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Lars-Göran Larsson

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Department of Economics
School of Business, Economics and Law at University of Gothenburg
Vasagatan 1, PO Box 640, SE 405 30 Göteborg, Sweden
+46 31 786 0000, +46 31 786 1326 (fax)
www.handels.gu.se info@handels.gu.se

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ON EXPECTED DEMAND FUNCTIONS WITHOUT UTILITY MAXIMIZATION.

BY LARS-GÖRAN LARSSON

Department of Economics and Statistics,

School of Business, Economics and Law

University of Gothenburg

Vasagatan 1 S 41180
Goteborg
Sweden

E-mail lars-goeran.larsson@economics.gu.se

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In this paper we assume that choice of commodities at the individual (household) level is made in the budget set and that the choice can be described by a probability density function.
Describing choice as random is less restrictive in a behavioural sense than to assume utility maximization. To emphasize that, we add a section of comments by "leading" economists on the descriptive value of utility maximization.
We find that for all positive continuous function f(x, y) the expected demand functions Ex (p_x, p_y, m), E y (p_x, p_y, m) *are homogeneous of degree zero in prices and income* (p_x, p_y, m) *and that "law of demand " (* the own price derivatives of the expected demand functions are negative) is valid.
We calculate (expected) demand functions, elasticity (classify goods as substitutes etc.) and consumer´s surplus just as in conventional theory. Functions used in examples include convex, constant, lower bounded and decreasing functions.

Why not keep descriptions as simple as possible?

Entia non sunt multiplicanda praetor necessitatem
Beings ought not to be multiplied except out of necessity

"Occam´s razor"

Encyclopedia Brittannica

KEYWORDS: Properties of expected consumer demand functions, Microeconomics, Consumer theory, Consumer behaviour, Choice described in random terms, Expected individual and market demand.
JEL classification: C60, D01, D11

1. INTRODUCTION AND MAIN RESULTS

The main purpose of this paper is to see what properties expected demand functions Ex ($p_x, p_y, m)$, E y ($p_x, p_y, m)$ have if we abandon the assumption of maximising utility functions. Here x and y are choice variables (commodities), ($p_x, p_y, m)$ prices and income and E denotes expected value.

We think maximization of utility functions is too complex and doubtful in a descriptive (behavioural) interpretation. That we are not alone in this belief can be seen in some comments on the mainstream ideas, extensions and alternatives that have been given (see section 3).

On some econometric tests of traditional theory see Klevmarken (1979). On another test of the neoclassical theory of consumer behaviour using an experimental approach (see Sippel (1997)).

A simple approach with less of optimal behaviour ("Occam´s razor").

To this end we assume that choice can be *described* as random in the budget set. We start by assuming a positive integrable continuous function f(x, y) which we convert into a probability density function p(x, y, a, b) defined in the budget set D= $\{(x, y) \text{ in } \mathrm{R}^2 : p_x x + p_y x_2 \leq m \text{ and } \mathrm{x} \geq 0, y \geq 0\}$, where, ($a = \dfrac{m}{p_x}$, $b = \dfrac{m}{p_y}$).
This density function is assumed to describe the choice of commodities in a bounded set (here the budget set).
We calculate in general form (and in specific examples) the expected demand functions Ex ($p_x, p_y, m)$, E y ($p_x, p_y, m)$ and study its properties.

The properties of expected demand functions in our continuous function f(x, y) approach are:
Demand functions are homogeneous of degree zero in prices and income ($p_x, p_y, m)$
Negativity ("Law of demand") is valid at the individual and market level for two choice variables. In other words the own price derivatives of the expected demand functions are negative.
In formal terms: The diagonal of the following price derivative matrix is negative.

$$\begin{pmatrix} Ex_{p_x} & Ex_{p_y} \\ Ey_{p_x} & Ey_{p_y} \end{pmatrix} . Here\ Ex_{p_x} = \frac{\delta E(x)}{\delta p_x} \text{ and so on.}$$

Budget balancedness (adding up) (see below) is not valid
E ($p_x x(p_x, p_y, m) + p_y y(p_x, p_y, m)) < m$
Note that the expected choice is found in the budget set and not on the budget line.

Proof of negativity and homogeneity is found in section 5.

There is no need to use a behaviourally and mathematically more complex approach to obtain these results (“Occam´s razor”).
A continuous probability density function describing choice is sufficient.

Comparison with the utility maximising approach

As an example of a behaviourally and mathematically more complex approach we present the traditional neoclassical utility maximising consumer model (in section 2.1) and extensions of it to prove “law of demand” (in section 2.2). We here briefly state the neoclassical results of section 2.1 and compare with our results

The *independent* properties of demand functions in expected form are:
Budget balancedness (adding up) E ($p_x x(p_x, p_y, m) + p_y y(p_x, p_y, m)) = m$
and negative semi definiteness of the symmetric substitution matrix.

$$\begin{bmatrix} Ex_{p_x} & Ex_{p_y} \\ Ey_{p_x} & Ey_{p_y} \end{bmatrix} + \begin{bmatrix} Ex \\ Ey \end{bmatrix} \begin{bmatrix} Ex_m & Ey_m \end{bmatrix}.$$

For details (see Jehle and Reny (2001 pp 82-83)).

As noted above we find that the first matrix has a negative diagonal (“law of demand”).We find it without using utility maximization and the substitution matrix. We also note that budget balancedness (adding up) (see above) is not valid

What we found using examples (in section 6).

The substitution matrix has a negative diagonal and is non symmetric.
The latter implies that symmetry property of the substitution matrix is not a general property (for all f(x, y)). As is well known the symmetry property of neoclassical theory comes from assumptions about the differentiability of the utility function.
There is an *inconsistency* between the random approach and the differentiability approach – as in example 3 section 6 we have a function f(x, y) = x y a Cobb-Douglas form where the neoclassical solution has a symmetric substitution matrix but the random approach with a lower bound has a non-symmetric substitution matrix.
See also Varian (1992) on the intuitivity of utililty maximization in section 3 (below).
Negative definiteness of the substitution matrix is *a stronger property* than negative semi definiteness as in the traditional theory.

Many types of choice behaviour. Since our general results so far are valid for any integrable continuous function f(x, y) we note the great variety of functions that can be used. Convex, decreasing and constant functions are such examples (see section 6). There is no need to only use increasing functions (“high consumption”= high probability for consumption near the budget line). Decreasing functions can describe “low consumption”. Note that constant functions (example 1)resembles constant utility but give own price negativity and a unique solution.

From independent to gross complementary goods by introducing lower bounds on choice.
We calculate numerical elasticities (own price and cross price) to classify goods as gross substitutes, independent or gross complements. For the same function we can find *changes in classification* from independent goods to gross complements by introducing lower bounds on choice. Furthermore note the *change in type of goods from substitutes to complements* in example 10 and 6. . Both have a linear form but example 10 has an increasing function " high consumption" and example 6 has an decreasing function "low consumption".

We can also study demand responses to changes in economic policy (taxes and subsidies). We can calculate consumer´s surplus for welfare analysis.

The content of the paper follows

In section 2.1 *Utility maximization* we present the neoclassical utility maximising model, its results and its assumptions to obtain these results.
In section 2:2 *Utility maximization and negativity* we present some assumptions added to the model to prove " the law of demand" (negativity of own price derivative). Notably concavity of the utility function plus a numerical condition is used (quasiconcavity is not sufficient).
In section 3.1 *On the descriptive value of utility maximization* comments from "leading economists" on utility maximization are added.
Section 3.2 *Separate description of behaviour from prescription of how to behave* contains some comments on the headline notably by (Frank 2008).
Section 4 comments on extensions of the random approach.
In section 5 we give our proof of negativity and homogeneity
Section 6 contains some examples.

2 TRADITIONAL THEORY AND "LAW OF DEMAND"

2.1. Utility maximization

In the traditional "rational" theory of choice based on maximization of an *increasing quasiconcave* utility function (u(x ,y)) subject to a budget constraint the derived demand functions($x(p_x, p_y, m), y(p_x, p_y, m)$) have certain *testable* properties. The demand functions are homogeneous of degree zero in the variables prices and income (p_x, p_y, m). The choice is mostly found on the budget line (the adding up property).Furthermore the substitution matrix of the demand function is symmetric and negative semi definite (see Jehle and Reny (2001)).
For good x (p_x, p_y, m) we have $\frac{\partial x}{\partial p_x} + x(p_x, p_y, m)\frac{\partial x}{\partial m} \le 0$

Traditional theory in expected form

To facilitate a comparison or our expected demand approach with traditional theory we note that the latter can be expressed in expected values. In testing and/or estimation we have

E(x) = $x(p_x, p_y, m) + E(\nu) = x(p_x, p_y, m)$,

where ν is a random error term with E(ν)=0 . This assumption gives the result that the expected demand function have the same slope as the ordinary demand function $Ex_{p_x} = x_{p_x}$, and so on for the other properties.

The *independent* properties of demand functions in expected form are:
Budget balancedness (adding up) E ($p_x x(p_x, p_y, m) + p_y y(p_x, p_y, m)) = m$
and negative semi definiteness of the symmetric substitution matrix.

$$\begin{bmatrix} Ex_{p_x} & Ex_{p_y} \\ Ey_{p_x} & Ey_{p_y} \end{bmatrix} + \begin{bmatrix} Ex \\ Ey \end{bmatrix} \begin{bmatrix} Ex_m & Ey_m \end{bmatrix}.$$

For details (see Jehle and Reny (2001 pp 82-83)).
The traditional approach is not without mathematical complexity.
Just take a look in any advanced textbook in microeconomics.

2.2. *Utility maximization and negativity*

A negative slope of demand (negativity) in relation to own price ($\frac{\partial x(p_x, p_y, m)}{\partial p_x} < 0$) is not valid in traditional theory without making further assumptions.

That the traditional theory holds the negativity result for important can be seen as it is formulated as a “law “(see Varian (2006)). “If the demand for a good increases when income increases, then the demand for that good must decrease when its price increases” (see Varian (2006 p 147)). The normal goods assumption $(\frac{\partial x}{\partial m} > 0)$ is sufficient to give negativity at the individual and market level (sum of individual demands**).**

The importance can also be seen in that papers are published in the highly ranked journal Econometrica. One paper (see Quah (2000 p 916 and p 912)) introduce convex indirect utility functions plus some *numerical* elasticity condition to prove negativity. Other proofs referred to in Quah (2000) use *concavity* of the direct utility function plus some *numerical* elasticity condition.

Focus on the expected law of demand at the aggregate (market) level can be found in Hildenbrand (1983) and Härdle, Hildenbrand and Jerison (1991)). The latter aggregate approaches look at density functions of household income distribution and focus less on “rational” choice.

The extended traditional approach is not without mathematical complexity either as can be seen in the quoted references.

3 COMMENTS ON THE TRADITIONALAPPROACH

3.1. *On the descriptive value of utility maximization*

As we saw there is no need to use a utility function and its maximization to find the above general and specific results. We think maximization of utility functions is too complex and doubtful in a descriptive interpretation. That we are not alone in this belief can be seen in some critical remarks against the mainstream ideas, extensions and alternatives that have been given.

Samuelson (1947) ("Nobel" price winner) (Atheneum edition 1970 p 117). After he concludes his chapter 6 on theory of consumer`s behaviour by stating its empirical meaning in difference and differential form (p 116) he comments " Many writers have held the utility analysis to be an integral and important part of economic theory. Some have even sought to employ its applicability as a test criterion by which economics might be separated from the other social sciences. Nevertheless, I wonder how much economic theory would be changed if either of two conditions above were found to be empirically untrue. I suspect, very little".

Varian (1992) After writing the properties of demand functions on page 99 in Varian 1992, he gives on p 123 the following comment
"this is a rather non intuitive result: a particular combination of price and income derivatives has to result in a negative semidefinite matrix. However it follows inexorably from the logic of maximizing behaviour."

Stigler (1961) ("Nobel" price winner) on price dispersion as one measure of ignorance about the market and the introduction of a probabilistic search theory.

Herbert Simon (1965) ("Nobel" price winner) on "bounded rational behaviour".

Barten (1969) on empirical tests of the traditional theory of demand.
According to Klevmarken (1979) the classical theory of demand is rejected.

Kahneman - Tversky (1974) (Kahneman "Nobel" price winner) on behavioural elements in economic choice.
We now have a new field of research in economics – behavioural economics.

Myerson (1999) ("Nobel" price winner) p 1069 "This assumption of perfect rationality is certainly imperfect as a description of real human behaviour"

3.2. *Separate description of behaviour from prescription of how to behave*

The extension of the traditional theory by introducing new constraints (time constraints, lower bounds, rationing) and also parameters in the utility function (taste, peer groups, prices) to make the theory more "realistic" could explain many (almost all) choices as "rational".
One question is: how can we test these extended theories?
A second question is: can we assume that the optimal model is explaining (describing) actual economic behaviour?
In a prescriptive interpretation of the optimal model someone else (economists, mathematicians etc.) calculate and give advice.

If economic behaviour is almost rational who would need prescriptions of optimal behaviour?

In the long run there is a "problem" maintaining neoclassical utility theory as a description of actual behaviour and at the same time give advice to improve behaviour.

To separate the study of consumer choice into a descriptive part (consumer behaviour) and normative (prescriptive) part might be useful in the future. The normative part may contain improvement of consumer choice by better information of prices and quality, optimal search theories of prices and quality, utility maximization with more complex preferences and constraints. The use of optimization methods in economic models reflects the important aspect of helping forming better decisions in the world. This does not necessarily mean that they are good models for explaining (describing) actual economic behaviour.

As a textbook of Robert H Frank puts it: "But even where economic models fail on descriptive grounds, they often provide useful guidance for decisions" (Frank 2008 p 6).

4 SOME EXTENSIONS OF THE RANDOM APPROACH

There is no "Giffen good" in this expected demand approach or in the examples. To allow for"Giffen good" some *new information is needed*. In traditional utility theory shift variables (s) are introduced in the utility functions u(x,y,s). Something similar could be used in the function f(x, y, s). If as an example, a theory is based on monetary variables like price dependent preferences in utility theory (see Kalman 1968) we can have price dependent functions $(f(x, y, p_x))$ and integrate them to find expected demand. The result can be demand functions which are non-homogenous in prices and income or have non-negative own price slopes.

Negativity might not be compatible with any choice behaviour, but if we know more then we know more.

Constraints such as *time restrictions* could also be used in the same manner perhaps together with the budget constraint. To find examples of expected demand and supply functions in consumer theory and theory of the firm where choice is made in bounded sets should be possible.

Recently, behavioural sciences and *behavioural economics* have supplied economics with examples of choice behaviour. It this choice behaviour can be expressed in specific density functions these functions can be integrated over the budget set to relate them to economic variables. Help to integrate more complex density functions can be done by experts. In waiting for a "final theory", which of course is an illusion, probability formulated choice gives a link between traditional theory of negativity and homogeneity and theories of behaviour within the budget set.

5. THE PROPERTIES OF EXPECTED DEMAND FUNCTIONS

We prove that expected demand functions have negative own price slope and are homogeneous of degree zero in prices and income. We use continuous density functions describing household choice of commodities in a bounded set .

5:1 One dimensional frequency function of choice p(x, a)

To find the frequency function p(x, a) we start by assuming a positive continuous integrable function f(x)>0 defined on the interval I = $[0,c]$. Put $a=\frac{m}{p_x}<c$. Define

$F(a)=\int_0^a f(x)dx$ and G(a)=$\int_0^a xf(x)dx$. We then have

$E(x)=\frac{G(a)}{F(a)}<a.$

F (a) is the area below the positive function f(x) in the interval (0, a) and the frequency function

p(x, a) = f(x)/F(a).Remember that a density function has the property $\int_A p(x)dx$ =1 .

Note that the *parameter* (a) is part of the frequency function and that (a) is a *variable* in the expected value function.

Properties of E(x)

Next we want to find some properties of E(x). We can use the chain rule of differentiation $\frac{\partial E(x)}{\partial p_x}=\frac{dE(x)}{da}\frac{\partial a}{\partial p_x}$

We take

$$\frac{dE(x)}{da}=\frac{1}{F(a)^2}\left(G'(a)F(a)-F'(a)G(a)\right)=\frac{1}{F(a)^2}(af(a)F(a)-f(a)G(a))$$

$$=\frac{f(a)}{F(a)^2}\left(a\int_0^a f(x)dx-\int_0^a xf(x)dx\right)>0$$

(1) $\frac{dE(x)}{da}>0$

We have found that the expected demand function have negative own-price slope for all continuous choice frequency functions.

$\frac{\partial E(x)}{\partial p_x}<0$

Homogeneity of degree zero in price and income is obvious since $a=\frac{m}{p_x}$ is homogeneous of degree zero in price and income.

Change of variable

For later use we *change variable*. Let x=au. We then find

$$E(x)=\frac{a^2\int_0^1 uf(a,u)du}{a\int_0^1 f(a,u)du}=\frac{a\int_0^1 uf(a,u)du}{\int_0^1 f(a,u)du}=\frac{G(a)}{F(a)}$$ where (a) is the Jacobian of the transformation.

For the change of variable formula (see Buck(1956 p244)).

We differentiate

(2) $$\frac{dE(x)}{da}=\frac{1}{F(a)^2}\left(G'(a)F(a)-F'(a)G(a)\right)>0$$

following the result (1)) above.
The latter result (2) will be useful when we turn to two dimensional choices.

5.2 *Choice in two dimensions*

To find the frequency function p(x, y, a, b) we start by assuming a positive continuous integrable function f (x, y)>0 defined on a set E, where $D\subset E$.
For the function f(x, y) we should integrate over the budget set
D=$\left\{(x,y) \text{ in } \mathrm{R}^2 : p_x x+p_y x_2\le m \text{ and } \mathrm{x}\ge 0, y\ge 0\right\}$

To make the calculations simpler we change variables
We put x=u$\frac{\mathrm{m}}{\mathrm{p_x}}$=au and y=v$\frac{\mathrm{m}}{\mathrm{p_y}}$=bv to integrate over the set
D′=$\left((\mathrm{u,v})\in R^2 : \mathrm{u}\ge 0,\ \mathrm{v}\ge 0, \mathrm{u+v}\le 1\right)$
We then have $\iint_D f(x,y)dxdy=ab\iint_{D'} f(x(u),y(v))dudv$,
where the Jacobian (= a b)of the transformation is taken outside the integral on the right side. For the change of variable formulas (see Buck (1956 p 244)).

Calculating E(x)

To find E(x) we first integrate f(x, y) over the area D'

$$ab\iint_{D'} f(x(u),y(v))dudv = ab\iint_{D'} f(u,v,a,b)dudv = ab\int_0^1 du\int_0^{1-u} f(u,v.a,b)dv.$$

We now put the last integral $\int_0^{1-u} f(u,v,a,b)dv=F(u,a,b)$ and the volume below f(x,y) is $ab\int_0^1 F(u,a,b)du$ .The next integral to be calculated is

$$aba\iint_{D'} uf(u,v,a,b)dudv = aba\int_0^1 udu\int_0^{1-u} f(u,v.a,b)dv = aba\int_0^1 uF(u,a,b)du$$

We now find $E(x) = \dfrac{aba\int_0^1 uF(u,a,b)du}{ab\int_0^1 F(u,a,b)du} = \dfrac{a\int_0^1 uF(u,a,b)du}{\int_0^1 F(u,a,b)du} = \dfrac{G(a,b)}{F(a,b)}$

Finding the properties of expected demand E(x)

Expected demand is homogeneous of degree zero in prices and income (p_x, p_y, m). This follows since $E(x) = \dfrac{G(a,b)}{F(a,b)}$ is a function of ($a = \dfrac{m}{p_x}, b = \dfrac{m}{p_y}$) both homogeneous of degree zero in prices and income.

The chain rule of differentiation helps us to find sensitivity in relation to price (p_x).

$$\frac{\partial E(x)}{\partial p_x} = \frac{\partial E(x)}{\partial a}\frac{\partial a}{\partial p_x}$$

Differentiate w r t a. and use the result (2) in one dimension

$\dfrac{\partial E(x)}{\partial a} > 0$. This in turn gives us $\dfrac{\partial E(x)}{\partial p_x} < 0$.

Negativity is valid for all continuous frequency functions p (x, y, a, b). The property is additive so the result is valid at the aggregate (market) level as well. Homogeneity was mentioned above.

6. EXAMPLES

Even if our general results are valid for all continuous functions f(x, y) we also illustrate the stochastic approach by presenting some examples. We deliberately choose many of them found in microeconomic textbooks. At the same time we find other properties of expected demand functions and compare them with traditional theory.

Each example is calculated as follows. For a given function f(x, y) its volume over the budget set is calculated. This volume is used to transfer f(x, y) into the density function p(x , y, a, b).To simplify the calculations we do a change of variables. Using the density function we calculate the expected demand E(x(a, b)) which then is differentiated to find various properties.

Example 1. A constant function.

Choosing a constant function can give us a uniform distribution.

Why use the uniform distribution?

The uniform distribution assumption might reflect our *ignorance* of what determines the choice (such as preferences) inside the bounded budget set. If we know that choice is taken place in the set we know that expected choice changes when the “walls” (budget line and lower bounds) of the set changes. A uniform distribution can also *approximate* a more complex function if we use a lower bound as in the example. The assumption also makes for *simple calculations* (used before in economics) and produces some results. To quote Krugman (2008): “express your ideas in the simplest possible model.”

This example has *no unique solution in utility theory (a constant utility)* and most solutions (the demand functions) are *independent* of price and income.

The following example shows *no such independence of economic variables* and is

$$E(x) = \frac{1}{3p_x}(m - p_y y_0) + \frac{2}{3}x_0$$

$$E(y) = \frac{1}{3p_y}(m - p_x x_0) + \frac{2}{3}y_0$$

the expected demand functions obtained by integrating a uniform distribution over a bounded set.

The set is bounded by the budget constraint and positive lower bounds $x \geq x_0$ $y \geq y_0$ where x_0 and y_0 can be seen as a function of shiftvariables s if one wishes (one can then use the chain rule to obtain sensitivity wrt s).

We name these lower levels “subsistence” levels, *S levels* for short .One early example on the use of lower levels is Stone-Geary preferences (Hey 2003 p 80).

Given the expected demand we can study properties such as own price and cross price derivatives, income derivatives, homogeneity and symmetry properties as in ordinary theory. In the example given we have

$$Ex_{p_x} < 0, Ex_m > 0, Ex_{p_y} < 0, Ey_{p_x} < 0, Ex_{p_y} \neq Ey_{p_x}$$

In words: Own prise derivative negative, cross price derivatives negative(x and y are gross complementary commodities), income derivative positive (normal good) .

The own price elasticity of demand is inelastic $El_{Expx} = \frac{\partial E(x) p_x}{\partial p_x E(x)} > -1$.

If no lower bound we have $E(x)=\frac{m}{3p_x}$, $E(y)=\frac{m}{3p_y}$.

Then x and y are independent commodities *Note that the introduction of a lower bound changes the character of the commodities from independent to complements.*

Symmetry of the substitution matrix is not a general property in the present approach as seen by introducing numerical values in this example.

Putting m=100, $p_x = 10$, $p_y = 5$, $x_0=5$ and $y_0=6$ we find

$E(x)=5\frac{2}{3}$, $E(y)=7\frac{1}{3}$

Average expenditure = 0,933m < m.

Note the relative closeness to m due to the tight lower border. Identify lower bounds and the choice and/or preferences inside the set matter less. The substitution matrix in the numerical example is *non symmetric and negative definite.*

$$\begin{bmatrix} \frac{-7}{30} & \frac{-1}{5} \\ \frac{-1}{3} & \frac{-2}{3} \end{bmatrix} + \begin{bmatrix} \frac{17}{3} \\ \frac{22}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{30} & \frac{1}{15} \end{bmatrix} = \begin{bmatrix} \frac{-4}{90} & \frac{8}{45} \\ \frac{-8}{90} & \frac{-8}{45} \end{bmatrix}.$$

$$\text{The own price elasticity } \mathrm{El}_{\mathrm{Exp_x}} = \frac{\partial E(x) p_x}{\partial p_x E(x)} = \frac{-7}{17}$$

$$\text{The cross price elasticity } \mathrm{El}_{\mathrm{Exp_y}} = \frac{\partial E(x) p_y}{\partial p_y E(x)} = \frac{-3}{17}$$

$$\text{The income elasticity } \mathrm{El}_{\mathrm{Exm}} = \frac{\partial E(x) m}{\partial m E(x)} = \frac{10}{17}$$

Note that the homogeneity condition in elasticities hold (sum of elasticities =0).
For elasticity forms in traditional theory (see Shone (1975 p 91)).

Table of examples

In the following table we present more examples. The table contains the used function f(x, y). It also contains numerical values calculated for $(p_x, p_y, m, x_0, y_0) = (10, 5, 100, 5, 6)$ and c=a + b=30. The change in consumer´s surplus (ΔCS) is calculated (triangular elasticity approximation) using a lower price of x(from 10 to 9).

We give expected demand, the utility maximising solution for each function f(x, y), own price elasticity, cross price elasticity and type of commodity following classification by cross price (gross complements etc.), average expenditure and change in consumer´s surplus.

The table is ordered following own price elasticities (from inelastic to elastic). Note especially the *change in type of goods from independent to complements* due to introduction of a lower bound for the same function. Example 1 and 2 plus example 3 and 4. Furthermore note the change in type of goods from substitutes to complements in example10 and 6. Both have a linear form but example 10 has an increasing function " high consumption" and example 6 has an decreasing function "low consumption". Example 12 in one dimension give us a " Luxury good".

Functional form of f(x, y)	Expec-ted demand	Utility maximizing solution	Own price elasticity	Cross price elasticity. Type of commodities	AE Average expenditure ΔCS Change in Consumer´s surplus

1 Constant-uniform with lower bounds f(x, y)=1	E(x)= 5,67 E(y)= 7,33	Infinite number of solutions	-0,41 Inelastic demand	-0,18 Comple-ments	AE=0,93m
2 Constant-uniform with no lower bounds	E(x)= 3,33 E(y)= 6,67	Infinite number of solutions	-1 Unit elastic demand	0 Independent	AE=0,67m ΔCS =3,5
3 Quasi-concave with lower bounds. Cobb-Douglas form f(x, y)=xy	E(x)=5,8 E(y)=7,6	x=5 y=10	-0,48 Inelastic demand	-0,20 Comple-ments	AE=0,96m
4 Quasi-concave with no lower bounds. Cobb-Douglas form f(x, y)= xy	E(x)=4 E(y)=8	x=5 y=10	-1 Unit elastic demand	0 Independent	AE=0,8m ΔCS =4,2
5 Linear proportions Perfect complements f(x, y)=min(x, y)	E(x)=4,1 7 E(y)= 6,67	x=6,67 y=6,67	-0,87 Inelastic demand	-0,13 Comple-ments	AE=0,75m ΔCS =4,35
6 Linear “low consumption” f(x, y)=c-x-y	E(x)= 3,33 E(y)= 5,83	x=0 y=0	-0,91 Inelastic demand	-0,08 Comple-ments	AE=0,63m ΔCS =3,48
7 “Quasilinear” f(x, y)= $\sqrt{x}+y$	E(x)= 2,86 E(y)=9,3	x=0,1 y=19,8	-0,93 Inelastic demand	0,08 Substitutes	AE=0,74m ΔCS =2,99
8 Quasi-concave Cobb Douglas form f(x, y)= $x^2 y$	E(x)=5 E(y)= 6,67	x=6,67 y=6,67	-1 Unit elastic demand	0 Independent	AE=0,83m ΔCS =5,25
9 Concave f(x,y)= $\sqrt{x}+\sqrt{y}$	E(x)=3,4 5 E(y)= 7,39	x=3,33 y=13,33	-1,05 Elastic demand	0.05 Substitutes	AE=0,71m ΔCS =3,63
10 Linear “High consumption” f(x, y)=x + y	E(x)=3,3 E(y)= 8,33	x=0 y=20	-1,16 Elastic demand	0,17 Substitutes	AE=0.75m ΔCS =3,49
11 Convex f(x, y)= x^2+y^2	E(x)=2,8 E(y)= 10,4	x=0 y=20	-1,45 Elastic demand	0,46 Substitutes	AE=0,8m ΔCS =3,0

12 Exponential f(x)= e^x	E(x)=9	x=10	-1,11 Elastic demand	Income elasticity 1,11 Luxury good	AE=0,9m

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