Estimating Okun's coefficient in the Swedish Economy

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Abstract

This study takes a look at Okun's law och tries to determine whether or not it is applicable to the Swedish economy. Okun's law is a verified statistical relationship between economic growth and unemployment, first described and popularised by Arthur in the early 1960's.

There has been a shortage of studies relating to Okun's law and the Swedish economy in recent years, and this study aims to fill this vacuum with more recent data and new approaches to estimating the coefficients in the relationship.

We examine three common versions of Okun's law: a growth version, a gap version and a dynamic version. Our results indicate that it is, to a certain extent, possible to apply the law to the Swedish data, with reservations toward how they are interpreted. An analysis of the regressions that have been run shows that the dynamic version produces the highest explanatory power, whereas the gap version produces the worst predictions.

Keywords: Okun's law, seasonal adjustments of data, GDP, NAIRU, unemployment rate, economic growth

Sammandrag

Den här studien behandlar Okuns lag och undersöker huruvida den går att tillämpa på den svenska ekonomin eller ej. Okuns lag är ett statistiskt påvisat samband mellan arbetslöshet och ekonomisk tillväxt, påvisat och populariserat av ekonomen Arthur Okun under början av 1960-talet.

Det råder en brist på studier som undersöker Okuns lag och dess användbarhet på den svenska ekonomin, ett vakuum som den här studien ämnar att fylla med hjälp av nya data och nya tillämpningar för att skatta sambandets koefficienter.

Studien undersöker tre vanliga former av Okuns lag, en tillväxt-, en "gap-" och en dynamisk version. Resultaten tyder preliminärt på att Okuns lag, inom vissa rimliga gränser, går att tillämpa på den svenska datan, med reservationer för hur dessa tolkas. En analys av de regressioner som har genomförts visar att den dynamiska modellen påvisar det största förklaringsvärdet av de tre versionerna, medan "gap"-versionen är starkt opålitlig.

Nyckelord: Okuns lag, säsongsjusteringar av data, BNP, NAIRU, arbetslöshet, ekonomisk tillväxt

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Preface

We would like to thank our thesis adviser, Lars-Göran Larsson, for his valuable input and insights into the topics this thesis describes and discusses. Special thanks to the economics department administration at Gothenburg School of Business, Economics and Law for their help with the details concerning this thesis.

"Doing econometrics is like trying to learn the laws of electricity by playing the radio."

Guy Orcutt, March 1983.

Källman, Marika and Nordell, Hugo
Swedish West Coast, March 2012

Glossary

- Autoregressive model – A type of random process that often is used to model and predict various types of natural phenomena.
- Bartlett kernel function – A function used in hypothesis testing
- Bias – A statistical difference in the population parameter of interest.
- Calendar effects – Effects caused by the different lengths of intervals
- Critical values – A value that somewhat is crucial for the experiment and is used as a fixed point to compare with.
- Coefficients – A multiplicative factor in some term of an mathematical expression.
- Confidence interval (CI) – An interval that is statistically estimated to indicate the reliability of the estimated result.
- Dynamic model – A regression model that incorporates more than one regressor.
- Durbin-Watson test - A test used to detect the presence of autocorrelation.
- Economic significance – Something that is of importance for economic theories and concepts.
- Explanatory variables – The variables that a model is dependent of, ergo the variables that explain the result.
- External validity – The validity of generalised inferences in experiments dependent on external factors, such as if the answers used in a random sample is trustworthy or not.
- First order lag – A relation often used to represent the dynamic response characteristics of a system.
- Gretl – The software used in this study to make all regression models.
- HAC – Heteroskedasticity and Autocorrelation. HAC standard errors are robust for the presence of these phenomenon.
- Heteroskedasticity – The existence of different variances between the residuals in an experiment.
- Hodrick-Prescott filter (HPF) – A filter in statistic software that is often used to estimate potential GDP.
- Homoscedasticity – The opposite of heteroskedasticity, i.e, the residuals all have the same variance.
- Internal validity - The validity of generalised inferences in experiments dependent on internal factors, such as if the model is correctly constructed.
- Multicollinearity – A statistical phenomenon where two variables are perfectly correlated.
- The Non-Acceleration Inflation Rate of Unemployment (NAIRU) - A state of equilibrium in the economy when inflation doesn't cause unemployment, or vice versa.
- Null hypothesis – A statement in a hypothesis test that can be rejected or not rejected depending on its significance.
- Organisation for Economic Co-operation and development (OECD) – The source of some of our statistical data.
- Ordinary least squares (OLS) – A method that is used to estimate the unknown parameters in a linear regression model.

- Omitted variable bias – Occurs when a model is created and incorrectly leaves out one or more important factor.
- Quantile-quantile plot – A probability plot used as a graphical method to compare two probability distribution.
- R-squared – The correlation of the outcomes in a regression model.
- Seasonal adjustments – Adjusting raw data to account for trends that arise at certain times every year that could potentially skew the results of a regression.
- Statistically significant – If the estimated value of a coefficient, for a given confidence level, can reject the null hypothesis.
- X12-ARIMA – Method (statistical software) used for seasonally adjusting data.
- Chi-square test – Goodness of fit test that describes how well the model fits the set of observations.

1 Introduction

In 1962 the economist Arthur Okun was given the task to examine the gains of the U.S. Gross National Product, GNP, in relation to reduction of unemployment by the John F. Kennedy Administration. Okun discovered that, from 1947 up until 1962, a 3 per cent reduction of the U.S. GNP was strongly related to an increase in unemployment by 1 per cent (Okun, 1962). This statistical finding has become the foundation of what is now referred to as Okun's law.

The relationship implies that a reduction in unemployment contributes to a much larger effect on output. According to Okun, quoted by Gylfason (1997, p. 2), this would "...stem from some or all of the following; induced increases in the size of the labor force, longer average weekly hours or greater productivity."

As a model, Okun's law assumes that all of the market related factors, which determine the relationship between GNP and unemployment, are included in the equation (Stock & Vogler-Ludwig, 2010, p. 35). Because the relationship states that the effect on output is larger than the effect on unemployment, as Okun himself stated, this means that the productivity of a country could be assumed to increase over time if the assumptions hold.

One version of Okun's law includes the NAIRU, in order to incorporate the effects of inflation. The NAIRU describes a state of equilibrium in the economy when inflation doesn't cause unemployment, or vice versa (for further information on the NAIRU, see Stock & Vogler-Ludwig, 2010). The way the two versions are related to each other can, somewhat loosely, be expressed as the percentage reduction of GNP for every level that unemployment rate exceeds the natural rate of unemployment.

Since 1962, the relationship has been analysed, discussed and utilised by economists all over the world. The most common application of Okun's law is forecasting changes in unemployment in relation to GDP.[1] Quoted by Gylfason (1997, p. 2), the Nobel laureate and economist James Tobin described Okun's law as "... one of the most reliable empirical regularities of macro economics."

1.1 Purpose

- To evaluate whether the basic principles of *"Okun's law"* from 1962 are applicable to – and reasonably valid for – the Swedish economy using well known econometric regression models within a set of specified assumptions about the Swedish labour market.

1.2 Related Studies

As a statistical relationship, Okun's law has been subjected to many revisions in the ever-changing macro-economy. One of the most common concerns in studies relating to the model is its simplicity, a linear relationship between GNP and unemployment. Though the model can be imprecise in a short perspective of time, in the long run the linear relationship is more or less accurate, according to Knotek (2007), who also writes that this is only valid if one disregards heavy business cycles, and it's therefore of great importance to view Okun's law as a rule of thumb, rather than an economical feature.

Both Okun's law, and NAIRU to a certain extent, are based upon a few relatively simple equations, yet still they've been able to capture most of the changes in important economic determinants, such as monetary, fiscal, social and industrial policies. All of those factors affect unemployment as well as the labor market and has an impact on taxes, the labor, monetary and trade market, which all should be a part of the regression (Stock & Vogler-Ludwig, 2010).

The most common conclusion that can be drawn from previous studies is that, confusingly enough and regardless of the other factors, the models provide a fair description of the relationship between unemployment and GNP. Though Stock & Vogler-Ludwig (2010) conclude that the old models require a high level of economical skill and experience in order for the coefficients to have any economical meaning, and should therefore stand aside for more dynamic models.

[1] GNP was the measure originally used by Okun, but most studies try to verify the relationship using the GDP instead.

1.3 An Application to Sweden

A previous study by Gylfason (1997) has inconclusively claimed that the Okun relationship can be applied to the Swedish market. Gylfason (1997) argues that through a detailed study of the relationship between GDP and unemployment in Sweden, more accurate forecasting models can be constructed; and that the more variables being included in the model, the more potential it has to be valid.

The importance of accurate models is also crucial for forecasting, and there are no recent studies looking at new variations of Okun's law being applied to the Swedish economy. A more specific model would hopefully mean better predictability, and hence, better forecasting. Most of the studies have only investigated Okun's relationship for the U.S. economy, whereas this study aims to see if the relationship can be applied specifically to the Swedish one.

2 Problem Analysis

In order to understand the basic concepts, as well as historical usage, of Okun's law it is essential to evaluate the most widely accepted econometric research about the law, how it's been estimated previously and what kind of problems most econometricians have run into while doing this. Among other things, this includes looking at the most commonly known versions of Okun's law and determining which one is most feasible for use within the scope of this study.

There is also a problem of where we should fetch the necessary raw data in order to estimate the regression models we will propose in this study, judging what kind of eventual adjustments that need to be done for it to be useful and how to adjust it. Without trustworthy data, none of the forthcoming conclusions we will try to draw from this study will have any chance of being valid. To solve this problem, we will have to look at methods of dealing with seasonal trends in data, how to filter those out, and ultimately choose one or several of these that we deem adequate for our needs.

Another potential problem stems from how we should analyse our results in a manner such that we cover any eventual doubts about them, but still try to draw serious conclusions. For instance, when examining the results and looking back at our assumptions about the Swedish labour market, and the econometric model itself: *were they all necessary and do they hold?*

3 Methodology

This section present and discusses how the study has been conducted, the choice of raw data, how it's been adjusted and potential problems with using it for this study's purpose. Several assumptions as well as delimitations are formally put to the letter, as to avoid potential confusion that may otherwise arise when looking at the results and the subsequent analysis and conclusion.

3.1 Assumptions

In addition to the standard mathematical assumptions that follow from using ordinary least square for regression, this study makes a series of assumptions about the nature of the data that is being used:

(1) Any eventual errors in the GDP data from Statistics Sweden are sufficiently small to not have a significant impact on the economic growth.

(2) The Swedish labour market is assumed to *not* be rigid in the long-run. This is essentially a necessary assumption in order for the regression results to be interpretable.

(3) The estimates of the structural NAIRU, as estimated by the OECD in 2011, are adequately accurate for use within the scope of this study.

(4) GDP is an economic indicator that is *at least* as good as the GNP to predict the change in unemployment over time.

3.2 Delimitations

First off, the study will not focus on explaining the mathematical foundation of econometrics. Rather, it will give brief, but accurate mathematical explanations and motivations that are then applied to estimating Okun's law.

Secondly, the study will not take into account any potential cointegration between the regressors, as this might require an introduction to instrumental variables.

Finally, the study will not aim to estimate the effects of labour market rigidity in Sweden. Instead, verbal reasoning together with the estimated effects of changes in the GDP will be used to assess this factor and how it may affect the regression results.

3.3 Raw Data

Statistics Sweden has a vast database of statistics about the Swedish economy, including unadjusted, quarterly GDP from 1950 an onwards in current prices.[2] For this particular study, unadjusted quarterly GDP in current prices from 1983 up until the last quarter of 2010 is utilized. The main reason for this is that quarterly GDP gives the shortest time period between data points, and is updated and revised by Statistics Sweden every time new GDP estimates are published. Other studies, such as Gylfason (1997), use annual GDP to provide estimates of Okun's law, but are then also exposed to the problem of few data points to estimate the equation. Although quarterly data is more prone to seasonal variations and trends (Shah et al., 2012), this can be filtered out using X12-ARIMA/TRAMO methods or a simple Hodrick-Prescott filter (see 3.3.1).[3]

It is also important to realise the shortcomings of the data used in this study. GDP, whether it is real or nominal, suffers from statistical discrepancies that could cause the standard errors in the regressions to be biased. Although some economists (Nalewaik, 2010) argue that the gross domestic income (GDI)[4] is a more accurate measure of economic growth, there are issues that might make the measure unreliable. As Rosnick & Baker (2011) points out, GDI is primarily measured through reported wage rates, which is relies heavily on self-reporting from the working

[2]For the Swedish reader, Statistics Sweden is *Statistiska Centralbyrån*.
[3]For a short description of how the Hodrick-Prescott filter works, see appendix B.
[4]GDI = GDP - Statistical Discrepancy.

population. Furthermore, there is less variance in the output measure, than the income one, in the quarterly data that this study is based upon (Rosnick & Baker, 2011, p. 1).

As for statistics about the unemployment rates for the quarters between the years 1983 til the end of 2010, the statistical database Eurostat[5] is being used. However, as there are many definitions of unemployment readily available, below is the definition set by Eurostat:

- Age 15-74;
- Who were without work during the reference week, but currently available for work; and
- Who were either actively seeking work in the past four weeks, or who had already found a job to start within the next three months.

In other words, the unemployment rate is based off of the ratio between unemployed people and the total economically active population. Note though that this does not necessarily constitute every person without a job, it is merely the number of people who fit the Eurostat's definition of unemployment. Furthermore, the unemployment data from Eurostat is already seasonally adjusted and calender effects corrected using X12-ARIMA algorithms, and will thus not be reworked for this study.[6]

The NAIRU estimates for the Swedish economy are calculated by the OECD and fetched from its online database.[7] The estimates are structural, which means that equilibrium models of wage rate and price level are being used to compute a time-invariant NAIRU equilibrium (Stock & Vogler-Ludwig, 2010). The NAIRU estimates are also annual for this study's examined time period. Therefore, for every quarter in a given year, the same NAIRU is being used. Also, all of the data is available upon request.

3.3.1 Adjustments to the GDP

As for the GDP data, several adjustments have been made to make it usable for the scope of this study. Firstly, as the GDP is in current prices, a GDP deflator is necessary to discount the data to one and the same year for a consistent measure of its value. The base year was set to 2005 and the deflator values are gathered from the World Bank's database.[8]

Secondly, an X12-ARIMA[9] correction was applied to the raw data using the econometrics software package Gretl[10]. This smoothes out the sig-saw shape of the quarterly data and removes any calendar effects and seasonal trends and produces data can be used to analyse actual economic growth from one time period to another. Figure 1 presents the non-adjusted versus the seasonally adjusted real GDP using 2005 as base year.

[5]See http://epp.eurostat.ec.europa.eu/portal/page/portal/employment_unemployment_lfs/data/database for a direct link to the downloads page.

[6]For a more thorough description of how Eurostat seasonally adjusts its data, see http://epp.eurostat.ec.europa.eu/statistics_explained/index.php/Short-term_business_statistics_-_seasonal_adjustment_methods.

[7]See http://stats.oecd.org/Index.aspx?QueryId=32456.

[8]See further http://data.worldbank.org/.

[9]See appendix A for a description of the algorithm.

[10]An open-source econometrics software available from gretl.sourceforge.net/.

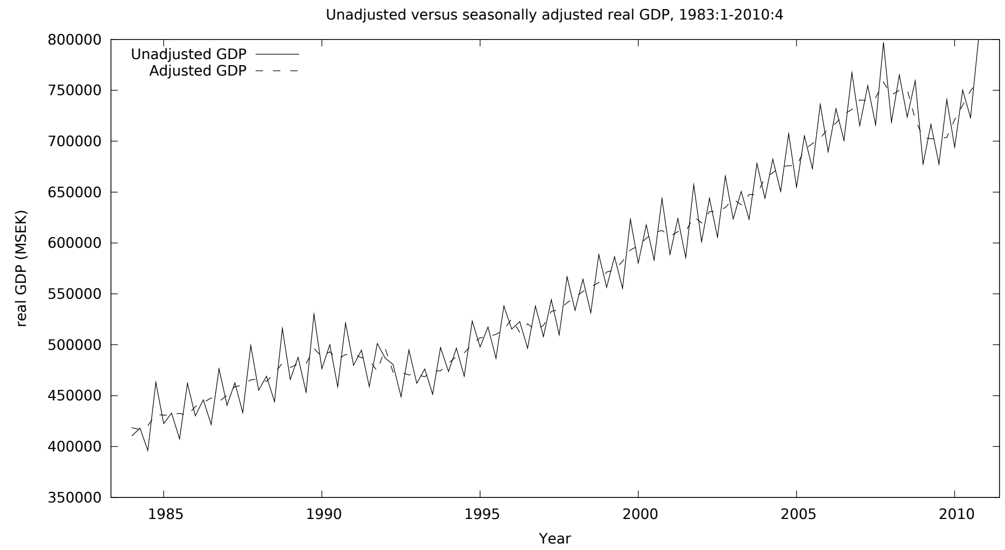

Figure 1: The non-adjusted real GDP versus the seasonally adjusted using X12-ARIMA in Gretl, 1983:1-2010:4. Base year: 2005. Sources: Statistics Sweden (2011).

Thirdly, annualised growth rates are computed by comparing the same quarters from the current year to the previous. The main reason as to why this study utilizes the year on year growth rate is that this makes it easier to compare the results to similar studies, and it further filters out any seasonal component that the X12-ARIMA algorithm might have missed.

As one of the examined versions of Okun's law is based on the aggregate output gap, a Hodrick-Prescott filter (briefly explained in appendix B) was applied to the GDP data to estimate potential GDP using Gretl. A thorough motivation for this choice can be found in a study by Razzak & Dennis (1999). The estimated potential GDP can be found in figure 2 below.

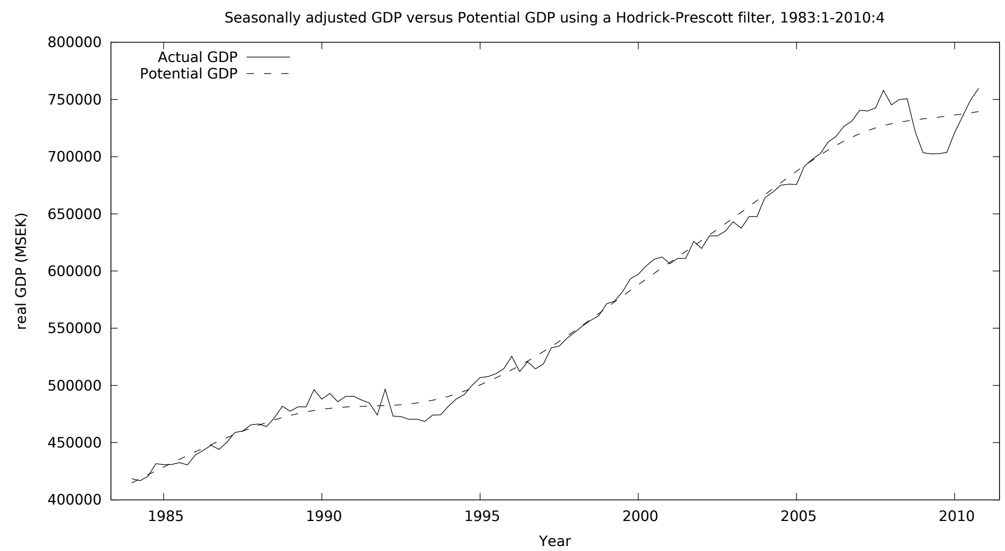

Figure 2: The seasonally adjusted actual GDP and the resulting time series after running it through a Hodrick-Prescott filter with $\lambda = 1600$, 1983:1-2010:4. Base year: 2005. Sources: Statistics Sweden (2011).

3.4 Criticism of Previous Studies

Previous studies of how reliable Okun's law in actuality is often lacks transparency in the sense that only results that are in line with the authors assumptions are presented. Gylfason (1997) does not present full scale tables of the entire regression results, nor does he express the magnitude of importance his choice of estimating potential GDP has. Although Gylfason in the end doesn't draw any bewildering conclusions, the consistent lack of test results makes it hard to assert how well his expanded version of Okun's law performs.

Furthermore, an overwhelmingly large number of the economists who have delved into Okun's observed relationship fail to discuss the potential of shocks between the dependent and independent variables. Some economists use changes in the rate of unemployment to predict changes in GDP or GNP, whereas others do it the other way round. The causality is essential if Okun's law is to provide us with any useful way to predict future states in the macroeconomy, and the persistent lack of discussing this issue does not help current research in the area.

The articles written by Knotek (2007) and Stock & Vogler-Ludwig (2010) are the two ones that are deemed most reliable, as they do in fact discuss the potential problems mentioned in the previous paragraph and in the problem analysis.

4 Okun's Law

As mentioned in section 1, Okun's original proposition stems from the use of gross national product, which is the GDP plus total capital gains abroad (investments and similar) minus foreign nationals' income in a given country. In the context of this study, GDP will be used instead of GNP for the following line of reasoning: *As unemployment and economical growth are the two concepts Okun's law tries to connect, and GDP is more closely related to the health of the local economy, than is GNP, it seems more natural to base the regressions off of GDP. Furthermore, foreign nationals working in Sweden contributes to total production output, and may also benefit from publicly funded unemployment insurances if they lose their job.*[11]

Three different versions of Okun's law are presented, together with formal hypotheses of the seemingly spurious relationship between unemployment and economic growth for each respective version.

4.1 The Growth Version

The first relationship proposed by Arthur Okun in 1962 is called the growth, or difference version and can be expressed as a simple linear regression equation to answer this question:

$$\Delta u_t = \beta_0 + \beta_1 \cdot \Delta Y_t + \varepsilon_t, \quad \text{where} \tag{1}$$

- Δu_t is the change in the unemployment rate between the current time period and the previous one,
- ΔY_t is the growth rate of ouput (GDP) in per cent between the current time period and the previous one,
- the coefficient β_0 is the percentage point change in the unemployment rate given zero growth for the coming time period,
- β_1 is the percentage point change in the unemployment rate for a one percentage point change in the output growth, also called *Okun's coefficient*; and
- $E(\varepsilon_t|\Delta Y_t) = 0$ for each time period t.

Intuitively, the coefficient β_1 should be *negative*, as a growing economy – in the long-run – ought to lead to a declining unemployment.

Formal hypothesis 1:

H_0: $\beta_1 \geq 0$. *The presumed linear relationship between unemployment and economic growth, as measured by the change in GDP, is non-negative.*

H_1 is then: $\beta_1 \geq 0$. *The relationship is negative.*

A graphical illustration of this thinking can be found in figure 3. Interestingly, the ratio β_0/β_1 can be economically be interpreted as the required level of economic growth to keep the unemployment rate constant over time, which is also one of the main reasons as to why this version of Okun's law is so popular among economists (Stock & Vogler-Ludwig, 2010).[12]

[11] See for instance the Swedish Public Employment Service, `http://www.arbetsformedlingen.se/download/18.4fd70913124390604db80001061/worksweden.pdf` p. 18.

[12] This follows quite easily from manipulating equation 1 on the right hand side.

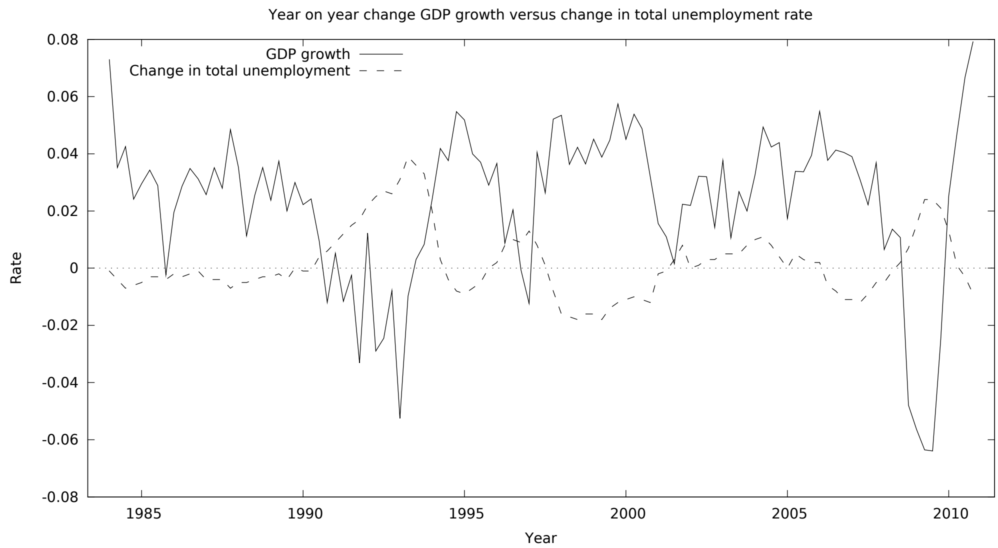

Figure 3: The year on year real growth of Swedish GDP versus the changes in total unemployment rate, 1983:1-2010:4. Both time series are have been adjusted using X12-ARIMA. Sources: OECD (2011) and Statistics Sweden (2011).

Furthermore, one of the most important advantages of the growth version is that, in order to regress Δu with respect to ΔY, all of the macroeconomic data needed is available to the public. Hence, only the coefficients need to be estimated.

4.2 The Gap Version

There are many different versions of the gap version of Okun's law, but they all try to assert the same thing, namely, *how is ouput and unemployment related?* Originally, Okun proposed the following gap equation:

$$(u_{a,t} - u_{n,t}) = \beta \cdot \frac{Y_{p,t} - Y_{a,t}}{Y_{a,t}} + \varepsilon_t, \quad \text{where} \tag{2}$$

- $Y_{a,t}, Y_{p,t}$ is the *actual* and *potential* output (GDP) respectively at time t, and the ratio is the GDP growth rate.
- $u_{a,t}, u_{n,t}$ is the *actual* and *natural* rate of unemployment. More specifically, $u_{n,t}$ is the NAIRU at time t within the scope of this study.
- β is the percentage point change in the unemployment gap for a one unit change in the output gap, and ε_t is the error term, where $E\left(\varepsilon_t | \frac{Y_{p,t}-Y_{a,t}}{Y_{a,t}}\right) = 0$.

Looking at equation 2, Okun's coefficient β ought to again be *negative*, as a high level of actual unemployment should be associated with a low level of actual output.

Formal hypothesis 2:

H_0: $\beta \geq 0$. *The relationship between the unemployment gap and the output gap is non-negative.*

H_1: $\beta < 0$. *The relationship is negative.*

Neither the NAIRU u_n, nor the potential output Y_p are directly measurable, and have to be estimated. This in particular is a problem that has spurred many economists, including Okun himself, into deriving other forms of Okun's law that are more feasible to estimate (Knotek, 2007, p. 76), such as the growth version discussed previously, and the more recent dynamic one that is discussed below. In figure 4, the structural Swedish NAIRU, as estimated by OECD in 2011, can be found together with the total unemployment rate over the period 1983:1-2010:4.

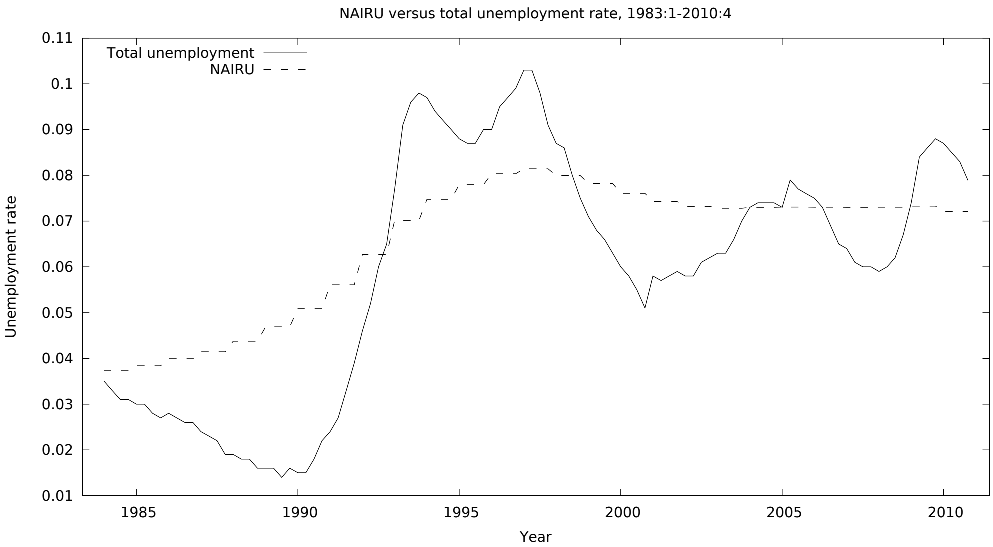

Figure 4: The structural Swedish NAIRU, as estimated by the OECD in 2011, versus the total unemployment rate of people aged 15-74, 1983:1-2010:4. Sources: OECD (2011).

Appendix C also provides a brief mathematical motivation for the linkages between the gap and growth versions of Okun's law using elementary algebra.

4.3 The Dynamic Version

As mentioned in previous sections, the simple interpretation of Okun's law is one of the main reasons as to why it's so attractive for policy makers (Knotek, 2007; Stock & Vogler-Ludwig, 2010). In the following section, the growth version of Okun's law is extended to include multiple explanatory variables, such as the lagged change in the unemployment rate and growth rate of GDP. The interpretation of the coefficients will, when performing the regression, will no longer have the same interpretation. Rather, they will be the individual effects of their respective regressor, *controlling* for the others. For a more in depth discussion of how to interpret multiple linear regression coefficients, see Stock & Watson (2006, p. 158).

A mathematical representation of the previous paragraph can be given by

$$\Delta u_t = \beta_0 + \beta_1 \cdot \Delta Y_t + \beta_2 \cdot \Delta Y_{t-1} + \beta_3 \cdot \Delta u_{t-1} + \varepsilon_t \tag{3}$$

where ΔY_{t-1} and Δu_{t-1} are one time period lagged regressors and $E(\varepsilon_t|\Delta Y_t, \Delta Y_{t-1}, \Delta u_{t-1}) = 0$. Under the standard multiple linear regression assumptions (see further Stock & Watson, 2006, p. 170-173) this model can readily be extended to include more time lags if necessary. Equation 3 is also referred to as a *autoregressive distributed lag* model, as it includes *at least* one time-lag of the dependent variable and lags of other regressors as well (Stock & Watson, 2006, p. 543).

Again, the coefficient on ΔY_t, i.e. Okun's coefficient, should be *negative* following the same reasoning as before. Following the same line of reasoning, the coefficient on ΔY_{t-1} should also be negatively related to the change in the unemployment rate, whereas the coefficient on Δu_{t-1} intuitively should be positive. The *dynamic* version of Okun's law thus opens up for the possibility of setting up hypothesis tests for the other coefficients as well, not just for Okun's coefficient.

Formal hypothesis 3:

H_0: $\beta_1 \geq 0$. *The relationship between the change in total unemployment rate and the change in GDP is non-negative.*

H_1: $\beta_1 < 0$. *The relationship is negative.*

Formal hypothesis 4:

H_0: $\beta_2 \geq 0$. *The relationship between the change in total unemployment rate and the change in the first order lag of GDP is non-negative.*

H_1: $\beta_2 < 0$. *The relationship is negative.*

Formal hypothesis 5:

H_0: $\beta_3 \leq 0$. *The relationship between the change in total unemployment rate at time t and its first order lag is negative.*

H_1: $\beta_3 > 0$. *The relationship is positive.*

Also – as concluded in many different studies (Knotek, 2007; Stock & Vogler-Ludwig, 2010; Barreto & Howland, 1993; Stephanides, 2006) of the European nations, Japan and the U.S. – the explanatory power of Okun's law seems to increase when regressing the dynamic version. However, the trade-off is that the coefficients are no longer easily interpreted, which could be a potential problem for policy makers (Knotek, 2007, p. 78).

5 Empirical Results

In this section, the three specifications of Okun's law given in section 4 are estimated and presented in accordance with standardised econometric methodology (see further Ziliak & McCloskey, 2004). The dependent variable is the year on year change in the total unemployment rate for both the *growth* and *dynamic* versions, whereas it is the unemployment gap for the *gap* version. The forthcoming sections are divided into each version of Okun's law separately, and the results are presented in equation form, tables and graphs. The full regression results can be found in appendices D, E and F. Graphs for the χ^2-test for normality and the quantile-quantile plots are also found in the attached appendices.

More results from the regressions are presented, than are subsequently analysed, and this is to ensure a certain scientific transparency and allow other researchers to critically examine the results and be able to draw their own conclusions about the estimated models if need be. Furthermore, tests for normality in the residuals are also presented, both through the χ^2-statistic, and through quantile-quantile plots. Note also that all three regression models of Okun's law have been estimated over the same sample period. This is to ensure that the information criterion presented in the tables are comparable across the regressions, and to make their subsequent analysis more coherent for the reader.

5.1 Growth Version

Equation 4 below presents the estimates to equation 1. The standard errors are computed using heteroskedasticity and autocorrelation robust methods with a time period lag of three (3) periods using a Bartlett kernel. A preliminary observation of the results do seem to fit the notion that the slope coefficient, $\beta_{\Delta Y_t}$, is negative. The standard error of the regression is noted by $\hat{\sigma}$.

Table 4 presents the entire regression results, including the unadjusted R^2 and Durbin-Watson test statistic for presence of autocorrelation in the residuals.

$$\widehat{\Delta u_{total,t}} = \underset{(0.0021021)}{0.00865471} - \underset{(0.050673)}{0.300308}\,\Delta Y_t \tag{4}$$

$$T = 108 \quad \bar{R}^2 = 0.4609 \quad F(1{,}106) = 35.122 \quad \hat{\sigma} = 0.0087756$$

(standard errors in parentheses)

From table 4 it is clear that both of the regression coefficients, the intercept and the slope, are statistically significant at the 1% significance level. This is also confirmed by the p-values. In addition, table 1 presents the 95% confidence interval for the regression coefficients. Notice that with 95% confidence the signs on either coefficient will not change.

Table 1: 95% Confidence interval for the growth version regression, $t(106, 0.025) = 1.983$.

Variable	Coefficient	95% confidence interval	
$\widehat{\beta_0}$	0.00865471	0.00448718	0.0128222
$\widehat{\beta_{\Delta Y_t}}$	−0.300308	−0.400772	−0.199845

The *growth* version regression's predictions, in-sample, are presented in figures 5 and 6, spanning over the period 1984:1-2010:4. The quantile-quantile plot of the *growth* version's residuals can be found in figure 10, which indicates how closely distributed to a normal distribution they are. Lastly, figure 11 is presented to illustrate the residuals' actual values, based on a Doornik & Hansen (2008) χ^2-test for normality. As the χ^2-statistic is 14.129, with a p-value of 0.0009, we cannot reject the null hypothesis that the residuals are in fact normally distributed.

Figure 5 shows how well the linear estimated regression fits the actual observations, whereas figure 6 presents how well the estimates match the observations over time.

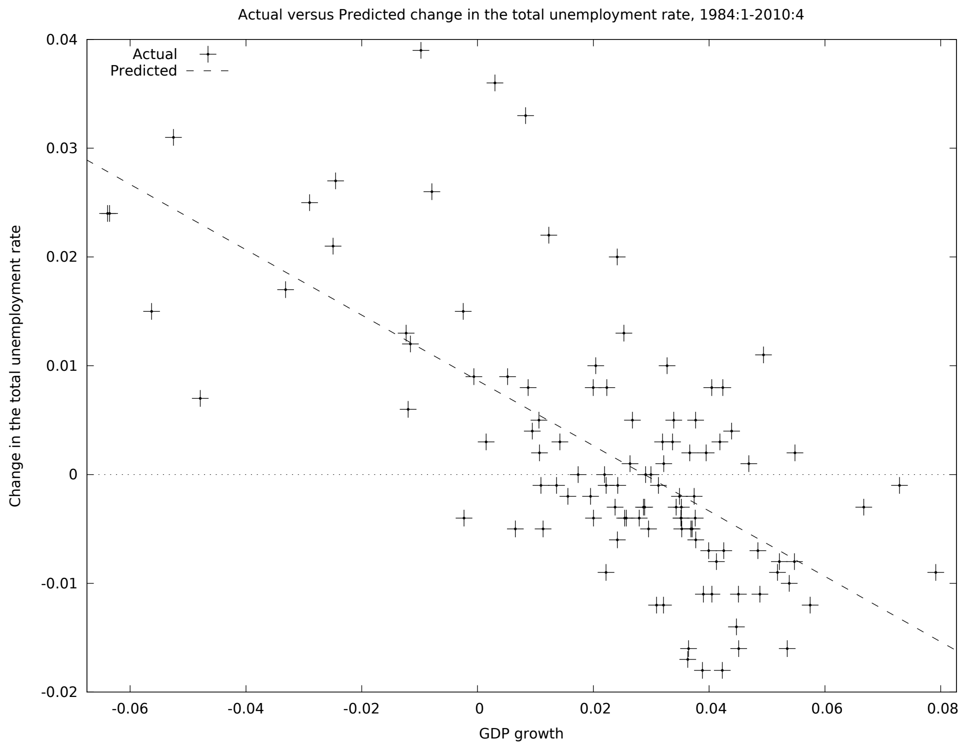

Figure 5: Actual observations versus fitted estimates from the growth version regression, 1984:1-2010:4.

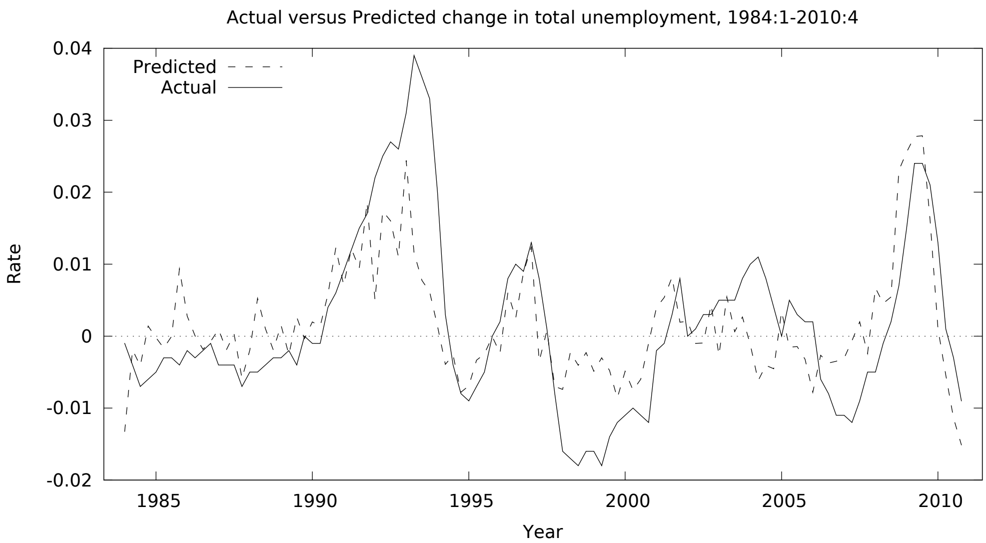

Figure 6: The in-sample predictions of the change in total unemployment rate using the growth version regression, 1984:1-2010:4.

5.2 Gap Version

The results from the gap version regression using Gretl are presented in equation 5 below, and table 5 in appendix E shows the full output. Furthermore, as with the *growth* version regression, the standard errors in parentheses are computed using heteroskedasticity and autocorrelation robust numerical methods under the Bartlett kernel. The coefficient $\widehat{\beta_{\frac{Y_{p,t}-Y_{a,t}}{Y_{a,t}}}}$ is statistically significant (and *negative*) at the 1% level with a t-ratio of -3.9131 (p-value of 0.0002).

$$\widehat{(u_{total,t} - u_{n,t})} = \underset{(0.11050)}{-0.432399}\,\frac{Y_{p,t} - Y_{a,t}}{Y_{a,t}} \tag{5}$$

$$T = 108 \quad \bar{R}^2 = 0.2118 \quad F(1,107) = 15.313 \quad \hat{\sigma} = 0.014756$$

(standard errors in parentheses)

As with the regression for the *growth* version of Okun's law, table 2 present a 95% confidence interval for the estimated coefficient with consistently negative estimates over the sampled time period.

Table 2: 95% Confidence interval for the gap version regression, $t(107, 0.025) = 1.982$.

Variable	Coefficient	95% confidence interval	
$\widehat{\beta_{\frac{Y_{p,t}-Y_{a,t}}{Y_{a,t}}}}$	-0.432399	-0.651451	-0.213347

As with the *growth* version regression results, the fitted values from the *gap* regression in figure 7 illustrates the negative relationship between the quarterly change in total unemployment rate and the change in cumulated quarterly GDP from the production side. Figure 8 shows how well the estimates fit the actual observations over the sampled time period, 1984:1-2010:4.[13] The graphs for the tests for normality, again using a χ^2-test, can be found in appendix E and figures 12 and 13. From these graphs, and the statistics presented in them, it is evident that the residuals are not normally distributed.

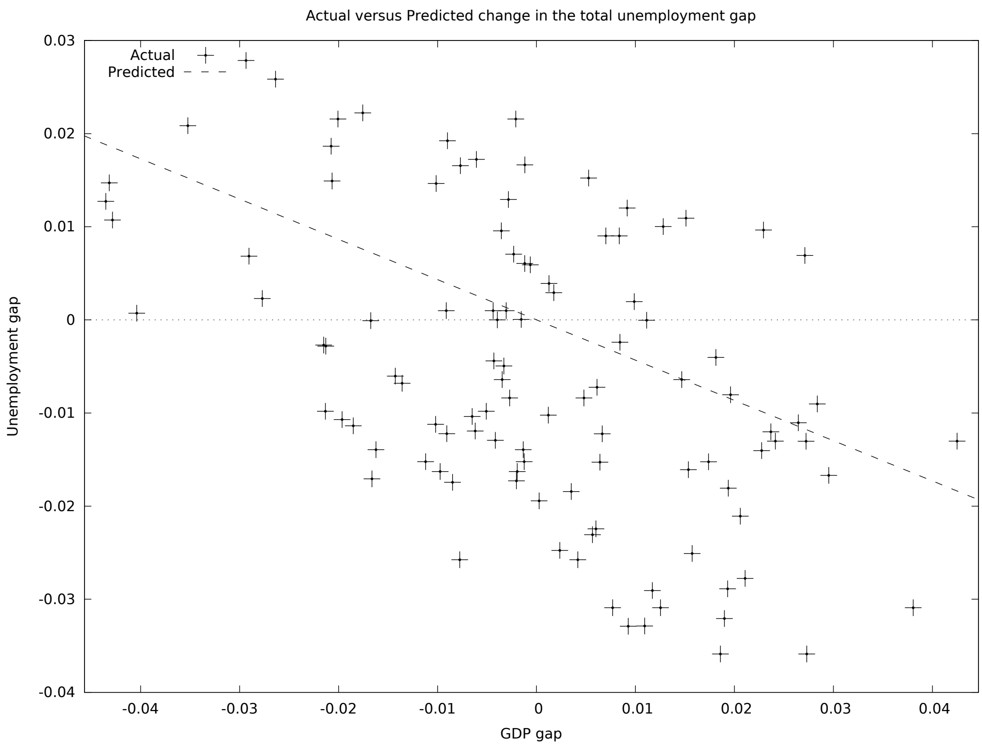

Figure 7: Actual observations versus fitted estimates from the gap version regression, 1984:1-2010:4.

[13] Again, note that these "predicted" or fitted values are derived from in-sample estimates.

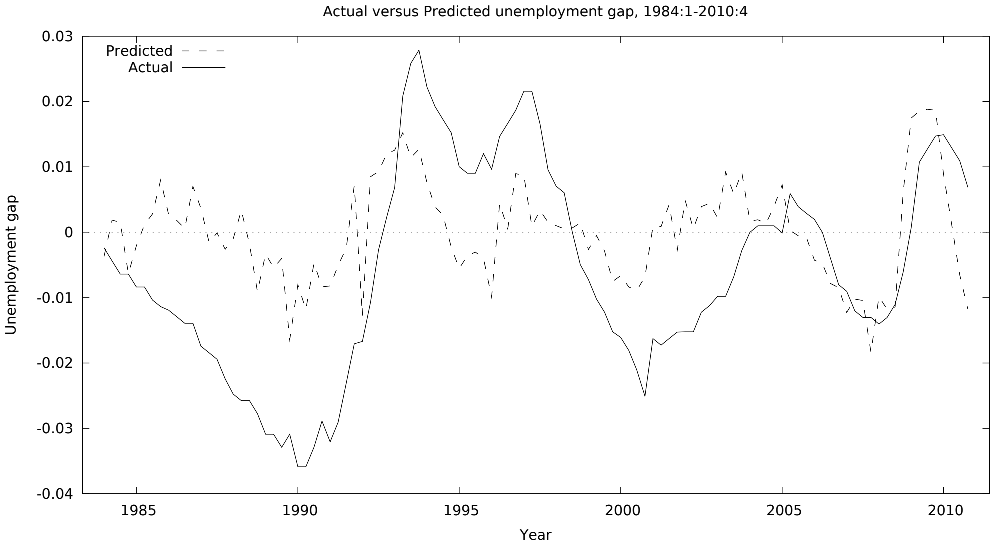

Figure 8: The in-sample predictions of the change in total unemployment rate using the gap version regression, 1984:1-2010:4.

5.3 Dynamic Version

The final regression, i.e. the *dynamic* version regression results are presented in this section. First, as with the previous regressions, the full regression output can be found in the attached appendices and table 6. Also, the graphs concerning the goodness-of-fit of the residuals to a normal distribution can be found in figures 14 and 15 respectively.

In equation 6 the results from the *OLS* regression using Gretl are presented. A first look at the signs do seem the fit the stated hypotheses of *negative* values on the GDP coefficients, and a positive relationship between the first order time lag of the change in total unemployment. All but the coefficient on the first order time lag of the GDP are statistically significant at the 1% level, as can be seen in table 6. However, all of the coefficients are well beyond the 5% significance level. The adjusted R^2 is 0.925593, which is roughly twice as large as the adjusted R^2 from the *growth* regression presented previously.

$$\widehat{\Delta u_{total,t}} = \underset{(0.00069747)}{0.00372030} - \underset{(0.017967)}{0.0849232}\,\Delta Y_t - \underset{(0.024449)}{0.0615367}\,\Delta Y_{t-1} + \underset{(0.051310)}{0.740933}\,\Delta u_{total,t-1} \tag{6}$$

$$T = 108 \quad \bar{R}^2 = 0.9256 \quad F(3{,}104) = 211.94 \quad \hat{\sigma} = 0.0032601$$

(standard errors in parentheses)

The 95% confidence intervals for each estimated coefficient can be found in table 3, and at both ends of each interval the signs are as expected. Notice that the coefficient $\beta_{\Delta Y_t}$ is a factor four (4) smaller than in the *growth* regression.

Table 3: 95% Confidence interval for the dynamic version regression,$t(104, 0.025) = 1.983$.

Variable	Coefficient	95% confidence interval	
$\widehat{\beta_0}$	0.00372030	0.00233718	0.00510341
$\widehat{\beta_{\Delta Y_t}}$	−0.0849232	−0.120553	−0.0492934
$\widehat{\beta_{\Delta Y_{t-1}}}$	−0.0615367	−0.110020	−0.0130530
$\widehat{\beta_{\Delta u_{total,t-1}}}$	0.740933	0.639184	0.842683

The time graph of the fitted values from the *dynamic* regression can be seen in figure 9 below, in-sample. No graph for the actual observations versus the fitted values, as presented previously, is presented as this regression is a multiple linear regression and a plot such as that would not make much sense.

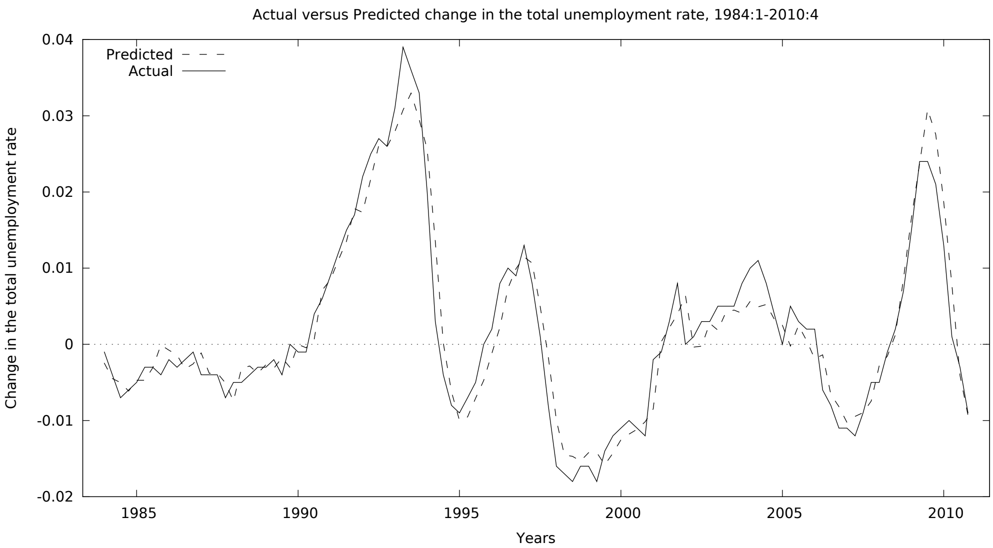

Figure 9: The in-sample predictions of the change in total unemployment rate using the dynamic version regression, 1984:1-2010:4.

6 Analysis

In the coming subsections, we will analyse and discuss the different parts of the estimated regressions of Okun's law, if Okun's coefficient is actually negative, as we think it is, and how good the three regressions are at explaining the variance in the change in total unemployment rate.

To start off, an analysis of the adjustments to the raw data, and how this might have affected the regression results is made. After that, we take a look at the *ordinary least squares* assumptions, which need to hold for the results to be valid. Next up we examine and answer the formal hypotheses stated in section 4, followed by a thorough analysis of the explanatory power of each regression and its economic significance.

6.1 The Raw Data

First off, as the raw data of the GDP is denominated in current prices, a GDP inflator (deflator) had to be used to inflate the aggregate output to the base year 2005. This computation is completely linear, but inflation is not constant over time, which introduces a statistical uncertainty that might make the standard errors in the regressions larger than they appear. In spite of this potential pitfall, this method of discounting value is very common among practitioners as well as scholars, and why this study chose to use it.

Secondly, the X12-ARIMA algorithm for smoothing out seasonal trends and calendar effects is *de facto* industry standard among almost every statistical bureau in the world, including Statistics Sweden. Therefore, it is believed that the results from using this methodology are are as good a of an approximation as any study of this kind could hope to achieve. There are, however, other known methods of adjusting raw data for seasonal trends, such as TRAMO/SEATS, but they mostly render equivalent results to X12-ARIMA.

Thirdly, the Hodrick-Prescott filter is the source of most of the uncertainty in the adjustments to the data in this study. In particular this is due to the fact that the filter is utilised to give an actual estimate of *potential* GDP in 2005 year's price level. As there are numerous ways of estimating *potential* output in an economy, not using the Hodrick-Prescott filter could render substantially different estimates. Previous studies of the Swedish economy (see further Gylfason, 1997) estimate potential GDP using a linear equation regressing with respect to time in years. We, on the other hand, believe that a non-constant change in *potential* GDP renders more believable results when they're based on actual data.

Finally, in addition to the concerns discussed in the previous paragraphs, the NAIRU poses a great load of potential trouble for the *gap* regression estimates. At first glance it isn't obvious why this might be, but consider this: we estimate a model by using estimates, rather than estimating a model using actually observed data. This means that the standard errors in the *gap* regression will be bigger than they are reported as by Gretl, just like using the Hodrick-Prescott filter. So, even before in-depth analysing the *gap* regression of Okun's law, we can be certain that it will be filled with uncertainties that make it unreliable.

6.2 The Least Squares Assumptions

In order to – with reasonable justification – discuss the economic implications of using Okun's law on the Swedish economy, the assumptions that follow from using *ordinary least squares* regression need to be analysed and assessed. This section is divided into each respective assumption and are then discussed and compared for the three estimated models adjacently.

6.2.1 The Residuals Have a Mean of Zero

This assumption is quite impossible to satisfy to its fullest theoretical extent in any empirical study, due to the fact that numerical computations will only approach zero, never reach it. Nonetheless, the tests for normality presented in the results for the three regressions, show (under the *central limit theorem*) that the conditional distribution of the residuals (error terms) given the regressors should be sufficiently close to a normal distribution with a mean of zero as the number of observations increase. A rule of thumb is that the number of observations should be over a hundred for the normality assumption to have any chance of holding.

It is evident from figures 11, 13 and 15 that the *growth* and *dynamic* version regressions' residuals are in effect zero. As for the residuals from the *gap* regression, they are clearly not normal, nor do they approach to having a mean of zero. Thus, the first assumption does not hold for the *gap* version. In effect, this means that any standard errors and results from the regression are unreliable at best and should be viewed with a high dose of skepticism.

The following assumptions will not take into account the *gap* version, as it not trustworthy.

6.2.2 $(X_{1,i}, X_{2,i}, \cdots, Y_i), \quad i = 1, \cdots, n$ are Independently and Identically Distributed

This assumption would automatically hold if the raw data was collected through standard random sampling. Unfortunately, when using economic data, random sampling is rarely an automatic feature of the data, and it is up to the economist to verify this. With this in mind, this study has collected the raw data from acknowledged sources, such as Eurostat, OECD and Statistics Sweden for all three estimated versions of Okun's law. All of these sources have extensive information about their respective methodologies for making certain that their data collection is in fact random. Therefore, we will not discuss this more in-depth, but rather assume that this assumption holds, as we lack the necessary tools to make a full assessment of this.

6.2.3 Large Outliers are Unlikely

Large outliers have the potential to yield misleading regression results, as one data point that is far beyond the rest of the observations can shift the intercept or change the gradient in the estimated models. This is mostly a problem when there is a lack of observations and each individual observation accounts for a relatively large part of the available information that is used to estimate the models. In this study, more than one hundred individual observations have been used for each of the three estimated versions of Okun's law, which means that one observation doesn't necessarily cause a large shift in the estimated coefficients. Therefore, this particular assumption should, to our standards, hold for all three regressions.

6.2.4 No Perfect Multicollinearity

This assumption is not as large of a problem as it used to be before the introduction of econometric software for estimating economic models, such as Okun's law. Perfect multicollinearity arises when two or more of the explaining variables have a correlation of one. This essentially means that the coefficients either implode or explode due to the computations of the standard errors involving incredibly small or large numbers. Gretl, the software package used for this study, has a built-in feature that warns the user when there is a risk of perfect multicollinearity present. As can be seen in all of the tables presented, Gretl has not found any reason to warn us of any multicollinearity.

6.3 Answering the Hypotheses

This part of the analysis will answer the formal hypotheses stated in section 4. For the *growth* and *gap* versions of Okun's law there are one formal hypothesis, although involving different coefficients. For the *dynamic* version there are three hypotheses of interest. For clarity, the hypotheses are re-stated below and answered accordingly using a 95% confidence level.

Formal hypothesis 1:

H_0: $\widehat{\beta_{\Delta Y_t}} \geq 0$. *The presumed linear relationship between unemployment and economic growth, as measured by the change in GDP, is non-negative.*

H_1 is then: $\widehat{\beta_{\Delta Y_t}} \geq 0$. *The relationship is negative.*

The *t*-statistic from the *growth* regression's $\beta_{\Delta Y_t}$ is -5.9264, which is to be compared to the critical value -1.983. As the hypothesis is one-sided, we conclude immediately that we can reject the null hypothesis in favour of the alternative.

Formal hypothesis 2:

H$_0$: $\hat{\beta} \geq 0$. *The relationship between the unemployment gap and the output gap is non-negative.*

H$_1$: $\hat{\beta} < 0$. *The relationship is negative.*

Again, looking at the computed *t*-statistic in table 5 we see that it is -3.9131, which is a long way from the critical value -1.982 (see table 2), and we reject the null in favour of the alternative. Do have in mind that this computation suffers from omitted variable bias, since the first linear regression assumption did not hold. Therefore, even if we reject the null based on the information we have, we cannot with great confidence say that this test provides any adequate evidence to support the alternative hypothesis.

Formal hypothesis 3:

H$_0$: $\widehat{\beta_{\Delta Y_t}} \geq 0$. *The relationship between the change in total unemployment rate and the change in GDP is non-negative.*

H$_1$: $\widehat{\beta_{\Delta Y_t}} < 0$. *The relationship is negative.*

The *t*-statistic for the first hypothesis concerning the *dynamic* version of Okun's law is on the equivalent coefficent to the one in formal hypothesis one (1), and is -4.7265. This is much farther to the left in the distribution, than is the critical value -1.983. Once again, we may reject the null in favour of the alternative.

Formal hypothesis 4:

H$_0$: $\widehat{\beta_{\Delta Y_{t-1}}} \geq 0$. *The relationship between the change in total unemployment rate and the change in the first order lag of GDP is non-negative.*

H$_1$: $\widehat{\beta_{\Delta Y_{t-1}}} < 0$. *The relationship is negative.*

As for the first order lag of the GDP regressor, the coefficient $\beta_{\Delta Y_{t-1}}$ has a *t*-statistic of -2.5169, which is *not* statistically significant at the 1% level, but *is* at the 5% level. We may reject the null in favour of the alternative at the 5% level based on the estimated standard errors.

Formal hypothesis 5:

H$_0$: $\widehat{\beta_{\Delta u_{total,t-1}}} \leq 0$. *The relationship between the change in total unemployment rate at time t and its first order lag is negative.*

H$_1$: $\widehat{\beta_{\Delta u_{total,t-1}}} > 0$. *The relationship is positive.*

Finally, the last formal hypothesis, and the only one that is principally different from the others, has a *t*-statistic of 14.4404. This is by far the largest *t*-statistic computed based on our sample size, and we can safely reject the null hypothesis in favour of the alternative, i.e., that the relationship between the change in total unemployment rate at time *t* is positively related to the previous time periods' change.

Summing up the answers to our formal hypotheses, we can see that our results are in line with the results from the other studies mentioned earlier in the study. Okun's coefficient is statistically significantly estimated to be *negative*, just as economic theory would have us believe.

6.4 The Regressions' Explanatory Power

As we mentioned when the results were presented, simply comparing the $\bar{R}^2$ between the three estimated versions of Okun's law will not suffice. This is, to repeat, due to the nature of the dependent variables that we are trying to fit our models to. The dependent variable $\Delta u_{total,t}$ is the same for the *growth* and *dynamic* versions, whereas $(u_{a,t} - u_{n,t})$ is the dependent variable for the *gap* version. This essentially means that a higher $\bar{R}^2$ for the *growth* regression, than for the *gap* one, doesn't necessarily mean that it is a better model, as they measure different things. Hence, we need to include economic judgement and reasoning in order to create a proper comparison between the three.

The *gap* regression suffers from many more sources of potential errors than the other two regressions do, and has already proven to be unreliable through the first linear regression assumption. Moreover, the *gap* regression has a relatively low $\bar{R}^2$ (not comparing to the other regressions' equivalent), which means that this regression explains only about one fifth of the variance in the unemployment gap over the sampled time period. Thus, nearly 80 per cent of the variance in the unemployment gap cannot be explained by the percentage difference between *potential* and *actual* GDP. The Durbin-Watson test statistic that was mentioned in the results (found in table 5) has a value of about 0.1576, compared to 0.5365 and 4.7303 for the *growth* and *dynamic* regressions respectively, indicating a strong presence of autocorrelation. Even though heteroskedasticity and autocorrelation robust methods for computing the standard errors were used, this affects the distribution of the error terms over the sampled time period.

Another problem with the *gap* regression, which we touched on briefly earlier, is the fact that we are using estimates in our dependent variable $(u_{a,t} - u_{n,t})$, not just for the regressor. This introduces uncertainty in the data that we're assuming is accurate, when in fact it is not. For these reasons, the *gap* version of Okun's law is highly unreliable and lacks any significant explanatory power. This uncertainty is further confirmed by the Doornik & Hansen (2008) χ^2-test we discussed previously. Essentially, this means that any economic significance from the *gap* regression will be difficult to assert, as we have concluded that the estimated coefficient's properties are biased.

It is easier to directly compare the *growth* regression with the *dynamic*, as they have the same dependent variable. The $\bar{R}^2$ increased by roughly a factor two (2) when we included the first order time lags of the year on year change in real GDP and change in total unemployment rate. Interestingly – and briefly mentioned in the results – the coefficient on the regressor ΔY_t drops from -0.300308 to -0.0849232, or -73 per cent. This is a remarkable result, as it effectively means that a one unit change in ΔY_t contributes to the change in the total unemployment rate considerably much less when controlling for the first order time lags. Looking at the coefficient on the $\Delta u_{total,t-1}$ also reveals an interesting observation, namely that the previous period's change in unemployment rate causes the majority of the change in this time period. This is also one of the main strengths of using ratios in the regressors, as they are unit less in the sense that we can directly compare how "powerful" the estimated coefficients are relative to each other. In addition, the $\bar{R}^2$ is approximately 0.9256 for the *dynamic* regression provides evidence that the *dynamic* model is better than the *growth* model at predicting the change in unemployment rate over the sampled time period.

The Durbin-Watson statistic is the most significant for the *dynamic* regression, which can be interpreted as the functional specification being good enough to filter out the autocorrelation as we're controlling for more than just the change in the GDP.

As for the economic significance of the *growth* regression, Okun's coefficient has a high economic significance, since the change in unemployment will be -0.30 percentage points for a one percentage point change in GDP. It doesn't look like much, but bear in mind that these percentages are related to a population size in the millions, which means that even a small change in unemployment or GDP growth will have a large economic effect in absolute monetary terms, even if estimated how large this effect will be, is difficult. Comparing this with the *dynamic* regression's economic significance, leads us to believe that it diminishes quite substantially, since Okun's coefficient is now only about one fourth as large.

7 Conclusion

Finally, we are ready to discuss whether or not our research meets the internal and external validity criteria that econometricians usually make use of when scrutinising studies of this kind Stock & Watson (see 2006, p. 327).

Without further ado, the differences in labour market regulations and policies across countries that give rise to different levels of labour movement rigidity, might have a strong impact on the relationship Okun originally proposed to the Kennedy Administration back in the 1960's. The U.S. labour market is not as rigid as the Swedish one, and including the fact that the U.S. utilizes minimum wages makes it easier for employers to "hire and fire" as demand for goods and services follow the business cycles. In Sweden, according to our estimates, unemployment could be viewed as lagging behind the business cycle, measured through economic growth (production side of GDP), and this could perhaps partly be explained by the presence of labour market rigidity. Furthermore, Sweden is not as labour intensive in its industries, as is the U.S., which could also provide a basis to why external validity in Okun's relationship might not hold.

There is also the problem of simultaneous causality between unemployment and economic growth. Okun's law is a statistical relationship, which disregards this issue, but when performing a regression with either the unemployment, or the GDP, as the dependent variable makes all the difference in the world. It is not a preposterous assumption to think that these both do affect each other and thus give rise to shocks between the observations. In other words, there are two equations that explain variances in each other. A change in GDP causes a change in the unemployment rate, which in turn causes a change in GDP, and so on and so forth. We cannot know for certainty how large this effect might be, but if it is present, we will either need to remove these shocks by introducing an instrumental variable, or pray that it is sufficiently small to not create any statistically significant impact on our results.

Apart from the problems with simultaneous causality bias, the computational errors in computing the aggregate output, as well as the unemployment rate, in an economy can introduce sample selection bias. It is assumed that Eurostat have taken every precaution to make sure that its estimate of the Swedish unemployment rate is randomly sampled, and that there are no miss-measured data points. For the observations of GDP, as provided by Statistics Sweden, there will always be a statistical discrepancy that could be big enough to create an economic growth from one quarter to the next where there in fact did not occur. As far as our sampled data goes, this is probably one of the major problems that we have no useful means of controlling for.

To summarise, and tie the analysis back to the purpose of this study, which is to assess whether Okun's law may be applied to the Swedish economy, we can state the following: *Okun's original relationship* <u>*cannot*</u> *be directly applied to the Swedish economy for the purpose of forecasting without taking into account labour market rigidity or removing the effects of simultaneous causality bias.* This is an issue that, to our knowledge, every study relating to Okun's law suffers from, but hardly any discuss. Roughly speaking and disregarding this issue, our study is as valid as the ones written by Gylfason (1997); Knotek (2007); Stock & Vogler-Ludwig (2010) and Prachowny (1993).

As a purely statistical relationship, however, we can use our estimates as a *rule of thumb* that together with other macroeconomic policy tools can help policy makers to make more informed decisions. If used in isolation, our estimates of Okun's law – as well as anyone else's – to predict changes in unemployment will almost certainly lead to precarious conclusions that *at best* are in the ball park, but definitely not accurate.

This study has not taken to any great measures to assess the actual impact of labour market rigidity when estimating Okun's coefficient, as Gylfason (1997) has tried to do, although his conclusions are not conclusive. Therefore, we suggest that any further research relating to Okun's law and the Swedish economy should try to incorporate the effects of labour market rigidity more extensively in order to increase the interpretative power of Okun's coefficient.

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Appendix

A X12-ARIMA

X12-ARIMA is a software (read: series of algorithms) developed by the U.S. Census Bureau that adjusts raw data for seasonal trends and calendar effects. ARIMA stands for *AutoRegressive Integrated Moving Average*, which refers to a "...a versatile family of models for modeling and forecasting time series data." [14]

To this study's authors' knowledge, this software is the industry standard among statistics bureaus around the world and is continually revised by the U.S. Census Bureau to ensure that the data adjustments are on par with existing research.

B The Hodrick-Prescott Filter

As first introduced for use within the field of econometrics, Nobel laureate Edward C. Prescott and economist Robert J. Hodrick proposed a mathematical decomposition of time series to separate cyclical components from raw data (Hodrick & Prescott, 1997). Applying the filter, according to Hodrick & Prescott (1997), yields a smoothed representation of the original time series that is now more sensitive to long-term fluctuations, rather than short term ones. Equation 7 below illustrates how the decomposition is done:

$$\min_{\tau_t} = \sum_{t=1}^{T} (S_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \left((\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})\right)^2, \tag{7}$$

where $\{S_t\}_{t=1}^{T}$ is the natural logarithm of the time series to be decomposed, and τ_t is the trend component such that $S_t = \tau_t + c_t$ for some cyclical component c_t. $\lambda > 0$ is a multiplier modifier that defines the trend component's sensitivity to short-term fluctuations, and for quarterly data a value of 1600 is commonly used (Ravn & Uhlig, 2002).

Looking at equation 7, it is apparent that there is a specific trend component τ_t that will minimize the expression, as the first term is simply the sum of squared deviations that will penalise c_t. The second term in the equation consists of the sum of squares of differences between the trend component's first and second order lags. Multiplying this with λ will weaken the variations in the growth rates between time periods, and as one can see, the higher the value of λ, the more weakening. Finally, also note that using the Hodrick-Prescott filter, given equation 7 above assumes that the time series being decomposed does *not* contain any seasonality.

C Connecting the Gap and Growth Versions of Okun's Law

Starting with the gap version, equation 2 can be re-written in the following way to arrive at the growth version of Okun's law:

$$\begin{aligned}
(u_a - u_n) = \quad & \beta \cdot \frac{Y_p - Y_a}{Y_a} = \beta \cdot \left(\frac{Y_p}{Y_a} - 1\right) \Rightarrow \text{(take annual differences)} \\
\Rightarrow \quad & \beta \cdot \Delta\left(\frac{Y_p}{Y_a}\right) = \beta \cdot \left(\frac{Y_p + \Delta Y_p}{Y_a + \Delta Y_a} - \frac{Y_p}{Y_a}\right) \\
\approx \quad & \beta \cdot \left(\frac{Y_a \Delta Y_p - Y_p \Delta Y_a}{Y_a Y_p}\right) = \beta \cdot \left(\frac{\Delta Y_p}{Y_p} - \frac{\Delta Y_a}{Y_a}\right) \\
\approx \quad & \Delta u_a - \underbrace{\Delta u_n}_{\text{Assume that } u_n \text{ constant}} \Rightarrow \Delta u_a \approx \underbrace{\beta \cdot \frac{\Delta Y_p}{Y_p}}_{=\beta_0 \text{ from equation 1}} + \beta_1 \cdot \frac{\Delta Y_a}{Y_a}
\end{aligned}$$

[14]See further: `http://www.census.gov/srd/www/x12a/glossary.html`.

D Regression Results, Growth Version

Table 4: Growth version model: OLS, using observations 1984:1–2010:4 ($T = 108$)

Dependent variable: $\Delta\widehat{u_{total,t}}$
HAC standard errors, bandwidth 3 (Bartlett kernel)

	Coefficient	Std. Error	t-ratio	p-value
$\widehat{\beta_0}$	0.00865471	0.00210206	4.1173	0.0001
$\widehat{\beta_{\Delta Y_t}}$	−0.300308	0.0506728	−5.9264	0.0000

Mean dependent var	0.001722	S.D. dependent var	0.011951
Sum squared resid	0.008163	S.E. of regression	0.008776
R^2	0.465893	Adjusted R^2	0.460854
$F(1, 106)$	35.12246	P-value(F)	3.90e–08
Log-likelihood	359.2287	Akaike criterion	−714.4574
Schwarz criterion	−709.0931	Hannan–Quinn	−712.2824
$\hat{\rho}$	0.723593	Durbin–Watson	0.536522

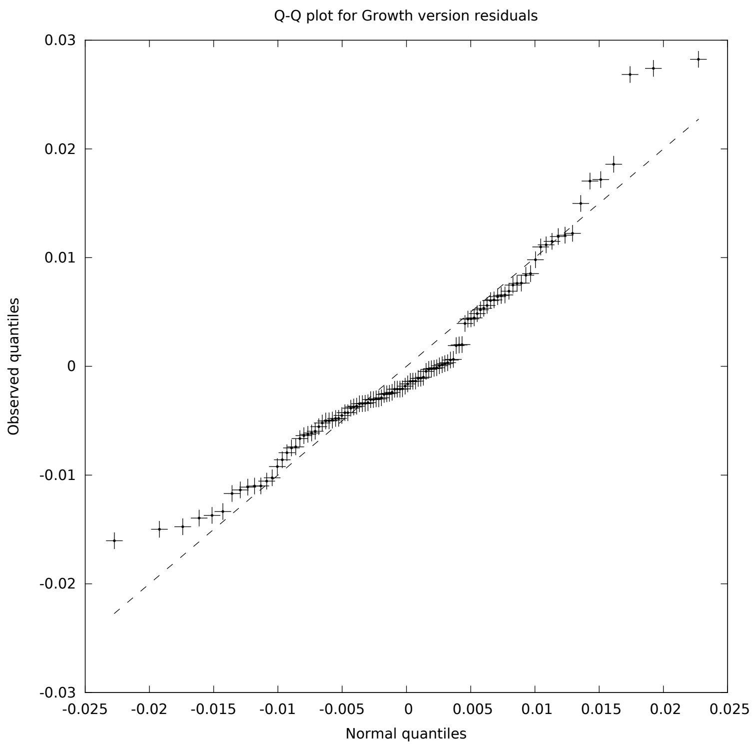

Figure 10: Quantile-quantile plot of the residuals for the growth version regression of Okun's law, 1984:1-2010:4.

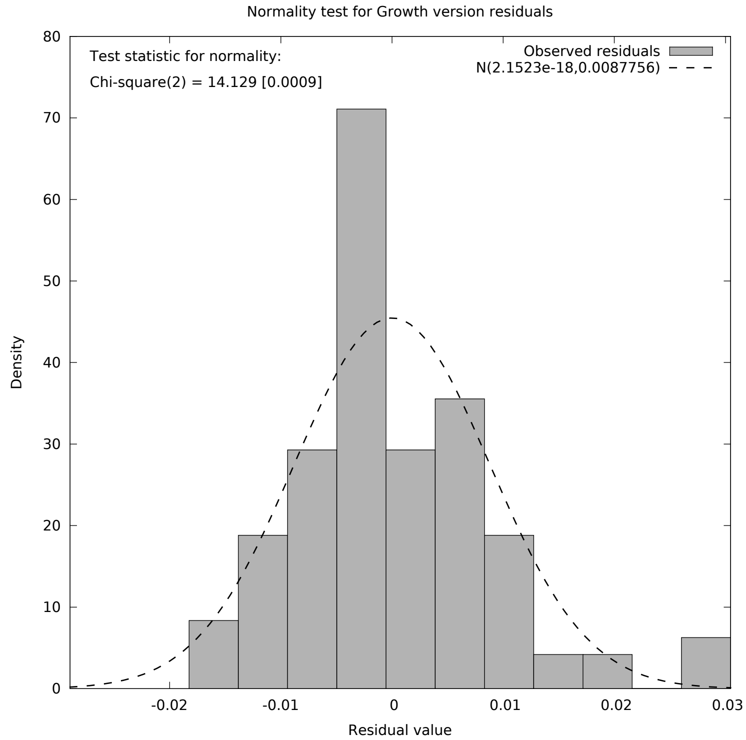

Figure 11: A Doornik & Hansen (2008) χ^2-test for normality of the residuals in the growth version regression, 1984:1-2010:4.

E Regression Results, Gap Version

Table 5: Gap version model: OLS, using observations 1984:1–2010:4 (T = 108).

Dependent variable: $(u_{total,t} - u_{n,t})$
HAC standard errors, bandwidth 3 (Bartlett kernel)

	Coefficient	Std. Error	t-ratio	p-value
$\widehat{\beta_{\frac{Y_{p,t}-Y_{a,t}}{Y_{a,t}}}}$	−0.432399	0.110499	−3.9131	0.0002

Mean dependent var	−0.005770	S.D. dependent var	0.015577
Sum squared resid	0.023297	S.E. of regression	0.014756
R^2	0.211801	Adjusted R^2	0.211801
$F(1, 107)$	15.31264	P-value(F)	0.000160
Log-likelihood	302.5987	Akaike criterion	−603.1974
Schwarz criterion	−600.5153	Hannan–Quinn	−602.1099
$\hat{\rho}$	0.927553	Durbin–Watson	0.157584

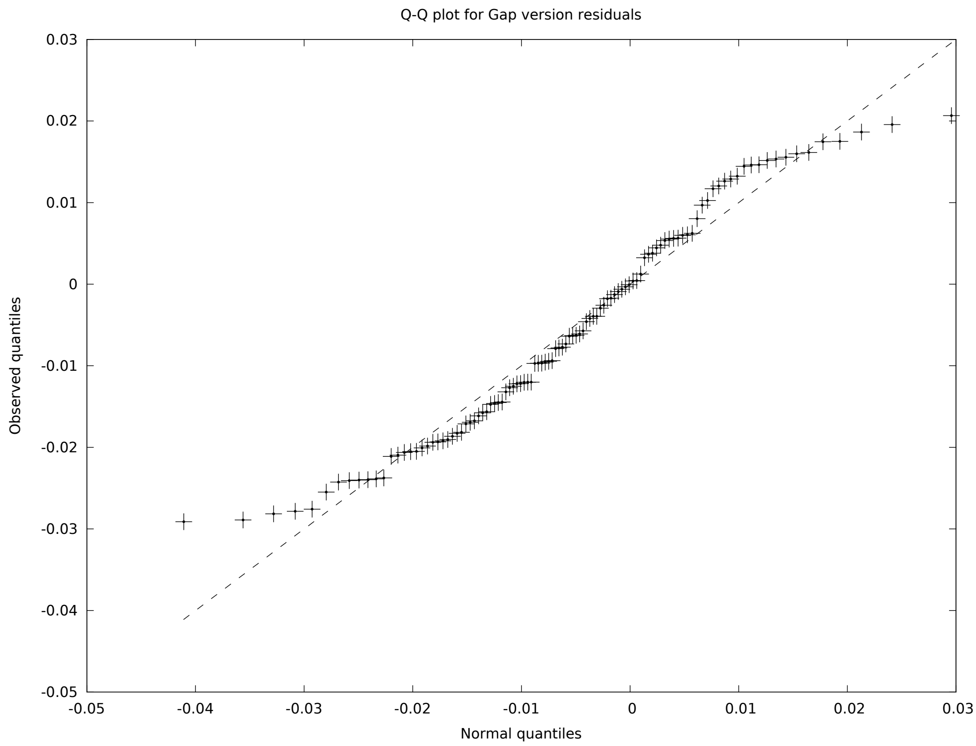

Figure 12: Quantile-quantile plot of the residuals for the gap version regression of Okun's law, 1984:1-2010:4.

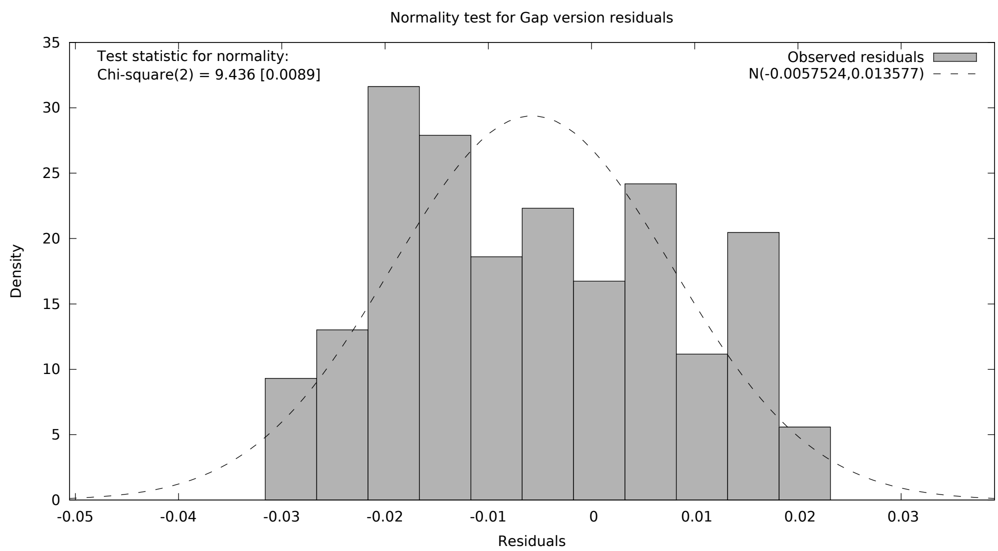

Figure 13: A Doornik & Hansen (2008) χ^2-test for normality of the residuals in the gap version regression, 1984:1-2010:4.

F Regression Results, Dynamic Version

Table 6: Dynamic version model: OLS, using observations 1984:1–2010:4 ($T = 108$)

Dependent variable: $\Delta u_{total,t}$
HAC standard errors, bandwidth 3 (Bartlett kernel)

	Coefficient	Std. Error	t-ratio	p-value
$\widehat{\beta_0}$	0.00372030	0.000697472	5.3340	0.0000
$\widehat{\beta_{\Delta Y_t}}$	−0.0849232	0.0179673	−4.7265	0.0000
$\widehat{\beta_{\Delta Y_{t-1}}}$	−0.0615367	0.0244492	−2.5169	0.0134
$\widehat{\beta_{\Delta u_{total,t-1}}}$	0.740933	0.0513099	14.4404	0.0000

Mean dependent var	0.001722	S.D. dependent var	0.011951
Sum squared resid	0.001105	S.E. of regression	0.003260
R^2	0.927679	Adjusted R^2	0.925593
$F(3, 104)$	211.9440	P-value(F)	3.73e–44
Log-likelihood	467.2011	Akaike criterion	−926.4022
Schwarz criterion	−915.6737	Hannan–Quinn	−922.0522
$\hat{\rho}$	0.387567	Durbin's h	4.730264

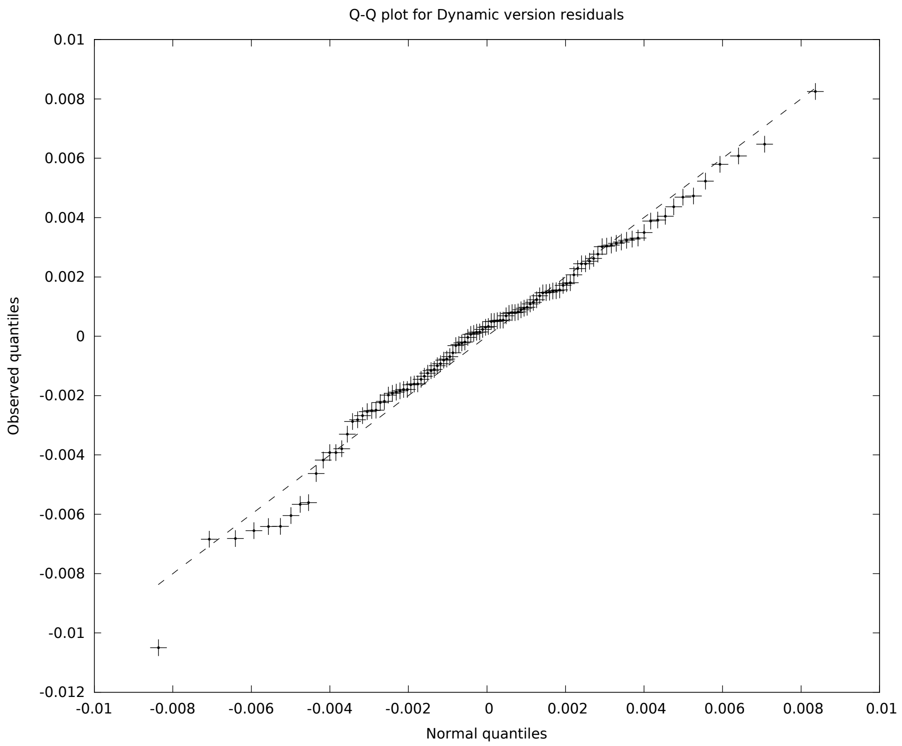

Figure 14: Quantile-quantile plot of the residuals for the dynamic version regression of Okun's law, 1984:1-2010:4.

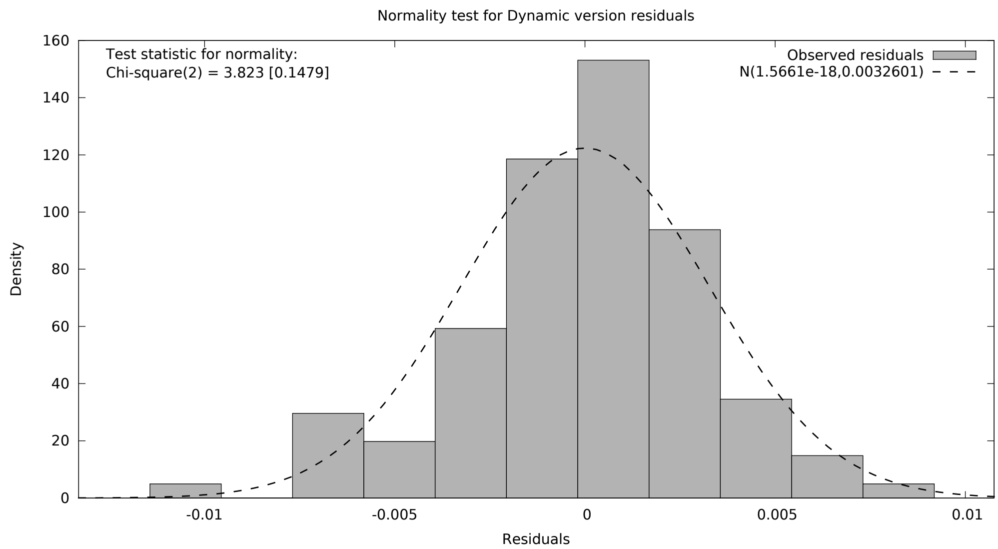

Figure 15: A Doornik & Hansen (2008) χ^2-test for normality of the residuals in the dynamic version regression, 1984:1-2010:4.