# An Analysis of Subordinated Debt in Banking: the Case of Costly Bankruptcy

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***Abstract***

This paper adopts a contingent-claim valuation framework to investigate the role of subordinated debt in alleviating the moral hazard problem in banking and providing the regulator with the information on the risk of bank assets. The incorporation of bankruptcy cost in the framework of the analysis provides some new evidence about the potential role of subordinated debt. The extent of market discipline of subordinated debt critically depends on its relative magnitude to senior debt and bankruptcy costs. Under specific conditions, subordinated debt prices are found to provide additional information about the value of bank assets relative to equity prices. The credibility of existing subordinated debt proposals is also discussed. The results indicate the critical role of the regulator's judgment in interpreting and acting upon information from subordinated debt prices.



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# I. Introduction

The effectiveness of market discipline in mitigating the moral hazard problem among banks has been a major policy issue for almost two decades. Academics, regulators, and public officials have extensively investigated the potential benefits of using the private sector to monitor and regulate bank risk-taking and suggested alternative market participants best situated to perform these functions.

Lately, market discipline has been explicitly recognized as one of the three mutually reinforcing pillars that allow banks and supervisors to evaluate properly the various risks that banks face (Basel Committee on Banking Supervision (2001))[1]. The main supposed advantage of market discipline according to its advocates is that market participants process information on banks more efficiently than do government regulators and have better incentives to gather and act upon this information due to financial incentives.

A large number of regulatory proposals[2] argue that introducing minimum subordinated debt requirements in banking may be a preferred method of imposing market discipline. According to these proposals, the requirement to issue an uninsured junior debt could potentially reach the following five objectives: (1) improve direct market discipline (2) augment indirect market discipline (3) improve depository institutions transparency and disclosure (4) increase the size of the financial cushion for the deposit insurer and (5) reduce regulatory forbearance (The Board of Governors of the Federal Reserve System and the Secretary of the U.S. Department of Treasury (2000)).

Direct market discipline is supposed to materialize through the effect of subordinated debt price reactions on the ex-ante risk-taking incentives of the depository institutions. The expected negative reaction of subordinated debt prices to

[1] The New Basel Capital Accord (Basel Committee on Banking Supervision (2001)) focuses on: minimum capital requirements, which seek to refine the measurement framework set out in the 1988 Accord; supervisory review of an institution's capital adequacy and internal assessment process; and market discipline, through effective disclosure to encourage safe and sound banking practices.

[2] The report by the Federal Reserve Study Group on Subordinated Notes and Debentures (1999) contains the detailed analysis of the proposals. The earliest ones are actually dated by 1983.

excessive risk-taking should effectively restrain managers of a depository institution from taking on those risks. Indirect discipline may come when government supervisors monitor secondary market prices of the subordinated debt or a bank's ability to issue this debt and react promptly to the extracted information. In addition, it is often argued that the information from the subordinated debt prices can be used in calculating risk-based deposit insurance fees.

Direct and indirect market discipline of subordinated debt is often regarded as an important complement of supervisory discipline because it is, among other things, more objective and incentive-unbiased. Moreover, market information can be utilized in allocation of scarce supervisory resources, in setting deposit insurance premiums, and in triggering prompt corrective action (Federal Reserve Study Group on Subordinated Notes and Debentures (1999)).

Recognizing the fact that subordinated debt prices are not a unique source of market information on banks, the authors of subordinated debt proposals stressed its advantages relative to other market instruments like common equity. Most frequently cited advantages are (1) alignment of incentives between subordinated debt holders and the deposit insurer (2) visible market signals provided by subordinated debt prices or yields and (3) incentives created for bankers to disclose risks to the market (SFRC 2000).

This paper adopts a contingent-claim valuation framework to investigate the role of subordinated debt in both alleviating the moral hazard problem in banking and providing the regulator with the information on the risk of bank assets.

Levonian (2000) showed that bank's shareholders benefit from the risk-shifting even when the discipline of subordinated debt is perfect (i.e. it is both instantaneous and complete). In other words, subordinated debt alone is never able to completely remove the risk-taking incentives of shareholders which underline the moral hazard problem in banking.

I extend the valuation framework adopted by Levonian (2000) by explicitly introducing bankruptcy costs in the model. The results show that the presence of bankruptcy costs improves the discipline of subordinated debt and under certain conditions results in complete alleviation of moral hazard.

The explicit modeling of bankruptcy costs provide some new evidence with respect to the informational role of subordinated debt prices. Gorton and Santamero (1990) and Levonian (2000) pointed out the important methodological issues related

to extracting information from subordinated debt yield spreads and showed that the straightforward comparison of these spreads in the cross-section of banks can be misleading. Levonian (2000) argues then that the necessity to use some form of contingent-claim valuation framework to interpret yield spreads on subordinated debt dissipates (at least in theory) a widely acclaimed comparative advantage of subordinated debt as a tool to differentiate among the risks of banks. The author continues arguing that, in fact, the same information can be extracted from equity prices using similar techniques.

This paper shows that the subordinated debt prices have "value added" relative to equity. In fact, the joint use of equity and subordinated debt prices can provide information on the magnitude of expected bankruptcy costs.

Overall theoretical analysis reveals that the interpretation and comparison across banks of information from the subordinated debt prices requires the regulator's judgment. The *discipline* of subordinated debt, the *amount of information* provided by subordinated debt and the *modeling framework* used to interpret that information all critically depend upon the amount of subordinated debt relative to senior debt and the magnitude of bankruptcy costs.

The remainder of the paper is organized as follows: Section II describes the stylized features of subordinated debt proposals. Section III develops the contingent-claim valuation framework of a subordinated debt and offers a look at the effect of subordinated debt's discipline on bank's risk-taking incentive. Section IV investigates issues related to extracting and interpreting the information from the subordinated debt prices. Section V discusses credibility of the existing mandatory subordinated debt proposals. Section VI provides a look at the potential noise in the subordinated debt prices due to speculative strategies. Section VII is a summary of the results and states the conclusions of the paper.

## II. The stylized features of the subordinated debt proposals

I do not intend to describe and analyze in detail various proposals to introduce a mandatory subordinated debt requirement in banking. Instead, I will state next some common features of these proposals and then continue with the more conceptual analysis of the issue.

The following are some common features of subordinated debt proposals:

- The typical proposed ratio of subordinated debt to total (risk-weighted) assets is two percent or higher.
- Different rules (regarding "tradability", allowed counterparties etc.) are proposed to apply to the large and small banks. Some proposals suggest making only banks over a certain size subject to the mandatory subordinated debt regulation.
- Qualifying debt should be subordinated in all regards to the deposits and other general creditors; it must be non-callable (some proposals though suggest to make it putable), unsecured, and non-convertible into equity capital.
- Maturity of qualifying subordinated debt is suggested to be at least one year and a portion of this outstanding debt has to be rolled over periodically.
- The subordinated debt securities should have a large denomination so only institutional investors would tend to buy it.
- The subsidiaries of the banking holdings should issue the subordinated debt separately.
- The issuing bank or bank's insiders would not be allowed to purchase any of its "own" subordinated debt.
- Some proposals allow subordinated debt to count for a greater proportion of bank's total capital relative to the current BIS capital rules.
- Finally, the adverse information from the subordinated debt prices (yield spreads) or the inability of a bank to issue subordinated debt should trigger some punitive regulatory action.

## III. Subordinated debt's discipline and bank's risk-taking incentive

The increase in the mean-preserving spread of a company's asset value distribution results in a well-known moral hazard problem of the levered firm. This increase in risk simply transfers the value from debtholders of a firm to its equityholders. The traditional methods employed by debtholders of the companies to deal with this problem include the increase in the required rate of return on their financial claims, incorporation of protective covenants in the debt contract, monitoring of companies' activities, and simple termination of the relationships with the companies. The main difference between a commercial banks' debtholders and the debtholders of other companies is that the former have weaker incentives to protect the value of their claims. The reason is the moral hazard effects of implicit or explicit guarantees provided to commercial banks by national governments. These guarantees basically imply that banks' debtholders will get repaid because the government will likely bail out banks should they become insolvent. Thereby, bank creditors have no incentive to engage in costly monitoring or to "exit" excessively risky banks.

Given the facts stated above, one intention of requiring a minimum level of subordinated debt in the capital structures of banks would be to create a class of debtholders with incentive to impose a discipline on banks.

There are several attributed conditions to the ability of subordinated debt to impose this discipline. First of all, governments would have to credibly commit not to bail out subordinated debtholders (hence equityholders). Second, unlike other classes of debtholders, subordinated debtholders should not be able to secure their claims by collateral or by keeping the maturity of the debt contract short.

In addition, requiring banks to issue subordinated debt basically intends to create a security the price of which can be used to infer market consensus on the risk of the bank's assets and to then charge the bank accordingly for the benefit of the safety net. The requirement of issuing subordinated debt securities periodically is intended to encourage the informational disclosure of banks and make them more transparent, further improving the information content of the market prices.

The arguments above are twofold: subordinated debt should be able to provide market discipline and the information from subordinated-debt prices can be used to provide supervisory discipline.

The following sections will develop a simple contingent-claim model to value subordinated debt, and use the results of the model to inspect the disciplining and informational role of subordinated debt prices.

## A. Contingent Claim Valuation of Subordinated Debt

### *No Bankruptcy Costs Case*[3]

A contingent-claim valuation approach can be applied to a bank's subordinated debt. The original approach of the contingent-claim valuation was developed by Black and Scholes (1973). Merton (1974) used it to price liabilities in the case of a single issue of nonconvertible debt. The model with multiple debt claims was derived by Black and Cox (1976).

The latter model does not take into account bankruptcy costs and assumes that a capital structure of a bank consists of equity and two types of debt. The value of the assets is random, and evolves over time as a martingale. The debt claims are assumed to be single-payment instruments earning continuously compounded rates of return. The debt claims are differentiated by their priorities and are assumed to have the same maturity date.

The payoffs of different claims depend on the bank value at the maturity of this claim. If the realized bank value ($V_T$) is greater than the promised repayment of the senior debt ($S$), then the senior debt is paid in full. Otherwise, the senior claimants (or their insurer) obtain the realized value of the bank and other claimants get nothing.

The value of the senior debtholders' claim, $D_S$, is therefore,

$$D_S = min\ [V_T, S] \tag{1}$$

[3] The presentation of the contingent claim valuation of the subordinated debt without bankruptcy costs is similar to the one in Gorton and Santamero (1990).

If the bank value at maturity is greater than the total amount of junior and senior debt *(S + J)* then the debtholders get repaid in full and equityholders receive the residual amount. If the value of the bank is greater than the promised payment on the senior debt but smaller than the total outstanding amount of debt claims ($S < V_T < S + J$) than the junior creditors receive the difference between the bank value $V_T$ and $S$. The value of the equityholders' claims ($E$) and junior creditor claims ($D_J$) therefore can be written as

$$D_J = max\ [min\ (V_T - S, J), 0] \quad (2)$$

$$E = max\ [V_T - (S + J), 0] \quad (3)$$

The payoffs to the claimholders for various realized asset values are summarized in Table 1.

**Table 1: Claimholders' Payoffs for Various Realized Asset Values at the Debt Maturity Date**

| | $S + J < V_T$ | $S < V_T < S + J$ | $V_T < S$ |
|---|---|---|---|
| **Senior debt** | $S$ | $S$ | $V$ |
| **Junior (subordinated) debt** | $J$ | $V_T - S$ | $0$ |
| **Equity** | $V_T - (S+J)$ | $0$ | $0$ |

Recognizing that payoff of the above claims resembles the payoffs of the put and call options, and using the assumptions of Black-Scholes model, Black and Cox (1976) showed that the current value of subordinated debt can be expressed as:

$$D_J = V\left[N(d_1) - N\left(\hat{d}_1\right)\right] - Se^{-R_f t} N(d_2) + (S + J)e^{-R_f t} N\left(\hat{d}_2\right) \quad (4)$$

where

$$d_1 = \frac{\left[\ln\left(\frac{V}{S}\right) + \left(R_f + \frac{s^2}{2}\right)t\right]}{s\sqrt{t}}$$

$$d_2 = d_1 - s\sqrt{t}$$

$$\hat{d}_1 = \frac{\left[\ln\left(\frac{V}{S+J}\right) + \left(R_f + \frac{s^2}{2}\right)t\right]}{s\sqrt{t}}$$

$$\hat{d}_2 = \hat{d}_1 - s\sqrt{t}$$

$s$ is the volatility of the logarithm of the value of the bank, $t$ is a time to maturity of the subordinated debt, and $N(\cdot)$ is the univariate cumulative normal distribution.

Equation (4) states that the value of subordinated debt is the difference between the value of a call option on the value of the bank with exercise price equal to the face value of senior debt ($S$), bought from senior debtholders, and a call option on the value of the bank with an exercise price equal to the face value of total debt ($S+J$), sold to equityholders:

$$D_J = C(V,S,t) - C(V,S+J,t) \tag{5}$$

Equation (4) can be rewritten in terms of the spread between the yield on subordinated debt ($R_J$) and the riskless rate ($R_f$) of the same maturity:

$$R_J - R_f = -\frac{\ln\left\{\frac{V}{J}e^{-R_f t}\left[N(d_1) - N\left(\hat{d}_1\right)\right] - \frac{D}{J}N(d_2) + \frac{S+J}{J}N(\hat{d}_2)\right\}}{t} \tag{6}$$

The risk premium $R_J - R_f$ in equation (6) is a function of leverage terms $V/J$ and $(S+J)/J$, as well as volatility $s$ and time to maturity $t$.

If we consider only single debt class (*J=0*) then the risk premium will take the following form[4]:

$$R - R_f = -\frac{\ln\left[\frac{V}{S}e^{-R_f t}N(-d_1) + N(d_2)\right]}{t} \tag{7}$$

Comparison of (6) and (7) reveals that the value of the junior debt behaves differently than does senior debt with respect to maturity ($t$), the volatility of the bank's assets ($s$), and the riskless rate ($R_f$). The response of the value of junior debt to the risk factors is ambiguous and depends on the value of the bank relative to the senior debt. When the value of the bank is close to the promised payment of the senior debt, the junior debt becomes effectively the residual claim on the bank and behaves like an equity claim. For the relatively high realizations of the bank value, the junior debt will behave like debt. In other words, junior debt is initially a convex function when the value of the bank is low and becomes a concave function when the bank value is high enough. In fact, there is a single inflection point[5], $\hat{V}$, given by

$$\hat{V} = \sqrt{S(S+J)} \cdot e^{-\left(R_f + \frac{s^2}{2}\right)t} \tag{8}$$

As opposed to senior debt, the default risk premium on junior debt is a decreasing function of $s^2$ when $V$ is less than $\hat{V}$ and is an increasing function of $s^2$ when $V$ is greater than $\hat{V}$ (Black and Cox (1976))[6]. In other words, subordinated debtholders, unlike senior debtholders, can benefit from the increase in asset volatility.

[4] $d_1$ and $d_2$ are defined as before with *J=0*

[5] This point can be found by equating the vegas of two options comprising the subordinated debt and solving the condition with respect to *V*.

***Bankruptcy Costs Cases***

The model above can be modified to account for bankruptcy costs. Bankruptcy costs are introduces as a value $K$ that is lost when the bank is liquidated or reorganized[7]. Bankruptcy is costly both because of direct costs and because of disruption in bank's activities. The transfer of the insolvent bank's assets is costly. The direct costs, such as legal costs, reflect costly verification of collateral value. Indirect costs include potential loss of bank-specific human capital resulting in a time delay before the assets are utilized (if possible) at their full efficiency level after the transfer of control. Like in Anderson and Sundaresan (1996), I assume that bankruptcy costs reduce the collateral value of assets in a liner fashion, so the later is simply the market value of assets net of bankruptcy costs, $V$-$K$[8]. Bankruptcy costs are incurred if the realized value of the bank's assets is below the total amount of debt ($S$+$J$) at the debt maturity[9].

The payoffs of different claims depend on the bank value at the maturity of these claims and the magnitude of bankruptcy costs, $K$.

When the magnitude of bankruptcy costs is smaller than the face value of subordinated debt, the junior claimants' payoff is positive only if the realized value of the bank's assets is greater than the sum of the face value of senior debt, $S$, and bankruptcy costs, $K$. When the opposite is true and the magnitude of bankruptcy costs is greater than the face value of subordinated debt, the junior claimants' payoff is positive only if the bank is solvent, i.e. the realized value of bank's assets is above the total amount of debt ($S$+$J$) at the debt maturity.

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[6] Gorton and Santamero (1990) provide the same result in the banking context.

[7] In U.S., the liquidation of bank assets has historically been used in situations where a merger was unavailable or too costly or where the bank's loss to the community would impose few local costs. Reorganization of the failed banks traditionally included a merger of all or part of the assets of the insolvent bank with a stronger healthy bank's assets. In U.S., with the exception of the systematic risk exemption the least-cost resolution strategy requires the Federal Deposit Insurance Corporation (FDIC) to employ the method that imposes most failure costs on the uninsured depositors (Saunders (1999)).

[8] Like in Anderson and Sundaresan (1996) I argue that other functional forms for collateral values can be accommodated at the cost of greater complexity.

[9] Unlike Anderson and Sundaresan (1996), this paper does not model the strategic debt service. Note that with the strategic debt service, the costliness of formal bankruptcy may induce creditors to accept deviations from contractual payments.

The payoffs to the claimholders for various realized asset values and alternative magnitudes of the bankruptcy cost are summarized in Table 2.

**Table 2: Claimholders' Payoffs for Various Realized Asset Values at the Debt Maturity Date (with bankruptcy costs)**

| | ***Case 1: J>K*** | | |
|---|---|---|---|
| | $S+J<V_T$ | $S+K<V_T<S+J$ | $V_T \leq S+K$ |
| **Senior debt** | *S* | *S* | *Max [Min[$V_T$-K, S]]* |
| **Junior (subordinated) debt** | *J* | *$V_T$- S-K* | *0* |
| **Equity** | *$V_T$ – (S+J)* | *0* | *0* |
| | | | |
| | ***Case 2: J≤K*** | | |
| | $S+J<V_T$ | $V_T \leq S+J$ | |
| **Senior debt** | *S* | *Max[Min [$V_T$-K, S],0]* | |
| **Junior (subordinated) debt** | *J* | *0* | |
| **Equity** | *$V_T$ – (S+J)* | *0* | |

The structure of payoffs in Table 2 indicate that the expression for the value of subordinated debt with costly insolvency will depend on the relative magnitude of the total promised amount of subordinated debt, *J,* and bankruptcy costs, *K*.

***Case 1*: K<J.**

When the bankruptcy costs are effectively subordinated to the junior claims, the modified equation for the value of subordinated debt, $D_J^{bc}$, becomes

$$D_J^{bc} = V\left[N(b_1) - N\left(\hat{d}_1\right)\right] - (S+K)e^{-R_f t}N(b_2) + (S+J)e^{-R_f t}N(\hat{d}_2) + Ke^{-R_f t}N(\hat{d}_2) \quad (9),$$

where

$D_J^{bc}$ is the value of subordinated debt of the bank in the presence of bankruptcy costs smaller than the face value of subordinated debt.

$$b_1 = \frac{\left[\ln\left(\frac{V}{S+K}\right)+\left(R_f + \frac{s^2}{2}\right)t\right]}{s\sqrt{t}}$$

$$b_2 = d_1 - s\sqrt{t}$$

$$\hat{d}_1 = \frac{\left[\ln\left(\frac{V}{S+J}\right)+\left(R_f + \frac{s^2}{2}\right)t\right]}{s\sqrt{t}}$$

$$\hat{d}_2 = \hat{d}_1 - s\sqrt{t}$$

In the presence of bankruptcy costs, the value of the bank's subordinated debt becomes the difference between the value of a call option on the value of the bank with exercise price *S*+*K* purchased from the senior debtholders and two sold call options. One of the sold call options is on the value of the bank with exercise price *S*+*J*, sold to equityholders and second is a digital call option on the value of the bank paying *K* if the realized firm value is greater than *S*+*J* and zero otherwise sold to senior debtholders:

$$D_J^{bc} = C(V,S+K,\ t\ ) - C(V,S+J,\ t\ ) + C^d(V,S+J,\ K,\ t\ ) \qquad (10)$$

The presence of bankruptcy costs changes the behavior of subordinated debt prices. The value of subordinated debt is driven by the relative values of three options and the convex region of the relationship between subordinated debt values and the value of bank's assets increases (see Figure 1 in Appendix for the illustration of this fact). The same phenomenon is illustrated by the relative magnitude of subordinated debt's vegas in the model with and without bankruptcy cost. Vegas are also changing depending on the value of the bank's assets (see Figure 2 in Appendix). In the presence of bankruptcy costs, the subordinated debtholders start behaving as residual claimants for realizations of bank asset values higher than in cases without bankruptcy costs.

***Case 2: K>J***

When the magnitude of bankruptcy costs is greater than the promised amount of subordinated debt, the holders of subordinated debt have a positive payoff only when the value of the bank's assets are greater than the total amount of debt outstanding. In this case the value of subordinated debt resembles the payoff of a digital call option on the value of the bank's assets with a strike price $S+J$, which pays $J$ if $V>S+J$ and zero otherwise.

The value of subordinated debt therefore can be expressed as:

$$D_J^{bc'} = C^d\left(V, S+J, K, t\right) = Je^{-R_f t} N(\hat{d}_2) \tag{11}$$

where

$D_J^{bc'}$ is the value of subordinated debt of the bank in the presence of bankruptcy costs greater than the face value of subordinated debt.

The behavior of the subordinated debt value in the case when $K>J$ remains similar to the one when $K<J$ (See Figure 5 and 6 in Appendix).

## B. The Discipline of the Subordinated Debt

To illustrate the potential effect of subordinated debt in providing market discipline, I will follow the framework developed in Levonian (2000) and extend it by incorporating the costs of bankruptcy into analysis. I make an extreme assumption that subordinated debtholders can perfectly and continuously adjust their claims in response to the equityholders' increase of asset volatility by making the total promised amount of subordinated debt depend upon asset volatility. Under this assumption, $J$ becomes a function of $s$ which I will write as $J(s)$. I will proceed then by investigating the reaction of equity value to the increase in asset volatility in the presence of this "perfect" discipline from subordinated debt. If I find that the equityholders will benefit from the increase in asset volatility under the extreme assumptions of the model, the market discipline of debt will be unlikely to materialize under less restrictive assumptions.

I will investigate and compare the discipline effect of subordinated debt in two models – with and without bankruptcy costs.

***No Bankruptcy Costs Case***

Equation (4) can be used to inspect the reaction of subordinated debt values to the increase in asset volatility.

In the absence of bankruptcy costs, and with the total promised amount of subordinated debt dependent upon asset volatility, the total effect on $D_J$ of a change in $s$ is:

$$\frac{dD_J}{ds} = \mathrm{S}\,e^{-R_f t} N'(d_2) - (S+J(s))e^{-R_f t}\frac{\partial N(\hat{d}_2)}{\partial \hat{d}_2}\frac{\partial \hat{d}_2}{\partial s} - J'(s)N(\hat{d}_2) \quad (12)$$

The value of equity claim on the bank is

$$E = VN\left(\hat{d}_1\right) - (S + J(s))e^{-rt} N\left(\hat{d}_2\right) \quad (13)$$

Hence, the reaction of equity to the increase in $s$ can be expressed as

$$\frac{dE}{ds} = (S+J(s))e^{-R_f t}\frac{\partial N(\hat{d}_2)}{\partial \hat{d}_2}\frac{\partial \hat{d}_2}{\partial s} + J'(s)N(\hat{d}_2) \quad (14)$$

The perfect discipline of subordinated debt in our context implies that $J(s)$ should adjust such that the value of subordinated debt, $D_J$, remains the same. Therefore, the following condition should hold:

$$\mathrm{S}\,e^{-R_f t} N'(d_2) = (S+J(s))e^{-R_f t}\frac{\partial N(\hat{d}_2)}{\partial \hat{d}_2}\frac{\partial \hat{d}_2}{\partial s} + J'(s)N'(\hat{d}_2) \quad (15)$$

Using condition (14) in (15), the reaction of equity to the increase in volatility becomes

$$\frac{dE}{ds} = S\,\mathrm{e}^{-\mathrm{R}_f t} N'(d_2) \tag{16}$$

The expression (16) is always positive and implies that in the presence of perfect market discipline from the subordinated debt, the equityholders still benefit from the increase in risk given that the senior debtholders do not adjust the value of their claims (Levonian (2000)). The discipline provided by the subordinated debt reduces the value transfer to the equityholders from the increase in risk by reducing the strike price of an implicit call option underlying the payoff of the equityholders from *S+J* to *S*.

***Bankruptcy Cost Cases***

***Case 1: K<J***

In the presence of bankruptcy costs which are less than the face value of subordinated debt, the reaction of subordinated debt value to the increase in volatility of assets is:

$$\frac{dD_J^{bc}}{ds} = (\mathrm{S}+\mathrm{K})\,\mathrm{e}^{-\mathrm{R}_f t} N'(b_2) - (S+J(s))\frac{\partial N(\hat{d}_2)}{\partial \hat{d}_2}\frac{\partial \hat{d}_2}{\partial s} + Ke^{-R_f t}\,\frac{\partial N(\hat{d}_2)}{\partial \hat{d}_2}\frac{\partial \hat{d}_2}{\partial s} - J'(s)e^{-R_f t} N(\hat{d}_2) \tag{17}$$

The reaction of equity remains the same as in (14) and the preservation of the subordinated debt value requires that

$$(\mathrm{S}+\mathrm{K})\,\mathrm{e}^{-\mathrm{R}_f t} N'(b_2) = (S+J(s))\frac{\partial N(\hat{d}_2)}{\partial \hat{d}_2}\frac{\partial \hat{d}_2}{\partial s} - Ke^{-R_f t}\,\frac{\partial N(\hat{d}_2)}{\partial \hat{d}_2}\frac{\partial \hat{d}_2}{\partial s} + J'(s)e^{-R_f t} N(\hat{d}_2) \tag{18}$$

Using condition (18) in (14) gives

$$\frac{dE}{ds} = (S+K)\, e^{-R_f t} N'(b_2) + K e^{-R_f t} \frac{\partial N(\hat{d}_2)}{\partial \hat{d}_2} \frac{\partial \hat{d}_2}{\partial s} \tag{19}$$

Equation (19) says that when the bankruptcy costs are smaller than the face value of subordinated debt, the benefits to the equityholders from the increase in volatility of assets depends on the relative magnitude of two terms: one, on the implicit call option of equity on senior debt with a strike price $S+K$, which is always positive and greater than the corresponding term in equation (16) because of increase in the strike price from $S$ to $S+K$; and the second, on the value of the digital call option on the bankruptcy cost; it is negative when the bank value is high and turns positive for lower bank values[10].

Unlike equation (16), equation (19) can be negative for higher realizations of bank assets values. Keeping other parameters constant, the larger amount of subordinated debt relative to senior debt increases the region in which $\frac{dE}{ds}$ is negative. Other things equal, higher costs of bankruptcy also increase the region where $\frac{dE}{ds}$ is negative.

***Case 2: K>J***

When the bankruptcy costs are greater than the face value of subordinated debt, the value of the latter is determined by equation (11) and the reaction of subordinated debt to the increase in asset volatility is:

$$\frac{dD_J^{bc'}}{ds} = J(s) e^{-R_f t} \frac{\partial N(\hat{d}_2)}{\partial \hat{d}_2} \frac{\partial \hat{d}_2}{\partial s} + J'(s) e^{-R_f t} N(\hat{d}_2) \tag{20}$$

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[10] $\frac{\partial N'(\hat{d}_2)}{\partial \hat{d}_2} \frac{\partial \hat{d}_2}{\partial s} = \frac{1}{\sqrt{2p}} e^{\frac{-(\hat{d}2-s\sqrt{t})^2}{2}} \left[ -\left( \frac{\ln(\frac{V}{S+J(s)}) + (R_f + \frac{s^2}{2})t}{s^2\sqrt{t}} \right) + \left( \frac{J'(s)}{(S+J(s))Vs\sqrt{t}} \right) \right]$

In order for the value of subordinated debt to remain the same, the following condition should hold:

$$J(s)e^{-R_f t}\frac{\partial N(\hat{d}_2)}{\partial \hat{d}_2}\frac{\partial \hat{d}_2}{\partial s} + J'(s)e^{-R_f t}N(\hat{d}_2) = 0 \qquad (21)$$

Using (21) in equation (14) gives the following expression for the reaction of equity to the increase in assets volatility:

$$\frac{dE}{ds} = Se^{-R_f t}\frac{\partial N(\hat{d}_2)}{\partial \hat{d}_2}\frac{\partial \hat{d}_2}{\partial s} \qquad (22)$$

Right-hand side of equation (22) can be either positive or negative depending on the parameter values. $\frac{dE}{ds}$ responds to different parameter values in a similar fashion as in the case where $K<J$.

Figures 3 and 7 in the Appendix illustrate the reaction of a bank's equity value to the increase in asset volatility in the models with and without bankruptcy costs in the case where the face value of subordinated debt adjusts instantaneously to the increase in risk such that the value of subordinated debt remains unaltered.

The model without bankruptcy costs shows that even if the subordinated debtholders perfectly adjust the required return on their claims, the fact that they benefit from the transfer of value from the senior debtholders makes the subordinated debt discipline imperfect. In other words, if the required return on senior debt does not adjust to the increase in volatility, both equityholders and bondholders may benefit from the decline of senior debt value (see Figure 3 and 7 in Appendix).

In the presence of bankruptcy costs, the subordinated debt discipline is dependent upon the relative amount of subordinated debt on the balance sheet of the bank, together with the magnitude of the bankruptcy costs. In the presence of perfect discipline from the subordinated debtholders, the larger amount of subordinated debt and higher magnitude of bankruptcy costs increases the region of bank asset values in which equityholders do not benefit from the increase in asset volatility. When the subordinated debt constitutes a small proportion of bank funding the equityholders do

not benefit from the increase in risk except when there are very large realizations of bank asset values. When the asset value of the bank gets lower, the equityholders's payoff from the increase in risk becomes positive even in the presence of perfect discipline from subordinated debtholders. The equityholders simply have to compensate the subordinated debtholders for the loss in value of two options which are part of the subordinated debt value: 1) the short position in call option on the value of assets with a strike equal to face value of total debt and 2) a loss in value of a digital option on the amount of bankruptcy costs[11].

The presence of bankruptcy costs reduces the gain of equityholders from the increase in risk (relative to "no bankruptcy costs case") when the vega of subordinated debt in the "bankruptcy costs case" is larger than in the "no bankruptcy costs case" and increases it otherwise (see Figures 2,3,6 and 7 in Appendix). Moreover, other things being equal, the higher the magnitude of bankruptcy costs, the lower the required compensation of subordinated debtholders in the region where equityholders benefit from the increase in risk.

## IV. Extraction of information from subordinated debt

The fact that the relationship of the subordinated debt spreads and asset risk is nonlinear and, more importantly, depends on the assets value complicates the straightforward comparison of subordinated debt spreads as an indicator of the asset risk (Gorton and Santomero, 1990).

Levonian (2000) uses a contingent-claim valuation framework (without bankruptcy costs) to show the difficulty in interpreting subordinated debt spreads and comparing them in the cross-section of banks. In order to see this, the information conveyed by the subordinated debt yield spreads can be compared to the information conveyed by the variables underlying public policy objectives. Two possible policy variables to consider are the liability of the provider of the safety net, and the probability of bank failure. In the presence of information on the market value of

[11] Note though that the digital call option's value decreases with the increase in risk for high realizations of banks' asset value and increases for the lower ones.

equity, a contingent-claim valuation framework can be used to obtain the values of the later variables.

Without taking into account bankruptcy costs, the liability of the safety net provider ($L$) is:

$$L = S\,e^{-R_f t}\,N\left[\frac{s\sqrt{t}}{2} - \frac{ln(\frac{V}{S}) + R_f t}{s\sqrt{t}}\right] - VN\left[-\frac{ln(\frac{V}{S}) + R_f t}{s\sqrt{t}}\right] \qquad (23)$$

The risk-neutral probability of bank failure ($P$) is:

$$P = N\left[\frac{s\sqrt{t}}{2} - \frac{ln(\frac{V}{S+J}) + R_f t}{s\sqrt{t}}\right] \qquad (24)$$

Using alternative capital structures and asset volatilities values, the equations (6), (23), and (24) can be solved simultaneously to obtain three important bits of information: the credit spread on subordinated debt, the default probability ($P$), and the insurance liability ($L$). Levonian (2000) showed, using numerical simulation, that the straightforward comparison of subordinated debt yields or spreads is impossible and misleading unless the capital structures of two banks are the same. Solvent banks with relatively smaller subordinated debt spreads (and lower asset volatility) can in fact have higher default probability, and narrower spreads can imply greater liability for the deposit insurer. The author argues then that the necessity to use some form of contingent-claim valuation framework to interpret the subordinated debt yield spreads evaporates (at least in theory) a widely acclaimed comparative advantage of subordinated debt as a tool to differentiate among the risks of the banks. The author continues arguing that, in fact, the same information can be extracted from equity prices using similar techniques.

What I will show next is that the last statement deserves an extra investigation. If we take into account bankruptcy cost, the risk-neutral probability of bank failure remains the same but the expression for the liability of the safety net provider changes

depending on the magnitude of the bankruptcy costs relative to the amount of subordinated debt.

***Case1: J>K***

When the magnitude of the bankruptcy cost is greater than the amount of subordinated debt, an expected liability of the deposit insurer is:

$$L^{bc} = (S+K)\, e^{-R_f t} N\left[\frac{s\sqrt{t}}{2} - \frac{ln(\frac{V}{S+K}) + R_f t}{s\sqrt{t}}\right] - VN\left[-\frac{ln(\frac{V}{S+K}) + R_f t}{s\sqrt{t}}\right] \tag{25}$$

where

$L^{bc}$ is an expected liability of the deposit insurer in the case where the face value of junior debt is greater than the amount of bankruptcy costs.

The insurer's liability is equivalent to a written put option on the value of bank assets with an exercise price of $S+K$. If the value of the bank at the maturity of debt is below the total amount of senior debt plus the bankruptcy costs and the bank is liquidated, the liability of insurer is positive and the insurer has to pay insufficient funds to the senior creditors and incur bankruptcy costs recovering the salvage value of the bank assets.

***Case2: J<K***

When the magnitude of the bankruptcy cost is smaller than the amount of subordinated debt, an expected liability of the deposit insurer is:

$$L^{bc'} = (S+K)\, e^{-R_f t} N\left[\frac{s\sqrt{t}}{2} - \frac{ln(\frac{V}{S+J}) + R_f t}{s\sqrt{t}}\right] - VN\left[-\frac{ln(\frac{V}{S+J}) + R_f t}{s\sqrt{t}}\right] \tag{26}$$

where

$L^{bc'}$ is an expected liability of the deposit insurer in the case where the face value of junior debt is less than the amount of bankruptcy costs.

The insurer's liability is equivalent to a written put option on the value of bank assets with an exercise price of $S+J$. If the value of the bank at the maturity of debt is below the total amount of debt and the bank is liquidated, the value of the subordinated debt is zero because the latter is smaller than the bankruptcy costs and the insurer has to pay senior creditors and incur bankruptcy costs recovering the salvage value of the bank assets[12].

A comparison of equations (23), (25), and (26) reveals that the liability of the safety net provider, calculated without taking into account bankruptcy costs, underestimates its true value (see Figure 4 and 8 for examples of the two respective cases).

The second observation is that the information on the value of the liability of the safety net provider cannot be estimated using equity prices alone if the magnitude of bankruptcy costs is unknown. The reason is that equity prices do not contain information about the bankruptcy cost-parameter $K$. Thus, the subordinated debt prices can be used together with equity prices to obtain the value of $K$ when the face value of subordinated debt exceeds the bankruptcy costs.

Using the standard option valuation approach, the value of a bank's equity ($E$) and the market value of assets are related by the following expression:

$$E = VN\left[\frac{ln\left(\frac{V}{S+J}\right)+(R_f+\frac{s^2}{2})t}{s\sqrt{t}}\right]-(S+J)e^{-rt}N\left[\frac{ln\left(\frac{V}{S+J}\right)-(R_f+\frac{s^2}{2})t}{s\sqrt{t}}\right] \quad (27)$$

[12] This paper does not consider the issues of insurer's ex-post incentives in liquidating the bank when $K>J$. This problem is a subject of another paper, which is a work in progress right now.

The relationship between asset and equity volatility is the following:

$$s = s_E \frac{E}{VN\left[\frac{ln\left(\frac{V}{S+J}\right)+(R_f+\frac{s^2}{2})t}{s\sqrt{t}}\right]} \tag{28}$$

The notations in (28) are the same as earlier in the paper except $s_E$, which denotes volatility of the firm's equity.

Together with the equations (9) for subordinated debt value $D_J^{bc}$, equations (27) and (28) contain four unknown parameters: $s_E$, $s$, $V$, and $K$.

There are two possible procedures to obtain the unknown parameters from the equations (9), (27) and (28). Both of them involve finding a procedure for estimating volatility of equity. If there exists a traded option on the bank equity, then the implied volatility from the option prices can be used as $s_E$. Alternatively, the historical volatility of equity can be used as a proxy for $s_E$. Knowing $s$ and $V$, the subordinated debt prices can be used to extract the value of bankruptcy costs $K$.

## V. Credibility of subordinated debt schemes

In order to investigate the contingent effects of subordinated debt schemes on regulatory behavior one can classify the proposals into rules and discretion-based (Maclachlan (2001)).

The proposals of Wall (1989) and Calomiris (1999) are examples of "rule-based proposals". Wall (1989) describes a scheme in which banks would have to maintain an over 4% ratio of putable subordinated debt to risk-adjusted assets. The simultaneous failures to comply with the regulatory ratio and to redeem the subordinated debt securities would trigger a bank closure. Calomiris (1999) outlines a scheme wherein the inability of a bank to refinance its subordinated debt at a predefined spread over risk-free interest rate would result in compulsory gradual reduction of its risky assets.

The proposal of Evanoff and Wall (2000) is an example of a discretion-based rule. The deterioration of the credit quality of the bank proxied by the subordinated debt yield spreads (and potentially augmented with other indicators of solvency) is supposed to trigger some form of corrective regulatory action[13].

At the first glance rule-based subordinated debt proposals seem to be the preferred method of dealing with the moral hazard problems associated with the bank safety net. The usual problem arising from this type of approaches is credibility[14]. If a bank bankruptcy is socially costly and there are possibilities that the institution is viable but has to be closed for not complying with an "excessively strict" rule, there is a good chance of a regulator relaxing this rule[15]. As usual with this type of problem, non-credible rule will result in investors' treating the rule-based policy as if it really were a discretionary policy.

Discretion-based policy avoids the credibility problems of a rule-policy but leads to the substantial fading out of the intended effects of market discipline through subordinated debt. In the Evanoff and Wall (2000) framework, the subordinated debt yield is just another (complementary) piece of information at the disposal of the regulator. The interpretation of this information and the choice of action are at the discretion of the regulator. The investors' focus on the regulatory closure rules instead of the bank's fundamentals is the main disadvantage of the discretion-based policy.

The frequent imbalances and noise in the financial markets[16], especially in times of system-wide financial distress, make it difficult to accurately set trigger points for mandatory supervisory intervention. As I have illustrated in previous sections, the interpretation and comparison across banks of information from the subordinated debt prices requires a regulator's judgment! The *discipline* of subordinated debt, the *amount of information* provided by the subordinated debt, and

[13] In particular, it should trigger the prompt corrective actions in the U.S.A. The fact that Prompt Corrective Action in the U.S. include both mandatory and discretionary provisions points out that one has to interpret the proposals classified as "discretion-based" in this paper as discretionary relative the proposals which are classified as "rule-based".

[14] Moreover, stricter rules (like Wall's proposals) result in larger credibility problems.

[15] The potential legal actions against regulator on part of the bank's shareholders are one example of the pressure to relax the rule.

[16] That noise includes changes in supply and demand of the bond market, links with the stock market volatility changes in perceived sector risk, and general macroeconomic conditions.

the *modeling framework* used to interpret that information critically depend on the amount of subordinated debt relative to senior debt and the magnitude of bankruptcy costs.

Overall, the analysis suggests that the presence of a sufficient amount of subordinated debt on the balance sheet of the banks can provide some useful information to regulators and somewhat improve the direct market discipline but whether this information can be used for setting up a credible intervention rule is at least unclear.

## VI. Strategic manipulation of subordinated debt prices

Another potential factor complicating interpretation of the subordinated debt information in the framework of current proposals is the portfolio strategies involving this subordinated debt and equity of a bank (Ely (2000)). The pitfall of the proposals is in the prohibition for banks, its holding companies or bank insiders to trade its own subordinated debt. This leaves a room for potential speculative strategies that would involve gradual build-up of the long position in the subordinated debt of a bank and the short-selling of bank's shares. Selling the large amount of a bank's subordinated debt in a relatively illiquid market would likely increase the debt yields. The rise in yields would result in a higher likelihood of regulatory intervention, which would depress the bank's stock price. At that time, the speculator would close out his short-position compensating for the losses made on the long debt position. In a situation where the bank is barred from buying its own subordinated debt, the only way left to neutralize the speculative attack is to buy its own shares. This latter strategy, though, would result in the weakening of the capital base of the bank.

Other portfolio strategies introducing the noise into subordinated debt prices are combinations of buying subordinated debt on margins and dampening it in a short sell.

Overall, the nature of the subordinated debt proposals and the plausible portfolio strategies of financial markets participants might result in dramatic changes of subordinated debt yields which would lack any fundamental information content. A forced intervention on part of the regulatory authority (especially in the presence of a

rigid rule) and the increased cost of recapitalization on part of banks would be the likely consequences of these types of developments. The possible attempts to prohibit or limit the speculative portfolio strategies would depress the liquidity of the subordinated debt market and are unlikely to succeed in a global market place (Ely (2000).

## VII. Summary and Conclusions

This study provides a theoretical analysis of mandatory subordinated debt proposals in banking.

The effect of market discipline of subordinated debt on bank's risk-taking incentives is analyzed in a contingent-claim valuation framework. The traditional firm valuation model with multiple debt claims is extended to incorporate the costs of bankruptcy. The market discipline of subordinated debt has been shown to differ significantly in the models with and without bankruptcy costs. In the absence of bankruptcy costs, the discipline of subordinated debt is imperfect and equityholders are *always* able to compensate subordinated debtholders for the increase in mean-preserving asset risk. In the presence of bankruptcy costs, there always exists a region of bank asset values where equityholders' payoff from the value transfer, net of subordinated debtholders' compensation, is less or equal to zero. The size of this region is positively related to the magnitude of bankruptcy costs and subordinated debt claim. Therefore, the market discipline of subordinated debt critically depends on the proposed magnitude of mandatory subordinated debt holdings and on the potential variation of bankruptcy costs across various bank types and countries.

This paper also shows that the subordinated debt prices can provide new information relative to equity prices when the bankruptcy costs are unknown and the subordinated debt claims exceed the bankruptcy costs. Equity prices together with subordinated debt prices (and preferably option prices of equity) can be used to extract the information on the unknown bankruptcy cost parameter. It is shown that knowing the magnitude of expected bankruptcy costs is necessary for calculating the variables underlying policy objectives. In particular, it is illustrated that the value of the deposit insurer's expected liability would be underestimated if the bankruptcy costs were not taken into account.

An overall theoretical analysis reveals that the interpretation and comparison across banks of information from the subordinated debt prices requires the regulator's judgment. The *discipline* of subordinated debt, the *amount of information* provided by subordinated debt, and the *modeling framework* used to interpret that information all critically depend upon the amount of subordinated debt relative to senior debt and upon the magnitude of bankruptcy costs.

The incorporation of bankruptcy costs into the analysis has direct implications for the size of the proposed mandatory ratios of subordinated debt. In particular, the current evidence on the magnitude of bankruptcy costs (Weiss (1990)) suggests that the average ration of direct bankruptcy costs to total assets is 2.8 percent which is already greater than the proposed ratios of mandatory subordinated debt holdings.

In the framework of our model, requiring banks to hold the subordinated debt in excess of bankruptcy costs would potentially improve the direct market discipline of subordinated debt and provide additional information on the value of bank assets.

Although the analysis suggests that the presence of a sufficient amount of subordinated debt on the bank balance sheet can provide some useful information to regulators and somewhat improve the direct market discipline, the usefulness of this information for setting up a credible intervention rule is, at least unclear. The later fact casts some doubt on the credibility of the existing subordinated debt proposals and highlights the need for more research in this area.

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# Appendix

Figures 1 to 4 are plotted using the following parameter values:

**S=70**

**J=30**

**K=20**

**$R_f$=10%**

***s*** **=20%**

***t*** **=1 year**

**Figure 1: The Value of Subordinated Debt with and without Bankruptcy Costs (K<J)**

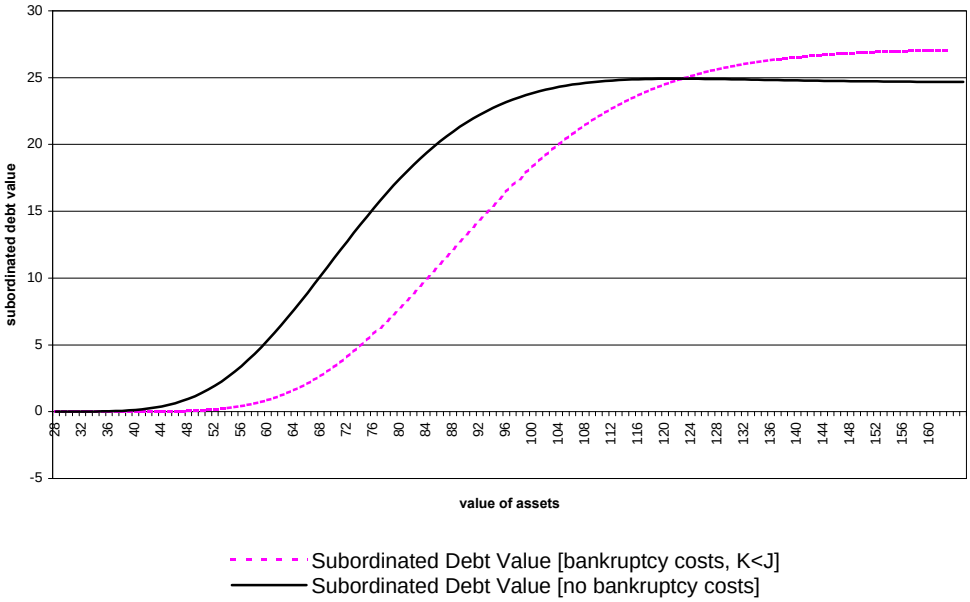


**Figure 2: Subordinated Debt's Vega with and without Bankruptcy Costs (K<J)**

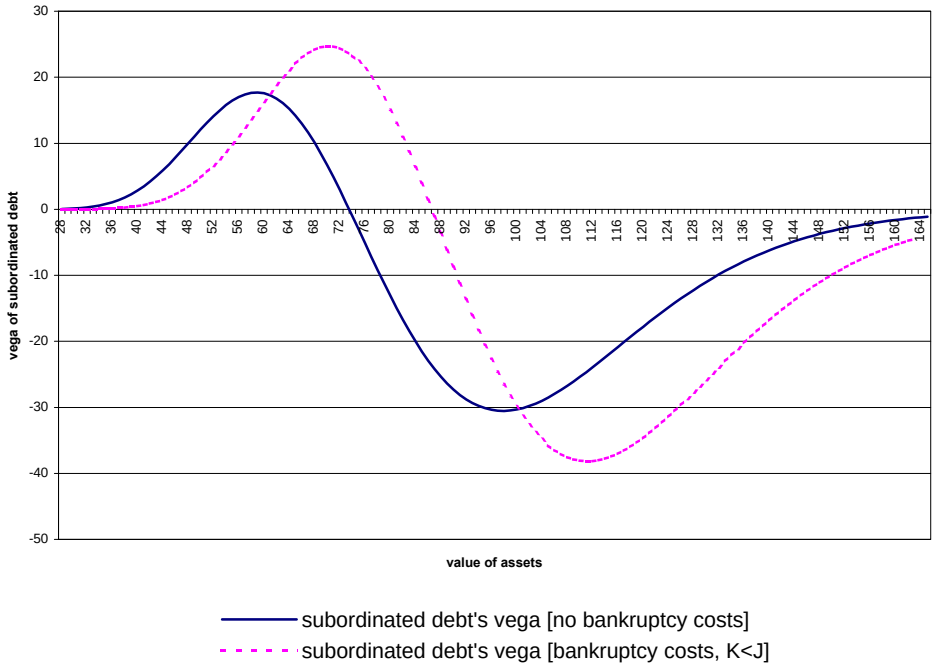



**Figure 3: Equity Value Reaction to Increase in Asset Risk with the Instanteneous Adjustments in Value of Subordinated Debt (K<J)**

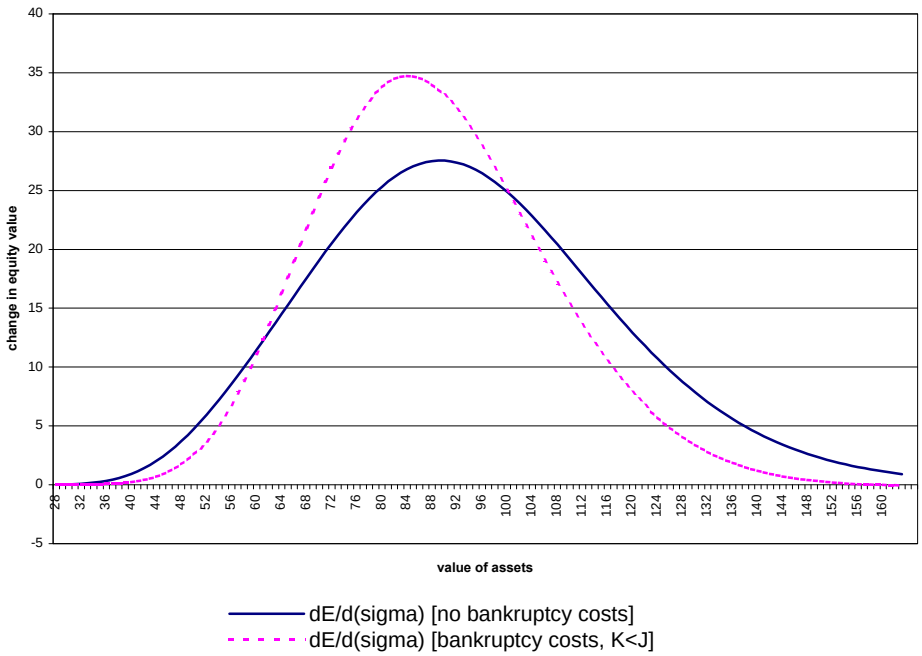
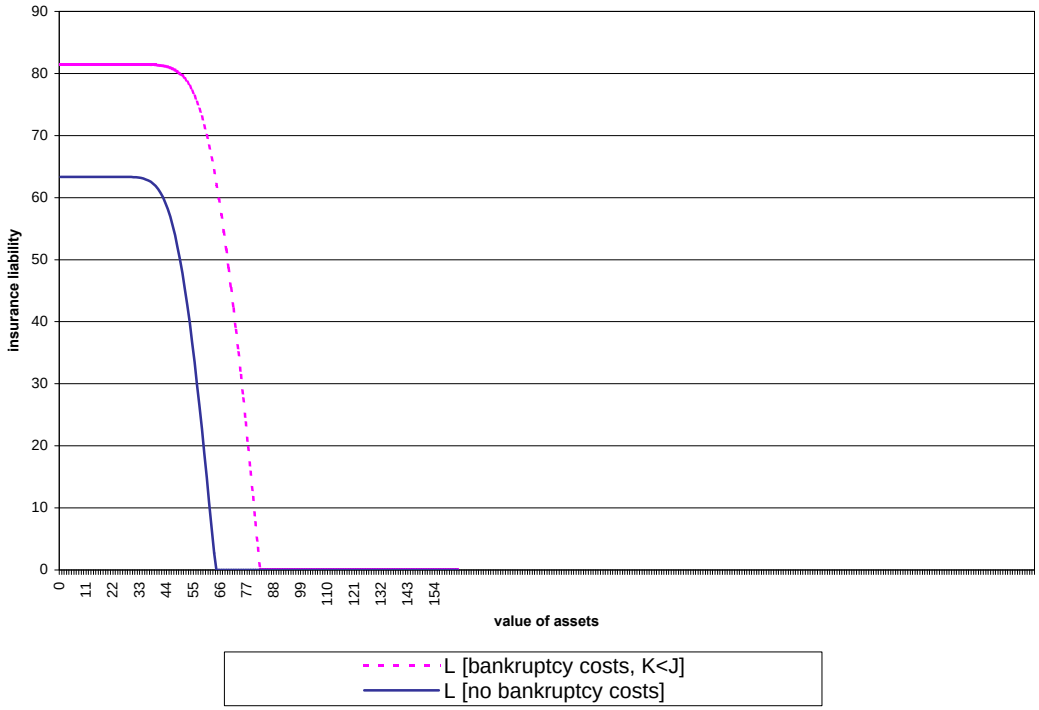


**Figure 4: Expected Liability of the Insurer (K<J)**



Figures 5 to 8 are plotted using the following parameter values:

**S=70**

**J=15**

**K=20**

**$R_f$=10%**

***s*** **=20%**

***t*** **=1 year**

**Figure 5: The Value of Subordinated Debt with and without Bankruptcy Costs (K>J)**

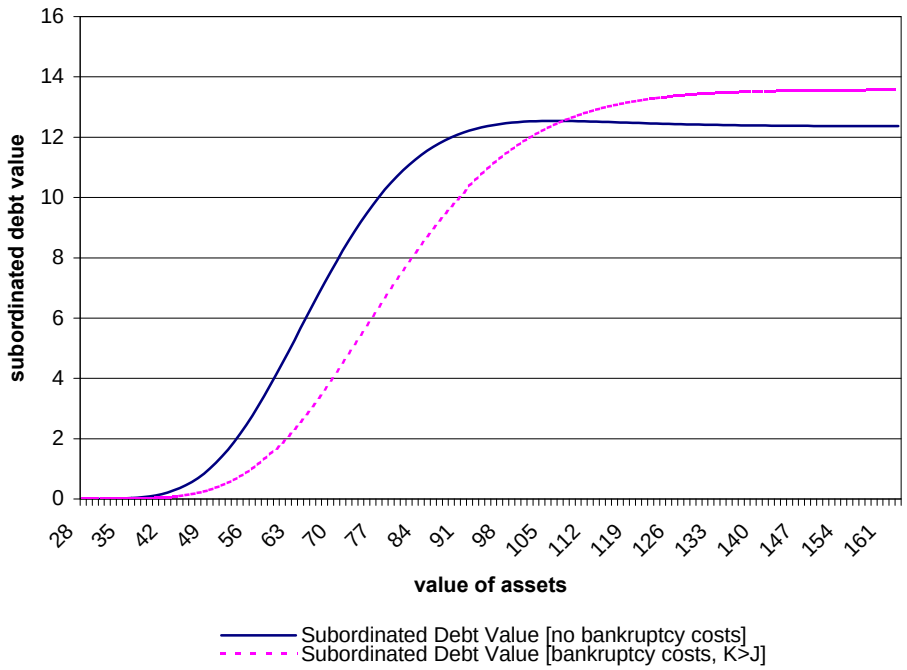


**Figure 6: Subordinated Debt's Vega with and without Bankruptcy Costs (K>J)**

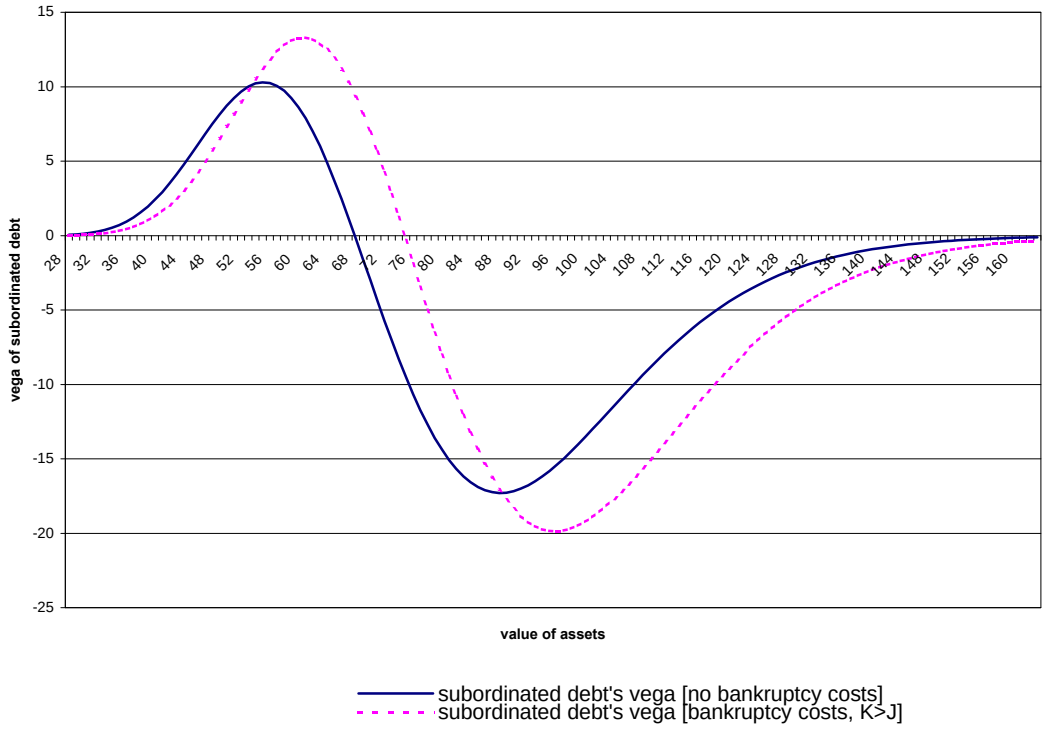



**Figure 7: Equity Value Reaction to Increase in Asset Risk with the Instanteneous Adjustments in Value of Subordinated Debt (K>J)**

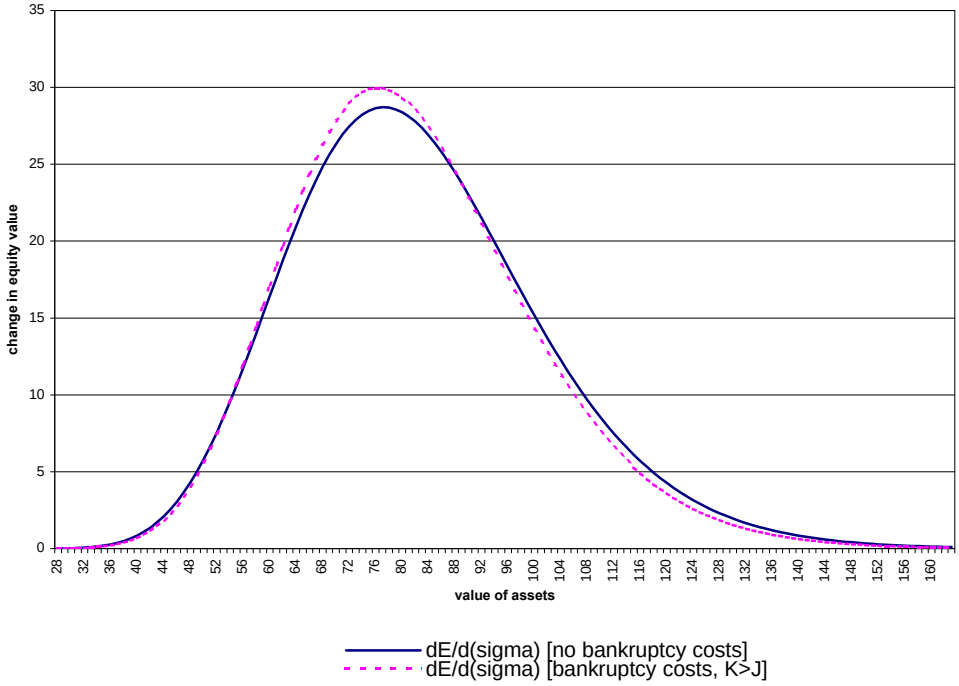


**Figure 8: Expected Liability of the Insurer (K>J)**