Does the Black-Scholes formula work for electricity markets? A nonparametric approach

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Working Papers in Economics no 101, July 2003

Abstract

Despite the high volatilities recorded for electricity prices, there seems to be little demand for options on electricity. One reason for the disinterest in electricity options could arise from uncertainty about how to price these options. This study uses recent econometric advances to nonparametrically estimate correct prices for electricity options and compare these to the Black-Scholes prices. The main finding is that although the nonparametric estimates deviate significantly from the Black-Scholes prices, it would be difficult to find an alternative parametric model that performs better. Thus, from a practical viewpoint, the Black-Scholes prices appear to be the best available.

JEL Classification: C14, C22, G13, L94

Keywords: Electricity markets, Nonparametric estimation, Option pricing

*The suggestions and comments provided by Peter C.B. Phillips, Andrew Jeffrey and Lennart Hjalmarsson have been very helpful. Jan Foyn and the other Nord Pool staff have also been very accommodating. Financial aid provided by Göteborg Energi AB's Foundation for Research and Development is gratefully acknowledged.

1 Introduction

The erratic behaviour of electricity spot prices on deregulated power markets has caught the attention of both researchers and market practitioners. During the last few years, financial markets trading derivative products on electricity have also emerged. Given the extremely high volatilities recorded for deregulated electricity prices (e.g. Johnson and Barz, 1999), one might expect a high demand for financial products to hedge electricity contracts. However, while the futures and forwards markets for electricity seem to perform reasonably well, at least on some electricity exchanges, the markets for options have so far not worked very well. There are hardly any electricity options actually traded despite the fact that they are readily available for those who are interested. This may seem somewhat surprising since options are the hedging instruments *par excellence* and deregulated electricity prices are among the most volatile prices ever observed.[1] Whether this failure of the options markets is due simply to a lack of interest in options by electricity traders or because of a distrust in the options markets themselves is hard to say. The distrust for the market could stem from the obvious liquidity problems of these instruments or from a suspicion of mispricing. Since the financial markets related to electricity do differ from traditional financial markets in certain important aspects, traders might be uneasy about how to price electricity options and lead them not to trade in them.

The aim of this paper is to help remedy this part of the problem by providing an empirical analysis on the pricing of electricity options. Since there are no reliable option prices available, the most dependable way to analyze option pricing on electricity contracts is to estimate models for the underlying assets and, from these, derive the corresponding option prices. Given that many of the existing options on electricity contracts are, in fact, options on electricity

[1] For example, Johnson and Barz (1999) report annualized volatilities in the range from 120% to 2600% for different electricity markets.

forwards rather than on the actual spot price, this involves modelling both electricity spot and forward prices.[2]

Theoretical models of electricity spot prices have received some attention in the literature and various combinations of stochastic volatility and jump processes have been proposed to model the highly volatile behaviour of electricity prices; e.g. Deng (1999), Johnson and Barz (1999), and Kamat and Oren (2000). Baker et. al. (1998) and Duffie et. al. (1999) also look at this modelling problem from the wider perspective of energy prices in general. Relatively little attention, however, has been directed to empirically testing any stochastic models of electricity prices. Knittel and Roberts (2001) try to identify the salient features of electricity spot prices based on hourly data from the Californian electricity market. Their findings show that traditional linear models are not capable of replicating the paths of spot prices. In addition, including jumps or a time-varying mean does not improve the forecast ability very much. Johnson and Barz (1999) find that mean reverting models with jumps fit the data better than non-mean reverting models, such as the geometric Brownian motion. There has been very little work done with regards to empirical examination of electricity forward prices. Bessembinder and Lemmon (2002) analyze forward prices in the context of an equilibrium pricing model and show that electricity forward prices are typically not unbiased estimators of the future spot price, due to the non-storability characteristic of electricity.[3] However, no serious attempt at modelling the dynamics of electricity forward prices has been done before. Given the findings of Knittel and Roberts (2001) in particular, and the lack of prior evidence on the dynamics of electricity forward prices, I will impose very few restrictions on the continuous-time models that I estimate in this study and use a nonparametric estimation approach that is robust to nonlinear behavior

[2] For instance, all officially traded options on Nord Pool, the Nordic power exchange, are on forward contracts.
[3] There are, of course, some ways to store electricity over limited amounts of time, such as batteries and hydro power dams.

in the data.

The main purpose of this paper is therefore to estimate nonparametric continuous time models for the spot price of electricity and various forward contract prices, using daily price data from the Nordic electricity exchange, Nord Pool, and to use these models to price electricity options. Since options are only traded on the forward contracts, and given the problems of pricing options on electricity spot prices due to the non-storability of electricity, I will only derive option prices on the forward contracts. The results from the models of the spot price are still of interest per se, as a comparison to the results from the forward markets.

In order to keep the analysis as general and robust as possible, a fully nonparametric estimation method is used.[4] This method, which is due to Bandi and Phillips (2001a), enables nonparametric estimation of both the drift and the diffusion component in a general diffusion model and allows for nonstationarity of the data. The estimation method also allows for the diffusion and drift components to be estimated independently of each other, treating the other function as a nuisance parameter. This is useful in the option pricing context considered here, since we are then often mainly interested in estimates of the volatility function. Furthermore, for the spot price specifications, the possibility of jumps as well as a form of stochastic volatility, where volatility is a function of temperature, is incorporated. The models thus estimated are compared to estimates of traditional parametric models. Specifically, I consider the geometric Brownian motion and both arithmetic and geometric mean reverting diffusions, with the allowance for jumps in the spot price case.

The original contributions of this study are thus to use nonparametric methods to estimate continuous time models of both spot and forward electricity prices, and to use the estimated models to price options on electricity. The empirical results show that in the context

[4] The difficulty of correctly parameterizing a diffusion model is well illustrated in Aït-Sahalia (1996) where he shows how many of the popular diffusion models for the spot interest rate process do not fit the data very well.

of univariate diffusion models, the volatility functions for both spot and forward prices exhibit nonlinear behaviours. The volatility functions for the different forward contracts behave in a fairly monotone fashion and might possibly be approximated by linear functions. The estimates of the volatility functions resulting from fitting either a geometric Brownian motion or an Ornstein-Uhlenbeck process to the forward price data provide a reasonable approximation to the nonparametric estimate.

The volatility function for the spot prices exhibits a non-monotonic behaviour, that may or may not be well approximated by a constant volatility function. I also fit a bivariate diffusion model of the spot prices and a measure of temperature in Nord Pool's market region. The results from this model show some nonlinearities in the diffusion function which could be large enough to make the use of linear models doubtful. Further, a clear relationship between temperature and the spot price is captured in the covariance function.

The option prices derived corroborate the results described above. The Black-Scholes prices (which are based on the assumption that the underlying price follows a geometric Brownian motion) are closer to the option prices based on the nonparametric estimates than the option prices obtained from the estimated Ornstein-Uhlenbeck process. Although the results are mixed, there is evidence that the Black-Scholes prices are significantly, in both a statistical and economical sense, different from the option prices resulting from the nonparametric models. Thus, there is some evidence that the Black-Scholes assumption of prices following a geometric Brownian motion might not be adequate when pricing options on Nord Pool. However, given the evidence from the nonparametric estimations, it would be difficult to conceive of a reasonable parametric model that gives a better fit than the geometric Brownian motion. Therefore, from a practical viewpoint, the Black-Scholes formula might still be the best available.

The rest of this paper is organized as follows. Section 2 presents models for electricity prices. Section 3 describes the econometric methods employed and section 4 gives the results from the estimations. In section 5, I calculate option prices and section 6 concludes. The appendices contain descriptions of the temperature data, bandwidth selection for the nonparametric estimators, a small Monte Carlo study on the accuracy of the nonparametric estimators employed in this paper, and a derivation of the option pricing formula for forward contracts.

2 Models for electricity prices

2.1 Univariate models

Since the main goal of this study is to price electricity options, it is natural to specify the models for electricity prices as continuous time diffusion processes, in line with the general literature on option pricing. The simplest specification fitting into this framework would be a univariate diffusion process of electricity prices, with the possibility of jumps added to account for the often occurring large movements in electricity spot prices. Thus our first model is:

$$dS_t = \mu_S(S_t)dt + \sigma_S(S_t)dB_t + dJ_t, \tag{1}$$

where S_t is the spot price of electricity, B_t is a standard Brownian motion, and J_t is an independent jump process. The jumps occur with intensity $\lambda_S(S_t)$ and each jump is independently normally distributed with mean zero and variance σ_y^2. The only restrictions imposed on the functions $\mu_S(\cdot)$ and $\sigma_S(\cdot)$, are that they are both time-homegenous functions and that equation (1) satisfies the regularity conditions detailed in Bandi and Nguyen (2001). Stationarity is not assumed; rather, the identifying assumption will instead be recurrence, a substantially weaker requirement.

The same model as in (1) will be used to model the forward prices of electricity, with the modification that the jump process, J_t, is removed since there are no observable jumps in the forward prices.[5] The non-parametric model of forward prices that will be estimated is thus:

$$dF_t = \mu_F(F_t)dt + \sigma_F(F_t)dB_t, \tag{2}$$

where F_t is the forward price at time t. The same restrictions as above apply to (2) as well.

To compare the results obtained from the estimation of (1) and (2) the following parametric models are also considered, where P_t is either the spot or forward price:

$$dP_t = \mu P_t dt + \sigma P_t dB_t, \tag{3}$$

$$dP_t = \kappa\left(\mu - \log P_t\right) P_t dt + \sigma P_t dB_t, \tag{4}$$

$$dP_t = \kappa\left(\mu - \log P_t\right) P_t dt + \sigma P_t dB_t + z P_t dq_t, \tag{5}$$

$$dP_t = \kappa\left(\mu - P_t\right) dt + \sigma dB_t, \tag{6}$$

$$dP_t = \kappa\left(\mu - P_t\right) dt + \sigma dB_t + z dq_t, \tag{7}$$

[5] While we could allow for the possibility of jumps in the forward price specifications, there are several good reasons not to. First, as mentioned in the text, there are no obvious jumps in the plotted forward prices. Second, given that the forward contracts, in effect, specify an average price for electricity over a relatively long period, three months and more, there is a natural smoothing of the forward prices relative to the spot prices. Thus, while the spot price tend to react quickly, and often dramatically, to new information, similar changes in the forward price should be very rare indeed. Last, as seen in Appendix C, jump-diffusions are much more difficult to estimate than pure diffusions and the resulting estimates are typically less precise. Faced with no compelling evidence of jumps, it seems better to choose a specification without jumps.

Equation (3) is the familiar geometric Brownian motion, equation (4) is a geometric mean reverting process, and equation (5) is a geometric mean reverting process with jumps. Equations (6) and (7) are arithmetic mean reverting processes without and with jumps, respectively. Equation (6) is also popularly referred to as the Ornstein-Uhlenbeck process. Model (4) is equivalent to using model (6) with the log-price. μ and σ are now constants and κ is the speed parameter for the mean reversion; it must be larger than zero for the process to be stationary. q_t is a jump process with arrival intensity λ and z is a normal random variable with mean μ_z and variance σ_z^2. These models will be referred to as the GBM, GMR, GMRJ, MR, and MRJ models, denoting geometric Brownian motion, geometric mean reversion, geometric mean reversion with jumps, mean reversion, and mean reversion with jumps, respectively.

2.2 A multivariate model for the spot prices

While (1) and (2) are very general in terms of univariate models, they might still be too restrictive to give a good description of electricity prices. There are inarguably some predictable components of electricity prices coming from sources other than past prices. Temperature, for example, is one driving factor of electricity demand which is, to a certain extent, predictable. It is important to remember, however, that all electricity markets differ from each other, and it is hardly feasible to capture all of them with the same model. For example, some of the highest demand for electricity in the Northern California market occurs during the warmest summer months when air conditioning is most needed. This is in stark contrast to the Nordic electricity market where electricity demand peaks during the coldest winter months when heating is most needed. Both of these features, though, could be captured by the same model, only using different parameters. Therefore, using a flexible approach like the nonparametric one relied upon here, the greater problem in specifying models for electricity prices becomes that of which explanatory variables to include. However, due to the nature of the forward markets, only shorter time series of the forward prices are available. Hence, it is not possible to include the forward prices in multivariate models since multivariate nonparametric methods require substantial amounts of data. This section therefore only deals with multivariate models for the spot price.

As mentioned above, temperature is a likely candidate to include in any model for electricity prices. Other components, proposed by Deng (2000), might be the prices of generating fuels, such as natural gas. In a region where hydro power is important, the level of water in the hydro power stations' reservoirs is most likely an important indicator for future electricity prices. However, given the problems of presenting and interpreting results from nonparametric regressions of three dimensions, or higher, it seems reasonable to limit ourselves to two-dimensional models. Of the potential variables affecting the spot price mentioned above, it seems that temperature would be the key one, at least with regards to short run dynamics and the determination of the volatility function for the spot prices. The other variables are probably more likely to affect the longer run dynamics and the drift of the spot price. Since option pricing is the main motivation behind this study and the volatility function plays a more vital part in option pricing than the drift, I use the temperature specification.[6] We thus end up with a bivariate diffusion model of electricity prices, S_t, and a measure of temperature T_t,

$$d\begin{pmatrix} S_t \\ T_t \end{pmatrix} = \begin{pmatrix} \mu_1(S_t, T_t) \\ \mu_2(T_t) \end{pmatrix} dt + \begin{pmatrix} \sigma_{11}(S_t, T_t) & \sigma_{12}(S_t,T_t) \\ 0 & \sigma_{22}(S_t, T_t) \end{pmatrix} d\mathbf{B}_t, \tag{8}$$

where $\mathbf{B}_t$ is a two-dimensional standard Brownian motion and (8) satisfies the regularity

[6] Another issue that could be of interest in modelling electricity prices is that of major metereological events, such as severe storms. Since these are partially forecastable they could be used to model the way prices react to such expected shocks. However, those kind of weather events are, if not nonexistent, at least extremely rare in the market region of Nord Pool and should be of no consequence to the modelling of electricty prices.

conditions given in Bandi and Moloche (2001). As in the univariate case above, we do not assume stationarity of the process, but only a form of recurrence.

A few observations on (8) are in order. First, the volatility of temperature is obviously not dependent on the electricity price, as $\sigma_{22}(\cdot)$ would indicate. But since this restrictions is not easy to implement in the econometric estimation of (8), I state the model in the form in which it will be estimated. Second, the reason that there are no jumps included in this specification is purely a matter of econometric convenience, as the theory to include the possibility of jumps in non-parametric estimation of multi-variate diffusion models is not yet developed. Fortunately, the effect of the omission of jumps in the bivariate model might be less severe than in the univariate case since the temperature process potentially has the ability to account for large moves in the price process. That is, if extreme moves in prices are typically associated with extreme moves in temperatures, then the need for a separate jump-process is reduced. Furthermore, as the Monte Carlo study in Appendix C shows, jumps are difficult to identify even in the univariate case and are probably more so in the multi-variate case. Thus, in an empirical multi-variate specification, jumps may be of little use. Lastly, we can also interpret (8) as a stochastic volatility extension of (1), where the volatility process is assumed to be a function of the temperature process.

Given the nonparametric specifications of (1), (2) and (8), the main modelling assumption, apart from which variables to include, is that the data follows a diffusion process. This, of course, is in itself a strong assumption. The results of Knittel and Roberts (2001) hint at the possibility that a more complicated structure might be needed, although their parametric models are less flexible than the nonparametric models I present above. They also use hourly observations of the prices, further complicating the model selection issue due to intra-daily effects. However, the general option pricing theory available today is based on prices following diffusion processes. It is important to derive option prices under these assumptions as a benchmark for any future results that might be derived using different specifications.

3 Econometric methods

3.1 Nonparametric estimation of diffusions

In order to estimate equations (1), (2) and (8), I use a nonparametric estimation technique for diffusions, originally developed in Bandi and Phillips (2001a) and further extended in Bandi and Nguyen (2001) and Bandi and Moloche (2001) to cover jump-diffusions and multivariate diffusions, respectively. What follows is a brief description of the estimators described in this literature, omitting all of the technical conditions that are detailed in the original papers.

3.2 Estimation of a scalar jump-diffusion

We assume that we observe a jump-diffusion process X_t at times $t = t_1, t_2, ..., t_n$ in the time interval $[0, T]$ and that the observations are at equally spaced intervals. Letting $\Delta_{n,T} = T/n$, we then have n observations $\left\{X_{\Delta_{n,T}}, X_{2\Delta_{n,T}}, ..., X_{n\Delta_{n,T}}\right\}$ of the process X_t at times $\{t_1 = \Delta_{n,T}, t_2 = 2\Delta_{n,T}, ..., t_n = n\Delta_{n,T}\}$. The proposed estimator for the kth infinitesimal conditional moment of the process X_t is:

$$\hat{M}^k(x) = \frac{1}{\Delta_{n,T}} \frac{\sum_{i=1}^{n-1} K\left(\frac{X_{i\Delta_{n,T}} - x}{h_n}\right)\left[X_{(i+1)\Delta_{n,T}} - X_{i\Delta_{n,T}}\right]^k}{\sum_{i=1}^{n} K\left(\frac{X_{i\Delta_{n,T}} - x}{h_n}\right)} \tag{9}$$

where $k \geq 1$, h_n is a bandwidth parameter, and $K(\cdot)$ is a kernel function.[7]

[7] The reason that we need to consider higer order moments in the jump-diffusion case follows from the fact that jump-diffusions, unlike pure diffusions, are not completely characterized by their first two moments; Bandi and Nguyen (2001).

From the conditional moment estimators, we can easily retrieve the quantities we are interested in, namely $\mu(\cdot)$, $\sigma(\cdot)$, $\lambda(\cdot)$ and σ_y^2. Suppose the sizes of the jumps are given by the random variable $y \overset{d}{=} N\left(0, \sigma_y^2\right)$. Then, following the procedure of Johannes (2000), we obtain the following estimates of σ_y^2, $\lambda(x)$, $\sigma^2(x)$, and $\mu(x)$:

$$\hat{\sigma}_y^2 = \frac{1}{n} \sum_{i=1}^{n} \frac{\hat{M}^6(X_{i\Delta_{n,T}})}{5\hat{M}^4(X_{i\Delta_{n,T}})}, \tag{10}$$

$$\hat{\lambda}(x) = \frac{\hat{M}^4(x)}{3\hat{\sigma}_y^4}, \tag{11}$$

$$\hat{\sigma}^2(x) = \hat{M}^2(x) - \hat{\lambda}(x)\hat{\sigma}_y^2, \tag{12}$$

and

$$\hat{\mu}(x) = \hat{M}^1(x). \tag{13}$$

The asymptotic properties of (9) are

$$\sqrt{h_n L(x)}(\hat{M}^k(x) - M^k(x)) \overset{d}{\to} N\left(0, \left(\int_{-\infty}^{\infty} (K(s))^2 ds\right) M^{2k}(x)\right), \tag{14}$$

as $n \to \infty, T \to \infty, \frac{T}{n} \to 0$, and $h_n \to 0$ such that $h_n^5 L\left(x\right) \overset{a.s.}{\to} 0 \ \forall x$.

$L(x)$ is the chronological local time of the process at point x and is estimated by

$$\hat{L}(x) = \frac{\Delta_{n,T}}{h_n^{local}} \sum_{i=1}^{n} K\left(\frac{X_{i\Delta_{n,T}} - x}{h_n^{local}}\right), \tag{15}$$

where h_n^{local} is a bandwidth parameter. For ease of notation, $L\left(x\right)$ will be reffered to simply as local time.

3.3 Estimation of a scalar diffusion

In order to estimate a scalar diffusion without jumps, as in equation (2), we can use the same estimator for the infinitesimal conditional moments that is described in (9). Only now, $\hat{\mu}(x) = \hat{M}^1(x)$ and $\hat{\sigma}^2(x) = \hat{M}^2(x)$. The asymptotic properties for these estimators are:

$$\sqrt{h_n L(x)}(\hat{\mu}(x) - \mu(x)) \xrightarrow{d} N\left(0, \left(\int_{-\infty}^{\infty} (K(s))^2 ds\right) \sigma^2(x)\right) \tag{16}$$

as $n \to \infty, T \to \infty, \frac{T}{n} \to 0$, and $h_n \to 0$ such that $h_n^5 L(x) \xrightarrow{a.s.} 0 \; \forall x$ and

$$\sqrt{\frac{h_n L(x)}{\Delta_{n,T}}}(\hat{\sigma}^2(x) - \sigma^2(x)) \xrightarrow{d} N\left(0, 4\left(\int_{-\infty}^{\infty} (K(s))^2 ds\right) \sigma^4(x)\right) \tag{17}$$

as $n \to \infty, T \to \infty, \frac{T}{n} \to 0$, and $\frac{h_n^5 L(x)}{\Delta_{n,T}} \xrightarrow{a.s.} 0 \; \forall x$.

3.4 Estimation of a multivariate diffusion

In estimating a multivariate diffusion without jumps, we assume the same setup as in the scalar case, only now X_t is assumed to be a d-dimensional diffusion process:

$$dX_t = \boldsymbol{\mu}(X_t)dt + \boldsymbol{\sigma}(X_t)\, d\mathbf{B}_t, \tag{18}$$

where $\mathbf{B}_t$ is a d-dimensional standard Brownian motion, $\boldsymbol{\mu}(\cdot)$ is a $d \times 1$ vector and $\boldsymbol{\sigma}(\cdot) = \{\sigma_{ij}(\cdot)\}_{1 \le i,j \le d}$ is a $d \times d$ matrix. Equation (8) is then a special case of (18) with $d = 2$, and $\boldsymbol{\mu}(\cdot)$ and $\boldsymbol{\sigma}(\cdot)$ written out explicitly. Further, let $\mathbf{a}(x) = \boldsymbol{\sigma}(x)\boldsymbol{\sigma}(x)'$. The estimators of $\boldsymbol{\mu}(x)$

and $\mathbf{a}(x)$ are:

$$\hat{\mu}(x) = \frac{1}{\Delta_{n,T}} \frac{\sum_{i=1}^{n-1} \mathbf{K}\left(\frac{X_{i\Delta_{n,T}}-x}{\mathbf{h}_n}\right)\left(X_{(i+1)\Delta_{n,T}} - X_{i\Delta_{n,T}}\right)}{\sum_{i=1}^{n} \mathbf{K}\left(\frac{X_{i\Delta_{n,T}}-x}{\mathbf{h}_n}\right)} \tag{19}$$

and

$$\hat{\mathbf{a}}(x) = \frac{1}{\Delta_{n,T}} \frac{\sum_{i=1}^{n-1} \mathbf{K}\left(\frac{X_{i\Delta_{n,T}}-x}{\mathbf{h}_n}\right)\left(X_{(i+1)\Delta_{n,T}} - X_{i\Delta_{n,T}}\right)\left(X_{(i+1)\Delta_{n,T}} - X_{i\Delta_{n,T}}\right)'}{\sum_{i=1}^{n} \mathbf{K}\left(\frac{X_{i\Delta_{n,T}}-x}{\mathbf{h}_n}\right)}, \tag{20}$$

where $\mathbf{h}_n$ is a bandwidth parameter and $\mathbf{K}(\cdot)$ is a kernel function.

In general, the notion of local time is not defined for multivariate diffusions. However, the multivariate extension of the estimator in equation (15) will still have well defined asymptotic properties and can be interpreted as an estimate of the time spent by the process in the vicinity of the spatial point x. The selection of the bandwidth $\mathbf{h}_n$, relies on this multivariate version of (15):

$$\hat{L}(x) = \frac{\Delta_{n,T}}{\mathbf{h}_n^{local}} \sum_{i=1}^{n} \mathbf{K}\left(\frac{X_{i\Delta_{n,T}} - x}{\mathbf{h}_n^{local}}\right). \tag{21}$$

Since confidence intervals for the multivariate estimates are difficult to display graphically, the asymptotic properties of (19) and (20) are omitted.

3.5 Estimation of the parametric models

The models in (3)-(7) are estimated by means of conditional maximum likelihood. The closed form maximum likelihood estimators for the parameters of a geometric Brownian motion and an Ornstein-Uhlenbeck process, obtained through 'exact' discretization, can be found in, for example, Gourieroux and Jasiak (2001), and will not be detailed here. To estimate equation (7), I use the discrete time approximation method suggested by Ball and Tourus (1983). They assume that, in each time interval, there either occurs one jump or no jump. A jump occurs with approximate probability λ and the resulting likelihood function is thus:

$$\begin{aligned} L &= \prod_{t=1}^{n} \lambda \exp\left(\frac{-\left(P_{t+1} - P_t - \kappa\left(\theta - P_t\right) - \mu_z\right)^2}{2\left(\sigma^2 + \sigma_z^2\right)} \right) \frac{1}{\sqrt{2\pi\left(\sigma^2 + \sigma_z^2\right)}} \\ &\quad + (1-\lambda) \exp\left(\frac{-\left(P_{t+1} - P_t - \kappa\left(\theta - P_t\right)\right)^2}{2\sigma^2} \right) \frac{1}{\sqrt{2\pi\sigma^2}}. \end{aligned} \tag{22}$$

Das (1998) provides a more thorough exposition of this estimation procedure. Finally, the geometric mean reversion models with and without jumps, are simply estimated by replacing the actual price with the log-price in the corresponding arithmetic mean reversion estimators.

4 Empirical results

4.1 A brief description of Nord Pool

The data that is used come from Nord Pool, the Scandinavian electricity exchange. Nord Pool is an international optional power exchange, where buyers and sellers from Denmark, Finland, Norway and Sweden trade. These four countries make up Nord Pool's market area.[8] The spot market for electricity at Nord Pool is a one-hour, one-day-ahead double-auction market. The resulting equilibrium-, or market-price is the so called *system price,* which is the market price for electricity in the whole region if there are no transmission constraints. In case of transmission constraints, the market area is divided into several price areas, with higher prices in the deficit areas and lower prices in the surplus areas. The prices I consider are daily system prices, which are calculated as the unweighted average hourly system price. The supply side of the market is characterized by a majority of the produced electricity coming from hydro power stations. Other sources of generation include nuclear plants, thermal (cogeneration) plants, and

[8] Nord Pool began operating on May 4, 1992, with Norway as the only member. Sweden later joined on January 1, 1996, Finland joined on March 1, 1999, and finally Denmark became a full member on July 1, 1999.

coal plants. Very little electricity is produced using oil.[9]

4.2 Estimation of univariate models for the spot price

To estimate equation (1), and the GBM, GMR, GMRJ, MR, and MRJ models for the spot price, I use a time-series of the daily system price at Nord Pool, spanning from May 4, 1992, to November 30, 2001. This yields a total of 3498 observations. Prices are given in Norwegian kronor (NOK) per MWh, where one U.S. dollar is approximately equal to 10 NOK, and are shown in figure 1. The plot reveals the typical characteristics of electricity prices, a strong seasonal component and frequent spikes in the price. The starting date for the time-series is the same as the day that Nord Pool started operating. The series thus includes every trading day at Nord Pool, up until recently. While hourly data are available, these would bring the added complication of intra-daily effects, which for electricity prices can be quite severe. Summary statistics of the data are given in table 1.

All estimates presented are on an annual basis, using 365-day years, since the spot market is open every day of the year. In the nonparametric estimation, this is achieved by setting $T = \#observations/365$ while in the parametric procedures, the estimates and standard errors are simply multiplied by 365. A Gaussian kernel is used and bandwidth selection is discussed in Appendix B.

The estimates of $\mu_S(\cdot)$, $\sigma_S(\cdot)$, and $\lambda_S(\cdot)$ from equation (1), using the system price, are shown in figure 2. Starting with the drift function, $\mu_S(\cdot)$, there is evidence of nonlinear behavior. The estimated drift is also everywhere positive, which is somewhat peculiar and goes against the standard assumption of mean reversion of electricity prices. As the Monte Carlo study in Appendix C shows, however, the drift in a jump-diffusion is difficult to estimate for

[9] For a survey of the Nordic electricity market, see Hjalmarsson (2001).

data stretching only a relatively short time span. There are also two economic explanations for this drift upward in the spot price. First, since the start of Nord Pool in 1992, there has been a gradual increase in market concentration on the supply side, enabling prices to deviate further from those in a perfectly competitive market.[10] Second, it is plausible to assume, and generally believed among practitioners, that the supply side participants have improved their skills over time to achieve higher prices.

The diffusion, or volatility function of the system price, $\sigma_S(\cdot)$, shows how volatility is a function of the price level; there is high volatility for low and high prices and low volatility for prices closer to the mean. This result is contrary to the volatility functions specified in most parametric models, which typically assume that volatility is a monotonic function of the price level. However, the magnitude of the change in volatility as a function of the system price is relatively small and it would seem that volatility might therefore be approximated by a constant.

The estimate of the jump intensity, $\lambda_S(\cdot)$, shows a pattern similar to the diffusion function. The estimated function indicates a jump frequency of about two jumps per year – an estimate that does not seem too far off when looking at figure 1. The estimate of σ_y is equal to 172.8. This implies that under the normality assumption imposed on the jumps here, approximately 95% of the jumps would be of a size less than 350 NOK/MWh.

Figure 3 shows confidence intervals for the estimates of the infinitesimal first and second moments. The confidence intervals are shown only for prices between 50 and 300 since the confidence bands spread out vastly close to 1 and 400, which are the bounds of the domain of the estimated functions in figure 2. Thus, most information in the graph would be lost if the confidence intervals were plotted over this domain.

None of the parametric models in equations (3)-(7) produce estimates that are close

[10] For example, Sweden, who joined in 1996, has a very concentrated power industry which would have contributed to an increased concentration on Nord Pool.

to the nonparametric ones just described. These estimates are shown in table 2. It is evident that any specification not including jumps gives estimates that are extremely different from the corresponding specification with jumps, and also very far from the nonparametric estimates. The results from the two parametric jump models, GMRJ and MRJ, are plotted in figure 4, together with the nonparametric estimates. Clearly neither of these are good approximations to the nonparametric estimates. The plot also highlights that volatility is better approximated by a constant function than a volatility function proportional to the price. This supports Johnson and Barz (1999) who also find some evidence of constant volatility of spot prices, using Nord Pool data. Interestingly, they find that the spot prices from the other electricity exchanges that they examine better fit geometric mean reverting models with volatility proportional to the price level.[11]

4.3 Estimation of univariate models for forward prices

Equation (2) and the GBM, GMR, and MR models are estimated using forward prices from Nord Pool. There are two kinds of forward contracts at Nord Pool, on which options are traded. First, there are the seasonal contracts, denoted FWV1, FWSO, and FWV2. FWV1 specifies delivery of electricity from January 1 through April 30, for a total of 2879 hours. FWSO specifies delivery of electricity from May 1 through September 30, for a total of 3672 hours. FWV2 specifies delivery of electricity from October 1 through December 31, for a total of 2209 hours. Second, there are the yearly contracts, FWYR, that specify delivery from January 1 through December 31, for a total of 8760 hours. These contracts, although stated in terms of electricity delivery, are purely financial, and are settled financially, not physically by actual electricity delivery. The contract volumes are in MW, and prices of the contracts are given in

[11] They examine data from Nord Pool, the UK, California, and Victoria, Australia.

NOK/MWh. Specific contracts are labeled with the year in which delivery takes place after the symbol. That is, a forward contract for, say, delivery in January through April 2001 is labeled FWV1-01.

The problem with forward prices is that they do not provide a long continuous time series. As figures 5-8 clearly show, there are several forward contracts traded on any given day. But since trade in the contracts stops just before the first day of delivery, there are no long time series available.[12] However, the longer of these series still contains around 600 observations, and using a single one of these series should still give reasonably accurate estimates when estimating equation (2). The parametric models are, of course, less dependent on a great number of observations. Therefore, I use the longest series available for each type of forward contract to estimate equation (2) and the GBM, GMR, and MR models. These are the series for FWV1-01, FWSO-01, FWV1-01, and FWYR-02.[13] All observations are daily closing prices. Summary statistics for these are given in table 1. The bandwidth selection for the nonparametric estimation is described in Appendix B. All estimates are presented as annual values, using a 250 day year, based on the number of actual trading days per year in the financial market.

Given the short time span of the available observations and that the main interest lies in option pricing, I only estimate the diffusion function for the four forward contracts, and not the drift function. The results from the estimation of equation (2) for FWV1-01, FWSO-01, FWV1-01, and FWYR-02, with 95% confidence intervals, are shown in figure 9. The nonparametrically estimated diffusion functions, $\sigma_F(\cdot)$, together with the estimates from the GBM and MR models are shown in figure 10; the GMR model produced nearly identical results to the GBM model and is not displayed graphically. It is not obvious from figure 10 whether the GBM model

[12] Not all forward contracts of the types described here are plotted in figures 5-8.

[13] FWV1-01 is traded between September 7, 1998, and December 29, 2000, for a total of 580 trading days. FWSO-01 is traded between September 7, 1998, and April 30, 2001, for a total of 662 trading days. FWV2-01 is traded between September 7, 1998, and September 28, 2001, for a total of 766 trading days. FWYR-02 is traded between March 1, 1999, and December 21, 2001, for a total of 706 trading days.

or the MR model provides the better approximation of the nonparametric volatility function, although the GBM model seems a little closer to the nonparametric estimates. Comparison of option prices from these models shows, however, that the GBM model typically is closer to the nonparametric estimates. The parametric estimates on the forward prices provide strikingly better approximations to the nonparametric ones, as compared to the models fitted to the system price. This could be due to the drift being better behaved for the forward prices than for the system prices, causing the maximum likelihood estimates of the volatility to be more accurate, or due to the greater difficulty of estimating diffusions with jumps in them. The local time estimates for the four forward contracts are shown in figure 11.

Finally, I perform a check for the representativeness of the forward contracts under consideration. The nonparametric estimates, together with the estimates from the GBM and MR models, of the volatility functions for the other forward contracts plotted in figures 5-8 are shown in figure 12. The graph shows that while there is no clear pattern for each forward contract's volatility function, the considered contracts at least do not seem to be outliers in any sense. The estimates are performed over the same range of prices as for the corresponding contract type considered above.

4.4 Estimating the bivariate diffusion

For the estimation of equation (8), I require, in addition to the system price, a measure of temperature for Nord Pool's market area. Since this is a fairly large geographical region, comprised of Denmark, Finland, Norway and Sweden, observations from multiple places are needed. Because these countries joined Nord Pool at different times, only the relevant member countries are considered at any given date. In order to arrive upon a representative measure of aggregate temperature, I used daily data from several observation points in each country. Within each country, the observations from each location were weighted by the size of the population living nearby and a country average was derived. The aggregate temperature measure used in my analysis is then simply the mean of the individual country averages. The aggregate temperature measure thus derived is shown in figure 13. A detailed description of the temperature data is found in Appendix A and summary statistics are shown in table 1.

Using the system price and the aggregate temperature, I estimate equation (8) using a product of univariate Gaussian kernels. Bandwidth selection is described in Appendix B. The functions are estimated over a somewhat smaller range of system prices, 50 to 300, compared to the univariate case since the data points are very scarce for values outside this range of prices in the two-dimensional space created by the system price and the aggregate temperature. The drift of the temperature is estimated separately using the estimator for the drift in a univariate diffusion model, since it does not depend on the spot price.

The estimates of $\mu_1(\cdot,\cdot)$ and $\mu_2(\cdot)$ are given in figure 14. The drift function for the system price, $\mu_1(\cdot,\cdot)$, has a shape quite different from the univariate estimate and the typical mean reversion pattern is more apparent here. There is also some variation when considering the system price drift in the temperature dimension. The drift function for the aggregate temperature, $\mu_2(\cdot)$, although independent of the system price, is plotted in the same three-dimensional manner as the the drift of the system price, for ease of comparison. The temperature drift shows a clear mean reversion pattern.

The estimate of $\sigma_{11}(\cdot,\cdot)$, shown in figure 15 seems to indicate that price volatility tends to be higher for higher prices and lower temperatures. The variation in price volatility, as a function of price and temperature, is much greater here than the corresponding variation in the univariate estimate. The estimate of $\sigma_{22}(\cdot,\cdot)$, also in figure 15, shows an increase in temperature volatility for lower temperatures, and little dependence on the system price.

The estimate of the covariance function, $\sigma_{12}(\cdot,\cdot)$, shown in figure 16, is almost always negative. This seems reasonable, since higher prices are likely to be associated with lower temperatures and vice versa. Furthermore, the covariance is greatest, in absolute numbers, for very low temperatures, which also makes sense, since the temperature is likely to have a greater impact on price volatility during the cold winter months when demand for heating electricity fluctuates with temperature than in the warmer summer months. The temperature does seem to play an important part in the evolution of electricity prices as shown by the estimate of the covariance function in figure 16.

5 Option pricing

5.1 Option pricing using the empirical models

In this section, I will use my estimates of equation (2) and the estimates of the GBM and MR model, fitted to the forward contracts, to derive prices of European call options. The GMR model will not be used here since it produced results very similar to that of the GBM model. Only options on the forward contracts are priced since these are the only types of options officially traded on Nord Pool. Considering options on financial forward contracts of electricity, rather than physical spot contracts, also circumvents the problem that the non-storability of electricity causes for the pricing of options on electricity spot market contracts. It is well known that this particular characteristic of electricity causes the normal arbitrage pricing arguments for option pricing to break down (e.g. Eydeland and Geman, 1999).

Note that while electricity spot prices exhibit strong seasonality the forward prices do not, the reason being that the forward prices specify the average price of electricity for a fixed period in time. That is, the forward price for the FWV1-01 contract is the average price for delivery of electricity during the first three months of the year. Thus, when pricing options on these forward contracts, the usual complication of seasonal patterns encountered for commodities does not occur and we can proceed as if the underlying contract was a forward contract on a normal financial asset.

I derive option prices through the risk-neutral valuation approach. This method relies on the fact that the price of a European call option with strike price X and exercise date T on an asset with price S_t that pays no dividends, is as follows:

$$c = e^{-\int_0^T r_t dt} E^* \left[\max \left(S_T - X, 0 \right) \right], \tag{23}$$

where r_t is the instantaneous discount rate and E^* is the expectation taken under the risk-neutral measure. The risk-neutral measure is the probability measure generated by the risk-neutral dynamics of S_t. As Black (1976) implicitly shows, the risk-neutral drift for futures and forwards under a nonstochastic interest rate is equal to zero. For completeness, this result is reproduced in Appendix D.

The Girsanov theorem (e.g. Steele, 2001) further tells us that the diffusion function will remain the same under the risk-neutral measure as under the objective probability measure. The corresponding risk-neutral dynamics of equations (2), (3), and (6) for the forward prices are thus:

$$dF_t^* = \sigma \left(F_t^* \right) dB_t^*, \tag{24}$$

$$dF_t^* = \sigma F_t^* dB_t^*, \tag{25}$$

$$dF_t^* = \sigma dB_t^*, \tag{26}$$

where B_t^* is a Brownian motion under the risk-neutral measure. We are thus able to identify the risk-neutral dynamics, equations (24)-(26), although we only observe the price-process under the objective, or data-generating, dynamics.

In order to price options on assets following the risk-neutral dynamics in equations (24)-(26), I simulate sample paths of these processes and calculate the options payoff for each of these sample paths and then take the mean over all the sample paths. The option prices thus obtained provide a numerical approximation to the true option prices given by the pricing formula, (23). I perform the simulations using Euler's scheme (e.g. Kloeden and Platen, 1992) with 10,000 repetitions and using a time interval of a half hour between each simulated observation. That is, for each day I simulate 48 observations, and thus for an option with maturity in, say, 91 days, the sample paths consist of $91 * 48 = 4368$ observations. I further use the antithetic variation reduction approach (e.g. Campbell et. al., 1997), which gives an effective number of repetitions of 20,000. The interest rate, r, is taken to be constant and equal to 5% on an annual basis. To account for outliers, I use a constant extrapolation outside the interval that the nonparametric estimates are evaluated. I use a constant extrapolation rather than a linear one since it is difficult to judge which way the estimated functions are going.

5.2 Option pricing results

The options traded on Nord Pool are of the European kind and they are traded on the four forward contracts examined above, the FWV1, FWSO, FWV2, and FWYR. I use the estimates from the nonparametric model, equation (2), and the GBM and MR models, fitted to the forward prices, to price European call options on the underlying forward contracts. In addition to the simulation methods described above, I also price options using the analytical

Black-Scholes formula, adapted to forward contracts as described in Appendix D.[14] This enables a check of the accuracy of the simulated prices, since the simulated GBM prices should be equal to the Black-Scholes prices. The volatility parameter used in the Black-Scholes formula is the historical estimate obtained from the GBM model. The strike prices for the FWV1, FWV2, and FWYR contracts are set to 140, 145, 150, 155, 160, and 115, 120, 125, 130, 135 for the FWSO contract.[15] The initial forward prices at time zero are set to 150 and 125 respectively.

The option prices for the four forward contracts are shown in tables 3-6. For each contract, four option prices are calculated: the analytical Black-Scholes price (BS), the simulated prices from the GBM and MR models, and the prices using the nonparametric estimates (NP). The BS and GBM prices should be the same, except for errors resulting from the simulation procedures. Judging from the results, the simulation errors are small and typically no more than a few hundreths of a Norwegian crown (NOK). It is also evident that the option prices obtained from the MR model are typically further away from the option prices obtained from the nonparametric model than those resulting from the GBM model. Given this, I will focus on comparing the option prices from the nonparametric model and the GBM model.

It would thus be desirable to test whether the option prices obtained from the nonparametric model are statistically significant from option prices obtained in the same nonparametric way under the condition that the underlying model is a geometric Brownian motion. In order to do so, I calculate confidence intervals for the nonparametric option prices, under the assumption that the underlying model is a geometric Brownian motion. Since it would be extremely difficult to actually derive analytical confidence intervals for the nonparametric option prices a Monte Carlo approach is used. The method can be summarized in six steps. (1) Calculate the

[14] This formula is often referred to as an option pricing formula for futures, but under nonstochastic interest rates, forward and future prices must be the same (e.g. Merton, 1990). As shown in Appendix D, this also leads to identical option prices on forwards and futures.

[15] The strike prices for the FWSO contract are set to these lower values since the FWSO forward prices are typically considerably lower than those of the other forward contracts.

maximum likelihood estimates for the parameters of a geometric Brownian motion fitted to the relevant price process, e.g. the FWSO-01 prices. (2) Simulate a geometric Brownian motion, using these maximum likelihood estimates as parameters. (3) Using the nonparametric estimator for diffusions (without jumps) described in Section 3.3, obtain the functional estimate of the diffusion function for the simulated geometric Brownian motion. The estimates are obtained over the same range of prices as in the actual estimation. (4) Calculate the option prices for relevant strike prices and expiration dates, using the estimated diffusion function and the option pricing methods described in Section 5.1. (5) Store the set of option prices thus obtained and repeat steps (2)-(5) a 'sufficient' number of times. (6) Find the upper and lower 2.5% quantile of the simulated option prices. These give the upper and lower bounds of the approximate 95% confidence interval for the nonparametric option prices under the assumption that the true underlying model is a geometric Brownian motion. The number of repetitions used is 250, and the option prices for each repetition is calculated using 1,000 simulated sample paths.[16]

A test of whether the NP option prices, as presented in tables 3-6, are statistically different from option prices derived under the assumption of an underlying geometric Brownian motion is to see whether the NP prices lie within the 95% confidence intervals obtained according to the procedure above. The confidence intervals are reported next to the NP estimates in tables 3-6. Of course, given the multiple testing involved here, 25 tests for each forward contract, the actual significance level of an overall test is much less than 95%, but the results should still be indicative.

The results presented in tables 3-6 show that the outcomes of the above test vary considerably from contract to contract. The FWV2-01 and FWSO-01 contracts reject the GBM model almost totally whereas the evidence from the FWV1-01 contract clearly are in favour of

[16] The small number of repetitions is due to the long computer time needed for these simulations. This will be remedied in the future.

the GBM model. The results from the FWYR-02 contract are more mixed. It is also reassuring that all the BS and GBM option prices are well within the 95% confidence interval.

It is clear that, for some contracts, there are statistically significant deviations from the BS option prices. Given the magnitude of the differences, these deviations should also be economically significant. Blind trust in the Black-Scholes formula, therefore, does not seem like a good approach when pricing options on Nord Pool. Hence, more accurate parameterizations than the geometric Brownian motion, for the diffusion processes driving the forward prices at Nord Pool might be needed. However, considering the shape of the nonparametric estimates of the volatility functions for the forward contracts presented in figure 10, it is difficult to think of a reasonable parameterization that would give a better fit than the linear one given by the GBM model. Thus, although the Black-Scholes formula does not produce entirely accurate option prices for the forward contracts considered here, it seems difficult to find a better alternative that would be of practical use.

6 Conclusion

In this paper, I have conducted a nonparametric analysis of electricity prices. I have fitted both a nonparametric jump-diffusion model of the system price at Nord Pool (the relevant spot market price for electricity in Denmark, Finland, Norway, and Sweden) and nonparametric diffusion models for some of the forward price processes. Furthermore, using the nonparametric estimates for the forward contracts, option prices for the forward contracts have been calculated.

The nonparametric estimates of the jump-diffusion process fitted to the system price exhibit some nonlinear behaviour, and parametric estimates from mean reverting processes with jumps provide poor approximations to the nonparametric estimates. Likewise, the functional estimates from scalar diffusions fitted to various forward prices show signs of nonlinearity, but the estimates from both the geometric Brownian motion and the mean reverting Ornstein-Uhlenbeck process provide decent approximations to the nonparametric estimates.

For some forward contracts the option prices based on the nonparametric estimates deviate rather substantially from the Black-Scholes option prices and are found to be statistically different from those obtained under the assumption of a geometric Brownian motion, i.e. the Black-Scholes assumption. The differences between the option prices derived from the nonparametric estimates and the Black-Scholes prices are also, in many cases, large enough to be of economic significance. These results thus indicate that more accurate option prices can be obtained than the Black-Scholes' ones. However, given the shape of the nonparametric estimates, it is difficult to think of any parametric model that would give a better approximation than the linear geometric Brownian motion, upon which the Black-Scholes formula is based. Therefore, from a practical viewpoint, the Black-Scholes option prices might be the best achievable.

A possible extension to the analysis performed in this paper would be to incorporate stochastic volatility. The estimation of a stochastic volatility model might be performed by using daily range data, as suggested by Gallant et. al. (1999). However, the inclusion of stochastic volatility would create a two-dimensional model, which typically requires more data to be accurately estimated. Thus, a more efficient usage of the available forward data would have to be conceived. Finally, there is the issue of how to price volatility, which might be difficult to solve without reliable option prices.

A Constructing the aggregate temperature measure

The aggregate temperature measure was created as a weighted average of several temperature observations from Denmark, Finland, Norway, and Sweden. Temperature data was collected from the following places and assigned the following weights based on population: Aalborg, Copenhagen, and Odense in Denmark with weights 12, 110, and 14 respectively; Helsinki, Turku, Tampere, and Vaasa in Finland with weights 56, 17, 20, and 6 respectively; Bergen, Kristiansand, Oslo, Stavanger, and Trondheim in Norway with weights 23, 7, 51, 11, and 15 respectively; Gothenburg, Malmö, and Stockholm in Sweden with weights 2, 1, and 6 respectively. A weighted country mean was then calculated for each individual country using the above weights. Finally, the aggregate temperature measure was calculated as an unweighted mean of the weighted individual country averages. However, Norway was the only participant at Nord Pool between May 4, 1992 and December 31, 1995, then Sweden joined on January 1 1996, Finland on March 1, 1999 and Denmark on July 1, 1999. Thus between May 4, 1992 and December 31, 1995 only the Norwegian country mean is used, between January 1, 1996 and February 28, 1999 the mean of Norway and Sweden is used, between March 1 1999 and June 30, 1999 the mean of Finland, Norway, and Sweden is used and, after that, the mean of Finland, Norway, Sweden, and Denmark is used.

B Bandwidth Selection

The choice of bandwidth is always a difficult question in nonparametric inference. This is particularly true for continuous time models where firm rules for bandwidth selection are still nonexistent. There are, however, guidelines that describe up to a constant of proportionality what bandwidths to use for the continuous time estimators employed in this paper.

In the univariate case, Bandi and Phillips (2001a) give the rule that the bandwidth for the local time estimator h_n^{ltime} should be set equal to

$$h_n^{ltime} = c^{ltime} \frac{1}{\log(n)} n^{-\frac{1}{2}} \tag{27}$$

where c_{ltime} is a constant of proportionality. Furthermore, Bandi and Nguyen (2001) state that the bandwidth sequence for the $p-$th infinitesimal moment of a jump-diffusion process can be set to

$$h_n^p(x) = \upsilon^p \log\left(\frac{1}{\hat{L}(x)}\right) \hat{L}(x)^{-\frac{1}{5}} \tag{28}$$

where υ^p is a moment specific constant of proportionality.

For the multivariate case, Bandi and Moloche (2001) recommend that the bandwidth sequence for the drift estimator be set equal to

$$h_n^{drift}(x) = c^{drift}\left(\frac{1}{\log \hat{L}(x)}\right) \hat{L}(x)^{-\frac{1}{d+4}} \tag{29}$$

and that the bandwidth sequence for the diffusion estimator be set equal to

$$h_n^{diff}(x) = c^{diff}\left(\frac{1}{\log\left(\hat{L}(x)/\Delta_{n,T}\right)}\right)\left(\hat{L}(x)/\Delta_{n,T}\right)^{-\frac{1}{d+4}} \tag{30}$$

where c^{drift} and c^{diff} are constants of proportionality and d is the dimension of the diffusion system. The sequences in (29) and (30) also work for the scalar diffusion case (without jumps), by setting $d = 1$.

Equations (28)-(30) show that the optimal choice of bandwidth for the corresponding estimators are of a local nature. A greater bandwidth is used in regions where the observations are less frequent, i.e. where the local time is relatively small. Thus, the estimate of the local time plays a vital role in the determination of the bandwidth sequences for the other estimators. Further discussion on bandwidth selection in continuous time can also be found in Bandi and Phillips (2001b).

Starting with the estimation of local time in the univariate case, the constant c^{ltime}

needs to be determined. Common to many rules of optimal bandwidth selection for discrete time estimators is that they scale the bandwidth with the standard deviation of the relevant data (e.g. Pagan and Ullah, 1999). I will follow the same rule when determining bandwidths for the estimators in this paper. Furthermore, in order to simplify the selection of bandwidths, I choose to set υ^p, c^{drift}, and c^{diff} in the scalar case equal to the standard deviation of the data. In the multivariate case, both c^{drift} and c^{diff} are set equal to $diag\left(S^{1/2}\right)$ where S is the sample covariance matrix, in accordance with the procedure for general nonparametric estimators outlined in Härdle and Linton (1994). Since there exist no stringent rules for bandwidth selection, the final choice of bandwidth must be somewhat subjective and the procedure of fixing υ^p, c^{drift}, and c^{diff} in accordance with the above rules thus essentially reduces the number of parameters to be chosen to one, namely c^{ltime}.

As a starting point, it seems natural to just set c^{ltime} equal to the standard deviation of the system price. The resulting estimates of the local time of the system price, and of the drift, diffusion and jump functions, using this bandwidth choice, is shown in figure 17. It is well known that the local time of a diffusion process need not be a smooth, well behaved function, and these estimates could therefore possibly be close to optimal. However, one of the main purposes of this paper is to illustrate what the characteristics of the drift, diffusion, and jump functions for a general jump-diffusion process for electricity prices are; it might be desirable to also get some more smooth estimates where the salient features of the estimated functions are more prominent. The estimates in figure 2 are obtained by setting c^{ltime} equal to 20 times the standard deviation of the system price. The scale and general shape of the estimated functions remain the same but it is easier to discern the salient features. The models concerning the forward prices also use c^{ltime} equal to 20 and the bandwidth sequence described in (30), subject to the conventions described above.

Of course, setting a constant of proportionality equal to 20, or 100 as is done below, might seem a bit of a stretch. This is probably true in this case as well, and the estimates of the local time arrived upon in this way are most likely not optimal. However, by over smoothing the estimates of the local time, which is not of any direct interest, I am able to get more easily interpreted estimates of the functions of interest. The need for over smoothing the local time estimates to get smooth estimates of the drift, diffusion, and jump functions stems from the dependence of the smoothing sequences in (28)-(30) on the local time estimate. If the local time estimate is very jagged, then the bandwidths for the other estimators will vary a lot from one point to another and most likely also the functional estimates at these points, thus creating non smooth estimates. It is important to note that although the estimates of the relevant functions become more smooth as the bandwidth for the local time estimator increases, they still retain their general shape. That is, the functional estimates are smoothed locally by increasing the bandwidth for the local time estimator, but not much globally. Thus, increasing the bandwidth drastically for the local time estimator has less of a distortive effect than similar increases in more traditional bandwidth selection would have.

Similarly, in the bivariate case, I set c^{ltime} equal to $diag\left(S^{1/2}\right)$ times a constant, where S is the sample covariance matrix of the system price and the temperature measure. However, setting c^{ltime} equal to $diag\left(S^{1/2}\right)$ produces very irregular estimates as seen in figure 18. Multiplying $diag\left(S^{1/2}\right)$ by 20 gives somewhat more regular estimates, shown in figure 19, but raising the multiplication factor to 100 gives a more clear outline of the prominent features of the objects of interest, as seen in figures 14-16.

C A Monte Carlo Study

To evaluate the effects of the different bandwidth choices outlined above and the accuracy of the diffusion estimators in general, I perform a small Monte Carlo experiment. The following models are considered:

$$dP_t = \kappa \left(\mu_1 - P_t\right) dt + \sigma_1 dB_t + dJ_t \tag{31}$$

and

$$dP_t = \kappa \left(\mu_2 - \log P_t\right) P_t dt + \sigma_2 P_t dB_t, \tag{32}$$

where J is a jump process with constant jump intensity λ and jump size $y \sim N\left(0, \sigma_y^2\right)$. The parameters are set to the following annual values:

$$\kappa = 3.65, \tag{33}$$

$$\mu_1 = 150, \tag{34}$$

$$\mu_2 = 5$$

$$\sigma_1 = 200, \tag{35}$$

$$\sigma_2 = 0.2, \tag{36}$$

$$\lambda = 3, \tag{37}$$

and

$$\sigma_y = 175. \tag{38}$$

The parameters in equations (31) and (32) are chosen so that the simulated processes exhibit similar volatility, jump intensity, and jump size as the system price and the forward prices, respectively.

Sample paths of 3500 daily observations are simulated for equation (31), and 600 daily observations for equation (32), which is approximately equal to the number of observations used in the empirical study in this paper. The simulation is done using the Euler scheme and the jumps are simulated using the 'coin-tossing' approach proposed by Ball and Tourus (1983). That is, for any given date there is a $\lambda/365$ probability of a jump occurring. 500 sample paths were generated and the means of the estimates for the drift, diffusion, jump intensity, and local time functions, and of σ_y, were calculated. The estimates are obtained for two different bandwidths for equation (31): c^{ltime} equal to the standard deviation of the simulated path and c^{ltime} equal to 20 times the standard deviation. Equation (32) is simulated using c^{ltime} equal to 20 times the standard deviation. The results from equation (31) are shown in figures 20 and 21 and those from equation (32) are shown in figure 22.

Concerning the issue of bandwidths, it is clear from figures 20 and 21 that the difference between the mean estimates of the drift, diffusion and jump intensity functions for the two bandwidths are small. The only noticeable difference is for the estimates of the local time, which is decidedly more jagged in figure 20 than in figure 21. Thus it seems fairly safe to over smooth the local time estimate somewhat to achieve smooth estimates of the other functions. The estimates of σ_y are equal to 158.95 and 158.99, corresponding to the estimates in figures 20 and 21, respectively. Thus, the estimates of σ_y are very close for the two different bandwidths, and also fairly close to the true value, 175.

The functional estimates suffer from two shortcomings. First, the drift is very poorly estimated. Given that the mean estimate of the drift function presented in figure 22, for the non-jump case, is fairly accurate, this problem seems to arise from the presence of the jumps. Second, the jump intensity is severely over estimated, which in turn causes an under estimation of the diffusion function (as can be seen from equation (12)). This second problem is a manifestation of the ever present problem when estimating jump diffusions: how to discern between the actual jumps in the process and the jumps caused by the discrete sampling of the process.

Figure 22 shows the estimation results from the 500 simulations of (32). It is evident that the estimation problem is much easier in the case without jumps. Even for such a small sample size the estimates are fairly accurate. The estimate of the drift function is now also much closer to the true value.

D Option pricing for forward contracts

Suppose that the forward prices, F_t, satisfies

$$dF_t = \mu F_t dt + \sigma F_t dB_t. \tag{39}$$

Denote the option price formula $c(F, X, t, T, r)$ and use Itô's lemma:

$$dc = \left(\frac{\partial c}{\partial F} \mu F + \frac{\partial c}{\partial t} + \frac{1}{2} \frac{\partial^2 c}{\partial F^2} \sigma^2 F^2 \right) dt + \frac{\partial c}{\partial F} \sigma F dB. \tag{40}$$

Create a portfolio that is short one option and long $\partial c / \partial F$ forward contracts and denote the value of this portfolio V. Since positions in forward contracts cost nothing, it follows that at any time, $V_t = -c_t$. The change in value of this portfolio is

$$dV = -dc + \frac{\partial c}{\partial F} dF = -\left(\frac{\partial c}{\partial t} + \frac{1}{2} \frac{\partial^2 c}{\partial F^2} \sigma^2 F^2 \right) dt, \tag{41}$$

which is nonstochastic. Thus, to rule out arbitrage,

$$dV = -\left(\frac{\partial c}{\partial t} + \frac{1}{2}\frac{\partial^2 c}{\partial F^2}\sigma^2 F^2\right) dt = rVdt = -rcdt. \tag{42}$$

That is, the option price c must satisfy

$$\frac{\partial c}{\partial t} + \frac{1}{2}\frac{\partial^2 c}{\partial F^2}\sigma^2 F^2 = rc, \tag{43}$$

with boundary condition $c = \max(0, F - X)$. The solution is given by

$$c = e^{-r(T-t)}\left[F\Phi(d_1) - X\Phi(d_2)\right], \tag{44}$$

where

$$d_1 = \frac{\log(F/X) + (\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \tag{45}$$

and

$$d_2 = d_1 - \sigma\sqrt{T-t}. \tag{46}$$

This is the same formula as derived by Black (1976) for options on futures contracts.[17] The derivation also shows that we can treat option pricing on forwards as equivalent to option pricing on an asset paying a dividend of r, which also means that when phrasing the option pricing problem in terms of a risk-neutral measure, the drift in forward prices under this measure must be zero.

[17] As pointed out by Duffie (1989) there is a distinction between pure and conventional futures options. The pure futures options requires daily settlements just like a futures contract, whereas the conventional futures option works like a normal option contract with a premium paid for the option at the time of purchase and at the time of exercise pays the buyer any excess of the underlying asset price over the strike price. All futures options referred to in this paper is of the conventional kind.

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Table 1: Summary statistics.

	System price (NOK/MWh)	Aggregate temperature(C^o)	FWV1-01 (NOK/MWh)	FWSO-01 (NOK/MWh)	FWV2 (NOK/M
Mean	136.79	7.38	152.50	126.82	
Std. Dev	66.53	7.14	14.53	18.78	
Min	1.48	-14.20	135.35	103.00	
Max	633.36	23.63	188.00	197.00	
No. of observations	3498	3498	580	662	

Table 2: Estimates for the parametric specifications in equations (3)-(7). The numbers in parentheses are the standard errors of the estimates.

Parameter	μ	σ^2	κ	λ	μ_z	σ_z^2
			Panel A. System price			
GBM	3.8882	7.5491				
	(0.89211)	(0.18051)				
GMR	4.7680	7.6526	9.9700			
	(0.01047)	(0.18549)	(1.4624)			
GMRJ	3.6558	0.9929	1.4378	61.0718	0.0280	0.10629
	(0.8756)	(0.0511)	(0.9256)	(4.129)	(0.0143)	(0.0083)
MR	136.79	104938	11.944			
	(1.1206)	(2550.4)	(1.6050)			
MRJ	93.598	20156	5.3348	28.441	8.5194	2813.5
	(10.269)	(739.67)	(0.8119)	(2.4242)	(3.3274)	(18.926)
			Panel B. FWV1-01			
GBM	-0.12297	0.01289				
	(0.07453)	(0.00076)				
GMR	5.0229	0.01299	2.1133			
	(0.00230)	(0.00077)	(1.3555)			
MR	152.50	294.30	2.0720			
	(0.34992)	(17.354)	(1.3421)			
			Panel C. FWSO-01			
GBM	0.11716	0.03387				
	(0.11310)	(0.00076)				
GMR	4.8333	0.03516	2.9292			
	(0.00301)	(0.00194)	(1.4962)			
MR	126.82	693.14	3.1688			
	(0.40643)	(38.340)	(1.5569)			
			Panel D. FWV2-01			
GBM	0.02002	0.02361				
	(0.08779)	(0.00121)				
GMR	5.0928	0.02371	1.1550			
	(0.00366)	(0.00121)	(0.87029)			
MR	164.24	745.86	1.1482			
	(0.65117)	(38.199)	(0.86771)			
			Panel E. FWYR-02			
GBM	0.05574	0.02106				
	(0.086367)	(0.00112)				
GMR	5.0198	0.02116	1.2288			
	(0.00349)	(0.00113)	(0.93517)			
MR	152.21	560.86	1.2486			
	(0.56402)	(29.926)	(0.94273)			

Table 3: Option prices for the FWV1-01 contract. The BS, GBM, and MR models denote the same models as in the main text. NP denotes the nonparametric model, equation (2). The last two columns contain the lower and upper 95 percent confidence intervals for the nonparametric option prices under the assumption that the true underlying model is a geometric Brownian motion. Entries with a * next to them indicates an option price outside these 95 percent confidence intervals.

Strike price	BS	GBM	MR	NP	95%-confidence intervals for NP under the GBM assumption	
Panel A. Time-to-maturity: 7 Days						
140	9.990	9.990	10.022	10.005	9.855	10.120
145	5.008	5.007	5.044	5.054	4.871	5.139
150	0.940	0.944	0.962	1.159*	0.786	1.078
155	0.016	0.015	0.014	0.027	0.002	0.030
160	0.000	0.000	0.000	0.000	0.000	0.000
Panel B. Time-to-maturity: 91 Days						
140	10.315	10.315	10.300	10.392	9.802	10.740
145	6.335	6.325	6.316	6.694	5.835	6.755
150	3.350	3.342	3.296	3.759*	2.793	3.717
155	1.491	1.481	1.395	1.638	1.026	1.750
160	0.551	0.551	0.462	0.560	0.265	0.670
Panel C.Time-to-maturity: 182Days						
140	10.976	10.963	11.168	11.088	10.246	11.577
145	7.443	7.429	7.593	7.822	6.784	7.954
150	4.678	4.659	4.759	5.102	3.960	5.126
155	2.709	2.697	2.721	2.937	2.028	3.085
160	1.442	1.431	1.392	1.511	0.895	1.630
Panel D. Time-to-maturity: 273 Days						
140	11.581	11.596	12.014	11.714	10.673	12.363
145	8.299	8.317	8.641	8.655	7.424	9.039
150	5.657	5.671	5.870	6.047	4.801	6.268
155	3.661	3.679	3.753	3.852	2.803	4.074
160	2.247	2.261	2.240	2.300	1.514	2.570
Panel E. Time-to-maturity: 364 Days						
140	12.111	12.177	12.547	12.218	11.044	12.889
145	9.003	9.073	9.343	9.282	7.933	9.747
150	6.451	6.519	6.653	6.742	5.435	7.140
155	4.452	4.522	4.514	4.582	3.428	5.037
160	2.959	3.024	2.902	2.956	2.047	3.364

Table 4: Option prices for the FWV2-01 contract. The BS, GBM, and MR models denote the same models as in the main text. NP denotes the nonparametric model, equation (2). The last two columns contain the lower and upper 95 percent confidence intervals for the nonparametric option prices under the assumption that the true underlying model is a geometric Brownian motion. Entries with a * next to them indicates an option price outside these 95 percent confidence intervals.

Strike price	BS	GBM	MR	NP	95%-confidence intervals for NP under the GBM assumption	
Panel A. Time-to-maturity: 7 Days						
140	9.991	9.990	10.083	10.005	9.744	10.220
145	5.069	5.068	5.239	5.021	4.870	5.365
150	1.272	1.264	1.536	1.075*	1.117	1.707
155	0.087	0.085	0.162	0.040*	0.043	0.249
160	0.001	0.001	0.006	0.000	0.000	0.016
Panel B. Time-to-maturity: 91 Days						
140	10.980	10.981	11.968	10.374*	10.446	12.499
145	7.355	7.379	8.419	6.446*	6.857	8.922
150	4.543	4.573	5.525	3.727*	4.044	5.998
155	2.554	2.574	3.351	1.971*	2.110	3.855
160	1.311	1.309	1.850	0.956	0.938	2.270
Panel C.Time-to-maturity: 182Days						
140	12.191	12.175	13.668	11.178*	11.444	14.571
145	8.964	8.950	10.435	7.661*	8.237	11.300
150	6.330	6.320	7.691	5.142*	5.582	8.683
155	4.289	4.273	5.447	3.332*	3.636	6.213
160	2.787	2.770	3.688	2.083*	2.218	4.385
Panel D. Time-to-maturity: 273 Days						
140	13.190	13.260	14.931	11.919*	12.262	16.185
145	10.179	10.245	11.875	8.646*	9.260	13.144
150	7.655	7.717	9.214	6.217*	6.783	10.409
155	5.610	5.678	6.971	4.375*	4.726	7.958
160	4.007	4.074	5.120	3.024*	3.249	6.086
Panel E. Time-to-maturity: 364 Days						
140	14.030	14.068	15.929	12.453*	13.106	17.531
145	11.169	11.213	13.014	9.360*	10.186	14.531
150	8.728	8.773	10.433	7.025*	7.726	11.971
155	6.696	6.736	8.203	5.222*	5.748	9.538
160	5.046	5.077	6.322	3.832*	4.144	7.636

Table 5: Option prices for the FWYR-02 contract. The BS, GBM, and MR models denote the same models as in the main text. NP denotes the nonparametric model, equation (2). The last two columns contain the lower and upper 95 percent confidence intervals for the nonparametric option prices under the assumption that the true underlying model is a geometric Brownian motion. Entries with a * next to them indicates an option price outside these 95 percent confidence intervals.

Strike price	BS	GBM	MR	NP	95%-confidence intervals for NP under the GBM assumption	
Panel A. Time-to-maturity: 7 Days						
140	9.991	9.990	9.938	9.991	9.812	10.180
145	5.051	5.050	5.037	5.012	4.903	5.292
150	1.202	1.185	1.277	0.996*	1.053	1.584
155	0.066	0.065	0.083	0.043	0.032	0.183
160	0.001	0.001	0.000	0.000	0.000	0.007
Panel B. Time-to-maturity: 91 Days						
140	10.822	10.733	11.200	10.134*	10.292	11.961
145	7.132	7.015	7.563	6.338*	6.616	8.326
150	4.282	4.155	4.679	3.572*	3.760	5.480
155	2.322	2.208	2.620	1.925	1.873	3.351
160	1.132	1.047	1.307	1.064	0.778	1.855
Panel C.Time-to-maturity: 182Days						
140	11.917	11.840	12.618	10.778*	11.199	13.721
145	8.636	8.548	9.324	7.537*	7.893	10.458
150	5.979	5.884	6.607	5.092*	5.269	7.629
155	3.949	3.865	4.460	3.464	3.239	5.433
160	2.486	2.410	2.838	2.400	1.798	3.658
Panel D. Time-to-maturity: 273 Days						
140	12.834	12.784	13.636	11.470*	11.936	14.936
145	9.776	9.709	10.532	8.526*	8.854	11.763
150	7.231	7.161	7.888	6.276	6.259	9.075
155	5.192	5.133	5.711	4.677	4.237	6.854
160	3.620	3.576	3.982	3.552	2.666	5.115
Panel E. Time-to-maturity: 364 Days						
140	13.610	13.628	14.678	12.091*	12.504	15.856
145	10.705	10.724	11.674	9.396*	9.584	12.896
150	8.244	8.263	9.073	7.280	7.072	10.344
155	6.216	6.233	6.874	5.723	5.102	8.041
160	4.591	4.610	5.078	4.554	3.466	6.251

Table 6: Option prices for the FWSO-01 contract. The BS, GBM, and MR models denote the same models as in the main text. NP denotes the nonparametric model, equation (2). The last two columns contain the lower and upper 95 percent confidence intervals for the nonparametric option prices under the assumption that the true underlying model is a geometric Brownian motion. Entries with a * next to them indicates an option price outside these 95 percent confidence intervals.

Strike price	BS	GBM	MR	NP	95%-confidence intervals for NP under the GBM assumption	
			Panel A. Time-to-maturity: 7 Days			
115	9.991	9.990	9.962	9.992	9.816	10.212
120	5.067	5.061	5.110	5.011	4.917	5.406
125	1.270	1.250	1.446	0.983*	1.115	1.867
130	0.087	0.080	0.139	0.032*	0.047	0.298
135	0.001	0.001	0.003	0.000	0.000	0.017
			Panel B. Time-to-maturity: 91 Days			
115	10.954	10.940	11.603	10.422*	10.518	12.370
120	7.333	7.314	8.097	6.477*	6.876	9.031
125	4.524	4.505	5.241	3.573*	4.016	6.254
130	2.559	2.541	3.141	1.821*	2.119	4.008
135	1.325	1.312	1.727	0.865*	0.948	2.395
			Panel C.Time-to-maturity: 182Days			
115	12.145	12.110	13.219	11.330*	11.434	14.530
120	8.931	8.877	10.012	7.807*	8.190	11.335
125	6.317	6.258	7.308	5.121*	5.609	8.595
130	4.295	4.249	5.139	3.293*	3.595	6.383
135	2.810	2.782	3.450	2.066*	2.158	4.593
			Panel D. Time-to-maturity: 273 Days			
115	13.130	13.052	14.423	12.132*	12.235	16.164
120	10.138	10.053	11.376	8.857*	9.240	13.121
125	7.638	7.550	8.754	6.266*	6.758	10.473
130	5.617	5.538	6.549	4.395*	4.750	8.232
135	4.035	3.963	4.749	3.055*	3.197	6.294
			Panel E. Time-to-maturity: 364 Days			
115	13.958	13.820	15.383	12.849*	13.003	16.939
120	11.120	10.970	12.470	9.733*	10.048	14.177
125	8.707	8.556	9.893	7.252*	7.616	11.699
130	6.704	6.562	7.680	5.395*	5.628	9.474
135	5.079	4.954	5.818	3.993*	4.037	7.476

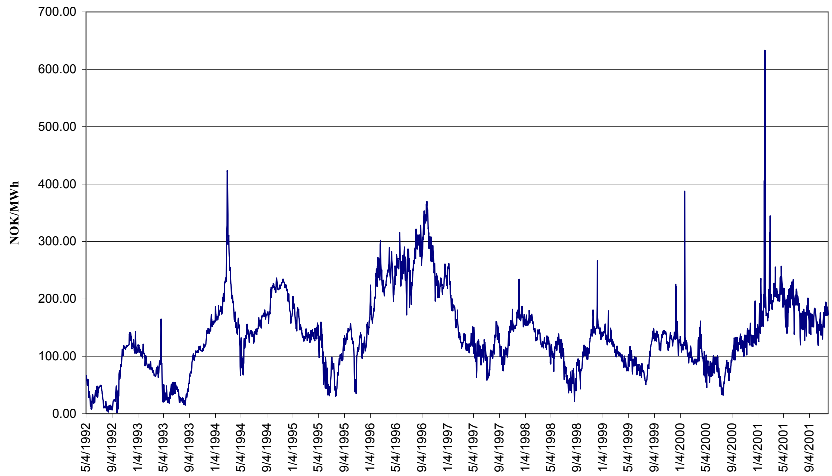


Figure 1: The system price at Nord Pool, in Norwegian kronor (NOK) per MWh, from May 4 1992 to November 30 2001.

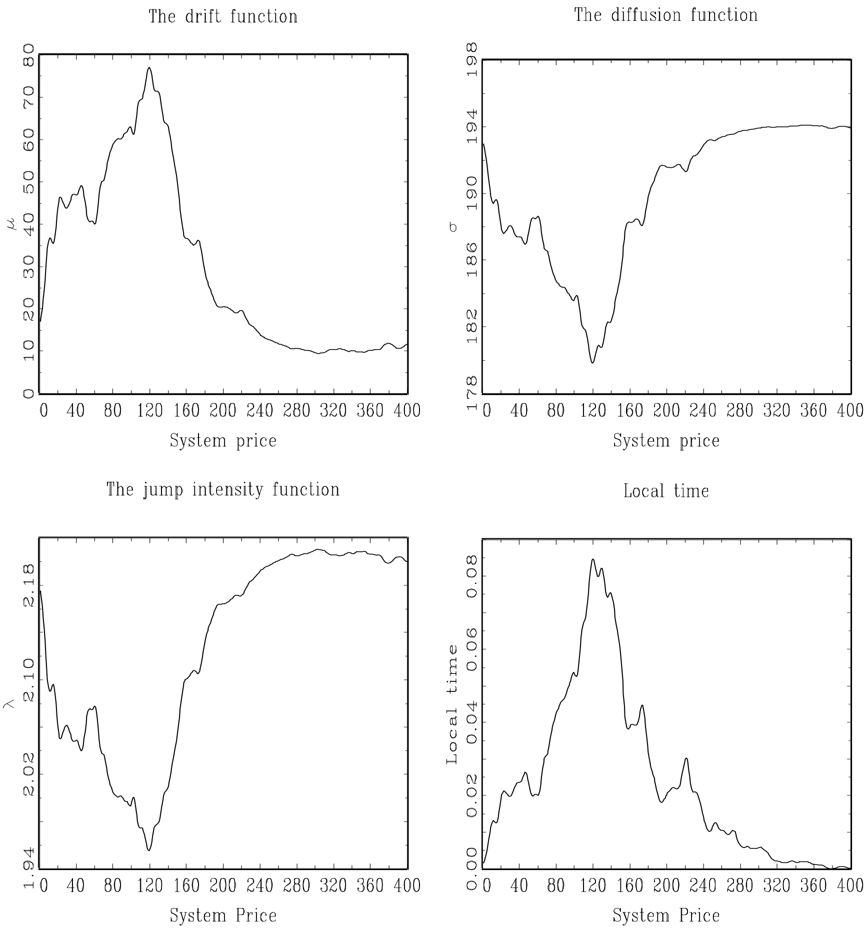


Figure 2: Estimates of the drift, diffusion, and jump intensity functions and the local time for the system price in the univariate jump diffusion model.

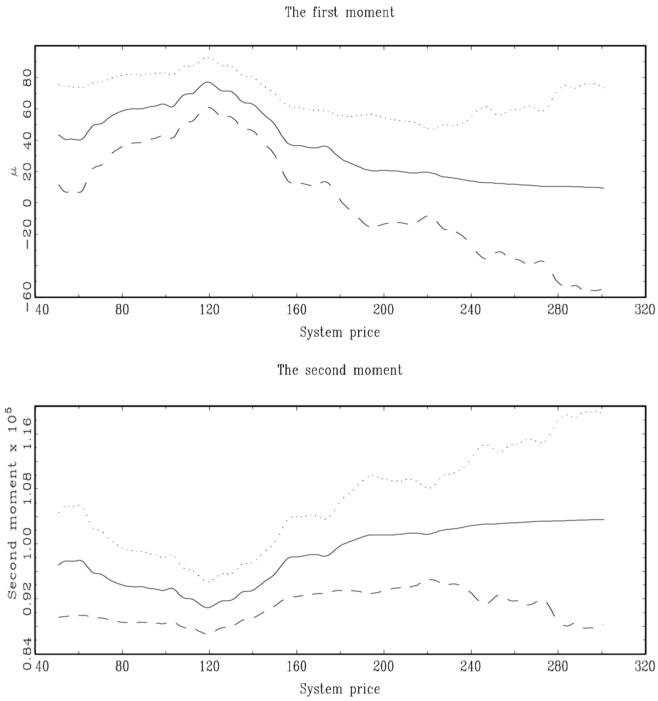


Figure 3: Confidence interval for the first moment and second moment of the system price in the univariate jump diffusion model. The first moment is also equal to the drift function.

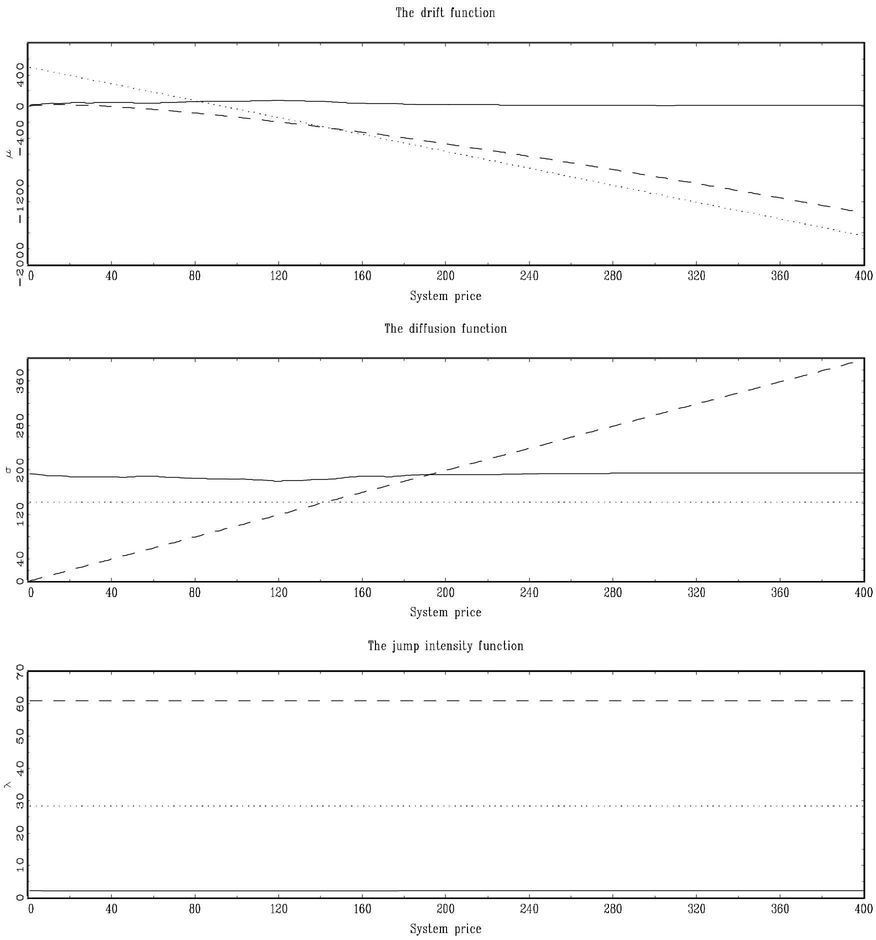


Figure 4: Comparison of the nonparametric estimates of equation (1) and the estimates of the GMRJ and MRJ models, for the system price. The solid lines are the nonparametric estiamtes, the dashed lines the GMRJ estimates, and the dotted lines the MRJ estimates.

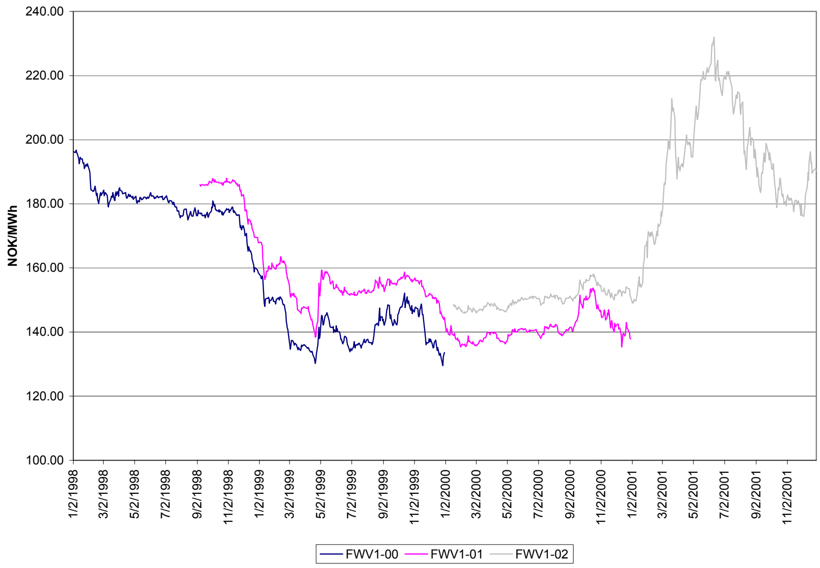


Figure 5: Price series for the FWV1-00, FWV1-01, and FWV1-02 forward contracts.

Figure 6: Price series for the FWSO-00, FWSO-01, and FWSO-02 forward contracts.

Figure 7: Price series for the FWV2-00, FWV2-01, and FWV2-02 forward contracts.

Figure 8: Price series for the FWYR-00, FWYR-01, and FWYR-02 forward contracts.

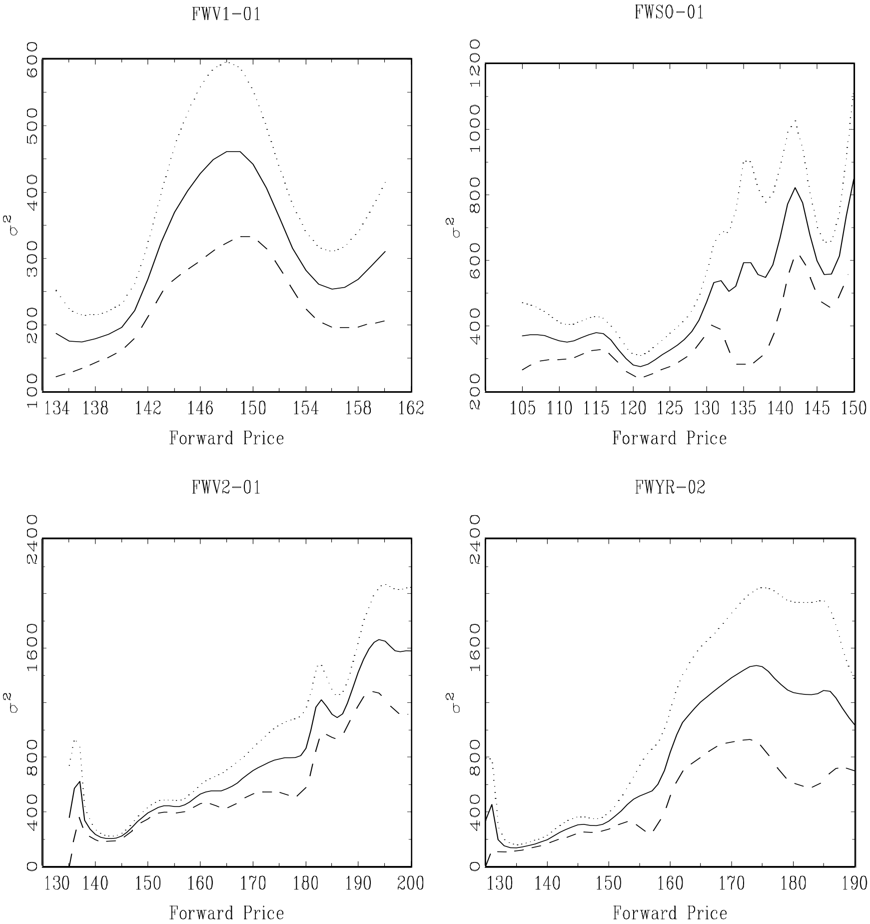


Figure 9: Estimates of the squared diffusion function, $\sigma_F^2(\cdot)$, for the four different forward contracts. The dotted and dashed lines are the upper and lower 95% confidence intervals, respectively.

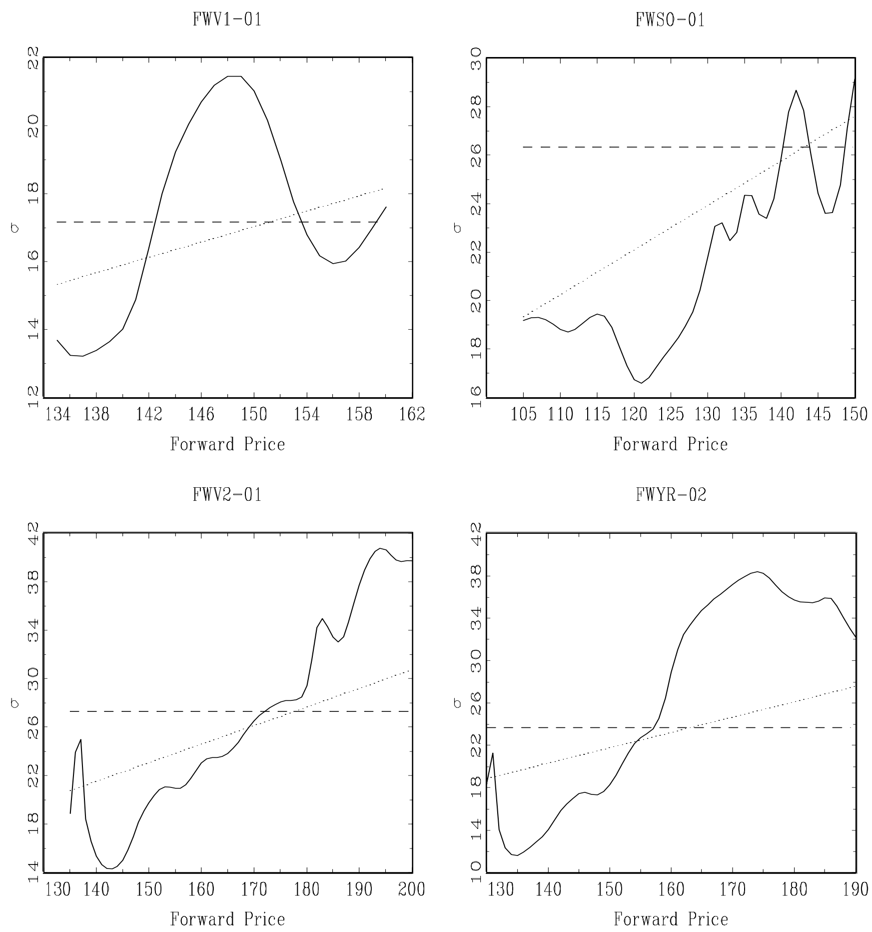


Figure 10: The solid lines are the nonparametric estimates of the diffusion function, $\sigma_F(\cdot)$, for the respective forward contracts, that is, the square root of the estimates shown in figure 9. The dashed lines are the estimated diffusion functions from the MR model and the dotted lines are the estimated diffusion functions from the GBM model.

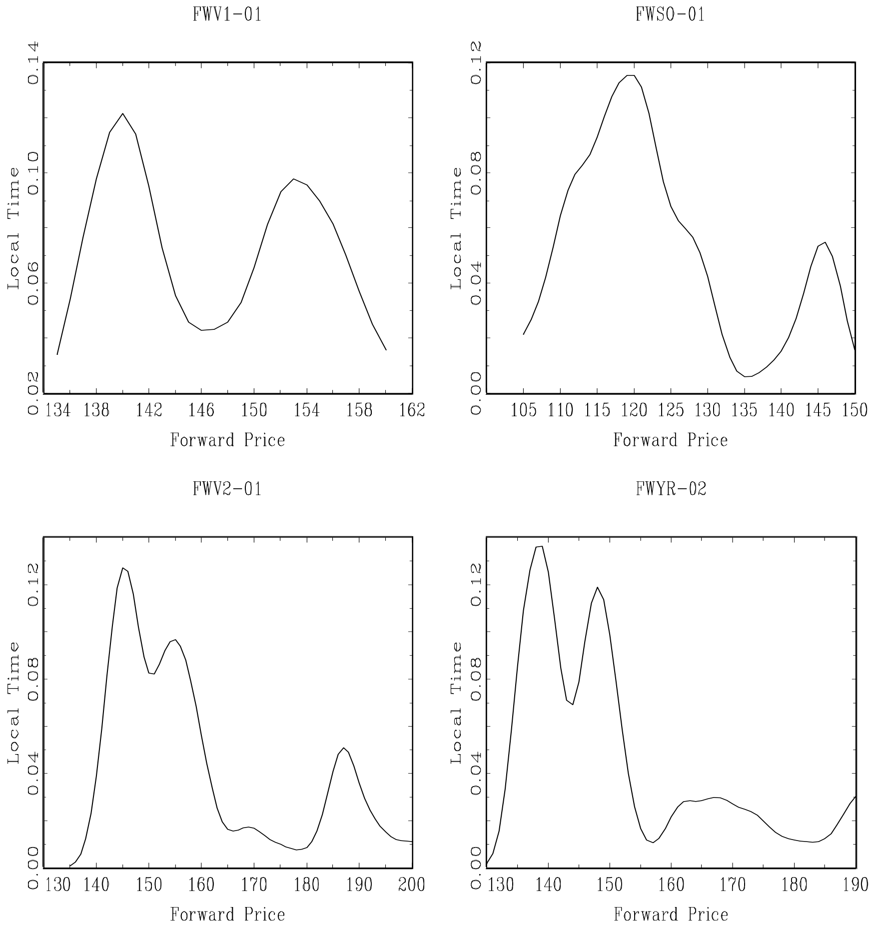


Figure 11: Estimates of the local time for the four forward contracts.

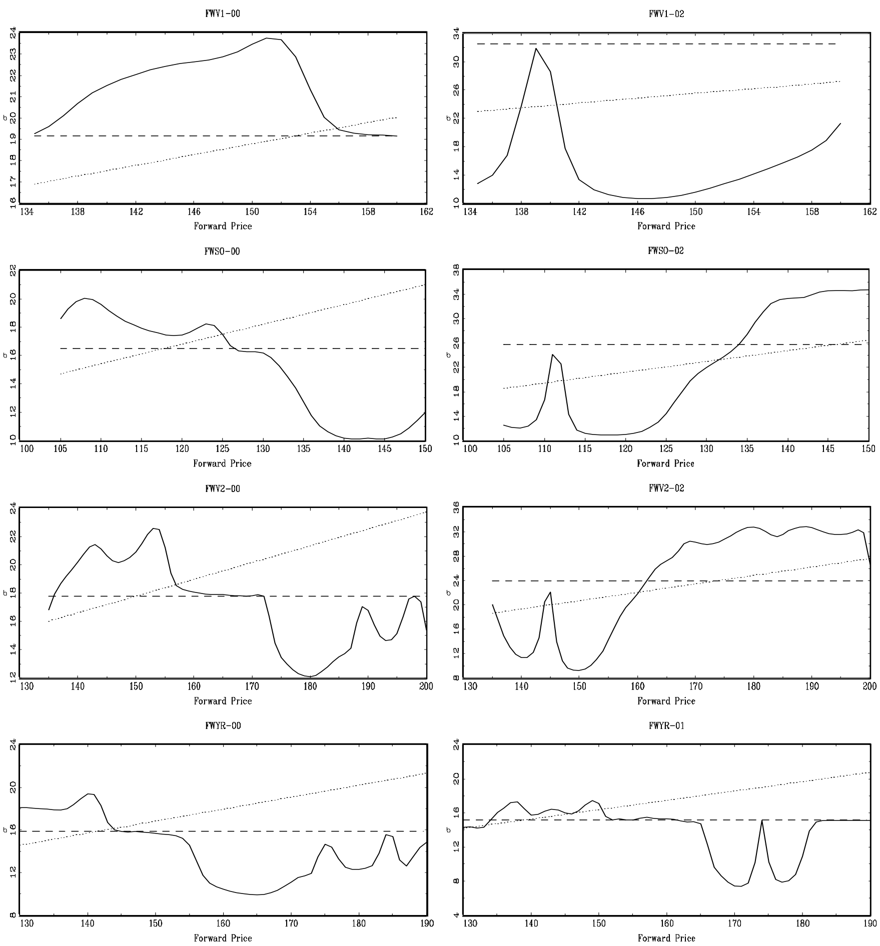


Figure 12: Estimates of the volatility functions for the other forward contracts. The solid lines are the nonparametric estimates of the diffusion function, $\sigma_F(\cdot)$, for the respective forward contracts. The dashed lines are the estimated diffusion functions from the MR model and the dotted lines are the estimated diffusion fucntions from the GBM model.

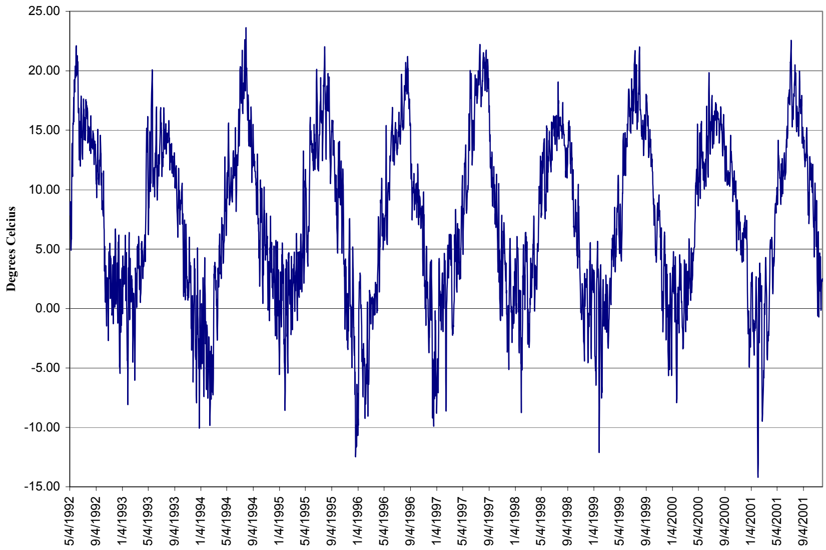


Figure 13: Aggregate temperature, May 4 1992 to November 30 2001.

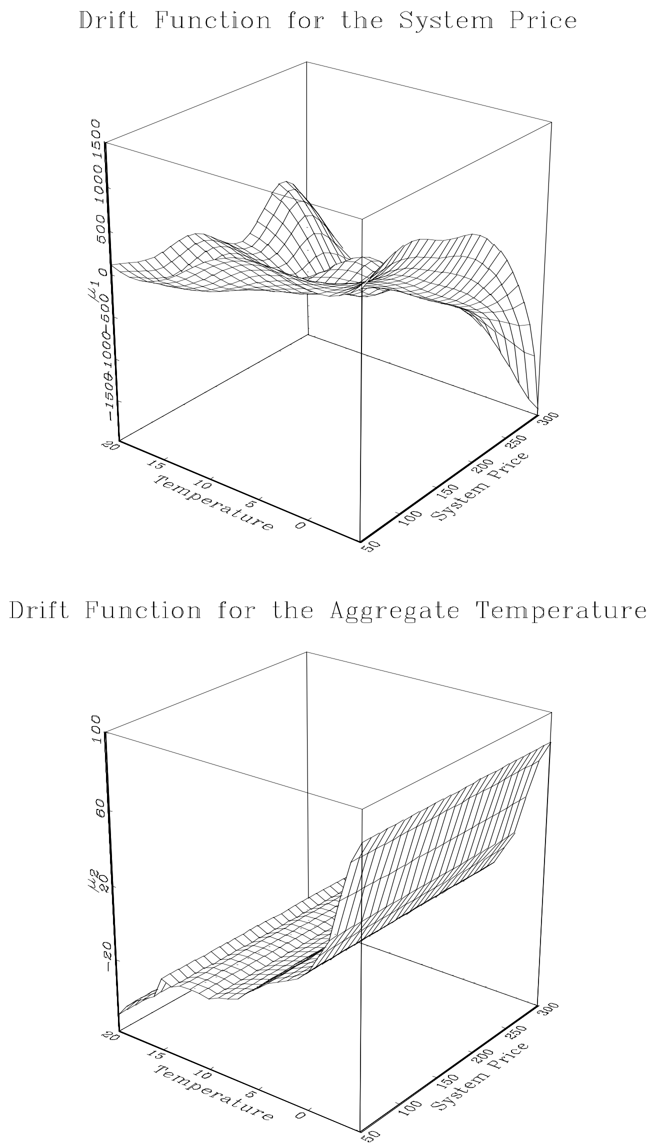
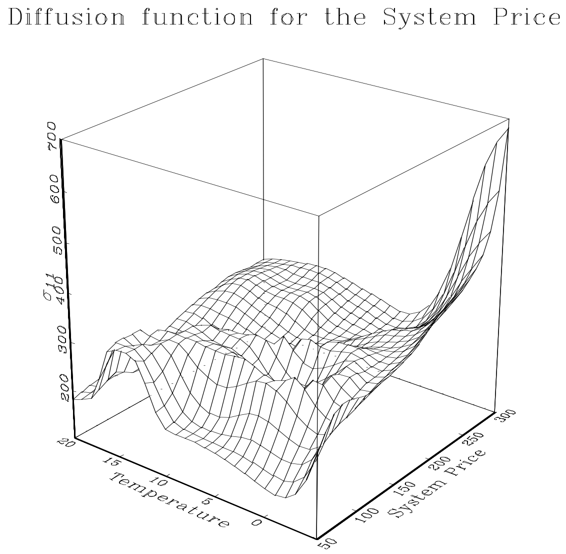


Figure 14: Estimates of the drift functions for the system price and the aggregate temperature in the bivariate diffusion model.



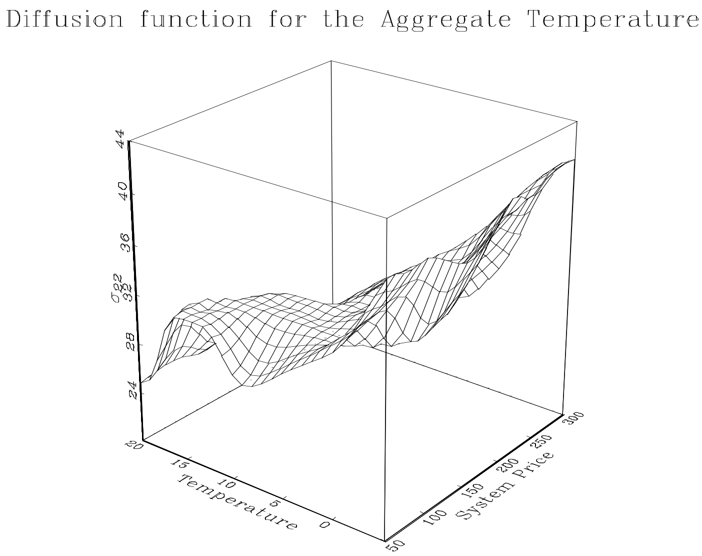


Figure 15: Estimates of the diffusion functions for the system price and the aggregate temperature in the bivariate diffusion model (σ_{11} and σ_{22} respectively).

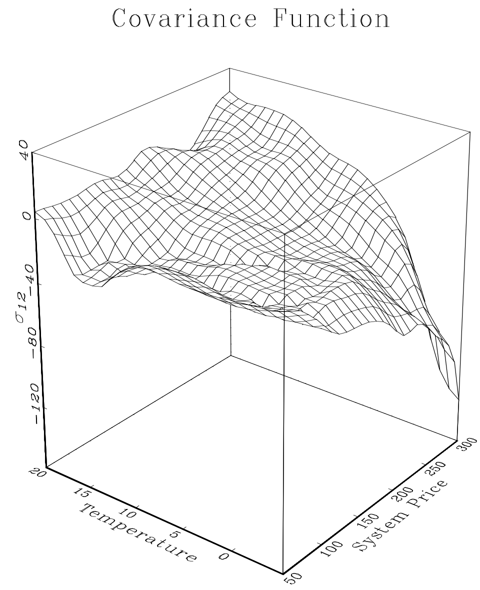


Figure 16: Estimate of the covariance function (σ_{12}) in the bivariate diffusion model.

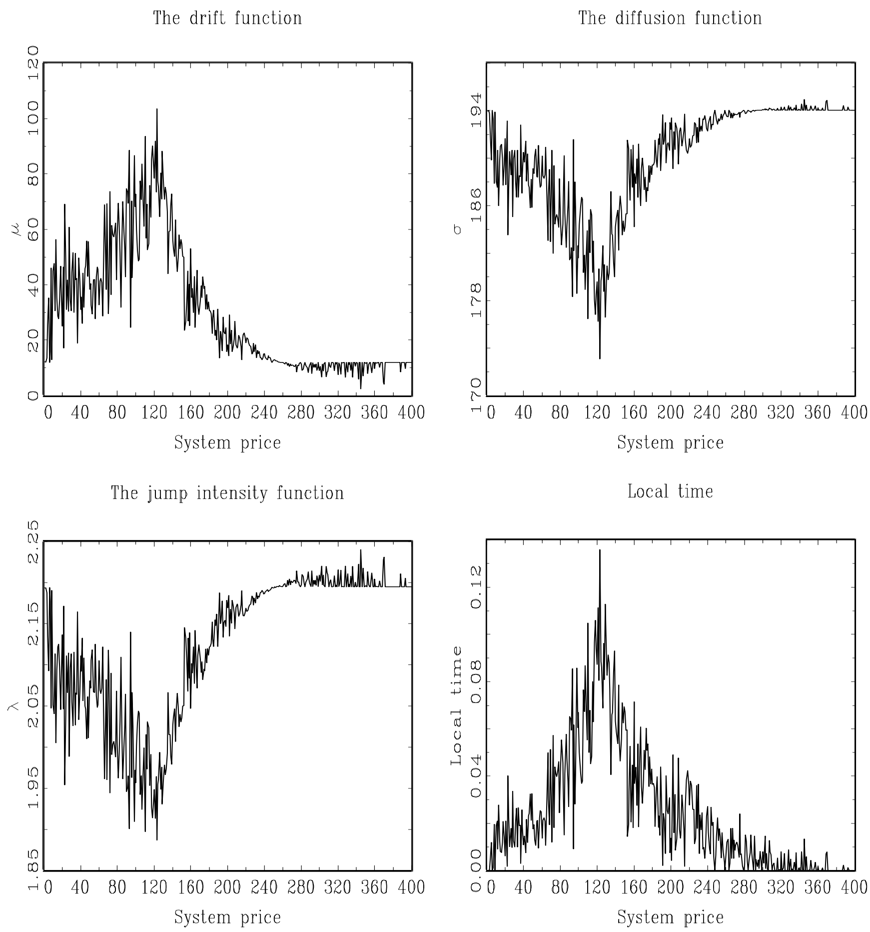


Figure 17: Estimates of the drift, diffusion, and jump intensity functions and the local time for the system price in the univariate jump diffusion model. The bandwidth parameter c^{ltime} is set equal to the standard deviation of the system price.

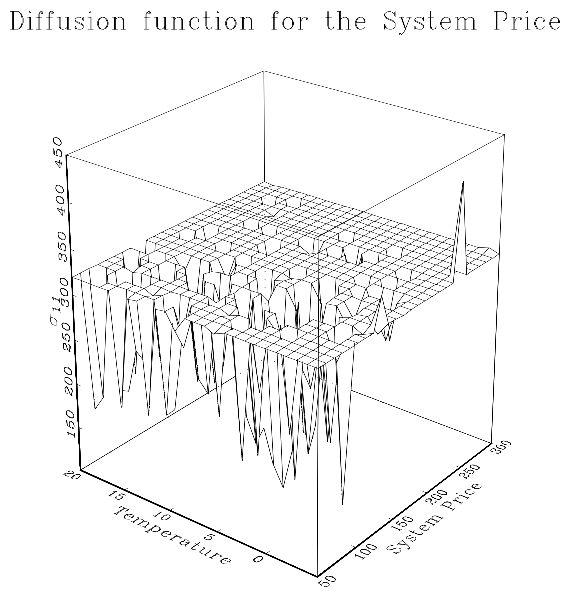


Figure 18: Estimate of the diffusion function for the system price in the bivariate diffusion model. The bandwidth parameter c^{ltime} is set equal to $diag\left(S^{1/2}\right)$, where S is the sample covariance matrix of the system price and the aggregate temperature.

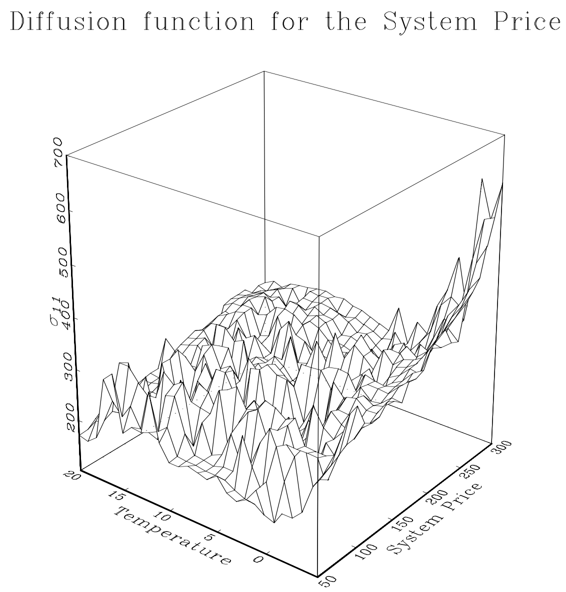


Figure 19: Estimate of the diffusion function for the system price in the bivariate diffusion model. The bandwidth parameter c^{ltime} is set equal to $20 * diag\left(S^{1/2}\right)$, where S is the sample covariance matrix of the system price and the aggregate temperature.

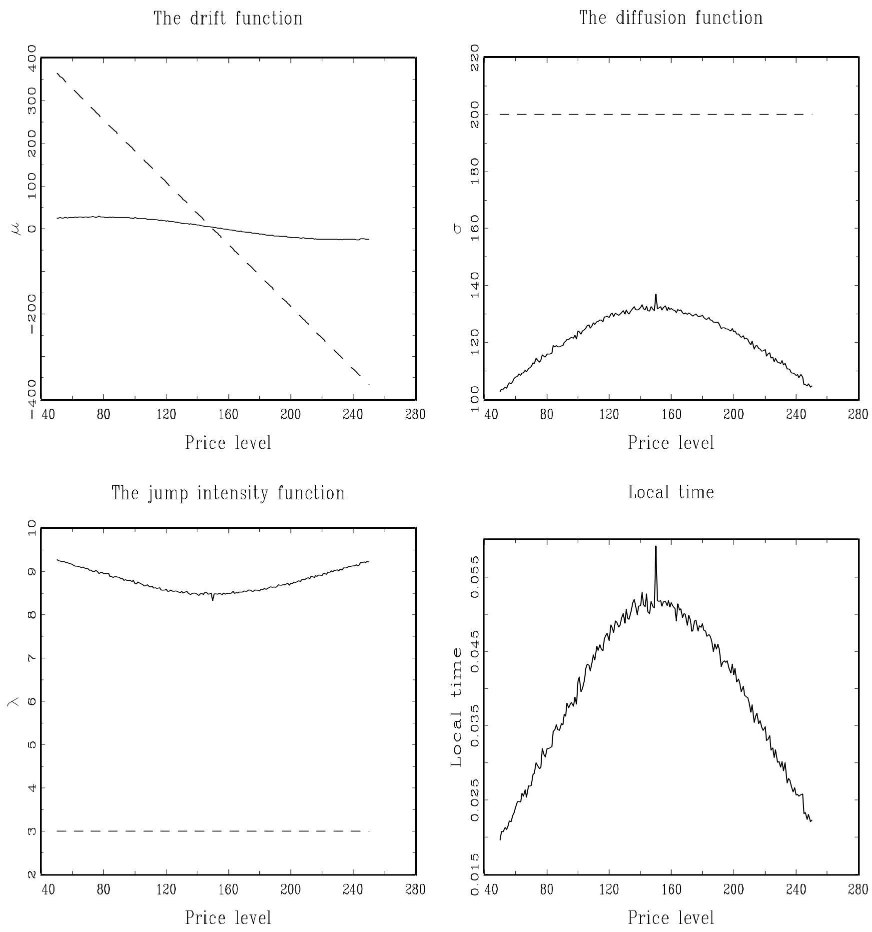


Figure 20: Results from the Monte Carlo study using equation (31). The bandwidth parameter c^{ltime} is set equal to the standard deviation of the generated data. The solid lines are the mean estimates of the respective functions and the dotted lines are the true functions. The mean estimates are obtained from 500 repetitions of sample paths, consisting of 3500 daily observations.

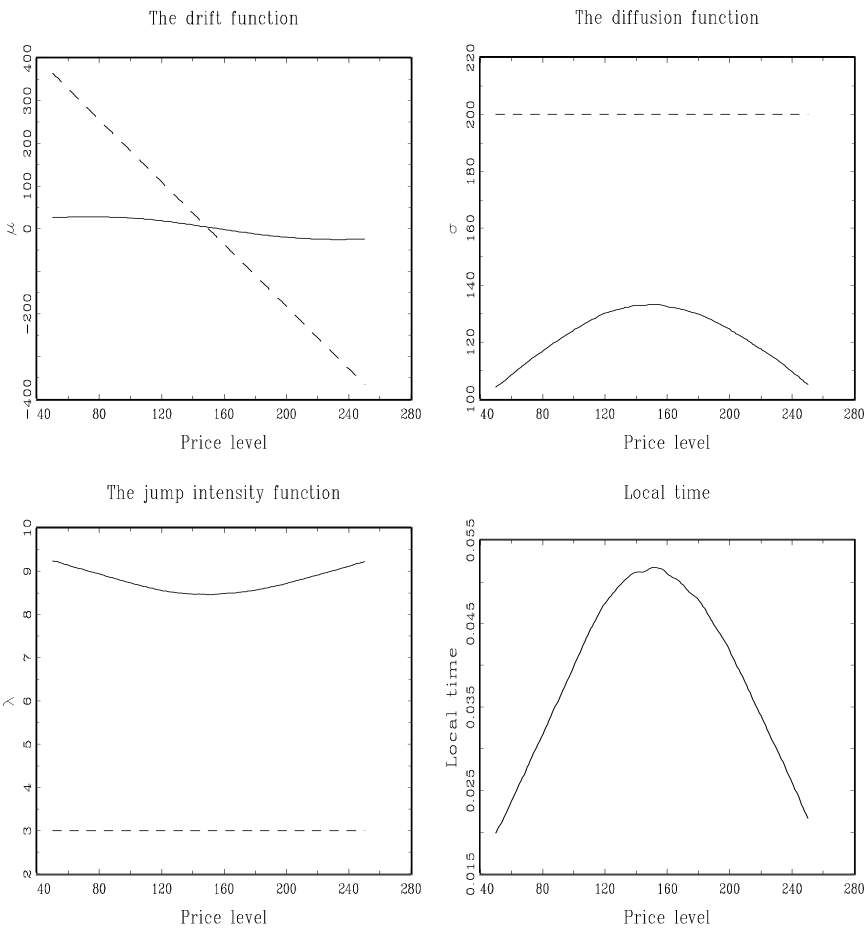


Figure 21: Results from the Monte Carlo study using equation (31). The bandwidth parameter c^{ltime} is set equal to 20 times the standard deviation of the generated data. The solid lines are the mean estimates of the respective functions and the dotted lines are the true functions. The mean estimates are obtained from 500 repetitions of sample paths, consisting of 3500 daily observations.

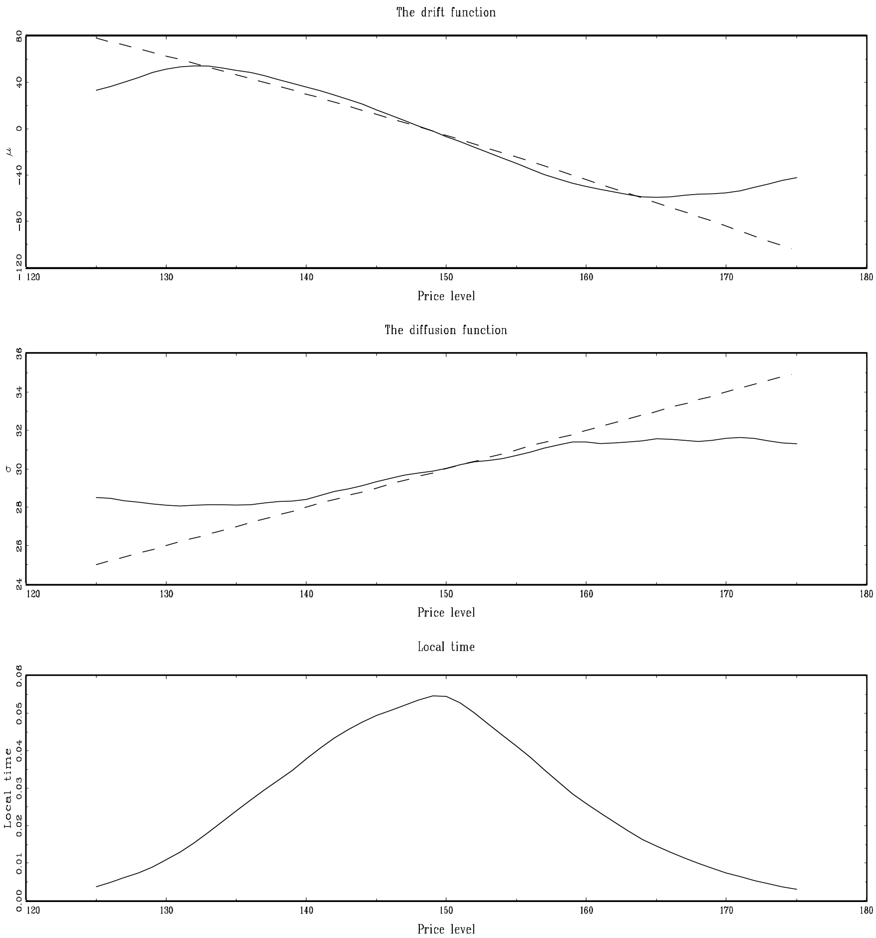


Figure 22: Results from the Monte Carlo study using equation (32). The bandwidth parameter c^{ltime} is set equal to 20 times the standard deviation of the generated data. The solid lines are the mean estimates of the respective functions and the dotted lines are the true functions. The mean estimates are obtained from 500 repetitions of sample paths, consisting of 600 daily observations.