot{\lambda} = (\rho + \delta)\lambda + a - 2DS - 2G(n - \gamma X). \tag{13}$$

If we take the time derivative of X given from Equation (12), we get:

$$\dot{X} = \frac{2G\dot{S}\gamma + \beta\gamma\dot{\lambda}}{2(B + C + A\gamma^2)}. \tag{14}$$

Since we know $\dot{\lambda}$ from Equation (13), λ from Equation (11), and $\dot{S}$ from Equation (4), we can substitute in for $\dot{\lambda}$, λ, and $\dot{S}$ in Equation (14), which means that $\dot{X}$ can be written as a function of X and S. Accordingly, the dynamic system can be summarized by two differential equations, $\dot{S}$ and $\dot{X}$. More specifically, the system that we are interested in solving is:

$$\begin{cases} \dot{X} = KS + (\delta + \rho)X + M \\ \dot{S} = \beta n - \beta\gamma X - \delta S, \end{cases} \tag{15}$$

where

$K = \frac{-2D\beta\gamma - 2G\gamma(2\delta+\rho)}{2(B+C+A\gamma^2)}$

$M = \frac{-a\gamma(\delta+\rho-\beta) - 2An\gamma(\delta+\rho) + b(\delta+\rho) - c(\delta+\rho) - 2Cy(\delta+\rho)}{2(B+C+A\gamma^2)}$.

The general solution to this system is given by:

$S(t) = S^* + R_1 e^{\mu_1 t} + R_2 e^{\mu_2 t}$

$X(t) = X^* + R_3 e^{\mu_1 t} + R_4 e^{\mu_2 t}$,

where R_i =constants, and μ_i =eigenvalues given from the system, and * refers to steady state values of S and X.

We now define the matrix Ω as

$$\Omega = \begin{bmatrix} (\delta + \rho) & (K) \\ (-\beta\gamma) & (-\delta) \end{bmatrix}.$$

The determinant is then equal to $\det \Omega = K\beta\gamma - \delta^2 - \delta\rho$.

The *trace* of Ω ($tr\Omega$) is equal to ρ, and the eigenvalues of the determinant Ω are

$\mu_{1,2} = \frac{1}{2}(\rho \pm \sqrt{\rho^2 + 4(-K\beta\gamma + \delta^2 + \delta\rho)})$.

If the determinant is negative, we know that we have two real, distinct, non-zero roots, while if the determinant is positive we can either have two real, distinct, non-zero roots ($\frac{(tr\Omega)^2 - 4\det\Omega}{4} > 0$), or we have that the eigenvalues are complex conjugates ($\frac{(tr\Omega)^2 - 4\det\Omega}{4} < 0$). As we proceed (see Section 3.2) we are able to show that $\det\Omega < 0$, given our assumption of habit formation, which in our case corresponds to a positive and a negative root, i.e.:

$\mu_1 = \frac{1}{2}(\rho - \sqrt{\rho^2 + 4(-K\beta\gamma + \delta^2 + \delta\rho)}) < 0$ and

$\mu_2 = \frac{1}{2}(\rho + \sqrt{\rho^2 + 4(-K\beta\gamma + \delta^2 + \delta\rho)}) > 0$.

3.1 Solution of the Model

Now we proceed with the solution to the system. Since $\mu_2 > 0$, we know that $R_2, R_4 = 0$ for $S(t)$ to tend asymptotically to S^*. Hence, the general solution of the system is reduced to

$$S(t) = S^* + R_1 e^{\mu_1 t}$$
$$X(t) = X^* + R_3 e^{\mu_1 t}.$$

To determine R_1 and R_3 we assume that we are in period $t = 0$. Then the initial value of $S(0) = S^* + R_1$, and hence $R_1 = S(0) - S^*$. Correspondingly $R_3 = X(0) - X^*$.

The solution of the system is then equal to:

$S(t) = S^* + (S(0) - S^*)e^{\frac{1}{2}(\rho - \sqrt{\rho^2 + 4(-K\beta\gamma + \delta^2 + \delta\rho)})t}$

$X(t) = X^* + (X(0) - X^*)e^{\frac{1}{2}(\rho - \sqrt{\rho^2 + 4(-K\beta\gamma + \delta^2 + \delta\rho)})t}$.

This means that if $S(0) < S^*$ ($S(0) > S^*$), then S grows (falls) over time towards S^*, and the same logic holds for X.

3.2 Phase-diagrams and Transitional Dynamics

A convenient property of the differential equations derived in Section 3 (15), is that they do not explicitly depend on time, and can therefore be used to illustrate the dynamics of the system. To be more specific: we have an autonomous system when time does not enter explicitly into a system of equations. This means that the time-derivatives do not change over time, but the solutions for S and X respectively *do* depend on time. Also, given some uniquely defined initial condition $(S(0), X(0))$, there is a corresponding solution curve (path or trajectory), $(S(t), X(t))$, that is unique for the given initial condition. As t varies over time, the system moves along a trajectory that depends only on the coordinates (S, X), and not on the time of arrival at that point (Shone, 1997). In a phase-diagram, the direction along the trajectory as time increases is illustrated by arrows. Shone lists three important properties for trajectories of autonomous systems which are important to bear in mind:

1. There is no more than one trajectory through any point in the phase plane.
2. A trajectory that starts at a point which is not a fixed point will only reach a fixed point in an infinite time period.
3. No trajectory can cross itself unless it is a closed curve. If it is a closed curve then the solution is a periodic one.

A fixed point, which in the economic literature is referred to as a steady state, is defined as a point where neither $S(t)$ nor $X(t)$ changes over time. To find the steady states for our model we need to solve for $\dot{S}(t) = 0$, and $\dot{X}(t) = 0$.

We have analytically solved for the dynamics in the model through the determinant and the eigenvalues, but for a further discussion it is illuminating to illustrate the discussion graphically using a phase-diagram. A phase-diagram is used to find out what happens if we deviate from the curves $\dot{S}(t) = 0$ and $\dot{X}(t) = 0$. If we are on the $\dot{S} = 0$ curve, S is constant. Then what happens to $\dot{S}$, and hence to S, if we go to the right (increasing X) or left (decreasing X) of the $\dot{S} = 0$ line? The same applies for the $\dot{X} = 0$ curve, for which we want to find out what happens to $\dot{X}$ if we are below (decreasing S) or above (increasing S) the $\dot{X}$ curve. Firstly we need to solve for $\dot{S}(t) = 0$, and $\dot{X}(t) = 0$.

If $\dot{S}(t) = 0$, then S is given by:

$$S = \frac{\beta(n - \gamma X)}{\delta}. \tag{16}$$

If $\dot{X}(t) = 0$, then S is given by:

$$S = \frac{M}{K} + \frac{\delta + \rho}{K} X. \tag{17}$$

Before we proceed, let us think about these two equations for a short moment. The second equation is crucial, since in this expression we define N (it is a function of X) either to be a complement or a substitute to S or to be unrelated over time. Given the quadratic utility function, we by assumption have a linear relationship between the habit stock and environmental quality. The relationship is either positive (past levels of environmental quality have a positive effect on the present level of environmental quality), negative (past levels of environmental quality have a negative effect on the present level of environmental quality), or the coefficient is equal to zero (past levels of environmental quality has no effect on the present level of environmental quality - independence). From the definition of habit formation (given by Pollak, 1970), we must have a negative relationship between the habit stock and the environmental bad (since this corresponds to a positive relationship between the habit stock and environmental quality), otherwise a higher level of past environmental quality would not imply a higher level of present environmental quality. Hence, for an increase in current level of environmental quality to increase future levels of environmental quality indicates, not only that $G > 0$, but also that we should have a positive relationship between the habit stock and consumption of the habitual good.[3] Since N depends negatively on the consumption of X it follows that to have habit formation we are going to make the crucial assumption that

$$\frac{\delta + \rho}{K} < 0.$$

[3]It should be noted that this is closely linked to the concept of adjacent complementarity. The distinction between adjacent and distant complementarity was developed by Ryder and Heal (1973). These two concepts are not trivial, but to quote the authors:

> "...a person with distant complementarity who expects to receive a heavy supper would tend to eat a substantial breakfast and a light lunch. A person with adjacent complementarity would tend to eat a light breakfast and a substantial lunch in the same circumstances." (Ryder and Heal, 1973, page 5).

In our model we assume a quadratic utility function, which implies that the habitual good and the habit stock can only have a strictly positive, strictly negative or unrelated relationship. But since we assume habit formation we must have a positive relationship, and hence by definition we have adjacent complementarity. Further, this implies that we disregard the possibility of distant complementarity. A more general habit formation model could allow for periods of decreasing consumption, following a period of increased consumption in the good that is habitual, but this is not possible in this setting given the functional form of the utility function.

From this we can conclude that $K < 0$, and hence that the $\det \Omega < 0$, since $\det \Omega = K\beta\gamma - \delta^2 - \delta\rho$. Given the assumption of a positive relationship between N and S we disregard the possibility of a high rate of time preference, since this would make $K > 0$, and possibly the $\det \Omega > 0$ (which could result in an unstable equilibrium).

The corresponding phase diagram is presented in Figure 1.

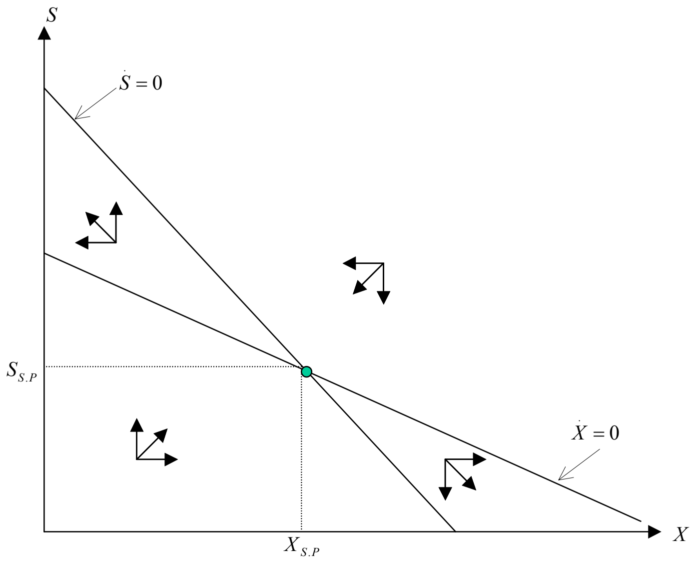

Figure 1. Phase-diagram in S and X-space.

The phase-diagram graphically illustrates the dynamics of the system derived in Section 3.1. Following the maximum principle, not all possible trajectories of the system are optimal. In Figure 2 below we have taken two examples of different $S(0)$: one high and one low. If we assume that the initial value of S is high, then the optimal trajectory hinges upon the choice of the control variable X at $t = 0$. Path I illustrates an optimal choice of $X(0)$, according to the maximum principle. The same argumentation can be made for the case when we assume a low initial value of $S(0)$. Choosing a high $X(0)$ would in this case result in the optimal path such as II. Also, the optimal paths fulfill the sufficient condition for optimality to hold (Theorem 1, Condition 4), i.e. $\lim_{t\to\infty}[\lambda(t)e^{-\rho t}(S(t)-S^{*}(t))]\geq 0$ for all admissible $S(t)$.

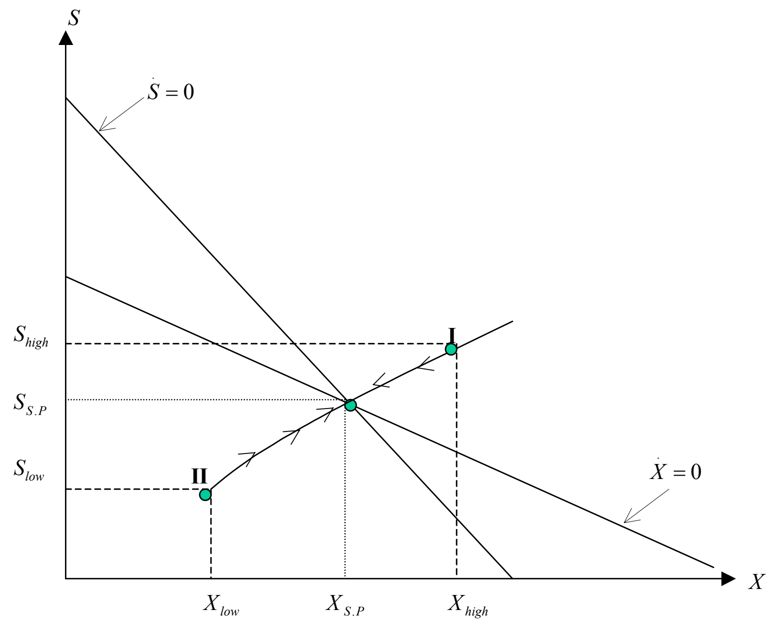

Figure 2. Optimal trajectories given initial condition on the habit stock.

It is worth noting that if we relax the assumption that $X>0$, and assume that $X\geqslant 0$, we could have a boundary solution where the $\dot{S}=0-curve$ cuts the $S-axis$. Hence, the optimal consumption of the environmental bad is equal to zero. The optimality of such a solution hinges upon whether the welfare over time is higher on such a path compared to a path towards the steady state discussed earlier. This is not a trivial comparison and is left for future research (for a discussion on comparisons of multiple steady states, see for example Deissenberg, Feichtinger, Semmler, and Wirl, 2001). We disregard this solution in this study, but acknowledge that such a solution could still exist, and would depend partly on the substitutability between goods X and Z. For example, in the case of gasoline where leaded and unleaded gasoline have been shown to be almost perfect substitutes, we could imagine that this solution would be preferred to a solution with consumption of both leaded and unleaded gasoline. A solution where the $\dot{X} = 0 - curve$ cuts the $X - axis$ $(S = 0)$ is impossible, since it violates the transversality condition $\lim_{t\to\infty} S(t) > 0$.

This rather elementary exercise points at the importance of the initial value of S, the habit stock of past environmental quality.

3.3 Steady State Consumption and Comparative Statics

Given what we have found so far we can solve for the optimal steady state consumption of X. In steady state we know that (16) must equal (17), and solving for X yields the following steady state consumption of X:

$$X^*_{S.P} = \frac{\frac{\beta n}{\delta} - \frac{M}{K}}{\frac{\delta+\rho}{K} + \frac{\beta\gamma}{\delta}}. \tag{18}$$

Solving correspondingly for S, N, and Z is straightforward:
$\dot{S}(t) = \beta N(t) - \delta S(t) = 0$ in steady state $\Rightarrow S(t) = \frac{\beta N(t)}{\delta} \Rightarrow$

$$S^*_{S.P} = \frac{\beta}{\delta}\left(n - \gamma\frac{\frac{\beta n}{\delta} - \frac{M}{K}}{\frac{\delta+\rho}{K} + \frac{\beta\gamma}{\delta}}\right),$$

$N(t) = n - \gamma X(t) \Rightarrow$

$$N^*_{S.P} = n - \gamma\frac{\frac{\beta n}{\delta} - \frac{M}{K}}{\frac{\delta+\rho}{K} + \frac{\beta\gamma}{\delta}},$$

$Z(t) = y - X(t) \Rightarrow$

$$Z^*_{S.P} = y - \frac{\frac{\beta n}{\delta} - \frac{M}{K}}{\frac{\delta+\rho}{K} + \frac{\beta\gamma}{\delta}}.$$

Since the focus of this paper is on habit formation, we would like to investigate the effect on steady state X $(X^*_{S.P})$ given a change in either G or β. We start by examining the effect of an increase/decrease in G (an increase in past levels of environmental quality increases the marginal utility of the present level of environmental quality) on optimal consumption of X.

Substituting in for M and K in $X^*_{S.P}$ yields the following expression:

$X^*_{S.P} =$
$\frac{2Dn\beta^2\gamma+2Gn\beta\gamma(2\delta+\rho)-b(\delta^2+\delta\rho)+c(\delta^2+\delta\rho)+2Cy(\delta^2+\delta\rho)+a\gamma\delta(-\beta+\delta+\rho)+2An\gamma(\delta^2+\delta\rho)}{2(D\beta^2\gamma^2+(B+C+A\gamma^2)(\delta^2+\delta\rho)+G\beta\gamma^2(2\delta+\rho))} =$
$\frac{\Theta}{2\Psi}$,

where $\Theta = 2Dn\beta^2\gamma + 2Gn\beta\gamma(2\delta+\rho) - b(\delta^2+\delta\rho) + c(\delta^2+\delta\rho) + 2Cy(\delta^2+\delta\rho) + a\gamma\delta(-\beta+\delta+\rho) + 2An\gamma(\delta^2+\delta\rho)$ and

$\Psi = D\beta^2\gamma^2 + (B+C+A\gamma^2)(\delta^2+\delta\rho) + G\beta\gamma^2(2\delta+\rho)$.

Also $\Theta, \Psi < 0$ (following from the sign of the determinant). Then we can write the derivative of $X^*_{S.P}$ with respect to G as:

$\frac{\partial X^*_{S.P}}{\delta G} = \frac{-\beta\gamma^2(2\delta+\rho)\Theta}{2\Psi^2} + \frac{4n\beta\gamma\delta+2n\beta\gamma\rho}{2\Psi}$.

We know that $\frac{-\beta\gamma^2(2\delta+\rho)\Theta}{2\Psi^2} > 0$, and $\frac{4n\beta\gamma\delta+2n\beta\gamma\rho}{2\Psi} < 0$. Hence, if $\frac{-\beta\gamma^2(2\delta+\rho)\Theta}{2\Psi^2} + \frac{4n\beta\gamma\delta+2n\beta\gamma\rho}{2\Psi} > 0$, then $\frac{\partial X^*_{S.P}}{\delta G} > 0$.

If we assume that $\frac{-\beta\gamma^2(2\delta+\rho)\Theta}{2\Psi^2} + \frac{4n\beta\gamma\delta+2n\beta\gamma\rho}{2\Psi} > 0$, then after some simplifications it follows that

$\frac{\Theta}{2\Psi}(= X^*_{S.P}) > \frac{n}{\gamma}$. But since this implies that $N < 0$ (remember that $N = n - \gamma X$), this cannot be true. Hence,

$\frac{\partial X^*_{S.P}}{\delta G} = \frac{-\beta\gamma^2(2\delta+\rho)\Theta}{2\Psi^2} + \frac{4n\beta\gamma\delta+2n\beta\gamma\rho}{2\Psi} < 0$.

The more habitual environmental quality the lower the optimal steady state consumption of the environmental bad (X). This is what we expect.

We now turn to the habit formation coefficient β. Using the same approach as above, we can write $\frac{\partial X^*_{S.P}}{\delta\beta}$ as:

$$\frac{\partial X^*_{S.P}}{\delta\beta} = -\frac{(2D\beta\gamma^2+G\gamma^2(2\delta+\rho))\Theta}{2\Psi^2} + \frac{4Dn\beta\gamma-a\gamma\delta+2Gn\gamma(2\delta+\rho)}{2\Psi}.$$

Hence $\frac{\partial X^*_{S.P}}{\delta\beta} \gtrless 0$ dependent on if

$$-\frac{(2D\beta\gamma^2+G\gamma^2(2\delta+\rho))\Theta}{2\Psi^2} + \frac{4Dn\beta\gamma-a\gamma\delta+2Gn\gamma(2\delta+\rho)}{2\Psi} \gtrless 0.$$

Going through some simplifications yields the following restrictions:

$\frac{\partial X^*_{S.P}}{\delta\beta} > 0$ *iff* $\frac{\Theta}{2\Psi} > \frac{n}{\gamma} - \frac{a\delta}{2\gamma(2D\beta+G(2\delta+\rho))}$ and

$\frac{\partial X^*_{S.P}}{\delta\beta} < 0$ *iff* $\frac{\Theta}{2\Psi} < \frac{n}{\gamma} - \frac{a\delta}{2\gamma(2D\beta+G(2\delta+\rho))}$.

Hence, if $2D\beta + G(2\delta+\rho) < 0$ then $\frac{\partial X^*_{S.P}}{\delta\beta} < 0$ (given the same argumentation as for $\frac{\partial X^*_{S.P}}{\delta G}$).[4]

If $2D\beta + G(2\delta+\rho) > 0$ then $\begin{cases} \text{if } X^*_{S.P} \text{ large } (N \text{ small}) \ \frac{\partial X^*_{S.P}}{\delta\beta} > 0 \\ \text{if } X^*_{S.P} \text{ small } (N \text{ large}) \ \frac{\partial X^*_{S.P}}{\delta\beta} < 0. \end{cases}$

We can conclude that the sign of $\frac{\partial X^*_{S.P}}{\delta\beta}$ hinges upon several parameters, and their relative size. But, if we face a situation where we have a large optimal steady state consumption of X, and $2D\beta+G(2\delta+\rho) > 0$, then a larger habit formation coefficient (i.e. that we increase the habit stock more for every given level of N) would indicate that the optimal level of steady state X increases. When $2D\beta + G(2\delta+\rho) < 0$ or $2D\beta + G(2\delta+\rho) > 0$ in combination with a small steady state consumption level of X, an increase in the habit formation coefficient would indicate a reduction of the optimal steady state consumption of X.

[4] From the positive relationship between N and S we know that $D\beta+G(2\delta+\rho) > 0$, but we do not know the sign of $2D\beta + G(2\delta+\rho)$.

4 The Decentralized Problem

4.1 The Problem of the Individual

Now we turn to the maximization problem of the individual. The individual consumption of X is *independent of time*, due to the assumption that individuals do not take into account that their consumption of X affects environmental quality, and hence affects the habit stock. The individual chooses to consume the exact same amount of X in every period. This is interesting from an analytical point of view since it allows us to exactly determine the start value of X and S, and given the initial conditions, the social planner needs to find the optimal path towards steady state, i.e. the optimal path of the environmental tax. The tax is equal to the Pigovian tax[5] (see e.g. Gruber and Köszegi, 2001; and Löfgren, 2002), but it is of interest to examine how it optimally should evolve over time.

The corresponding maximization problem for the individual is written as (note that the individual does not take into account the negative effect of the consumption of X on the environment and therefore we can treat the individual problem as a static problem):[6]

$$U^{ind} = a(\bar{N} - S) + bX + cZ + dS + A\bar{N}^2 + BX^2 + CZ^2 + DS^2 + 2G\bar{N}S.$$

Taking the derivative of U^{ind} with respect to X yields the following steady state consumption of X for the individual, when no tax is imposed:

[5] The tax is equal to the shadow price or marginal damage of the externality (Pigou, 1946).

[6] For the chosen parameters, the utility function can be shown to be concave with respect to X, as we would require.

$$X_{ind} = \frac{-b + c + 2C(v + y)}{2(B + C)}.$$

When we impose a tax ($\tau = 1 + r$) on the consumption of good X, the first order condition for the individual corresponds to

$$X_{ind} = \frac{-b + c\tau + 2C\tau(v + y)}{2(B + C\tau^2)}$$

and hence consumption of X strictly decreases in τ.[7]

4.2 Optimal Taxation

Again our problem can be illustrated by a phase-diagram. Given that the individual consumption of X is lower than the optimal consumption of X, we know that environmental quality is worse than optimal. But we can only speculate whether or not the initial value of S is lower or higher than the optimal S. The initial value on the habit stock crucially depends on time, i.e. how long time individuals have consumed the environmental bad. What we do know is that we have a problem of optimal environmental taxation including two different dynamics and three different initial taxes (see Figure 3 below). Hence, to illustrate what the implementation of an optimal tax would look like, we refer to three different levels on the initial habit stock (i, ii, and iii). These three levels all have different implications on the time-path of the optimal tax.

[7] The derivative of X_{ind} with respect to τ, $\frac{\partial X_{ind}}{\partial \tau} = \frac{B(c+2C(v+y))-C\tau(-2b+\tau(c+2C(v+y))}{2(B+C\tau^2)^2}$. If we assume an interior solution then $B < C\tau^2$ must hold, and then the derivative can be shown to be negative. Hence, X strictly decreases in τ (assuming an interior solution).

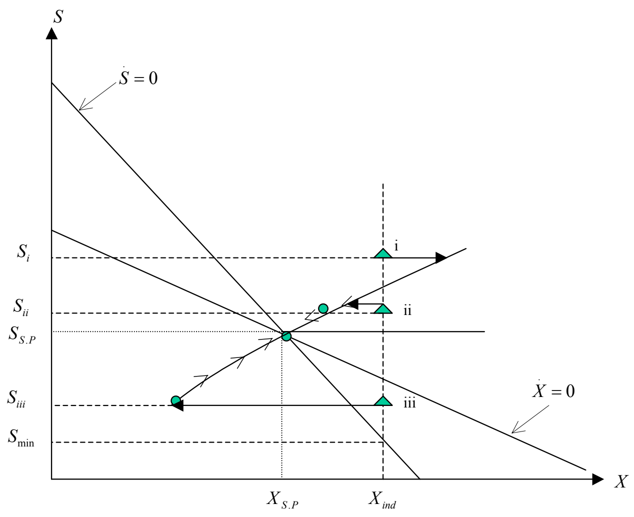

Figure 3. Optimal environmental tax path given initial conditions on the habit stock.

The arrows in Figure 3 represent the shift in consumption going from the individual's maximization problem to the social planner's maximization problem. From a visual inspection of Figure 3 we can conclude that the more the habit stock degenerates over time, the more likely the social planner is to face a situation such as (iii). Since we have shown that the consumption of X for the individual strictly decreases in tax, we find that given a habit stock *lower* than the optimal habit stock, the optimal response from a social planner would be to implement a tax (corresponding to a decrease in X), and then decrease the tax level over time (increase X) until steady state is reached. For a given habit stock *higher* than the optimal habit stock, the optimal response from a social planner would be either of two options: (a) situation (ii) corresponds to implementing a tax (decrease in X), and then increase the tax level over time (decrease X) until steady state is reached, or (b), situation (i) corresponds to implementing a *subsidy* initially, and then an increasing tax (level) until steady state is reached. Note that the habit stock can only be degenerated to $S_{\min}$ and that this point would correspond to an individual steady state, where both the habit stock and consumption of X are constant. Hence, following from Figure 3 we know that $\tau(0) > \tau^*$ if $S(0) < S^*$, and $\tau(0) < \tau^*$ if $S(0) > S^*$, and could even be a subsidy ($\tau(0) < 1$). Furthermore, $\dot{\tau} < 0$ if $S(0) < S^*$, and $\dot{\tau} > 0$ if $S(0) > S^*$.

We now formalize our discussion on the time-path of an optimal environmental tax. For optimality to hold, the social planner should set the tax so that the optimal path of X is equal to the individual's choice of X at every point in time.

The optimal path of X is given by $X(t) = X^* + (X(0) - X^*)e^{\mu_1 t}$,where $\mu_1 = \frac{1}{2}(\rho - \sqrt{\rho^2 + 4(-K\beta\gamma + \delta^2 + \delta\rho)})$. The social planner should set the optimal environmental tax so that $X_{ind} = X^* + (X(0) - X^*)e^{\mu_1 t} \Rightarrow$

$\frac{-b+c\tau+2C\tau(v+y)}{2(B+C\tau^2)} = X^* + (X(0) - X^*)e^{\mu_1 t}$. Solving for τ yields the following optimal tax path:

$$\begin{aligned}\tau(t) = {} & \frac{1}{2(2Ce^{\mu_1 t}(X^0 - X^*) + 2CX^*)} \\ & [c + 2C(y+v) - ((-c - 2C(y+v))^2 \\ & -4(b + 2BX^* + 2Be^{\mu_1 t}(X^0 - X^*))(2CX^* + 2Ce^{\mu_1 t}(X^0 - X^*)))^{\frac{1}{2}}]\end{aligned} \tag{19}$$

We have already shown the effect of G (remember that $G = \frac{1}{2}u_{NX}$, which should be interpreted as: an increase in past levels of environmental quality increases the marginal utility of the present level of environmental quality) on steady state consumption of X. It was shown that $\frac{\partial X^*}{\partial G} < 0$. This indicates that the optimal tax path is affected by G in a non-trivial way (both through X^* and μ_1). It is straightforward to show that $\frac{\partial \mu_1}{\partial G} > 0$, which can be interpreted as an effect on the transition speed towards the steady state tax. The larger the G, the larger (i.e. the less negative) is μ_1, the longer time it takes for the tax to approach the steady state tax (compared in levels). The same holds for the habit formation coefficient, β.

From Equation (19) we note the following:

The steady state tax ($t \to \infty$) and initial tax ($t = 0$) have the same "form":

$\tau(0) = \frac{1}{4CX(0)}[c + 2C(y+v) -$
$\sqrt{(-c - 2C(y+v))^2 - 8(b + 2BX(0))(CX(0))}]$

and

$\tau^* = \frac{1}{4CX^*}[c + 2C(y+v) -$
$\sqrt{(-c - 2C(y+v))^2 - 8(b + 2BX^*)(CX^*)}]$.

Hence, we find that the initial tax and steady state tax depends on habit formation through the optimal choice of X at time $t = 0$ (there is only one optimal choice of X given the initial value on the habit stock) and through steady state consumption of X.

It can also be shown that the optimal environmental tax in *steady state* is equal to the optimal environmental tax in a static case, i.e. disregarding the dynamic property of the habit stock.

5 Concluding Remarks

Habit formation has not, to our knowledge, been studied in an environmental framework before, and in this paper, using a simple model, we show that habit formation crucially affects optimal environmental taxation in a dynamic setting.

Even if we know that an environmental bad should be taxed using a tax equal to the Pigovian tax, it is of interest to study how such a tax evolves over time, and also how optimal consumption levels are affected by the assumption of habit formation. We show that when individuals experience habit formation in environmental quality, the time-path of the optimal tax critically hinges upon the initial value of the habit stock. There are three alternative paths: one corresponding to an increasing tax over time and two corresponding to a decreasing tax over time, with the initial tax being either a tax or a subsidy. The initial value of the habit stock depends on how long individuals have consumed the environmental bad. The longer the time, the lower the corresponding habit stock. Furthermore, habit formation affects the optimal tax path both in level and transition speed towards steady state levels. The transition speed towards the steady state levels is affected negatively by an increase in habit formation, while the level of the optimal tax is affected non-trivially by changes in habit formation. Also, the optimal consumption of the environmental bad decreases in habit formation through G. Hence, the stronger the habit, the higher the optimal level of environmental quality.

It should also be noted that the dynamics are highly dependent on two factors: firstly the concavity of the utility function with respect to the habit stock, and secondly how strong the habit formation is. The more diminishing the utility with respect to the habit stock, and the stronger the habit, the more likely that the model is unstable. We disregard the case of such strong habits and very concave utility function with respect to the habit stock, but acknowledge the explanatory power it has for addiction.

An evident extension of this paper is to incorporate habit formation in a growth-environment framework. In an article by Shieh et al. (2000), addiction and growth are studied, and the authors find that addiction has an effect on the steady state growth rate (the effect depends on the properties of the addiction), but the authors do not specifically consider environment, nor taxation. Another area for future research is to empirically test for the character of habit formation in environmental quality, i.e. what determines the reference point, and how persistent the habit is.

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