Nominal Wage Flexibility in a Monetary Union

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Abstract

Membership in a monetary union reduces the possibilities to counteract fluctuations in productivity by monetary policy. One condition for entrance not to lead to adverse unemployment performance is that wages are flexible with respect to productivity. Here I show that, depending on workers' risk aversion, the incentive for workers to choose more nominal wages flexibility may increase after entrance in a monetary union. The reason is that if nominal wages are fixed in long-term contracts, the abolishment of exchange rates decreases the risk in real wages. On the other hand, the common monetary policy increases the employment risk. Assuming that individuals' preferences do not change, the institutional change in monetary policy may increase wage flexibility in a monetary union.

Keywords: New open-economy macroeconomics, Nominal wage flexibility, Optimal wage setting, Monetary unification

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1 Introduction

Entrance in a monetary union affect many fields in the entering economy. During recent years many papers have been written with the objective to capture the effect the creation of a monetary union has on wage pressure in the economies involved, and hence on the equilibrium level of unemployment.[1] The prime focus has been on how the interaction between wage setters and the monetary authority changes in a monetary union where one central bank is to coexist and interact with many different institutional settings for wage formation. In general, the argument has been that absence of national currencies and national central banks will have two effects on wage formation: (i) Wage restraint in each country might be lower since higher wages in one particular country will no longer "automatically" lead to tighter monetary policy, even if the central bank targets inflation. In a monetary union the wage setters may seek to externalize too high wage increases in one country on all other countries in the union. (ii) The elasticity of labor demand increases, since there are no longer any national monetary policy instruments to compensate for national fluctuations in productivity. This would be an incentive to lower wages, or alternatively, to make them more flexible. These two effects work in opposite directions. The possibility to externalize too high wage increases creates an upward pressure on the nominal wages, while increased elasticity of labor demand may reduce wage pressure. The question of which one is the dominating is an empirical issue still to be solved.

In the present paper I will focus on wage formation in a small open economy entering into a monetary union. An often-discussed condition for entrance not to lead to adverse employment effects is that the wage increases in the entering economy must adjust to the average level of wage increases in the monetary union. As a consequence of the effect (i) discussed above, however, this adjustment is only a necessary condition, that does not guarantee that unemployment in the entering economy will not increase after entrance in the monetary union.[2] Given that labor mobility is too low within the

[1] See for example Iversen (1998) and Cukierman and Lippi (1999)

[2] Iversen and Soskice (1998) analyze the effect the creation of EMU will have on the average wage level in the union in a model with German wage setters as leaders in the sense that wage setters in other EMU countries will follow the wage increases reached in Germany. The replacement of the Bundesbank by the ECB may, however, reduce wage restraint in Germany. If that is the case, given that the Germans are wage leaders, Iversen and Soskice argue that the average wage level in Europe will increase, leading to worse

monetary union, the presence of possibly asymmetric productivity shocks calls for another condition; that wage setting becomes more flexible. The purpose of this paper is to examine the hypothesis that the gain for workers from a more flexible wage setting, where the nominal wage is more strongly connected to productivity, is higher within a monetary union that outside.

The idea underlying the paper comes from the observation that entrance in a monetary union implies a shift in the sharing of risks connected to wage setting between workers and firms. The common currency reduces the real wage risk that has to be carried by workers. On the other hand, entrance in a currency union may increase the risks connected to fluctuations in productivity, since with a national currency and a national central bank it has been possible to use monetary policy to counteract country-specific productivity shocks. If this increased productivity risk has to be carried by workers or firms depends on how the labor contracts are written. If long-term contracts determine both nominal wages and employment, then the productivity risk has to be carried by the firms. That case is not discussed in this paper. The more relevant case is when nominal wages are determined in long-term contracts, while employment is unilaterally determined by the firms after the realization of the productivity shocks. The productivity risk is then carried by the workers, since firms can adjust their labor demand after shocks have realized. It is this case that will be analyze here in a stochastic general equilibrium model.

This work differs from from much earlier work modelling the possible interaction between monetary policy and wage formation in the sense that monetary policy will not be treated as endogenous to wage setting. The interaction between wage setters and the central bank does not have the nature of an "inflation game" and the intention is *not* to, given the institutional feature of the wage setting system, find any optimal degree of conservatism of the central bank, that has been the case in most earlier studies.[3] Instead, monetary policy is modelled as a response rule to possibly asymmetric productivity shocks, with the objective to assure that the economy always is in a post shock equilibrium. Given the rules, institutional setting and limitations governing monetary policy the objective is to study how wage formation should adjust. The degree of stickiness in nominal wages is therefore endogenous to monetary policy and the exchange rate regime.

unemployment scores.

[3]See for example Cukiermann and Lippi (2000)

The main result is that the incentive for workers to choose flexible wages, in the sense that wages are set after the realization of productivity shocks, changes if the economy enters a monetary union. The direction depends critically on the individuals' preferences (in particular the degree of risk aversion) and on the size of the entering economy.

In the model two sequences of events are compared. They differ with respect to the length of wage contracts. Nominal wages, set by the individuals are either preset *(sticky)* one period in advance[4] or *flexible* in the sense that nominal wages are set after the realization of the productivity shocks. Note, however, that this implies that the choice to set sticky or flexible wages is made before the shocks realize. Employment is determined by the firms after the productivity and monetary shocks have realized. Workers supply the amount of labor demanded by the firms at either the preset or the flexible nominal wage. With flexible wages, the individuals know what amount of labor the firms will demand and can choose their nominal wage optimally. In the case of preset nominal wages, however, it might be the case that unexpected productivity disturbances give that the firms demand a different level of employment than the one that is *ex ante* optimal for the individual at the posted wage, leading the economy away from the competitive equilibrium.

Earlier work modelling wage formation as endogenous to monetary policy includes Holden (2001), using the idea when he analyzes how the optimal degree of co-ordination among wage setters may change depending on the degree of conservatism with the central bank. In the context of a monetary union, Siebert and Sutherland (1999) study the changed incentives among policy makers to undertake labor market reform before and after a country has joined a monetary union. They find that the more uncorrelated shocks are among the memberstates, the higher are the policy makers' incentives to increase labor market flexibility. If shocks are to a large extent correlated, however, monetary union membership may lower the incentives for reforms. Similar results, in the sense that the incentives for reforms can be either increasing or decreasing in a monetary union, are obtained by Saint-Paul and Bentolila (2000). More similar to the present paper, in the sense that focus is on the changed incentives among wage setters and not among politicians, is

[4]The assumption that *all* nominal wages are set one period in advance is a simplification. A more realistic assumption would be to assume that nominal wages are set in staggered long-term contracts, but that would render an intractable model for our purposes. For a microfounded general equilibrium model with overlapping nominal contracts, see Bergin and Feenstra (2001)

Calmfors and Johansson (2002). By utilizing a stylized small open economy model they conclude that membership in a monetary union would increase the incentive for nominal wage indexation. Also, joining a monetary union would induce wage setters to reduce the lenght of the nominal wage contracts.

The paper is organized as follows. Section 2 and 3 present the stochastic general equilibrium model. The main results are derived in Section 4. Section 5 discusses the results and gives proposals for extensions of the model. The paper ends with the concluding Section 6.

2 A Stochastic General Equilibrium Model

In order to study the wage setters' incentive to set more flexible wages in a monetary union I will employ a stochastic general equilibrium model, similar to the models developed by Obstfeld and Rogoff (2000, 2002). The choice of model is motivated by a couple of reasons. Firstly, the model is possible to solve analytically and therefore offers the advantage to make explicit welfare comparisons between the flexible and sticky wage cases, respectively. Secondly, the model is fully utility-based why the results can be derived without using *ad hoc* assumptions on individuals' or monetary authorities' loss functions. Thirdly, the model incorporates individuals attitude towards risks. This implies that when choosing the optimal nominal wage, the individuals possibly will not set the wage to its certainly-equivalent value, but instead consider the risks incorporated.

2.1 Assumptions

The prime objective of this paper is to study the consequences for wage setting in a small, open economy that considers to enter a monetary union (MU). We therefore assume a two-country world, where the Home economy is of size $n \in (0,1)$ and where the MU economy is of size $1-n$. Utility maximizing individuals are indexed i and i^* over the unit interval, where $i \in (0,n]$ are residents in the Home country and where $i^* \in (n,1]$ are MU residents. Each individual supplies differentiated labor input and sets its optimal nominal wage. Monopolistically competitive firms produce an array of final goods out of the differentiated labor input. There are two sectors in both economies. One nontraded sector where differentiated goods indexed on the interval $[0,1]$ are produced and one traded sector. For convenience Home tradables are indexed on the interval $[0, n]$, while MU tradables are indexed on the interval $(n, 1]$. Money supply in each economy is set by the central banks. There are two types of shocks in the model, productivity shocks and monetary shocks.

2.1.1 Individual utility

A Home individual of type i obtains utility from consumption and disutility from working. As is convenient in microfounded macroeconomic models individuals also obtain utility from holding real balances $\left(\frac{M}{P}\right)$. The utility function for period t of a Home individual i is therefore assumed to be given by

$$U_t(i) = \frac{1}{1-\delta} C_t(i)^{1-\delta} + \chi \log \frac{M_t(i)}{P_t} - K_t L_t(i), \tag{1}$$

where $C(i)$ is consumption and $L(i)$ is labor supply of individual i. The parameters $\delta > 0$ and $\chi > 0$ are assumed to be equal across individuals and across countries, with δ being the coefficient of relative risk aversion. K_t is an exogenous shock to the willingness of supplying labor that in this model will represent a productivity shock.[5] The nominal budget constraint for individual i over the period is given by

$$M_t(i) + P_t C_t(i) = M_{t-1}(i) + P_t T_t + W_t(i) L_t(i), \tag{2}$$

where $W_t(i)$ is the nominal wage for worker i and T_t is lump-sum tax transfers to the individual from the government. Since the model is single-period, time indexes will be dropped in the remaining part of the paper.

2.1.2 Firms

Each Home firm j is a price taker on the labor market. It produces output $Y(j)$, using differentiated labor input $L(i, j)$. For Home firms operating in

[5]This way to model productivity shocks is common in microfounded macroeconomic models. The standard alternative is to model the productivity shock as a technological shift in the production function, but the formulation in (1) is superior in many ways: The fast diffusion of technology makes it reasonable to think that the level of technology does not differ among industrialized countries considered here. Instead other factors, such as the general knowledge level of the population and/or fiscal policy (taxation) factors may be as important, and, most important, asymmetric between countries. In fact, even factors such as for example the general health condition of the population may induce asymmetric affects on the willingness to supply labor.

the traded sector the production function is given by

$$Y_H(j) = L_H(j) \quad \text{where} \quad L_H(j) = \left(\int_0^n L_H(i,j)^{\frac{\phi-1}{\phi}} di \right)^{\frac{\phi}{\phi-1}}. \tag{3}$$

In (3), $L_H(i,j)$ is labor input of type i used in sector H and $\phi > 1$ is the constant elasticity of substitution between the different types of labor input. Firms operating in the nontraded sector have an identical production function (with subscript N instead of H). Since productivity is assumed not to differ between the traded and the nontraded sector, nominal wages will be equal in the two sectors. Aggregating over the individual nominal wages gives the minimum wage of producing one unit of output as

$$W = \left(\int_0^1 W(i)^{1-\phi} di \right)^{\frac{1}{1-\phi}}.$$

For a given nominal wage, the demand for labor of type i is found by letting the marginal cost of acquiring one more unit of labor input of type i equal the predetermined wage of this input $W(i)$. The resulting firm j demand for labor of type i is given by the standard

$$\frac{L_H(i,j)}{L_H(j)} = \left(\frac{W(i)}{W} \right)^{-\phi}. \tag{4}$$

It is assumed that $\phi > 1$, why the demand for individual i's labor is inversely related to the ratio of the nominal wage chosen by individual i to the the aggregate wage, W.

2.1.3 Prices and Demand

As in Obstfeld and Rogoff (2000, 2002) overall (per capita) real consumption for a Home individual, C, is an index of tradable and non-tradable products:

$$C = \left(\frac{C_T}{\gamma} \right)^{\gamma} \left(\frac{C_N}{1-\gamma} \right)^{1-\gamma}. \tag{5}$$

Allowing for different sizes of the two countries, Home preferences over tradable products are

$$C_T = \left(\frac{C_H}{n} \right)^{n} \left(\frac{C_F}{1-n} \right)^{1-n},$$

where the consumption subindexes for C_H, C_F and C_N are defined analogous to (3) with a constant and identical elasticity $\theta > 1$.[6] The overall price index in Home is defined as $P = P_T^{\gamma} P_N^{1-\gamma}$. The price index for tradable consumption C_T is $P_T = P_H^n P_F^{1-n}$, where price indexes for Home and Foreign goods, respectively, are defined as

$$P_H = \left[\left(\frac{1}{n} \right) \int_0^n P(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, P_F = \left[\left(\frac{1}{1-n} \right) \int_n^1 P(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.$$

It is assumed that the law of one price holds for all individual goods. Taken together with the assumption that Home and Foreign residents have identical preferences this implies that the power purchasing parity must hold for the consumer prices in the two countries:

$$P = SP^*,$$

where S is the nominal exchange rate. The derivations of total demand for a typical Home produced good h, $C(h)$, and a representative Foreign produced good f, $C(f)$, are standard. The resulting demands for $C(h)$ and $C(f)$ are, respectively,

$$C(h) = \left(\frac{P(h)}{P_H} \right)^{-\theta} \left(\frac{P_H}{P} \right)^{-1} C^W, \quad C(f) = \left(\frac{P(f)}{P_F} \right)^{-\theta} \left(\frac{P_F}{P} \right)^{-1} C^W$$

where C^W is world *per capita* consumption.

2.1.4 Goods market clearing

In single-period general equilibrium models, goods markets always clear. Taking account of the differing populations in the two countries, total output supply of tradables equals total domestic demand when

$$n[nP_T C_T + (1-n) S P_T^* C_T^*] = n P_H Y_H \ ,$$
$$(1-n)[nP_T C_T + (1-n) S P_T^* C_T^*] = (1-n) P_F Y_F^* \ .$$

Combining these conditions implies that the following relation must hold

$$Y_H P_H = Y_F P_F.$$

The domestic markets for non-tradables clear when demand equal domestic supply: $C_N = Y_N$ and $C_N^* = Y_N^*$. These market clearing conditions for

[6] The full expressions are presented in Appendix A.1

non-tradables imply that $P_TC_T = P_HY_H$ and $SP_T^*C_T^* = P_FY_F$ and hence, in equilibrium, per capita consumption of tradables are always equal in the two countries

$$C_T = C_T^*.$$

The consumption of non-tradable goods, however, do not have to be equal in the two countries, and hence C and C^* do not have to be equal. However, when we use the price of tradables as a numeraire and define the total spending measured in units of tradables as

$$Z = C_T + \left(\frac{P_N}{P_T}\right) C_N,$$

then it is possible to show[7] that the total per capita spending measured in units of tradables is equal in the two economies:

$$Z = Z^*. \tag{6}$$

2.2 Equilibrium

Individuals maximize utility by setting the nominal wage. At the end of period $t-1$ each individual has to choose between setting the nominal wage sticky or flexible. In the sticky wage equilibrium wage setters write contracts at the end of period $t-1$ determining the nominal wages $W(i)$ and $W^*(i)$ in period t. Since it is assumed that wage setters are not allowed to change the wage for period t after the productivity shocks have realized, each wage setter choosing to set a preset nominal wage will set a wage that hedges towards the risks the individual faces. The uncertainty wage setters face is fully captured by the exogenous productivity shocks and the response from the monetary authority these productivity shocks may trigger. The shocks change the individuals' optimal level of consumption and labor supply. When nominal wages are predetermined in a contract, the individuals cannot change their wage after the shocks have realized, while they are free to change their behavior instantaneously in the flexible wage case. After the productivity shocks have realized, workers supply the amount of labor demanded by firms[8] and

[7]Note that by dividing the expression for C_T by C_N one obtains $\frac{P_N}{P_T} = \left(\frac{1-\gamma}{\gamma}\right)\frac{C_T}{C_N}$.

[8]This assumption, that workers supply the amount of labor that is profit maximizing for the firms at the decided nominal wage is relevant for sufficiently small shocks. For

the monetary authority set money supply as a response to these productivity shocks with the objective to maximize the utility of the representative individual. Prices in the model are allowed to be fully flexible.

2.2.1 Wage setting by the individuals

Each Home individual i sets the nominal wage for period t in period $t-1$ in order to maximize expected utility. The maximization of (1) subject to the budget constraint (2) gives the first order conditions for optimal nominal wage as

$$E\left\{KL(i)\right\} = \left(\frac{\phi-1}{\phi}\right) W(i) E\left\{\frac{1}{P}\frac{L(i)}{C(i)^{\delta}}\right\}. \tag{7}$$

In (7) the expectations operator, E, is defined as $E(X) = E\left(X_t | I_{t-1}\right)$, where X_t is any stochastic variable and where I_{t-1} is the information set available to the agents in period $t-1$. This first order condition for optimal preset wages says that the *ex ante* expected disutility from suppling labor (the left hand side) must equal the expected utility the individual obtains from higher wage revenue (the right hand side).

2.2.2 Equilibrium money demand

Real money balances enter directly in the individual utility function. Individual money demand in optimum is then found by maximizing equation (1) with respect to real money balances, $\frac{M(i)}{P}$, subject to the budget constraint (2). The resulting expression for optimal holding of real balances is

$$\frac{M(i)}{P} = \chi C(i)^{\delta}. \tag{8}$$

It is through this relation that we later on in the paper will see how the central bank can affect the economy, by setting the nominal money supply, M.

2.2.3 Price setting

It is assumed that firms operate in an environment characterized by monopolistic competition. In Appendix A.2 I demonstrate the standard result, that

large shocks it may be the case that the disutility from working at the ongoing nominal wage exceeds the marginal utility obtained from wage income. Therefore, this model is only true for minor shocks.

the profit maximizing price for a representative Home firm j is to set the price as a mark-up over the wage the firm has to pay to the workers in the firm: $P(j) = \left(\frac{\theta}{\theta-1}\right) W(i,j)$. Assuming symmetry among firms and individuals, the price levels of domestically produced goods in the Home and Foreign countries, respectively, can be written[9]

$$P_H = P_N = \left(\frac{\theta}{\theta-1}\right) W, \qquad P_F^* = P_N^* = \left(\frac{\theta}{\theta-1}\right) W^*. \tag{9}$$

In the case of sticky wages the *overall* price level in the two countries can be affected by monetary policy through fluctuations in the nominal exchange rate.

2.3 Solutions for utility

One of the advantages of using a microfounded stochastic general equilibrium model is that the model allows to explicitly derive expressions for expected welfare under different monetary regimes. Therefore, it is possible to analytically determine what consequence entrance in a monetary union has for the incentives of wage setters to choose flexible nominal wages.

In order to derive the expected welfare from the two possible wage setting alternatives we have to find a tractable expression for societal welfare. By using our definitions of Home and Foreign spending:[10] $PC = P_T Z$ and $P^*C^* = P_T^* Z^*$ the optimal price equations in (9) total consumption in Home an Foreign can be written

$$C = \left(\frac{W^*S}{W}\right)^{(1-n)(1-\gamma)} Z, \qquad C^* = \left(\frac{W^*S}{W}\right)^{-n(1-\gamma)} Z^*. \tag{10}$$

Furthermore, using the first order conditions for sticky nominal wages (7) and the Home budget constraint $P_H Y_H + P_N Y_N = P_H L = PC$ we see that it

[9]The assumption that the law of one price holds then give that
$P_H^* = \frac{1}{S}\left(\frac{\theta}{\theta-1}\right) W = \frac{P_H}{S}, \qquad P_F = S\left(\frac{\theta}{\theta-1}\right) W^* = SP_F^*$

[10]Since

$$\begin{aligned} PC &= P_H Y_H + P_N Y_N \\ &= P_T C_T + P_N C_N = P_T \left(C_T + \frac{P_N}{P_T} C_N\right) = P_T Z. \end{aligned}$$

is possible to express the disutility from working in terms of consumption as

$$E\{KL\} = \psi E\left\{C^{1-\delta}\right\}, \qquad E\{K^*L^*\} = \psi E\left\{C^{*(1-\delta)}\right\}, \tag{11}$$

where $\psi = \frac{(\theta-1)(\phi-1)}{\theta\phi}$.[11]

When calculating expected utility in single-period general equilibrium models with "money-in-the-utility" utility functions it simplifies calculations substantially to assume that the direct utility obtained from holding of real money is negligible ($\chi \to 0$). Substituting equation (11) into the equation for expected utility for an Home individual (1) we then obtain

$$E(U) = E\left\{\frac{C^{1-\delta}}{1-\delta} - KL\right\} = \Psi E\left\{C^{1-\delta}\right\},$$

where the term $\Psi \equiv \left(\frac{\theta\phi-(1-\delta)(\theta-1)(\phi-1)}{\theta\phi(1-\delta)}\right)$ arises due to the assumed monopoly power of individuals in wage setting and of firms in price setting. By (10), expected welfare in the Home country is therefore given by

$$E(U) = \Psi E\left\{\left(\frac{W^*S}{W}\right)^{(1-n)(1-\gamma)(1-\delta)} Z^{1-\delta}\right\}. \tag{12}$$

Similarly, the expected utility for an individual in Foreign can be expressed:

$$E(U^*) = \Psi E\left\{\left(\frac{W^*S}{W}\right)^{-n(1-\gamma)(1-\delta)} Z^{1-\delta}\right\}. \tag{13}$$

These two expressions show that the division of consumption goods into tradables and non-tradables introduces the terms of trade (and hence the nominal exchange rate) as a factor determining expected welfare.

Lemma 1 *The smaller the Home economy is relative to the Monetary Union, the more important the real exchange rate as a determinant of expected welfare in the Home economy.*

Proof. The real exchange rate can be written

$$\frac{SP^*}{P} = \left(\frac{W^*S}{W}\right)^{1-n}$$

[11]The full derivations of equations (10) and (11) are in Appendix A.3

The proposition follows directly from substituting this into equations (12) and (13) above. ■

Note that when all goods are tradables ($\gamma = 1$) then international risk sharing is complete and welfare is determined solely by expected spending. The term Ψ will not be affected by whether nominal wages are preset or flexible, since increased wage flexibility in this model does not correspond to increased competitiveness in the labor market.

To close the model and to find a channel for monetary policy to have an impact on expected welfare we have to find explicit expressions for the terms of trade, $\frac{W^*S}{W}$ and normalized spending, Z as functions of the exogenous variables.

2.3.1 Expected welfare with flexible wages

We start with the benchmark case, when wages are flexible implying that monetary policy is redundant. In Appendix B.1 the following expressions for the terms of trade and for spending are derived, defining the flexible wage equilibrium

$$\left(\frac{W^*S}{W}\right)^{flex} = \left(\frac{K}{K^*}\right)^{-\frac{1}{1-(1-\gamma)(1-\delta)}} \tag{14}$$

and

$$Z^{flex} = \left[\frac{(\phi-1)(\theta-1)}{\phi\theta K^n K^{*(1-n)}}\right]^{\frac{1}{\delta}}, \tag{15}$$

where superscript *flex* represents the flexible equilibrium value of a variable.

The model contains two types of shocks; productivity shocks (Z) and monetary shocks (M). It is assumed that these shocks are log-normally distributed. If $\kappa \equiv \log K$, $m \equiv \log M$ then $\kappa \sim N(0, \sigma_\kappa^2)$ and $m \sim N(0, \sigma_m^2)$. This assumption is convenient for a couple of reasons: It assures that in the model "level" productivity (K) and monetary expansion (M) only take positive values and it makes it possible to derive an explicit closed-form solution.[12] In the analysis ahead it will be assumed that the both mean and

[12]This is due to the desirable property of any log-normally distributed variable X that

$$E(X^a) = \exp\left\{aEx + \frac{a^2}{2}\sigma_x^2\right\}.$$

variance productivity are equal in the two economies. That is, we assume $E\kappa = E\kappa^*$ and $\sigma_\kappa^2 = \sigma_{\kappa^*}^2$.

When analyzing two-country models it is useful to divide the productivity shocks in an idiosyncratic part, that measures the difference between the countries and a common, world-wide shock.

Definition 2 *The "world", common productivity shock is defined as a weighted average of the shocks in each country,*

$$K_w = K^n (K^*)^{1-n} \quad \Leftrightarrow \quad \kappa_w \equiv n\kappa + (1-n)\kappa^*,$$

while the idiosyncratic or country-specific productivity shock is defined as the difference between the productivity shocks in each economy,

$$K_i = \frac{K}{K^*} \quad \Leftrightarrow \quad \kappa_i \equiv \kappa - \kappa^*.$$

Note that since $K > 1$ implies a higher than expected aversion with the Home individuals to supply labor (a negative productivity shock), $\kappa^i > 0$ refers to the case when the Home economy is hit by a negative idiosyncratic productivity shock. The assumption that $E\kappa = E\kappa^*$ implies that $E\kappa^i = 0$.

Substituting expressions (14) and (15) into equation (12), expected utility with flexible nominal wages equals[13]

$$\begin{aligned} E(U)^{flex} &= \Psi \left[\frac{(\phi-1)(\theta-1)}{\phi\theta}\right]^{\frac{1-\delta}{\delta}} \exp\left\{-\frac{(1-\delta)E\kappa}{\delta} + \frac{(1-\delta)^2}{2\delta^2}\sigma_{\kappa_w}^2 \right. \\ &\qquad \left. + \frac{(1-n)^2(1-\gamma)^2(1-\delta)^2}{2\left[1-(1-\gamma)(1-\delta)\right]^2}\sigma_{\kappa_i}^2\right\} \\ &= \Psi \exp\left\{\frac{(1-\delta)\omega}{\delta}\right\}, \end{aligned} \tag{16}$$

where the parameter ω is a constant.[14] Therefore, since neither spending, z, nor the nominal exchange rate, s, enter the expression for welfare under wage

[13]Note that, since $K^i = \frac{K}{K^*}$, the covariance between K^i and any variable X can be expressed $\sigma_{\kappa_i x} = \sigma_{\kappa x} - \sigma_{\kappa^* x}$ Similarly, since $K^w = K^n K^{*(1-n)}$, we can write the covariance between K^w and X as $\sigma_{\kappa_w x} = n\sigma_{\kappa x} + (1-n)\sigma_{\kappa^* x}$. For further calculations with the productivity terms, see Appendix A.3.3.

[14]The parameter is defined as

$$\omega \equiv \log\left[\frac{(\phi-1)(\theta-1)}{\phi\theta}\right] - E\kappa + \frac{(1-\delta)}{2\delta}\sigma_{\kappa_w}^2 + \lambda,$$

flexibility, from (16) it follows that with perfectly flexible wages, monetary policy cannot improve on expected welfare. Nominal wages are set after the realization of the productivity shock, and hence there are no wage contracts stopping the workers from optimally supplying their desired level of labor *ex post*. Remember that the expectations operator enters expression (16) since the individual has to choose to set wages flexible in period $t-1$.

2.3.2 Expected welfare with sticky wages

Given that the exogenous variables in the model are log-normally distributed, expected spending, Ez, and expected terms of trade, $E\tau$, are log-normally distributed as well. With the definition $E\tau \equiv w^* - w + Es$ for the expected log terms of trade we can write expected utility with sticky wages, equation (12), as

$$E(U) = \Psi \exp \left\{ (1-\delta)\, Ez + (1-n)(1-\gamma)\,(1-\delta)\, E\tau + \frac{(1-\delta)^2}{2}\sigma_z^2 \right.$$
$$\left. + \frac{(1-n)^2(1-\gamma)^2\,(1-\delta)^2}{2}\sigma_s^2 + (1-n)(1-\gamma)\,(1-\delta)^2\,\sigma_{sz} \right\}, \qquad (17)$$

where of course $\sigma_s^2 = \sigma_\tau^2$ and $\sigma_{sz} = \sigma_{\tau z}$ since nominal wages now are preset.

In order to undertake explicit comparative welfare analysis we therefore need to solve for mean log terms of trade, $E\tau$ ($\equiv w^* + Es - w$) and mean log spending measured in units of tradables, Ez.[15] By the symmetry of the production functions for tradables and non-tradables (3) it follows that total labor demand in Home is $L = Y_H + Y_N$. Note also from the national income constraint, $PC = P_H C_H + P_N C_N$, that it is possible to write labor demand as

$$L = Y_H + Y_N = \left(\frac{W^* S}{W}\right)^{1-n} Z. \qquad (18)$$

Using (18) we can rewrite the Home wage first order condition (7) as

$$\left(\frac{W}{W^*}\right)^{(1-n)[1-(1-\gamma)(1-\delta)]} = \left(\frac{\phi\theta}{(\phi-1)\,(\theta-1)}\right) \frac{E\left\{KS^{(1-n)}Z\right\}}{E\left\{S^{(1-n)(1-\gamma)(1-\delta)}Z^{1-\delta}\right\}}. \qquad (19)$$

---

with

$$\lambda = \frac{(1-n)^2\,(1-\gamma)^2\,(1-\delta)\,\delta}{2\,[1-(1-\gamma)\,(1-\delta)]^2}\sigma_{\kappa_i}^2.$$

[15]All calculations underlying the results in section 2.3.2 are in Appendix A.3.2.

Combining this with the MU counterpart to this expression[16] we get the following equilibrium condition

$$\left(\frac{W}{W^*}\right)^{1-(1-\gamma)(1-\delta)} = \frac{E\left\{KS^{(1-n)}Z\right\}E\left\{S^{-n(1-\gamma)(1-\delta)}Z^{1-\delta}\right\}}{E\left\{K^*S^{-n}Z\right\}E\left\{S^{(1-n)(1-\gamma)(1-\delta)}Z^{1-\delta}\right\}}. \tag{20}$$

By equations (19) and (20) it is indeed possible to find a simultaneous solution for $E\tau$ and Ez. A log-linearization of equation (20) gives $E\tau$ as

$$E\tau = \frac{-1}{1-(1-\gamma)(1-\delta)}\left\{\frac{1-2n}{2}\sigma^2_{\kappa_i} + \sigma_{\kappa_w s} + (1-2n)\,\sigma_{\kappa_i s} + \sigma_{\kappa_i z} + \left[1-(1-\gamma)(1-\delta)^2\right]\sigma_{sz} + \frac{(1-2n)}{2}\left[1-(1-\gamma)^2(1-\delta)^2\right]\sigma^2_s\right\}. \tag{21}$$

By taking the log of (19) and then combining it with (21) we obtain an expression for expected spending when wages are set one period in advance as

$$Ez = \frac{1}{\delta}\left\{\omega - \lambda - \frac{(1-\delta)}{2\delta}\sigma^2_{\kappa_w} - \frac{1}{2}\sigma^2_{\kappa_w} - \frac{n\,(1-n)}{2}\sigma^2_{\kappa_i} - n(1-n)\sigma_{\kappa_i s} - \sigma_{\kappa_w z} - \frac{n(1-n)}{2}\left(1-(1-\gamma)^2(1-\delta)^2\right)\sigma^2_s - \frac{1-(1-\delta)^2}{2}\sigma^2_z\right\}, \tag{22}$$

where ω and λ are defined in footnote 16 above.

Monetary policy affect the variances and covariances involved in equations (21) and (22) and thereby also mean wages and prices. For example, the term $\sigma_{\kappa_i z}$ in equation (21) represents the covariance between the country specific productivity shock and spending (measured in units of tradables). If world spending is high at the same time as the Home economy is hit by a negative productivity shock ($\sigma_{\kappa_i z} > 0$), then expected marginal utility from consumption is low exactly when the expected marginal utility loss from supplying labor is high, inducing the Home wage setters to set a higher nominal wage. Similarly, if the term $\sigma_{\kappa_i s}$ in (22) is positive, then Home individuals' increase nominal wages since real wages are low at the same

[16]The MU counterpart to equation (19) is given by

$$\left(\frac{W}{W^*}\right)^{-n[1-(1-\gamma)(1-\delta)]} = \left(\frac{\phi\theta}{(\phi-1)\,(\theta-1)}\right)\frac{E\left\{K^*S^{-n}Z\right\}}{E\left\{S^{-n(1-\gamma)(1-\delta)}Z^{1-\delta}\right\}}.$$

time as the disutility from effort is unexpectedly high. This increase in Home nominal wages in turn lowers the value of mean (real) spending.[17]

Using these derived expressions for $E\tau$ and Ez in (17) the expected welfare in Home when wages are sticky can be written

$$
\begin{aligned}
E(U) = \Psi \exp \Bigg\{ \frac{1-\delta}{\delta} \Bigg[& \omega - \lambda - \frac{1}{2\delta}\sigma_{\kappa_w}^2 - \frac{n(1-n)}{2}\sigma_{\kappa_i}^2 \\
& - \frac{n(1-n)\left(1-(1-\gamma)^2(1-\delta)^2\right)}{2}\sigma_s^2 - n(1-n)\sigma_{\kappa_i s} - \sigma_{\kappa_w z} \Bigg] \\
& - \frac{(1-\delta)}{2}\sigma_z^2 + \frac{(1-n)^2(1-\gamma)^2(1-\delta)^2}{2}\sigma_s^2 - \delta(1-n)(1-\gamma)(1-\delta)\sigma_{sz} \\
& - \frac{(1-n)(1-\gamma)(1-\delta)}{1-(1-\gamma)(1-\delta)} \Bigg[\frac{1-2n}{2}\sigma_{\kappa_i}^2 + \sigma_{\kappa_w s} + (1-2n)\sigma_{\kappa_i s} \\
& + \sigma_{\kappa_i z} + \frac{(1-2n)}{2}\left[1-(1-\gamma)^2(1-\delta)^2\right]\sigma_s^2 \Bigg] \Bigg\} \qquad (23)
\end{aligned}
$$

Therefore, the expected utility when wages are set one period before the shocks realize equal the expected utility with flexible wage and a number of uncertainty terms. It is illustrative to write the above expression as

$$
E(U) = \Psi \exp \left\{ \frac{(1-\delta)\,\omega}{\delta} - (1-\delta)\,\Lambda \right\} \qquad (24)
$$

where Λ is a function of the variance and covariance terms in (23), which in turn are functions of monetary policy. It is assumed that the monetary policy objective is to target the flexible solution, which in terms of equation (24) is equivalent to set money supply in a way that would make $\Lambda = 0$.

3 Monetary Policy

The use of this somewhat algebra abundant model makes it possible to undertake explicit welfare analysis of alternative monetary regimes. In the present paper I will use this property of the model and examine the welfare consequences from two different wage setting schedules in a country that enters into a monetary union.

[17] Note that if the Home economy makes up a small fraction of the total world economy ($n \to 0$) the effect from an increase in Home wages on mean spending disappears.

The presence of nominal rigidities in an economy may result in that unexpected disturbances move the economy away from equilibrium.[18] The working instrument to bring back the economy to an *ex post* equilibrium is monetary policy. The case when discretionary monetary policy is used to create monetary surprises in order to boost the economy away from some natural rate of growth or unemployment is not considered. Instead focus will be on monetary rules, where monetary policy is a tool to compensate for the productivity shocks.

The objective of the Home monetary authority is to target the outcome that would have been without wage rigidities. That allocation is identical to the competitive equilibrium, but for the distortions arising from the monopolistic competition in wage and price setting. Therefore, I do not specify any inflation target for the central bank. If the model was extended to be multi-period it would be possible to undertake the analysis with a central bank that is inflation targeting[19] but that would make the model even more complicated. Furthermore, if we assumed inflation targeting was the prime objective of the monetary authorities in Home and Foreign, but with some interest in stabilization of the real economy, an assumption that the Home and MU central banks gave equal weight to the real economy would be enough to get the same results as we get in this simpler single-period model.

In order to study how monetary policy can influence the equilibrium *ex post* we have to find expressions for the post-shocks realized spending and terms of trade for the case when wages are preset. These are derived from the individual first order conditions with respect to holding of nominal (or real) money, equation (8) and its MU counterpart. By taking an average of the two[20] realized spending can be expressed as

$$z = \frac{1}{\delta}\left(nm + (1-n)\,m^*\right) - \frac{1}{\delta}\left(nw + (1-n)w^*\right) - \frac{1}{\delta}\log\left(\frac{\theta}{\theta-1}\right) - \frac{1}{\delta}\log\chi, \quad (25)$$

where it is assumed that $\chi = \chi^*$. The realized nominal exchange rate is derived in a similar fashion by calculating the difference between the Home

[18] The model in this paper also includes distortions that arise due monopolies in the wage and in the product markets. We assume that it is not the task of the monetary authority to compensate for these distortions.

[19] See for example the paper by Benigno (2001).

[20] The average is found by multiplying the Home and Foreign money demand equation (8) (or adding them if the money demand equations are in logs).

and MU versions of (8) giving us

$$s = \frac{m - m^*}{1-(1-\gamma)(1-\delta)} + \frac{(1-\gamma)(1-\delta)}{1-(1-\gamma)(1-\delta)}(w - w^*)\,. \tag{26}$$

The benchmark case for monetary policy in most single-period general equilibrium is a situation where the central banks target the allocations that would have been the outcome under complete wage flexibility.[21] Next, we will solve for optimal monetary policy under two different institutional regimes: monetary independence and monetary union.

3.1 Monetary Independence

Let a hatted variable represents the surprise part of the variable: $\hat{m} \equiv m - Em$. Equalizing the log versions of the flexible nominal exchange rate outcome, given by equation (14) to the outcome under wage rigidity, given by equation (26), we obtain the following condition that must hold for to equalize the flexible to the sticky solution,

$$\frac{1}{1-(1-\gamma)(1-\delta)}(\hat{\kappa}^* - \hat{\kappa}) = \frac{1}{1-(1-\gamma)(1-\delta)}(\hat{m}^* - \hat{m})\,,$$

since in equation (26), $\hat{w}^* = \hat{w} = 0$. Similarly, equalizing the innovation in realized spending in the flexible and the sticky case, equations (15) and (25), gives

$$-\frac{1}{\delta}(n\hat{\kappa} + (1-n)\hat{\kappa}^*) = \frac{1}{\delta}(n\hat{m} + (1-n)\hat{m}^*)\,.$$

Combining these two conditions, it is straightforward to solve for the monetary feed-back rules the two central banks should adopt to assure that the sticky wage allocations equal the flexible wage allocations *ex post*. The rule for the Home central bank is,

$$\hat{m} = -(1-n)(\hat{\kappa} - \hat{\kappa}^*) - (n\kappa + (1-n)\kappa^*).$$

The corresponding rule for the MU central bank is

$$\hat{m}^* = n(\hat{\kappa} - \hat{\kappa}^*) - (n\kappa + (1-n)\kappa^*).$$

[21] One might consider other strategies for monetary policy, for example, that the central banks seeks to maximize expected utility of a representative individual. Equilibrium in the model is then given by the Nash equilibrium. Since the objective with this paper is to study the behavior of wage setters, I will only consider the simplest (in terms of algebra) rule for monetary policy: Target the flexible equilibrium.

To simplify notation, its useful to write the monetary policy feed-back rules as

$$\hat{m} = -\beta_i \hat{\kappa}_i - \beta_w \hat{\kappa}_w, \tag{27}$$

and

$$\hat{m}^* = \beta_i^* \hat{\kappa}_i - \beta_w^* \hat{\kappa}_w, \tag{28}$$

where β_i and β_w (β_i^* and β_w^*) are parameters describing monetary policy in Home (Foreign). In terms of the monetary policy parameters in (27) and (28) we can describe the optimal monetary policy rules for the Home and Foreign central banks as

$$\beta_w = \beta_w^* = 1 \tag{29}$$

$$\beta_i = (1-n), \quad \beta_i^* = n. \tag{30}$$

Since n represents the size of the Home economy, we can draw the conclusion that the smaller the Home economy is, the less should the Foreign central bank respond to asymmetric productivity shocks and the more should the Home central bank accommodate the idiosyncratic productivity shocks.

3.1.1 Monetary union

The ultimate objective is to compare the monetary independence case to the monetary union. When the Home economy has entered into the monetary union, the common central bank (CCB) has to decide upon a rule for union wide money supply. Since a monetary union implies a common currency, the nominal exchange rate can no longer be used as an adjustment mechanism for the relative prices. In this section we will redo the same analysis as above, but now with the nominal exchange rate fixed. For simplicity, assume $S = 1$. With a fixed nominal exchange rate, the expressions corresponding to equations (14) and (15) are

$$\left(\frac{W^*}{W}\right)^{flex} = \left(\frac{K^*}{K}\right)^{\frac{1}{1-(1-\gamma)(1-\delta)}} \tag{31}$$

and

$$Z^{flex} = \left[\frac{(\phi-1)(\theta-1)}{\phi\theta K^n K^{*(1-n)}}\right]^{\frac{1}{\delta}} \tag{32}$$

These equations together define the flexible wage equilibrium in a monetary union. The *ex post* level of consumption when wages are preset in a monetary union is, from (25) given by:

$$\hat{z} = \frac{1}{\delta}\hat{m}^{MU}, \tag{33}$$

where $\hat{m}^{MU}$ is the unexpected part of union wide money supply. Therefore, taking the logs of (32) and equalizing it to the sticky allocation given by (33) we get the following simply rule governing monetary policy in a monetary union:

$$\hat{m}^{MU} = -\left(n\ \hat{\kappa} + (1-n)\,\hat{\kappa}^{*}\right) = -\hat{\kappa}^{w} \tag{34}$$

Therefore, in a monetary union the CCB will only target the common productivity shock. In terms of the monetary policy parameters, monetary policy in the monetary union is given by

$$\beta_w^{MU} = 1, \quad \beta_i^{MU} = 0$$

Since in a monetary union there are no longer any nominal exchange rate available why there is no longer any instrument available for the central bank to compensate for the possible asymmetry of productivity shocks, the competitive equilibrium is no longer attainable for the case when wages are preset. This leaves us with the following proposition:

Proposition 3 *Given the existence of idiosyncratic productivity shocks, the competitive equilibrium is not attainable in a monetary union when wages are preset.*

Proof. We have seen that the rule for monetary policy that would always restore the competitive equilibrium (the flexible solution) *ex post* is given by $\beta_w = \beta_w^{*} = 1$, $\beta_i = 1 - n$ and $\beta_i^{*} = n$. In a monetary union, the exchange rate is fixed, why the nominal exchange rate can no longer move in order to restore equilibrium in presence of idiosyncratic productivity shocks. Therefore, as long as the nominal wages are set before the realization of the shocks, the nominal exchange rates can no longer change the relative prices that would have been necessary if idiosyncratic productivity shocks hit the union economy. ■

4 Utility comparisons

The objective of the paper was to study if the incentive for wage setters to choose flexible wages changes once a country enters a monetary union. The way to answer this question is to compare the expected utilities from flexible and sticky wages outside and inside a monetary union, respectively.

Denoting the difference in expected utility between the flexible and the sticky equilibria as

$$\Delta \equiv \frac{E(U)^{flex}}{E(U)^{sticky}} \tag{35}$$

we can compare this difference when the Home economy does not enter the MU, Δ^{non-MU}, with the case when it does, Δ^{MU}.

Consider first the case with monetary independence, Δ^{non-MU}. Using the expressions for s and z (equations (25) and (26)) above, the variance and covariance terms entering Home expected utility $E(U)$ can be written in terms of the monetary policy parameters β_w, β_i, β_w^* and β_i^* as

$$\hat{s} = -\left[\frac{\beta_i + \beta_i^*}{1-(1-\gamma)(1-\delta)}\right]\hat{\kappa}_i - \left[\frac{\beta_w - \beta_w^*}{1-(1-\gamma)(1-\delta)}\right]\hat{\kappa}_i \tag{36}$$

$$\hat{z} = -\left[\frac{n\beta_i - (1-n)\beta_i^*}{\delta}\right]\hat{\kappa}_i - \left[\frac{n\beta_w + (1-n)\beta_w^*}{\delta}\right]\hat{\kappa}_w \tag{37}$$

Substituting in the optimal policy rules, we may rewrite the variances and covariances entering the expression for expected utility as

$$\sigma_{\kappa_i z} = 0, \quad \sigma_{\kappa_w z} = -\frac{1}{\delta}\sigma_{\kappa_w}^2, \quad \sigma_z^2 = \frac{1}{\delta^2}\sigma_{\kappa_w}^2, \quad \sigma_{sz} = 0$$

$$\sigma_{\kappa_i s} = -\frac{1}{1-(1-\gamma)(1-\delta)}\sigma_{\kappa_i}^2, \quad \sigma_{\kappa_w s} = 0, \quad \sigma_s^2 = \frac{1}{[1-(1-\gamma)(1-\delta)]^2}\sigma_{\kappa_i}^2$$

Using these, expected utility with preset wages under monetary independence reduces to

$$\begin{aligned} E(U)^{sticky,non-MU} &= \Psi \exp\left\{\frac{(1-\delta)}{\delta}\left[\omega - \lambda - \frac{(1-\delta)}{2\delta}\sigma_{\kappa_w}^2\right]\right. \\ &\quad \left. + \frac{(1-\delta)^2}{2\delta^2}\sigma_{\hat{\kappa}_w}^2 + \frac{(1-n)^2(1-\gamma)^2(1-\delta)^2}{2[1-(1-\gamma)(1-\delta)]^2}\sigma_{\hat{\kappa}_i}^2\right\} \\ &= \Psi \exp\left\{\frac{(1-\delta)\,\omega}{\delta}\right\} \end{aligned} \tag{38}$$

Proposition 4 *As long as the Home economy is not a member of the monetary union, the individuals are indifferent between setting flexible or sticky nominal wages.*

Proof. Using the equations (16) and (38) the value of Δ^{non-MU} is given by

$$\Delta^{non-MU} = \frac{E\left(U\right)^{flex}}{E\left(U\right)^{sticky}} = 1. \tag{39}$$

Hence, the $t-1$ expected utilities from choosing sticky or flexible nominal wages are identical and, assuming away potentially different costs, the individuals are indifferent. ■

If the Home economy enters the monetary union, then the monetary policy parameters are $\beta_i^{MU} = 0$ and $\beta_w^{MU} = 1$ and the nominal exchange rate is fixed (assume $S = 1$), so that $\sigma_s^2 = \sigma_{\kappa_i s} = \sigma_{\kappa_w s} = 0$. Expected utility with preset wages therefore reduces to

$$\begin{aligned} E(U)^{sticky,MU} &= \Psi \exp\left\{ \frac{(1-\delta)}{\delta} \left(\omega - \lambda - \frac{n\left(1-n\right)}{2} \sigma_{\kappa_i}^2 \right.\right. \\ &\qquad \left.\left. - \frac{\delta\left(1-2n\right)\left(1-n\right)\left(1-\gamma\right)}{1-(1-\gamma)(1-\delta)} \sigma_{\kappa_i}^2 \right) \right\}. \end{aligned} \tag{40}$$

Given that the individuals' preferences and that the structure of the productivity shocks do not change, expected welfare with flexible wages is not affected by the move from monetary independence to monetary union. The corresponding difference to (39) when Home is a member of the monetary union is therefore given by

$$\begin{aligned} \Delta^{MU} &= \frac{E\left(U\right)^{flex,MU}}{E\left(U\right)^{sticky,MU}} \\ &= \frac{\Psi \exp\left\{ \frac{(1-\delta)\omega}{\delta} \right\}}{\Psi \exp\left\{ \frac{(1-\delta)}{\delta} \left\{ \omega - \lambda - \frac{n(1-n)}{2} \sigma_{\kappa_i}^2 - \frac{\delta(1-2n)(1-n)(1-\gamma)}{1-(1-\gamma)(1-\delta)} \sigma_{\kappa_i}^2 \right\} \right\}} \\ &= \frac{\exp\left\{ \frac{(1-\delta)\omega}{\delta} \right\}}{\exp\left\{ \frac{(1-\delta)\omega}{\delta} - \frac{(1-\delta)}{\delta} \left\{ \lambda + \frac{n(1-n)}{2} \sigma_{\kappa_i}^2 + \frac{\delta(1-2n)(1-n)(1-\gamma)}{1-(1-\gamma)(1-\delta)} \sigma_{\kappa_i}^2 \right\} \right\}} \\ &= \exp\left\{ \frac{(1-\delta)}{\delta} \left\{ \lambda + \frac{n\left(1-n\right)}{2} \sigma_{\kappa_i}^2 + \frac{\delta\left(1-2n\right)\left(1-n\right)\left(1-\gamma\right)}{1-(1-\gamma)(1-\delta)} \sigma_{\kappa_i}^2 \right\} \right\}. \end{aligned} \tag{41}$$

Proposition 5 *Entrance in a monetary union implies a change in incentives for wage setters to choose to set flexible nominal wages. If the incentive increases or decreases depends critically on the relative risk aversion.*

(i) *If* $0 < \delta < 1$*, the loss in expected utility from choosing preset instead of flexible nominal wages is higher if the country enters into a monetary union* $\left(\Delta^{non-MU} - \Delta^{MU} < 0\right)$.

(ii) *If* $\delta > 1$ *the loss in expected utility from choosing preset instead of flexible nominal wages is lower if the country enters into a monetary union* $\left(\Delta^{non-MU} - \Delta^{MU} > 0\right)$.

Proof. From equations (39) and (41) above we have the following relation:

$$\begin{aligned} & \Delta^{non-MU} - \Delta^{MU} \\ = & \; 1 - \exp\left\{\frac{(1-\delta)}{\delta}\left\{\lambda + \frac{n(1-n)}{2}\sigma^2_{\kappa_i} + \frac{\delta(1-2n)(1-n)(1-\gamma)}{1-(1-\gamma)(1-\delta)}\sigma^2_{\kappa_i}\right\}\right\} \\ = & \; 1 - \exp\{\Omega\} \end{aligned} \quad (42)$$

That is, in order to determine the sign of the difference $\Delta^{non-MU} - \Delta^{MU}$ we have to consider the value of Ω. If $\Omega > 0$, then $\Delta^{non-MU} - \Delta^{MU} < 0$, implying that the loss from setting preset wages is bigger in a monetary union. If $\Omega < 0$, then $\Delta^{non-MU} - \Delta^{MU} > 0$ implying that the incentive to set nominal wages before the realization of the productivity shocks actually increases once the Home economy enters the monetary union. It turns out that the most illustrative way to show illustrate the result is by doing simple numerical simulations of the model. In Table 4.1 below I have simulated equation (42) for different values of the relative risk aversion δ. As long as $\delta < 1$, the utility loss from choosing preset wages is larger in a monetary union, why the incentive for workers to choose wage flexibility is higher. For $\delta > 1$ the result is the opposite. ■

Table 4.1 show numerically the value of $\Delta^{non-MU} - \Delta^{MU}$ for different values of δ and n. In the calculations I have assumed that $\gamma = 0.6$ and that $\sigma^2_{\kappa_i} = 0.1$.

Table 4.1. The value of $\Delta^{non-MU} - \Delta^{MU}$

	n			
δ	**0.05**	**0.10**	**0.20**	**0.50**
0.2	-0.062	-0.063	-0.064	-0.054
0.5	-0.027	-0.025	-0.022	-0.013
0.9	-0.004	-0.004	-0.003	-0.001
1	0	0	0	0
2	0.022	0.019	0.015	0.005
4	0.034	0.030	0.022	0.006

The most decisive parameter for the outcome is δ. Combining equation (labordemandind) and (labordem) we see that with nominal wages fixed, the expected labor demand is given by,

$$E\{L\} = \left(\frac{W^*}{W}\right)^{1-n} E\{Z\}.$$

Thus, expected labor supply is proportional to expected spending. Since Z is log-normally distributed $E\{Z\} = \exp\left\{Ez + \frac{1}{2}\sigma_z^2\right\}$. The effect of spending variability, σ_z^2, on expected level spending is therefore

$$\frac{\partial E\{Z\}}{\partial \sigma_z^2} = \left(-\frac{(1-\delta)}{2}\right) \exp\left\{Ez + \frac{1}{2}\sigma_z^2\right\}$$

The sign of this derivative is determined by the coefficient of relative risk aversion. If $\delta < 1$, then the expected level of spending, and hence expected labor supply is decreasing in the variability of z. For $\delta > 1$, however, expected labor demand is increasing in σ_z^2. Without the nominal exchange rate as an instrument available to hinder unexpectedly high disutility from effort, highly risk avert wage setters will hedge against the possibility that marginal utility from consumption is low at the same time as the marginal disutility from effort is high by setting a higher preset nominal wage.

The main difference between this model and the model in Obstfeld and Rogoff (2002) is the generalization that countries need not be of equal size. The results in Table 4.1 indicate that country size does not determined the sign of the difference, but it does affect the magnitude.

Corollary 6 *For the relevant values* $0.05 < n < 0.20$ *the difference* $\Delta^{non-MU} - \Delta^{MU}$ *is marginally affected by country size. Note, however, that the main results generally are more pronounced given that the countries are not of equal size. The conclusion is that country size is of some importance and ignoring this by assuming* $n = 0.50$*, may bias the results.*

Proof. Given by the numerical simulations in Table 4.1. ■

5 Discussion

I regard the most important contribution of this paper to be that it introduces a new approach to study wage setting, by employing an explicitly microfounded macroeconomic model, where individuals' attitudes towards risk are decisive for the way in which wages are written.

In the literature, the most commonly discussed ways to increase nominal wage flexibility are either to include indexation clauses in the wage contracts, or to shorten the contract lengths. Although not aiming to explicitly study the optimal length of wage contracts the spirit of this paper has more in common with the contract length literature. The model does not incorporate any costs from increasing nominal wage flexibility. Assuming a higher cost from setting flexible rather than sticky wages could explain the observed dominance of sticky contracts, despite the neutrality result in Proposition 4. Note also that the gain in expected utility from increasing wage flexibility after monetary union membership for values $0.5 < \delta < 1$ decreases with country size. Introducing a "physical" cost connected to wage flexibility could therefore produce the outcome that wage flexibility increases if the Home country is sufficiently small, while the opposite is true if the Home country is larger.

The shock structure of the model and the assumption of rule based monetary policy implies that the paper has little in common with the wage indexation literature. One possible way to include demand shocks in the model is to extend the model by including fiscal policy.[22] An independent fiscal policy in a monetary union could either have the rule of an alternative stabilizing tool, thereby possibly decreasing the need for wage flexibility. Alternatively, one could incorporate demand shocks in the model by assuming opportunistic fiscal policy. With demand shocks present, the study of wage indexation would be possible.

The present model assumes identical individuals why, in fact, trade unions are redundant. An interesting extension of the model would be to assume that agents are heterogenous with respect to, for example, initial wealth and risk aversion. This would create a rationale of unions.

Finally, monetary policy here is modelled in the simplest possible way. Alternative policies of the central banks are that they either cooperatively or non-cooperatively maximize welfare. With the Home country relatively small, the result in Proposition 1 could give scope for a "beggar-thy-neighbor" strategy, where the Home central bank would set monetary policy to move the nominal exchange rate in a favorable way for the Home individuals. This

[22]For a suggestion of how to incorporate fiscal policy in the "new open-economy macroeconomics" framework, see Obstfeld and Rogoff (1998) or Corsetti and Pesenti (2001).

alternative strategy is not explicitly dealt with here, since in would merely strengthen the results obtained under flexible targeting central banks. If the Home central bank acts as to utilize the nominal wage contracts to maximize Home welfare, the increase in incentives to choose nominal wage flexibility after a monetary union would be even stronger.

6 Conclusions

One implication of the creation of a monetary union is that monetary policy becomes impotent as an instrument to prevent from idiosyncratic productivity shocks sending the economy to disequilibrium. In the present paper I have shown that this, depending on agents risk aversion, implies a changed incentive for wage setters to set wages that follow the fluctuations in productivity. In the stochastic general equilibrium model with sticky wages, originally developed by Obstfeld and Rogoff (2000), used here the national central bank in the small entering Home economy optimally acts procyclically to idiosyncratic productivity shocks. When then Home economy enters the monetary union, this possibility disappears leading to that idiosyncratic shocks may force the workers to supply a non-optimal amount of labor given the contracted nominal wage. If the labor supply is included in labor contracts, then the productivity risks lie with the firms. Entrance in a monetary union in this case increases the incentive for firms to increase wage flexibility.

An important question in the discussion about monetary unions is whether idiosyncratic productivity should lead to real or nominal fluctuations. The results derived in this paper show that entrance in a monetary union most probably implies a stronger incentive to increase nominal flexibility for the agents that carry the risk connected to real fluctuations.

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A Demand and prices

A.1 Demand

All consumption subindexes are symmetric and defined as

$$C_H = \left[\left(\frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n C(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

$$C_F = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 C(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

$$C_N = \left[\int_0^1 C(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

where θ is the constant elasticity of substitution between goods produced in different firms within a country.

A.2 Optimal price setting

The problem for the profit maximizing, monopolistically competitive Home firm to maximize profits given demand:

$$\max_{P(h)} \left[\frac{P(h)}{P} - \frac{W}{P} \right] \left(\frac{P(h)}{P_H} \right)^{-\theta} \left(\frac{P_H}{P} \right)^{-1} C^W$$

where we have used the relation $Y(h) = L(h) = C(h)$. The first order conditions reduces to

$$(1-\theta)\, P(h)^{-\theta} P_H^{\theta-1} C^W + \theta W P(h)^{-\theta-1} P_H^{\theta-1} C^W = 0$$

Simplifying and rearranging we get the result that the optimal price set by firm j in the Home country is

$$P(h) = \left(\frac{\theta}{\theta-1} \right) W$$

In a symmetric equilibrium, the price levels for domestically produced goods in Home and MU will be $P_H = \left(\frac{\theta}{\theta-1}\right) W$ and $P_F^* = \left(\frac{\theta}{\theta-1}\right) W^*$

A.3 Solutions for utility:

Derivation of equation (10):

$$\begin{aligned} C &= \frac{P_T}{P} Z = P_T^{1-\gamma} P_N^{-(1-\gamma)} Z = \left(P_H^n P_F^{(1-n)} \right)^{(1-\gamma)} P_N^{-(1-\gamma)} \\ &= \left(\frac{\theta}{\theta - 1} \right)^{-(1-n)(1-\gamma)} W^{-(1-n)(1-\gamma)} \left(\frac{\theta}{\theta - 1} \right)^{(1-n)(1-\gamma)} (W^* S)^{(1-n)(1-\gamma)} Z \\ &= \left(\frac{W^* S}{W} \right)^{(1-n)(1-\gamma)} Z \end{aligned}$$

Derivation of equation (11). From the Home budget constraint $P_H (Y_H + Y_N) = P_H L = PC$. Hence, using the Home first order conditions for wage setting we can write

$$\begin{aligned} E\{KL\} &= \left(\frac{\phi - 1}{\phi} \right) W E \left\{ \frac{1}{P} \frac{L}{C^\delta} \right\} \\ &= \left(\frac{\phi - 1}{\phi} \right) W E \left\{ \frac{1}{P} \frac{P}{P_H} C^{1-\delta} \right\} = \left(\frac{(\phi - 1)(\theta - 1)}{\phi \theta} \right) E \left\{ C^{1-\delta} \right\} \end{aligned}$$

A.3.1 Expected welfare with flexible wages

First, note that $C = \frac{P_H}{P} L$. Also, express the Home price level as:

$$\begin{aligned} P &= P_T^\gamma P_N^{1-\gamma} = P_H^{\gamma n} P_F^{\gamma(1-n)} P_N^{1-\gamma} \\ &= \left[\left(\frac{\theta}{\theta - 1} \right) W \right]^{1-\gamma+n\gamma} \left[\left(\frac{\theta}{\theta - 1} \right) W^* S \right]^{\gamma(1-n)} \\ &= \left(\frac{\theta}{\theta - 1} \right) W \left(\frac{W^* S}{W} \right)^{\gamma(1-n)}. \end{aligned}$$

Similarly, the MU price level can be expressed as $P^* = \left(\frac{\theta}{\theta - 1} \right) W^* \left(\frac{W}{SW^*} \right)^{n\gamma}$.The first-order conditions for wage setting in the case of flexible wages is

$$\frac{W}{PC^\delta} = \left(\frac{\phi}{\phi - 1} \right) K \Leftrightarrow \frac{W}{P^{1-\delta} P_H^\delta} = \left(\frac{\phi}{\phi - 1} \right) K L^\delta.$$

Using the derived expression for P above we can rewrite this expression as

$$\left(\frac{W^* S}{W} \right)^{-(1-\delta)(1-n)\gamma} = \left(\frac{\phi \theta K}{(\phi - 1)(\theta - 1)} \right) L^\delta. \quad \text{(A.1)}$$

Similarly, using the MU budget constraint, $C^* = \frac{P_F^*}{P^*}L^*$ the MU counterpart to (A.1) is

$$\left(\frac{W^*S}{W}\right)^{n(1-\delta)\gamma} = \left(\frac{\phi\theta K^*}{(\phi-1)(\theta-1)}\right)(L^*)^{\delta}. \tag{A.2}$$

To complete the solution of the model, we have to use a relation between L and Z and between L^* and Z. Again turning to the resource constraint, we see that $P_T Z = PC = P_H L$. Hence

$$L = \frac{P_T}{P_H}Z = \frac{P_H^n P_F^{1-n}}{P_H}Z = \left(\frac{W^*S}{W}\right)^{1-n} Z. \tag{A.3}$$

The MU counterpart to this is

$$L^* = \left(\frac{W^*S}{W}\right)^{-n} Z. \tag{A.4}$$

Substituting (A.3) into (A.1) the flexible equilibrium spending, measured in units of tradables, level can be expressed as

$$Z = \left[\frac{\phi\theta K}{(\phi-1)(\theta-1)}\right]^{-\frac{1}{\delta}} \left(\frac{W^*S}{W}\right)^{-\frac{(1-n)[1-(1-\gamma)(1-\delta)]}{\delta}}. \tag{A.5}$$

Now, substituting (A.4) into (A.2) and using the above expression for $Z = Z^*$, we obtain the expressions (14) and (15) in the main text.

Substituting (14) and (15) into (12) gives an expression for expected welfare when wages are flexible as

$$
\begin{aligned}
E(U) &= \Psi E\left\{\left(\frac{K}{K^*}\right)^{-\frac{(1-n)(1-\gamma)(1-\delta)}{1-(1-\gamma)(1-\delta)}}\left[\frac{(\phi-1)(\theta-1)}{\phi\theta K^n\left(K^*\right)^{1-n}}\right]^{\frac{1-\delta}{\delta}}\right\} \\
&= \Psi\left[\frac{(\phi-1)(\theta-1)}{\phi\theta}\right]^{\frac{1-\delta}{\delta}} E\left\{\left(\frac{K}{K^*}\right)^{-\frac{(1-n)(1-\gamma)(1-\delta)}{1-(1-\gamma)(1-\delta)}}\left[\frac{1}{K^n\left(K^*\right)^{1-n}}\right]^{\frac{1-\delta}{\delta}}\right\} \\
&= \Psi\left[\frac{(\phi-1)(\theta-1)}{\phi\theta}\right]^{\frac{1-\delta}{\delta}} \exp\left\{\frac{(1-n)^2(1-\gamma)^2(1-\delta)^2}{2\left[1-(1-\gamma)(1-\delta)\right]^2}\sigma^2_{\kappa_i}\right. \\
&\qquad \left. + \frac{(1-\delta)^2}{2\delta^2}\sigma^2_{\kappa_w} - \frac{(1-\delta)}{\delta}E\kappa\right\} \\
&= \Psi\exp\left\{\frac{1-\delta}{\delta}\left[\log\left(\frac{(\phi-1)(\theta-1)}{\phi\theta}\right) + \frac{(1-\delta)}{2\delta}\sigma^2_{\kappa_w} - E\kappa + \right.\right. \\
&\qquad \left.\left. + \frac{\delta(1-n)^2(1-\gamma)^2(1-\delta)}{2\left[1-(1-\gamma)(1-\delta)\right]^2}\sigma^2_{\kappa_i}\right]\right\} \\
&= \Psi\exp\left\{\frac{(1-\delta)}{\delta}\omega\right\}
\end{aligned}
$$

with the parameter ω is defined as

$$
\omega \equiv \log\left[\frac{(\phi-1)(\theta-1)}{\phi\theta}\right] - E\kappa + \frac{(1-\delta)}{2\delta}\sigma^2_{\kappa_w} + \lambda,
$$

with

$$
\lambda = \frac{\delta(1-n)^2(1-\gamma)^2(1-\delta)}{2\left[1-(1-\gamma)(1-\delta)\right]^2}\sigma^2_{\kappa_i}.
$$

A.3.2 Expected welfare with sticky wages

Start by deriving the expressions (19) and (20) in the main text. Substituting the expressions $L = \left(\frac{W^*S}{W}\right)^{1-n} Z$, $C = \frac{P_H}{P}L$ and $P = \left(\frac{\theta}{\theta-1}\right) W \left(\frac{W^*S}{S}\right)^{\gamma(1-n)}$ into the first order conditions for nominal wage setting we get:

$$
E\left\{K\left(\frac{W^*S}{W}\right)^{1-n} Z\right\} = \left(\frac{\phi-1}{\phi}\right) W E\left\{\frac{\left(\frac{W^*S}{W}\right)^{(1-\delta)(1-n)} Z^{1-\delta}}{\left(\frac{\theta}{\theta-1}\right) W \left(\frac{W^*S}{W}\right)^{\gamma(1-n)(1-\delta)}}\right\}
$$

Simplifying this expression one obtains expression (19) in text.

The derivation of equation (21). Taking the logs of equation (20) gives

$$\begin{aligned}
&[1-(1-\gamma)(1-\delta)]\,(w-w^{*}) = E\kappa + (1-n)Es + Ez- \\
&\quad - n(1-\gamma)(1-\delta)Es + (1-\delta)Ez - E\kappa^{*} + nEs - Ez- \\
&\quad - (1-n)(1-\gamma)(1-\delta)Es - (1-\delta)Ez + (1-n)\sigma_{\kappa s} + \sigma_{\kappa z} \\
&\quad + (1-n)\sigma_{sz} - n(1-\gamma)(1-\delta)^{2}\sigma_{sz} + n\sigma_{\kappa^{*}s} - \sigma_{\kappa^{*}z} + n\sigma_{sz} \\
&\quad - (1-n)(1-\gamma)(1-\delta)^{2}\sigma_{sz} + \frac{1}{2}\sigma_{\kappa}^{2} + \frac{(1-n)^{2}}{2}\sigma_{s}^{2} + \frac{1}{2}\sigma_{z}^{2} \\
&\quad + \frac{n^{2}(1-\gamma)^{2}(1-\delta)^{2}}{2}\sigma_{s}^{2} + \frac{(1-\delta)^{2}}{2}\sigma_{z}^{2} - \frac{1}{2}\sigma_{\kappa^{*}}^{2} - \frac{n^{2}}{2}\sigma_{s}^{2} - \frac{1}{2}\sigma_{z}^{2} \\
&\quad - \frac{(1-n)^{2}(1-\gamma)^{2}(1-\delta)^{2}}{2}\sigma_{s}^{2} - \frac{(1-\delta)^{2}}{2}\sigma_{z}^{2}
\end{aligned}$$

Remember that $E\kappa = E\kappa^{*}$ and that $E(w^{*}+s-w) = E\tau$. Therefore, we can simplify the expression above as

$$\begin{aligned}
&-[1-(1-\gamma)(1-\delta)]\,E\tau = \frac{1}{2}\left(\sigma_{\kappa}^{2} - \sigma_{\kappa^{*}}^{2}\right) + (1-n)\,\sigma_{\kappa s} + n\sigma_{\kappa^{*}s} + \sigma_{\kappa z} - \sigma_{\kappa^{*}z} \\
&\qquad \left[1-(1-\gamma)(1-\delta)^{2}\right]\sigma_{sz} + \frac{(1-2n)}{2}\left[1-(1-\gamma)^{2}(1-\delta)^{2}\right]\sigma_{s}^{2} \qquad \text{(A.6)}
\end{aligned}$$

Which is the same as equation (21) in the main text. Similarly, log-linearizing the expression (19) in the text gives

$$\begin{aligned}
&(1-n)\,[1-(1-\gamma)(1-\delta)]\,(w-w^{*}) = \\
&\quad \log\left[\frac{\phi\theta}{(\phi-1)(\theta-1)}\right] + E\kappa + (1-n)\,[1-(1-\gamma)(1-\delta)]\,Es + \delta Ez + \\
&\quad \frac{1}{2}\sigma_{\kappa}^{2} + \frac{(1-n)^{2}}{2}\left[1-(1-\gamma)^{2}(1-\delta)^{2}\right]\sigma_{s}^{2} + \frac{1-(1-\delta)^{2}}{2}\sigma_{z}^{2} + \\
&+ (1-n)\sigma_{\kappa s} + \sigma_{\kappa z} + (1-n)\left[1-(1-\gamma)(1-\delta)^{2}\right]\sigma_{sz} \qquad \text{(A.7)}
\end{aligned}$$

Substituting in the expression for the expected log terms of trade (A.6) and simplifying one obtains expression (25).

A.3.3 The productivity shocks

From the *definitions* of the common and idiosyncratic productivity shocks, note that we can express the MU and Home productivity shocks as

$$K^{*} = \frac{K_{w}}{(K_{i})^{n}} \quad \text{and} \quad K = K_{w}\,(K_{i})^{1-n}$$

Therefore, for example $KS = K_w (K_i)^{1-n} S$ and $K^* S = \frac{K_w}{(K_i)^n} S$ why we must have that

$$\sigma_{\kappa s} = \sigma_{\kappa_w s} + (1-n)\,\sigma_{\kappa_i s} \quad \text{and} \quad \sigma_{\kappa^* s} = \sigma_{\kappa_w s} - n\sigma_{\kappa_i s}$$

From the definitions of the productivity shocks also follow that

$$\begin{aligned} \sigma_{\kappa}^2 &= \sigma_{\kappa_w}^2 + (1-n)^2 \sigma_{\kappa_i}^2 \\ \sigma_{\kappa^*}^2 &= \sigma_{\kappa_w}^2 + n^2 \sigma_{\kappa_i}^2 \end{aligned}$$

B Monetary policy

B.1 The welfare outcomes when the CBs target the flexible solution

First note that the using equations (36) and (37) in the main text, the variances and covariances entering the terms $E(U)$ and $E(U^*)$ can be expressed as:

$$\sigma_z^2 = \frac{(n\beta_i - (1-n)\,\beta_i^*)^2}{\delta^2}\sigma_{\hat{\kappa}_i}^2 + \frac{(n\beta_w + (1-n)\,\beta_w^*)^2}{\delta^2}\sigma_{\hat{\kappa}_w}^2$$

$$\sigma_s^2 = \left[\frac{\beta_i + \beta_i^*}{1-(1-\gamma)(1-\delta)}\right]^2 \sigma_{\hat{\kappa}_i}^2 + \left[\frac{\beta_w - \beta_w^*}{1-(1-\gamma)(1-\delta)}\right]^2 \sigma_{\hat{\kappa}_w}^2$$

$$\sigma_{\kappa_w z} = -\left[\frac{n\beta_w + (1-n)\,\beta_w^*}{\delta}\right]\sigma_{\hat{\kappa}_w}^2, \qquad \sigma_{\kappa_w s} = -\left[\frac{\beta_w - \beta_w^*}{1-(1-\gamma)(1-\delta)}\right]\sigma_{\hat{\kappa}_w}^2$$

$$\sigma_{\kappa_i z} = -\left[\frac{n\beta_i - (1-n)\,\beta_i^*}{\delta}\right]\sigma_{\hat{\kappa}_i}^2, \qquad \sigma_{\kappa_i s} = -\left[\frac{\beta_i + \beta_i^*}{1-(1-\gamma)(1-\delta)}\right]\sigma_{\hat{\kappa}_i}^2$$

$$\begin{aligned} \sigma_{sz} = {} & \frac{n\beta_i^2 - (1-2n)\,\beta_i\beta_i^* - (1-n)\,\beta_i^{*2}}{\delta\,[1-(1-\gamma)(1-\delta)]}\sigma_{\hat{\kappa}_i}^2 \\ & + \frac{n\beta_w^2 + (1-2n)\,\beta_w\beta_w^* - (1-n)\,\beta_w^{*2}}{\delta\,[1-(1-\gamma)(1-\delta)]}\sigma_{\hat{\kappa}_w}^2 \end{aligned}$$

Note that if substituting in the monetary policy response rules, $\sigma_{sz} = 0$.[23] The other terms become

$$\begin{aligned}
\sigma_{\kappa_i z} &= 0, \qquad \sigma_{\kappa_w z} = -\frac{1}{\delta}\sigma^2_{\kappa_w}, \qquad \sigma^2_z = \frac{1}{\delta^2}\sigma^2_{\kappa_w} \\
\sigma_{\kappa_i s} &= -\frac{1}{1-(1-\gamma)(1-\delta)}\sigma^2_{\kappa_i}, \qquad \sigma_{\kappa_w s} = 0, \qquad \sigma^2_s = \frac{1}{[1-(1-\gamma)(1-\delta)]^2}\sigma^2_{\kappa_i}
\end{aligned}$$

Substituting in these expressions in the expression for Ez:

$$\begin{aligned}
Ez = \frac{1}{\delta}\Bigg\{ & \omega - \frac{(1-\delta)}{2\delta}\sigma^2_{\kappa_w} - \lambda - \frac{1}{2}\sigma^2_{\kappa_w} - \frac{n(1-n)}{2}\sigma^2_{\kappa_i} \\
& - \frac{n(1-n)}{2}\frac{1-(1-\gamma)^2(1-\delta)^2}{[1-(1-\gamma)(1-\delta)]^2}\sigma^2_{\kappa_i} - \frac{1-(1-\delta)^2}{2\delta^2}\sigma^2_{\kappa_w} \\
& + \frac{n(1-n)}{1-(1-\gamma)(1-\delta)}\sigma^2_{\kappa_i} + \frac{1}{\delta}\sigma^2_{\kappa_w}\Bigg\} \\
= {} & \frac{1}{\delta}\left\{\omega - \lambda - \frac{(1-\delta)}{2\delta}\sigma^2_{\kappa_w}\right\}. \qquad \text{(B.1)}
\end{aligned}$$

---

[23]

$$\begin{aligned}
\sigma_{sz} &= \frac{n\beta_i^2 - (1-2n)\beta_i\beta_i^* - (1-n)\beta_i^{*2}}{\delta[\delta(1-\gamma)+\gamma]}\sigma^2_{\hat{\kappa}_i} \\
&\quad + \frac{n\beta_w^2 + (1-2n)\beta_w\beta_w^* - (1-n)\beta_w^{*2}}{\delta[\delta(1-\gamma)+\gamma]}\sigma^2_{\hat{\kappa}_w} \\
&= \frac{n(1-n)^2 - (1-2n)n(1-n) - (1-n)n^2}{\delta[1-(1-\gamma)(1-\delta)]}\sigma^2_{\hat{\kappa}_i} \\
&\qquad + \frac{n + (1-2n) - (1-n)}{\delta[1-(1-\gamma)(1-\delta)]}\sigma^2_{\hat{\kappa}_w} \\
&= (1-n)\frac{n(1-n) - (1-2n)n - n^2}{\delta[1-(1-\gamma)(1-\delta)]}\sigma^2_{\hat{\kappa}_i} + 0 \\
&= (1-n)\frac{n - n^2 - \left(n - 2n^2\right) - n^2}{\delta[1-(1-\gamma)(1-\delta)]}\sigma^2_{\hat{\kappa}_i} + 0 \\
&= 0
\end{aligned}$$

Substituting them into the expected terms of trade, $E\tau$:

$$\begin{aligned}
E\tau &= \frac{-1}{1-(1-\gamma)(1-\delta)}\left\{\frac{1-2n}{2}\sigma_{\kappa_i}^2 + (1-n)\,\sigma_{\kappa_w s} + (1-n)^2\,\sigma_{\kappa_i s} + n\sigma_{\kappa_w s} - n^2\sigma_{\kappa_i s}\right. \\
&\quad \left. + \frac{(1-2n)}{2}\left[1-(1-\gamma)^2(1-\delta)^2\right]\sigma_s^2\right\} \\
&= \frac{-1}{1-(1-\gamma)(1-\delta)}\left\{\frac{1-2n}{2}\sigma_{\kappa_i}^2 - \frac{(1-n)^2}{1-(1-\gamma)(1-\delta)}\sigma_{\kappa_i}^2 + \frac{n^2}{1-(1-\gamma)(1-\delta)}\sigma_{\kappa_i}^2\right. \\
&\quad \left. + \frac{(1-2n)\left[1-(1-\gamma)^2(1-\delta)^2\right]}{2\left[1-(1-\gamma)(1-\delta)\right]^2}\sigma_{\kappa_i}^2\right\} \\
&= \frac{-1}{1-(1-\gamma)(1-\delta)}\left\{\frac{(1-2n)\left[1-(1-\gamma)(1-\delta)\right]^2}{2\left[1-(1-\gamma)(1-\delta)\right]^2}\sigma_{\kappa_i}^2 - \frac{2(1-2n)\left[1-(1-\gamma)(1-\delta)\right]}{2\left[1-(1-\gamma)(1-\delta)\right]^2}\sigma_{\kappa_i}^2\right. \\
&\quad \left. + \frac{(1-2n)\left[1-(1-\gamma)^2(1-\delta)^2\right]}{2\left[1-(1-\gamma)(1-\delta)\right]^2}\sigma_{\kappa_i}^2\right\} \\
&= 0 \qquad \text{(B.2)}
\end{aligned}$$

Substituting the derived expressions (B.1) and (B.2) into the general expression for $E(U)$ in the main text, equation (17), one obtains equation (38) in text.

B.2 The welfare outcomes when the CCB targets the flexible solution

In a monetary union, the nominal exchange rate is fixed, implying that $\sigma_s^2 = \sigma_{\kappa_i s} = \sigma_{\kappa_w s} = 0$, and since the central bank cannot react to idiosyncratic productivity shocks by changing the relative prices, we have that $\beta_i^{MU} = 0$, implying that $\sigma_{\kappa_i z} = 0$.

The expressions for $E(U)^{sticky,MU}$, Ez and $E\tau$ under wage rigidity then become

$$E(U) = \Psi \exp\left\{(1-\delta)\,Ez + (1-n)\,(1-\gamma)\,(1-\delta)\,E\tau + \frac{(1-\delta)^2}{2}\sigma_z^2\right\}$$

with the terms for Ez and $E\tau$ now becoming:

$$\begin{aligned}
Ez = \frac{1}{\delta}\Big\{ & \omega - \frac{(1-\delta)}{2\delta}\sigma_{\kappa_w}^2 - \lambda - \frac{1}{2}\sigma_{\kappa_w}^2 - \frac{n\,(1-n)}{2}\sigma_{\kappa_i}^2 \\
& - \frac{1-(1-\delta)^2}{2\delta^2}\sigma_{\kappa_w}^2 + \frac{1}{\delta}\sigma_{\kappa_w}^2 \Big\} \\
= \frac{1}{\delta}\Big\{ & \omega - \frac{(1-\delta)}{2\delta}\sigma_{\kappa_w}^2 - \lambda - \frac{n\,(1-n)}{2}\sigma_{\kappa_i}^2 \Big\}
\end{aligned}$$

and

$$\begin{aligned}
E\tau &= \frac{-1}{1-(1-\gamma)(1-\delta)}\left\{\frac{(1-2n)}{2}\sigma_{\kappa_i}^2 + \sigma_{\kappa_i z}\right\} \\
&= \frac{-\,(1-2n)}{1-(1-\gamma)(1-\delta)}\sigma_{\kappa_i}^2
\end{aligned}$$

Combining these, one gets equation (40) in text.