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Investing in commodities

Will commodity futures enhance risk-adjusted return in efficient portfolios?

Bachelor thesis, Financial Economics, 15 HP

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Abstract

With this paper we intend to investigate what kind of benefits there are by adding commodity futures to a well-diversified portfolio. Since the last fifteen years the commodity speculation has grown tremendously, which partially can be explained by that commodities exposes the investor to certain factors other than an investment in equities. According to our calculations the commodity futures have outperformed stocks during our research period, which partially could be explained by the increasing demand of physical commodities in developing countries e.g. India & China (Akey, 2005). By constructing different portfolios consisting of equities and corporate bonds we could investigate whether our portfolios will benefit from commodity futures and how this will vary over different levels of risk. By using monthly data; 2002-2012, we have concluded that by adding commodity futures to efficient portfolios, the return-to-volatility increases. We have established that gold is a superior investment among the commodity futures, during our sample period and geographical area. We have also concluded that a Norwegian portfolio consisting of stocks and bonds benefit more, in terms of return to volatility, than a Swedish portfolio by adding commodity futures.

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1 Introduction

In the last years using commodity futures as an alternative investment in portfolios for diversification purpose has become very popular. With new instruments and derivatives using all kinds of commodities as underlying assets, it is nowadays as easy for an individual speculating investor to trade commodity derivatives, as it is for an institutional investment bank with decades of knowledge within the industry. The vast majority of investors using commodity futures are institutional or commercial users that are hedging against price movements in order to reduce financial loss. The other part of investors using futures in their portfolios is most likely individuals who are speculating in the price movements of the commodity (Investopedia, 2012). Speculating in price movements will demand that the future contract is needed to be closed out before the maturity date; otherwise the speculator might end up with 100 barrels of oil in physical form.

However, one can argue that commodity futures can add diversification benefits to portfolios because of many different reasons. With this paper we intend to investigate these benefits from a perspective based on modern portfolio theory and to provide the reader with empirical results based on historical data.

Stocks, mutual funds and bonds belong to the more traditional asset classes, while commodity futures together with hedge funds and private equity form a more alternative way to invest funds (UBS, 2011). Combining different asset classes give rise to different covariance relationships. Why we found it interesting and important to see how commodity futures affect efficient equity portfolios is because of the covariance between commodity futures and stocks/bonds often tend to be low, and *if* this relationship could be exploited to improve portfolio performance. Another aspect of the importance is that commodity investing, has grown rapidly in volume the latest years and our results might provide an explanation for this increase (CBOE Futures Exchange, 2012). In the following section we will present earlier studies on the field. The question we want to answer follows; will commodity futures increase the risk-adjusted return in efficient portfolios using assets from the Swedish and Norwegian equity markets? Why we compare two different markets is to open up for analysis and see if our hypotheses differ between the two areas. We intend to compare the results from the Swedish stock market with the results from the Norwegian stock market. Our study will be based on years 2002-2012 to receive an "up-to-date" research within the field, and during a more concentrated sample period than many of the previous studies mentioned below.

1.1 Earlier empirical work

This section explains the relevance of the thesis contents and shows earlier research results from different academic sources. The research about combining efficient portfolio with specific commodities has been done many times before and a majority of the studies has reached the same conclusions, which is that commodities will provide the efficient portfolio with a higher average return and a lower average variance/standard deviation.

Several researchers argue that due to the low correlation between commodities and stocks/bonds, the diversified portfolio will receive a lower standard deviation (Georgiev, 2001), or significant return enhancements at all levels of risk (Jensen, Johnson, & Mercer, 2000). According to previous empirical results; by combining a diversified portfolio consisting of U.S. stocks and bonds with a commodity index, it reduced the standard deviation by 0.90 percent while the Sharpe ratio was maintained (Georgiev, 2001). The study tested the same approach with a global portfolio, then the standard deviation was reduced by 0.50 percent and the Sharpe ratio did slightly improve (Georgiev, 2001).

There are several studies that have reached the same results, that the Sharpe ratio will be higher or maintained with a lower standard deviation. Another study concluded that an equally weighted portfolio consisting of a commodity index and S&P 500 assets, received a higher compounded return and lower standard deviation compared to a stand-alone investment of S&P 500 assets, the data that was used stretched from 1969 to 2004 (Erb & Campbell, 2006). The conclusion was that because of the negative correlation between the assets and low transaction costs the combined portfolio was more efficient (Erb & Campbell, 2006).

This covariance relationship does often add benefits to the portfolio such as higher risk-adjusted return and lower risk. Including commodity futures to an efficient portfolio during periods of restrictive monetary policy, enhances return at all levels of risk. While during periods of expansive monetary policy, including futures to efficient portfolios has no return enhancements at all (Jensen, Johnson, & Mercer, 2000). However we will not investigate the effect of monetary policy on commodity investment in this paper.

1.2 Hypotheses

A discussion will be established further on in this paper, we have the intention to answer some hypothesizes regarding commodity futures, these are essential in order to find answers and conclusions regarding our general hypothesis; *will commodity futures enhance risk-adjusted return in efficient portfolios?*

There have been many discussions and speculation regarding gold future investments, and investors share different point of a view. Earlier empirical work shows that a combination with stocks/bonds & commodity futures will increase the return-to-volatility, since the gold price has increased greatly since the last ten years (Figure A1 in Appendix), we believe it will be an outstanding investment.

Commodity prices are mainly set by market supply and demand; since the Norwegian economy is highly based on the oil industry it will probably have high correlation with the commodity indices. Therefore we believe that the Norwegian portfolio will not benefit as much as the Swedish portfolio, because of the fact that the correlation between assets and commodities in Sweden will be generally lower, thereof a higher return-to-volatility will be expected.

During the 21st century the global equity markets have suffered from major financial crises. At the same time emerging economies like China and India started to expand their supply of physical commodities in order to develop infrastructure etc. This has increased the general commodity market price and therefore it is possible for speculating investors to make huge profits. Due to a higher demand of physical commodities and big losses at the global financial markets, we believe that a stand-alone investment in commodities will be superior in terms of return-to-volatility compared to other assets like stocks and bonds during our research period.

The facts and issues presented above are the foundation of the three hypotheses that we have constructed. As earlier mentioned, in order to create a discussion regarding the main question, we need a base to proceed from and these hypotheses will accomplish that.

1.2.1 Hypothesis 1

Gold futures will outperform the other commodity futures in terms of return-to-volatility.

1.2.2 Hypothesis 2

The Norwegian portfolio will *not* benefit as much as a Swedish portfolio by adding commodity futures, in terms of return-to-volatility.

1.2.3 Hypothesis 3

A stand-alone performance of commodity futures during our research period will be better than stocks and bonds on average.

2 Theory

2.1 Modern Portfolio theory

Modern Portfolio Theory assumes investors to be risk-averse i.e. if an investor can choose between two portfolios with the same return; he will choose the one with lowest variance or risk. A risk-averse investor will avoid adding risky securities to the portfolio, if not compensated by higher returns depending on the degree of risk-aversion (Investopedia). Before we can construct a portfolio we have to anticipate future returns of securities and also the variance of the returns. Assuming a risk-averse investor, we will think of future expected return as a *wanted* thing, and variance of the return as *unwanted*. A risk-averse investor wishes to maximize the future expected return and to minimize variance of returns i.e. the risk (Markowitz, 1952).

Further there is an incentive for the investor to diversify among assets and to maximize the expected return. The investor should therefore diversify funds over the assets resulting in the highest expected return (Markowitz, 1952, p. 79). The portfolio with the highest expected return is not necessarily the one with lowest variance, which is what the investor would try to achieve. The portfolio with highest expected return might also have a high variance of the expected returns and therefore the investor can reduce variance by giving up some of the expected return. The E-V rule (expected return – variance) impose that the investor will chose a portfolio that increases the E-V relationship i.e. the portfolio with minimum variance given the expected return, or the maximum expected return given the variance (Markowitz, 1952, p. 82). In line with the E-V rule, the investor will choose a portfolio somewhere on the efficient frontier and can maximize the trade-off between expected return and variance at some point along this frontier.

Combining these facts we can conclude that there is a purpose of diversification among assets which will result in the highest expected return – variance relationship. There are many ways to measure the return-variance relationship, but in this paper we will focus on the Sharpe-Ratio and the mean variance optimization concept to construct portfolios.

2.2 Sharpe Ratio

The Sharpe ratio measures the reward-to-volatility ratio investing in a risky asset over a risk-free asset. In other words the ratio provides you with the portfolio's excess return per unit of risk. It is a risk-adjusted measurement that allows you to compare different assets with different risk and therefore it is a good measurement for portfolio evaluation. The Sharpe-ratio is calculated by dividing the risk-premium (the expected return of the portfolio subtracting the risk-free rate) with the portfolios standard deviation. The Sharpe ratio could get a negative value, but then the asset is underperforming the risk-free rate. (Bodie, Kane, & Marcus, 2011, pp. 161, 234)

$$SR = \frac{\bar{R}_p - R_f}{\sigma_p}$$

2.3 Portfolio optimization

The goal within modern portfolio theory is to optimally allocate your invested funds between different assets. The mean-variance optimization (MVO) is a quantitative analysis tool, which takes the risk to volatility measure into account when allocating resources. The target is to maximize the mean return for the portfolio at the lowest level of risk, or to minimize the level of risk at a given level of return. Optimization is highly dependent on the covariance between the assets, so an asset used for hedging purposes must have negative correlation with the assets itself. If a portfolio has less than perfectly correlated assets it will always provide better risk-return relationship than holding the individual assets by themselves. As the correlation becomes more negative the greater the gains in efficiency of the portfolio are. (Bodie, Kane, & Marcus, 2011, p. 232)

2.3.1 Optimal risky portfolio

The optimal risky portfolio is a combination of risky assets that gives the best risk-return trade-off (Bodie, Kane, & Marcus, 2011, p. 224). At the point where the Capital Allocation Line (CAL) tangents the efficient frontier we find our optimal risky portfolio, and also the highest possible Sharpe-ratio of the portfolio.

$$Max \frac{(R_P - r_f)}{\sigma_p} \; subject\ to \sum w_N = 1$$

N symbolizes the number of assets in the portfolio.

2.3.2 Portfolio Variance

The variance of a portfolio consisting of 2 risky assets could be calculated through:

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(R_1R_2)$$

However, as the number of assets increases the number of covariance terms increases rapidly and the matrix grows with one grade for each asset added. The variance for a portfolio consisting of n assets could be written as WvW^T where W is the column vector containing the different weights of the assets, V is the covariance matrix of the assets and W^T is the transpose of the matrix W (Pareek, 2009).

$$Portfolio\ variance = [w_1 \dots w_n]\ x \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_n^2 \end{bmatrix} x \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$

One can simplify this optimization problem using Excel add-ins which we present in Appendix 11.1.

2.4 Commodities

A commodity is a physical item that is usually used as an input to produce other goods or services. However, a commodity is as well a traded financial asset on the commodity exchange markets in the same way as other financial securities e.g. bonds and stocks (Investopedia, 2012).

The spot prices of commodities are mainly determined by supply and demand. For instance, during the financial crisis in 2007 when the global economy went into a bad recession the price on rice went up significantly (The World Bank, 2012). This kind of scenario could be explained by; during that time period the consumers had less cash-flow and therefore they will consume cheaper products. However, if a lot consumers act the same way, the demand on rice will go up which will result into that the price will increase.

2.5 Commodity investment vehicles

2.5.1 Exchange Traded Funds (ETF)

An easy way to get access to the commodity markets is to invest in an Exchange traded commodity fund. Funds like these may be iShares GSCI Commodity-Indexed Trust Fund (iShares, 2012) or PowerShares DB Commodity Index Tracking Fund (Deutsche Bank, 2012). These funds have the objective to track a commodity index related to each fund. Buying shares in an exchange traded index tracking fund allows the investor to "buy the market" within a single investment. Instead of investing in many single commodity futures, the investor can use an ETF to receive a diversification among many sectors of commodities.

2.5.2 Mutual Funds

Another was for an investor to get easy access to commodity markets is to invest in a mutual fund. A mutual fund pools together funds from many investors to invest in assets such as stocks, bonds and other derivatives e.g. commodity futures (Investopedia, 2012).

2.5.3 Equities in commodity based companies

The most common or traditional way for investors to get access to the benefits of commodity exposure is to invest in companies which operate in a commodity intense industry (Jensen & Mercer, 2011, pp. 3-4). E.g. investing in Lundin Mining will give the investor exposure to precious metals prices, or investing in Statoil will give you exposure to oil prices. It is important to point out that this kind of investment is not only dependent on commodity prices, but also the performance of the company itself. Even if oil prices are rising, it does not necessarily mean that the Statoil stock will rise.

2.5.4 Future contract

A future contract is a standardized contract that allows the investor to buy or sell an asset in the future at a pre-determined price i.e. the future price (Hull, 2011). Generally these transactions are made on the future exchange and the underlying assets are commonly a commodity or another kind of financial instrument. As mentioned before, these contract are standardized which implies that certain requirements must have been established, e.g. the quality of the asset, the amount, the delivery date and location of the delivery (Hull, 2011).

Speculating investors heavily trade this kind of contracts, however the actual delivery of the underlying asset rarely happens. Investors tend to close out their position before the maturity of the contract, in order to make profits (Hull, 2011). The difference between the actual spot price of the asset, and futures price of the contract is called the *basis*. At the expiration date of the contract the basis should be zero if the no arbitrage condition holds, but before the expiration the basis may be both positive and negative which exposes the investor to a kind of risk, basis risk (Hull, 2011).

2.6 Goldman Sachs Commodity Index (GSCI)

"The S&P GSCI is a composite index of commodity sector returns representing an unleveraged, long-only investment in commodity futures that is broadly diversified across the spectrum of commodities" (Goldman Sachs, 2013). The index is divided into five different commodity-types (Energy, Industrial-metals, Precious-metals, Agriculture and Livestock) where Energy is the highest weighted with almost 80% of the index (Morningstar, 2007). The index is world-production weighted which means that the weight in the index of each commodity is determined by the average produced quantity of each commodity the last five years.

2.7 Test for normally distributed returns

The mean variance optimization assumes the returns to be normally distributed i.e. that there is no skewness in the distribution of returns. Testing for this assumption it is possible to use Shapiro-Wilk test for normality by first stating a null hypothesis and an alternative hypothesis.

$H_0 = The\ returns\ are\ normally\ distributed$
$H_1 = The\ returns\ are\ not\ normally\ distributed$

If the p-value of the test is lower than the chosen significance level (5%) it is not possible to reject the null hypothesis, i.e. one can conclude that the data is normally distributed. A test statistic (W) close to one indicated that the data is normally distributed as well. The test statistic for Shapiro-Wilk test is

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$x_{(i)}$ is the ith order statistic (smallest number in the sample)
$\bar{x}$ is the sample mean
a_i constants are given by $(a_1, \ldots, a_n) = \frac{m^T V^{-1}}{(m^T V^{-1} V^{-1} m)^{1/2}}$
$m = (m_1, \ldots, m_n)^T$ m_n are the expected values of the *order statistics* of independent and identically distributed random variables sampled from the standard normal distribution.
V is the covariance matrix of the order statistics

2.8 Criticism of MPT

As with many theories, modern portfolio theory is based on certain assumptions to make the model applicable in practice. Sometimes assumptions can be fully possible to achieve in theory but not in the real world. Examples of these assumptions are that stock returns generally follow a normal distribution, which has been proven not true and which we later on provide evidence with the same conclusions for (Fama, Jan, 1965). MPT assumes all investors to be price-takers and cannot affect stock prices. In reality it is possible to affect prices by selling or buying enough amounts of an asset to affect market prices up or down. Other assumptions such as investors are rational and have access to the same information have been criticized as well (Elton, Gruber, & Busse, 2004). Further on, critique of the Capital Asset Pricing Model has been presented showing that mean-variance efficiency of the market portfolio and CAPM equations are equivalent and that any mean-variance efficient portfolio will satisfy the CAPM equation. The market portfolio, which is vital in the CAPM equation, is not possible to achieve; it would include every possible asset in the world such as stocks, bonds, precious metals, jewelry or anything with value. This portfolio is not observable and therefore investors often use market indices as a proxy for this portfolio which leads to a discussion about the validity of CAPM (Roll, 1977).

Many financial theories are based on assumptions that may be impossible to achieve in practice, but to be able to use the theories one must simplify observations in the real world. If we would take every little thing into account when making a model, the model would not be applicable, since there would be an infinite number of inputs. What we can do when trying to model real world scenarios is to take into account as much as possible, without making the model to complicate.

However, one can argue that if the model is not applicable in practice, or that is based on assumptions which cannot be satisfied, why make the model in the first case? And this is of course not a problem designated to only modern portfolio theory, but to all models describing the real world.

3 Data

3.1 Stocks

To construct two standard portfolios for each country we first needed to select our stocks in the portfolios. We have chosen fifteen major companies operating in different sectors from each of the stock markets, which should represent a well-diversified stock and bond portfolio (Table A 1,Table A 2). We have used monthly historical price data during time period 2002-2012, which we received from Bloomberg. We have chosen monthly data because of lack of observations from the daily data. This should not affect our results, because we calculate an average over the sample period. In addition to our selection of stocks and corporate bond funds, we have chosen to add commodity future contracts; gold, oil, coffee, rough rice, silver and cooper to evaluate whether we are able to increase the value of our portfolios without any additional risk exposure.

3.2 Commodities

Our selection of futures was based on the most heavily traded commodities by today's measure. We also considered using commodity futures from different sectors to investigate if there was a general pattern for all commodities.

Table 1 shows how our chosen commodity future contracts are measured relatively to their future price (CME Group, 2013), we have not considered the contract size. For instance, if you wanted to buy a future contract with copper, one COMEX contract contains the amount of 37,500 pounds. This was essential to further on calculate the expected return of each commodity.

Table 1

	Price	Amount
Gold (GC1)	US dollars	100 troy ounces (≈ 3 110 gram)
Oil (CO1)	US dollars	per barrel (≈ 159 liters)
Coffee(KC1)	US cents	per pound (≈ 454 gram)
Copper	US cents	per pound (≈ 454 gram)
Silver(SI1)	US dollars	troy ounce (≈ 31 gram)
Rough Rice (RR1)	US dollars	100 pounds (≈ 45 kg)

3.3 Corporate Bond Funds

Having some troubles finding data on corporate bonds over our full sample period, we decided to use two major mutual funds investing in corporate bonds as a proxy. Since *Carnegie* corporate bond fund invest mainly in Nordic corporate bonds (Carnegie Investment Bank AB, 2012), we found it appropriate to use it in our Swedish portfolio and our Norwegian portfolio. *SEB fund 5 – SEB corporate Bond Fund*, invests in corporate bonds in OECD countries, the fund receives its interest rate risk from the Swedish market (Morningstar, 2013).

3.4 Risk-free rate

The Swedish risk-free rate was calculated from a 3 month Government Bond (GSGT3M Index). The Norwegian risk-free rate was calculated in the same way, with a 3 month Government bond (GNGT3M) but it was issued by the Norwegian government instead. The data was collected through Bloomberg, both of the rates represents the same time period as the other chosen assets, i.e. 2002-2012. We considered using other proxies for the risk-free rate such as STIBOR and NIBOR, which are the Inter Bank Offered Rates within each country. However, we ended up with more appropriate values using the 3 month bond; the STIBOR rate was surprisingly high in our opinion.

Nevertheless, the risk-free rate in Norway exceeded the Swedish by almost 0.7% at an annually basis. This will of course affect our results in terms of the expected return for each created portfolio, because the general risk premium in Norway will be lower.

In a previous study the authors examined older data from a larger sample period (1973-1997) (Jensen, Johnson, & Mercer, 2000). Our approach is to study more up-to-date data over a shorter sample period since commodity trading has grown massively the last 15 years.

4 Methodology

To begin with we wanted to evaluate how important the role of commodity futures was in a well-diversified Swedish stock and bond portfolio, over different levels of risk. By important we mean higher risk-adjusted return in terms of the Sharpe ratio. To get a comparable result we added a Norwegian portfolio consisting of stocks and bonds which allowed us to gather certain differences and similarities between the two markets.

We began by computing an index with 2002 as starting point to see graphically the correlation between the Swedish and Norwegian stock markets, and the GSCI Index. In Figure 2 we used the GSCI Index as a proxy for all our commodity futures since it would get a better overview of the correlation between the two Nordic markets and the commodity returns. The graph shows clearly that the OSEBX index is more correlated with the GSCI Index, and the OMXS30 is somehow correlated with GSCI Index but only weakly.

4.1 Bloomberg add-ins

As mentioned earlier our data comes from Bloomberg and was added in Excel using the Bloomberg add-in. To collect the monthly stock prices we used "*Historical end of day wizard*" and specified which asset we wanted data from. We used the "Last Price" (PX_LAST) which for equities is the last price provided by exchange and for futures the last price traded until the settlement price is received.

4.2 Optimal Portfolio Allocation

The focus in our methodology part is mainly based on portfolio optimization, where we intend to maximize the Sharpe-ratio. To be able to use our data we had to first calculate the monthly returns of each asset from its price and also the standard deviation and average return which is used in the optimization part. From the monthly return data we used the Data analysis tool in Excel to build a covariance matrix for each of our portfolios including the new commodity.

In order to construct our efficient portfolios based on modern portfolio theory with maximized risk-adjusted return, we used the formulas presented in the theory part, but had Excel making the calculations since the portfolio variance would contain a huge number of terms. This calculation is a trivial task when only using two assets, but as the number of assets increases, the number of covariance terms increases rapidly. To simplify these calculations we used Excel VBA code and also the Solver function. The target was to maximize the Sharpe-ratio by changing the weights for the assets in our portfolio, under the constraint that the sum of the weights could not be larger than 100%, e.g. we don't allow leveraged positions.

We received the optimal weights among our chosen assets, but we wanted to see how the weights changed over different levels of risk. Initially we thought about creating the optimal risky portfolio and the minimum variance frontier for each of the stock/commodity portfolios. However we received unexpectedly low values for the monthly standard deviation and therefore we decided to look further on how the expected return and Sharpe-ratio evolved over different levels of risk. Still maximizing the Sharpe ratio but adding constraints to the solver that the cell containing standard deviation should be equal to certain values ranging from 0,5% to 5% of monthly standard deviation. This procedure was repeated for each combination of commodity/portfolio.

4.3 Test for normally distributed returns

In order to investigate whether the returns of each asset class were normally distributed we computed the Shapiro-Wilk test for normality using Stata.

5 Results

In this part we will summarize our results from the optimal allocation of assets from the methodology section. The results were almost as we expected in theory. We did not allow for short-selling when maximizing the Sharpe Ratio subject to the constraints, which caused some asset weights to be equal to zero. Since we did not allow for short-selling we did neither receive any leveraged positions among our chosen assets because of the sum of the weights in our portfolio should be equal top 100%. Therefore our results will be based on no short-sell positions or leverage positions.

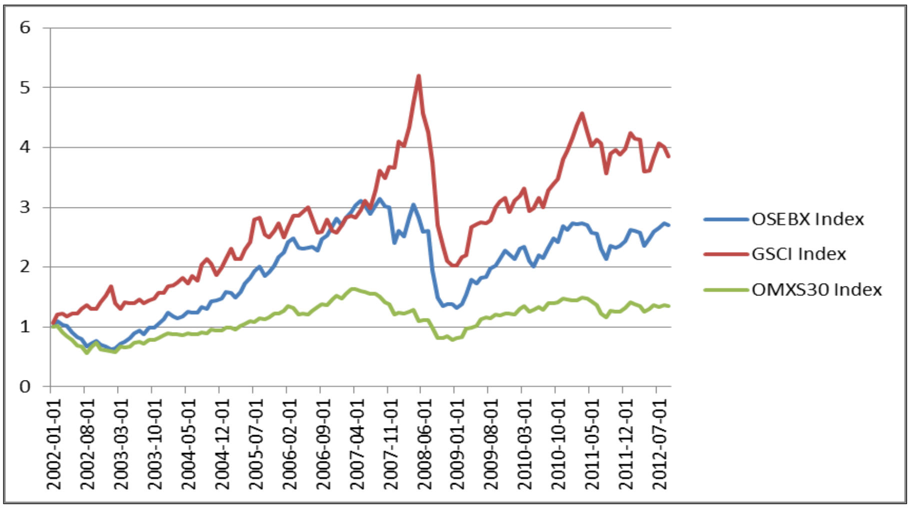

Figure 1

This evidence does support Hypothesis 2 that the Norwegian portfolio would not benefit as much from adding commodity futures as the Swedish one.

Table 2

	SPGSCI Index	OMX Index	OSEBX Index
SPGSCI Index	1,00		
OMX Index	0,17	1,00	
OSEBX Index	0,46	0,77	1,00

The correlation matrix also provides evidence that the Norwegian stock market is more correlated with the commodity index GSCI.

5.1 Swedish results

First we will present the results based on the Swedish portfolio.

5.1.1 Asset Allocation

In Table 3 we present the asset allocation of the Swedish portfolio consisting of stocks, government bonds and corporate bonds (Henceforth referred to as Swedish standard portfolio).

Table 3

	Swedish standard portfolio		
Portfolio std dev	Stocks	Govt. Bonds (3m)	Corporate bond
0,25%	3,23%	64,04%	32,73%
0,50%	6,25%	30,43%	63,32%
1,00%	14,69%	0,00%	85,31%
1,50%	23,82%	0,00%	76,18%
2,00%	32,41%	0,00%	67,59%
2,50%	40,82%	0,00%	59,18%
3,00%	49,13%	0,00%	50,87%
3,50%	57,39%	0,00%	42,61%
4,00%	65,62%	0,00%	34,38%
4,50%	73,83%	0,00%	26,17%
5,00%	82,02%	0,00%	17,98%

Table 4 displays the asset allocation of the Swedish standard portfolio combined with commodity futures (Henceforth referred to as Swedish basket portfolio)

Table 4

	Swedish portfolio with commodity futures			
Portfolio std dev	Commodity Futures	Stocks	Govt. Bonds (3m)	Corporate bond
0,25%	2,30%	3,13%	64,62%	29,94%
0,50%	4,74%	5,84%	32,21%	57,21%
1,00%	10,58%	12,59%	0,00%	76,83%
1,50%	16,63%	19,74%	0,00%	63,63%
2,00%	22,44%	26,85%	0,00%	50,71%
2,50%	28,21%	33,82%	0,00%	37,98%
3,00%	33,92%	40,70%	0,00%	25,37%
3,50%	39,61%	47,55%	0,00%	12,84%
4,00%	45,27%	54,37%	0,00%	0,36%
4,50%	45,70%	54,30%	0,00%	0,00%
5,00%	46,25%	53,75%	0,00%	0,00%

Figure 3 is a comparison of the performance of the two Swedish portfolios over different levels of risk. We can see that the Swedish basket portfolio generates higher return at every given level of risk than the Swedish Standard portfolio.

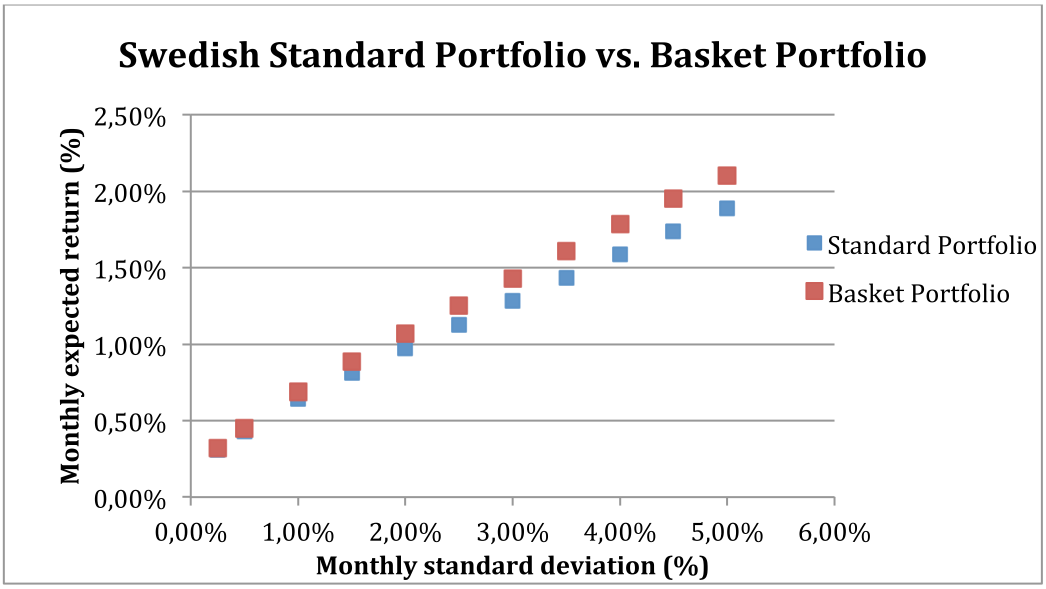

Figure 2

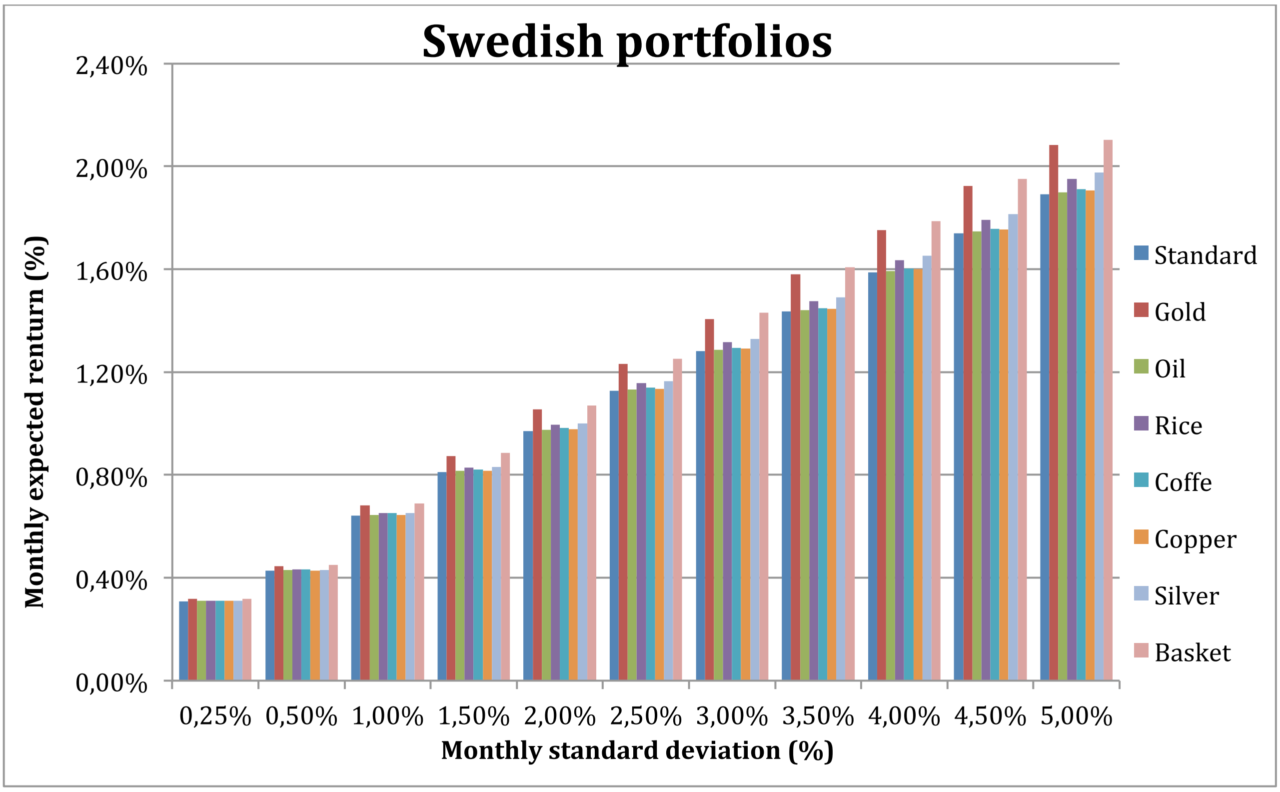

Figure 3

Figure 4 was created as evidence to show that by adding a single commodity future to your portfolio, you will receive a higher return per unit of standard deviation. However, as expected, a low magnitude of the standard deviation will result in insignificant differences compared to the standard portfolio. By observing the Figure 3, it is clear that by adding gold futures, the return remarkably exceed the other chosen futures. The Standard portfolio including gold futures performs almost as well as the Basket portfolio.

5.2 Norwegian results

In this section the results from the Norwegian portfolio allocation will be presented.

5.2.1 Asset allocation

Table 5 displays an overview of the asset allocation of the Norwegian portfolio consisting of Norwegian stocks, government bonds and Nordic Corporate bonds (Henceforth referred to as Norwegian standard portfolio)

Table 5

	Norwegian standard portfolio		
Portfolio std dev	Stocks	Govt. Bonds (3m)	Corporate bond
0,25%	2,63%	66,64%	30,73%
0,50%	4,77%	33,11%	62,12%
1,00%	10,87%	0,00%	89,13%
1,50%	16,92%	0,00%	83,08%
2,00%	22,62%	0,00%	77,38%
2,50%	28,20%	0,00%	71,80%
3,00%	33,90%	0,00%	66,10%
3,50%	39,57%	0,00%	60,43%
4,00%	44,85%	0,00%	55,15%
4,50%	50,48%	0,00%	49,52%
5,00%	56,10%	0,00%	43,90%

Table 6 displays the asset allocation of the Norwegian portfolio consisting of Norwegian stocks, government bonds, Nordic corporate bonds and commodity futures (Henceforth referred to as Norwegian basket portfolio).

In Table 6 we received some unexpected results that the weight of commodity futures in the Norwegian portfolio became generally high. We expected the weight of futures in the Norwegian portfolio to be less than the Swedish portfolio since the Norwegian stock index has higher correlation with the GSCI index.

Table 6

	Norwegian portfolio with commodity futures			
Portfolio std dev	Commodity Futures	Stocks	Govt. Bonds (3m)	Corporate bond
0,25%	3,56%	1,04%	72,00%	23,39%
0,50%	7,64%	1,73%	40,24%	50,40%
1,00%	17,44%	2,78%	0,00%	79,79%
1,50%	28,07%	4,67%	0,00%	67,27%
2,00%	37,88%	6,72%	0,00%	55,39%
2,50%	47,57%	8,70%	0,00%	43,73%
3,00%	57,16%	10,66%	0,00%	32,18%
3,50%	66,69%	12,58%	0,00%	20,73%
4,00%	76,21%	14,52%	0,00%	9,27%
4,50%	82,75%	17,25%	0,00%	0,00%
5,00%	79,65%	20,35%	0,00%	0,00%

The performance of the Norwegian Basket portfolio does clearly outperform the Norwegian Standard portfolio. As the level of risk increases, the larger the difference between the Standard portfolio and the Basket portfolio gets and the more weights in futures are requested well.

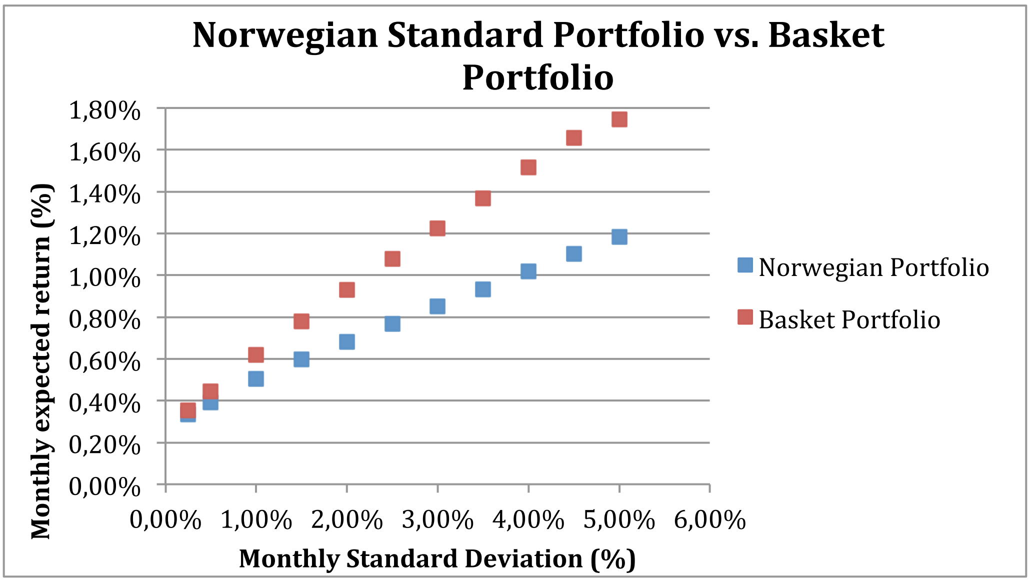

Figure 4

At a 3% monthly standard deviation the standard portfolio will have 0,851% in monthly return, and the basket portfolio will have a return of 1,224%. This means that the Basket portfolio would have 44% higher return than the Standard portfolio consisting of Norwegian stocks and Nordic corporate bonds. We have also concluded that the Sharpe ratio is higher for the Basket portfolio at each level of standard deviation.

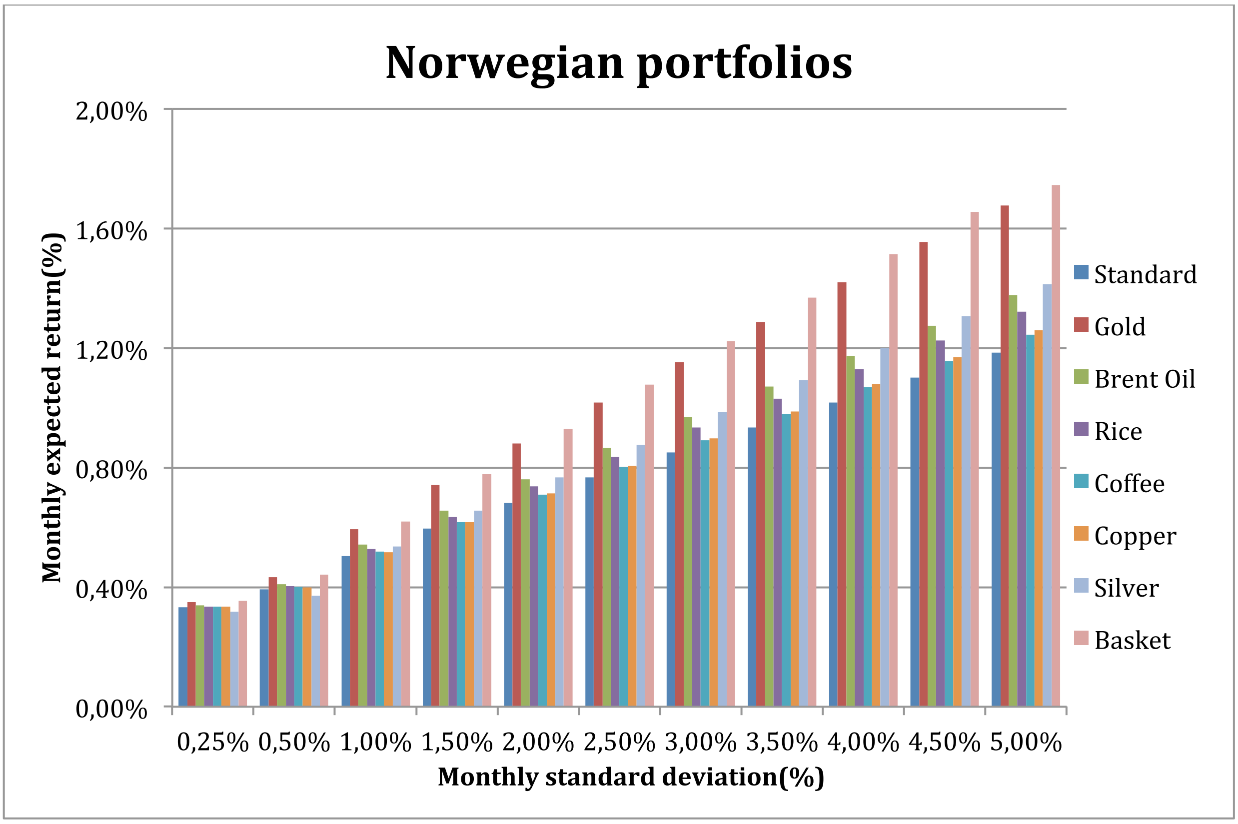

Figure 5

One can obviously see that at any given level of risk the Basket portfolio outperforms all other combinations of assets. At a 5 % level of monthly standard deviation, gold futures requests almost 40% of the allocation of the basket portfolio which was not very surprisingly. What surprised us though is that Brent Oil futures allocate 15% of our optimal basket portfolio, since Oil futures have high correlation with many Norwegian companies producing oil products. On the other hand, when maximizing the allocation of Norwegian stocks, we did not receive any weights in oil based companies.

6 Analysis

Because of the fluctuations within the global economy, investors seek different alternatives to reduce their exposure to risk. Since the financial crisis in 2007, many precautions were made in order to withstand another crisis. There has been a debate regarding speculators being a key reason for the financial crisis 2007. A result of this was the Volcker Rule which restricted US commercial banks from speculating in certain risky investments, such as commodity derivatives, using deposits to trade on its own account (Financial Times, 2012). However, from a portfolio diversification point-of-view, commodity futures can be very useful as an investment instrument.

As mentioned earlier in this paper, in general, commodity futures are quite uncorrelated with the market i.e. as Table 2 indicates the correlation between SPGSCI and OMX30 is 0.17. Why does this correlation generally differ from other more traditional asset classes? Because of the fact that commodity prices are mainly set up by global supply and demand and equity markets are highly dependent on the performance of specific industries and sectors. For instance, if an economic recession occurs within a country it is likely that an equity index e.g. OMX30, will perform poorly but this does not necessarily mean that commodity prices will decrease. Hence, we can state that the correlation between commodities and equities is low. We have concluded within this paper that by adding commodity futures to a stock and bond portfolio the investor will receive a higher risk-adjusted return, at least within out research period and our chosen geographical areas.

Still, a lot of previous studies have tried to evaluate the historical performance of commodity futures as a stand-alone investment with different results and conclusions. In a study based on 1973-1997 data, the performance of commodity futures as a stand-alone investment is concluded to be *inferior* to other asset classes (Jensen, Johnson, & Mercer, 2000). However, in a new updated study based on 1970-2009 data , evidence is provided that the stand-alone performance of commodity futures has higher returns but higher standard deviations as well, which resulted in a risk-return relationship in line with the equity markets (Jensen & Mercer, 2011, pp. 6-11).

In our results we have established that the commodity futures have in general been a *superior* investment to the equity markets during the 21st century. The general return of commodity futures has been higher than the equities but the standard deviations have also been higher, basically we received the same results as previous studies (Jensen & Mercer, 2011). On a risk-adjusted return basis the commodity futures during our research period have been superior to the equities with a few exceptions.

Since our study is based on the latest ten years (2002-2012), it is not unexpected that we achieve these results. Prior to year 2000, the commodity prices have been relatively low (Center for Futures Education, Inc., 2013), but since increasing demand in emerging economies like China and India, the commodity prices have risen substantially. In a previous study (Jensen & Mercer, 2011) the authors used a larger sample period including both less and more volatile years of commodity prices, which explains why their results indicates that commodity futures have been a *relatively* good stand-alone investment.

In a previous study they used a different time period than this paper, but our data is even more updated and it is concentrated on the Scandinavian markets instead of American markets (Jensen & Mercer 2011). However, there is some kind of a pattern that commodity futures in general as a stand-alone investment the latest 10 years has been a relatively good investment compared to stocks. Though, commodity futures are more commonly used as a portfolio component rather than a stand-alone investment, because it is a good option for investors who want to diversify their portfolio in order to reduce market risk.

Overseeing the results in the previous section, we can clearly state for both countries that by adding several commodity futures to the standard portfolio, the return-to-volatility relationship will increase. It was expected that both portfolios would benefit from this additional investment, in terms of lower risk and higher return. However, it was interesting that the Norwegian portfolio requested more weights in commodity futures when optimizing the portfolio allocation than the Swedish portfolio. Initially we thought that the Swedish portfolio would have a higher proportion of futures than the Norwegian, because of the relatively low correlation between OMX30 and the GSCI, which is stated in Hypothesis 2. What we did not consider was that the stocks in the Swedish portfolio were a good investment opportunity compared to the Norwegian stocks, in terms of return-to-volatility.

Observing Table 3 and 5, we can point out as the standard deviation increases; the Swedish portfolio requests significantly more weights in stocks compared to the Norwegian portfolio at all specified levels of risk. When we stated the hypotheses in the beginning of the paper, we did not think about the individual performance of each stock, but selected the stocks from each equity market representing a well-diversified portfolio. In general the Swedish stocks performed far better in terms of return-to-volatility compared to the Norwegian stocks. Therefore, by comparing Table 4 and 6 we can clearly see that the Swedish portfolio requested less percentage of commodity futures. Though, both of the Swedish portfolios had a higher Sharpe ratio than the Norwegian portfolios on every level of risk. This could be explained by; during this specific time period the Swedish stocks performed better compared to the Norwegian stocks. A reason for this might be that during the financial crisis in 2007, oil spot price decreased by 68 % June – December (The World Bank, 2013) which affected the Norwegian economy and equity market more than the Swedish equity market. This can be observed from Figure 1 that during 2008 the GSCI (which contains almost 80% weight in oil) fell by 55 %, and the OSEBX index fell by 47 %.

It is also important to acknowledge that the Norwegian risk-free rate was almost 0.1% higher on a monthly basis (1.2 % annually) i.e. this results in a lower risk premium, which reduces the Sharpe ratio. To state the importance of this matter we used the Norwegian risk-free rate when calculating the Swedish basket portfolio to see what would happen with the Sharpe-Ratio. If both countries would have the same risk-free rate our results would be the other way around, that the Norwegian portfolios would have higher Sharpe-ratios than the Swedish portfolios. This calculation was made only in illustrating purpose to show the significance of the risk-free rate and its effect on the Sharpe-Ratio.

From figure 3 and 5 it is obvious that the Norwegian portfolio benefits more than the Swedish portfolio in terms of increased return. For instance, by adding commodity futures to the Norwegian standard portfolio at a 3% level of risk, as an investor you will receive almost 44% higher return. During the same circumstances concerning the Swedish portfolio, the investor will only get 12% higher return.

The conclusions about Hypothesis 1 regarding gold were correct. We observed from the results that gold took a major weight among the futures in both of our portfolios when maximizing the risk-adjusted return. On a 5% monthly standard deviation gold stood for 40% of the weight in the Norwegian Basket portfolio. When comparing the Basket portfolio with the Standard portfolio including gold futures, we can observe Figure 6 and state that the risk-return tradeoff is very similar. So why not use only gold to increase the return of your portfolio? One might argue that gold will provide a good diversification to a portfolio consisting of stocks and bonds by overseeing this historical data, but for diversification purposes this kind of strategy would be far from the best option. Investors seeking alternative investments like commodity futures in order to diversify their portfolios should not limit themselves to one single asset. By invest in a broader spectrum of commodities or in a commodity index, the investor will receive a better diversification within the portfolio. We have concluded that there is a slightly better risk-return reward by investing in a basket of commodity futures rather than by using only gold futures in the portfolio.

Through this paper it is important to clarify that we have excluded a lot of reality factors concerning commodity trading. We have not included factors like: transaction cost, commission, broker fees or taxes. Because the access to this kind of information is limited and including those factors in the calculations would have been very difficult. It is also necessary to mention that we have no leverage or shortage position in any single case. If considering these costs for trading single commodity futures, a private investor with a limited amount of funds would not benefit as much from the investment because of a major part of the return will be reduced by fees. A better and cheaper way for a private investor to receive diversification among many commodities would be to invest in an index tracking ETF and pay a small percentage annual fee.

7 Conclusions

As mentioned earlier through the paper, adding commodity future contracts to the portfolio is a relatively new phenomenon and since the last 15 years the trading volumes has exploded at the future markets. In 2011, the commodity futures trading volume at CBOE Futures Exchange increased by 174 % (CBOE Futures Exchange, 2012) compared to 2010, corresponding to several millions of future contracts.

By analyzing our results we can conclude that two out of the three hypotheses we stated were correct. A few of our assumptions and predictions were accurate, but as stated we did not consider the individual performance of each asset and how it would affect the proportion of commodities within the portfolio.

We provide certain evidence that the stand-alone performance of commodities have been superior to equities during our sample period, and we believe this is mainly due to increasing demand from emerging economies and an unchanged supply in the world as a whole. By our measurements and sample period we can conclude that gold has been a superior commodity investment during the last decade in terms of higher return and lower risk compared to the others. However, it is not efficient "to put all of your eggs in the same basket", a combination of commodity futures are therefore the optimal choice in terms of reducing the overall exposure to risk.

It is not likely for an investor in the real world to invest 80% of their funds in commodities and the rest in equities. One of the purposes of this paper was to display what happens with the allocation of each asset over different levels of risk**.** When constructing our optimal risky portfolios we received an allocation of 2.7% of future contracts in the Swedish Basket portfolio and 12.1% in the Norwegian basket portfolio. This might represent a more traditional allocation of commodities in portfolios as many studies has concluded before, that the optimal allocation of commodity futures in efficient portfolios usually vary between 5 to 10% (Du, 2005, pp. 192-194)

Comparing two countries within the same geographical area was essential in order to see if we could find a pattern regarding the benefits of adding commodity futures to a diversified portfolio. We concluded that there is a pattern for both countries in terms of that the return-to-volatility increased, but the Swedish Basket portfolio got a higher Sharpe ratio than the Norwegian portfolio at every specified level of risk. On the other hand, overall the Norwegian equities performed poorly in comparison to the Swedish which might explain why the Norwegian Basket portfolio surprisingly requested a lot more allocation of the commodities.

Whether commodity futures will provide high returns in the upcoming years or not is highly uncertain and it is a subject which is analyzed on a daily basis. Though, commodities will probably continue to provide benefits of diversification to equity portfolios in the future.

8 Methodology critique

In this paper we are aware of that some of our data and calculations might not reflect a real world scenario. We are aware of that the corporate bond funds that we have used might not be an optimal choice as a proxy to represent actual corporate bonds, but this was a last solution in order to receive a diversification beyond only stocks. Through the statistic Shapiro-Wilks test, we have noticed that the return data we collected from Bloomberg.com was not normally distributed. Since, mean-variance-optimization assumes that the returns of the assets are normally distributed; this may cause biasness within our results. However, usually the stock returns are not normally distributed, in other terms there is some skewness of the distribution among the returns (Ford, 2012). As we mentioned earlier within this paper, we have excluded a lot of reality factors regarding commodity trading; transactions cost, commissions and taxes. If these factors were included, we would get different results, especially regarding the return of each asset.

9 Further research

There are a few excluded factors we could have used in order to get more reliable results. A good extension of this paper would be if we included factors like transaction costs, taxes and commissions. It would also be interesting to investigate how the monetary policy could affect the allocation and efficiency of commodity futures, as our inspiring paper did (Jensen & Mercer, 2011). Then we could see if we got any similar results as they did, and compare both papers if there were certain differences, that would be a great addition to the analysis. It would have also been interesting to see what would happen with the allocation of the assets if we added other securities like; real estate's investment trusts and currencies. We are highly confident that the results would differ a lot and that the allocation would be completely reallocated.

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11 Appendix

In the appendix we will present tables and figures which are not suited in the thesis, but still provide necessary information regarding the results.

Figure A1 is a graphical illustration, where Gold Futures and the Goldman Sachs Commodity Index have been indexed with 2002-01-01 as starting value. From the figure we can see that gold futures have outperformed GSCI, which we use as a comparison for the rest of our commodities. This supports Hypothesis 1.

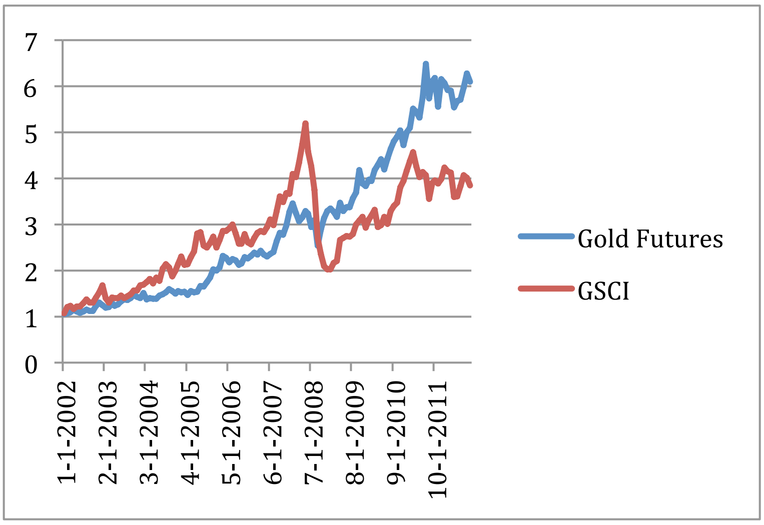

Figure A 1

HMB (Hennes & Mauritz AB)	apparel retail
VOLVB (Volvo AB)	construction and farm machinery; heavy trucks
ASSAB (Assa Abloy AB)	building products
TEL2B (Tele2 Ab)	telecom
SEBA (SEB AB)	commercial bank
MTGB (Modern Times Group AB)	broadcasing
INVEB (Investor AB)	multi-sector holdings
SWMA (Swedish Match AB)	tobacco
TLSN (Telia Sonera AB)	telecom
AZN (Astra Zeneca AB)	pharmaceuticals
ERICB (Ericson AB)	telecom
NDA (Nordea Bank AB)	commercial bank
SCAB (Svenska Cellulosa AB)	paper products
BOL (Boliden AB)	diversified metals and mining
LUPE (Lundin Petroleum AB)	petroleum

Table A 1

Table A1 displays our chosen Swedish company stocks and which sector they operate within.

STL (Statoil ASA)	petroleum
NHY (Norsk Hydro ASA)	petroleum
ORK (Orkla ASA)	multi-sector holdings
TEL (Telenor ASA)	telecom
ATEA (Atea ASA)	IT
NSG (Norske Skogsindustrier ASA)	paper products
DNB (Den Norske Bank ASA)	commercial bank
GOD (Goodtech ASA)	electric utility
SCI (Scana Industrier ASA)	industrial
TOM (Tomra ASA)	recycling
SUBC (Subsea7 ASA)	petroleum
EVRY (EVRY ASA)	information technology
BON (Bonheur ASA)	holding company
RCL (Royal Caribbean Cruisses ASA)	hospitality, tourism
TGS (TGS Nopec Geophysical ASA)	geoscience data

Table A 2

Table A2 displays our chosen Norwegian company stocks and which sector they operate within.

11.1 Excel and VBA

To simplify our optimization problems we used Excel and Visual Basics for Microsoft Applications, which is a programming language. The purpose for using VBA in this paper is to reduce the vast amount of calculations which would have been necessary to compute the portfolio variance from a covariance-matrix of 17 assets. To receive *the portfolio variance* it is possible to use the VBA-Code (People.Brunel, 2012).

$$= ((MMULT\left(MMULT(WEIGHTS; COVARIANCEMATRIX); TRANSPOSE(WEIGHTS)\right))$$

This VBA-Code will multiply the column vector with the covariance matrix multiplied by the transpose of the column vector. To receive the portfolio return we used the $= \left(MMULT\left(AVERAGE\ RETURN; TRANSPOSE(WEIGHTS)\right)\right)$

Weights are in this case the chosen weights from the optimal allocation and Average return is the average return of each asset over a certain period.

11.2 Test statistics for Shapiro-Wilk test for normality

In this part of the appendix we will summarize our tests from Stata and SPSS Figure A 2 is a histogram from Stata over the distribution of stock returns, which are not exactly normally distributed. To get an exact result of the normality test we have to observe the p-values for the Shapiro-Wilk test.

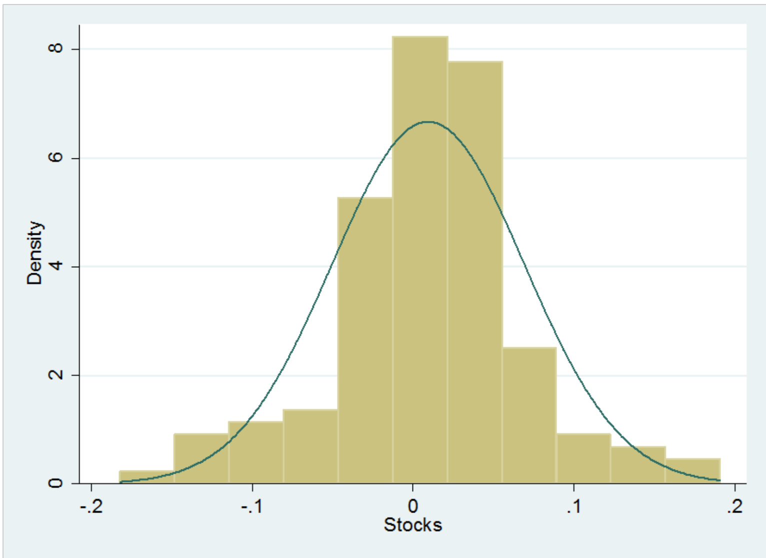

Figure A 2

Figure A 3 is a Q-Q plot of the stock returns from SPSS

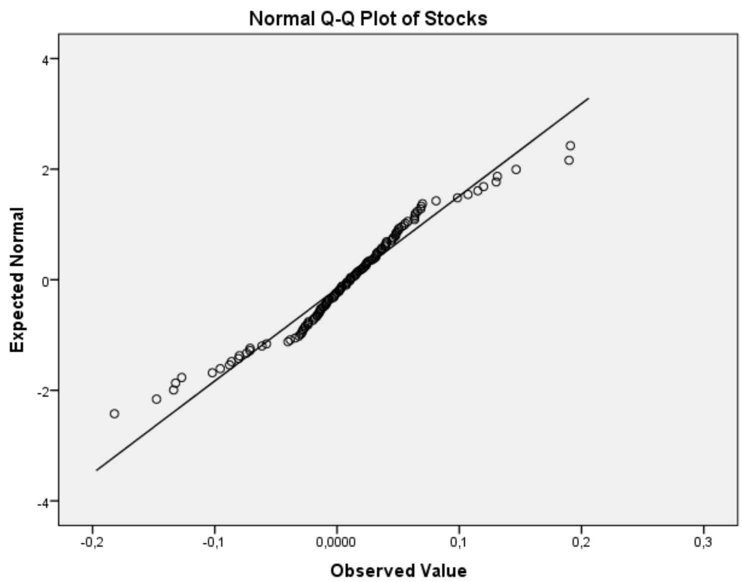

Figure A 3

Figure A 4 is a histogram from Stata over the corporate bond returns distribution. The histogram shows that returns could be taken as normally distributed, however to conclude whether this statement or not is true we have to rely on the p-values.

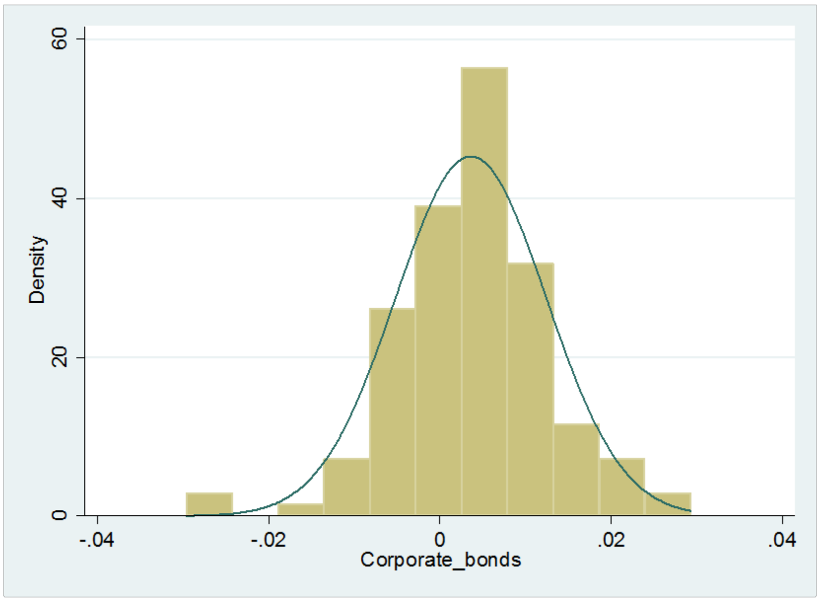

Figure A 4

Figure A 5 is a Q-Q plot of the corporate bonds return from SPSS

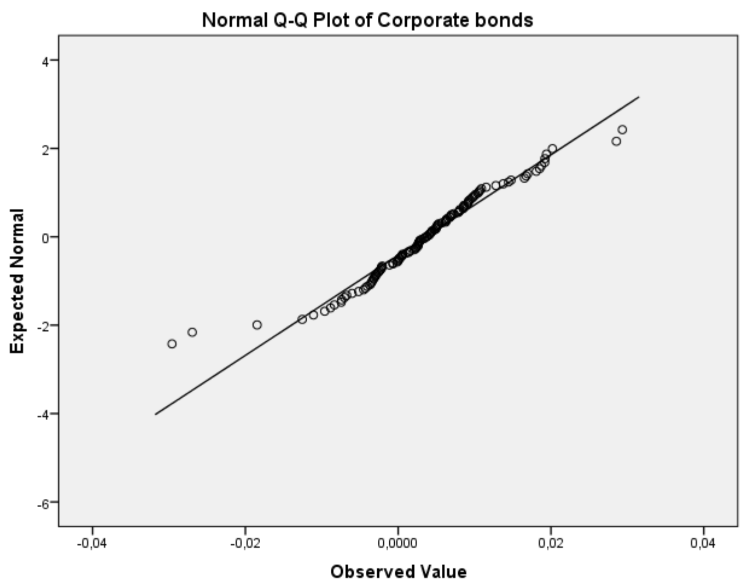

Figure A 5

Figure A 6 is a histogram from Stata displaying the commodity futures returns distribution. From the histogram we cannot conclude that the returns are normally distributed and again, we need to rely on the p-value

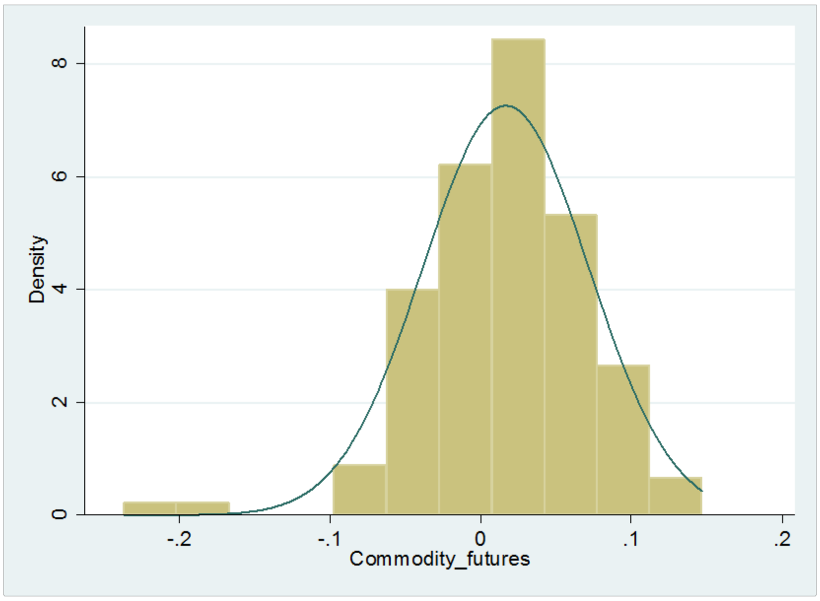

Figure A 6

Figure A 7 is a Q-Q plot of the commodity futures returns from SPSS

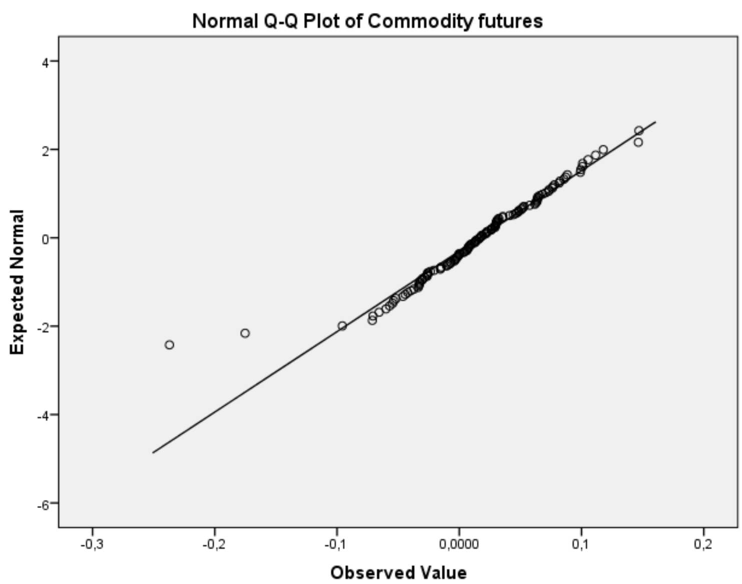

Figure A 7

Table A 3 is a summary of statistics used in the Shapiro-Wilk test for normality

	Summary statistics				
Variable	Obs	Mean	Std. Dev.	Min	Max
Stocks	129	0.0094631	0.0598137	(-)0.1821368	0.190926
Coroporate Bonds	129	0.0036416	0.0088096	(-)0.0296038	0.0293444
Commodity Futures	129	0.0165823	0.0549259	(-)0.2371066	0.1469634

Table A 3

Table A 4 contains the test statistics from the Shapiro-Wilk test. Observing the table we can see that the test statistic W is close to one which could explain that the returns are normally distributed. However to be sure about this statement, we need to observe the p-values which in this case are lower than the chosen level of significance (5%) i.e. we can reject the null hypothesis that the returns are normally distributed.

	Shapiro-Wilk W test for normal data			
Variable	Obs	W	V	Sig (Prob>z)
Stocks	129	0.96173	3.916	0.00107
Coroporate Bonds	129	0.96033	4.058	0.00082
Commodity Futures	129	0.95046	5.650	0.00013

Table A 4