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Modelling and Pricing Contingent Convertibles

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Abstract

Recent years financial turbulence has energized implementation of comprehensive regulatory standards on bank capital adequacy. Regulators demand more capital with loss absorbance properties and the Contingent Convertible bond, CoCo, has become an increasingly popular way to gather such capital. A CoCo automatically and mandatorily converts to loss absorbing regulatory capital before the point of non-viability and hence instantly improves the capital structure of the distressed bank. To date, only a few CoCos have been issued but we expect the market to grow rapidly with new and tighter regulation of bank capital adequacy. This thesis examines how pricing of such blended debt and equity products can be modelled. We examine existing methods, develop them further to improve the estimation precision and present a real world implementation on Swedbank's potentially upcoming issue of CoCos. The application also includes sensitivity analysis to further understand the dynamics of the different methodologies.

Key words: Contingent Convertible Bonds, Contingent Convertible Capital, CoCos, Pricing, Credit Derivatives, Equity Derivatives, Credit Default Swaps (CDS), Bond Pricing

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1 Introduction

1.1 Background

One of the most efficient drivers of new economic legislation is the occurrence of financial turmoil. The recent years' events are no exceptions and have energized the implementation of comprehensive regulatory standards on bank capital adequacy, mostly through the Basel III accord. In fact, the very reason for the founding of the Basel Committee for Banking Supervision was a bankruptcy of a large German bank in 1974, which led to large economic aftermaths for several international institutions as the bank's payments were ceased.

Following the financial turbulence beginning in 2008, supervising authorities around the world realized the current capital adequacy requirements might not be sufficient enough to keep the banking and financial sector resilient against such events. To strengthen the requirements regulators are advocating an increase of capital that can be used to absorb losses (BIS, 2009a). Common equity and retained earnings are clear examples of such capital and is together often referred to as Core Tier 1 Capital. Accordingly, it is the equityholders of struggling firms that take the hit under financial distress while debtholders sail through almost unharmed. Given the firm does not file for bankruptcy, debtholders will receive payments and even if default is realized, partial redemption of their investment is often possible. There exist forms of capital where holders of debt can convert their capital into equity under predetermined conditions, such as convertible bonds. While this type of hybrid capital under some circumstances can be included, in addition to the equity, when measuring a bank's capital adequacy, it is hard to imagine an investor who voluntary would convert his/her relatively "safe" debt into risky equity when the firm is under distress. Hybrid capital are securities that inhabits certain properties of debt but are subordinated standard debt and therefore resembles equity in terms of claims rights. An important distinction from equity and one of the main reasons for its popularity in recent years is that the payments to investors holding hybrid capital are treated as interest payments and are thus tax deductible (IFC, 2008).

Regulators are now demanding more loss absorbency within hybrid capital and they are further starting to strengthen laws governing capital requirement and solvency ratios. Capital with loss absorbing properties is simply defined as money or investments that can be used to cover sudden unexpected losses. The ambition is to make the holders of hybrid capital contribute more in the recovery phase and thus spreading the cost of financial distress to more claimants than the equityholders. One of the latest amendments to Basel III (January 2011) stipulate that only hybrid capital with full loss absorbency should be counted as regulatory capital.[1]

[1] Deutsche Bank Research May 23, 2011

In 2002, Flannery introduced a new type of bond, which he called a "Reverse Convertible Debenture (RCD) that would automatically and mandatorily convert into equity when a company's market capital ratio falls under a predetermined level. He argues that the trade-off within capital structure decision would be maximized using this new instrument. It would inhabit the positive tax advantages of debt while also act as a buffer in financially distressed situations. Even though equity shares will be diluted if conversion takes place, companies near insolvency usually turn to shareholders for more money through rights offers, which will dilute their investments anyway. This new financial instrument did not receive much attention at the time, but got a revival following the financial crisis.

In the wake of the new regulatory changes that has been announced, a demand for new types of hybrid capital has emerged. The Basel Committee for Banking Supervision has, after all the bailouts since 2008, realized the old hybrid capital did not ensure enough capital support in bad times. On January 13th last year, they issued a press release, adding new paragraphs to the Basel III accords. The amendment stated new capital requirements to *"ensure that all classes of (regulatory) capital instruments fully absorb losses at the point of non-viability before taxpayers are exposed to loss"* (BIS, 2011). This amendment to the Basel III capital rules of December 2010 was only targeted at internationally active banks. Furthermore, they also state that all non-common equity capital of Tier 1 or Tier 2 type must have a provision that requires this capital to convert into equity or suffer write-downs if the bank approaches default. A more detailed definition of regulatory capital can be found in Section 1.5. The Basel Committee further mark that relevant authorities in each country will be in charge of deciding how suitable initiation events of such conversions should be designed.

1.2 Contingent Convertible Bonds

Clearly, financial supervisors around the world have now turned their eyes towards the forgotten instrument Flannery (2002) suggested and regulators are now very positive towards it and advocate its implementation. The new regulations through Basel III resulted in the creation of a new class of convertible bonds; the Contingent Convertible bond or CoCo, which in its basics is exactly what Flannery (2002) introduced. The CoCo is a form of convertible, but unlike the standard convertible bond, this new version automatically converts into equity or suffers a write-down when a set of conditions is met, often called "trigger events". Certain parts of a firms' outstanding debt will thereby automatically transform into equity when the risk of insolvency is imminent and the CoCo will serve as a buffer, helping the equity to absorb losses.

The contingent convertible is a subordinated debt instrument with ordinary coupon payments and if the trigger is not set off before the maturity date, the CoCo will mature as a standard bond and redeemed as usual. Forcing investors to convert their bonds into equity when the trigger threshold is violated, will serve as an equity injection right at the moment when the firm is failing to meet its minimum capital requirements.

Financial institutions around the globe have met massive criticism from both regulators but also from domestic and foreign civil citizens. Regulators and governments have been forced to bail out banks in order to prevent unstoppable systemic crises. These bailouts have cost the governments, and i.e. the taxpayers, enormous amounts and several financial institutions have undoubtedly taken on too much risk without protection. The financial industry should be enough capitalized to withstand these events and therefore regulators are promoting the use of contingent capital.

On June 6 2012, the European Commission proposed new and tighter EU-wide recovery and resolution rules to prevent massive state-aid programs in the future (EU Commission, 2012). The framework comprise of 3 resolutions where one directly relates to contingent capital. The governmental bail-out programs should be minimized and be replaced by bail-ins, where the banks recapitalize themselves. All banks must have a minimum percentage of their liabilities structured such that they are eligible as bail-in tools. Bail-ins are formed so that they are written-off or converted into shares once the bank reaches non-viability. Further, the bail-ins must also have a clear trigger point. The Commissioner Michel Barnier commented in the press release: *"The financial crisis has cost taxpayers a lot of money (. . .) We must equip public authorities so that they can deal adequately with future bank crises. Otherwise citizens will once again be left to pay the bill, while the rescued banks continue as before knowing that they will be bailed out again."* (EU Commission, 2012). The EU Commission has thus taken a distinct stand on how to proceed from the events of 2008-2009. Contingent capital, irrespective of being called CoCos or bail-ins, will clearly be an important and compulsory part of the future.

1.3 Previous studies

As mentioned earlier, Flannery (2002) introduced the concept of CoCos, i.e. convertibles that mandatorially convert into equity when the point of non-viability is imminent. The instrument did not get much attention at the time but has recently gained considerably in popularity after the financial crisis. Due to the rise in attention, more academics have turn their research towards the question of how to calculate the value of a CoCo. In 2010 Pennacchi developed a structural credit risk model to price such instruments under the assumption that the return on a bank's assets follows a jump-diffusion process. Pennacchi (2010) argues that letting a bank's assets have the possibility to "jump" better captures the effect of a financial crisis. He concludes that contingent capital would be a feasible low-cost method of mitigating financial risk if the trigger is set high enough. A structural model for pricing CoCos is also presented in Albul et al. (2010) where he studies the model together with capital structure decisions. Glasserman and Nouri

(2010) analyze how CoCos with a Capital Ratio trigger with partial and on-going conversion can be priced. They model the CoCo such that just enough debt is converted into equity so that minimum capital requirement is fulfilled every time the threshold is breached. Glasserman and Nouri (2010) derive closed-form expressions to value such instruments when firms' assets follow a geometric Brownian motion. Corcuera et al. (2010) price CoCos under the assumption that the underlying risky asset dynamics are given by an exponential Lèvy process which allows for jumps (as in Pennacchi (2010)), but also heavy tails. In 2011, Hilscher and Raviv studied the effect of introducing CoCos on the balance sheet and find that it can, if designed properly, reduce the probability of default. They model the dynamics and value of CoCos by expanding the well-known Merton (1974) model to work with this new debt instrument. We throughly asses this model further in Section 4.3 where we also suggest improvements. In Madan and Schoutens (2011) the authors introduced a fundamental model where CoCos are priced using conic finance techniques. The authors argue that the trigger event should not be designed around an accounting measure, since it does not take into account risky liabilities on the balance sheet. Spiegeleer and Schoutens (2011) developed two methods of pricing contingent convertibles, both using existing derivatives. The first is based on equity derivatives and a standard bond whilst the other prices a necessary yield spread on the CoCo using the dynamics of credit default swaps. We carefully examine both these methods in Section 4.1 and 4.2.

1.4 Outline of this thesis

In the wake of the new regulatory standards proposed by EU and the Basel Committee, an investigation and analysis on how to model the value of CoCos is of great interest for both investors and regulatory authortities. Hence, we are in this paper we are going to study CoCos in detail. Our aim is to gain deeper knowledge about the pros and cons as well as an understanding on how to price these instruments. We will go through three models on how to price CoCos thoroughly and also investigate the sensitivity of the chosen pricing frameworks. The models will be discussed and analyzed and we also propose several potential modification of the models as we believe they can be improved. Furthermore, in Mars 2012 one of the largest banks in Sweden, Swedbank, declared its interest in issuing CoCos (Swedbank, 2012). In Section 5.4 we will therefore investigate how such a hypothetical CoCo issued by Swedbank could be constructed and priced. We limit our study to focus on CoCos which convert into equity and hence leave modelling of *write-down* CoCos for further studies. The rest of our thesis is structured as follows, in Section 2, we go through the building parts of CoCos and in Section 3 we present and describe the structure of some of the CoCos issued so far. Futhermore, in Section 4 we carefully study the three chosen pricing frameworks and the modifications to increase accuracy will be presented. In Section 5 the sensitivity analysis will be conducted and finally in the last two Sections 5.5 and 6, we summarize and conclude our work. To begin with, we will in Section 1.5 give a brief introduction the contents of the banking supervision laws in the Basel Accords.

1.5 Regulatory Capital in the Basel Accords

Many references will be made to the Basel Accords throughout this thesis as why a short summery of the content will be presented in this section. Here we closely follow Part 1, Section I in BIS (2010). When the first regulatory financial adequacy requirements on the banking sector were introduced in 1988 the calculations to find capital ratios was relatively easy and straightforward. As time went by, the accords evolved along with the calculation principle and today, with the third accords - Basel III - the number of calculations needed to compute the regulatory financial required levels of core capital has risen to millions (Haldane, 2011). The computations are now executed by sophisticated software on computers and "normal" investors will not be able to perform them on a piece of paper. The goal with this is, according to the Basel Committee, to seek greater risk sensitivity. Due to the complexity, we will not attempt to explain them deeply but give a brief comment on its elements and structure. Total regulatory capital consists of the sum of:

1. Core Tier 1 Capital (going-concern capital) which consists of:
    - Common Equity Tier 1
    - Additional Tier 1
2. Tier 2 Capital (gone-concern capital)

Common Equity Tier 1 will consist of common equity, retained earnings and, if present, fully paid non-cumulative perpetual preferred stock. Furthermore, subordinated, perpetual and unsecured capital can also be included in Additional Tier 1. In Tier 2 Capital (supplementary capital) a financial institution can include for example undisclosed reserves, revaluation reserved, general provisions, subordinated term debt and hybrid debt capital instruments, i.e. CoCos, but standard convertibles are no longer allowed to be included. A last key component in measuring capital adequacy as depicted by the Basel Accords is the measure of Risk-weighted assets (RWA). The RWA measure weighs the assets by their riskiness and an asset gets less weight the more secure it is. For example, an AAA sovereign debt will have weight of 0% whereas a below B-rated bond will count as 150% of the book value. Risk-weighted assets is thus a measure of how much uncertainty of future payments that the asset base contains.

Making use of the above definitions, the capital ratios can be calculated and the current requirements for financial institutions is as follows:

- Common Equity Tier 1 Capital must be at least 4.5% of RWA
- Core Tier 1 Capital must be at least 6.0% of RWA
- Total regulatory capital must be at least 8.0% of RWA

These are the Basel III requirements in brief, which all international banks must at least satisfy. But on top of this, national agencies are allowed to set other requirements as long as they are at least the size of what Basel III stipulate. For a more detailed report on the Basel III accords and its components see BIS (2010) with amendments in January 2011.

2 The Contingent Capital framework

In the following section we will describe and analyze the structure and components that underlie a CoCo. The two main properties that will follow are the importance of well-defined trigger events and the different aspects regarding the conversion. In Section 2.1 we describe what the trigger is and how it can be defined. Further, Section 2.2 describes how the conversion mechanism works and the relationship between held CoCos and received equity.

2.1 The trigger

The trigger event must be defined so that it is clear for the buyer under what circumstances the automatic conversion will occur. Otherwise it would be difficult for a potential investor to assess a sound price and the underlying risk in his/her investment. There are generally four different types of triggers. In Subsection 2.1.1 we examine triggers with an accounting-based condition. Moreover, in Subsection 2.1.2 and 2.1.3 we study market and regulatory-based triggers, while in Subsection 2.1.4 scrutinize multi-variate triggers.

2.1.1 Accounting trigger

An accounting trigger is when the conversion is triggered by an accounting measure, e.g. Core Tier 1 Capital ratio or another solvency ratio. In this case the conversion is executed when the issuing company's capital ratio breaches a certain level, say for example 7%. Even though CoCos are a relatively new breed of hybrid capital, several scholars have studied which trigger that is best suited to protect individual companies but also the financial industry as a whole. In IMF (2011) we find examples of benefits from the accounting trigger such as easy to understand, relatively easy to price and that all figures are fully disclosed. However, there are several complications with using an accounting-based trigger. Since accounting reports generally are, at most, published publicly quarterly, the conversion will occur with a lagged effect and will thus not reflect the current status of the issuing firm. When conversion occurs it may already be to late. For example, Bear Stearns, Lehmann Brothers, Wachovia and Merrill Lynch all reported capital ratios above 8% at the break of the financial crisis in 2008 (Spiegeleer and Schoutens, 2011). Another concern is that using book values might incentivize manipulation by CoCo issuers to either force conversion to improve their company's funding or avoid conversion to prevent influential shareholders from getting their shares diluted (Bloomberg, 2009).

Furthermore, as Haldane (2011) argues, using today's calculation principle for the regulatory capital ratio, it would be close to impossible for investors to by their own calculate the current regulatory capital ratio. Haldane (2011) is openly critical against accounting triggers and brings forward a simple but very to-the-point survey. He collected a sample of 33 large international banks during the crisis in 2008. The panel was partitioned into two sub-panels; crisis banks that were subject to governmental intervention and non-crisis banks free from any form of intervention.

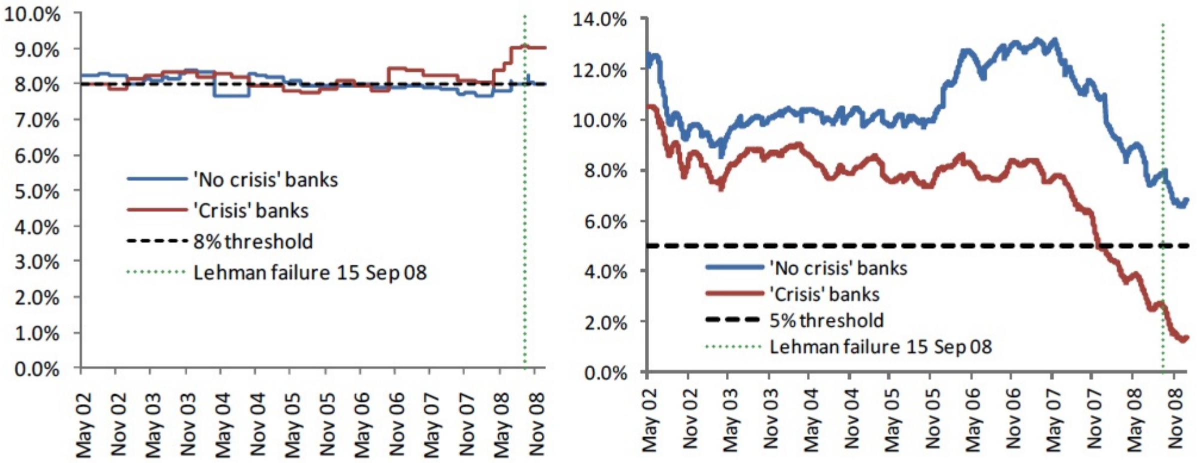

Figure 1: The Core Tier 1 ratio (left) and market capitalization-to-book value of assets (right) for 33 international banks during 2002 to 2008, Haldane (2011).

Figure 1 displays the result of the study in Haldane (2011). In the left subplot of Figure 1, the banks' Core Tier 1 Capital ratio is shown. The difference between the two panels is almost indistinguishable and the non-crisis banks even had lower capital ratios at the break of the crisis. In the right subplot of Figure 1, market capitalization-to-book value of assets (market value of the company divided by book value of assets) is presented. Using the latter measure, banks that were subject to governmental help showed evidence of distress almost a year before Lehman's bankruptcy. Haldane (2011) concludes that using an accounting trigger might not serve to reinforce distressed banks but instead lead to triggering CoCos in banks that does not need reinforcing and thereby "wasting" the protection. The author argues that as signaling tool, a market-based trigger is preferable over an accounting-based.

2.1.2 Market trigger

When the conversion is triggered by an underlying observed market factor it is said to have a market-based trigger. The conversion will in this situation occur when for example the issuing firm's share price falls below a predetermined threshold level or if a credit default swap (CDS) spread on the company rises above a certain level. CDSs will be explained carefully in Subsection 4.1.2 but principally it is an insurance against default that a debt investor, or a bare speculator, can buy. Many academics advocate the use of a market trigger (see for example Flannery (2005), Haldane (2011) or Calomiris and Herring (2011)) since it provides clarity and transparency for the buyer of a CoCo. It also reflects the current financial situation of the issuing firm in a superior way over an accounting trigger. It could in this way better help to prevent the need for government interventions for "too big to fail" entity in distress.

There is however a concern about market manipulation associated with market-based triggers. For example, if conversion is initiated by a stock price falling below a threshold level, a hostile investor, holding large quantities of CoCos, can force a conversion by (short) selling a large amount of shares on a day with small trading volumes. The debtholdings will then convert to equity and the investor will gain increased control in the company. This manipulation will have higher probability of succeeding if the share price already is close to the conversion point. In situations like this it would not be the actually financial status of the issuing firm that caused the conversion but instead other market forces and this would thus contradict the main purpose of CoCos; financially stable institutions. This problem can to some extent be avoided by letting the trigger point consist of a moving average price over a set of trading days (Flannery, 2005). Market indicators as trigger could also be in the form of CDS spreads if applicable. The CDS spread also inhabits the advantage that it is forward looking rather than a snapshot at a single time. But one disadvantage could be that prices for CDSs are less transparent than equity prices in the sense that CDS spreads are harder to find than equity prices for a non-professional investor.

2.1.3 Regulatory trigger

With a regulatory trigger, a governmental authority, e.g., a Financial Supervision Authority (FSA), would initiate the conversion. The FSA would act when they believe the distress of a certain institution could affect the whole economy substantially. A regulatory trigger would eliminate the issue of lagged data associated with accounting triggers along with the problem of market manipulation related to market triggers. In return it puts a heavy burden on the assigned monitoring agency. It would demand a great deal of judgement and discretion from that agency. The fact that the decision of conversion is put in the hands of an authority might be an uncertainty factor for investors of CoCos. It could be complicated to quantify the probability of conversion when the decision lies solely in the hands of the financial authority. Valuation of CoCos with regulatory trigger could thereby also be a problem.

2.1.4 Multi-Variate triggers

For a conversion of a CoCo with a multi-variate trigger to occur several trigger events must be fulfilled. A multi-variate trigger could for example consist of one trigger that indicates the situation in the economy on a macro level and one trigger that reflects the status of the issuing firm. For the macro trigger to be realized, the regulatory authority would need to step in and declare "a state of emergency" (French et al., 2005). This would once again put a great burden in the hands of the regulators. The micro trigger could consist of either an accounting trigger or a market trigger. Thus, the micro trigger will be prone to the same issues as with a single bank specific trigger, i.e lagged effects or market manipulation. The benefit with a multi-variate trigger is that it provides a broad-based recapitalization for the banking-system while allowing for differences among the banks. However, IMF (2011) writes that it is a construction prone to create mixed signals of the status of the economy among the regulators and the market. Meaning that it puts regulatory judgement against the market-perceived believes about the gravity of systemic distress.

2.2 Aspects of Conversion

In the following section we will present the features governing the conversion process in a CoCo. We will follow the notations and outline of Spiegeleer and Schoutens (2011). When a conversion occurs, it is crucial that the condition on how it is conducted is predefined and clear. These conditions generally consist of two parts. First, the conversion fraction, the fraction of the CoCo that is converted to equity once the trigger point is breached. Secondly the type, a CoCo can be converted into a predefined number of shares or the number of shares received can be determined by the share price at the time of conversion. Instead of converting into equity, CoCos can also fully or partially be written off at the trigger event. This will be discussed more in Subsection 2.2.2.

2.2.1 Conversion fraction

The fraction of a CoCo that could be converted into equity is entitled α, where $0 \leq \alpha \leq 1$. Hence, the total "money amount" up for conversion is so forth the nominal amount (face value) of the CoCo N times the conversion fraction; $N \cdot \alpha$. A CoCo with $\alpha = 1$ is therefore called a "full CoCo", since the entire investment is transferred to equity. The fraction $(1 - \alpha)$ could either be paid back in cash or written off upon conversion. Since all CoCos are more or less tailor-made for each issue, no CoCo will look the same and all possible features will therefore be hard cover in this thesis. As the examples written in Section 3 demonstrates, and to the best of our knowledge, all issued CoCos with conversion into equity have $\alpha = 1$.

2.2.2 Conversion Ratio and Price

When the potential conversion is initiated, the CoCo can either convert into a predetermined number of shares or it can convert into a variable amount of shares determined by the issuing company's share price around the time of conversion. The number of shares an investor receives per CoCo after a conversion is an important variable and will subsequently be called the conversion ratio C_r. Instead of converting into equity, the CoCo can also suffer a *write-down* on the face value, but as said in the beginning, this thesis will primarily focus on the pricing of *equity-converting* CoCos.

Dividing the conversion amount αN with the conversion ratio C_r yields the conversion price C_p i.e. the implied price the investor "pays" for the new equity shares

$$C_p = \frac{\alpha N}{C_r}. \tag{1}$$

If the conversion price C_p is known when the CoCo is issued, one can use Equation (1) to calculate how many shares a CoCo holder gets per CoCo by

$$C_r = \frac{\alpha N}{C_p}.$$

In both conversion types the implied price C_p a CoCo holder has to pay for the converted shares is essential for pricing the CoCo. The three most common conversion prices are the share price at the time of the issuance, the share price on the time of the trigger event or the share price on the trigger date but with a floor. Let us now discuss these three cases and their implications in more detail.

A conversion price equal to the share price on the CoCo issue date will benefit existing shareholder. If conversion is imminent, the issuer would already be in a precarious situation and the share price would be significantly lower than at the time of the issue. If the conversion amount is a pre-decided number of shares the conversion ratio will be low and the existing shareholders stake in the company will not be drastically diluted.

If the conversion price equals the share price on the "trigger day" the situation will be reversed. The conversion price will be low and the conversion ratio high and thus will the existing shareholders get their shares significantly diluted. The CoCo holders are however better off since with the newly purchased equity, they will get a potential upside or at least shares to the market price.

A third alternative is a conversion price with a floor denoted S_{floor}. It would be constructed so that the conversion price C_p would be the highest value of the price at the trigger day S^* and a minimum level S_{floor}, that is

$$C_p = \max(S^*, S_{floor}).$$

This alternative protects existing shareholders from dilution since the CoCo investors will never be able to buy the newly issued shares for less than S_{floor}.

There is the possibility that the CoCo is converted into a fixed number of shares and if this is combined with variable conversion price, the conversion fraction α will not be fixed. If the CoCo does not convert to equity, the fraction that will be written off is simply equal to α.

3 Issued CoCos

Since CoCos so far have been a rare and almost exotic instrument, a brief summery will be written about some of the most circumscribed CoCos that have been issued to date. In Section 3.1 to 3.5 a brief summery of CoCos issued by five banks will be given whereafter in Section 3.6 and 3.7 a short summery of capital adequacy requirement laws in Sweden and Switzerland will be addressed.

3.1 Lloyds Banking Group

First out in implementing Contingent Convertibles was Lloyds in late 2009 (Lloyds, 2009). Instead of issuing more capital, Lloyds made an offer to existing bondholders to convert their ordinary bonds to CoCos. The interest was immense and Lloyds ended up raising over £9 billion in new regulatory capital through a number debt exchange offers (Lloyds, 2009). Since investors where able to convert many different securities into these so called "Enhanced Capital Notes" the outstanding maturity differs but most of them are, if not converted, redeemed in 10 to 20 years and bears a coupon ranging from 6 to 16%. The notes are mandatory converted into a fixed number of shares if Lloyds' Core Tier 1 ratio falls below 5% and the total face value is transferred to equity ($\alpha = 1$). The notes thereby holds an accounting-based trigger mechanism which only can be observed when the bank releases financial statements.

3.2 Rabobank

Rabobank first issued contingent capital in March 2010 and this issue raised about EUR 1.25 billion in new capital (Rabobank, 2010). But instead of converting to equity, this security will suffer a *write-down* to 25% of par once the equity ratio falls below 7% of the bank's risk-weighted assets (BIS, 2009b). As such, this contingent convertible is more of a hybrid contingent bond since it does not convert into anything else. Rabobank is a privately held cooperative and hence a market trigger based on its share prices is not applicable as why they incorporated an accounting-based version. The bank itself called this new issue a "Senior Contingent Note" and it was, except for the trigger write-down contingency, an ordinary 10 year bond yielding a coupon of Libor + 3.5%.

In January 2011 a new issue raised about $2 billion in new capital. The new CoCo will suffer a write down of 75% once the trigger is initiated, while the trigger event is defined as in the previous issue (BIS, 2009b).

3.3 Credit Suisse

On February 24 2011 Credit Suisse announced that they would implement a targeted issue of contingent capital, named "Buffer Capital Notes", to two specific investors, Qatar Holding LLC and The Olayan Group (Credit Suisse, 2010). These two big investors agreed to exchange their Tier 1 capital notes issued by Credit Suisse in 2008 for this new issue of CoCos. The issue comprised of two parts, one $3.5 billion issue bearing coupon of 9.5% and one CHF 2.5 billion issue with a coupon of 9.0%. In line with what Lloyds and Rabobank used, Credit Suisse also chose an accounting trigger. The entire face value of the notes will convert into a predetermined number of shares if the group's reported Basel III common equity ratio falls below 7%. But, the conversion trigger is two-folded, the Swiss regulatory agencies has the power to force a direct conversion if they believe Credit Suisse is near insolvency and would otherwise need governmental support. The conversion price is not set in advance but determined by the highest value of $20, CHF 20 and the average price of the shares over a trading period preceding the conversion date.

3.4 Bank of Cyprus

Four days after Credit Suisse announcement, Bank of Cyprus declared their issue of CoCos, or what they called *Convertible Enhanced Capital Securities* (Bank of Cyprus, 2011). This specific issue is a hybrid between a standard convertible bond and a CoCo. A holder can choose to convert his/her investment into equity at their discretion before May 2016. The issue is perpetual but after May 2016, only mandatory conversion can take place. The mandatory conversion can be set off by several triggers, including if Core Tier 1 ratio falls below 5% and by the discretion of the Central Bank of Cyprus. The bank can further, in consultation with the Central Bank, decide when to convert. The conversion price is set to $C_p = \max(S_{floor}, C_{p(mandatory)})$, where S_{floor}=EUR 1 and $C_{p(mandatory)}$ is set as $\min(3.3, 0.8 \cdot$ (average share price 5 days before conversion)$)$. The whole value of the CoCo will be converted ($\alpha = 1$) and the number of shares received is equal to the principal amount (EUR 1.0) on each CoCo divided by C_p. Until 2016, the CoCo will bear a coupon of 6.5% whereafter the coupon rate will float with 6-month Euribor+3%. The complexity of the conversion possibilities will make it difficult to find a closed-form solution of the value, but a value for the CoCo may still be computed using Monte Carlo simulations.

3.5 UBS

In early 2012, another Swiss bank, UBS, announced the implementation of Basel III loss absorbing subordinated notes, the total issue size is not yet exactly determined but around $2 billion will be issued. It is though decided that the loss absorbing conversion is triggered when common equity capital ratio falls below 5%. Like Rabobank's issue, this contingent capital will not be converted into ordinary shares upon triggering but instead the entire debt will be totally and permanently written off.

3.6 Switzerland

It is not a coincidence that two of the banks that have, or are about to, issue CoCos are Swiss Banks. In mid 2011 Swiss Financial Market Supervisory Authority, FINMA, announced tightened regulatory requirements for the largest "too-big-to-fail"-banks (FINMA, 2011). In cooperation with the Swiss National Bank and federal government, they acted rapidly to increase the resilience of the Systematically Important Financial Institutions (SIFIs) of the country, to limit the economic impact of crises in the international financial system. FINMA (2011) stated several criteria for to be counted as a SIFI resulting in that two Swiss banks was elected by FINMA; UBS and Credit Suisse. FINMA introduced higher solvency and liquidity requirements than in the baseline Basel III, the banks are obliged to have 19% in total regulatory capital of RWA, whereof at least 10%-points in pure common equity is needed. On top of that, the Swiss additional conservation buffer of 8.5% of RWA may consist of CoCos to a maximum of 3% where eligible CoCos must have a high trigger at 7.5% Core Tier 1 Capital or above. FINMA has to approve all CoCos being issued so that the instruments are defined such that they are triggered when re-stabilizing is crucial. We believe that the new and tighter rules has lead the mentioned Swiss banks to issue contingent capital as it would be much more expensive to issue pure equity.

3.7 Sweden

Along with Switzerland, Sweden has one of the most rigorous capital adequacy requirement laws in Europe. On November 25th 2011, Swedish financial supervising authority (Finansinspektionen), Ministry of Finance and the Central Bank (Riksbanken) issued a joint press release stating the Swedish interpretation of Basel III that will be ordained on the four largest SIFI's in Sweden (Riksbanken, 2011). A total regulatory capital ratio of at least 10% will be introduced on the 1st of January 2013 and raised to 12% the following year. Of these, at least 7% have to be in Core Tier 1 capital.

In late March 2012 Swedbank, as the first Swedish bank, announced its interest in issuing CoCos as the board on the same day gave their approval for the new instrument (Swedbank, 2012). *"Our expectation is that it is going to be a compulsory part of the capital structure for banks in Europe"* said Swedbank's director of communications Thomas Backteman. The announcement from Swedbank was welcomed by the Swedish Minister of Finance Anders Borg who said that CoCos will be a powerful and useful instrument for Swedish banks (DI, 2012-03-30). No further details of the new issue has been announced and Swedbank is awaiting response on how supervising authorities within Sweden will handle CoCos. We will in Section 5.4 show how a potential issue from Swedbank could be priced.

Finansinspektionen have yet to comment on the usage of and how to handle CoCos but given the current requirements for Basel III we believe banks will be more or less forced to consider alternative and cheaper ways of gathering regulatory capital.

4 Pricing of CoCos

In the remaining part of this thesis, we will present different modelling approaches on how to calculate the price and yield spread of a CoCo. There exist several valuation methods for this task but we have decided to focus on three different models. In Section 4.1 and 4.2 we will present the Credit Derivative and the Equity Derivative approach, introduced by Spiegeleer and Schoutens (2011). In both these sections we will follow the outline and notations of the original authors. In Section 4.3 we present a structural framework that models the balance sheet structure by assuming that the company's assets follow a stochastic process. This approach builds on the Merton (1974) based work of Hilscher and Raviv (2011) but we also introduce a new way of modelling conversion. Furthermore, the model also outlines a method on how to integrate coupon payments into a structural model. To the best of our knowledge, this kind of CoCo model has not been studied before.

4.1 Credit Derivative approach

In this section we will examine the Credit Derivative approach. This model uses the imbedded dynamics of credit derivatives to price CoCos and was developed by Spiegeleer and Schoutens (2011). We are throughout this section going to follow Spiegeleer and Schoutens (2011) work and notations, but present a more detailed description of the underlying factors of the model than in the original paper. The Credit Derivative approach uses the theory of credit risk modelling with default intensities by an application of regular credit default swap (CDS) pricing. We will in Subsection 4.1.1 present the foundation of intensity-based credit risk modelling by following work of Lando (2004) and Herbertsson (2011). Furthermore, in Subsection 4.1.2 and 4.1.3 we make a in-depth description of the dynamics of a CDS and how it can be priced, by following the outline in Herbertsson (2011). Last, in Subsection 4.1.5 and 4.1.6 we will show how a CDS can be used to compute the credit spread of a CoCo.

4.1.1 Foundation of intensity-based credit risk modelling

The basic intuition behind intensity-based credit risk modelling is that given an "information" $\mathcal{F}_t$, the probability for a company to default on its payment obligations within the time period t to $t + \Delta t$ is the intensity λ_t multiplied with the time distance Δt. Thus, this is the conditional default probability in time period $[t, t + \Delta t]$ given the "information" $\mathcal{F}_t$, mathematically expressed as

$$\mathbb{P}[\tau \in [t, (t + \Delta t)|\mathcal{F}_t] \approx \lambda_t \Delta t \quad \text{if } \tau > t$$

where τ denotes the random time of default. The process λ_t is often modelled as the intensity for a Cox process, or a "doubly stochastic Poisson process". This type of modelling lets the default intensity λ_t be decided by "exogenous" state variables, as can be seen below.

We assume a probability space of $(\Omega, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ where $\mathbb{P}$ could either be the risk-neutral probability measure in an arbitrage free world or the actual real probability. The default intensity λ_t is driven by a d-dimensional stochastic process consisting of different states, $X_t = (X_{t,1}, X_{t,2}, \ldots, X_{t,d})$, while $\mathcal{G}_t^X$ is the filtration generated by X_t, that is $\mathcal{G}_t^X = \sigma(X_s; 0 \leq s \leq t)$. If we let λ be a non-negative measurable function where $\lambda : \mathbb{R}^d \to [0, \infty]$, then λ_t is modelled as the stochastic process $\lambda_t = \lambda(X_t(\omega))$. Next, let E_1 be a random exponential distributed variable with parameter 1 and independent of the process X_t. We then define the default time τ as

$$\tau = \inf \left\{ t \geq 0 : \int_0^t \lambda(X_s) ds \geq E_1 \right\}. \tag{2}$$

Hence, τ is defined as the first time the increasing process of $\int_0^t \lambda(X_s) ds$ breaches the random level of E_1.

From Equation (2) it is possible to find an expression for the survival probability, $\mathbb{P}[\tau > t]$, which is how likely it is for a company to not default on its obligations up to time t. To find $\mathbb{P}[\tau > t]$ we follow the derivations in Herbertsson (2011). If we begin by assuming that $u \geq t$, we can by using Equation (2) find the conditional survival probability on "information" $\mathcal{G}_u^X$

$$\begin{aligned}\mathbb{P}\left[\tau > t \mid \mathcal{G}_u^X\right] &= \mathbb{P}\left[\int_0^t \lambda(X_s)ds < E_1 \mid \mathcal{G}_u^X\right] \\ &= 1 - \mathbb{P}\left[\int_0^t \lambda(X_s)ds \geq E_1 \mid \mathcal{G}_u^X\right].\end{aligned} \tag{3}$$

Since E_1 and $\mathcal{G}_u^X$ are independent, the process $\int_0^t \lambda(X_s)ds$ is $\mathcal{G}_u^X$-measurable and the cumulative distribution function of an exponential distributed variable with parameter 1 is $F_{E_1} = 1 - e^{-x}$, Equation (3) can be rewritten as

$$\mathbb{P}\left[\tau > t \mid \mathcal{G}_u^X\right] = \exp\left(-\int_0^t \lambda(X_s)ds\right). \tag{4}$$

From Equation (4) the derivation for the non-conditional survival probability follows as:

$$\begin{aligned}\mathbb{P}[\tau > t] &= \mathbb{E}\left[\mathbf{1}_{\{\tau > t\}}\right] \\ &= \mathbb{E}\left[\mathbb{E}[\mathbf{1}_{\{\tau > t\}} \mid \mathcal{G}_u^X]\right] \\ &= \mathbb{E}\left[\mathbb{P}[\tau > t \mid \mathcal{G}_u^X]\right] \\ &= \mathbb{E}\left[\exp\left(-\int_0^t \lambda(X_s)ds\right)\right].\end{aligned} \tag{5}$$

The formula for the survival probability in Equation (5) will be very useful when calibrating the conversion intensity for a CoCo.

There are several processes that can be used to model the default intensity λ_t. For example, one can use a Vasicek, Cox-Ingersoll-Ross or a Markov process (Herbertsson, 2011). In this thesis we will model the default intensity λ_t as a deterministic function of time, both by letting λ_t be a constant but also by assuming that λ_t is piecewise constant in time. Assuming that λ_t is a deterministic function will simplify the calculation of a CDS in the following sections substantially, since $\mathbb{E}\left[\exp\left(-\int_0^t \lambda_s ds\right)\right] = \exp\left(-\int_0^t \lambda_s ds\right)$ in this setting.

Going back to the pricing of CoCos, in the Credit Derivative approach, credit default swaps (CDS) are used to calibrate the default intensity λ_t. How this is done exactly is examined in Subsection 4.1.6 but first we need to describe what a CDS is and how intensity-based credit risk modelling relates to it. This is done in Subsection 4.1.2 and 4.1.3 by closely following the outline presented in Herbertsson (2011).

4.1.2 Credit Default Swaps

In this subsection we will give a detailed description of how a credit default swap (CDS) is defined and structured. Begin by considering a company **C** that has issued a bond, with the random default time of τ. Furthermore, consider another company, **A**, that wants to buy protection against the event that company **C** defaults on its obligations within T years, on the nominal amount of N units of currency. For this the *protection buyer* **A** turns to a third company, **B**, the *protection seller*. Company **B** agrees to underwrite the risk of company **C** defaulting and in return for this the *protection buyer* **A** will pay a quarterly premium $\frac{S(T)N}{4}$ to **B**. The premium is paid until the CDS matures at time T unless the default occurs prior to T, i.e. $\tau < T$, whereupon the *protection seller* **B** pays the *protection buyer* **A** the nominal insured amount times the loss ratio of **C**, that is $N\ell$. Illustrated in Figure 2 is the structure of a CDS contract and in Figure 3 we show the cash flows of a CDS for two different scenarios.

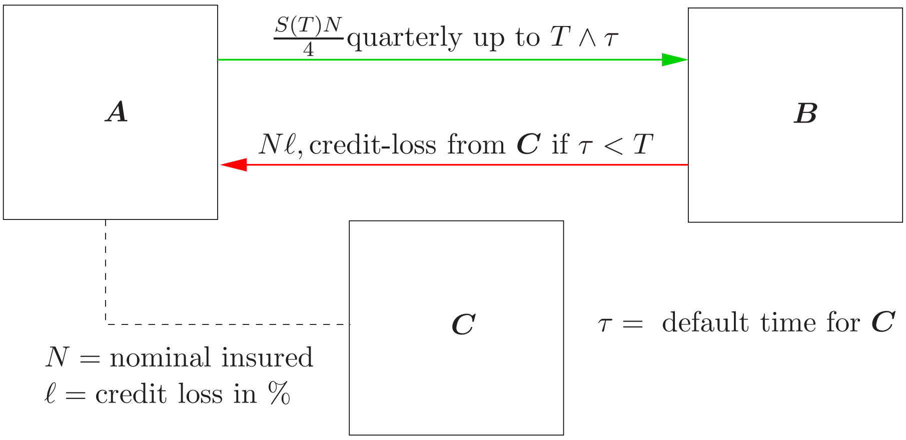

Figure 2: Structure of a CDS contract, Herbertsson (2011).

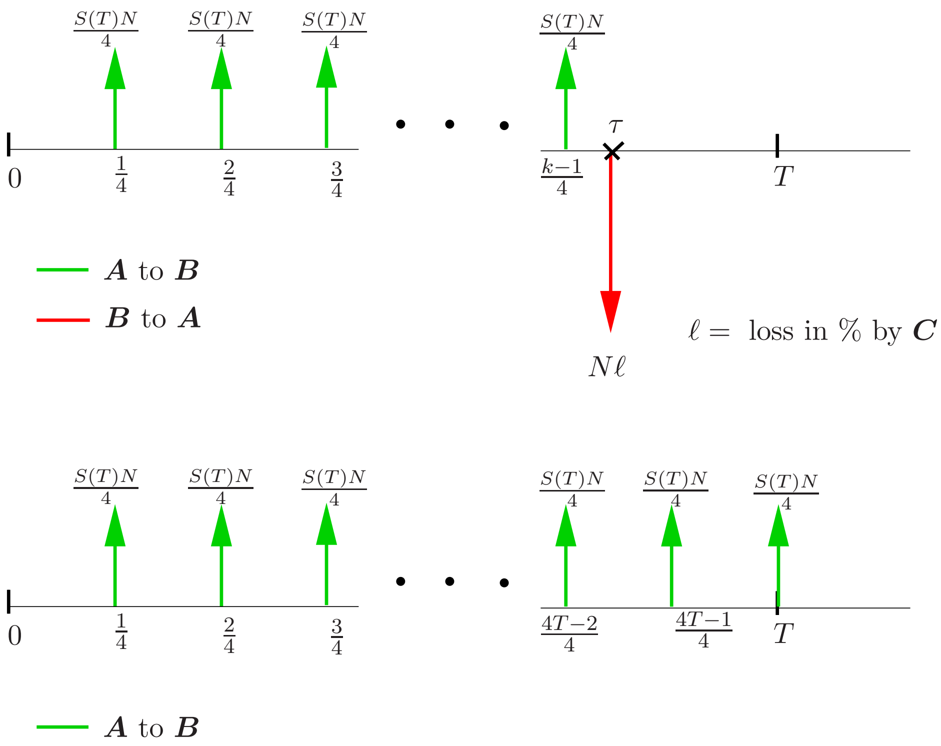

Figure 3: Two different cash flow scenarios in a CDS contract: No default before time T (bottom) and default before time T (top), Herbertsson (2011).

4.1.3 The CDS spread in a general model

The quarterly premium the *protection buyer* pays the *protection seller* in a CDS contract is often denoted as the CDS spread. In this subsection we will present a general formula for the CDS spread, which is model independent. Then, we will in Subsection 4.1.4 show how it could be computed using intensity-based credit risk modelling. Both these subsections will be presented by following the outline of Herbertsson (2011). The CDS spread is set so that the expected discounted cash flows is equal for both the *protection buyer* and the *protection seller* at the time the contract is entered. This is illustrated in the general formula below

$$S(T) = \frac{\mathbb{E}[\mathbf{1}_{\{\tau \le T\}} D(\tau)(1-\phi)]}{\sum_{n=1}^{n_T} \mathbb{E}\left[D(t_n)\frac{1}{4}\mathbf{1}_{\{\tau > t_n\}} + D(\tau)(\tau - t_{n-1})\mathbf{1}_{\{t_{n-1} < \tau_i \le t_n\}}\right]}. \tag{6}$$

$S(T)$ is the spread over the annual interest rate the *protection buyer* has to pay on the nominal amount insured, ϕ is the recovery in case of a default and $D(t)$ is the discount factor.

In Equation (6) the nominator is usually called the default leg since it represents the expected cash flow from the *protection seller* and the denominator is often entitled the protection leg and thus constitutes the expected cash flow from the *protection buyer*. The term $D(t_n)\frac{1}{4}\mathbf{1}_{\{\tau > t_n\}}$

symbolizes the discounted premiums and the term $D(\tau)(\tau - t_{n-1})\mathbf{1}_{\{t_{n-1}<\tau_i\le t_n\}}$ accounts for the accrued interest of these premiums.

The rather complicated expression from Equation (6) can however be simplified by making a few assumptions. First, we assume that the interest rate is a deterministic function of time, $D(t) = \exp\left(-\int_0^t r(s)ds\right)$, which is independent of the time of default τ. Here $r(s)$ denotes the risk-free interest rate. Furthermore, we also make the assumption that the loss, $\ell = 1-\phi$ is constant. Moreover, the probability of default before time t is given by $F(t) = \mathbb{P}[\tau \le t]$ and $f_\tau(t)$ is the density of τ, that is $f_\tau(t) = \frac{dF(t)}{dt}$. By using the above assumptions and notations, Equation (6) can be rewritten into the following expression

$$S(T) = \frac{(1-\phi)\int_0^T D(t)f_\tau(t)dt}{\sum_{n=1}^{4T} D(t_n)(1-F(t_n))\frac{1}{4} + \int_{t_{n-1}}^{t_n} D(s)(s-t_{n-1})f_\tau(s)ds}. \tag{7}$$

If we assume that the interest rate is constant and continuously compounded, we can substitute $D(t)$ with the continuous discounting factor $D(t) = e^{-rt}$. The CDS spread in Equation (7) can simplified even further by adding two additional assumptions:

1. Ignoring the accrued interest term

2. At the default time τ the loss is paid at time $t_n = \frac{n}{4}$, i.e. at the end of the quarter instead of immediately at τ.

These two assumptions, together with the assumption of constant loss and interest rates makes it possible to rewrite Equation (7) to the following expression

$$S(T) = \frac{(1-\phi)\sum_{n=1}^{4T} e^{-r\frac{n}{4}}(F(\frac{n}{4}) - F(\frac{n-1}{4}))}{\sum_{n=1}^{4T} e^{-r\frac{n}{4}}(1-F(\frac{n}{4}))\frac{1}{4}}, \tag{8}$$

for more details see e.g. in Herbertsson (2011).

4.1.4 The CDS spread in intensity-based models

To be able to calculate the default probability we can use intensity-based credit risk modelling, described in Subsection 4.1.1. Recall from Equation (5) that the survival probability can be written as $\mathbb{P}[\tau > t] = \mathbb{E}\left[\exp\left(-\int_0^t \lambda(X_s)ds\right)\right]$ and the default probability will simply be $\mathbb{P}[\tau \le t] = 1 - \mathbb{P}[\tau > t]$. If we make the assumptions that λ_t is a deterministic function of time t, $\mathbb{P}[\tau \le t]$ can be written as

$$\mathbb{P}[\tau \le t] = 1 - \exp\left(-\int_0^t \lambda(s)ds\right). \tag{9}$$

Consider a function $\lambda(t)$ which is piecewise constant between the fixed time points $t_1, t_2, \ldots, t_j$ and thereafter constant. Then, $\lambda(t)$ is given by

$$\lambda(t) = \begin{cases} \lambda_1 & \text{if} \quad 0 < t \leq t_1 \\ \lambda_2 & \text{if} \quad t_1 < t \leq t_2 \\ \vdots & \vdots \\ \lambda_j & \text{if} \quad t_{j-1} < t \end{cases} \tag{10}$$

from Equation (9) and (10) the default probability is then given by

$$\mathbb{P}[\tau \leq t] = \begin{cases} 1 - e^{-\lambda_1 t} & \text{if} \quad 0 < t \leq t_1 \\ 1 - e^{-\lambda_1 t_1 - \lambda_2 (t - t_1)} & \text{if} \quad t_1 < t \leq t_2 \\ \vdots & \vdots \\ 1 - e^{-\lambda_1 t_1 - \lambda_2 (t_2 - t_1) \ldots - \lambda_j (t - t_{j-1})} & \text{if} \quad t_{j-1} < t. \end{cases} \tag{11}$$

To describe the parameters $\lambda_1, \ldots, \lambda_j$ which, constitutes the piecewise constant function $\lambda(t)$, we will sometimes for convenience write the whole vector of intensities as $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_j)$.

By assuming that λ is constant instead of piecewise constant $\lambda(t)$, the default probability can be computed by

$$\mathbb{P}[\tau \leq t] = 1 - e^{-\lambda t}. \tag{12}$$

With a constant λ, in conjunction with all the previous assumptions we can simplify Equation (8) substantially and then write the formula for the CDS spread as

$$S(T) = 4\Big(1 - \phi\Big)\left(e^{\frac{\lambda}{4}} - 1\right) \tag{13}$$

for proof of how to go from Equation (8) to (13) see Herbertsson (2011). If λ is "small" we can use a Taylor-expansion, as follows

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots,$$

and rewrite the expression in Equation (13). Since $\frac{x^n}{n!} \approx 0$ for $n \geq 2$ when x is small, we can use that $e^x \approx 1 + x$, which in Equation (13) yields that $e^{\frac{\lambda}{4}} \approx 1 + \frac{\lambda}{4}$ and the CDS spread $S(T)$ can thus be simplified to

$$S(T) \approx (1 - \phi)\lambda. \tag{14}$$

Equation (14) can also be proven if the fee $S(T)$ is paid continuously, see e.g. O'Kane (2008).

4.1.5 Connecting the CDS and the CoCo

In this subsection and in Subsection 4.1.6 we will derive a price for a CoCo using the dynamics of a CDS. This will be done by following the outline presented in Spiegeleer and Schoutens (2011).

We begin to illustrate the connection between a CDS and a CoCo by introducing a new parameter, λ_{CoCo}. This parameter is the conversion intensity and is related to the dynamics of the default intensity parameter for the corresponding company that has issued the CoCo. The conversion intensity will be greater than the default intensity, $\lambda_{CoCo} \geq \lambda$, since a conversion must occur before a default and might even help to prevent one. So, by using the conversion intensity of the CoCo, we can create a CDS that defaults if the CoCo converts. The recovery of the CoCo ϕ_{CoCo} will be the decided by the choice of conversion price C_p and the share price at the trigger time S^*, that is

$$\phi_{CoCo} = \frac{S^*}{C_p}. \tag{15}$$

If S^* and C_p are constants, i.e. the CoCo is converted into a predetermined number of shares and has a market trigger, and if we assume $\lambda_{CoCo}(t)$ is piecewise constant between the time points $t_1, t_2, \ldots, t_j$ and thereafter constant, the credit spread for a CDS-CoCo can be computed by using Equation (8) with $F(t)$ given by Equation (11) and where ϕ is replaced by ϕ_{CoCo} given in Equation (15). Since CoCos pay annual coupons we will use a CDS-CoCo with annual premiums, instead of quarterly which is standard for CDS contracts. Note that with a CDS-CoCo that pays annual premiums assumption **1** will get weaker but it is still used for convenience. If we on the other hand assume that λ_{CoCo} is constant we can use Equation (13) and write the credit spread of a CDS-CoCo with annual payments as

$$S_{CoCo}(T) = (1 - \phi_{CoCo})(e^{\lambda_{CoCo}} - 1) \tag{16}$$

and if we assume that λ_{CoCo} is "small" , we can use Equation (14) and the spread for a CDS-CoCo will thus be given by

$$S_{CoCo}(T) \approx (1 - \phi_{CoCo})\lambda_{CoCo}.$$

This means that the loss for a CoCo investor in the case of conversion will be

$$\ell_{CoCo} = N\left(1 - \phi_{CoCo}\right) = N\left(1 - \frac{S^*}{C_p}\right) \tag{17}$$

where N is the nominal amount invested. From Equation (17) we can notice that if $C_p = S^*$ the loss will be 0. This is because the CoCo holder, in the case of a conversion, is allowed to buy equity for a market price, thus will the conversion only lead to that the CoCo holder's nominal value in CoCos is transferred to equity.

4.1.6 Computing the credit spread of CoCo

Finally, we need to calibrate the conversion intensity λ_{CoCo}, to do this we will continue to follow the outline of Spiegeleer and Schoutens (2011), but also Herbertsson (2011). The conversion intensity is calibrated through modelling the probability of conversion up to time T, $p^*(T)$. However, modelling $p^*(T)$ is a highly complicated task if the trigger is accounting or regulatory-based. It is difficult to model both the behaviour of regulatory authorities and the movement of the Core Tier 1 capital ratio. But in the case of a market trigger, modelling and computing $p^*(T)$ are viable to conduct in a creditable way.

If we assume that the share price follows a geometric Brownian motion (GBM), we can use the Black-Scholes approach of computing the value of a barrier option, to estimate the conversion probability, p^*. More specific, by following the outline of Su and Rieger (2009), we can model p^* such that it represent the probability that a GBM, starting at S_0, touches the barrier S^* before time T. The threshold value S^* is also defined as the trigger level for the CoCo. Then, $p^*(T)$ can be computed as:

$$p^*(T) = \Phi\left(\frac{\ln\left(\frac{S^*}{S_0}\right) - \mu T}{\sigma\sqrt{T}}\right) + \left(\frac{S^*}{S_0}\right)^{\frac{2\mu}{\sigma^2}} \Phi\left(\frac{\ln\left(\frac{S^*}{S_0}\right) + \mu T}{\sigma\sqrt{T}}\right). \tag{18}$$

For more details see Su and Rieger (2009). Here Φ is the cumulative probability density function for a normally distributed random variable, q is the continuous dividend yield, r is the continuous interest rate and σ is the volatility of the share price. Further, μ is the drift parameter of the long-term equilibrium share price and is given by

$$\mu = r - q - \frac{\sigma^2}{2}. \tag{19}$$

By using the Black-Scholes probability of touching a barrier, $p^*(T)$, we can calibrate a value for the conversion intensity for a CoCo, λ_{CoCo}. Recall that in accordance with Equation (5) and (12) the default probability for company with constant default intensity is

$$\mathbb{P}[\tau \leq t] = 1 - e^{-\lambda_{CoCo} \cdot t} \tag{20}$$

and with piecewise constant $\mathbb{P}[\tau \leq t]$ it is given by

$$\mathbb{P}[\tau \leq t] = \begin{cases} 1 - e^{-\lambda_{CoCo,1} \cdot t} & \text{if} \quad 0 < t \leq t_1 \\ 1 - e^{-\lambda_{CoCo,1} \cdot t_1 - \lambda_{CoCo,2} \cdot (t - t_1)} & \text{if} \quad t_1 < t \leq t_2 \\ \vdots & \vdots \\ 1 - e^{-\lambda_{CoCo,1} \cdot t_1 - \lambda_{CoCo,2} \cdot t_2 \ldots - \lambda_{CoCo,j}(t - t_{j-1})} & \text{if} \quad t_{j-1} < t. \end{cases} \tag{21}$$

Since all the variables in Equation (18) are known, or can be calibrated, we can set the probability of touching the barrier before time T equal to the default probability from Equation (20) and solve for λ_{CoCo}. This is illustrated with a constant conversion intensity below

$$\begin{aligned} \mathbb{P}[\tau \leq T] = 1 - \exp(-\lambda_{CoCo} \cdot T) &= p^*(T) \\ \lambda_{CoCo} &= -\frac{\ln(1 - p^*(T))}{T}. \end{aligned} \tag{22}$$

With the calibrated value for λ_{CoCo} we are now able to compute a value for the CoCo spread. This could be done either by using the exact method as in Equation (8):

$$S_{CoCo}(T) = \left(1 - \frac{S^*}{C_p}\right)\left(e^{\lambda_{CoCo}} - 1\right)$$

and if we assume that λ_{CoCo} is "small" we can use Equation (14), that is

$$S_{CoCo}(T) \approx \left(1 - \frac{S^*}{C_p}\right)\lambda_{CoCo}.$$

If we consider a CoCo with piecewise constant conversion intensity the procedure gets more complicated and $\boldsymbol{\lambda_{CoCo}} = (\lambda_{CoCo,1}, \lambda_{CoCo,2}, \ldots, \lambda_{CoCo,j})$ have to be solved by bootstrapping. Bootstrapping refers to a methodology where one first computes a value for $p^*(t_1)$ using Equation (18) and thereafter solves for $\lambda_{CoCo,1}$, by using Equation (22). Given $\lambda_{CoCo,1}$, the bootstrapping then continuous to find $\lambda_{CoCo,2}$ by computing $p^*(t_2)$ and then solving for $\lambda_{CoCo,2}$ using that

$$p^*(t_2) = 1 - e^{-\lambda_{CoCo,1} \cdot t_1 - \lambda_{CoCo,2} \cdot (t_2 - t_1)},$$

which implies that

$$\lambda_{CoCo,2} = -\frac{\ln\left(1 - p^*(t_2)\right) + \lambda_{CoCo,1} \cdot t_1}{(t_2 - t_1)}.$$

Next, $\lambda_{CoCo,3}$ can be obtained in a similar way, by using

$$\lambda_{CoCo,3} = -\frac{\ln\left(1 - p^*(t_3)\right) + \lambda_{CoCo,1} \cdot t_1 + \lambda_{CoCo,2} \cdot (t_2 - t_1)}{(t_3 - t_2)}$$

and thus, it also possible to find values for $\lambda_{CoCo,4} \ldots \lambda_{CoCo,j}$ by using the same method. By inserting all our calibrated conversion intensities into Equation (8) we can compute the CoCo spread for a company with piecewise conversion intensity.

4.2 Equity Derivative approach

Instead of considering CoCos as fixed income securities and pricing them through credit default swaps, one can look at it from the equity side. The following section will describe and analyze the Equity Derivative approach, introduced by Spiegeleer and Schoutens (2011). We will closely follow the notations from Spiegeleer and Schoutens (2011) in order to simplify for readers who have read the first article. In the Credit Derivative approach, we derived the necessary yield spread over an appropriate risk-free interest rate an investor would demand in order to be compensated for the possibility of a loss when the trigger event is satisfied. The intuition behind the Equity Derivative approach is that common and existing equity derivatives are added together to replicate the payoff structure of a CoCo. In order to value the CoCo using equity derivatives, the CoCo is split into different parts, which will all sum up to the total value of the CoCo. In Subsection 4.2.1, 4.2.2 and 4.2.3 we will describe the payoff structure of a zero-coupon CoCo and how it can be replicated by using a zero-coupon bond and *knock-in* forwards. Further, Subsection 4.2.4 show how to handle the coupons payments by the use of binary barrier options. Finally, Subsection 4.2.5 present closed-form solutions to the equity derivatives presented in Subsection 4.2.1-4.2.4 by using the methods presented in Hull (2009) and Rubinstein and Reiner (1991).

4.2.1 Payoff - Zero-Coupon CoCo

We start by deriving the value of a CoCo that bears no coupon, a zero-coupon CoCo (ZCC). After this, we will replicate the coupon payments along with the value of a potential conversion into equity, with equity derivatives such that the sum replicates the payoff of a CoCo. Let N be the face value of the zero-coupon CoCo. Note that N also is the amount which will be redeemed as the CoCo matures given no trigger event before T. A fraction α of N will be converted to equity while $(1-\alpha)$ of N will be paid out in cash to the investor upon conversion. With these definitions we can set up a payoff scheme for the zero-coupon CoCo, given the different states that can occur

$$\text{Payoff}_{CoCo} = \begin{cases} (1-\alpha)N + \frac{\alpha N}{C_p} S^* & \text{if converted} \\ N & \text{if not converted} \end{cases} \tag{23}$$

where S^* is the share price on the trigger day. If the CoCo is triggered, the investor will have a long position of $\frac{\alpha N}{C_p}$ shares in the company plus a position in cash equal to $(1-\alpha)N$. To formalize the equations further we introduce a trigger indicator function, $\mathbf{1}_{\{trigger\}}$. Define τ as the trigger time and T as maturity, then $\mathbf{1}_{\{trigger\}} = \mathbf{1}_{\{\tau \leq T\}}$. The random variable $\mathbf{1}_{\{\tau \leq T\}}$ is thus 1 if τ occurs during the period $[0, T]$ and 0 otherwise. Hence,

$$\mathbf{1}_{\{\tau \leq T\}} = \begin{cases} 1 & \text{if } \tau \leq T \\ 0 & \text{otherwise.} \end{cases} \tag{24}$$

By inserting Equation (24) into Equation (23) and using Equation (1) we can write the payoff from a CoCo, Payoff_{CoCo}, as

$$\begin{aligned} \text{Payoff}_{CoCo} &= \mathbf{1}_{\{\tau > T\}} N + \left[(1-\alpha)N + \frac{\alpha N}{C_p} S^*\right] \mathbf{1}_{\{\tau \leq T\}} \\ &= N + \left[\frac{\alpha N}{C_p} S^* - \alpha N\right] \mathbf{1}_{\{\tau \leq T\}} \\ &= N + [C_r S^* - \alpha N] \mathbf{1}_{\{\tau \leq T\}} \\ &= N + C_r \left[S^* - \frac{\alpha N}{C_r}\right] \mathbf{1}_{\{\tau \leq T\}} \\ &= N + C_r [S^* - C_p] \mathbf{1}_{\{\tau \leq T\}}. \end{aligned} \tag{25}$$

4.2.2 Replicating a CoCo with no coupons

Equation (25) clearly demonstrate how the value of the ZCC is either equal to the face value N or N plus the random value of the may-to-come purchase of C_r shares. From the expression within the square brackets, we see that an investor is better off the smaller the difference is between S^* and C_p since normally $S^* - C_p < 0$. Further, note that throughout this thesis we think of the conversion event as that the CoCo investor pays for the equity using the face value N. That is, if a conversion takes place, the CoCo is assumed to be redeemed and the investor receive N whereafter he/she immediately buys equity shares and pays C_p for each share. As derived in Spiegeleer and Schoutens (2011), the value of the last component in Equation (25) can be approximated using *knock-in* forwards written on the shares of the company, with a strike price equal to the conversion price C_p and knock-in level equal to trigger level S^*. The intuition behind the use of *knock-in* forwards is that if the CoCo converts (into equity), the CoCo investor will use the face value N to exercise the forward contract and buy C_r shares for C_p. To be more specific, if $C_p > S^*$, the CoCo investor has in some sense bought the shares for an overprice. The investor is thereby, from the conversion date, long the company's equity. Spiegeleer and Schoutens (2011) argues that if you buy *knock-in* forwards that is knocked in at the same time the CoCo is converted, it is a good valuation approximation of the equity part of the CoCo. The *knock-in* forwards' maturity will match the maturity of the CoCo. At the conversion point, the investor is knocked into the forward contract and commits to buy shares at maturity for the same value as the face value received from the bond. Thereby canceling out the redemption payment from the bond which should not be paid since the CoCo has been converted from a bond to equity. Define ZCB_t as a standard zero-coupon bond and by adding the *knock-in* forwards, a zero-coupon CoCo can thus be written as

$$ZCC_t = ZCB_t + \textit{Knock-in} \text{ Forwards.} \tag{26}$$

where the zero-coupon bond can be priced through standard bond pricing, e.g. $ZBC_t = e^{-r(T-t)}N$.

However, to replicate the conversion with *knock-in* forwards creates a discrepancy with the real world. Upon conversion, one will effectively hold C_r forward contracts (for every CoCo held) written on the underlying bank's share and a zero-coupon bond. The idea is that the proceeds from the face value of the zero-coupon bond will be used to exercise the forward contract at maturity. Defining S_f as the shares forward price at the time of conversion, this position will be worth $C_r[S_f - C_p] + N$ but since $C_r \cdot C_p = N$ it can be simplified to $C_r \cdot S_f$. Hence, you effectively hold a position worth $C_r \cdot S_f$ and not $C_r \cdot S^*$ as it should.

4.2.3 The *Knock-in* forward

By following Spiegeleer and Schoutens (2011) we model a position in a *knock-in* forward as taking a long position in a *down-and-in* call option and going short a *down-and-in* put option, both being so called barrier options (Hull, 2009). Similar to the structure of a contingent convertible, a barrier option is an option whose payoff depends on whether the underlying asset reaches a certain level during the time to maturity (Hull, 2009). In the Equity Derivative approach, *down-and-in* options are used which means that they only come into life if the underlying asset falls below a predetermined level. Going back to Equation (25), we recall that

$$\text{Payoff}_{CoCo} = N + C_r[S^* - C_p]\mathbf{1}_{\{\tau \leq T\}}, \tag{27}$$

and we can now do the following observation

$$\begin{aligned}(S^* - C_p)\mathbf{1}_{\{\tau \leq T\}} &= \max(S^* - C_p, 0)\mathbf{1}_{\{\tau \leq T\}} - \max(C_p - S^*, 0)\mathbf{1}_{\{\tau \leq T\}} \\ &= \max(S^* - C_p, 0)\mathbf{1}_{\{\tau \leq T\}} + \min(S^* - C_p, 0)\mathbf{1}_{\{\tau \leq T\}}.\end{aligned} \tag{28}$$

For further explanation of Equation (28), see Corcuera et al. (2010), p.8. A visualization and description of the connection in Equation (28) is displayed in Figure 4. Hence, from Equation (28) we see that using *down-and-in* options to replicate a forward enables an investor to use the Black-Scholes framework to value the position. To be valid instruments, both options must have the same exercise price K as the CoCo's conversion price C_p and the same barrier as the CoCo's trigger level S^*. The payoff structure, given that the *knock-in* barrier has been touched, of these two combined options and the forward, respectively, is displayed in Figure 4.

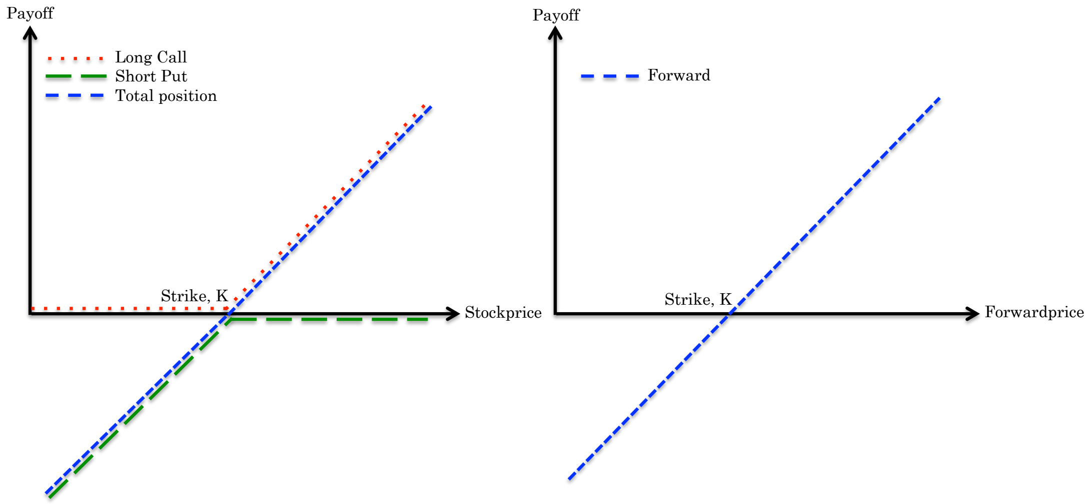

Figure 4: The payoff function of a long *knock-in* call and a short *knock-in* put option (left) and a *knock-in* forward (right), given that the *knock-in* level has been breached.

As can be seen in Figure 4, the payoff structure of the two figures are identical. Note that if $C_p > S^*$ (strike is smaller than barrier), the short put will be *in-the-money* but the long call *out-of-the-money*, hence the investor will be knocked in at the negative part of the payoff function. By using this approach, if the conversion is triggered, the investor is assumed to end up with options written on the underlying share and not the actual share itself as he or she should. This produces a minor flaw in the model but as Spiegeleer and Schoutens (2011) argues, this can under some circumstances be ignored due to its relatively small impact error. The value of a position in an option will only differ if the company is expected to pay dividends under the life of the option. Paying dividends after the trigger has been set off is highly unlikely as the financial position of the company, by definition, is not at its best. Considering the voting rights a position in the ordinary shares give while options do not, this method is further exposed to flaws but the monetary value of voting rights is tough to approximate.

The formula for calculating the value of a barrier option is derived from the standard Black-Scholes model where the underlying stock price follows a log-normal process but with modifications to adjust for the specific dynamics of the barrier. We will in Subsection 4.2.5 go into more detail how to value barrier options, but for now define the value of the option portfolio consisting of a short barrier put and a long barrier call, as V_t^{op}. Next, we will consider CoCos with coupons, which is a bit more difficult to evaluate.

4.2.4 Capturing the coupons

By exchanging the zero-coupon bond in Equation (26) to a standard corporate bond that pays coupons, denoted CB_t, we get the following pricing formula for a CoCo

$$P_t^{CoCo} = CB_t + V_t^{op}. \tag{29}$$

The corporate bond can be priced using the standard bond pricing formula

$$CB_t = \sum_{i=t}^{T} c_i e^{-rt_i} + Ne^{-r(T-t)}. \tag{30}$$

Note that in Equation (29) we include all future coupons which is incorrect since if the CoCo hits the trigger, the coupon payments will cease to be delivered and this feature will reduce the price. To capture this, Spiegeleer and Schoutens (2011) suggested the use of binary *down-and-in* barrier options, or more specifically *down-and-in cash-or-nothing* barrier options. A barrier *cash-or-nothing* call is a binary option that pays a predetermined amount of cash Q at maturity T if the barrier H has been reached during the life of the option and zero otherwise. We are seeking an option which cancels out any coupon payment received after a conversion has taken place. Consider a situation where one have sold a barrier *cash-or-nothing* call with barrier $H = S^*$, $Q = \text{Coupon}(c_i)$ and maturity equal to the time of next coupon payment. If the CoCo has not been converted by the next coupon, the barrier level will not have been touched either and the option will mature worthless. But if the CoCo has converted, the option barrier will also have been breached and the *cash-or-nothing* call will come into life. The short position means the seller (you) will have to pay the buyer Q and since $Q = c_i$, the coupon will be canceled out. For each coupon left until maturity, there is a matching short position in an barrier *cash-or-nothing* call with barrier level H equal to S^*. Hence, at each coupon payment there is an option maturing which cancels out the value of each coupon received after the CoCo has been converted. This will reduce the overestimation of the CoCo value that Equation (29) produces. In Subsection 4.2.5 we present a closed-form expression for the value of *cash-or-nothing* call options.

We have now, by using equity derivatives and a coupon paying corporate bond, replicated the payoff structure of a CoCo and the value is thus

$$P_t^{CoCo} = CB_t + V_t^{op} + \sum [\text{Short barrier } \textit{cash-or-nothing} \text{ Options}], \tag{31}$$

where CB_t is given by Equation (30). Furthermore, recall that V_t^{op} denotes the value of a portfolio consisting of a short *down-and-in* put option and a long *down-and-in* call option. In order for the reader to value a CoCo, we will now present closed-form equations for the components in Equation (31).

4.2.5 Computing the price of a CoCo

Barrier options

The equation to estimate the value of a barrier option depend on if the barrier is set below or above the strike price. As it is unlikely that the strike price C_p is set below the trigger level we will forthcoming rely on the assumption that $C_p \geq barrier$. If it would have been the other way around, the CoCo holder would upon conversion be able to buy the shares for less than the market price and the value of the position would be larger than the original CoCo position. Barrier option pricing uses a restricted density function since the option only comes into existence under some circumstances. The barrier option is further affected by the relative size of the strike price to the barrier level. Pricing *down-and-in* calls when the barrier S^* is smaller than or equal to the strike price C_p is done by integrating the payoff of a plain vanilla call option with the restricted density function for all possible stock prices starting from C_p to infinity. The value can further be broken down into two components; the payoff of a corresponding vanilla option if the stock price reaches the barrier and the rebate if the barrier is never reached by the stock price (Ali, 2004).

Following the notations in Hull (2009), we define H as the barrier level, which in our case is interpreted as the stock price level that triggers the conversion, that is S^*. Furthermore, q is the dividend yield, r is the risk-free interest rate, K is the strike price which in our case thus is interpreted as the conversion price C_p. Then, by recalling the first term of Equation (28) we write the payoff of a *down-and-in* call denoted c_T^{di} as

$$c_T^{di} = \max(S_T - K, 0)\mathbf{1}_{\tau \leq T}.$$

Note that $\tau = \inf \{t \geq 0 : S_t \leq H\}$. Next, by taking expectations, the formula for c_0^{di} at time 0 is given by

$$c_0^{di} = e^{-rT}\mathbb{E}\left[\max(S_T - K, 0)\mathbf{1}_{\tau \leq T}\right]$$

Using the Black-Scholes dynamics one can show that for $0 < t \leq T$ the closed-form formula for calculating a *down-and-in* call option is given by (see e.g. Hull (2009) p. 559)

$$c_t^{di}(S_t, H, K) = S_t e^{-q(T-t)}(H/S_t)^{2\lambda}\Phi(y) - Ke^{-r(T-t)}(H/S_t)^{2\lambda-2}\Phi\left(y - \sigma\sqrt{(T-t)}\right)$$

where

$$\lambda = \frac{r - q - \frac{\sigma^2}{2}}{\sigma^2} \quad \text{and} \quad y = \frac{\ln(H^2/(S_t K))}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{(T-t)} \tag{32}$$

and $\Phi(\cdot)$ is the cumulative normal distribution. Further, S_t is given by $S_0 e^{(r-q-1/2\sigma^2)+\sigma W_t}$ where W_t is a Brownian motion. By following the notations used above, the value of a *down-and-in* put p^{di} at maturity T is given by (see e.g. Hull (2009) p. 559)

$$p_T^{di} = \max(K - S_T, 0)\mathbf{1}_{\tau \le T} \Rightarrow p_0^{di} = e^{-rT}\mathbb{E}\left[\max(K - S_T, 0)\mathbf{1}_{\tau \le T}\right].$$

Again, by using the Black-Scholes dynamics for $0 < t \le T$ the closed-form formula for calculating a *down-and-in* put option is given by (see e.g. Hull (2009) p. 559)

$$\begin{aligned}
p_t^{di} &= -S_t\Phi(-x_1)e^{-q(T-t)} + Ke^{-r(T-t)}\Phi\big(-x_1 + \sigma\sqrt{(T-t)}\big) \\
&\quad + S_t e^{-q(T-t)}(H/S_t)^{2\lambda}[\Phi(y) - \Phi(y_1)] - Ke^{-r(T-t)}(H/S_t)^{2\lambda-2}\cdot \\
&\quad \Big[\Phi(y - \sigma\sqrt{(T-t)}) - \Phi(y_1 - \sigma\sqrt{(T-t)})\Big]
\end{aligned}$$

where y is given as in Equation (32) and

$$x_1 = \frac{\ln(S_t/H)}{\sigma\sqrt{(T-t)}} + \lambda\sigma\sqrt{(T-t)} \quad \text{and} \quad y_1 = \frac{\ln(H/S_t)}{\sigma\sqrt{(T-t)}} + \lambda\sigma\sqrt{(T-t)}. \tag{33}$$

We are replicating the value of a *knock-in* forward with a portfolio consisting of a short *down-and-in* put and a long *down-and-in* call such that the total value of the portfolio equal the value of the call minus the value of the put. The number of such portfolios needed, depends on C_r. More specific, we need one portfolio for every share one get after conversion. Thus, the total value of this option portfolio, denoted V_t^{op}, is given by

$$\begin{aligned}
V_t^{op}(S_t, H, K) =& \frac{\alpha N}{C_p}\left[c_t^{di} - p_t^{di}\right] = C_r\left[c_t^{di} - p_t^{di}\right] \\
=& C_r\Big[Se^{-q(T-t)}(H/S_t)^{2\lambda}\Phi(y) - Ke^{-r(T-t)}(H/S_t)^{2\lambda-2}\Phi\left(y - \sigma\sqrt{(T-t)}\right) \\
&+ S\Phi(-x_1)e^{-q(T-t)} - Ke^{-rT}\Phi\left(-x_1 + \sigma\sqrt{(T-t)}\right) \\
&- Se^{-q(T-t)}(H/S_t)^{2\lambda}\left[\Phi(y) - \Phi(y_1)\right] \\
&+ Ke^{-r(T-t)}(H/S_t)^{2\lambda-2}\left[\Phi\left(y - \sigma\sqrt{(T-t)}\right) - \Phi\left(y_1 - \sigma\sqrt{(T-t)}\right)\right]\Big].
\end{aligned} \tag{34}$$

Equation (34) can be simplified to the following expression

$$\begin{aligned}V_t^{op}(S_t, H, K) =& C_r\Big[S_t e^{-q(T-t)}(H/S_t)^{2\lambda}\Phi(y_1) + S\Phi(-x_1)e^{-q(T-t)} \\ &- Ke^{-r(T-t)}(H/S_t)^{2\lambda-2}\Phi\left(y_1 - \sigma\sqrt{(T-t)}\right) \\ &- Ke^{-r(T-t)}\Phi\left(-x_1 + \sigma\sqrt{(T-t)}\right)\Big]. \end{aligned} \tag{35}$$

where λ is given as in Equation (32), and x_1 and y_1 are defined by Equation (33). Remember that H is interpreted as the barrier level for the CoCo S^* and the strike price K as the conversion price C_p. Both the equation for a *down-and-in* put and a *down-and-in* call presented above and their interpretations can be found in numerous financial litteratures covering exotic options. We will leave the derivations aside and for interested readers we refer to e.g. Hull (2009) or Su and Rieger (2009).

Barrier *cash-or-nothing* options

The value of a barrier *cash-or-nothing* call is the cash amount Q times the probability that payment will occur, modelled in the Black Scholes framework and presented in e.g. Rubinstein and Reiner (1991). The option pays Q at the maturity of the option as we include the subscript $_{t_i}$ (there exist similar options that pays the fixed amount at the exact time of touching the barrier). Define $CN_{t_i}^{di}$ as a *cash-or-nothing* call, thus

$$\begin{aligned}CN_{t_i}^{di}(Q, S^*, t) &= Qe^{-r(t_i-t)}[\Phi(x_i - \sigma\sqrt{(t_i-t)}) + (S^*/S)^{2\lambda-2}\Phi(y_i - \sigma\sqrt{(t_i-t)})] \\ &= Qe^{-r(t_i-t)} \cdot \mathbb{P}[\tau \leq t_i]\end{aligned}$$

where τ is the conversion time and

$$x_{1i} = \frac{\ln(S_t/S^*)}{\sigma\sqrt{(t_i-t)}} + \lambda\sigma\sqrt{(t_i-t)} \quad \text{and} \quad y_{1i} = \frac{\ln(S^*/S_t)}{\sigma\sqrt{(t_i-t)}} + \lambda\sigma\sqrt{(t_i-t)} \tag{36}$$

note that t always is smaller than the t_i valued. The above derivations can be found in Rubinstein and Reiner (1991) along with many other variants of binary options and their valuations. In our setting, $Q = c_i$, hence to cancel the faulty coupon payments Equation (29) generates, one should subtract the sum of all short binary options, each maturing at every coupon payment, that is

$$\sum_{i=1}^{k} CN_{t_i}^{di}(c_i, S^*, t) = \sum_{i=1}^{k} c_i e^{-r(t_i-t)} \cdot \mathbb{P}[\tau \leq t_i] \tag{37}$$

Using equity derivatives and a standard corporate bond, we have thus derived the price of a contingent convertible bond with coupons, given by

$$P_t^{CoCo} = CB_t + V_t^{op}(S_t, S^*, C_p) - \sum_{i=1}^{k} CN_{t_i}^{di}(c_i, S^*, t), \tag{38}$$

where $V_t^{op}(S_t, S^*, C_p)$ is given by Equation (35), CB_t in Equation (30) and $\sum_{i=1}^{k} CN_{t_i}^{di}(c_i, S^*, t)$ is given by Equation (37) above.

4.3 Merton approach

A third way of pricing CoCos is to use a structural model. In a structural model a firm's assets are assumed to follow a random process, most commonly a geometric Brownian motion, which is then used to value the CoCo using, for example option-pricing theory. This has been done for example by Pennacchi (2010), Albul et al. (2010) and Hilscher and Raviv (2011). We will in this section use the Merton (1974) inspired work of Hilscher and Raviv (2011) as a foundation of our approach. Furthermore, we will also extend the model by Hilscher and Raviv (2011) by adding inspiration from the Equity Derivative approach from Section 4.2 (Spiegeleer and Schoutens, 2011) and to some extent from Madan and Schoutens (2011) and also, to the best of our knowledge, by our own ideas. In Subsection 4.3.1 and 4.3.2 we present the model, by closely following the work of Hilscher and Raviv (2011). Further, in Subsection 4.3.3 we outline a new way of replicating conversion and, by partly borrowing ideas from Spiegeleer and Schoutens (2011), display a way to add coupons in a Merton (1974) setting. We also derive some new results and show how the conversion can be replicated in the Merton approach. Finally, in Subsection 4.3.4 we describe how to compute the price of a CoCo using this framework.

4.3.1 Setting of the model

We start by considering a firm (tentatively a bank) with a balance sheet consisting of three types of assets. Firstly, standard debt, which in this case consists of zero-coupon bonds with maturity T, face value FV^{SD} and market value MV_t^{SD}. Secondly, one type of CoCo issued by the firm which has face value FV^{CoCo} and as the standard debt, maturity T. The holders of the standard debt claims will be senior bondholders whereas the holders of CoCos will be junior bondholders. The remaining part of the assets will consist of residual equity claims E_t. We denote the market value of the bank's assets with V_t, and accordingly, $V_t = MV_t^{SD} + MV_t^{CoCo} + E_t$. Here we further assume that the bank's capital structure will be constant throughout the maturity of the CoCo, i.e. the bank will not take on any more debt or issue more equity.

4.3.2 Balance sheet structure

We first consider the standard debt issued by the bank, which will either pay FV^{SD} if the bank does not default on its obligations before time T or it will pay H^{SD} if the default occurs before the claims matures at T. Here H^{SD} is the threshold asset value where default occurs. Thus, the random time of default τ_{SD} is defined as the first time t the asset value V_t breaches the threshold value of H^{SD}

$$\tau_{SD} = \inf\left\{t > 0 \mid V_t \leq H^{SD}\right\}. \tag{39}$$

In accordance with Black and Cox (1976) the threshold value will be slightly lower than the value of the standard debt since it is unreasonable that the financial authorities would be able to force the bank into declaring bankruptcy at the moment its assets hits the value of its debt. This would demand too much monitoring, which is costly, so a more reasonable assumption is that default occurs when the asset value is "a bit" below the value of the standard debt. Thus, $H^{SD} = FV^{SD}(1-\gamma)$ where $0 \leq \gamma \leq 1$. Next, consider the CoCo part of the liabilities. We know that a CoCo with an accounting trigger usually converts if a capital ratio breaches a threshold level. We denote this threshold level for a CoCo with H^{CoCo}. As in Equation (39) the conversion time τ_{CoCo} for a CoCo is defined as the first time the asset value breaches the threshold level of H^{CoCo}

$$\tau_{CoCo} = \inf\left\{t > 0 \mid V_t \leq H^{CoCo}\right\}.$$

The threshold level H^{CoCo} is decided by the terms in the CoCo contract and is given by

$$H^{CoCo} = (1+\beta)\left(FV^{SD} + FV^{CoCo}\right) \tag{40}$$

where β is a function of the terms set up in the contract. A higher β implies that the probability of conversion is higher relatively to a lower one. Note that the conversion of a CoCo will always occur prior to the default of the bank, so that $\tau_{CoCo} < \tau_{SD}$, which implies this $H^{CoCo} > H^{SD}$ relationship between the threshold levels. If a CoCo converts, the CoCo holder will receive a share of the equity in the underlying company. The fraction of the equity a CoCo holder will own in the case of conversion is denoted with θ, where $0 \leq \theta \leq 1$. The equity value is viewed as a residual of the asset value V_t and the face value of the standard debt FV^{SD}. Thus, at the time of conversion τ_{CoCo}, a CoCo holder will receive the equity value of

$$\begin{aligned}\text{conversion payoff} &= \theta E_{\tau_{CoCo}} \\ &= \theta\left(H^{CoCo} - FV^{SD}\right).\end{aligned}$$

We now introduce a method to compute the fraction θ. Recall from Section 2.2 that a CoCo can either be converted into a predetermined number of shares or by trading the face value into stocks by a variable share price. Consider a CoCo that is converted into a predetermined number of shares. Then an investor holding all CoCos will receive a fraction θ of the issuing company's equity, given by

$$\theta = \frac{FV^{CoCo}/C_p}{\text{Outstanding shares} + FV^{CoCo}/C_p}. \tag{41}$$

If we instead have a CoCo that is converted with a variable conversion price, the number of shares received would no longer be fixed and one would only be able to compute the expected value of θ. Throughout this section we will disregard CoCos with variable conversion price.

4.3.3 Pricing the CoCo

To find a closed-form solution for the price of a CoCo, we replicate the payoff dynamics of a CoCo using several types of barrier options. This will be done partly by following the method presented in Hilscher and Raviv (2011), but also by introducing a new way of replicating the conversion event. We will also display a method on how to add coupons into Hilscher and Raviv (2011) original model by using ideas from Equity derivative approach (Spiegeleer and Schoutens, 2011) described in Section 4.2. To the best of our knowledge, these ideas has not been treated in the literature before. From now on, we will thus diverge from the model presented in Hilscher and Raviv (2011) and develop our own model. This subsection will be focused on how to replicate a CoCo in the Merton approach, using barrier option, while we in Subsection 4.3.4 describe how these options can be priced using the Black-Scholes dynamics.

To be able to replicate a CoCo, we first need to recall its payoff structure. A CoCo, without considering the coupons, will either pay the face value FV^{CoCo} at maturity T if the trigger event has not happened before maturity. Otherwise it will pay a share in the underlying company's equity at the trigger event τ_{CoCo}. Thus, the value of a CoCo, not paying any coupons, can be written as

$$V_t^{CoCo} = FV^{CoCo}\mathbf{1}_{\{\tau_{CoCo}>T\}} + \theta E_{\tau_{CoCo}}\mathbf{1}_{\{\tau_{CoCo}\leq T\}} \tag{42}$$

where θ is the fraction of the equity value a CoCo holder will receive if conversion occurs before maturity. Recall from Subsection 4.3.2 that the equity value in the Merton approach is computed as a residual of the asset value and the face value of the total debt. If conversion occurs the debt value represented by CoCos will vanish from the balance sheet and the equity value will be given by $E_{\tau_{CoCo}} = (V_{\tau_{CoCo}} - FV^{SD})$. If a CoCo is designed with an accounting trigger, the asset value at the time τ_{CoCo} will be fixed and embodied by the threshold level H^{CoCo}, defined in Equation (40). Equation (42) can thus be rewritten to

$$V_t^{CoCo} = FV^{CoCo}\mathbf{1}_{\{\tau_{CoCo}>T\}} + \theta\left(H^{CoCo} - FV^{SD}\right)\mathbf{1}_{\{\tau_{CoCo}\leq T\}}. \tag{43}$$

To replicate the payoff structure of a CoCo we start by considering the dynamic of the first term in Equation (43), $FV^{CoCo}\mathbf{1}_{\{\tau_{CoCo}>T\}}$, i.e. the payoff for a CoCo without including the conversion part. This is also visually illustrated in Figure 5 below.

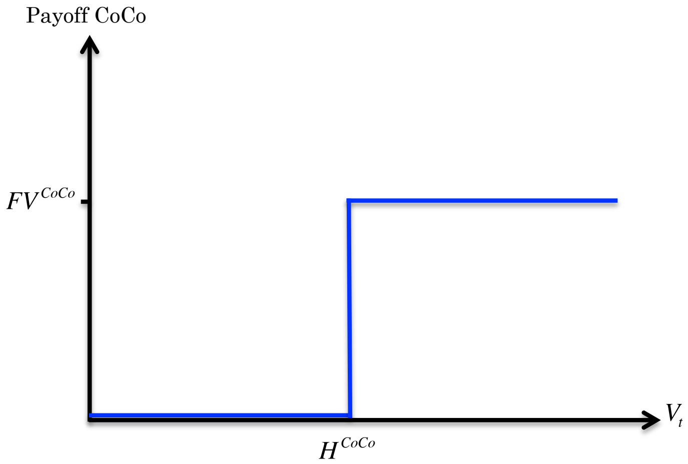

Figure 5: CoCo payoff excluding conversion

By studying Figure 5 one might notice that it resembles the payoff of a binary barrier option, more specifically a *cash-or-nothing*, *down-and-out* call. A binary barrier *cash-or-nothing* call option pays a fixed amount at maturity, if the underlying does not reach a predetermined barrier before maturity. In our figuration, we will include a *down-and-out* option, which is terminated if the underlying asset falls below a threshold level. To replicate the payoff of a CoCo that is not converted, the binary barrier option will pay the face value FV^{CoCo} at time T if the asset value does not fall below the threshold level of the CoCo H^{CoCo} and zero otherwise. Hence the first term of Equation (43) will be replicated with a binary barrier *down-and-out cash-or-nothing* call written on the company's assets with a barrier of H^{CoCo}. We define this option as $CN_T^{do}\left(FV^{CoCo}, H^{CoCo}\right)$.

To replicate the second term in Equation (43), $\theta\left(H^{CoCo} - FV^{SD}\right)\mathbf{1}_{\{\tau_{CoCo}\leq T\}}$, the payoff if conversion occurs, we will diverge from Hilscher and Raviv (2011) and present a new approach on how to capture the dynamics of this scenario. The payoff of a CoCo's conversion part is illustrated in Figure 6 below.

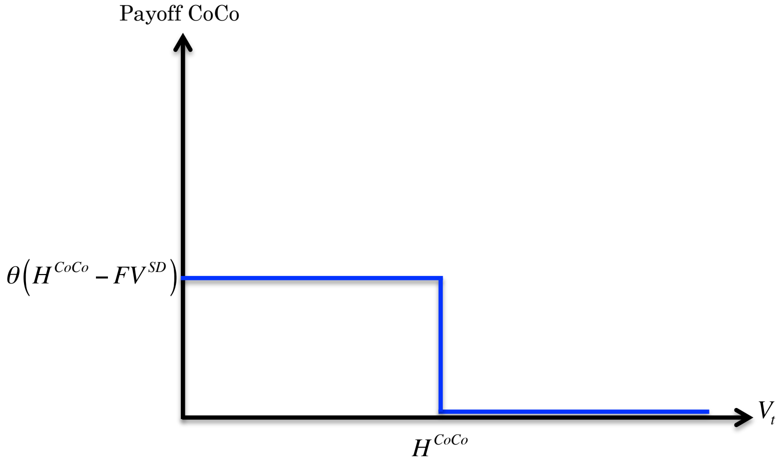

Figure 6: Payoff of a CoCo's conversion part

In Hilscher and Raviv (2011) the authors use a spread position of two European barrier call options. But since the payoff of an european option only is realized at maturity while a CoCo holder receives his/her shares at the time of conversion we have instead chosen to use a binary barrier option. This option will be a *down-and-in cash-at-hit* call that pays the fixed cash amount $\theta\left(H^{CoCo} - FV^{SD}\right)$ when the barrier H^{CoCo} is breached, if this happens before the maturity T and zero otherwise. The trigger time is defined as τ_{CoCo}. The price of this option will be denoted with $CN^{di}_{\tau_{CoCo}}\left(\theta\left(H^{CoCo} - FV^{SD}\right), H^{CoCo}\right)$. A weakness with this replication is that instead of shares a CoCo holder receives an amount of cash equal to the value of the shares he/she should have got. But if we make the simple assumption that the market for the underlying share is liquid, a CoCo holder would be indifferent to receive the shares or the money since he/she would be able to buy or sell the shares instantaneously to the market price.

To capture the dynamics of the coupon payments we take inspiration from the Equity Derivative approach, described in Section 4.2. But instead of using options that cancel out coupon payments after conversion, we now add options that pays the coupon amount until the barrier is hit. Adding coupons does however complicate the Merton model substantially, see e.g. p. 27-29 in Lando (2004). The money for paying the coupons needs to be gathered from somewhere on the balance sheet. This could be done by the equityholders paying the coupons out of their own pocket and thus, reducing the asset value. Another way is to issue new equity and thereby dilute the existing shareholders' stocks. A third alternative could be to issue new debt but since we have assumed the debt to be constant this option is disregarded. If the company decides to issue new equity the share in the company θ CoCo holders receive upon conversion will no longer be fixed as considered in Subsection 4.3.2 and thereby creates a problem in our valuation. Since we believe that equityholders only would choose to issue new equity in a situation of substantial financial distress, we argue that it is unlikely that new equity will be raised before the CoCo has converted and we will thereby not consider this option. The last alternative is to use existing capital in the company the pay the coupons. This will decrease the asset value V_t and thus increase the probability of conversion. However, since CoCos generally is a small part of the total debt we believe that ignoring the fact that coupons payment decrease V_t would have a small or minimal impact on the price. For example, Credit Suisse CoCo issue presented in Section 3.3, yields a annual coupon payment of

$$\$3.5 \text{ billions} \cdot 9.5\% + \text{CHF } 2.5 \text{ billions} \cdot 9.0\% \approx \text{CHF } 0.53 \text{ billions}$$

which only is 0.05% of the total asset value or 0.6% of the cash entry on the balance sheet as of Q1 2012. Hence, we believe adding coupons with a portfolio of binary barrier option could be a good approximation for the value of a CoCo's coupon payments and not result in a to large erroneous valuation.

The type of option we add is a binary barrier *down-and-out* call option written on the asset value, that pays a fixed amount equal to the coupon as long as the barrier is not breached. We denote this option with $CN_{t_i}^{do}$. Unlike the *down-and-in cash-at-hit* option, pays this option the fixed amount at maturity. So if we have a CoCo with 10 years maturity and annual coupon, we will have 10 options, each with maturity at one of the coupon dates, denoted by t_i. These options will have a barrier of H^{CoCo} and pay the coupon at maturity, if the asset value does not breach the barrier level.

If we combine the binary barrier option portfolio and the two types of binary barrier options presented above, with Equation (43) we can write the formula for payoff of a CoCo as

$$\begin{aligned} V_t^{CoCo} &= CN_T^{do}\left(FV^{CoCo}, H^{CoCo}\right) \\ &+ CN_{\tau_{CoCo}}^{di}\left(\left[\theta\left(H^{CoCo} - FV^{SD}\right)\right], H^{CoCo}\right) \\ &+ \sum_{i=1}^{T} CN_{t_i}^{do}\left(c_i, H^{CoCo}, t_i\right). \end{aligned} \tag{44}$$

4.3.4 Applying the Black-Scholes dynamics

To be able to calculate the price of a CoCo using the Merton approach we need closed-form solutions for binary barrier options. This is presented in Rubinstein and Reiner (1991) where the authors use the Black-Scholes dynamics to price several types of binary options. In this subsection we will display the pricing formulas for the two types of binary option that is included in Equation (44). Starting with the first term, the value of a binary *down-and-out*, *cash-or-*

nothing barrier option, that pay off at the maturity T, is given by

$$CN_T^{do}(Q, H) = Qe^{-rT}\Phi(x_1 - \sigma\sqrt{T}) - Qe^{-rT}(H/S_0)^{2\lambda-2}\Phi(y_1 - \sigma\sqrt{T}) \tag{45}$$

where

$$x_1 = \frac{\ln(S_0/H)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}, \quad y_1 = \frac{\ln(H/S_0)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \quad \text{and} \quad \lambda = \frac{r - q - \frac{\sigma^2}{2}}{\sigma^2}.$$

Here H is the barrier, which is in our case will be equal to the conversion barrier, H^{CoCo}. Furthermore, Q is the cash received at maturity if the barrier is not breached before time T, which here is equal to the face value of the CoCo FV^{CoCo}. Also recall that the underlying S_0 is the asset value at time 0, V_0, and σ is the asset volatility.

The second term in Equation (44), the value of a binary *down-and-in*, *cash-at-hit* call barrier option can be calculated by

$$CN_{\tau_{CoCo}}^{di}(Q, H) = Q(H/S_0)^{a+b}\Phi(z) + (H/S_0)^{a-b}\Phi(z - 2b\sigma\sqrt{T}) \tag{46}$$

where

$$a = \frac{\mu}{\sigma^2}, \quad b = \frac{\sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2} \quad \text{and} \quad z = \frac{\ln(H/S_0)}{\sigma\sqrt{T}} + b\sigma\sqrt{T}$$

again, H is the conversion threshold for the CoCo, H^{CoCo}, the payoff Q will in this case be the money's worth of the shares a CoCo holder receives upon conversion, $\theta\left(H^{CoCo} - FV^{SD}\right)$ and S_0 is the asset value at time 0, V_0. Further, μ is defined as in Equation (19), r is the risk-free interest rate and σ is the asset volatility.

The third term to value is the binary barrier options of the same type as in Equation (45) but that pays the value of the coupon c_i at maturity t_i if the barrier H^{CoCo} has not been breached. Since we need one such option for every coupon, we will have a portfolio with as many options as there are coupons, each with maturity at the date of the coupon it represents. The value of this portfolio will thus be the sum of all these options. By using Equation (45) and (46) we can calculate the value of a CoCo V_t^{CoCo} by the method presented in Equation (44) as follows

$$
\begin{aligned}
V_t^{CoCo} &= FV^{CoCo}e^{-rT}\Phi(x_1-\sigma\sqrt{T}) \\
&- FV^{CoCo}e^{-rT}\left(H^{CoCo}/V_0\right)^{2\lambda-2}\Phi(y_1-\sigma\sqrt{T}) \\
&+ \theta\left(H^{CoCo}-FV^{SD}\right)^{a+b}\Phi(z)+(H^{CoCo}/V_0)^{a-b}\Phi(z-2b\sigma\sqrt{T}) \\
&+ \sum_{i=t}^{T}\left[c_ie^{-rt_i}\Phi(x_{1i}-\sigma\sqrt{t_i})-c_ie^{-rt_i}(H^{CoCo}/V_0)^{2\lambda-2}\Phi(y_{1i}-\sigma\sqrt{t_i})\right].
\end{aligned} \tag{47}
$$

We have thus in Equation (47) derived a new model to value a CoCo by combining the work of Hilscher and Raviv (2011), Spiegeleer and Schoutens (2011) and Rubinstein and Reiner (1991) with our own ideas. Next, we will test the models presented in Section 4 through applications with real numbers.

5 Model application

Using the theoretical framework presented in the previous sections, we now apply the models with realistic data to put them into context. We will present a fictive CoCo and in Section 5.1 to 5.3 apply the three approaches on this CoCo. Each model will, after the application, be analyzed concerning how the pricing models works and how sensitive they are against changes in the underlying input variables and assumptions. Making use of the models, we will further apply them on a potential issue by Swedbank, see e.g., Swedbank (2012). The parameters of the fictive CoCo that will be used in the models are presented in Table 1.

Variable		**Comment**
N	100	Face value
α	1	Entire value is converted
S_0	100	Share price of bank on valuation day
σ	29%	Yearly average volatility on OMXS30 last 5 years
T	10y	The CoCo will mature in 10 years
r	1.94%	10y Swedish Government Bond (Riksbanken, 2012-04-04)
S^*	35	Share price at which the conversion will take place
q	2.00%	Dividend yield
C_p	65	Middle point between S^* and S
c_i	6.00%	Annual coupon rate

Table 1: Parameters of the fictive CoCo used in subsequent sections.

5.1 Credit Derivative approach

In this section we calculate the yield and price of a CoCo using the Credit Derivative approach presented in Section 4.1. We first, in Subsection 5.1.1, compute the credit spread of a CoCo with constant conversion intensity. Then, in Subsection 5.1.2, the CoCo spread of a CoCo assuming piecewise constant conversion intensity is calculated. Furthermore, in Subsection 5.1.3 we convert the calculated CoCo spreads into CoCo price. Finally, in Subsection 5.1.4 we analyze how sensitive the pricing model is to changes in the underlying variables.

5.1.1 Computing the CoCo spread - constant λ_{CoCo}

First recall that the approximation formula for a CoCo's credit spread in Spiegeleer and Schoutens (2011) is, for a company with constant conversion intensity, given by

$$S_{CoCo}(T) = (1 - \phi_{CoCo})\lambda_{CoCo} \tag{48}$$

while the exact version is

$$S_{CoCo}(T) = \Big(1 - \phi_{CoCo}\Big)\Big(e^{\lambda_{CoCo}} - 1\Big) \tag{49}$$

where ϕ_{CoCo} is the recovery ratio at conversion and λ_{CoCo} is the constant conversion intensity. All variables needed to compute the credit spread for our fictive CoCo, using the Credit Derivative approach, are given in Table 1. The idea of the Credit Derivative approach is to construct a fictive CDS that defaults at the same point as the CoCo converts into equity. Thus, we first need to calibrate the conversion intensity for the CoCo. To do this, we need to estimate the probability of conversion, $p^*(T)$. This assessment is done by using the Black-Scholes framework for calculating the probability to hit a specific barrier S^* before time T, that is

$$p^*(T) = \Phi\left(\frac{\ln\left(\frac{S^*}{S_0}\right) - \mu T}{\sigma\sqrt{T}}\right) + \left(\frac{S^*}{S_0}\right)^{\frac{2\mu}{\sigma^2}} \Phi\left(\frac{\ln\left(\frac{S^*}{S_0}\right) + \mu T}{\sigma\sqrt{T}}\right) \tag{50}$$

where

$$\mu = r - q - \frac{\sigma^2}{2}.$$

Here r is the risk-free continuous interest rate, q is the continuous dividend yield, σ is the volatility, S^* is the trigger level and S_0 is the share price at time 0 and T is the years to maturity. We can then solve for λ_{CoCo} using the conversion probability for a CoCo-CDS with constant conversion intensity

$$\mathbb{P}[\tau \leq T] = p^*(T) = 1 - \exp(-\lambda_{CoCo} \cdot T)$$

so that

$$\lambda_{CoCo} = -\frac{\ln(1 - p^*(T))}{T}. \tag{51}$$

Inserting the variables from Table 1 into Equation 50 and 51 yields the following values for $p^*(T)$, λ_{CoCo} and ϕ_{CoCo}.

Variable		**Comment**
$p^*(10)$	40.41%	Probability that the stock price will hit S^* before T
λ_{CoCo}	0.0518	Conversion intensity for the CoCo
ϕ_{CoCo}	53.85%	Recovery, S^*/C_p

Table 2: Conversion probability and intensity

Hence, using the Black-Scholes framework for barrier option pricing the likelihood for the stock price of hitting $S^* = 35$ during the next ten years is 40.41%, which yields a conversion intensity parameter for the CoCo of $\lambda_{CoCo} = 0.0518$. The recovery rate for this CoCo is estimated to 53.85%. Given the calibrated parameter λ_{CoCo}, we can now use Equation (48) or Equation (49) to find the CoCo spread in the case represented by Table 1. Inserting λ_{CoCo} returns a CoCo spread of

$$S_{CoCo}(10) = (1 - 0.5385) \cdot 0.0518 = 238.96 \text{ bp}$$
$$S_{CoCo}(10) = (1 - 0.5385) \cdot (e^{0.0518} - 1) = 245.26 \text{ bp.} \tag{52}$$

The difference between the two spreads is quite small, so the approximate formula appears to good for a quick and dirty "back of the envelope" calculation. Hence, if we apply the Credit Derivative method to our fictive bond, we are able estimate the value of its credit spread to 245.26 bp. To display this in the form of an annual continuous yield we need to add the risk-free interest rate, which in this case is equal to 1.94%. Thus, given the environment our fictive CoCo is represented in, it needs to have an annual yield of

$$Y_{CoCo} = 1.94 + 2.45 = 4.39\%. \tag{53}$$

5.1.2 Computing the CoCo spread - piecewise constant $\lambda_{CoCo}(t)$

In this subsection we will calculate the credit spread of a CoCo with piecewise constant conversion intensity. We will first show how $\boldsymbol{\lambda_{CoCo}} = (\lambda_{CoCo,1}, \ldots, \lambda_{CoCo,n})$ can be calibrated by bootstrapping and then present the computation for the CoCo spread. We have in this application made the assumption that $\lambda_{CoCo}(t)$ is piecewise constant and can take on four different values depending on t, more exactly

$$\lambda_{CoCo}(t) = \begin{cases} \lambda_{CoCo,1} & \text{if} \quad 0 \leq t \leq 3 \\ \lambda_{CoCo,2} & \text{if} \quad 3 < t \leq 5 \\ \lambda_{CoCo,3} & \text{if} \quad 5 < t \leq 7 \\ \lambda_{CoCo,4} & \text{if} \quad 7 < t. \end{cases} \tag{54}$$

From Equation (21) we know that the conversion probability $\mathbb{P}[\tau \leq t]$ for a CoCo with piecewise constant conversion intensity is given by

$$\mathbb{P}[\tau \leq t] = \begin{cases} 1 - e^{-\lambda_{CoCo,1} \cdot t} & \text{if} \quad 0 \leq t \leq 3 \\ 1 - e^{-\lambda_{CoCo,1} \cdot 3 - \lambda_{CoCo,2} \cdot (t-3)} & \text{if} \quad 3 < t \leq 5 \\ 1 - e^{-\lambda_{CoCo,1} \cdot 3 - \lambda_{CoCo,2} \cdot 2 - \lambda_{CoCo,3} \cdot (t-5)} & \text{if} \quad 5 < t \leq 7 \\ 1 - e^{-\lambda_{CoCo,1} \cdot 3 - \lambda_{CoCo,2} \cdot 2 - \lambda_{CoCo,3} \cdot 2 - \lambda_{CoCo,4} \cdot (t-7)} & \text{if} \quad 7 < t. \end{cases} \tag{55}$$

We can solve for $\boldsymbol{\lambda_{CoCo}} = (\lambda_{CoCo,1}, \lambda_{CoCo,2}, \lambda_{CoCo,3}, \lambda_{CoCo,4})$ with bootstrapping using Equation (50), so that $\mathbb{P}[\tau \leq T] = p^*(T)$. This will be done by first computing $p^*(3)$ and then solving for $\lambda_{CoCo,1}$ using Equation (55). Next, we calculate $p^*(5)$ and solve for $\lambda_{CoCo,2}$ and so forth. A more detailed description of this type of bootstrapping is presented in Subsection 4.1.6. When bootstrapping is conducted on the fictive CoCo from Table 1 the vector $\boldsymbol{\lambda_{CoCo}}$, displayed in Table 3, is obtained.

t	$p^*(t)$	$\lambda_{CoCo,i}$	For
3	0.0608	0.0209	$0 \leq t < 3$
5	0.1731	0.0637	$3 \leq t < 5$
7	0.2781	0.0679	$5 \leq t < 7$
10	0.4041	0.0640	$7 \leq t$

Table 3: Probability of conversion at each t and piecewise calibrated conversion intensities $\lambda_{CoCo,i}$, for $i = 1, \ldots, 4$.

By the vector $\boldsymbol{\lambda_{CoCo}}$ we can compute the CoCo spread using the general formula for a CDS spreads given in Equation(8). The formula is presented again below

$$S(T) = \frac{(1 - \phi_{CoCo}) \sum_{i=1}^{T} e^{-rt_i} (F(t_i) - F(t_{t_i - 1}))}{\sum_{i=1}^{T} e^{-rt_i} (1 - F(t_i))} \tag{56}$$

here r is the risk-free interest rate and $F(t_i) = \mathbb{P}[\tau \leq t_i]$, which is computed by Equation (55). Recall that the nominator in Equation (56) is often called the default leg and denominator the premium leg. Using $\boldsymbol{\lambda_{CoCo}}$ from Table 3 we receive the default and premium leg presented in Table 5. Table 4 displays the preceding computations.

Time t_i	**Default prob.** $F(t_i)$	**Surv. prob.** $1 - F(t_i)$	**Prob. of default in** $[t_{i-1}, t_i]$ $F(t_i) - F(t_{i-1})$	**Disc. fact.** e^{-rt_i}
1	2.07%	97.93%	2.07%	98.09%
2	4.10%	95.90%	2.03%	96.19%
3	6.08%	93.91%	1.99%	94.35%
4	11.88%	88.12%	5.79%	92.53%
5	17.31%	82.69%	5.43%	90.76%
6	22.74%	77.26%	5.43%	89.01%
7	27.81%	72.19%	5.07%	87.30%
8	32.28%	67.71%	4.47%	85.62%
9	36.48%	63.52%	4.20%	83.98%
10	40.41%	59.59%	3.94%	82.37%

Table 4: Calculation of the components in Equation (56).

Time t_i	**Premium leg** $e^{-rt_i}(1 - F(t_i))$	**Default leg** $e^{-rt_i}(F(t_i) - F(t_{t_i-1}))$
1	0.9605	0.0094
2	0.9225	0.0090
3	0.8861	0.0086
4	0.8154	0.0247
5	0.7505	0.0228
6	0.6877	0.0223
7	0.6302	0.0204
8	0.5798	0.0177
9	0.5334	0.0163
10	0.4908	0.0150
Sum	**7.2570**	**0.1662**

Table 5: Default and premium leg.

By dividing the default and premium leg from Table 5 with each other, we estimate a credit spread for our fictive CoCo of

$$S_{CoCo}(10) = \frac{\sum_{i=1}^{T} \text{default leg}_i}{\sum_{i=1}^{T} \text{premium leg}_i} = \frac{0.1662}{7.2570} = 228.96 \text{ bp}$$

which is roughly 16 bp lower than with constant conversion intensity given in Equation (52). A credit spread of 228.96 bp corresponds to an annual yield of

$$Y_{CoCo} = 1.94 + 2.29 = 4.23\%. \tag{57}$$

5.1.3 From CoCo yield to CoCo price

In this subsection we will transform the calculated yield from Equation (53) and (57) into a price of the CoCo. To rewrite the yield to a price we need to discount all the future cash flows with the yield rate. The calculation is illustrated in the equation below

$$\text{Price} = \sum_{i=1}^{T} \text{CF}_i e^{(-Y \cdot t_i)}. \tag{58}$$

where CF_i is the cash flows from the CoCo every year and Y is the yield. So, given the yield from Equation (53) and (57) together with the characteristics of our fictive CoCo from Table 1, face value $FV = 100$ and coupon rate $c_i = 6\%$, one can estimate two prices for the CoCo. The two estimated prices are given in Table 6 and is equal to 112.37% of the face value for constant conversion intensity and 113.41% for a CoCo with piecewise constant conversion intensity. The calculations are presented in more in detail in Table 6.

	$PV(CF_i) = CF_i e^{-Y_i t_i}$	
t_i	Piecewise λ_{CoCo}	Constant λ_{CoCo}
1	5.75	5.74
2	5.51	5.50
3	5.29	5.27
4	5.07	5.04
5	4.86	4.83
6	4.66	4.62
7	4.46	4.43
8	4.28	4.24
9	4.10	4.06
10	69.44	68.64
Sum	**113.41**	**112.37**

Table 6: Yield to price when using the Credit Derivatives approach for a CoCo given by Table 1.

5.1.4 Sensitivity - Credit Derivative approach

We will in this subsection conduct several sensitivity tests where we let variables vary while keeping the others constant to analyze the CoCo spread as function of some of the parameters. This is done both by assuming piecewise constant and constant conversion intensity using Equation (56) and (49).

When working with financial models there is always uncertainty concerning the input variables that are used when calibrating the parameters in a model. By varying the input data in the model, one can get a grasp of the models sensitivity as function of the calibrated parameters.

In the Credit Derivative approach the calibrated variables are the equity volatility σ and the continuous dividend yield q. These are both crucial variables when calculating p^*. Figure 7 displays how the CoCo spread is affected by changes in equity volatility, including the effect of different levels of conversion price C_p. From Figure 7 we see that the CoCo spread is increasing in volatility. Hence, with higher σ the probability of conversion increases and a CoCo investor would require a higher yield. Furthermore, we also observe that for a given level of volatility the CoCo spread is increasing in conversion price. In the Credit Derivative approach, the recovery ratio ϕ_{CoCo} is determined by the conversion price C_p together with the trigger level S^*. A higher conversion price would therefore imply a lower ϕ_{CoCo} and thus a higher CoCo spread, as we saw in Figure 7. When $S^* = C_p = 35$, the fictive CoCo's recovery is 100%. With a recovery of 100%, the CoCo investor does not loose any money upon conversion since the entire value of the investment is only transferred to equity. Hence, the CoCo is then risk-free with a spread equal to zero.

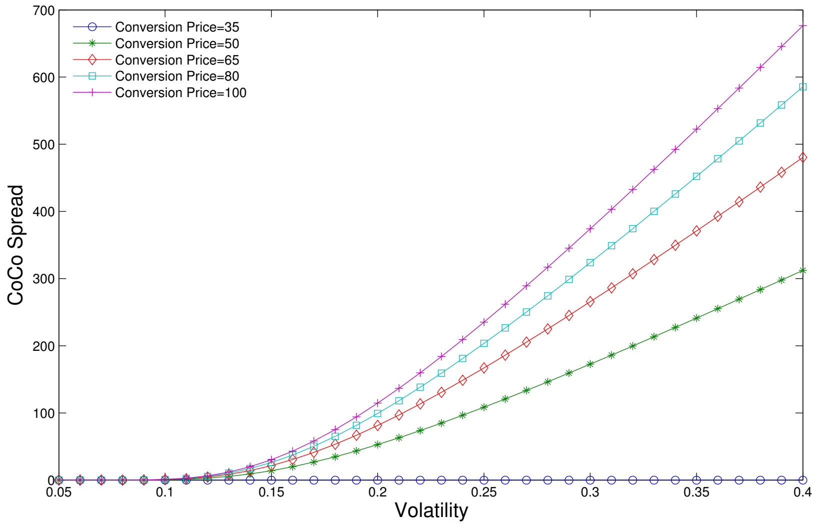

Figure 7: CoCo spread in the Credit Derivatives approach as a function of annual equity volatility σ for different levels of conversion price C_p

Figure 8 displays the CoCo spread as function of the dividend yield q and equity volatility σ. We note that the CoCo spread is increasing in both q and σ, which in both cases appears reasonable. If a bank pay higher dividends the bank's equity decreases and with that so does the share price. A lower share price in turn increases the probability of conversion and thus leads to a higher credit spread.

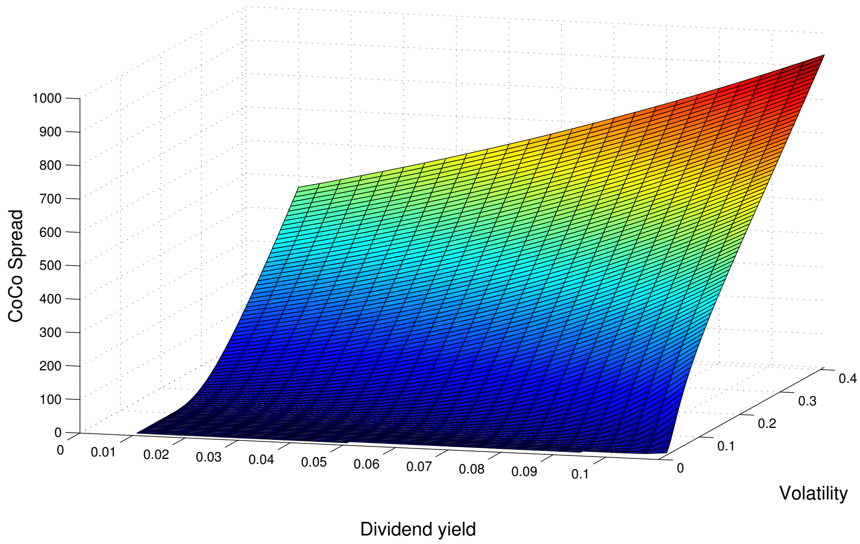

Figure 8: CoCo spread in the Credit Derivatives approach as a function of annual dividend yield q and volatility σ.

To capture the dynamics of the model we have also chosen to illustrate how the CoCo spread is affected by the maturity. For this we in Firgure 9 examine four different scenarios of equity volatility σ. We notice that for high volatilities the CoCo spread is first increasing then decreasing, which might seem odd but this relationship will also be present when we do sensitivity analysis in the Merton approach in Subsection 5.3.4. Plotting yield as function of time, generally results in so-called yield curves, and this figure can to some extent be interpreted as such a plot. The shape closely resembles what you usually find in yield curves, with yields close to zero for small maturities (less risk) and thereafter an increasing function until a max point.

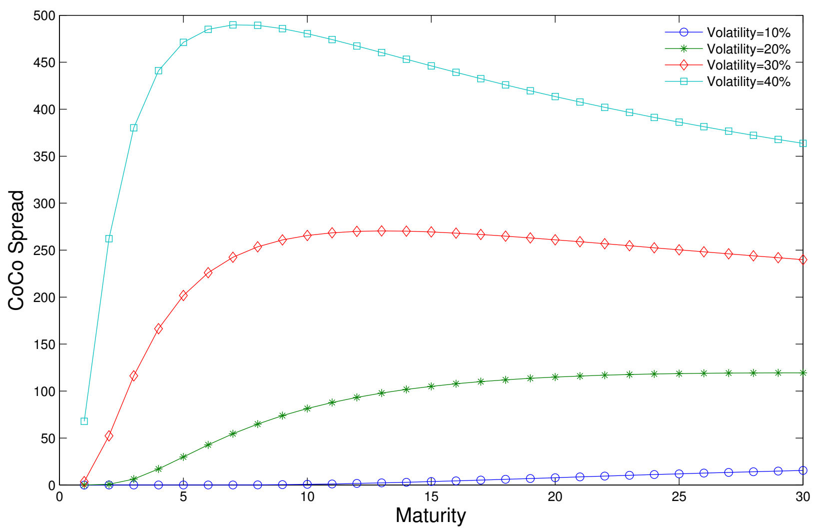

Figure 9: CoCo spread in the Credit Derivatives approach as a function of maturity T for different levels of volatility σ

Figure 10 shows the difference between assuming constant or piecewise constant conversion intensities for three different conversion prices. As we saw in Figure 8, the spread is increasing in volatility but we also find that the difference in spread between the two conversion intensities is not large.

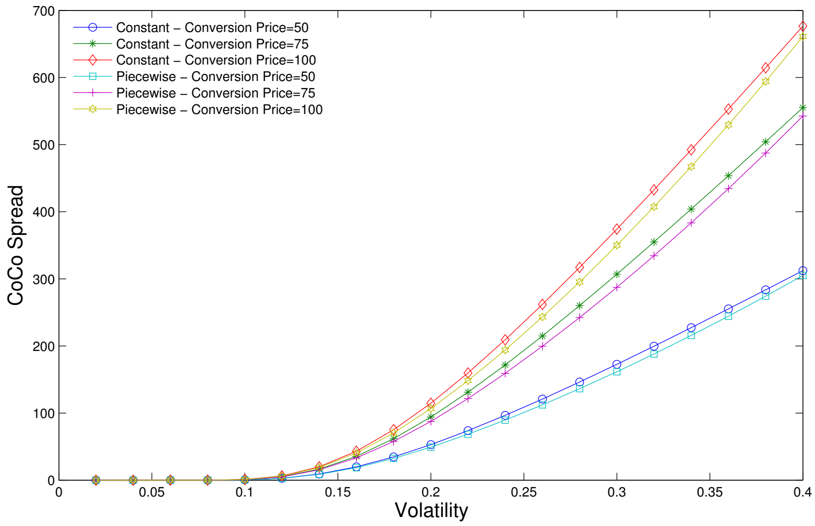

Figure 10: CoCo spread in the Credit Derivatives approach as a function of the annual volatility σ for different levels of conversion price C_p and with both constant λ_{CoCo} and piecewise constant conversion intensity $\boldsymbol{\lambda_{CoCo}}$.

5.2 Equity Derivative approach

In this section we price a fictive CoCo using the Equity Derivatives approach presented in Section 4.2. A sensitivity analysis of the pricing model will be performed in Subsection 5.2.2. Recall that in the Equity Derivative approach, the value of a CoCo is broken down into three parts, all of which sum up to the total value of the instrument. Recalling Equation (38) from Section 4.2, the price of a CoCo is given by

$$P_t^{CoCo} = CB_t + V_t^{op}(S_t, S^*, C_p) - \sum_{i=1}^{k} CN_{t_i}^{di}(c_i, S^*, t),$$

where CB_t is a corporate bond with the same maturity and coupon rate as the CoCo. Further, $V_t^{op}(S_t, S^*, C_p)$ is the value of the option portfolio as a function of the stock price S_t, barrier S^* and strike C_p. Remember, the option portfolio should capture the value of the equity received after conversion, modelled by a long position in a *down-and-in* call option and a short position in a *down-and-in* put option. The last term $\sum_{i=1}^{k} CN_{t_i}^{di}(c_i, S^*, t)$ are the binary barrier options that cancels out any coupon received after conversion. Further recall that the model attempts to replicate the payoff structure of a CoCo by using these derivatives. Using the numbers in

Table 1 we can now valuate the three components separately and thereafter the total value of the CoCo.

The Corporate bond

The value of a standard corporate bond CB_t with maturity T and the properties as in Table 1 can be computed by (Hull, 2009)

$$CB_t = Ne^{-r(T-t)} + \sum_{i=t}^{T} c_i e^{-rt_i}.$$

Table 7 displays all cash flows from the corporate bond CB_t as well as the present value of each cash flow. By applying the standard bond pricing formula, we conclude this bond to be valued at 136.38% of par at time 0.

Down-and-in options

By using Equation (35) we can calculate the value of a portfolio consisting of a long position in a *down-and-in* call and a short position in a *down-and-in* put. The value of this portfolio is equal to -10.02 and in our setting there are 1.54 such portfolios included since $C_r = \frac{\alpha N}{C_p} = \frac{1 \cdot 100}{65} = 1.54$, meaning that one gets 1.54 shares per CoCo. The option portfolio is thus worth $-10.02 \cdot 1.54 = -15.41$.

Binary options

By applying Equation (37), we can calculate the sum of all the barrier *cash-or-nothing* options that are added to cancel out all the faulty coupons paid after an eventual conversion. Since there is a total of ten coupons, there is an equal amount of binary options, all of which value is presented in Table 8, $CN_{t_i}^{di}$ denotes the present value of each option expiring at the matching coupon payment date. Since these options should cancel out any coupon payment after conversion, one has to sell the options such that the value is equal to

$$-\sum_{i=1}^{k} CN_{t_i}^{di}(c_i, S^*, 0) = -10.22 = -10.22 \tag{59}$$

t_i	CF_i	e^{-rt_i}	PV
1	6	98.08%	5.88
2	6	96.19%	5.77
3	6	94.35%	5.66
4	6	92.53%	5.55
5	6	90.76%	5.45
6	6	89.01%	5.34
7	6	87.30%	5.24
8	6	85.62%	5.14
9	6	83.98%	5.04
10	106	82.37%	87.31
		Sum	**136.38**

Table 7: The present value of all cash flows from the Corporate bond.

t_i	x_{1i}	y_{1i}	$CN^{di}_{t_i}$
1	3.76	-3.48	0.00
2	2.76	-2.36	0.10
3	2.34	-1.84	0.34
4	2.10	-1.52	0.64
5	1.94	-1.30	0.94
6	1.83	-1.13	1.22
7	1.75	-0.99	1.46
8	1.68	-0.88	1.67
9	1.64	-0.78	1.84
10	1.60	-0.69	2.00
		Sum	**10.22**

Table 8: Barrier *down-and-in cash-or-nothing* options, where the parameters x_{1i} and y_{1i} are given by Equation (36).

5.2.1 Total value of the fictive CoCo

Combining the results in Table 7 and 8 with Equation (59), we conclude that, when using the Equity Derivative approach, the value of the fictive CoCo presented in Table 1 is given by

$$\begin{aligned} P_0^{CoCo} &= CB_0 + V_0^{op}(S_t, S^*, C_p) - \sum_{i=1}^{k} CN_{t_i}^{di}(c_i, S^*, 0) \\ &= 136.38 - 15.41 - 10.22 = 110.75. \end{aligned}$$

Next, we proceed to examine the Equity Derivative approach, this time by conducting sensitivity analyses.

5.2.2 Sensitivity - Equity Derivative approach

Once we have calculated the price of a CoCo using the Equity Derivative approach, we continue to analyze how sensitive the price is to changes in the underlying variables. The plots in Figure 11 and 12 demonstrates how P_t^{CoCo} varies with changes in the conversion price C_p, equity volatility σ and dividend yield q, respectively. As seen in Figure 11, the value of a CoCo is inversely related to the price the investor has to pay for the equity in the event of conversion. This is in line with what we have discussed before, that is, an investor aims for an as low conversion price as possible. One aspect not covered in this model is the potential upside a low conversion price may give. Obviously, the more shares an investor owns the more he/she earns if the stock price rises after a conversion. Hence, a low conversion price benefits the holder in two ways as why P_t^{CoCo} is decreasing in C_p with decreasing speed. Furthermore, as already seen in the Credit Derivative approach the CoCo price is also decreasing in dividend yield as displayed in Figure 12.

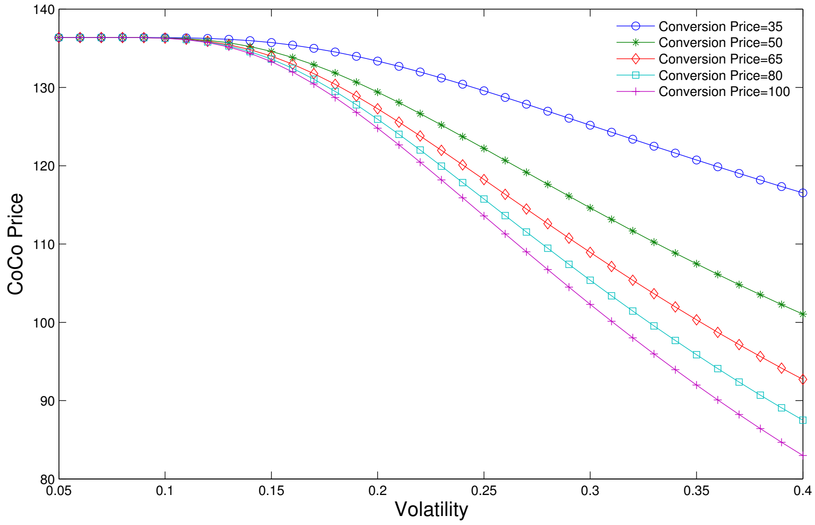

Figure 11: CoCo price in the Equity Derivative approach as a function of volatility σ for different levels of conversion price C_p.

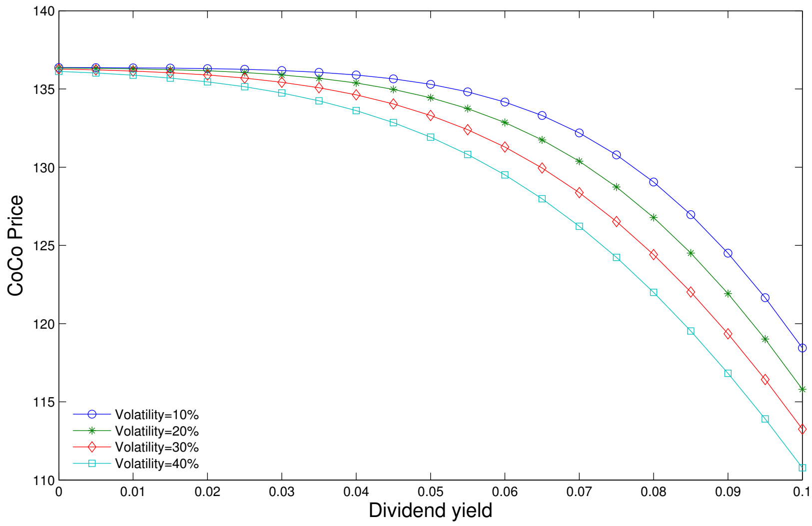

Figure 12: CoCo price in the Equity Derivative approach as a function of dividend yield q for different levels of volatility σ.

In Figure 13 we display the CoCo price for different levels of volatility as function of time to maturity. As can be seen in Figure 13, the CoCo price is decreasing in σ, that is, as volatility increases, the value of the CoCo decreases. When σ converges to zero, the CoCo price converges to the price of a corresponding standard corporate bond CB. This is due to the fact that as σ decreases to zero, the probability of the share price ever reaching the conversion level also approaches zero. If the probability of conversion is zero, or close to zero, the CoCo will be priced as a standard corporate bond. When σ increases to high levels, the options will increase in value and the short position in options will thereby lower the price to well under par. Furhermore, by examining Figure 14 we find that the CoCo price is positively related to the coupon level. This is expected as a CoCo holder will receive a higher annual return. One further notice is that when coupons are less then the value of dividend yield and risk-free interest, the price falls below par since the CoCo yields less than an alternative investment should.

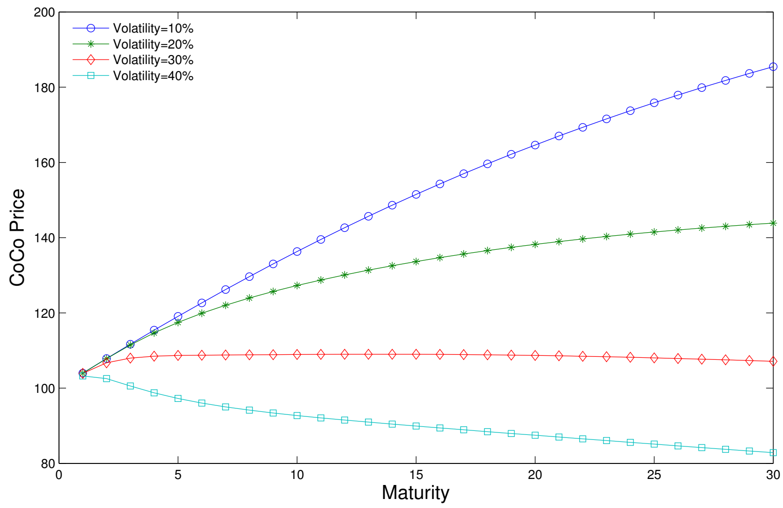

Figure 13: CoCo price in the Equity Derivative approach as a function of maturity T for multiple volatilities.

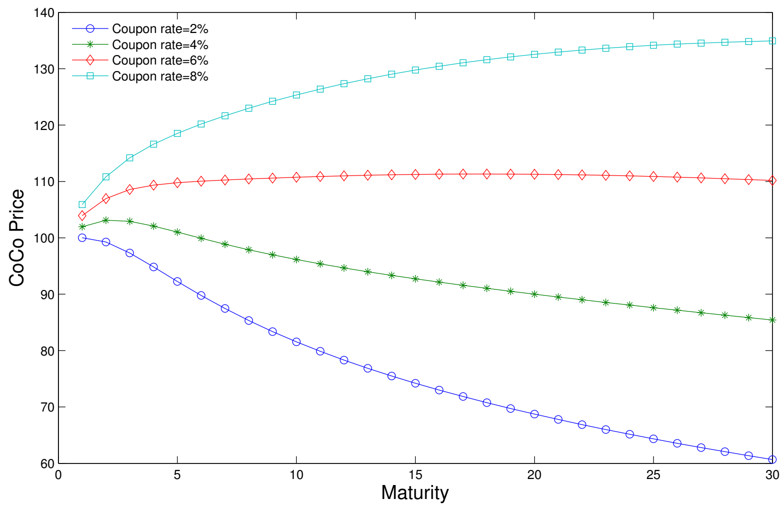

Figure 14: CoCo price in the Equity Derivative approach as a function of maturity T for multiple coupon rates.

Figure 15 displays the CoCo price as function of both the dividend yield q and the equity volatility σ computed in the Equity Derivative approach. Furthermore, Figure 16 shows the

CoCo price as function of annual equity volatility σ and coupon rate C_r. The surfaces show the same result as in Figure 14 and 12 but displayed in 3D which improves the visibility of the relationships.

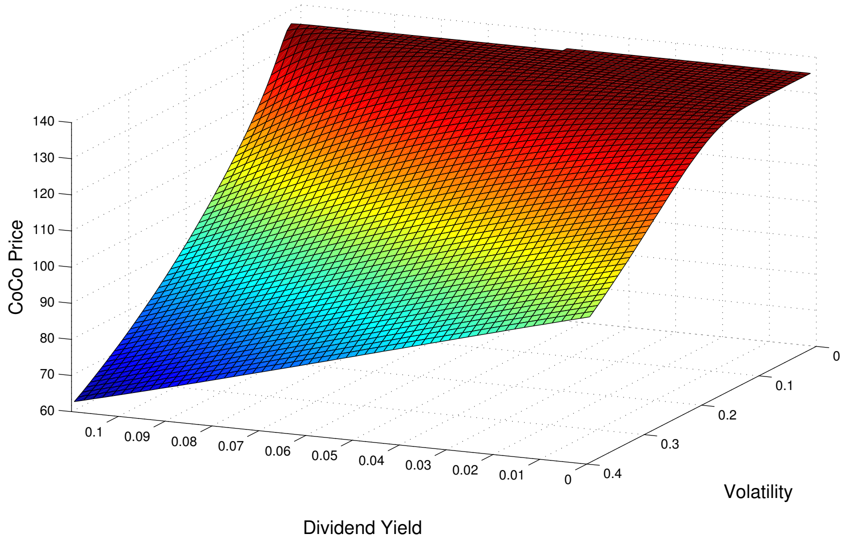

Figure 15: CoCo price in the Equity Derivative approach as a function of volatility and dividend yield

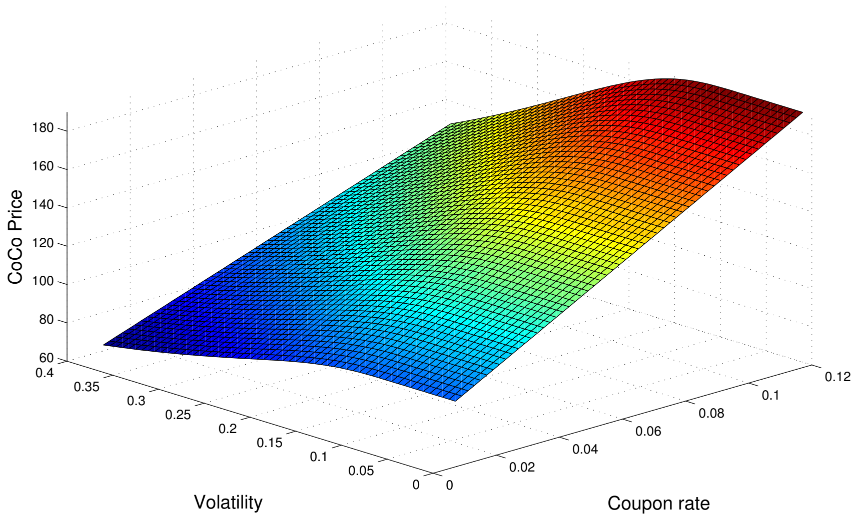

Figure 16: CoCo price in the Equity Derivative approach as a function of volatility and coupon rate

Comparing the result from Section 5.2 and 5.1, we find many similarities. Or rather, we find that the plots are roughly inversely related to each other since the figures display yield and price. First, the CoCo spread is positively related to volatility while the CoCo price is inversely related, which is expected since yield and price are inversely related to each other. The same holds for the trigger level S^*, in the equity approach (credit approach), the CoCo price (yield) is at the beginning a decreasing (increasing) function of S^* and then reach a point where it starts to increase (decrease).

5.3 Merton approach

In this section we will calculate the value of a fictive CoCo using the Merton approach described in Section 4.3. The fictive CoCo will be similar to the CoCo presented in Table 1 (and used in Section 5.1 and 5.2), but since the Merton approach uses, to a large extent, a different set of input variables, a completely identical CoCo would be impossible to construct. The variables used in the application of the Merton approach is presented in Table 9.

Variable		**Comment**
E_0	400	Market value of equity at time 0
# shares	150	Number of outstanding shares before conversion
σ_E	29%	Annual equity volatility
FV^{CoCo}	100	Face value of CoCos
FV^{SD}	550	Face value of the standard debt
H^{CoCo}	700	Conversion barrier for all CoCos
α	1	Entire value is converted
T	10y	Maturity of all CoCos and standard debt
r	1.94%	10y Swedish Government Bond (Riksbanken, 2012-04-04)
c_i	6.00%	Annual coupon rate
The following variables will be calibrated in Subsection 5.3.2		
V_t		Market value of assets at time t
σ_V		Volatility of assets
θ		Fraction of equity CoCo holder receives after conversion

Table 9: Fictive CoCo used in the Merton approach

We will need to calibrate a value for V_t, the market value of asset at time t, σ_V the annual asset volatility and θ the fraction of the bank a CoCo holder will own if the CoCos convert. This will be done explicitly in Subsection 5.3.2 followed by the actual calculations of the price in Subsection 5.3.3. Prior to this, we will recap how one can compute the CoCo price using the Merton approach.

5.3.1 Structure of the Merton approach

The Merton approach of pricing a CoCo is closely related to the original Merton model (Merton, 1974), a CoCo application was first developed by Hilscher and Raviv (2011) but this thesis has introduced some modifications to the original authors' work. In the Merton approach the issuing bank's assets is modelled by a GBM. By this assumption we can use the Black-Scholes dynamics to price a CoCo by replicating its payoff using binary barrier options written on the asset value. For a more detailed description of the replication see Subsection 4.3.3, in this subsection we will just state that a CoCo can be replicated using

1. **One *cash-or-nothing-at-maturity* binary barrier *down-and-out* call**
   that pays FV^{CoCo} at maturity T if the asset value does not fall below the barrier H^{CoCo} before T.

2. **One *cash-or-nothing-at-hit* binary barrier *down-and-in* call**
   that pays $\theta(H^{CoCo} - FV^{SD})$ if the asset value fall below the barrier H^{CoCo} before the maturity T. The option pays the cash amount exactly when hitting the trigger level, that is at time τ_{CoCo}.

3. **A portfolio of *cash-or-nothing-at-maturity* binary barrier *down-and-out* calls**
   with one option for each coupon, paying c_i at maturity t_i if the asset value does not fall below the barrier H^{CoCo}, where i goes from t to the maturity of the CoCo, T.

By using the notations in Equation (44) from Subsection 4.3.3 we can write the value of a CoCo as

$$
\begin{aligned}
V_t^{CoCo} &= CN_T^{do}\left(FV^{CoCo}, H^{CoCo}\right) \\
&+ CN_{\tau_{CoCo}}^{di}\left(\left[\theta\left(H^{CoCo} - FV^{SD}\right)\right], H^{CoCo}\right) \\
&+ \sum_{i=1}^{T} CN_{t_i}^{do}\left(c_i, H^{CoCo}, t_i\right).
\end{aligned} \tag{60}
$$

where each row in Equation (60) represent the option numbered 1, 2 and finally the option portfolio enumerated with 3. How Equation (60) can be quantified using the Black-Scholes framework is derived in Subsection 5.3.3. But first will we show how the missing variables in Table 9 can be calibrated.

5.3.2 Calibration in the Merton approach

To be able to compute the CoCo price using the Merton approach we need to calibrate a value for the market value of assets V_t, the annual asset volatility σ_V and the fraction of equity θ a CoCo holder will receive upon conversion. To calibrate V_t and σ_V we will use a method presented in Crosbie and Bohn (2003). In this method one solve for V_t and σ_V simultaneously by using Equation (61) and (62), displayed below. Solving non-linear equation systems is preferably done with a computer software and we have implemented this using MatLab. The logic behind the estimation of V_t using the Merton model is that the value of equity E_t can be seen as a call option written on the company's assets, with strike equal to the total face value of debt $D = FV^{SD} + FV^{CoCo}$. By following Crosbie and Bohn (2003), one can calibrate both parameters by simultaneously solving these equations

$$E_t = V_t\Phi(d_1) - e^{-rT}D\Phi(d_2) \tag{61}$$

$$\sigma_E = \frac{V_t}{E_t}\Delta\sigma_V \tag{62}$$

where

$$d_1 = \frac{\ln(V_t/D) + (r + \frac{\sigma_V^2}{2})T}{\sigma_V\sqrt{T}} \qquad d_2 = d_1 - \sigma_V\sqrt{T}$$

and Δ denotes the considered time period, which in this case is annually and thus $\Delta = 1$. If we solve Equation (61) and (62) simultaneously using Table 9, we receive a value for the assets of $V_t = 924.78$ and for the asset volatility $\sigma_V = 12.54\%$.

Next, we will calibrate a value for θ, the fraction of the bank's equity a CoCo holder will own if conversion occurs. This will be conducted as outlined in Subsection 4.3.2. Recall from Equation (41) that θ can be computed by this formula

$$\theta = \frac{FV^{CoCo}/C_p}{\text{Outstanding shares} + FV^{CoCo}/C_p} \tag{63}$$

where C_p denotes the conversion price. Since the value of equity at conversion, $E_{\tau_{CoCo}}$ is decided by the threshold level, H^{CoCo} and the face value of the debt, FV^{SD}, we have that

$$E_{\tau_{CoCo}} = H^{CoCo} - FV^{SD} = 700 - 550 = 150. \tag{64}$$

The number of shares outstanding is given in Table 9 and if we combine this with the equity value given in Equation (64) it will yield a share price at conversion S_τ of 1. Remember that we in the application of the Mertion approach, have deviated from the variables in Table 1 due to the differences in structure. But, to emulate the previous applications we will set the ratio between trigger level S^* and conversion price C_p equal. This ratio represents the "overprice" one pays for the shares when the CoCo converts. Thus,

$$\text{Overprice} = \frac{C_p}{S^*} = \frac{65}{35} = 1.857.$$

By this ratio, we can solve for the conversion price in this setting C_p^M when the share price at conversion S_τ^M equals 1.

$$1.857 = \frac{C_p^M}{S_\tau^M} \Rightarrow C_p^M = 1 \cdot 1.857 = 1.857. \tag{65}$$

Thus by inserting the conversion price in Equation (65) into Equation (63) we can compute a value for the fraction θ by

$$\theta = \frac{100/1.857}{150 + 100/1.857} = \frac{53.85}{100 + 53.85} = 26.42\%.$$

Hence, the CoCo holder (who is assumed to hold the entire issue of CoCos) will after conversion own 26.42% of the bank's equity, with the dilution effect included. All the variables calibrated in this subsection are summarized in Table 10.

Variable		**Comment**
V_0	924.78	Asset value at time 0.
σ_V	12.54%	Annual asset volatility.
θ	26.42%	The fraction of equity a CoCo holder will receive upon conversion.

Table 10: Calibrated variables in the Merton approach

5.3.3 Calculating the CoCo price in the Merton approach

We now have all the the variables needed to compute the price of our fictive CoCo using the Merton approach. Recall from Equation (60) that this will be done by calculating the value of two options and one option portfolio, all written on the assets of our bank and where each option being of binary barrier type. To price these options we will use the models presented in Rubinstein and Reiner (1991) and described in Subsection 4.3.4. As said, the value of a CoCo is given by

$$\begin{aligned} V_t^{CoCo} &= CN_T^{do}\left(FV^{CoCo}, H^{CoCo}\right) \\ &+ CN_{\tau_{CoCo}}^{di}\left(\left[\theta\left(H^{CoCo} - FV^{SD}\right)\right], H^{CoCo}\right) \\ &+ \sum_{i=1}^{T} CN_{t_i}^{do}\left(c_i, H^{CoCo}, t_i\right). \end{aligned} \tag{66}$$

The first component in Equation (66), the binary barrier call option that pays FV^{CoCo} at maturity T if the asset value does not fall under the barrier H^{CoCo}, is priced using the following formula

$$CN_T^{do} = FV^{CoCo}e^{-rT}\Phi(x_1 - \sigma_V\sqrt{T}) - FV^{CoCo}e^{-rT}(H^{CoCo}/V_0)^{2\lambda-2}\Phi(y_1 - \sigma\sqrt{T}) \tag{67}$$

where

$$x_1 = \frac{\ln(V_0/H^{CoCo})}{\sigma_V\sqrt{T}} + \lambda\sigma_V\sqrt{T}, \quad y_1 = \frac{\ln(H^{CoCo}/V_0)}{\sigma_V\sqrt{T}} + \lambda\sigma_V\sqrt{T} \quad \text{and} \quad \lambda = \frac{r - \frac{\sigma_V^2}{2}}{\sigma_V^2}.$$

By using the values from Table 9 and 10 we estimate the price as

$$CN_T^{do}\left(H^{CoCo}, FV^{CoCo}\right) = 40.06 \tag{68}$$

The second term in Equation (66) consist of a binary barrier call option that pays the value of the shares a CoCo holder receives at conversion. More specifically, it pays the conversion value in cash at the time the barrier H^{CoCo} is breached by the asset value, if this occurs before the maturity T. The value of this option is given by

$$\begin{aligned} CN^{di}_{\tau_{CoCo}}\left(\left[\theta\left(H^{CoCo} - FV^{SD}\right)\right], H^{CoCo}\right) =& \left[\theta\left(H^{CoCo} - FV^{SD}\right)\right]\Big[(H^{CoCo}/V_0)^{a+b}\Phi(z) \\ &+ (H^{CoCo}/V_0)^{a-b}\Phi(z - 2b\sigma_V\sqrt{T})\Big] \end{aligned} \tag{69}$$

where

$$a = \frac{\mu}{\sigma_V^2}, \quad b = \frac{\sqrt{\mu^2 + 2r\sigma_V^2}}{\sigma_V^2} \quad \text{and} \quad z = \frac{\ln(H^{CoCo}/V_0)}{\sigma_V\sqrt{T}} + b\sigma_V\sqrt{T}$$

If we insert the input from Table 9 and 10 we get a value of

$$CN^{di}_{\tau_{CoCo}}\left(\left[\theta\left(H^{CoCo} - FV^{SD}\right)\right], H^{CoCo}\right) = 14.23 \tag{70}$$

The third term in Equation (66) is a binary barrier option portfolio where each option is of the type *down-and-in*, *cash-at-maturity* and can thus be priced by Equation (67). Each option represent one coupon payment and hence, pays the coupon amount c_i at the maturity t_i if the asset value has not fallen below the barrier H^{CoCo} sometime during the option's lifetime. The value of each option and the total value of the portfolio is displayed in Table 11.

t_i	x_{1i}	y_{1i}	$CN^{do}_{t_i}$
1	2.44	-2.00	5.72
2	1.88	-1.26	5.07
3	1.66	-0.91	4.47
4	1.54	-0.68	3.99
5	1.48	-0.51	3.60
6	1.44	-0.37	3.28
7	1.41	-0.26	3.01
8	1.40	-0.17	2.78
9	1.39	-0.09	2.58
10	1.39	-0.01	2.40
		Sum	**36.89**

Table 11: Option portfolio - Merton approach

If we insert the results from Equation (68) and (70) and Table 11 into Equation (66) we get the following price for our fictive CoCo

$$V_0^{CoCo} = 40.05 + 14.16 + 36.89 = 91.18$$

which is thus 91.18% of the par value. We have now by using real numbers showed how one can estimate the value of a CoCo. This result rely heavily on the same assumption as in the Merton (1974) model, since the model presented above is an extension of that. Comparing this result with the other models' will not provide any additional understanding as the approaches widely differs.

5.3.4 Sensitivity - Merton approach

In this subsection we will analyze how the CoCo price, calculated by the Merton approach, responds to changes in some of the underlying variables. This is done by the visual illustrations presented in Figures 17, 18, 19, 20 and 21. Figure 17 displays the CoCo price as a function of the annual asset volatility σ_V for different levels of initial asset value V_0. We can notice that the CoCo price is a decreasing function of asset volatility, which appear to be reasonable since if the asset volatility increases the CoCo gets riskier and an investor would thus want to pay a lower price for it. Also with lower initial asset value the distance to conversion gets smaller and hence the conversion probability increases while the price decreases. To further illustrated the relationship between asset volatility, initial asset value and CoCo price we are in Figure 18 displaying this relationship in a 3D setting.

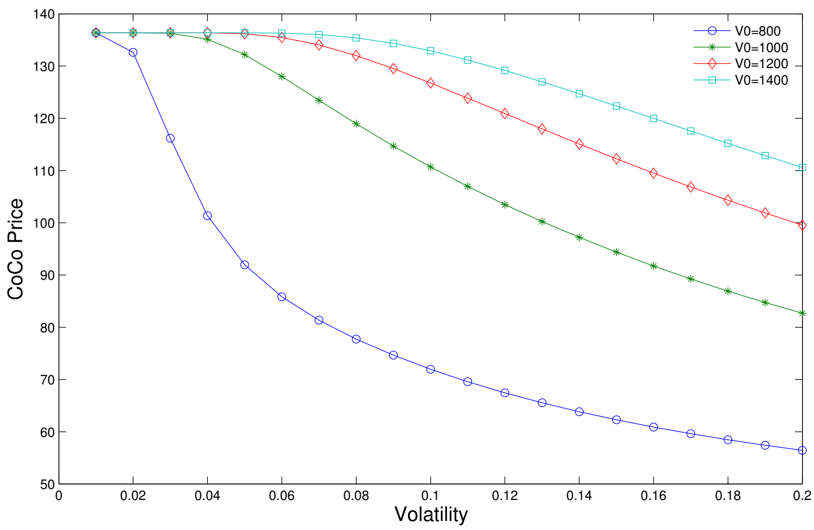

Figure 17: CoCo price in the Merton approach as a function of volatility σ_V for different levels of initial market value of asset V_0

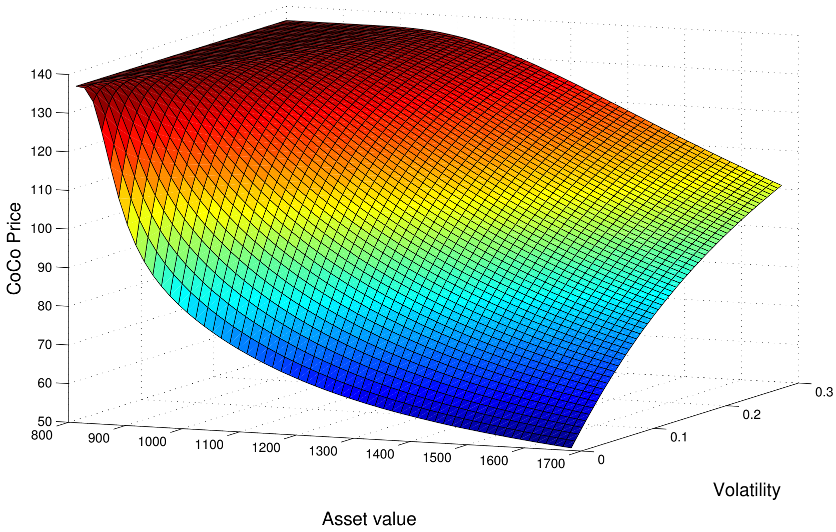

Figure 18: CoCo price in the Merton approach as a function of volatility σ_V and the initial market value of asset V_0

Figure 19 shows how the CoCo price reacts to changes in the coupon rate in four different scenarios regarding the share in the underlying company θ an investor receives upon conversion. This function is linearly increasing with the coupon rates. We also note that there is parallel shifts upwards in the function when θ changes. This means that the larger part of the company a CoCo holder would own if conversion occurs, the more the CoCo holder is prepared to pay for this position. This seems valid due to that the more shares a CoCo investor receives upon conversion the smaller the actual loss of conversion becomes. Hence, the larger part of the underlying company a CoCo investor gets, the smaller risk he or she takes and the higher price is the CoCo investor prepared to pay.

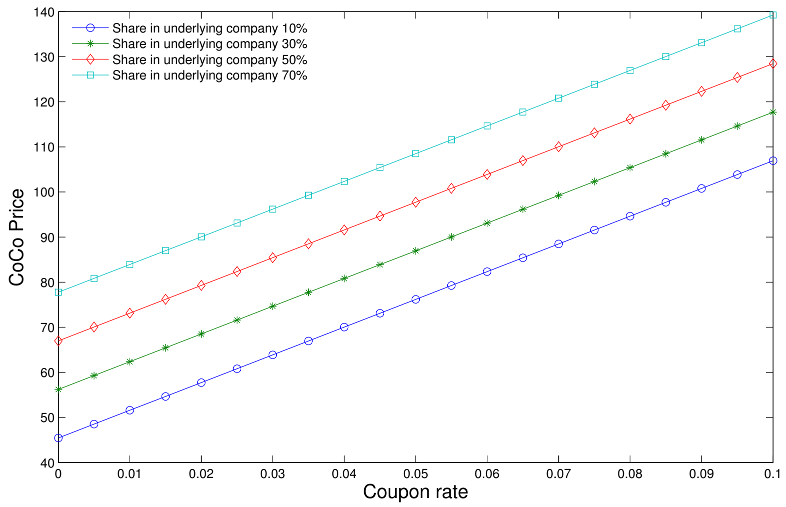

Figure 19: CoCo price in the Merton approach as a function of coupon rate for different level of the fraction of equity θ that a CoCo investor receives upon conversion.

Figure 20 and 21 presents the CoCo price as a function of the maturity, for CoCo with four different levels of asset volatility and coupon rates. One might notice that Figure 20 decreasing for small Ts and increasing for larger Ts apart from when $\sigma_V = 5\%$. This structure is due to the relatively high coupon of 6%. With a lower coupon rate we see a curve that flattens out and that does not increase with higher maturities, this is illustrated in Figure 21. This can be explained by the fact that with a high coupon rate the expected cash flow increase from getting one extra coupon is higher than the expected cash flow decrease created by the higher probability of conversion, $\mathbb{P}[\tau > t_i]C_i > [\mathbb{P}[\tau \leq t_i] - \mathbb{P}[\tau \leq t_{i-1}]]\,\theta E_\tau$.

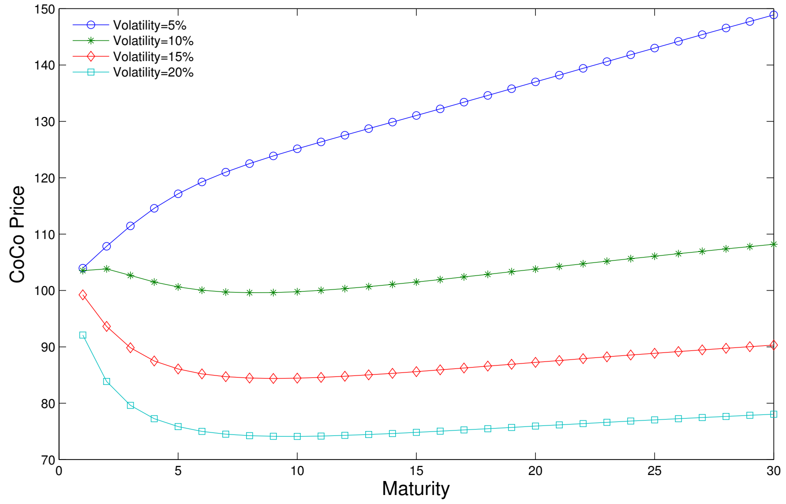

Figure 20: CoCo price in the Merton approach as a function of Maturity T with different levels of σ_V.

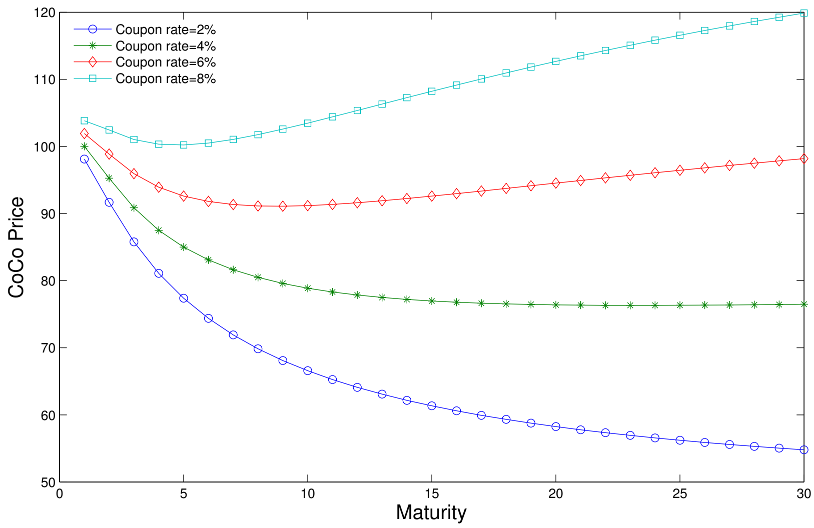

Figure 21: CoCo price in the Merton approach as a function of Maturity T with different levels of coupon rate.

5.4 Application on Swedbank

On March 27 2012, Swedbank, one of the largest banks in Sweden, announced they will consider issuing CoCos the coming years. Given this, it is of great interest for potential investors of Swedbank's CoCo to have a grasp of how such Contingent Convertible could be priced. To this end, we will use the two first models presented in previous sections to compute prices and yield of a potential issue. As detailed terms of the attended issue are undisclosed or yet to be determined, our application will build on several assumptions regarding the structure of the issue. Since we in Section 5.1 and 5.2 have thoroughly gone through how to value a CoCo with real numbers, we will in this section assume that the reader will follow our outline without the guidance of repeating the underlying formulas and theory.

We start with the very core of contingent convertibles, the trigger. Since CoCos with regulatory trigger is by definition very hard to model and it is also very hard to perfectly model the full dynamics of accounting triggers, the following section will therefore assume that a potential future issue from Swedbank will have a market-based trigger on the share price.

Beginning in 2007, Swedbank's share price started to decrease and with the collapse of Lehman Brothers in 2008, the share continued to fall. During the first months of 2009, the share traded at a tenth of the top notations in 2007. Due to this turbulence, Swedbank announced just a few months after Lehman they would participate in the Swedish governmental guarantee program (Swedbank, 2008). Under this program they issued a total of SEK 420 billion of debt (Swedbank, 2010) and with that, the share reached a low point of around SEK 20 before it started to recover. We will in our application design a trigger event such that the CoCo converts to equity when the share price falls below 30. We argue that at this level, Swedbank would need an injection of equity capital if governmental aid is to be avoided. In Table 12 we define all variables that is needed for pricing a CoCo issued by Swedbank.

Variable		**Comment**
S_0	101	Share price as of 2012/04/04
N	100	Face value of CoCo
σ	43%	Yearly average equity volatility since 2003
T	10 y	Assumed to be issued 2012/04/04 with maturity in 10 years
r	1.94%	Swedish government 10y bond as of 2012/04/04
S^*	30	Share price at which conversion will occur, trigger level.
q	5.95%	Dividend yield 2011
C_p	65	Middle point between S^* and S

Table 12: Parameters for a CoCo issued by Swedbank

If the CoCo does not convert into a fixed number of shares and the total number of received shares is determined by a variable conversion price, existing shareholders would be pleased if $C_p = S_0$, as dilution is minimized. Remember that S_0 is the share price on the day the CoCo is issued. On the other hand, CoCo investors prefer C_p as close to S^* as possible, since they receive more shares and the potential upside is maximized. Since it is difficult in advance to predict which way Swedbank will go, we have set $C_p = \frac{S+S^*}{2} = 65$, which can be seen as a "middle way" alternative. The conversion ratio is thus assumed to be 100/65=1.54, or 1.54 shares per converted CoCo.

5.4.1 Swedbank application with Equity Derivatives approach

To estimate a possible coupon rate one can use a current standard bond issued by Swedbank with matching maturity and add an "appropriate" spread over the existing coupon rate. Although this might be more or less guessing, it gives some idea of where to start your own estimate. Another possibility is to calibrate the coupon rate from the Equity Derivatives approach; what coupon rate does Swedbank have to provide for a CoCo with the properties as in Table 12 to be issued at par? Using this, we can back out an implied coupon that Swedbank has to give investors. With a conversion price equal to 65 and by setting the CoCo price equal to 100, and solving for c_i, we estimate that the potential issue must bear a coupon rate of 10.13% to be issued at par when using the Equity Derivative approach. In Figure 22 the result is display graphically, the figure further present how the par-coupon rate varies with the choice of conversion price. By letting $C_p = S^* = 30$, the CoCo investors are better off as why he/she "demands" a lower coupon rate just above 5%. The opposite holds for conversion prices above 65.

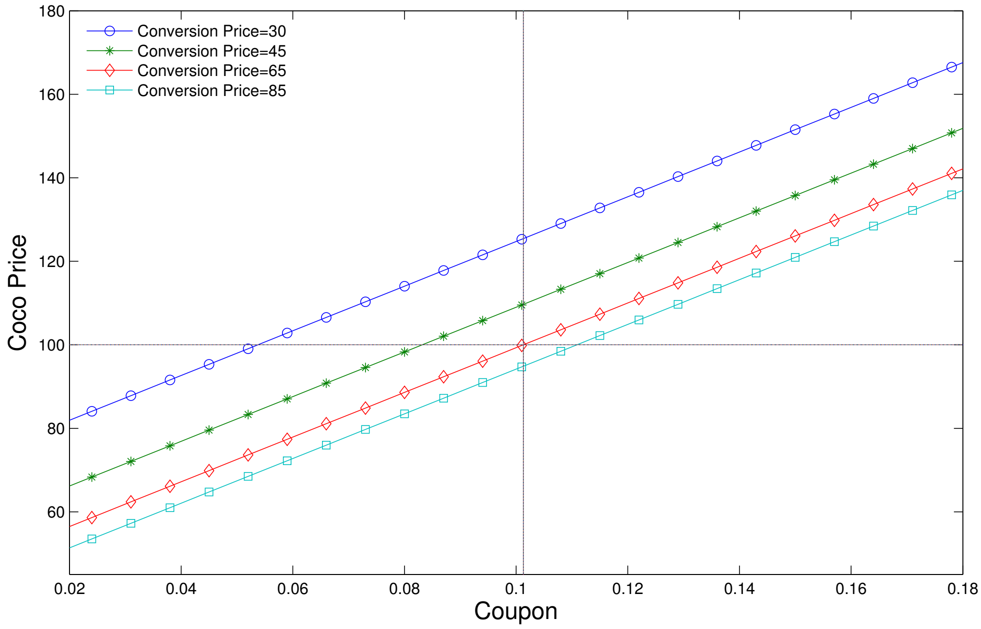

Figure 22: Swedbank application, CoCo Price as a function of coupon rate c_i for different conversion price C_p.

Using the CoCo presented in Table 12 with the estimated coupon rate of 10.13% we can now begin to value the different parts in the Equity Derivative approach, starting with the corporate bond and the barrier *cash-or-nothing* options that cancels out coupons received after conversion has occurred. Although we know what the price will be, these tables are displayed for transparency.

t_i	CF_i	e^{-rt_i}	PV
1	10.13	0.98	9.93
2	10.13	0.96	9.74
3	10.13	0.94	9.56
4	10.13	0.93	9.37
5	10.13	0.91	9.19
6	10.13	0.89	9.02
7	10.13	0.87	8.84
8	10.13	0.86	8.67
9	10.13	0.84	8.51
10	110.13	0.82	90.71
		Sum	**173.55**

Table 13: Swedbank Corporate bond and its cash flows.

t_i	x_{1i}	y_{1i}	$CN^{di}_{t_i}$
1	2.94	-2.7	0.11
2	2.17	-1.82	1.00
3	1.84	-1.42	2.13
4	1.66	-1.17	3.14
5	1.53	-0.99	3.94
6	1.45	-0.85	4.57
7	1.39	-0.74	5.05
8	1.34	-0.65	5.42
9	1.31	-0.58	5.70
10	1.28	-0.51	5.91
		Sum	**-36.97**

Table 14: Barrier *down-and-in cash-or-nothing* options (Swedbank).

By applying Equation (35) we can also compute the value of the barrier *down-and-in* option portfolio. The value of such a portfolio in our Swedbank setting is equal to -23.78, but now there are 1.54 such portfolios included since $\frac{\alpha N}{C_p} = \frac{1 \cdot 100}{65} = 1.54$. The total option portfolio is thus worth $-23.78 \cdot 1.54 = -36.58$.

Hence, by making use of the Equity Derivative approach, an investor can calibrate an appropriate coupon rate to use in his or hers valuation, before the actual specific terms of the CoCo has been announced. By using the par-coupon rate and Equation (38), the value of each of the components in the Equity Derivative approach is

$$173.55 - 36.58 - 36.97 = 100. \tag{71}$$

In Figure 23 the movement of the Swedbank share price together with five different versions of the corresponding fictive CoCo price are displayed. The CoCo is assumed to be issued 1^{st} January 2000 and its price is re-calibrated daily with the current market share price of Swedbank. In order for the CoCo not to mature within the data period, we have extended the maturity to 20 years. The price is re-calibrated every day for every trigger level S^* by using Equation (38). Furthermore, in Figure 23 the event of a trigger level being breached and the CoCo thereby converted into equity is represented by the price droping to zero. One can see in Figure 23 that a high trigger level of 80, the CoCo converted as early as July 2002. All fictive CoCos except the one with $S^* = 10$ would have been converted after the tremendous fall in share price in 2007-2009.

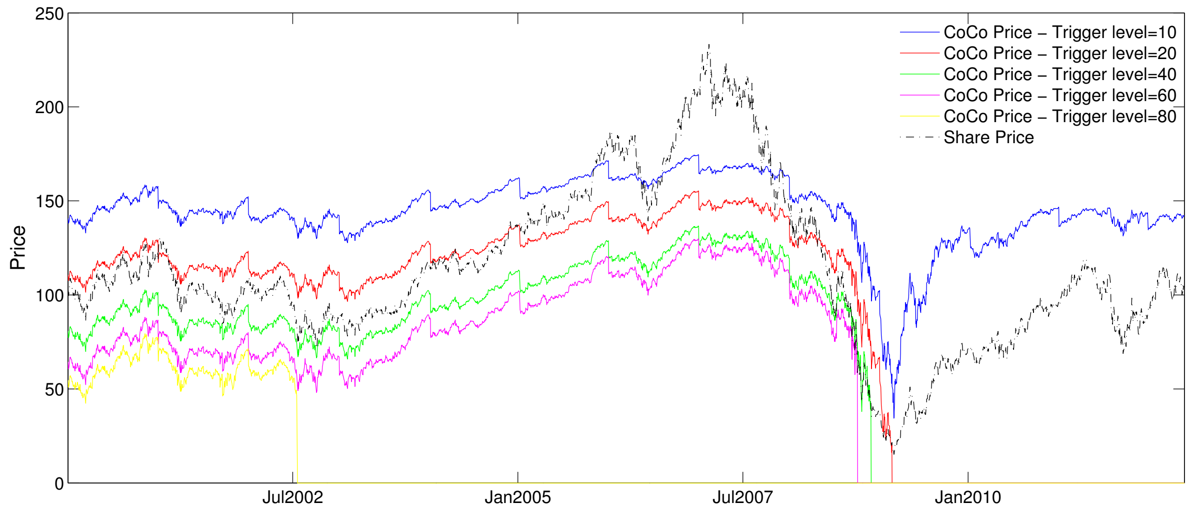

Figure 23: Swedbanks share price from 2000 to spring 2012 with corresponding CoCo price for five different trigger levels.

5.4.2 Swedbank application with Credit Derivatives approach

If we use the Credit Derivative approach to calculate the value of Swedbank's potential CoCo we get a slightly differing result. First of all, the Credit Derivative approach only returns a yield value, which has to be converted into a price for it to be possible to compare the two methods. Secondly, it is only in the conversion from yield to price that the coupon rate from Figure 22 is used. This might be two of the reasons for the slightly different results. To calculate the CoCo spread we follow the method from Section 5.1 and apply the values from Table 12 as inputs. The result of this computation is presented in Table 15.

Variable	**Result**
p^*	70.89%
λ_{CoCo}	0.12
ϕ_{CoCo}	46%
CoCo-spread	674.88 bp
Yield	8.69%

Table 15: Swedbank application; Credit derivative approach

If we convert the CoCo spread of Swedbank's CoCo to a price, we end up with a price above par even though we apply the par coupon rate. The calculation of all the present value of the CoCo's cash flows are displayed in Table 16. A yield of 8.69% leads to a price of 106.73% of the face value. As the coupon rate is higher than the yield, a price above par is expected.

t_i	CF	e^{-Yt_i}	PV
1	10.13	0.98	9.29
2	10.13	0.96	8.51
3	10.13	0.94	7.80
4	10.13	0.93	7.16
5	10.13	0.91	6.56
6	10.13	0.89	6.01
7	10.13	0.87	5.51
8	10.13	0.86	5.05
9	10.13	0.84	4.63
10	110.13	0.82	46.19
		Sum	**106.73**

Table 16: From yield to price (Swedbank)

5.5 Analysis

In this section we will analyze the three models presented in Section 4 as well as comparing the results from Section 5.1 to 5.3. We aim at distinguishing important structural differences and how they affect each result. This will also create important insights for a CoCo investor that wants to hedge his/her position. A mutual factor entering into every model is the Black-Scholes framework. All assumptions behind that framework will thus also enter into the models presented within this thesis. We are well aware of the limitations when modelling prices through the Black-Scholes. For example, it assumes that returns over time intervals are independent and normally distributed while in the real world asset price data often inhabits heavy-tailed returns, volatility clustering and long-term correlated returns. The fact that constant volatility is used in the models is a underlying weakness. Due to the "non-fat" tails of the Gaussian normal distribution that the model relies on will in most cases underestimate the probability of large share price movements, and thereby also underestimate the conversion probability within CoCos. However, the benefits of the Black-Scholes framework is that all expressions are given by analytical closed-form expressions.

Regarding the Credit Derivative model, when calibrating the conversion intensity λ it is assumed that the conversion probability is a deterministic function of time. By modelling λ_t with a stochastic process allowing for jumps, one could capture the problem with fat-tails and jumps in asset prices in a more satisfying way. But assuming a deterministic conversion intensity variable is far more practical from an implementation point of view. Furthermore, the common assumptions in CDS estimation to ignore the accrued premium term and that the loss is paid at the end of the default quarter, does simplify the computations but also reduce the accuracy. Especially when we have annual cash flows. Disregarding the accrued premium will decrease the cash flows from the *protection buyer* slightly and thus marginally overestimate the spread. Moving the payments until the end of the default quarter yields a larger discount factor on the cash flows from the *protection seller*, and hence, overestimates the credit spread. However, also recall that a CoCo is de facto not a CDS, so the above simplifications do not need to be so imperfect after all.

In the Equity Derivative model, one replicates the equity part by using *knock-in* forwards. Although the shape of the combined payoff function make them valid substitutes to the actual stock, they are not perfect replicates. First, as CoCos most probably will be clustered among large institutional investors, the proportion of equity share held after conversion can be substantial. They could thereby to some extent control the company if the stake is large enough and this fact is completely ignored if the investor is assumed to end up with forwards. Furthermore, to replicate the conversion with *knock-in* forwards creates yet another discrepancy with a real CoCo. Upon conversion, one will effectively hold C_r forward contracts written on the underlying share and a zero-coupon bond. As we mentioned in Subsection 4.2.2 this yields an erroneous value of the equity part as the position in the model is worth $C_r S_f$ and not $C_r S^*$ as it should be. Also, as the forwards will have matching maturity of the original CoCo, the longer it is between conversion and maturity the larger is the uncertainty factor regarding the difference in the future share price and the spot will be. To address this issue we propose a slight alteration of Spiegeleer and Schoutens (2011) Equity Derivative model. By exchanging the bond with a barrier *cash-or-nothing* call that pays N at the CoCos maturity if the share price does not fall below barrier S^* before T, we can model the conversion into equity more accurately. This option is denoted $CN_T^{do}(N, S^*)$. Further, by changing the *knock-in* forward against C_r barrier *down-and-in asset-or-nothing at hit* calls, we model the conversion with an option that pays the underlying share if the barrier S^* is breached before T. This option is denoted with $AN_\tau^{di}(S, S^*)$. We suggest the coupons to be replicated as in our modified Merton model. That is, with one *down-and-in cash-or-nothing at maturity* call with barrier S^* for every coupon payment. These options pays the coupon c_i as long as the barrier S^* is not breached before t_i. Adding the three components together will thus yield the following replicating value of a CoCo

$$P_t^{CoCo} = CN_T^{do}(N, S^*) + AN_\tau^{di}(S, S^*) + \sum_{i=1}^{T} CN_{t_i}^{do}(C_i, S^*, t_i).$$

Regarding the Merton (1974) based model, it inhabits the same underlying flaws as Black-Scholes. Among other things, it assumes the asset value to follow a stochastic process (GBM) that in most cases underestimates the probability of tail-events. In our setting, we model the path of assets through time, but a more realistic approach, when working with CoCos, might be to model the behaviour of the risk-weighted assets. Furthermore, calibrating market value of asset and asset volatility simultaneously with Equation (61) and (62) gives a good picture of the current risk situation but is sensitive to quick changes in market leverage (Crosbie and Bohn, 2003). As the investor in our model is assumed to end up with cash instead of stocks, the model requires a liquid market. Even though developed markets are fairly liquid, to assume that an investor could sell a large stake without cost is rather far-fetched and an assumption like this will probably lead to an overestimation of the value.

6 Conclusions

We have in this thesis analyzed three different approaches on how to price CoCos. The Credit Derivative - and the Equity Derivative approach introduced by Spiegeleer and Schoutens (2011) and the Merton (1974) based model of Hilscher and Raviv (2011) have been thoroughly examined to understand the underlying components. We have gained deeper knowledge on how to model and replicate the dynamics of Contingent Convertibles and by this been able to develop and present new versions for potentially better accuracy. The models presented will not only be helpful tools when valuing CoCos but also give insight to investors seeking to hedge investments in CoCos. We have in this thesis only used geometric Brownian motions to model the path of the underlying assets in the models. This is very handy since it provides us with closed-form solutions, but it may be more realistic to use e.g. Lèvy process to model asset or stock price dynamics. This is a topic for future research.

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