Negative Externalities in Day Care: Optimal Tax Policy Response*

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Abstract

Systematic pediatric evidence shows that the morbidity rates for children in day care are increasing in the group size. Sick children are usually cared for at home by parents. This creates a negative externality of parents' labor force participation. The social optimum implies lower group size than the non–intervention market equilibrium. We study the optimal tax policy. The cost of labor force participation should be increased. This can be done by either or both a *tax* on day care services and a home care *allowance*. The cost of providing day care should be decreased by a *subsidy* to entrepreneurs running day care centers. This policy will decrease the group size. It is, however, not necessarily the case that this will decrease labor force participation.

JEL: D1, D62, H23, I12, J13 and J21.

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1 Introduction

Most working parents have probably faced the following situation: The child is ill in the morning and must stay at home instead of attending the regular out–of–home day care. Parents are aware of the financial consequences of this; while attending the child at home one parent will probably lose some income.[1] The loss will depend on, i.a., the availability of social insurance and alternative forms of child care.[2] Such costs are taken into account when parents decide whether or not to participate in the labor force.[3] There may, however, also be social cost associated with participation in the labor force. We focus on one particular social cost. Children attending day care centers are ill more often than children cared for in other ways:[4]

> "Children in child care centers, especially those younger than 36 months of age, appear to have greater numbers of reported illnesses and greater numbers of illness and bed days than children cared for in their own homes. These findings persist in multiple studies, despite differences in study design, method of collecting the major variables, and the location of the study."
> Landis and Chang (1991, p. 710)

The medical reason for this is that children are in close contact, they use the same toys, the same hygienical facilities, etc. (Laborde et al., 1994). Hutchinson (1992) reviews several studies. They show that the probability of infectious organism transmission is lower if day care is organized in smaller groups. Transmission is particularly low if children are cared for in same–age groups. It does not matter, on the other hand, if day care centers are large or small.[5] Collet et al. (1994a,b), however, report that the infection risk is higher in small day care centers than in child care at home. An explanation for this may be that children are in larger groups in small day care centers compared to at home. At the same time, the risk is lower in large day care centers than in small. The reason may be that children often are divided into homogenous age groups in large day care centers. Also, larger day care

[1] In general not only parents but also older siblings may take care of a sick child. As an example Pitt and Rosenzweig (1990) studies how child morbidity affects the allocation of time within the family. This may give rise to difference in human capital accumulation between siblings of different sexes.

[2] In some countries alternative forms of care exist for sick children; see for instance Landis and Chang (1991); Giebink (1993); Giebink et al. (1994).

[3] Other private costs are expenditure on medical care, medicine etc.

[4] See Landis and Chang (1991) and the references quoted therein, Hutchinson (1992) and references quoted therein, Möttönen and Uhari (1992), Osterholm et al. (1992), references quoted in Reves and Pickering (1992), Thacker et al. (1992), Reves et al. (1993), Jorm and Capon (1994), Louhiala et al. (1995), and Holmes et al. (1996).

[5] Hutchinson (1992) recognizes other ways of infection control. One is specialization of certain functions of the staff, e.g., changing diapers versus preparing food.

centers are more often built specially for their purpose. The problems of hygiene has, therefore, received special attention during the design.[6]

There may, however, exist positive effects of illness. Reves et al. (1993) and Collet et al. (1994a,b) report that longer time in day care increases the protection against repeated infections. This is the result even after controlling for age. Early infections may protect against allergies in later life. Krämer et al. (1999) find that children who start attending day care centers at a young age have fewer allergies later on in life. The comparison group are children who started at an older age. These findings, however, only apply for children with no siblings.

In this paper we are concerned with these indirect effects of higher risks to be ill for children attending out–of–home day care. Our main assumption is that increased group size in day care implies a higher risk that a child will become ill. The assumption is based on the evidence reported above, If parental labor force participation increases, *ceteris paribus*, the number of children in each group in the day care centers will increase. This will increase the risk for each child to become ill. On the other hand, if the number of day care centers increases, *ceteris paribus*, the number of groups in day care will increase. This will decrease the number of children in each group. The risk of becoming ill will decrease for each child.

We also assume that parents and day care center entrepreneurs neglect these indirect effects. This means, on the one hand, that labor force participation is associated with a negative externality. On the other hand, the reducing the group size at day care centers is associated with a positive externality. Our focus is on the character of the optimal Pigouvian taxes and subsidies that are implied by the social optimum. In our model, however, the Pigouvian solution and the second best tax solution coincide.

Our first main results concerns labor force participation in the social optimum. The cost of labor force participation for the marginal participating parental household should be raised above the private cost of participation. This will make households take the negative externality that their participation causes into account. Second, the cost of running day care centers should be reduced below the private cost. Entrepreneurs will then take the positive externality of starting a day care center into account. The group size is smaller in the social optimum than the non–intervention market equilibrium, but labor force participation need not be smaller.

[6]Cordell et al. (1997) reports higher morbidity rates for day care homes (12 or less children) than for day care centers (more than 12 children). There is, however, no control for group size within day care centers. Data are based on reports from the day care facility. It is likely that policies regarding keeping mildly ill children at home differs between the two day care categories. This affects not only reported but also actual morbidity rates. Parents of illness–prone children may tend to enroll at day care centers which may not require such children to stay home. This may create a selection bias.

The simplest way to implement a policy to reach the social optimum is through a Pigouvian tax on day care services. This is equivalent to a tax on labor force participation in our model. Second, there should be a Pigouvian subsidy for running day care centers. Such a Pigouvian policy will in our model exactly balance the government's budget. It is, however, possible to use any combination of a tax on day care services and a home care allowance. This allowance is only paid to households not participating in the labor force. The only requirement is that the cost of participating in the labor force reaches the social optimum. This policy will decrease the group size. It is, however, not necessarily the case that this will decrease labor force participation as the number of day care centers may increase.

These results can be compared with traditional economic arguments for the *subsidizing or publicly providing* day care: Several contributions have studied the relation between availability of day care and parents decision to enter the labor force. Bergstrom and Blomquist (1996) model the increase in the tax base that higher labor force participation of parents leads to. Increased labor force participation may also reduce equilibrium wages, this is modelled by Lundholm and Ohlsson (1998). Blomquist and Christiansen (1995) focus on the distributional aspects of public provision of good such as day care. In all these contributions some subsidy or public provision is Pareto efficient. These contributions do not, however, study whether the intervention should be directed to the demand or supply side.

This paper focuses an additional argument for public intervention in the day care market. We conclude that there are reasons for giving households incentives to demand less day care. Day care suppliers should be encouraged to reduce group size.

Pediatricians have discussed how to deal with the problems created by the increased illness of children at day care centers. Landis and Chang (1991), for example, discusses several measures to deal with these problems. One objective of these measures is to solve the problem of day care for sick children. This, however, reduces the cost of labor force participation of parents. Parents may get stronger incentives to participate in the labor force. These measures may, therefore, aggravate the problem of negative externalities.

A second objective is to reduce the spread of the infectious diseases. This reduces morbidity rates and, therefore, improves social welfare. Measures with this objective concern changes in how day care is produced. This type of measures may, therefore, be more in line with the economic conclusions of this paper.

The paper continues with Section 2 that introduces the model. It also describes the labor force participation decision and the decision to run day care centers. The social optimum is discussed in Section 3. In Section 4 we interpret this optimum in terms of a structure of an optimal (Pigouvian) policy. The second best policies are discussed in Section 5. Section 6 concludes.

2 Model

2.1 The supply of day care

Consider price taking entrepreneurs who differ in their ability to run day care centers efficiently. This ability is measured by a fixed cost parameter $\gamma \in [0, \infty)$. The distribution of fixed costs is described by the cumulative distribution function $R : [0, \infty) \to [0, 1]$ defined by $R(\gamma)$. We assume that R is continuously differentiable and strictly increasing on its entire support. This means that R has a density function r such that $r(\gamma) > 0 \ \forall \gamma \in [0, \infty)$. Entrepreneurs will choose day care capacity (i.e., the number of children at their day care center) to maximize profits π. There is only one group at each day care center. Capacity will, therefore, equal group size g. Profits are given by[7]

$$\pi = \begin{cases} 0 & \text{if } g = 0 \text{ and} \\ \upsilon_p g - \frac{1}{2} g^2 - \gamma & \text{if } g > 0, \end{cases} \tag{1}$$

where υ_p is the producer price for day care for one child, $\upsilon_p g$ is total revenues at capacity g, and $\frac{1}{2} g^2$ is the variable cost at capacity g. Let g^* be the profit maximizing choice given that some day care is produced. The first order condition is $g^* = \upsilon_p$ and the profit function is $\frac{1}{2} \upsilon_p^2 - \gamma$, for entrepreneurs who decide to run day care centers. Hence, only the most efficient entrepreneurs will decide to be on the day care market. Let $g(\upsilon_p, \gamma)$ be the profit maximizing day care capacity at the producer price υ_p for entrepreneur γ so that

$$g(\upsilon_p, \gamma) = \begin{cases} 0 & \forall \gamma > \tilde{\gamma} \text{ and} \\ g^* = \upsilon_p & \forall \gamma \leq \tilde{\gamma}, \end{cases} \tag{2}$$

where the cost threshold for entry, $\tilde{\gamma} := \frac{1}{2} \upsilon_p^2$, is given by zero profits for the marginal profit maximizing entrepreneur. $R(\tilde{\gamma})$ entrepreneurs will, therefore, enter the day care market at any given producer price υ_p. Each day care center has the capacity to receive υ_p children. The total supply of day care, at the market price υ_p, can be written as $S(\upsilon_p) := R(\tilde{\gamma})\, g^* = R\left(\frac{1}{2} \upsilon_p^2\right) \upsilon_p$.

2.2 The demand for day care

Consider risk–neutral families with one child in which one spouse works full time. The other spouse either participates in the labor force or stays at

[7] In reality there may be a distinction between out–of–home day care and no–nanny day care (i.e., a single person cares for a child outside the home). In this model we make no distinction between the two. We focus on the group size in day care; i.e., a day care worker takes care of several children in a day care center. Only if there are sufficiently many day care centers, the group size decreases close to nanny day care (or for that case, home day care). The negative externality will then disappear.

home taking care of the child. If the child becomes ill the second spouse stays home temporarily and takes care of the child.[8] If the second spouse does not participate in the labor force he gets utility from staying home with his child. The monetary measure if this utility is $z > 0$. If instead the second spouse participates in the labor force the family earns the income $y \in [0, \infty)$. At the same time there is a loss of $z > 0$.[9] The distribution of income is described by the cumulative distribution function $F : [0, \infty) \to [0, 1]$ defined by $F(y)$. We assume that F is continuously differentiable and strictly increasing on its entire support. This means that F has the density function f such that $f(y) > 0 \; \forall y \in [0, \infty)$.

The family commits itself to purchase full time day care service for the child if the second spouse participates in the labor force. The consumer price of day care is v_c. Let $x \in 0, 1$, where $x = 1$ denotes that he participates in the labor force and $x = 0$ that he does not participate. The payoff of the labor participation decision is described by a function $U : \{0, 1\} \to \mathbb{R}_+$ defined by

$$U(x) = \begin{cases} z & \text{if } x = 0 \\ py - v_c & \text{if } x = 1, \end{cases} \tag{3}$$

where p is the probability that the child is well. Our main assumption is that this probability is determined by the average group size in day care g. This is defined as the ratio between the total number of children in day care and the total number of day care centers. We make the following assumption regarding the probability:

Assumption 1. *The probability p that the child is well is a function $p : [1, \infty) \to [0, 1]$ defined by $p(g)$ such that $p(1) = 1$ and $\lim_{g \to \infty} p(g) = 0$.*

Each household has to make its decision given its expectation of the group size, g^e. We assume that this expectation is common to all households. A household with income y will choose whatever alternative, $x(y)$, that maximizes its payoff given its expectation g^e; i.e.,

$$x(y) = \begin{cases} 0 & \text{if } p(g^e)y - v_c < z \text{ and} \\ 1 & \text{if } p(g^e)y - v_c \geq z. \end{cases} \tag{4}$$

The expected income $p(g^e)y$ is strictly increasing in y. There exists a unique, finite and strictly positive income threshold for labor force participation for every probability $p(g^e) \in (0, 1]$. This threshold, $\tilde{y}(g^e, v_c + z)$, is defined by

$$p(g^e)\tilde{y} - v_c - z = 0, \tag{5}$$

[8] We abstract from that in reality there is always some ambiguity whether the child is ill or not. Even sick children may in reality attend day care. The day care center may have an economic incentive to provide day care for a child even if the child has an acute illness. We also abstract from the possibility that parents may become ill. Reves and Pickering (1992) stresses that many diseases easily transmit to parents and day care personnel.

[9] Alternatively, $z > 0$ can be interpreted as a fixed real cost of labor force participation.

such that all households with income below $\tilde{y}(g^e, \upsilon_c + z)$ will not participate in the labor force. All household with higher income than $\tilde{y}(g^e, \upsilon_c + z)$ will participate in the labor force. In the case of $p(g^e) \to 0$ then $\tilde{y} \to \infty$ by Assumption 1. If $p(g^e) = 1$ then $\tilde{y} = z + \upsilon_c$. In the latter case the share of households $F(\upsilon_c + z)$ will not participate in the labor force. Therefore, threshold income is a function $\tilde{y} : [0, \infty) \to [\upsilon_c + z, \infty)$, defined by $\tilde{y}(g^e, \upsilon_c + z)$.

Suppose that the second spouse decides to participate in the labor force. The expected probability of working will be high if the expected group size is low. The threshold income will be low. This implies, for a given number day care centers, that the actual group size will be very high. This is an inconsistency.

The demand for day care, given the consumer price υ_c and expected group size g^e, is

$$D(g^e, \upsilon_c + z) = 1 - F(\tilde{y}) = 1 - F\left(\frac{\upsilon_c + z}{p(g^e)}\right).$$

Group size expectations are self–fulfilling when the expected group size is equal to the actual group size, $g^e = g^*$. The group expectations need to be self–fulfilling for day care demand to be based on rational expectations. There exists a unique self–fulfilling expected group size since threshold income $\tilde{y}$ is monotone in expected group size. It follows from equation (5) and Assumption 1 that $\frac{d\tilde{y}}{dg^e} = -\frac{p'\tilde{y}(g^e, \upsilon_c + z)}{p(g^e)} > 0$. Hence

$$g^e = g^* = \upsilon_p, \tag{6}$$

taking the first order condition the profit maximizing entrepreneurs into account. The demand for day care will, therefore, be

$$D(\upsilon_p, \upsilon_c + z) = 1 - F\left(\frac{\upsilon_c + z}{p(\upsilon_p)}\right).$$

2.3 Equilibrium in the day care market

Equilibrium in the day care market is given by the market clearing condition. The quantity demanded, given the consumer and producer prices, must equal the quantity supplied, given the producer price. Putting the different parts together reveals that we have a model with three equations. The system is

$$1 - F(\tilde{y}) = R(\tilde{\gamma})\upsilon_p, \tag{7a}$$

$$p(\upsilon_p)\tilde{y} - \upsilon_c - z = 0, \tag{7b}$$

$$\frac{1}{2}\upsilon_p^2 - \tilde{\gamma} = 0, \tag{7c}$$

the day care market equilibrium condition (7a), the indifference condition for the marginal household (7b), and the zero profit condition for the marginal entrepreneur (7c). Hence we have a system with three equations and four endogenous variables $(\tilde{y}, \tilde{\gamma}, v_p, v_c)$. It remains to specify how producer and consumer prices are related. Once this is added, the number of equations equals the number of endogenous variables.

Suppose first that there is no difference between consumer and producer prices ($v_c = v_p := v$). The system now contains three endogenous variables and three equations. It is well–defined. We can collapse it into one equilibrium condition which can solved for the equilibrium price v:

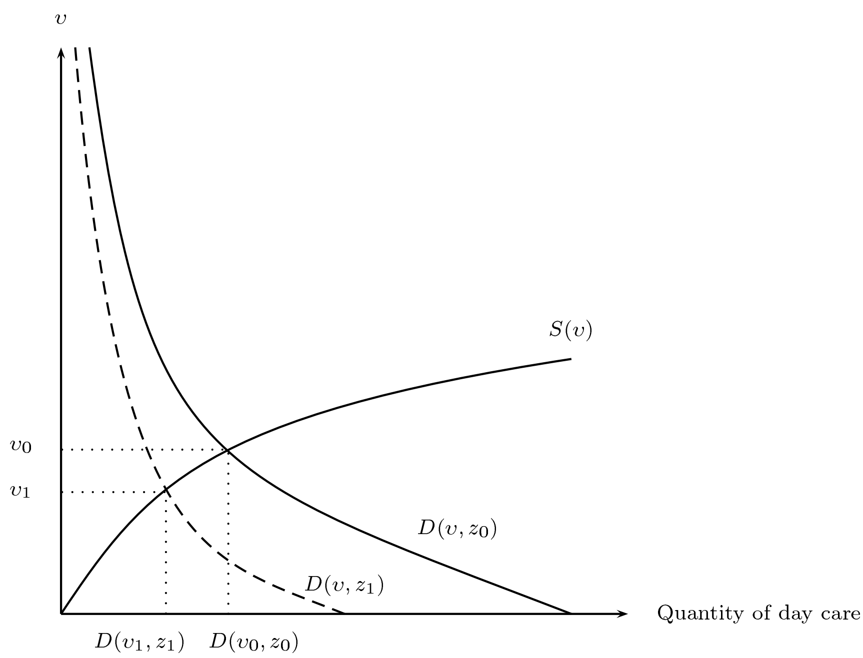
$$D(v, z) = S(v) \quad \Longleftrightarrow \quad 1 - F\left(\frac{v+z}{p(v)}\right) = R\left(\frac{1}{2}v^2\right)v. \tag{8}$$

There is a positive supply relation between the number of places in day care and the (producer) price of day care. There will be no supply if $v = 0$. Hence, we have $\frac{\partial S}{\partial v} = R(\tilde{\gamma}) + r(\tilde{\gamma})v > 0$ with $S(0) = 0$. Also, there is a negative demand relationship between the number of children in day care and the price of day care, $\frac{\partial D}{\partial v} = -f(\tilde{y})\frac{\partial \tilde{y}}{\partial v} < 0$. If $v \to \infty$ there will be no demand for child care, that is $D(v, z) \to 0$. Also, when $v = 0$ it follows from (5) that $\tilde{y} = v + z > 0$. Hence, we will always have a unique, strictly positive and finite equilibrium price. The monetary measure of the utility of staying home with the child $z > 0$ has a negative impact on labor force participation. This means that an increase in this income reduces the demand for day care, $\frac{\partial D}{\partial z} < 0$.

It is instructive to draw a figure in the quantity–price space; see Figure 1. This figure is drawn under the assumption that the distributions F and R are uniform and that the utility of staying home is z_0. Relaxing this assumption the demand (supply) function will still have a (negative) positive slope, but it need not be concave (convex) everywhere. The equilibrium price for day care is determined by the equilibrium condition $D(v_0, z_0) = S(v_0)$, where v_0 denotes the equilibrium price. The only exogenous variable that will change the equilibrium is the home utility value z. If this value increases, the equilibrium price and quantity will go down. Not participating in the labor force becomes more attractive. This decreases demand for day care. This is illustrated in Figure 1 where z_0 increases to z_1. This results in the dashed demand $D(v, z_1)$ and the lower equilibrium price v_1.

Suppose instead that consumer and producer prices may differ. This may, for instance, be because there exists a tax. We can conceptually think of the producer price v_p as an exogenous variable whereas the consumer price v_c is endogenous. The model can be reduced to a two–variable system with v_c and $\tilde{\gamma}$ as endogenous variables. It consists of the equilibrium condition (7a) and the first order condition for the marginal household (7b). Suppose that we differentiate the system with respect to the producer price v_p. This means that we ask which labor force participation and consumer

Figure 1: Non–intervention equilibrium in the day care market ($z_1 > z_0$).



price are consistent with any given point on the day care supply curve. The equilibrium condition yields a negative trade–off between threshold income and the producer price. It is

$$\frac{d\tilde{y}}{dv_p} = -\frac{1}{f(\tilde{y})}\left(R(\tilde{\gamma}) + rv_p^2\right) < 0. \tag{9}$$

where we have suppressed any indices indicating that we are dealing with equilibrium prices and quantities. If the producer price increases more day care will be supplied. It is necessary for equilibrium that the threshold income decreases so that the demand keeps up with the supply.

The trade–off (9) can be used when solving the first order condition for the marginal household for the trade–off between consumer and producer prices. It is

$$\frac{dv_c}{dv_p} = -\frac{p}{f(\tilde{y})}\left(R(\tilde{\gamma}) + rv_p^2\right) + p'\tilde{y} < 0. \tag{10}$$

This trade–off is also negative. If the producer price increases it is necessary for equilibrium that the consumer price decreases. A lower consumer price will increase demand, matching the increased supply that the higher producer price will lead to.

3 Social optimum

We now study the social optimum of the model. We assume that the policy maker is utilitarian. Households and firms are risk neutral. This means that the policy maker is not concerned with insurance and redistribution aspects of the problem. Any optimum that we find does, therefore, not define a unique welfare maximizing income distribution over states of nature and over households and entrepreneurs. The distribution that is implied by an optimal allocation is one of several possible distributions of *contingent income claims*. Each of these distributions will contain different degrees of social insurance and redistribution. We will study the distribution which is the result of no explicit social insurance and redistribution mechanism.

The utilitarian policy maker's measure of social welfare has three components. First, there is the utility of own child care. Second, there is the social value produced by second spouses who participate on the labor market. This value is assumed to correspond to their labor income. Third, there is the social cost of providing the day care that makes it possible for second spouses to work. Social welfare W is measured by

$$W(\tilde{y}, \tilde{\gamma}, g) := \int_0^{\tilde{y}} z dF(y) + \int_{\tilde{y}}^{\infty} p(g) y dF(y) - \int_0^{\tilde{\gamma}} \left(\frac{1}{2}g^2 + \gamma\right) dR(\gamma). \tag{11}$$

In a first best situation, the policy maker chooses three quantities to maximize social welfare. First, the policy maker chooses the number of households that participate on the labor market. This is done by choosing the threshold income $\tilde{y}$. The second quantity is the number of entrepreneurs (day care centers). This is chosen be deciding the cost threshold $\tilde{\gamma}$. Finally, the policy maker decides the group size g. This is done subject to the equilibrium condition $E(\tilde{y}, \tilde{\gamma}, g) := (1 - F(\tilde{y})) - R(\tilde{\gamma})g = 0$. The Lagrangian function to this problem is

$$L(\tilde{y}, \tilde{\gamma}, g, \lambda) = W(\tilde{y}, \tilde{\gamma}, g) - \lambda E(\tilde{y}, \tilde{\gamma}, g), \tag{12}$$

where $\lambda > 0$ is the Lagrange multiplier. Both cumulative distribution functions R and F are continuous and strictly increasing. This means that an interior solution's first order conditions, after some simple manipulations, can be written

$$\frac{\partial L}{\partial \tilde{y}} = 0 \quad \Longrightarrow \quad p(g^s)\tilde{y}^s - z - \lambda^s = 0, \tag{13a}$$

$$\frac{\partial L}{\partial \tilde{\gamma}} = 0 \quad \Longrightarrow \quad \frac{1}{2}(g^s)^2 + \tilde{\gamma}^s - \lambda^s g^s = 0, \tag{13b}$$

$$\frac{\partial L}{\partial g} = 0 \quad \Longrightarrow \quad \int_{\tilde{y}^s}^{\infty} p'(g^s)\, dF(\tilde{y}^s) - R(\tilde{\gamma}^s)\, g^s + \lambda^s R(\tilde{\gamma}^s) = 0, \tag{13c}$$

$$\frac{\partial L}{\partial \lambda} = 0 \quad \Longrightarrow \quad (1 - F(\tilde{y})) - R(\tilde{\gamma})g = 0, \tag{13d}$$

where Lagrange multiplier λ^s is interpreted as the shadow price of day care at the social optimum. Throughout superindex s denotes values at the social optimum. From (13c) we get

$$\lambda^s = -\frac{\int_{\tilde{y}^s}^{\infty} p'(g^s)\, dF(y)}{R(\tilde{\gamma}^s)} + g^s. \tag{14}$$

The first right hand side term measures the total income loss that the negative externality causes when the group size is increased marginally divided by the number of entrepreneurs. The second term, g^s, is the marginal private cost of production for day care. The shadow price of day care at the social optimum can be written as

$$\lambda^s := g^s(1+m), \tag{15}$$

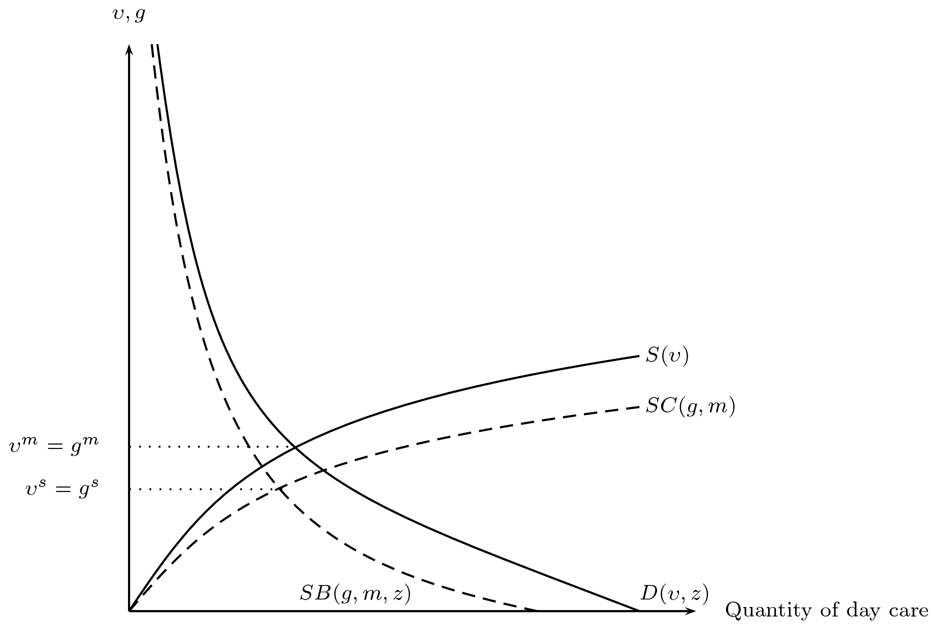
where m is a monetary measure of the negative externality for the average household participating on the labor market. This is the average loss of income per child in day care when group size is increased. It is defined by

$$m := -\frac{p'(g^s)\int_{\tilde{y}^s}^{\infty} y dF(y)}{g^s R(\tilde{\gamma}^s)}. \tag{16}$$

The average monetary loss per child can be interpreted as a markup on the marginal private cost that gives the marginal social cost of day care.

The first order conditions (13b) and (13c) related to the supply of day care can be manipulated to yield $\tilde{\gamma} = \left(\frac{1}{2} + m\right) g^2$ for arbitrary group sizes. Using this we can define the social marginal cost of supplying day care as $SC(g) := R\left((\frac{1}{2} + m)g^2\right) g$. Social costs are increasing in g. Also, as long as there is an externality cost, the social cost schedule will be below the supply schedule in the quantity–group size space; see Figure 2. The first order conditions (13a), related to the demand for day care, and (13c) can be manipulated to yield $\tilde{y} = \frac{(1+m)g+z}{p(g)}$. The social benefit of providing day care

Figure 2: Social optimum and market equilibrium.



can be defined as $SB(g,z) := 1 - F\left(\frac{(1+m)g+z}{p(g)}\right)$ for arbitrary group sizes. As long as there is an externality cost, the social benefit schedule will be below the demand schedule in the quantity–group size space; see Figure 2.

This means that marginal households and entrepreneurs in the social optimum should behave according to

$$p\left(g^s\right)\tilde{y}^s - z - g^s = mg^s, \tag{17a}$$

$$(1+m)\left(g^s\right)^2 = \frac{1}{2}\left(g^s\right)^2 + \tilde{\gamma}^s \tag{17b}$$

which can be compared to (7b) and (7c). The private net benefit of a marginal household participating in the labor force should equal the externality cost of participation in the social optimum. The social benefit of a marginal day care center should equal the marginal private cost. The reduction of externality costs should be included in the social benefits in addition to the revenues of the entrepreneurs.

3.1 General case

Figure 2 shows that the social benefit schedule is below the demand schedule and that the social cost schedule is below the supply schedule. This means that the group size in the social optimum is lower than the group size in the non–intervention market equilibrium.

The gain from participating in the labor force must be strictly positive in the social optimum for the marginal household. On contrary this gain is zero in the non–intervention market equilibrium. The marginal household should take the negative externality into account when deciding to participate in the labor force. For this reason, fewer households should participate. The price of day care will, on the other hand, be lower. This will work in opposite direction, more households will participate. The combined effect on labor force participation is, therefore, ambiguous.

The firm with the highest fixed cost that supplies on the day market must have costs exceeding market revenues in the social optimum. In the non–intervention market equilibrium profits for the marginal firm are zero. The marginal entrepreneur should take the positive externality that the group size decreases and, therefore, the probability that parents can work is increased. For this reason, more entrepreneurs should run day care centers. The price of day care will, on the other hand, be lower. This will work in opposite direction, fewer entrepreneurs will run day care centers. The combined effect on the number of entrepreneurs and day care centers is, therefore, ambiguous.

To sum up, the group size should be lower in the social optimum. It is, however, not necessarily the case that labor force participation should be lower. It may very well be that the number of day care centers should increase.

We can also prove this more rigourously:

Proposition 1. *The social optimum implies a lower group size compared to the non–intervention market equilibrium.*

Proof. We start by noting that the producer price is equal to the group size; i.e., $\upsilon_p = g$. We now want to show that the group size in the social optimum is lower than than in the non–intervention market equilibrium, which is equivalent to show that the social optimum implies a lower producer price, i.e., $\upsilon_p^s < \upsilon_p^m$ where super indices s and m indicate social optimum and unregulated market outcome. By equation (7c) and equations (13b) – (13c) we know that

$$R(\tilde{\gamma}^m) = R\left(\frac{1}{2}\left(\upsilon_p^m\right)^2\right) \text{ and } R(\tilde{\gamma}^s) > R\left(\frac{1}{2}\left(\upsilon_p^s\right)^2\right). \tag{18}$$

Therefore, by equation (7a)

$$\begin{aligned} 1 - F(\tilde{y}^s) - (1 - F(\tilde{y}^m)) = R(\tilde{\gamma}^s)\upsilon_p^s - R(\tilde{\gamma}^m)\upsilon_p^m > \\ > R\left(\frac{1}{2}\left(\upsilon_p^s\right)^2\right)\upsilon_p^s - R\left(\frac{1}{2}\left(\upsilon_p^m\right)^2\right)\upsilon_p^m. \end{aligned} \tag{19}$$

Suppose now that $\upsilon_p^s \geq \upsilon_p^m$. We note that both R and F are strictly increasing on their supports. Since $R\left(\frac{1}{2}\upsilon^2\right)$ is strictly increasing in υ, the assumption that $\upsilon_p^s \geq \upsilon_p^m$ implies, by equation (19), that $\tilde{y}^s \leq \tilde{y}^m$. However, equations (7b) and (13a) – (13c) imply

$$p(\upsilon_p^s)\tilde{y}^s - p\left(\upsilon_p^m\right)\tilde{y}^m > \upsilon_p^s - \upsilon_p^m. \tag{20}$$

Note also that $\upsilon_p^s \geq \upsilon_p^m$ implies $p(\upsilon_p^s) \leq p(\upsilon_p^m)$ by Assumption 1. Equation (20), therefore, implies $\tilde{y}^s > \tilde{y}^m$. This is a contradiction and, therefore, the conclusion $\upsilon_p^s < \upsilon_p^m$ follows. □

3.2 Limiting case

We will continue by discussing a limiting case with more clear results. Here it is possible to determine the difference between the social optimum and the non–intervention market equilibrium also for labor force participation and the number of entrepreneurs. In this case we can show that labor force participation is lower and the number of day care centers is higher in the social optimum. To see this suppose that the externality m for the *average* household is a constant. Total differentiation of the first order conditions (13a)– (13d) and some manipulations yield

$$p(g^s)d\tilde{y} = (1 + m + \tilde{m})dg + g^s dm, \tag{21a}$$

$$d\tilde{\gamma}^s = (1 + 2m)g^s dg + (g^s)^2 \, dm, \tag{21b}$$

$$R(\tilde{\gamma}^s)dg = -f(\tilde{y}^s)d\tilde{y} - g^s r(\tilde{\gamma}^s)d\tilde{\gamma}, \tag{21c}$$

where $\tilde{m} := -p'(g^s)\tilde{y}^s$ is the monetary measure of the negative externality for the *marginal* household participating on the labor market. This is the loss inflicted on the marginal household. The negative externality is more severe for the average than the marginal household because the average household has higher income.

Suppose that $p' = 0$ initially so there is no externality. This means that $m = \tilde{m} = 0$. The social optimum will coincide with the non–intervention market equilibrium. Now suppose that p' increases marginally. Consequently m and $\tilde{m}$ will also increase. This experiment only changes the social optimum, whereas the non–intervention market equilibrium is unaffected. Individual decision makers behave as there is no externality in the non–intervention market equilibrium. We can solve the system (21a) – (21c) for the effects on the social optimal quantities of small changes in the average externality. It is possible to show that

$$\frac{dg}{dm} < 0, \quad \frac{d\tilde{\gamma}}{dm} > 0 \text{ and } \frac{d\tilde{y}}{dm} > 0, \tag{22}$$

As an approximation, this implies that the group size should be reduced. But now we also get the clear results for the cost threshold and the income threshold. Both should be increased. This will increase the number of entrepreneurs and day care centers while the labor force participation decreases.

4 Tax instruments and policy implementation

The optimal solution calls for an increase in the opportunity cost of participating in the labor force for households. It also calls for a decrease in the opportunity cost of running day care centers for entrepreneurs. Increasing the cost for households can be done by introducing a tax on day care services. Another possibility is to introduce a home care allowance in the form of a fixed sum payment to parents who do not participate in the labor force. Decreasing the cost for entrepreneurs can be implemented through subsidy to entrepreneurs who run day care centers. Any such policy implementing the optimal solution is self–enforcing. Only the most productive households participate in the labor force and only the most efficient entrepreneurs run day care centers.

Suppose now that the policy maker chooses to use a tax on day care services τ, an allowance to households not participating in the labor force α, and a subsidy to entrepreneurs running day care centers κ. Equations (7b)–(7c) can now, for an optimal policy solution, be rewritten as

$$p\left(g^{s}\right)\tilde{y}^{s}-(1+\tau^{s})\upsilon_{p}^{s}-z-\alpha^{s}=0, \tag{23a}$$

$$\frac{1}{2}\left(\upsilon_{p}^{s}\right)^{2}+\kappa^{s}-\tilde{\gamma}^{s}=0, \tag{23b}$$

where the consumer price is $\upsilon_{c}^{s}=(1+\tau^{s})\upsilon_{p}^{s}$. Combining with (13a)–(13d) and using the fact that $g^{s}=\upsilon_{p}^{s}$ gives us

$$\tau^{s}+\frac{\alpha^{s}}{g^{s}}=m, \tag{24a}$$

$$\kappa^{s}=m\left(g^{s}\right)^{2}. \tag{24b}$$

We can now formulate a result that follows directly from equations (24a) and (24b) since $m>0$:

Proposition 2. *The policy $(\tau^{s},\alpha^{s},\kappa^{s})$ that implements the social optimum implies that $\kappa^{s}>0$ and that at least one of the pair (τ^{s},α^{s}) is strictly positive.*

The most natural first best policy implementation is, therefore, a tax on marginal households and a subsidy to marginal entrepreneurs, but with no home care allowance. If the tax is reduced, however, introducing a home care allowance, so that (24a) holds, is also consistent with a first best allocation.

But what will be the outcome if all households pay a day care tax and all entrepreneurs receive a subsidy? The policy maker has to handle possible budget surpluses or deficits generated. However, in this model there are no income effects on the households' labor force participation decision. Non–labor income earned by a household does not affect the decision to participate in the labor force. Any tax revenues that a first best policy generates or requires can, therefore, be disposed of or generated through lump sum transactions with all households. This will not change the first order condition for the social optimum. A traditional second best tax problem does, therefore, not exist in this model. Since this policy is the same for all households and entrepreneurs it is reasonable to assume it is feasible.[10]

Let us, therefore, compute the consequences for the public budget of first best policies. Suppose that all households are treated in the same way and that all firms also are treated in the same way. Using $g^s = \upsilon_p^s$, the budget balance, B, is

$$B := \left(1 - F\left(\tilde{y}^s\right)\right) g^s \tau^s - \alpha^s F\left(\tilde{y}^s\right) - \kappa^s R\left(\tilde{\gamma}^s\right) = -\alpha^s, \tag{25}$$

where the last equality is derived from plugging in the expressions for τ^s and κ^s from (24a) and (24b).

Hence, any policy given by (24a) and (24b) such that $\alpha^s = 0$ implies a balanced budget, i.e., $B = 0$. The most reasonable policy is to tax day care with $\tau^s > 0$ and subsidize day care entrepreneurs with $\kappa^s > 0$. At the same time there is no home care allowance, $\alpha^s = 0$. This policy has the appeal that it also balances the budget. Suppose that a home care allowance $\alpha^s > 0$ is used in a first best policy. The resulting budget deficit can be financed by a lump sum tax on all households equal to α^s. The tax on day care for participating households is lower compared to an optimal policy without a home care allowance. In addition, however, all households participating in the labor force will have to pay the lump sum tax α^s. Such a tax will, therefore, not affect labor force participation decisions. Such a balanced budget policy is, however, equivalent to an optimal policy without a home care allowance.

5 Regulation

It does not exist a traditional second best optimal tax problem in our model. We have so far assumed that the public sector has a sufficient number of policy tools to reach the social optimum. But suppose that the taxes and subsidies required by first best are not feasible. It may still be possible for the public sector to improve social welfare by regulating the group size at day care centers. This is clearly a second best policy. The policy maker would prefer to affect households and entrepreneurs independently with two different policy instrument. Here it is assumed that there is only one policy instrument available to affect households and entrepreneurs.

Suppose that the policy maker regulates the group size. Day care entrepreneurs maximize profits. The producer price is, therefore, regulated to

[10] Below, however, we study how regulating the group size that can be done if such taxes are unfeasible.

equal the group size. The total supply of day care will, in effect, be determined by the zero profit constraint on marginal entrepreneurs. Households are charged a price lower than in the non–intervention market equilibrium. Demand will, therefore, be higher. But there will be not be enough supply to meet this higher demand. Supply will be the limiting factor on the market. It is not possible for all households that want to participate in the labor market to do so. There will be excess demand at the regulation optimum. Day care will have to be rationed.[11] This should be possible as it is very difficult to resell day care services obtained. We assume that day care is allocated to households with high willingness to pay for day care. This is equivalent to an assumption that the policy maker controls the consumer price.

The cost threshold of entrepreneurs decreases if the regulator decreases the group size, i.e., $-\frac{\partial \tilde{\gamma}}{\partial g} = -g < 0$. Effective demand for day care is determined by supply, technically via the equilibrium condition. Decreased group size, therefore, has to imply lower effective demand and lower labor force participation. This means that the income threshold will increase, i.e., $-\frac{\partial \tilde{y}}{\partial g} = \frac{r(\tilde{\gamma})g^2 + R(\tilde{\gamma})}{f(\tilde{y})} > 0$.

Group size is chosen to maximize social welfare given by (11). This is done subject to the market equilibrium condition and the cost threshold condition $\tilde{\gamma} = \frac{1}{2}g^2$. The Lagrangian function to this problem is

$$M(\tilde{y}, \tilde{\gamma}, g, \lambda_1, \lambda_2) = W(\tilde{y}, \tilde{\gamma}, g) - \lambda_1 E(\tilde{y}, \tilde{\gamma}, g) - \lambda_2 \left(\frac{1}{2} g^2 - \tilde{\gamma} \right). \tag{26}$$

An interior solution's first order conditions can after some manipulations be written as

$$\frac{\partial M}{\partial \tilde{y}} = 0 \quad \Longrightarrow \quad p(g^r)\tilde{y}^r - z - \lambda_1^r = 0, \tag{27a}$$

$$\frac{\partial M}{\partial \tilde{\gamma}} = 0 \quad \Longrightarrow \quad -\left(\frac{1}{2} (g^r)^2 + \tilde{\gamma}^r - \lambda_1^r g^r \right) r(\tilde{\gamma}^r) + \lambda_2^r = 0, \tag{27b}$$

$$\frac{\partial M}{\partial g} = 0 \quad \Longrightarrow \quad \int_{\tilde{y}^r}^{\infty} p'(g^r) y dF(y) - (g^r - \lambda_1^r) R(\tilde{\gamma}^r) - \lambda_2^r g^r = 0 \tag{27c}$$

in addition to $\partial M / \partial \lambda_i = 0$ $i = 1, 2$. Note, however, that for $i = 2$ we have that $\tilde{\gamma}^r = \frac{1}{2} (g^r)^2$. Solving for the shadow price of day care at the regulation optimum we get

$$\lambda_1^r = -\frac{\int_{\tilde{y}^r}^{\infty} p'(g^r) \, dF(\tilde{y}^r)}{R(\tilde{\gamma}^r) + (g^r)^2 r(\tilde{\gamma}^r)} + g^r. \tag{28}$$

[11] There have been excess demand situations for day care in many countries.

There is a difference compared to the first best optimum for the following reason: Any reduction in the group size of day must take place along the supply curve. Total supply (and accordingly also effective demand) of day care will, therefore, be reduced. At the social optimum, the optimal group size can be determined independently of the optimal number of day care centers. This is not the case here. When there is only one policy instrument available, reducing the group size (and the producer price) will have two effects. The supply of day care will decrease, first, as the supply from day care centers that stay in business will go down. This is captured by the first term $R(\tilde{\gamma}^r)$ in the denominator. But there is a second effect that the policy maker has to take into account. The number of day care centers will be reduced. This is captured by the second term $(g^r)^2 r(\tilde{\gamma}^r)$ in the denominator; i.e., the previous supply of day care of firms leaving the market.

The shadow price of day care at the regulation optimum can be written as

$$\lambda_1^r = g^r (1 + n) \tag{29}$$

where n is a monetary measure of the negative externality. This is the total loss of income when group size is increased divided by the total change in supply of day care when the group size is increased. It is defined by

$$n := -\frac{\int_{\tilde{y}^r}^{\infty} p'(g^r)\, y dF(y)}{g^r \left(R(\tilde{\gamma}^r) + r(\tilde{\gamma}^r)(g^r)^2 \right)}. \tag{30}$$

We can, therefore, define the markup on the marginal private cost that gives the marginal social cost of day care in a similar way as in the social optimum.

Combining the first order conditions, we can obtain the two equations corresponding to (17a)–(17b). In the regulation optimum, the marginal households and entrepreneurs should behave according to

$$p(g^r)\tilde{y}^r - z - g^r = ng^r, \tag{31a}$$

$$(g^r)^2 = \frac{1}{2}(g^r)^2 + \tilde{\gamma}^r. \tag{31b}$$

The private net benefit of a marginal household participating in the labor force should equal the externality cost of participation in the regulation optimum. This is similar to what we found for the social optimum, see (17a). The social benefit of a marginal day care center equals the private benefit. It should equal the marginal private cost at the regulation optimum. This reflects our assumed constraint that the supply is determined by the zero profit condition for entrepreneurs. As a consequence externality costs will not affect the number of day care centers.

Note also that λ_2^r measures the negative impact on social welfare of this constraint. It satisfies

$$\lambda_2^r = -nr(\tilde{\gamma}^r)(g^r)^2 < 0. \tag{32}$$

We can also show that the following results regarding the regulation optimum hold:

Proposition 3. *The regulation optimum implies (i) a lower group size, (ii) fewer day care centers, and (iii) lower labor force participation compared to the non–intervention market equilibrium.*

Proof. The proof of part(i) is analogous to the proof of Proposition 1. Hence $\upsilon_p^r < \upsilon_p^m$ and $\tilde{\gamma}^r < \tilde{\gamma}^m$ because of the cost threshold condition. There will be fewer day care centers at the regulation optimum and part (ii) follows. Also, with lower group size and fewer day care centers, supply will be lower. Market equilibrium requires that $\tilde{y}^r > \tilde{y}^m$. Labor force participation will be lower, which gives part (iii). □

A comparison between the regulation optimum and the non–intervention market equilibrium gives very precise results, a similar comparison between the regulation optimum and the social optimum does not. There is a natural reason for this; the first best optimum may imply higher or lower labor force participation and more or fewer entrepreneurs compared to the market outcome. But what about the group size? We know that in both cases the solution is lower than in the non–intervention market equilibrium. But in the regulation optimum group size is reduced through a reduction of supply; i.e., the marginal entrepreneur will have a lower fixed cost (be more efficient) but will also have a lower marginal cost (operate at a lower group size). Demand is rationed at the supplied quantity. The social benefit of reducing group size in the regulation case is, therefore, not as high as in case first best policy instruments are available. This gives an argument for the group size in the regulation optimum to be higher than in the first best optimum.

6 Concluding remarks

Children at day care centers with larger child groups are ill more frequently than children at day care centers with smaller groups. Sick children are usually cared for at home by parents. This creates a negative externality of parents' labor force participation. At the same time, entrepreneurs' decision to decrease group size has positive externalities. We show that the social optimum implies lower group size than the non–intervention market equilibrium.

We have studied the optimal Pigouvian policy. The cost of labor force participation should be increased. This can be done by either or both a *tax* on day care services and a home care *allowance*. Moreover, the cost of providing day care should be decreased by a *subsidy* to entrepreneurs running day care centers. This policy will decrease the group size. It is, however, not necessarily the case that this will decrease labor force participation as the number of day care centers may increase.

Some features of the real problem have been neglected in the present analysis: In this paper we focus on a negative externality. We ignore problems of social insurance and income redistribution. There are interesting and natural extension of the present analysis. One is to investigate how social insurance interacts with the optimal incentives to participate in the labor force studied here. We have also only addressed one type of externality in day care. There are, of course, other factors and other externalities that may lead to different conclusions. One example: Here day care is assumed just to care for children while parents are working. Day care can be important in contributing to human capital accumulation. Day care may also contribute to increase the tax base in the economy.

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