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Reducing Portfolio Risk Using Volatility

A risk-return examination of the addition of VIX and VIX futures contracts to an equity portfolio

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This thesis examines the effects of adding volatility, as represented by the CBOE Volatility Index (VIX) and VIX futures contracts, to a stock portfolio in terms of portfolio risk and portfolio return. The study is based on statistical properties as well as Markowitz's modern portfolio theory, with support from previous research conducted by Hill (2013), Szado (2009), and Daigler and Rossi (2006). We find that volatility can be used to reduce risk in a stock portfolio, and in many cases also increase expected portfolio return. These findings are in line with previous mentioned research.

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List of Abbreviations

CBOE – Chicago Board Options Exchange

VIX – CBOE Volatility Index

VXX – iPath S&P 500 VIX Short-term Futures ETN

VXZ – iPath S&P 500 VIX Mid-term Futures ETN

OTM – Out of the money

ATM – At the money

1. Introduction

When the Volatility Index was introduced by the Chicago Board Options Exchange in 1993, investors for the first time got a reliable measurement of the important volatility. Ever since the introduction, the demand for tradable products connected to this index has been high, resulting in the inception of VIX futures contracts and VIX options in 2004 and 2006. The importance and possible usefulness of these instruments was realized during the financial crisis of 2007 and 2008, where even assets considered as safe showed a positive correlation to the market returns. In 2009, exchange-traded products tracking the VIX began trading. Now investors had an even bigger opportunity to diversify their portfolios with an asset that was negatively correlated with market returns, a correlation that actually increases during times of financial turmoil.

The purpose of this thesis is to examine volatility and the possibilities to diversify and reduce the risk of a stock portfolio using exchange-traded products tied to volatility. This is done by examining statistical properties and by constructing efficient frontiers consisting of equity and volatility. Similar research has been done, namely in Joane Hill's paper "The Different Faces of Volatility Exposure in Portfolio Management" (2013), Edward Szado's "VIX Futures and Options – A Case Study of Portfolio Diversification During the 2008 Financial Crisis" (2009), and Daglier's and Rossi's "A Portfolio of Stock and Volatility" (2006). The research conducted in these papers serve as a basis of our thesis. In particular we investigate if volatility can be used to reduce risk in a portfolio consisting of equity and bonds. This is answered by looking at the following questions: Do different sources of volatility generate different results? Is one source of volatility better for risk reduction purposes than another? Is volatility a good tool for risk reduction compared to other common assets, such as gold? We find that volatility can be used to reduce risk in a stock portfolio, and in many cases also increase expected portfolio return. These findings are in line with previous mentioned research.

The rest of the thesis is organized as follows. In Section 2 we will discuss theory and previous research related to our thesis, with the objective of providing a theoretical framework, helping the readers understanding the thesis. Here, Subsection 2.1 discusses the basics of volatility and its properties. Furthermore, Subsection 2.2 introduces the Volatility Index (VIX), and in 2.3 we are explaining the concepts of futures contracts related to the VIX. Subsection 2.4 and Subsection

2.5 is covering the relationship of the returns on VIX and S&P 500 and on gold and S&P 500, respectively.

In Section 3 we will introduce our method. The section goes into and discusses our scientific approach and how the research has been conducted. We will also introduce the theories the results and analysis are based upon. In Subsection 3.1 we present theories regarding the statistical properties that are used to interpret the empirical findings. Next, in Subsection 3.2 we present the properties and framework behind the modern portfolio theory that are also used to interpret the empirical findings.

Thereafter, in Section 4, we present our empirical findings. Section 4 is divided into Subsection 4.1, covering our findings in terms of statistical properties, and in Subsection 4.2 where we use portfolio performance evaluation and modern portfolio theory to interpret and analyze the empirical findings. This is followed by a conclusion in Section 5.

2.   Theory

In this section we introduce theory and previous research related to this thesis. We begin by examining volatility in Subsection 2.1, and then the Volatility Index (VIX) in Subsection 2.2 including a numerical derivation of the old and the new VIX. In Subsection 2.3 we explain the concepts of futures contracts related to the VIX. We are rounding off this section by examining the connection between volatility and market returns as well as gold and market returns.

2.1 Volatility

In this subsection we will examine the concept of volatility. It covers a brief explanation of the historical and the implied volatility.

Volatility is the common term referred to when talking about risk in the financial markets. Highly volatile stocks are often seen as risky, and the volatility has often been found at high levels before and during financial crises. In addition, this measurement of risk has for several

years been an important number for a number of policymakers, such as the Federal Reserve and Bank of England (Nasar, 1991).

Volatility is roughly defined as the standard deviation of asset pricing data (Demeterfi et al., 1999, p. 1). Thus, volatility is measuring the deviations from the mean price of an asset, such as a stock or an index. An important distinction is to be made between historical volatility and implied, or expected, volatility (Poon and Granger, 2003, p. 480).

An important characteristic of volatility is the seemingly negative correlation to returns, especially in times of market turmoil (Hill, 2013, p. 10-11). This relationship means that as market volatility increases, market returns fall. If this relationship holds, interesting diversification and risk reduction strategies arise.

In the presence of historical pricing data, historical volatility can be calculated. This measurement is used in useful financial analysis tools, such as the Sharpe ratio and in the capital asset pricing model. Historical volatility can be used as a forecast by assuming that the volatility over the coming time period will be the same as that same time period's actual volatility. Another way of forecasting volatility is by calculating a so called implied volatility (Poon and Granger, 2003, p. 480).

The implied volatility is the market's forecasted volatility over a given time period. This forecasted volatility can, for example, be observed by examining option prices using the Black-Scholes formula for option pricing. By inverting the Black-Scholes formula, we can obtain the volatility that generates a model price that will coincide with the corresponding market price for a fixed maturity and strike price. Hence, if we want to observe the implied volatility of the Standard & Poor's 500 Index (S&P 500) over the coming month, we use S&P 500 options with maturity in one month with the same, fixed strike price (Canina and Figlewski, 1993, p. 659-662).

2.2 Volatility Index

In this subsection we introduce the Volatility Index (VIX). It covers the history of VIX, as well as numerical derivations of the old and the new VIX. The numerical derivation in Subsection 2.2.2 is directly taken from "Whaley, Robert E. (2000), The Investor Fear Gauge, p. 12-17" and the numerical example in Subsection 2.2.3 is directly taken from "The CBOE Volatility Index - VIX, p. 1-19".

2.2.1 The Fear Index

The VIX was first introduced in 1993 by the Chicago Board Options Exchange. It was designed to measure the implied 30-day volatility on ATM S&P 100 index option prices, but later changed to using S&P 500 Index options as a base. This index has grown to become the major benchmark for U.S. stock market volatility, and is often referred to as the "fear index" (Chicago Board Options Exchange Technical Notes C, 2009, p. 2). Figure 1 displays the pricing movements of VIX since 1993.

Figure 1: VIX historical pricing chart. Source: Bloomberg.
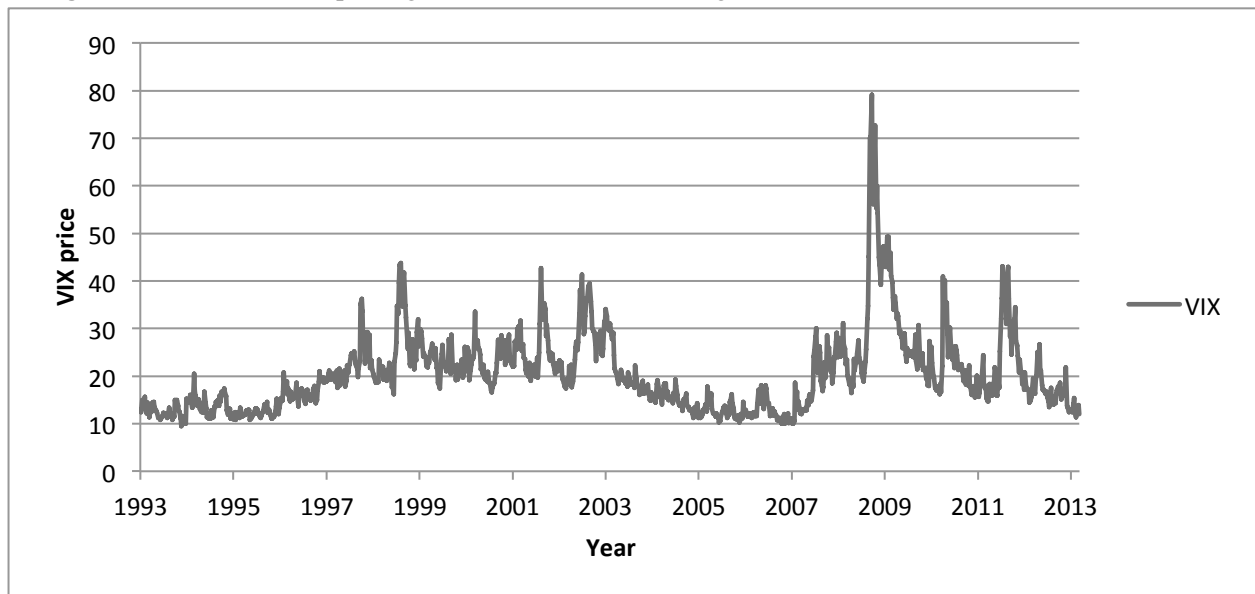
The index is expressed as expected percentage moves of the S&P 500 index (both up and down) over the next 30 days, annualized for one standard deviation (Williams, 2013, p. 1-2). Standard deviation is commonly used with normally distributed data and measures the distribution around

the mean. Generally, 68 percent of the sample will be within one standard deviation away from the mean. About 95 percent of the sample will be within two standard deviations away from the mean (Narasimhan, 1996). Numerically, this means that if the current level of VIX is at 35, the market expects the S&P 500 index to stay within 2.92 percent (35 percent divided by 12 months) of its current value in the next 30 days, about two thirds of the times (remember that two thirds is about equal to one standard deviation). Hence, with 68 percent probability, S&P 500 will in the next 30 days deviate at most 2.92 percent from its current value. In this case, with S&P 500 index at a hypothetical level of $1500, the market expects this level, 30 days from now to be within the range of $1456 to $1544 (Williams, 2013, p. 1-2).

As can be seen from Figure 1, the value of VIX is somewhat centered around 20. The general guideline according to many investors and traders is that a value over 30 indicates a worried market with a volatile, possibly negative outlook for the stock market (Pepitone, 2013). A VIX level of below 20, on the other hand, is considered as a calm and stable stock market. The VIX's ability of measuring the stock market's general perception of future volatility is one reason for the index being called "the fear index" – it shows when investors are scared (Pepitone, 2013).

2.2.2 The Old VIX

This subsection aims to explain the old VIX and also shows how to derivate it.

The old VIX is derived and explained by Robert E. Whaley in his paper "The Investor Fear Gauge". This subsection is to a large extent taken from this paper, and the derivation of the old VIX is directly taken from this paper, see p. 12-17 in Whaley (2000).

When introduced in 1993, the VIX was calculated by using the implied volatilities on eight near the money options, nearby, and second nearby OEX option series. The implied volatilities are then weighted so that the VIX reflects the implied volatility of a 30-calendar day ATM option.

The implied volatilities are calculated by using an option pricing model. The Black-Scholes option pricing model from 1973 is widely used. The parameters in the model are volatility, current index level, the options exercise price and time expiration, the risk free rate of interest and the cash dividends that will be paid during the option's life. All the parameters, except for the volatility, are easy to observe.

The VIX is based on trading days. If the time to expiration on the option is measured in calendar days, the implied volatility has to be transformed to trading-day basis, that is, $N_t = N_c - 2 * int(\frac{N_c}{7})$. Where N_t denotes the number of trading days to expiration, N_c denotes the number of calendar days to expiration, and $int(x)$ is the integer value of x.

To generate the "trading-day implied volatility rate" of the option, the "calendar-day implied volatility rate" of the option is multiplied by the factor of the square root of number of calendar days to expiration and the square root of number of trading days to expiration, that is, $\sigma_t = \sigma_c(\sqrt{N_c}/\sqrt{N_t})$.

The nearby OEX options series is defined as the series with the shortest time to expiration, but at least eight days. The second nearby OEX options series is the series of the next adjacent contract month.

The eight near the money options underlying the VIX are four nearby call and put options and four second-nearby call and put-options. The four nearby options are divided into two put and call options that have an exercise price just below the current index level, and two put and call options that have an exercise price just above the current index level. The next nearby options are divided in the same way. The implied volatilities of the eight options are presented in Table 1, where X_l denotes the lower exercise price, and X_u the upper exercise price.

Table 1: Implied volatility of the eight near the money options underlying VIX. Source: Whaley, Robert E. (2000).

Exercise price	Nearby (1)		Second nearby (2)	
	Call	Put	Call	Put
$X_l < S$	$\sigma_{c,1}^{X_l}$	$\sigma_{p,1}^{X_l}$	$\sigma_{c,2}^{X_l}$	$\sigma_{p,2}^{X_l}$
$X_u \geq S$	$\sigma_{c,1}^{X_u}$	$\sigma_{p,1}^{X_u}$	$\sigma_{c,2}^{X_u}$	$\sigma_{p,2}^{X_u}$

The next step in the calculation is to average the eight put and call options implied volatilities into four categories:

$$\sigma_1^{X_l} = \frac{\left(\sigma_{c,1}^{X_l} + \sigma_{p,1}^{X_l}\right)}{2}$$

$$\sigma_1^{X_u} = \frac{\left(\sigma_{c,1}^{X_u} + \sigma_{p,1}^{X_u}\right)}{2}$$

$$\sigma_2^{X_l} = \frac{\left(\sigma_{c,2}^{X_l} + \sigma_{p,2}^{X_l}\right)}{2}$$

$$\sigma_2^{X_u} = \frac{\left(\sigma_{c,2}^{X_u} + \sigma_{p,2}^{X_u}\right)}{2}$$

When the four volatilities are calculated, the next step is to generate ATM volatilities for each maturity. Here σ_1 denotes the ATM nearby average volatility, and σ_2 denotes the ATM second nearby average volatility.

$$\sigma_1 = \sigma_1^{X_l}\left(\frac{X_u - S}{X_u - X_l}\right) + \sigma_1^{X_u}\left(\frac{S - X_l}{X_u - X_l}\right)$$

$$\sigma_2 = \sigma_2^{X_l}\left(\frac{X_u - S}{X_u - X_l}\right) + \sigma_2^{X_u}\left(\frac{S - X_l}{X_u - X_l}\right)$$

When the ATM average volatility for each maturity is calculated, the last step is to create a 30-calendar day implied volatility. The 30 calendar days are converted into trading days: 30 calendar days equals 22 trading days. Let N_{t_1} denote the number of trading days to expiration of the nearby contract, and N_{t_2} denote the number of trading days to expiration of the second nearby contract. Then the old VIX is defined as $VIX = \sigma_1\left(\frac{N_{t_2} - 22}{N_{t_2} - N_{t_1}}\right) + \sigma_2\left(\frac{22 - N_{t_2}}{N_{t_2} - N_{t_1}}\right)$.

2.2.3 The New VIX

This subsection aims to explain what the new VIX is and how it is calculated. The calculation is shown by a hypothetical example that is directly taken from "The CBOE Volatility Index – VIX" (Chicago Board Options Exchange Technical Notes C, 2009, pp. 1-19).

In 2003, CBOE and Goldman Sachs updated the index using another way of calculating the expected volatility, in a way that is widely used by financial theorists and volatility traders. The new VIX is based on the S&P 500 (SPX) index instead of S&P 100 (SPO), and the expected volatility is calculated by averaging the weighted prices of SPX put and call options (Chicago Board Options Exchange, Technical notes C, 2009, p. 1-19). Thereby the VIX is no longer dependent on an option pricing model, unlike the old VIX, where the expected volatility is derived from an option pricing model such as Black- Scholes (Szado, 2009, p. 11).

Just like the original VIX, the new measures the implied volatility over a 30-day period. The options underlying the index are near term and second near term put and call options. And the near term options are required to have at least one week to expiration. When the expiration date is getting closer than 7 days, the VIX rolls to the second and third SPX contract months. (Chicago Board Options Exchange, Technical notes C, 2009, p. 1-19)

The generalized formula used to calculate the new VIX, presented in "The CBOE Volatility Index - VIX", is given by

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2,$$

where the summation goes from the lower bound (OTM put options with strike prices smaller than K_0) to the upper bound (OTM call options with a strike price larger K_0). The upper and the lower bounds are set when there are two consecutive options with a bidding price equal to zero. This will be further examined in Table 3. The properties in the equation above are as follows,

$$\sigma = \frac{VIX}{100} \text{ so that } VIX = \sigma * 100$$

$T = $ Time to expiration

$F = $ Forward index level derived from index option prices

$K_0 = $ First strike below the forward index level

$K_i = $ Strike price of ith OTM option; a call if $K_i > K_0$ and a put if $K_i < K_0$, both put and call if $K_i = K_0$.

$\Delta K_i = $ Interval between strike prices – half the difference between the strike on either side of K_i:

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$$

(Note: ΔK for the lowest strike is the difference between the lowest strike and the next higher strike, and ΔK for the highest strike is the difference between the highest strike and the next lower strike.)

R = Risk-free interest rate

$Q(K_i)$ = The midpoint of the bid-ask spread for each option with strike K_i

The calculations, the examples, tables and data in this subsection are directly taken from "The CBOE Volatility Index - VIX". The calculations present a hypothetic example of how the new VIX is calculated.

The VIX measures time to expiration in calendar days, and to adjust it to the precision required by most professional option and volatility traders, the calendar days are divided into minutes. The time to expiration is calculated by the following formula

$$T = \frac{\left(M_{current\ day} + M_{settlement\ day} + M_{other\ days}\right)}{minutes\ in\ a\ year}$$

where

$$M_{current\ day} = minutes\ until\ midnight\ of\ current\ day$$

$$M_{settlement\ day} = minutes\ from\ midnight\ until\ the\ time\ for\ calulation\ on\ SPX\ settlement\ day$$

$$M_{other\ days} = minutes\ in\ the\ days\ between\ current\ day\ and\ settlement\ day.$$

In the hypothetical example the near term option has 9 days to expiration, the next term option has 37 days to expiration, and the time for the calculation is 8.30 am. Then the time to expiration for the short term T_1 and the next term T_2 is calculated as

$$T_1 = \frac{(930 + 510 + 11520)}{525600} = 0.0246575$$

$$T_2 = \frac{(930 + 510 + 51840)}{525600} = 0.1013699$$

where

$$M_{current\ day}\ for\ both\ T_1 and\ T_2\ = 15.5 * 60 = 930$$

$$M_{settlement\ day}\ for\ both\ T_1 and\ T_2\ = 8.5 * 60 = 510$$

$$M_{other\ days}\ for\ T_1\ = 24 * 60 * 8 = 11520$$

$$M_{other\ days}\ for\ T_2 = 24 * 60 * 36 = 51840.$$

The next step in determining the VIX is to select which options that will be the subject for the calculations. The selected options are out-of-the money SPX puts and calls that are centered on an ATM strike price. The options are required to have a bidding price that is larger than zero.

To determine the forward SPX price level, the strike price where the absolute difference between the put and call is the smallest is identified. This is done for both the near term, and the second near term option, as seen in Table 2.

Table 2: Identifying the right strike price. Source: Chicago Board Options Exchange, Technical Notes C, 2009, p. 1-19).

Near term option				Next-term options			
Strike price	Call	Put	Difference	Strike Price	Call	Put	Difference
900	48.95	27.25	21.70	900	73.60	52.8	20.80
905	46.15	29.75	16.40	905	70.35	54.7	15.65
910	42.55	31.70	10.85	910	67.35	56.75	10.60
915	40.05	33.55	6.50	915	64.75	58.9	5.85
920	**37.15**	**36.65**	**0.50**	**920**	**61.55**	**60.55**	**1.00**
925	33.30	37.70	4.40	925	58.95	63.05	4.10
930	32.45	40.15	7.70	930	55.75	65.4	9.65
935	28.75	42.70	13.95	935	53.05	67.35	14.30
940	27.50	45.30	17.80	940	50.15	69.8	19.65

When the strike price is identified, the following formula will give the appropriate forward price. A forward price is determined for both the near term and next term contract.

$$F = Strike\ price + e^{RT}(call\ price - put\ price)$$

where R is the risk free rate of interest, in this case equal to 0.0038. Numerically, this is

$$F_1 = 920 + e^{(0.0038*0.0246575)} * (37.15 - 36.65) = 920.50005$$

$$F_2 = 920 + e^{(0.0038*0.0246575)} * (61.55 - 60.55) = 921.00039.$$

Next, we identify the strike price K_0 that is just below the forward price for both the near term and next term contract. As seen in Table 2, it is 920 for both the near term and next term options, although that does not have to be the case.

The next step is to select OTM put options with strike prices smaller than K_0, and OTM call options with strike prices larger than K_0. (Only options with a bidding price larger than zero. Once there are two consecutive options with bid-prices below zero, the following options will be excluded). This can be seen in Table 3.

Table 3: Select the correct OTM call and put options. Source: Chicago Board Options Exchange, Technical Notes C, 2009, p. 1-19).

Put strike	Bid	Ask	Include		Call Strike	Bid	Ask	Include
200	0.00	0.05			1215	0.05	0.50	YES
250	0.00	0.05			1220	0.05	1.00	YES
300	0.00	0.05			1225	0.00	1.00	NO
350	0.00	0.05	NO		1230	0.00	1.00	NO
375	0.00	0.10	NO		1235	0.00	0.75	
400	0.05	0.20	YES		1240	0.00	0.50	
425	0.05	0.20	YES		1245	0.00	0.15	
450	0.05	0.20	YES		**1250**	**0.05**	**0.10**	

Table 4 contains the upper and lower bound options, and the strike price K_0 used to compute the VIX. Both the call and the put option are selected at K_0 to calculate an average, while there is only one option, either a put or call, selected at the other strikes.

Table 4: Mid-quote prices, Source: Chicago Board Options Exchange, Technical Notes C, 2009, p. 1-19).

Near term Strike	Option Type	Mid-quote Price*		Next-term Strike	Option Type	Mid-quote Price*
400	Put	0.125		200	Put	0.325
425	Put	0.125		300	Put	0.30
.	.	.		.	.	.
915	Put	31.70		915	Put	58.90
920	**Put/Call Avarage**	**36.90****		**920**	**Put/Call Avarage**	**=61.05*****
925	Call	33.30		925	Call	58.95
.	.	.		.	.	.
1215	Call	0.275		1155	Call	0.725
1220	Call	0.525		1160	Call	0.60

* Mid-quote Price = (Bid+Ask)/2
** 36.90 = (37.15+36.65)/2
*** 61.05 = (61.55+60.55)/2

The volatilities for the near term options σ_1^2 and the next term options σ_2^2 underlying the VIX are presented in the formulas below:

$$\sigma_1^2 = \frac{2}{T_1} \sum \frac{\Delta K_i}{K_i^2} e^{RT_1} Q(K_i) - \frac{1}{T_1}\left[\frac{F_1}{K_0} - 1\right]^2$$

$$\sigma_2^2 = \frac{2}{T_2} \sum \frac{\Delta K_i}{K_i^2} e^{RT_2} Q(K_i) - \frac{1}{T_2}\left[\frac{F_2}{K_0} - 1\right]^2$$

The contribution of a single option to VIX is proportional to ΔK and the options price. For example, the contribution of the near term put option, with a strike of 400 is calculated as follows:

$$\frac{\Delta K_{400\,put}}{K_{400\,put}^2} e^{RT_1} Q\left(K_{400\,put}\right) = \frac{(425 - 400)}{400^2} * e^{0.0038*0.0246575} * (0.125) = 0.0000195$$

The same calculation is done for every option, and the values are summed and multiplied with $\frac{2}{T_1}$ for the near term options, and $\frac{2}{T_2}$ for the next term options. The calculated values are presented in Table 5.

Table 5: Contribution of a single option to VIX. Source: Chicago Board Options Exchange, Technical Notes C, 2009, p. 1-19).

Near term Strike	Option Type	Mid-quote Price*	Contribution		Next-term Strike	Option Type	Mid-quote Price*	Contribution
400	Put	0.125	0.0000195		200	Put	0.325	0.0008128
425	Put	0.125	0.0000173		300	Put	0.30	0.0002501
.	.	.	.		.	.	.	.
915	Put	33.55	0.0002004		915	Put	58.90	0.0003519
920	**Put/Call Avarage**	**36.90****	**0.0002180**		**920**	**Put/Call Avarage**	**61.05*****	**0.0003608**
925	Call	33.30	0.0001946		925	Call	58.95	0.0003446
.	.	.			.	.	.	
1215	Call	0.275	0.0000009		1155	Call	0.725	0.0000027
1220	Call	0.525	0.0000018		1160	Call	0.60	0.0000022
$\frac{2}{T_1} \sum \frac{\Delta K_i}{K_i^2} e^{RT_1} Q(K_i)$			**0.4727799**		$\frac{2}{T_2} \sum \frac{\Delta K_i}{K_i^2} e^{RT_2} Q(K_i)$			**0.3668297**

*Mid-quote price = (Bid+Call)/2
** 36.90 = (37.15+36.65)/2
***61.05 = (61.55+60.55)/2

The next step is to calculate $\frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$ for the near term and the next term:

$$\frac{1}{T_1}\left[\frac{F_1}{K_0} - 1\right]^2 = \frac{1}{0.0246575}\left[\frac{920.00039}{920} - 1\right]^2 = 0.0000120$$

$$\frac{1}{T_2}\left[\frac{F_2}{K_0} - 1\right]^2 = \frac{1}{0.1013699}\left[\frac{921.00039}{920} - 1\right]^2 = 0.0000117$$

Now the volatilities can be completed:

$$\sigma_1^2 = \frac{2}{T_1}\sum\frac{\Delta K_i}{K_i^2}e^{RT_1}Q(K_i) - \frac{1}{T_1}\left[\frac{F_1}{K_0} - 1\right]^2 = 0.4727799 - 0.0000120 = 0.4727679$$

$$\sigma_2^2 = \frac{2}{T_2}\sum\frac{\Delta K_i}{K_i^2}e^{RT_2}Q(K_i) - \frac{1}{T_2}\left[\frac{F_2}{K_0} - 1\right]^2 = 0.3668297 - 0.0000117 = 0.3668180$$

The last step is to calculate the VIX. It is achieved by calculating a 30-day weighted average of the volatilities, and multiply it with 100.

$$VIX = 100 * \sqrt{\left\{T_1\sigma_1^2\left[\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}}\right] + T_1\sigma_1^2\left[\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right]\right\} * \frac{N_{365}}{N_{30}}}$$

$$= \sqrt{\left\{0.0246575 * 0.4727679\left[\frac{53280 - 43200}{53280 - 12960}\right] + 0.1013699 * 0.3668180\left[\frac{43200 - 12960}{53280 - 12960}\right]\right\} * \frac{525600}{43200}}$$

$$= 0.612179986$$

$$VIX = 100 * 0.612179986 = 61.22$$

According to this hypothetic example that is directly taken from "The CBOE Volatility Index - VIX", the VIX is at 61.22, which according to the discussions in Subsection 2.2 is considered as a high level, possibly meaning that investors are worried about the future movements of the stock market.

2.3 VIX Futures

In this subsection we will explain the concepts of futures contracts related to the VIX. We will discuss the pricing of VIX futures, and introduce the exchange-traded products that are covered in our results and analysis, and further explain the concept of negative roll yield.

Like many other indexes, one cannot directly invest in the VIX. However, it is possible to trade derivatives with the VIX as underlying asset through VIX futures and VIX options, launched by the CBOE in 2004 and 2006 respectively (Chicago Board Options Exchange Technical Notes A, 2013, p. 2). This subsection will examine the characteristics of VIX futures.

To understand how VIX futures work we need to remember that the VIX is measuring the implied volatility of the S&P 500 index over the next 30 days. Therefore, when buying a VIX futures contract, we think that the market expects the future volatility to be higher at the time of expiration. Hence, if we are buying a VIX futures contract that expires in May, we think that the market, at maturity, expects the volatility of the S&P 500 index to be higher than implied by the futures contract value (Wise Stock Buyer, 2013). A futures contract is, according to Hull (2002), defined as "an agreement between two parties to buy or sell an asset at a certain time in the future for a certain price" (Hull, 2002, p. 5). Therefore, in order to make money from buying VIX futures contracts (and any other futures contract as well), we want the price of the underlying asset in the future to be higher than the price specified in our futures contract.

The value of the VIX futures contract is calculated by multiplying the level of the VIX futures contract with $1000. The futures contract is settled in cash, meaning that the difference between the initial value of the futures contract is compared to a Special Opening Quotation (SOQ) of VIX (Chicago Board Options Exchange Technical Notes A, 2013).

2.3.1 Pricing of VIX Futures

The pricing of these futures contracts are made out of expectations of the future levels of volatility and the VIX. It is of interest to examine the relationship between the VIX, VIX futures prices and maturity. Because of the uncertainty and difficulty in determining future levels of implied volatility, futures contracts with longer maturity are often priced higher than futures

contracts with shorter maturity (Hill, 2013, p. 12-13). This is seen in Figure 2, where the VIX spot level as of April 5th along with the respective quoted levels of VIX futures contracts with nine maturities are plotted.

Figure 2: VIX spot and futures prices on April 5, 2013. Source: Bloomberg.
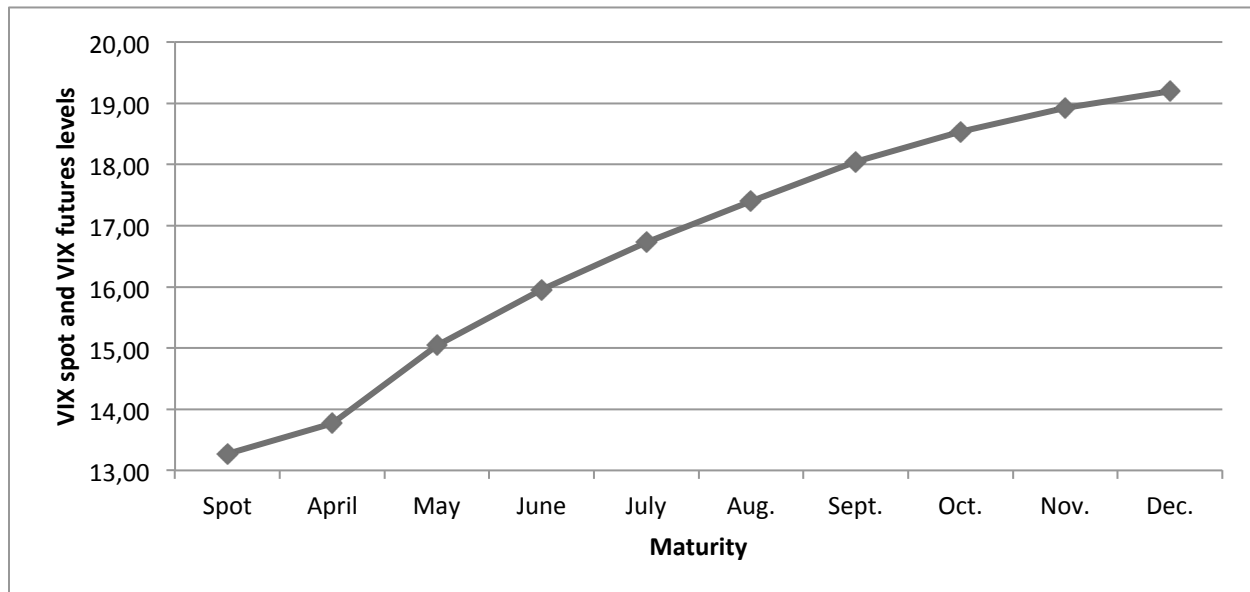
As we can see, the VIX spot level as of April 5th was about 13.30, the VIX futures contract with maturity in April had a quoted level of about 13.80, the contract expiring in May was quoted at about 15 and so on. That is, the futures price is higher than the spot price meaning that the futures price is converging downward to the spot price (Hull, 2002, p. 31). This situation is called contango, and is vital in the further examination of VIX futures contracts. Note also that the VIX term structure is not always in contango. However, as discussed in Subsection 2.3.3, this is common.

2.3.2 Exchange-Traded Products (ETPs)

Exchange-traded products ("ETPs") are, as the name reveals, financial products listed on stock exchanges. These products are tracking an underlying value, such as an index or a commodity (Wells Fargo Technical Notes). There are different types of ETPs, such as exchange-traded funds ("ETFs") and exchange-traded notes ("ETNs"). The former is similar to a traditional open-ended fund whereas the latter is more similar to debt securities (Credit Suisse Technical Notes).

If an investor is buying the VIX futures contract expiring in November, it means that the bought contracts are settled in cash at maturity in November, unless the investor sells the contracts before this maturity date. This poses some problems if the investor wishes to hold VIX futures contracts for an unlimited time, since his originally purchased contracts will expire at maturity. To avoid this, the investor can adjust his portfolio of VIX futures contracts by selling contracts near maturity and buying contracts with longer maturity. Imagine an investor wanting to hold VIX futures contracts for an unlimited time. In the beginning of April, he buys two VIX futures contracts: one expiring and June and one expiring in July. If he keeps his portfolio unadjusted, his position will expire in July since both of his contracts will be settled in cash by then. Therefore, when approaching June, the investor can sell the contract expiring in June and buy a VIX futures contract with maturity in August. This way, the investor is postponing the date of expiry. In other words, he is rolling his position forward.

Another way for the investor of holding this position in VIX futures over an unlimited time is to invest in ETPs consisting of VIX futures, and thus avoid having to roll the futures contract position manually.

Deng, McCann and Wang (2012) examined four of the most heavily traded VIX futures-based ETPs, among them the iPath S&P 500 VIX ST Futures ETN (VXX) and the iPath S&P 500 VIX MT Futures ETN (VXZ). These ETNs are based on two benchmark VIX futures indices, the short-term and mid-term S&P 500 VIX Futures Indexes. The difference between these two indices is that they are based off VIX futures contracts with different maturities; either short-term futures contracts or mid-term futures contracts. The indexes are computed by each day rolling over a fraction of all VIX futures contracts with shortest maturity to contracts with longer maturity. That is, the index is rebalanced each day by selling some number of futures contracts near maturity and buying contracts with longer maturity. The short-term index is using first and second month futures, meaning that each day the index is selling VIX futures with maturity in the first month and buying VIX futures with maturity in the second month. The same goes for the mid-term index, but instead of one and two month maturities, this index is using fourth, fifth and sixth month VIX futures contracts (Barclays Technical Notes, 2012).

These ETNs' trading volumes have been increasing rapidly since the financial crisis of 2008, as displayed by Figure 3.

Figure 3: Daily trading volume of VXX in millions of dollars. Source: Bloomberg.
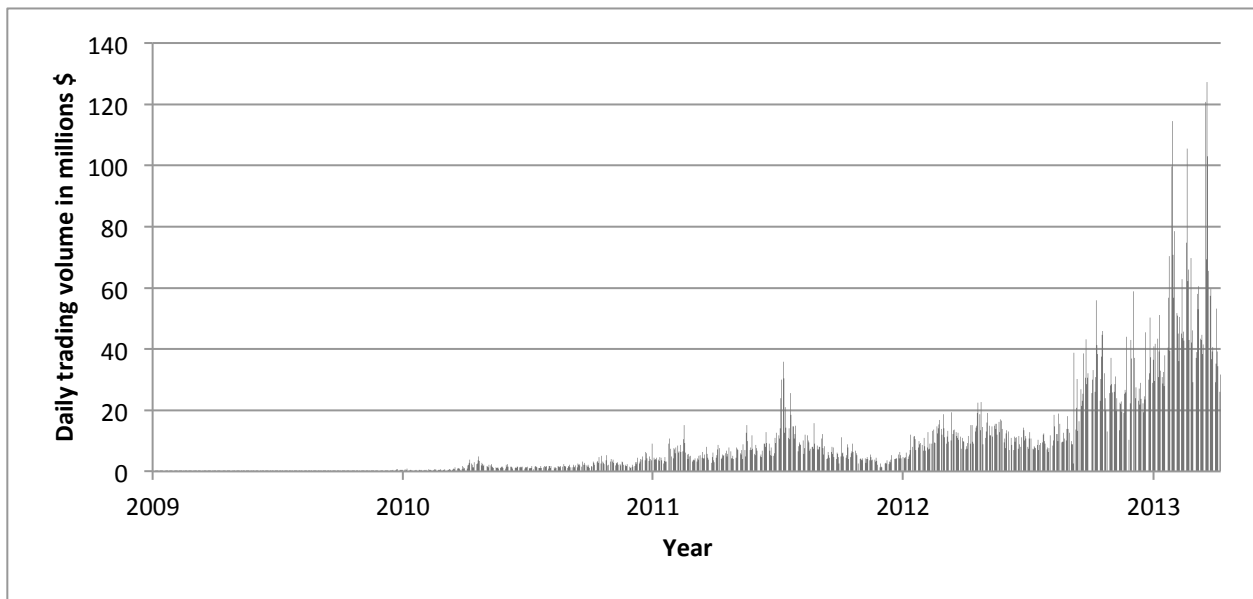
A possible explanation for the pattern in Figure 3 is the increased correlation between seemingly uncorrelated assets during the financial crisis of 2007 and 2008, forcing investors to look for other assets to diversify their portfolios with (Szado, 2009, p. 2-3). More traditional assets may have been considered good for diversification purposes before the crisis, but did not perform well enough during the actual crisis. As investors realized this, the trading volumes of other assets (VXX, for instance) increased.

2.3.3 Negative Roll Yield

The problem with VXX and VXZ is a phenomenon called negative roll yield, which is present when the VIX futures curve is in contango (Bernal, 2012). Contango is a situation where the futures price is above the expected spot price at maturity. That is, investors are generally willing to pay a premium for the underlying asset to be delivered at maturity instead of buying it in the spot market right now (which may be due to storage costs and so on). This means that the futures price, over time, has to converge downward to the spot price since the futures contracts will behave more like the spot price when approaching maturity. An underlying asset with these properties typically has a futures curve with a positive correlation, that is, an increasing curve between price and maturity, just as Figure 2 (Hull, 2002, p. 31). What then happens when the rolling takes place is that there is a negative spread between the value of the selling contract and

the buying contract, making us lose that amount. This is extra crucial when looking at the long term performance of VXX against VIX, as illustrated by Figure 4 (Bernal, 2012),

The negative roll yield is making the long-term performance of VXX significantly different from the performance of VIX. However, Figure 4 shows that the short-term performance of VXX is mirroring that of VIX. The reason for the differing performances long-term is the accumulated negative roll yield suffered when selling and buying futures contracts with different maturity, as described in the example in Subsection 2.3.2. Since the different levels of the two curves in Figure 4 is getting bigger with time, this shows that the VIX term structure more often than not is in contango, as discussed above in Subsection 2.3.1. Imagine a scenario with an opposite term structure: instead of losing money when rolling over the futures contracts, we earn money when doing so. This would lead to a situation where VXX instead is outperforming VIX, which clearly is not the case in Figure 4. Therefore, the conclusion that the VIX term structure often is in contango can be drawn.

Figure 4: Indexed performance of VIX and VXX. Source: Bloomberg.
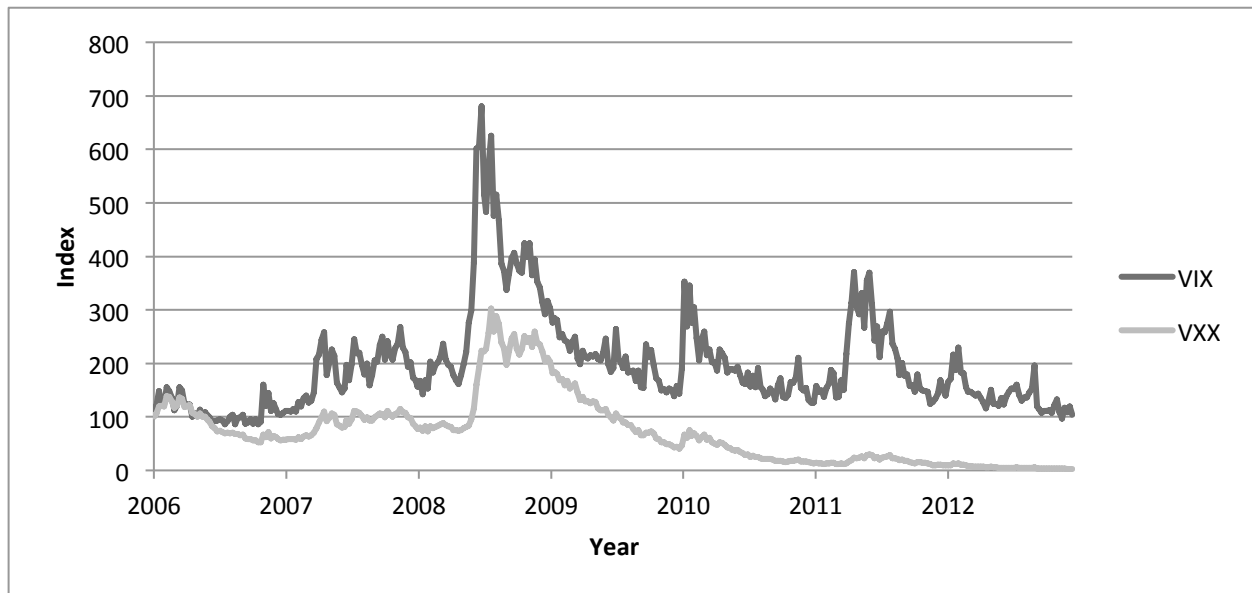
2.4 VIX and Market Returns

In this subsection we will briefly evaluate the historical relationship of the VIX and the S&P 500 returns.

Figure 5: Indexed VIX and S&P 500 returns. Source: Bloomberg.
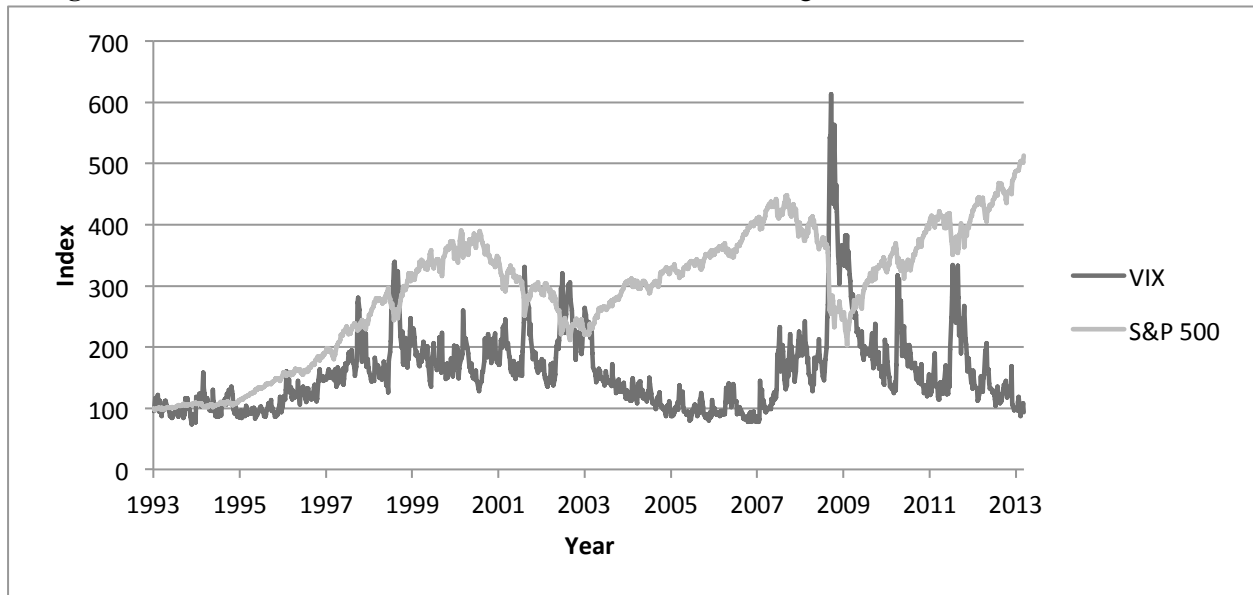

Figure 5 shows the movements of VIX and S&P 500 Index since the inception of VIX in 1993. It is quite clear that the returns of S&P 500 have a negative correlation to volatility and the VIX, that is, the two indexes have a tendency to move in opposite direction. For instance, when S&P 500 drops, the VIX increases – at least in an average sense. This is obvious when isolating times of crisis: in 2007 and 2008 we see an upward spike in the VIX at the same time as S&P 500 showed a sharp fall. The years leading up to the financial crisis of 2007 and 2008 has the inverted pattern: inclining S&P 500 and declining VIX.

2.5 Gold and Market Returns

In this subsection we will briefly evaluate the historical relationship between gold and S&P 500 returns.

Gold is an asset considered as a safe haven in times of market distress because of the actual value of the asset. The logic behind this is that investors sell of their stock and buy gold or gold futures because of this attribute. Consequently, the price of gold will increase due to the increased demand. With this logic, the price increase of gold would be extra present in times of market downturns (Seeking Alpha, 2012, p. 1-2). Figure 6 below shows price data of the SPDR S&P 500 ETF Trust ("SPY") and SPDR Gold Trust ("GLD"). The latter is an exchange-traded fund giving investors a relatively cheap way to invest in gold without actually buying the physical gold (Fontevecchia, 2012). This ETF has grown to become one of the most heavily traded ETFs on the market, according to ETF Database (ETF Database, 2013).

Figure 6: Indexed S&P 500 and gold returns. Source: Bloomberg.

Figure 6 shows that the correlation in fact seems to be negative during the financial crisis of 2007 and 2008. Some questions are raised regarding the correlation both before and after the financial crisis, where the correlation at first glance seems to be positive.

3. Methodology

In this section we introduce our method, discuss our scientific approach and how the study has been conducted. We will also introduce the theories that the results and analysis are based upon.

The main purpose of examining whether volatility can be used to reduce risk in a portfolio will be based on two broad parts: statistical properties and portfolio construction. This methodology is used in the papers inspiring this thesis, namely in Joane Hill's paper "The Different Faces of Volatility Exposure in Portfolio Management" (2013), Edward Szado's "VIX Futures and Options – A Case Study of Portfolio Diversification During the 2008 Financial Crisis" (2009), and Daglier's and Rossi's "A Portfolio of Stock and Volatility" (2006). The research conducted by Hill (2013) and Szado (2009) is used as a basis and inspiration for our empirical findings in terms of statistical properties. Portfolio construction is undertaken in way mainly seen in Daigler and Rossi (2006), and also Szado (2009).

3.1 Data

Pricing data for the following assets or indexes are used:

- SPDR S&P 500 ETF (SPY), which is an ETF tracking the S&P 500 index.
- CBOE Volatility Index (VIX).
- Short-term S&P 500 VIX Futures Index (SPVIXSTR), which is the index of which the iPath S&P 500 short-term VIX futures ETN (VXX) is based upon.
- Mid-term S&P 500 VIX Futures Index (SPVIXMTR), which is the index of which the iPath S&P 500 mid-term VIX futures ETN (VXZ) is based upon.
- SPDR Gold Trust (GLD). This is a tradable ETF tracking the gold price.

The data above is weekly and obtained from Bloomberg. It is divided into three time periods for purpose of calculation:

- Total time period: All data, ranging from 2006-05-05 to 2013-04-12.
- Crisis: The financial crisis in 2007-2008, where pricing data between 2007-08-10 and 2009-03-06 is used. This is illustrated by the space between the two vertical lines in Figure 7.
- Post-crisis: The period after the crisis, between 2009-03-06 and 2013-04-12. This is shown to the right of the second vertical line in Figure 7.

Figure 7: Time periods. Source: Bloomberg.
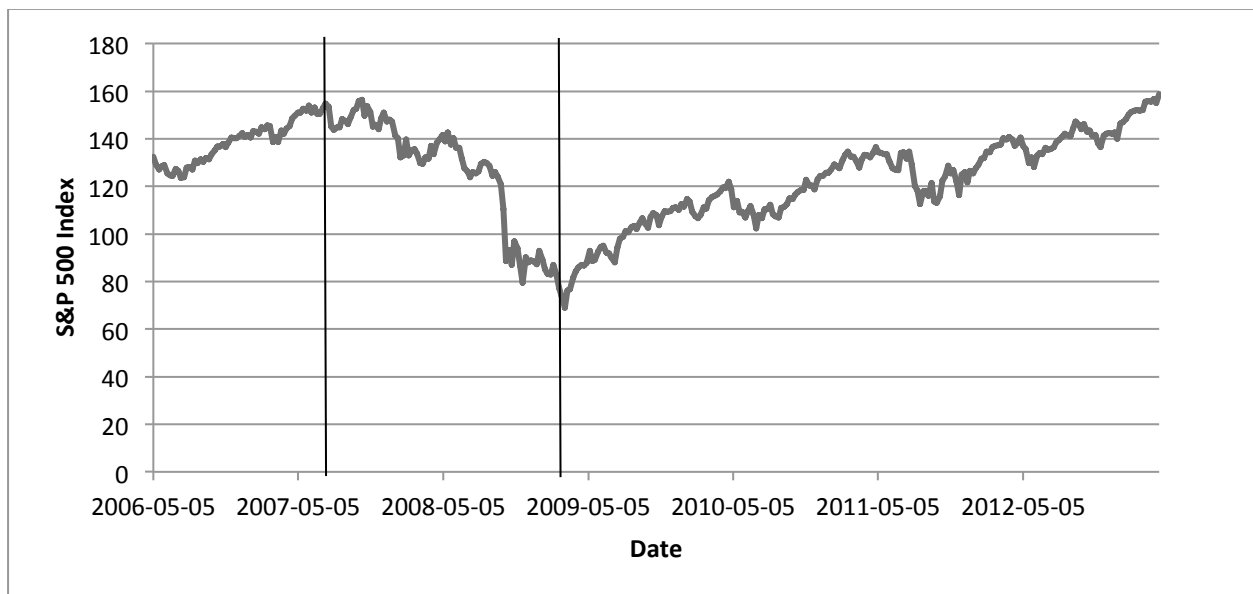
3.2   Statistical Properties

In this subsection we will illustrate the statistical properties that are used to interpret and analyze the empirical findings in Subsection 4.1. The different techniques presented in this subsection can all be found in in Hill, Joane (2013) and Szado, Edward (2009), two of the three papers that we base our thesis on. We will give examples continuously throughout the whole subsection of how the different techniques have been used in previous research related to our thesis.

3.2.1 Correlation Coefficient

The correlation coefficient is simply the linear dependence between two different variables. In our case it is the linear dependence between two assets' returns. The correlation coefficient can range between negative one and positive one. Where a correlation of positive one means that the return of the two assets moves in the same direction 100 percent of the time, and a correlation of negative one means that the two assets' returns moves in the opposite direction 100 percent of the time. If the correlation is zero, the returns of the assets are completely uncorrelated (Berk and DeMarzo, 2011, p. 333-334).

The correlation coefficient $\rho_{X,Y}$ is calculated by dividing the covariance of the two assets' returns by the product of the two assets' standard deviations, that is,

$$\rho_{X,Y} = \frac{COV(X,Y)}{\sigma_X * \sigma_Y}.$$

The correlation coefficient is a standard statistical property used in finance, and it has been used in previous research related to our thesis, see p. 3 in Szado, Edward (2009) and p. 14 in Hill, Joane (2013).

The calculations of the correlation coefficients will be performed in Microsoft Excel.

3.2.2 Beta Coefficient

The beta coefficient which will be used in this thesis is the beta derived from the original Capital Asset Pricing Model ("CAPM"), which builds on the modern portfolio theory developed by Harry Markowitz. According to the CAPM, the expected return on any asset $E(R_i)$ can be determined by adding the risk free rate of interest R_f to the product of the assets market beta β_{mi} and the premium per unit of beta risk $E(R_m) - R_f$, where $E(R_m)$ is the expected return on the market portfolio. Then the expected return is given by $E(R_i) = R_f + \beta_{mi}(E(R_m) - R_f), i = 1, \ldots, N)$ (Eugene F. Fama and Kenneth R. French, 2004, p.29).

The expected return on the market portfolio implies the expected return on every asset available in the world. This is impossible to estimate, so instead of the return on the market portfolio it is common to use an appropriate benchmark, such as the S&P 500 (Eugene F. Fama and Kenneth R. French, 2004, p. 25-26).

The market beta β_{mi} is the covariance of the individual assets return and the market return $Cov(R_i, R_m)$ divided by the variance of the market return $\sigma^2{}_m$. Hence the market beta is given by $\beta_{mi} = \frac{Cov(R_i, R_m)}{\sigma^2{}_m}$ (Eugene F. Fama and Kenneth R. French, 2004, p.28).

When we calculate the betas presented in our empirical findings, we will use two different benchmarks for the market portfolio, the S&P 500 and the VIX. Calculating the betas this way, we will be able to find the individual assets sensitivity to S&P 500 and the VIX. For example, the VXZ beta to VIX is calculated as $\beta_{VIX,VXZ} = \frac{Cov(R_{VXZ}, R_{VIX})}{\sigma^2{}_{VIX}}$. The same technique can be seen in previous research related to our thesis, see p. 15 in Hill, Joane (2013). The beta calculations will be performed in Microsoft Excel.

We will also calculate two types of rolling betas: the rolling beta of VIX, VXX and VXZ to S&P 500 and the rolling beta of VXX and VXZ to VIX. This will enable us to track the changes over time and thereby display the betas stability over time, first between the volatility-based instruments and the market and then between VXX and VXZ and VIX.

The rolling betas will be calculated as follows: first we calculate a beta using the returns for week 1 to 13, and then we will recalculate the beta using the returns for week 2 to 14 and so on. Thereby, we will obtain a 3-month beta for every week from 2006-08-04 to 2013-04-12. The betas will then be presented in a graph that displays the changes over time. These calculations will be performed in Microsoft Excel. This is used to display the beta stability over time, and can be seen in previous research related to our thesis, see p.16 in Hill, Joane (2013).

3.2.3 Volatility

The term volatility used in our thesis is simply the standard deviation of the individual assets historical returns. The standard deviation is calculated as $stdev(R_i) = \sqrt{\frac{\sum_{i=1}^{N}(R_i - \bar{R})^2}{N}}$. In finance, historical volatility is often referred to as the standard deviation of the historical returns (Berk and DeMarzo, 2011, p. 301-302)

We will calculate the volatilities for the five different assets (S&P 500, VIX, VXX, VXZ, gold) using historical returns (weekly) for the three different time periods referred to in Subsection 3.1. The numbers will then be presented in a staple chart, which will aim to illustrate the differences

among the five assets' volatilities, and if there are any different features across the three time periods. All the calculations will be performed in Microsoft Excel.

3.3 Portfolio Construction

In this subsection we will explain and elaborate the techniques used to analyze and interpret empirical findings in Subsection 4.2. We will also give examples of how these techniques have been used in some previous research related to our thesis.

Here, we will examine the data using modern portfolio theory, developed by Harry Markowitz (Edwin and Gruber, 1997, p. 2). This framework for portfolio management is based on the mean and variance of the portfolio, that is, the expected return and risk. By holding variance constant and maximizing expected return, or holding expected return constant and minimizing variance, a so called efficient frontier can be formed. Along this frontier, we find the portfolios with the smallest risk for each level of return (Edwin and Gruber, 1997, p. 2).

In our analysis, we focus on a portfolio called the minimum-variance portfolio. This portfolio has the smallest risk of all portfolios along the efficient frontier (Clarke et al., 2006, p. 1-2). Daigler and Rossi use a similar methodology in their paper "A Portfolio of Stocks and Volatility" (2006).

In order to explain how these efficient frontiers are calculated we consider a portfolio consisting of two assets; asset 1 and asset 2. These assets have returns R_1 and R_2, respectively. Furthermore, w_1 and w_2 are denoting the portfolio weights of asset 1 and asset 2 in percent, meaning that if $w_1 = 1$, 100 percent of the portfolio consists of asset 1. Furthermore, let σ_i denote the standard deviation of the return R_i for asset i, $i = 1,2$. We also let ρ_{12} denote the correlation coefficient between R_1 and R_2. Recall that $E(X)$ denotes the expected value of the random variable X. We are now ready to state the formulas for portfolio risk and portfolio expected return, as

$$E(R_P) = w_1 E(R_1) + w_2 E(R_2) \qquad (1)$$

$$\sigma_P = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_1 w_2 \rho_{12} \sigma_1 \sigma_2. \qquad (2)$$

The efficient frontier is made by altering w_1 and w_2. By letting w_1 take on values between 0 and 1 (and remembering that $w_1 + w_2 = 1$), we will obtain different portfolio returns and different

standard deviations according to Equation (1) and Equation (2). These numbers can be graphed in a risk-return matrix, ultimately forming the efficient frontier.

We are constructing four different efficient frontiers. In all cases, asset number one is S&P 500 (SPY), representing a well-diversified stock portfolio. Asset two will vary among the Volatility Index (VIX), the short-term VIX futures ETN (VXX), the mid-term VIX futures ETN (VXZ) and gold (GLD). We will find the minimum variance portfolios for each efficient frontier using Solver in Microsoft Excel, solving for the w_2 that will minimize portfolio risk according to Equation (2). These minimum variance portfolios are then compared to a portfolio consisting of only S&P 500. This comparison will be based on the risk-return relationship, where we can conclude diversification benefits if the minimum variance portfolio is less risky than the all S&P 500 portfolio, without having a significantly smaller expected return. This will be done for each time period outlined in Section 4.1. This will result in twelve different efficient frontiers.

Also, Equation (2) illustrates the positive effects on risk of constructing a portfolio with negatively correlated assets (Daigler and Rossi, 2006, p. 101). Since all factors in Equation (2) are positive (in this example we are assuming no negative weights, and we also know that the variance of returns cannot be negative) except for the correlation. With a negative correlation, the term $w_1 w_2 \rho_{12} \sigma_1 \sigma_2$ will therefore also be negative, which will reduce portfolio risk. Thus, holding everything but the correlation coefficient fixed, we will obtain the lowest standard deviation if the correlation coefficient is at its minimum: negative one.

To further illustrate the properties of the calculated minimum variance portfolios, we compare these portfolios to three benchmark portfolios. Here, we begin by isolating each time period, calculating efficient frontiers and minimum variance portfolios according to above. This will result in four minimum variance portfolios for each time period, where asset number two will be VIX, VXX, VXZ and gold, respectively (and S&P 500 will be asset one). In each case, we calculate a portfolio with the weights used to form these minimum variance portfolios. This will lead to a situation where, for each time period, we have four portfolios: S&P 500 and VIX, S&P 500 and VXX, S&P 500 and VXZ, and S&P 500 and gold. The weights used in the portfolios will vary since they are depending on the weights generated by calculating the minimum variance portfolios. These four portfolios will then be compared to three benchmark portfolios

(100 percent S&P 500, 90 percent S&P 500 and 10 percent Treasury bills, and 75 percent S&P 500 and 25 percent Treasury bills) on basis of the following:

- Annualized returns: We average the weekly returns and then annualize this number by the formula below.

$$Annualized\ return = (1 + Average\ weekly\ return)^{52} - 1$$

- Annualized standard deviation: Standard deviation of the weekly returns data is annualized according to the formula below.

$$Annualized\ standard\ deviation = (Standard\ deviation * 52)^{0,5}$$

- Maximum drawdown: This is a risk measurement telling us how much an investor could possibly lose during a time period, if he buys the asset at the peak and sells it at the bottom. Thus, the maximum consecutive price drop in between a peak and a trough. (Burghardt et al., 2003, p. 1) The calculations can be found in Appendix 2.

- Percent up weeks and down weeks: How many percent of the weeks during the time period that the portfolio gained value and lost value, respectively.

- Sharpe ratio: The Sharpe ratio is a risk measurement telling us how much excess returns we get in a portfolio per unit of risk for that portfolio (Sharpe, 1994, p. 49-53) We will use the risk-free rate in order to calculate excess returns, meaning that the formula used is:

$$Sharpe\ ratio = \frac{R_p - R_f}{\sigma_p}$$

- Treynor ratio: Treynor ratio is similar to Sharpe ratio in that we are comparing excess return to portfolio risk. Instead of dividing excess returns by portfolio standard devation, we divide it by portfolio beta (Bhardwaj, 2012). That is, instead of using total risk Treynor ratio is using market risk (also referred to as systemic risk), often considered to be non-diversifiable (Schwarcz, 2008, p. 198).

$$Treynor\ ratio = \frac{R_p - R_f}{\beta_p}$$

This is generating in total three tables (crisis, post-crisis and total time period) with seven portfolios in each table (three benchmark portfolios and four calculated minimum variance portfolios).

Furthermore, we proceed to undertake a sensitivity analysis on the calculated minimum variance portfolios. The purpose behind this is to see how a small change in the time period affects the risk and return of the minimum variance portfolio. If the estimated risk and return is varying substantially over time, we can conclude that these instruments are volatile and that their behavior is especially hard to predict. Here, we use a window of twelve weeks that we will move along the time period with four weeks each time. Thus, we start by using the first twelve weeks in the sample, that is, twelve weeks from 2006-05-05. From these weeks we estimate average return and standard deviation of returns of asset one (always S&P 500) and asset two (the examined asset; VIX, VXX, VXZ or gold), and also the covariance between the two assets during these twelve weeks. For instance, if we are performing a sensitivity analysis of the minimum variance portfolio with S&P 500 as asset one and VIX as asset two, we compute average return and standard deviation of returns for these assets, along with the covariance of returns between the assets. We then use Microsoft Excel to calculate the weights in the portfolio that minimizes portfolio risk, and proceed to use these weights to calculate portfolio return and portfolio standard deviation according to Equation (1) and Equation (2) on page 25. Next, we move this window of twelve weeks forward by four weeks. Thus, we start on 2006-06-09 and use the following twelve weeks to perform the same calculations. This is done all the way throughout all obtained data, generating a total of 86 different minimum variance portfolios: each calculated by using data from twelve weeks rolled forward by four weeks at a time. Next, these 86 resulting portfolio returns and portfolio standard deviations will be graphed to illustrate the sensitivity of the estimated parameters with respect to the time period and data used when performing these estimations. In total, we will perform four sensitivity analyses; one for VIX, VXX, VXZ and gold, respectively.

3.4   Methodology Critique

It is especially interesting for an investor to reduce portfolio risk in a financial crisis; therefore it is important to mention his or her abilities of doing so. The problem boils down to the investor's ability of predicting a financial crisis. Note that VIX, VXX and VXZ are measuring the implied volatility, meaning that when the market is predicting a financial crisis, the value of these indexes will be high. Therefore, the investor needs to predict the financial crisis before the market does so.

In this thesis, we predicted the financial crisis in a way only possible with known pricing data. It is slightly unrealistic to assume that investors have the ability of predicting the crisis the way we did. Our time period representing the financial crisis begins early, capturing the large returns in VIX, VXX and VXZ that happened before the actual crisis began. An investor is more likely to buy a volatility-based instrument later than we did, therefore possibly missing out on some of the large returns captured by our calculations. As a result of this, the performance of VIX, VXX and VXZ tends to be biased during the time period covering the financial crisis, compared to a realistic scenario.

Nevertheless, the main purpose of this thesis is not to discuss whether one can predict a financial crisis, but rather how to study the possibilities of using volatility as a tool for risk reduction in a stock portfolio.

4.   Empirical Findings

In this section we will present our empirical findings. In Subsection 4.1 we use the statistical properties discussed in Subsection 3.2 to analyze and interpret the data. In Subsection 4.2 we use the modern portfolio theory discussed in Subsection 3.3 to interpret and analyze the data.

4.1   Statistical Properties

In this subsection we present and analyze our empirical findings in terms of statistical properties. Subsection 4.1.1 covers the correlation coefficient, and Subsection 4.1.2 examines the beta of our examined assets. Subsection 4.1 is then rounded off with an analysis of the examined instruments' respective volatilities.

4.1.1 Correlation Coefficient

Table 6: Correlation to S&P 500.

Correlation to S&P 500	VIX	VXX	VXZ	Gold
Total time period	-0.72	-0.73	-0.68	0.04
Crisis	-0.76	-0.72	-0.64	-0.14
Post-crisis	-0.73	-0.75	-0.72	0.20

Table 6 presents a strong negative correlation between the VIX and S&P 500 during all three periods. Note that VXX and VXZ follow the same pattern with a strong negative correlation to S&P 500 during all three time periods. Notable is also that the VIX has an increasing negative correlation to the S&P 500 during the crisis, while VXX and VXZ display a decreasing negative correlation during the same period.

The gold ETF stands out with a weaker correlation the S&P 500 and with a bigger variation in the correlation coefficients. The correlation to the S&P 500 is weakly positive measured over the total time period, but increasingly negative during the crisis, and increasingly positive after the crisis. Such dynamics can be desirable: it indicates that gold is rising with the stock market in times of economic growth, but also rising when the stock market is suffering from a financial crisis. However, the correlation of gold to S&P 500 is relatively close to zero during all three time periods, which indicates that the correlation is weak and that the performance of gold relative to S&P 500 is unpredictable.

The correlation of the volatility based instruments, VIX, VXX and VXZ to the stock market, though, is around -0.7, indicating a strong negative relationship. Also, the stability of these

numbers over all three time periods further shows the consistent performance of volatility relative to the stock market.

The strong negative correlation between these instruments and the stock market is a key characteristic for risk reduction purposes. This is in line with Subsection 3.3, in which we illustrated the importance of negative correlation in order to reduce the risk of a portfolio. Therefore, according to the calculated correlations, volatility could be an appropriate tool for minimizing risk in a portfolio.

Table 7: Correlation to VIX.

Correlation to VIX	VXX	VXZ	Gold
Total time period	0.86	0.73	-0.04
Crisis	0.86	0.75	0.17
Post-crisis	0.86	0.77	-0.12

Table 7 presents the VIX future indices' and gold's ability to track the movements in the VIX. VXX displays the same positive correlation coefficient of 0.86 during all three periods, while VXZ shows a weaker correlation that varies over the three periods. Gold was positively correlated to VIX during the financial crisis, and negatively correlated after the crisis. This is in line with the theory about gold working as a "safe haven" for investors, and a possible explanation for this phenomenon.

We detect a resemblance between the correlation coefficients presented in Table 7 with the ones presented in Table 6. By switching the coefficients signs in either table we get similar results. The other main takeaways from these numbers are the consistent strong correlation among VIX, VXX and VXZ.

4.1.2 Beta Coefficient

In many ways it is more interesting to examine the beta rather than the correlation. With the beta, we do not only get the correlation, but also the magnitude of the correlation. When examining the four assets' beta to S&P 500, the sensitive nature of these instruments was revealed.

Table 8: Beta to S&P 500.

Beta to S&P 500	VIX	VXX	VXZ	Gold
Total time period	-3.61	-2.30	-1.12	0.04
Crisis	-2.55	-1.60	-0.84	-0.13
Post-crisis	-4.23	-2.78	-1.34	0.18

The betas presented in Table 8 show a negative relationship between the S&P 500 and the three volatility-based instruments during all three periods. That is consistent with the correlation coefficients presented in the Subsection 4.1.1. The highest betas are obtained after the financial crisis, and the lowest are obtained during the financial crisis. Although all the betas display a negative relationship to S&P 500, they are quite different. The VIX is way more sensitive to shifts in the S&P 500 than VXX and VXZ. A one percent shift in the S&P 500 during the crisis would lead to a 2.55 percent shift in the opposite direction in the VIX, but only a 1.6 percent shift in VXX, and a 0.84 percent shift in the VXZ.

The sign of the gold ETF betas varies between the different time periods, with a negative beta during the crisis, and positive betas during the other two periods. The betas are in fact almost mirroring the correlation coefficients presented in the first Subsection 4.1.1.

By looking at Table 8, we notice the large betas of VIX to S&P 500. These numbers can be used to illustrate the high volatility of VIX. For instance, during the total time period, this beta was -3.61, meaning that the expected drop in VIX is 3.6 percent for every 1 percent drop in the S&P 500. During the post-crisis period, this number was -4.23. Regardless of time period, VIX is therefore quite sensitive to movements in S&P 500.

Table 9: Beta to VIX.

Beta to VIX	VXX	VXZ	Gold
Total time period	0.53	0.24	-0.01
Crisis	0.57	0.29	0.05
Post-crisis	0.55	0.25	-0.02

The betas presented in the Table 9 show us the two future indices and the gold ETF's sensitivity to changes in the VIX. As we can see VXX is more sensitive than VXZ, and the gold ETF

displays almost no sensitivity at all. This is telling the same story as Table 8: VIX is more volatile than VXX and VXZ, and significantly more volatile than gold.

4.1.3 Volatility

This subsection aims to illustrate the differences among the five assets' volatilities and if there are any different features across the three time periods.
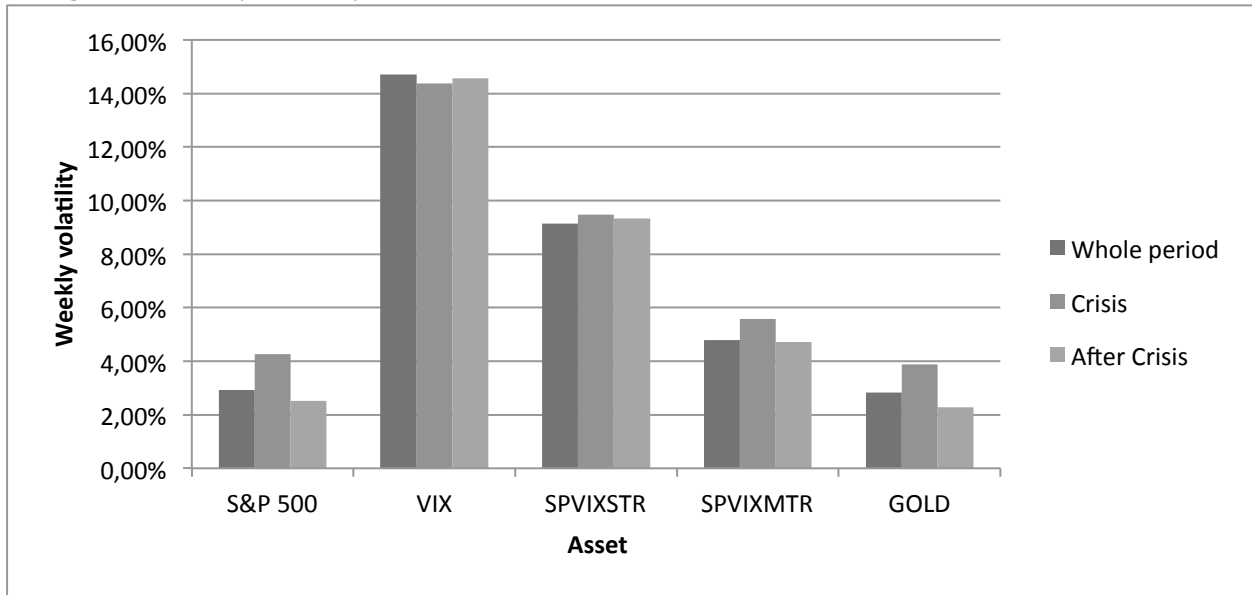
Figure 8: Weekly volatility.

Figure 8 displays the five assets' volatility during the three time-periods. The method behind the calculation is further explained in Subsection 3.2.3.

The VIX is by far the most volatile asset and with little variation across the three periods. This is confirming what the previous subsection hinted: the volatility-based instruments are sensitive and highly volatile. Notable is that the VIX actually has its least volatile period during the crisis, implying a relatively small variation in the returns during a time period where returns usually vary the most. This becomes evident when looking at the other assets' volatilities during the three time periods in Figure 8. In all other cases the volatility is largest during the crisis. This is a feature that is unique for the VIX in this study. Note that VXX is almost twice as volatile as VXZ.

The gold ETF and S&P 500 are very similar, they are least volatile, but have also the greatest variation across the three periods.

Table 10: Weekly gains and losses.

Ten Largest Weekly Gains in VIX (%)						Ten Largest Weekly Losses in the VIX (%)					
Week Ending in	VIX	VXX	VXZ	Gold	S&P	Week Ending in	VIX	VXX	VXZ	Gold	S&P
2010-05-07	85.7	41.8	21.5	2.5	-6.4	2013-01-04	-39.1	-28.3	-11.9	-0.0	4.5
2007-03-02	75.9	25.3	4.7	-5.9	-4.6	2007-08-24	-30.1	-16.4	-9.3	1.7	2.5
2008-10-10	55.0	37.8	9.4	0.7	-19.8	2011-03-25	-26.7	-14.6	-5.4	0.6	2.8
2010-01-22	52.5	9.0	4.3	-3.3	-3.9	2011-07-01	-24.8	-18.3	-10.2	-0.9	5.6
2011-07-29	44.1	13.3	1.8	1.4	-3.9	2008-10-31	-24.3	-0.7	-1.5	-1.2	11.3
2007-07-27	42.6	12.7	9.2	-3.2	-5.5	2007-03-09	-24.3	-7.3	-3.9	0.8	1.5
2009-10-30	37.8	13.8	4.9	-0.9	-4.2	2008-11-28	-23.9	-14.4	-9.7	1.9	13.3
2011-03-23	33.2	22.7	8.7	-9.2	-6.6	2007-03-21	-23.8	-16.7	-9.3	3.4	2.5
2010-04-30	32.7	15.3	6.8	1.9	-2.5	2010-05-14	-23.7	-8.7	-6.2	1.8	2.4
2008-06-06	32.1	13.9	6.6	1.8	-0.3	2007-03-23	-22.9	-15.9	-4.1	0.8	3.5

Table 10 is illustrating the movements of the VIX relative to that of the four other assets, VXX, VXZ, gold and S&P 500. We picked the ten weeks were the VIX gained the most, and then presented the returns for the four other assets, for the same ten weeks. The same is done in the right part of Table 10, but with the ten largest losses.

From Table 10, the negative correlation between VIX and S&P 500 becomes clear: in all of the ten weeks where VIX gained the most, S&P 500 fell. This is proving the supposed negative correlation from looking at Figure 5. The same can be seen in the weeks where VIX lost the most: S&P 500 gained in all cases. However, the numbers are rather unstable. An 86 percent gain in VIX happened during a week where S&P 500 only lost about 6.4 percent, whereas the week S&P 500 lost a horrific 20 percent, the VIX only gained about 55 percent. We can see the same pattern with negative correlation for VXX and VXZ to S&P 500 as well, while the movements of gold relative to the movements in S&P 500 are more unpredictable.

Figure 9 and Figure 10 shows the rolling 3-month betas for the volatility based instruments over time. The technique used to calculate the rolling betas are further explained in Subsection 3.2.2.

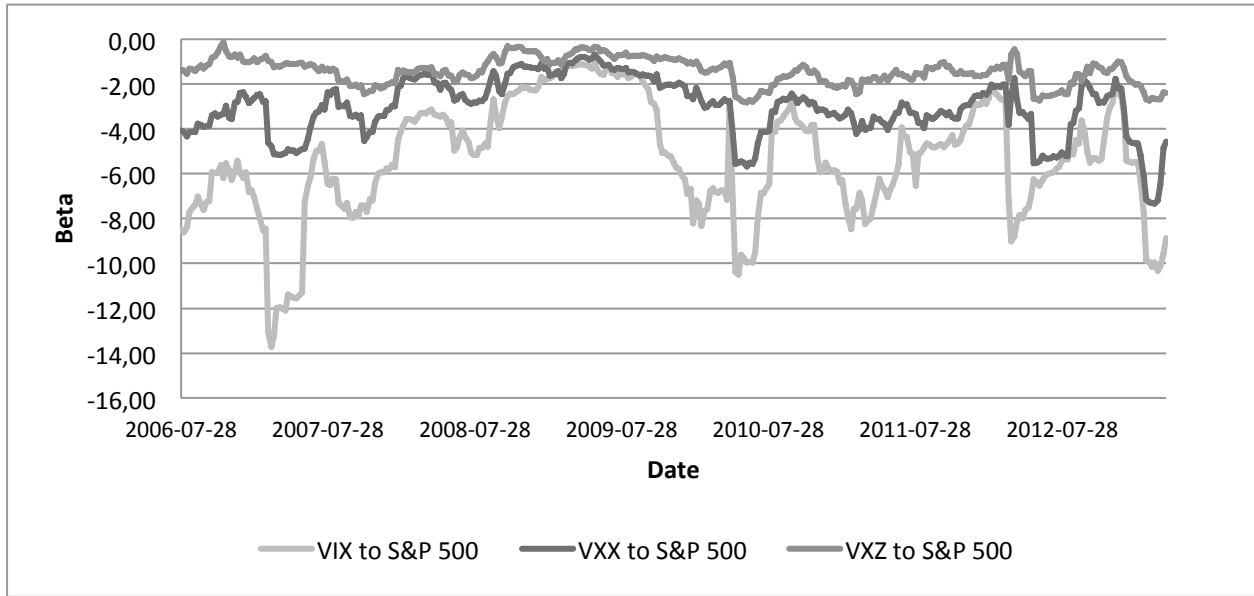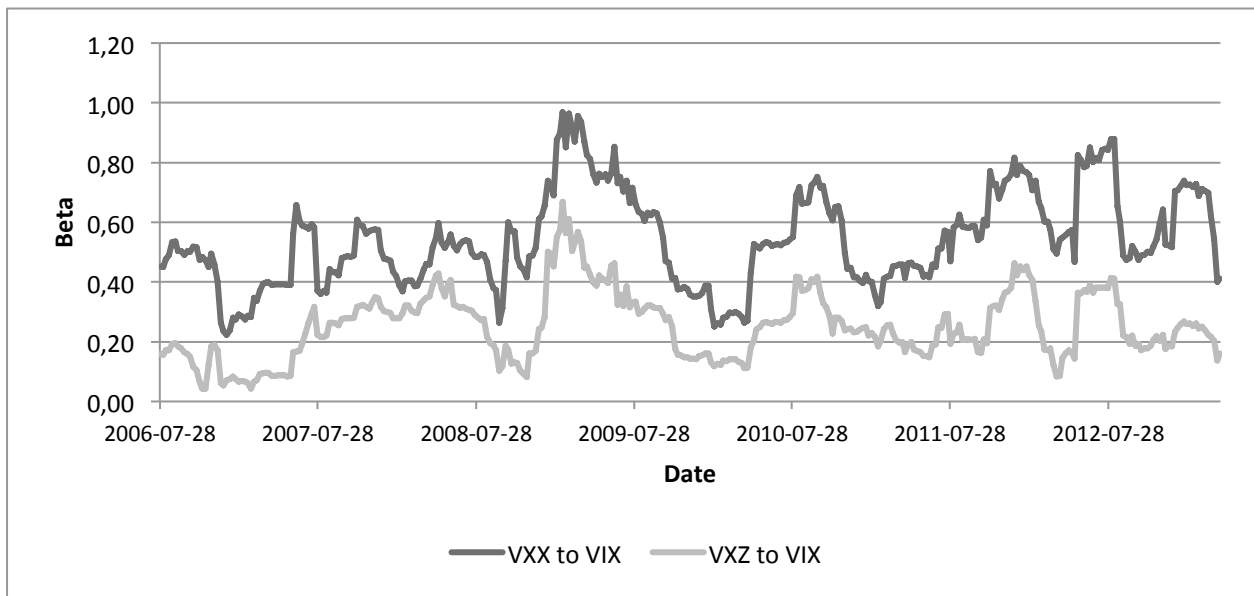**Figure 9:** Rolling beta to the market.

Figure 10: Rolling beta to VIX.

Tables 6 to 10 along with Figures 8 to 10 illustrate one important characteristic of the VIX and the volatility-based instruments: their volatilities. One of the main impacts of high volatility in this case is that it is difficult to construct a portfolio without systematic risk, that is, a portfolio with a portfolio beta equal to zero – a so called zero-beta portfolio. This is even hard when using historical data, because of the behavior of highly volatile instruments. Therefore, serious doubts are raised about the possibilities of constructing riskless portfolios using these instruments, when

perfect pricing information is not available. This can be illustrated using the VIX beta to S&P 500 of -2.55 during the crisis. Statistically, a 1 percent decline in the S&P 500 should result in a 2.55 percent incline of the VIX. By allocating about 40 percent (= 1/2.55) of the portfolio to VIX, the declines in S&P 500 should be offset by simultaneous gains in VIX. However, with a volatile instrument and therefore a volatile beta, this is not easy. First, the betas of all individual observations will be different from the beta over the total time period. This is the result of the high volatility and non-perfect correlation between the two assets. Second, the volatility-based instruments' betas to the market will vary over time, as shown in Figure 9.

The implications of these findings are that, even though using the correct beta, the high volatility of VIX will make it hard to construct a zero-beta portfolio. Also, the beta is far from consistent over time, making it difficult to find the correct beta. Overall, this makes us question the possibilities of using VIX and volatility to perfectly reduce risk in a stock portfolio. Nevertheless, the desirable statistical characteristics of all examined assets are suggesting that it is possible to reduce risk, even though not perfectly.

4.2   Portfolio Construction

In this subsection, we construct portfolios according to Markowitz's modern portfolio theory. Different efficient frontiers are calculated in a way presented in Subsection 3.3, and presented here in Subsection 4.2.1 and Subsection 4.2.2. In Subsection 4.2.1, these efficient frontiers are shown in four figures: one each for VIX, VXX, VXZ and gold. Each figure in Subsection 4.2.1 will have three efficient frontiers, representing the three different time periods: total (2006-2013), crisis (2007-2008), and post-crisis (2009-2013). In Subsection 4.2.2, we present these efficient frontiers in terms of time periods instead, enabling us to compare the performance of different assets during the same time period. Here, the performances of the portfolios are also summarized in tables. In Subsection 4.2.3 we proceed to perform a sensitivity analysis in terms of risk and return for each of the four assets.

In Subsection 4.2.2 we will compare some of our results to the results in Szado, Edward (2009). His results are only based on data collected during the time period he calls the financial crisis (March 2006 to December 2008), while we collected data from a larger time period (May 2006 to

April 2013) divided into three time periods, total time period, crisis and post crisis. The crisis (August 2007 to Match 2009) is the time period that matches Szados time period the best, therefore it will be natural to use our results from that period, presented in Figure 16, to compare to Szado's results.

4.2.1 Asset Classes

In Figure 11 to Figure 16 the following notations will be used: squares are representing a portfolio consisting of only S&P 500, triangles are representing a portfolio consisting of only the examined asset class, and the circles are representing the resulting minimum variance portfolio. For the circle showing the minimum variance portfolio, the weight in S&P 500 is varying depending on the characteristics of the examined asset class.

In Figure 11, the VIX and its possible diversification properties are examined. Figure 11 shows the results for the three time periods, total (2006-2013), crisis (2007-2008), and post-crisis (2009-2013). A risk-reducing effect can be seen in all time periods, and in all cases the expected portfolio return increased or stayed the same as compared to a portfolio with only S&P 500. This improved risk-return relationship becomes especially clear by isolating the crisis, where shifting around 15 percent of the portfolio from S&P 500 to VIX would have cut the risk by half and cut losses by two thirds. This is made possible by the negative correlation and beta, once again proving the possible positive diversification effects. It is also worth noting that in all cases, the optimal allocation from a risk-reducing view is to allocate around 15 percent to VIX.

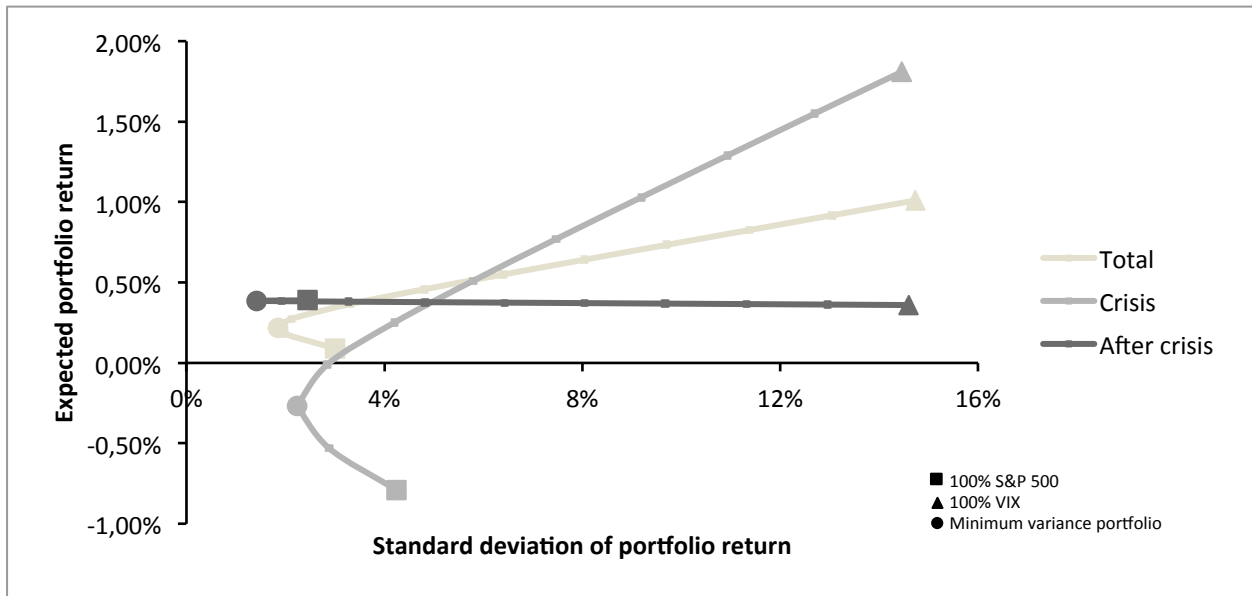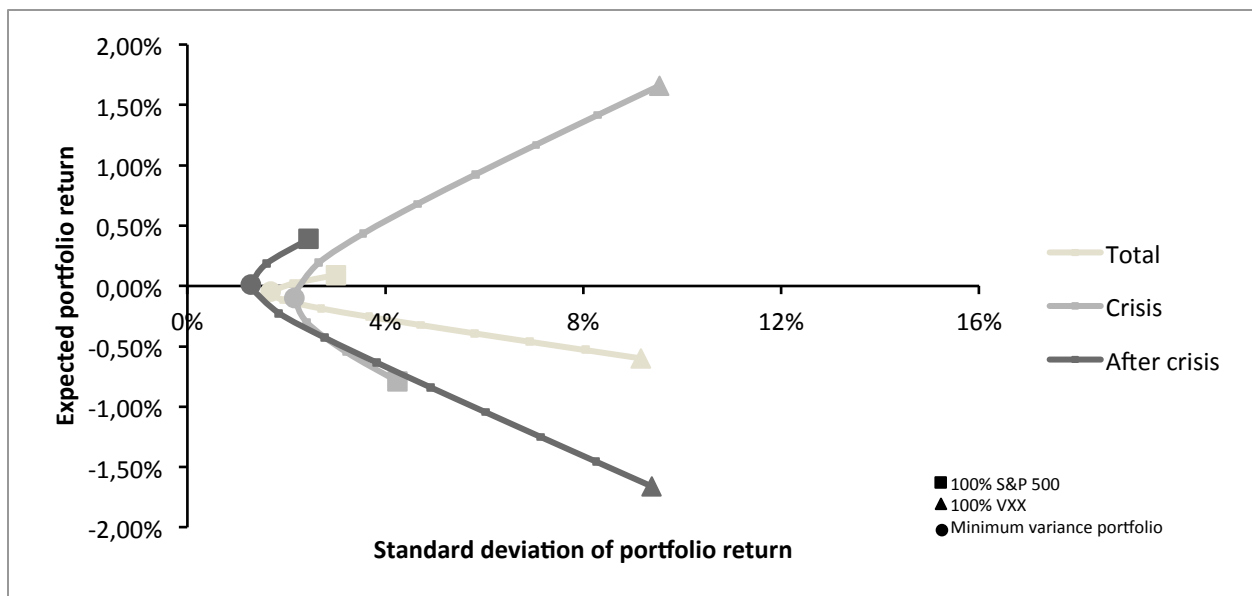**Figure 11:** VIX and S&P 500.

Figure 12 shows the efficient frontiers for S&P 500 with the asset class VXX. The risk-reducing effect is, just as above, present in all three time periods. During the financial crisis, VXX outperformed the S&P 500.

Figure 12: VXX and S&P 500.

The same results are shown in Figure 13, where S&P 500 is combined with the midterm VIX futures ETN: VXZ. The portfolio risk is reduced in all time periods, and the positive effects on portfolio return is shown during the crisis where VXZ also outperformed the S&P 500.

Figure 13: VXZ and S&P 500.
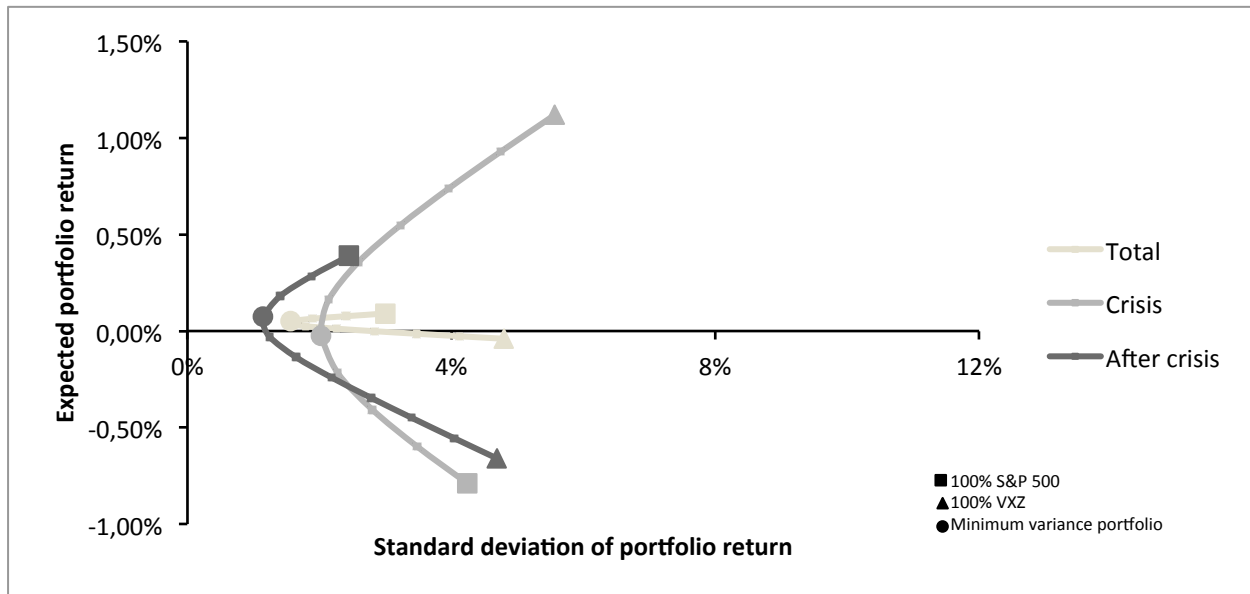
According to Figure 12 and Figure 13, VXX and VXZ do generally not show as positive effects as VIX in terms of risk reduction and increased returns. During the financial crisis, however, VXX and VXZ proved to be better than VIX, both in terms of risk reduction and in terms of increased returns. Even though risk is reduced by adding VXX and VXZ in the other time periods, this is compensated by a simultaneous decrease in return. Thus, positive diversification effects in form of reduced risk is present, but the pleasant increase in return cannot be proved for VXX and VXZ. The logic behind this finding is that VIX outperformed these instruments during the examined time period, and with a non-perfect correlation and low betas the returns of VXX and VXZ were lower. This can be explained by Figure 4, showing the negative effect of contango on the performance of VXX relative to that of VIX. Thus, the negative roll yield is making VXX underperform compared to VIX.

In Figure 14, S&P 500 and gold is examined. Gold is showing similar effects to those of VXX and VXZ. Risk is lowered in all cases, but return is suffering, especially in the post-crisis period. The biggest difference in expected return can be seen in the efficient frontier representing the total time period (2006-2013), where gold outperformed S&P 500 significantly. This becomes

very clear by looking at Figure 6. During the crisis and the post-crisis time periods, however, the performance in terms of return of gold and S&P 500 in Figure 6 seems similar, which is also proved in Figure 14 below. One noteworthy difference between the volatility based instruments and gold is that in order to reach the minimum variance portfolio one must allocate around 50 percent of the portfolio to gold instead of the 15 percent needed in the other portfolios. On the other hand, gold is significantly less volatile than the volatility based instruments, which is desirable.

Figure 14: Gold and S&P 500.
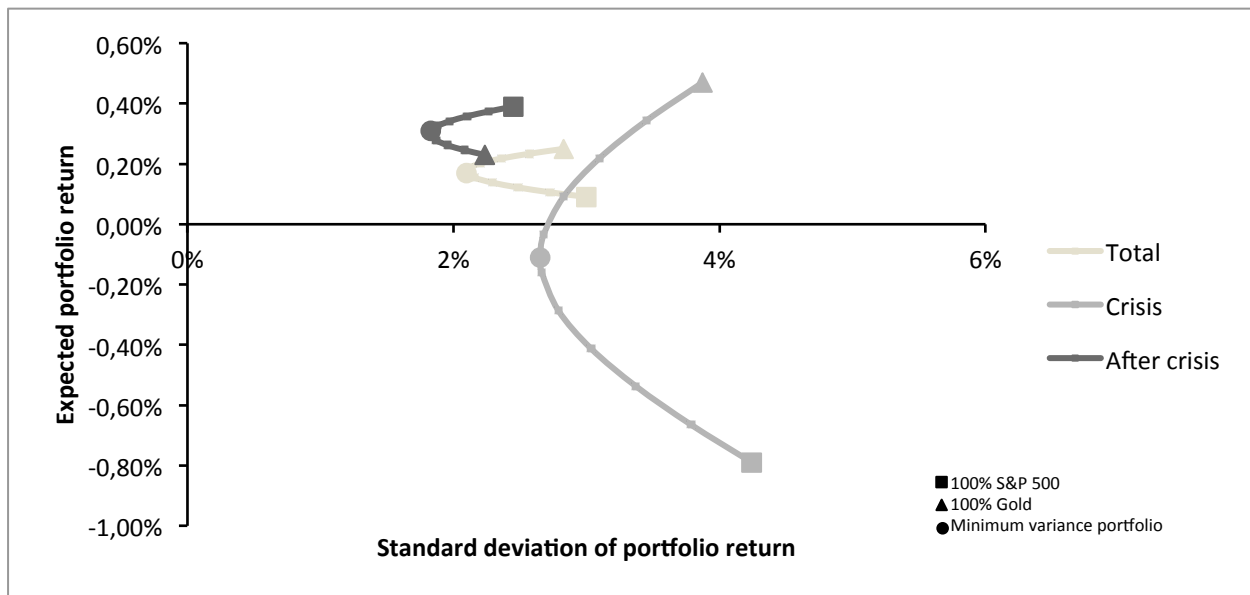
4.2.2 Time Periods

The risk reducing effect of adding VIX, VXX, VXZ or gold to a well-diversified stock portfolio is proved further in Figures 15 through 17. Here, the time periods have been isolated to show the relative performance of these instruments during each scenario.

The performance and optimal asset allocation during the total time period is shown in Figure 15 below. Adding about 15 percent of VIX to a portfolio consisting of S&P 500, that is the minimum variance portfolio, would not only decrease the risk, but also increase the expected portfolio return. By adding about 50 percent of gold to a portfolio of S&P 500, which also is the minimum variance portfolio, would yield about the same results. By adding VXX and VXZ the risk is reduced, but the expected portfolio return is also reduced. By looking at the triangles,

representing a portfolio consisting of only the examined asset, it becomes clear that VIX, VXZ and VXX are more volatile than both S&P 500 and gold.

Figure 15 is confirming that VIX is outperforming all other examined assets. By allocating around 15 percent to VIX during this period, returns would double and risk would have been cut in half. Adding more VIX to this portfolio would increase returns even more, but also sharply increase the portfolio risk because of the volatile nature of VIX. This is not the case when looking at gold, which had about the same volatility as S&P 500 during this time period. By further examining Figure 15, the relatively bad performance of VXX and VXZ is well-illustrated. By using VXX instead of VIX, for instance, the investor would reduce portfolio risk with the same number in both cases. However, expected portfolio return would be 0.22 percent weekly when adding VIX compared to -0.05 percent weekly when adding VXX. The numbers may seem small, but it is important to remember that this is expected weekly returns.

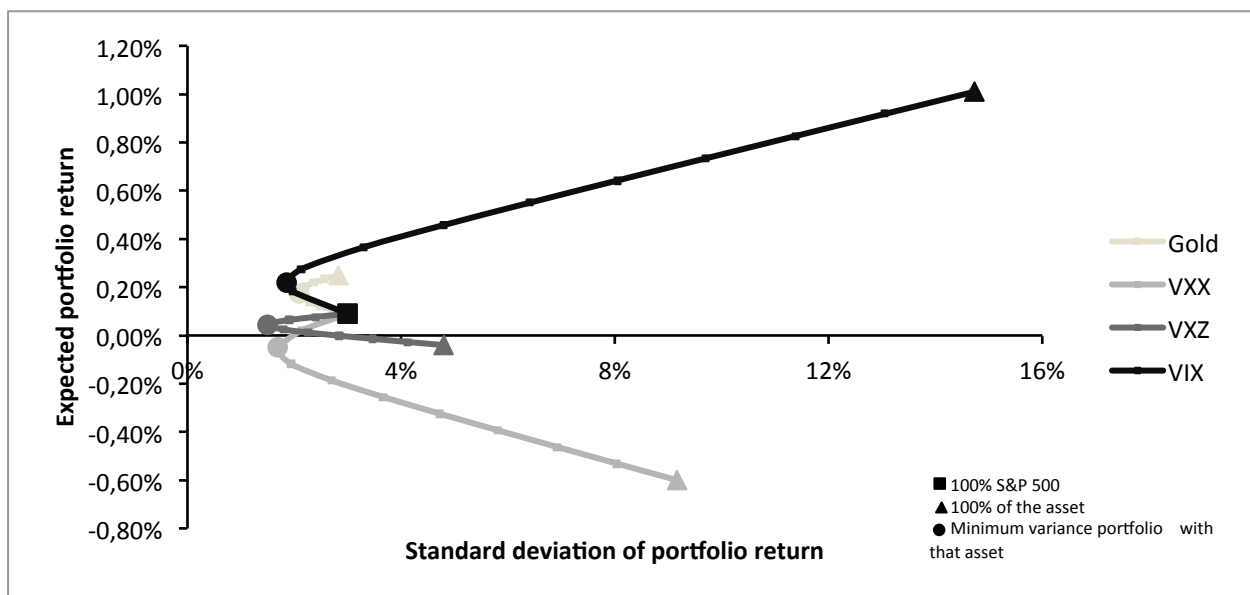**Figure 15:** Total time period (2006-05-05 to 2013-04-12).

Table 11 below is displaying a summary over the performance of the minimum variance portfolios shown in Figure 15. So in Table 11 we see that VIX is performing well, where a 14 percent position in VIX would increase returns by seven percentage points on an annual basis compared to a portfolio with only equity. Also, risk is substantially reduced. It also becomes evident that VXX and VXZ are not performing nearly as well as VIX during the total time period. However, the performance of gold is rather similar to that of VIX.

Table 11: Portfolios during total time period (2006-05-05 to 2013-04-12).

	100% Equity	90% Equity/10% Bonds	75% Equity/25% Bonds	78% Equity/22% VXX	64% Equity/36% VXZ	47% Equity/53% Gold	86% Equity/14% VIX
Annualized Returns	5.08 %	4.70 %	4.14 %	-2.91 %	2.50 %	9.93 %	12.28 %
Annualized St. Dev.	21.06 %	18.95 %	15.80 %	11.43 %	10.36 %	14.94 %	12.77 %
Maximum Drawdown	-55.91 %	-51.69 %	-44.75 %	-34.13 %	-21.70 %	-28.35 %	-35.42 %
% Up Weeks	54	54	54	46	50	58	60
% Down Weeks	46	46	46	54	50	42	40
Sharpe Ratio	0.24	0.25	0.26	-0.25	0.24	0.66	0.96
Treynor Ratio	0.04	0.04	0.04	-0.16	0.05	0.18	0.31

During the crisis, shown in Figure 16, all assets showed similar behavior. All minimum variance portfolios have negative expected returns, except for VXZ with an expected return of 0.01 percent weekly. The difference in standard deviation between a portfolio full of VIX, VXZ or VXX (the triangles) compared to the volatility of the S&P 500 portfolio (the square) is not as big compared to the other time periods. Unlike the other time periods, VIX performed badly compared to the other examined assets. The most interesting asset during this time period is VXZ. With starting-point in a portfolio consisting of S&P 500, risk is reduced in half and returns are improved from -0.8 percent weekly to 0.01 percent weekly by adding about 30 percent VXZ to the portfolio. Similar, though slightly worse, results can be seen by adding VXX. This is an important finding that proves the positive effects of buying volatility-based instruments in times of financial market turmoil.

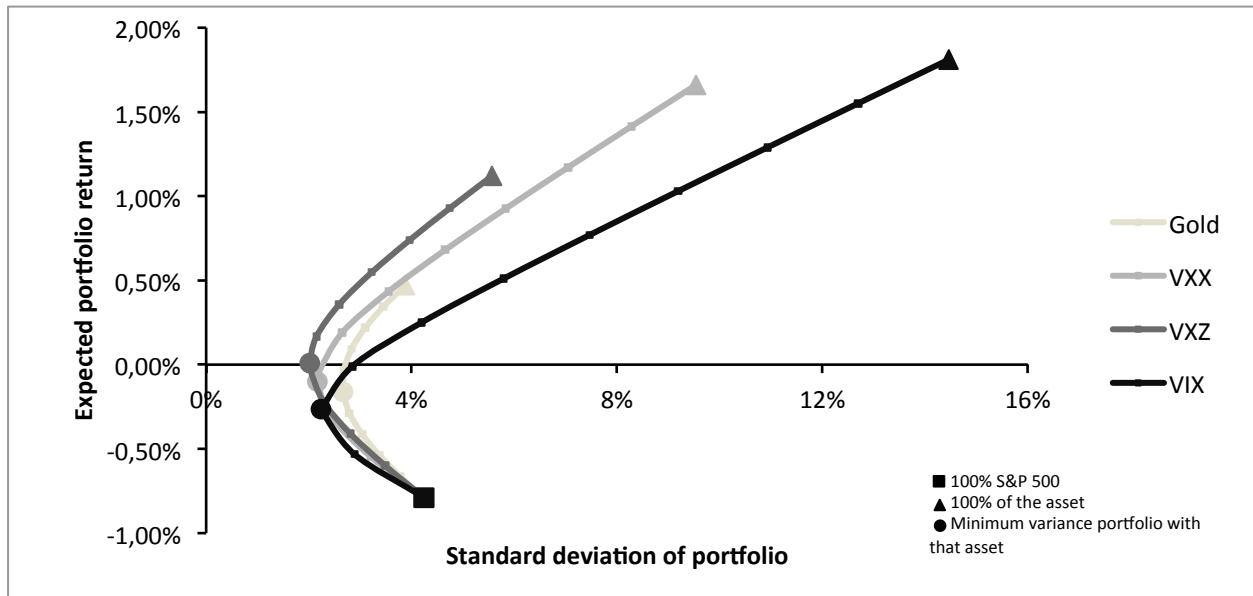**Figure 16:** Crisis (2007-08-10 and 2009-03-06).

Table 12 is showing further proof of the good performance of VXZ during the crisis. Another takeaway is the risk reducing effect seen by adding any volatility-based instrument to the stock portfolio. Risk, measured by annualized standard deviation, is cut in half compared to the all-equity portfolio, regardless of instrument added.

Table 12: Portfolios during crisis (2007-08-10 and 2009-03-06).

	100% Equity	90% Equity/10% Bonds	75% Equity/25% Bonds	72% Equity/28% VXX	58% Equity/42% VXZ	46% Equity/54% Gold	80% Equity/20% VIX
Annualized Returns	-33.74 %	-30.81 %	-26.20 %	-5.19 %	0.66 %	-5.58 %	-13.03 %
Annualized St. Dev.	30.81 %	27.73 %	23.11 %	15.67 %	14.70 %	19.19 %	16.26 %
Maximum Drawdown	-55.91 %	-51.69 %	-44.75%	-18,05 %	-16.07 %	-28.25 %	-27.24 %
% Up Weeks	42	42	42	39	49	53	41
% Down Weeks	58	58	58	61	51	47	59
Sharpe Ratio	-1.09	-1.11	-1.13	-0.33	0.04	-0.29	-0.80
Treynor Ratio	-0.34	-0.35	-0.36	-0.21	0.00	-0.16	-0.47

As explained in the introduction in Subsection 4.2, we chose to focus on the minimum variance portfolios based on the data obtained during the crisis, presented in Figure 16, to compare our results to the ones Szado makes in his article "VIX Futures and Options – A Case Study of Portfolio Diversification During the 2008 Financial Crisis" (2009). We chose to do so is because we used different time periods than Szado, and the time period we used that matches the time

period Szado used the best, is the one we call the crisis. This means that there are no comparisons made between Szado's results and Figure 15 (total time period) and Figure 18 (post-crisis). It is important to keep in mind though that the same technique is used to create every efficient frontier (from Figure 11 to Figure 18); the only difference is the time periods.

The results from the crisis presented in Figure 16 are in many ways in line with the results in Exhibit 10 in Szado, Edward (2009), although there are some issues comparing the exact results.

Szado's results are based on data collected during the time period he calls the financial crisis. That time period reaches from March 2006 to December 2008, while our crisis reaches from August 2007 to March 2009. The different time periods makes it harder to compare the exact results.

We could have picked the same time period as Szado, but we decided to base our decision on our data only, meaning that we looked at the historical data and detected a time period where S&P 500 went from high to low. The time period that we call the crisis is presented in Figure 17 below (the span between the vertical lines), and our total time period reaches from May 2006 to April 2013. On page 3 in Szado, Edward (2009), he motivates his choice of time period by starting at the time when VIX options were introduced.

Figure 17: Crisis (2007-08-10 and 2009-03-06). Source: Bloomberg.
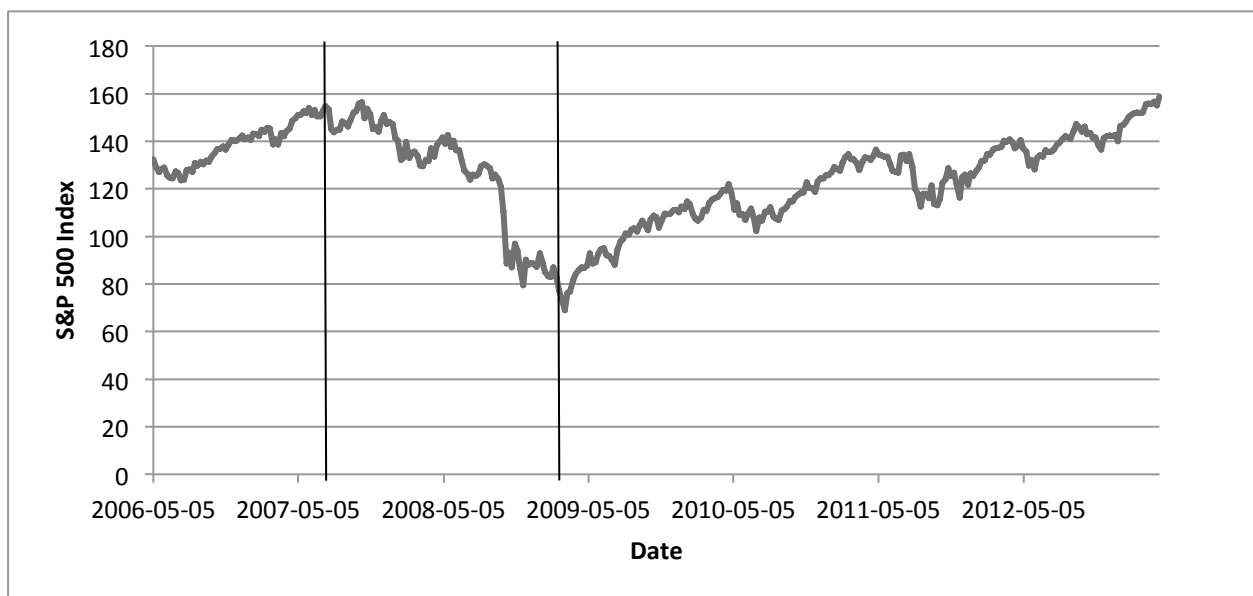
Another problem comparing our results with Szado's, is that in Exhibit 10 in Szado, Edward (2009), he uses a portfolio that differs from our portfolios in some ways. When we are only allocation between S&P 500 and the volatility based instruments, he is allocating between a portfolio consisting of "stocks/bonds/alternatives" and "VIX futures". He does not clearly state what type of VIX futures he is using in Exhibit 10, but earlier he has used one month VIX futures, and assuming he is being consistent this is also the case in Exhibit 10. However, taking the time period and portfolio differences into consideration, we can still make some useful comparisons.

As we stated earlier in this subsection, the results from our minimum variance portfolios presented in Figure 16 are in line with the efficient frontier presented in Exhibit 10 in Szado, Edward (2009).

The minimum variance portfolio presented in Exhibit 10 in Szado, Edward (2009), has a 10 percent allocation to VIX futures, and thereby a 90 percent allocation to stocks/bonds/alternatives. The risk-return relationship is a -2.5 percent expected return along with a 4 percent standard deviation. The portfolio on the efficient frontier, consisting of 100 percent stocks/bonds/alternatives has an estimated expected return of -5 percent, and an estimated standard deviation of 11.5 percent. He thereby states that a 10 percent allocation to VIX reduces the risk by more than half, and doubles the expected return.

The efficient frontiers we presented in Figure 16 are all very similar to the one Szado presents in Exhibit 10 (in this part we ignore the portfolio consisting of gold and S&P 500 and only focus on the volatility based instruments VIX, VXX and VXZ). Opposed to the result in Exhibit 10 in Szado, Edward (2009), a 10 percent allocation to any of the volatility-based instruments is not enough to construct a minimum variance portfolio, instead an allocation of between 20 and 30 percent is necessary (about 20 percent allocation to VIX, or about a 30 percent allocation to either VXX or VXZ) . What is common though is that an allocation to any of the volatility-based instruments decreases the standard deviation and increases the expected return.

According to our results, a 20 percent allocation to VIX reduces the standard deviation by about half and more than doubles the expected return. A 30 percent allocation to VXX decreases the standard deviation by more than half and increases the expect return from -0.8 percent to almost

zero. A 30 percent allocation to VXZ reduces the standard deviation by more than half and increases the expected return from -0.8 percent to 0.01. These results are very similar to the one in Exhibit 10 in Szado, Edward (2009), and we can state almost the same conclusion that an allocation to VIX or a VIX future reduces the risk by almost half and for the worst asset (VIX) doubles the expected return.

Our constructed efficient frontiers covering the post-crisis time period are shown in Figure 18, where VIX and S&P 500 showed similar return patterns. This is illustrated by a nearly straight line with a minimum variance portfolio with similar expected return as the S&P 500 portfolio, but with risk cut by one third. The performance of VXX and VXZ is not near that of VIX. Gold is once again less volatile than the other assets, and shows a small risk-reducing effect. The relative performance of the instruments is similar to the performance in Figure 15. VIX, followed by gold, is superior regarding returns. Risk reducing effects, however, are shown regardless of added instrument.

Figure 18: Post-crisis (2009-03-06 and 2013-04-12).
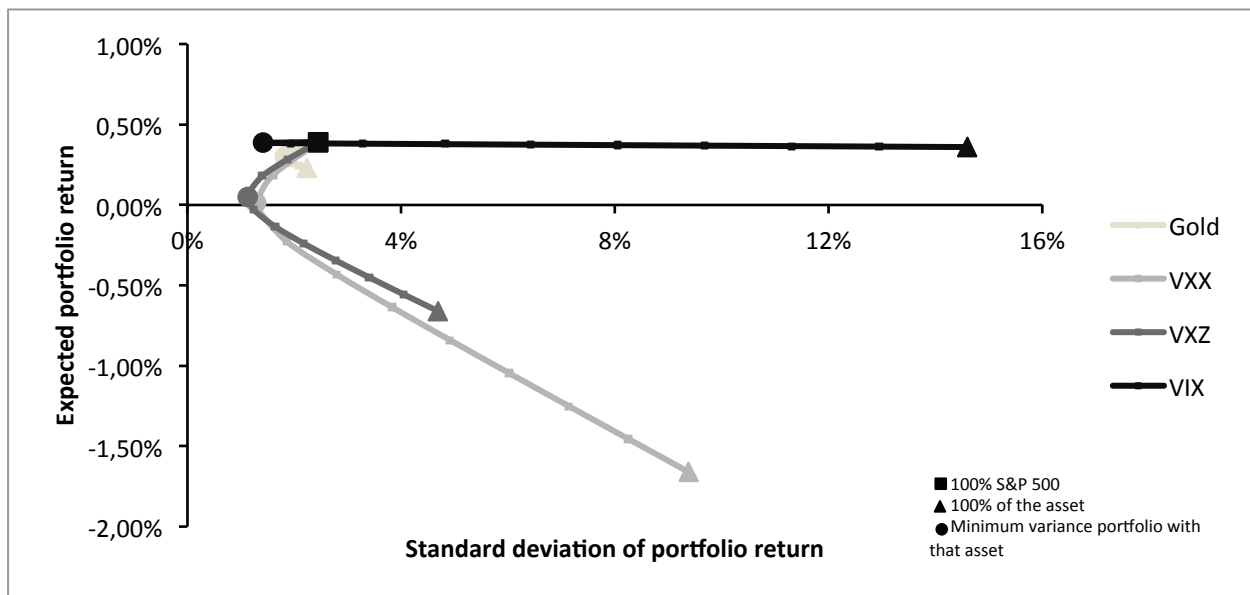
Once again, Table 13 shows proof of the strong performance of VIX. The returns are not noticeably affected by the addition of 12 percent VIX compared to the all-equity portfolio, but risk (i.e. annualized standard deviation) has been cut from 18 percent to 11 percent on an annualized basis. The relatively poor performance of VXX and VXZ also becomes evident.

A risk-reducing effect is found, but the returns of the VXX and VXZ portfolios are substantially lower than the all-equity portfolio.

**Table 13:** Portfolios during post-crisis (2009-03-06 and 2013-04-12).

	100% Equity	90% Equity/10% Bonds	75% Equity/25% Bonds	82% Equity/18% VXX	68% Equity/32% VXZ	44% Equity/56% Gold	88% Equity/12% VIX
Annualized Returns	22.31 %	19.89%	16.34 %	1.02 %	2.83 %	16.87 %	22.06 %
Annualized St. Dev.	18.22 %	18.76 %	13.67 %	9.89 %	8.86 %	13.35 %	10.93 %
Maximum Drawdown	-17.44 %	-15.78 %	-13.26 %	-17.44 %	-18.51 %	-9.51 %	-8.72 %
% Up Weeks	59	59	59	49	50	56	63
% Down Weeks	41	41	41	51	50	44	37
Sharpe Ratio	1.22	1.06	1.20	0.10	0.32	1.26	2.02
Treynor Ratio	0.21	0.21	0.20	-0.01	0.06	0.29	0.56

4.2.3 Sensitivity Analysis

Here, the assets' sensitivity with regards to the time period chosen is shown. As explained in Subsection 3.3, we start by using the first twelve weeks of data, proceeding to roll this window forward by four weeks. Hence, the data used for the estimations corresponding to the date 2006-09-01 is the twelve immediate weeks before this date. For each time period, one minimum variance portfolio will be estimated. The standard deviation and expected portfolio return corresponding to the minimum variance portfolio for each twelve week window is then graphed.

In Figure 19, this is done for VIX. The sensitivity with respect to the chosen time period is mostly made clear during the financial crisis, with spikes in opposite direction during the beginning of 2009. This suggests that small changes in the time period result in large changes in expected return and portfolio standard deviation, illustrating the sensitivity of the estimated minimum variance portfolios with respect to the time period and data used. By shifting the data forward by 8 weeks can substantially change the statistical properties of the minimum variance portfolio. For instance, the minimum variance portfolio corresponding to the twelve weeks ending 2009-03-06 has an expected return of -1.23 percent and a standard deviation of 2.28 percent. The same portfolio for the twelve week time period ending 2009-04-03 has an expected return of -0.68 percent and a standard deviation of 2.50 percent.

Figure 19: Sensitivity analysis for VIX.
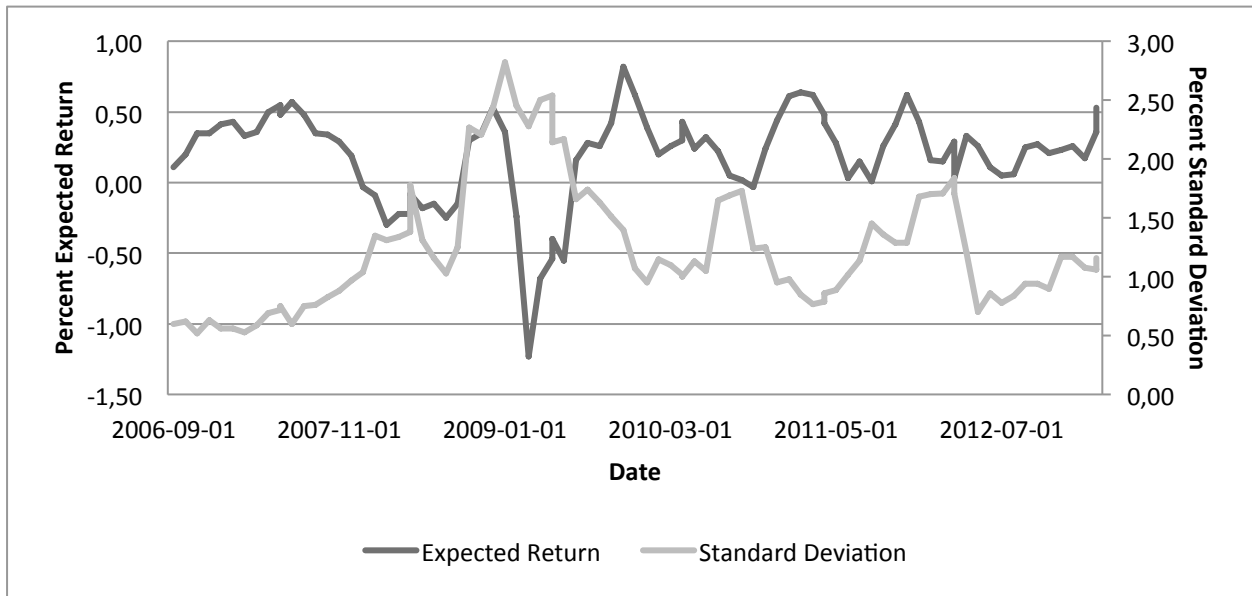
The same pattern is shown in Figure 20. Unlike Figure 19, though, the spikes during the financial crisis move in the same direction: increased returns and increased standard deviation. However, the volatile nature of the instrument is illustrated here as well. The importance of the chosen time period is once again evident.

Figure 20: Sensitivity analysis for VXX.
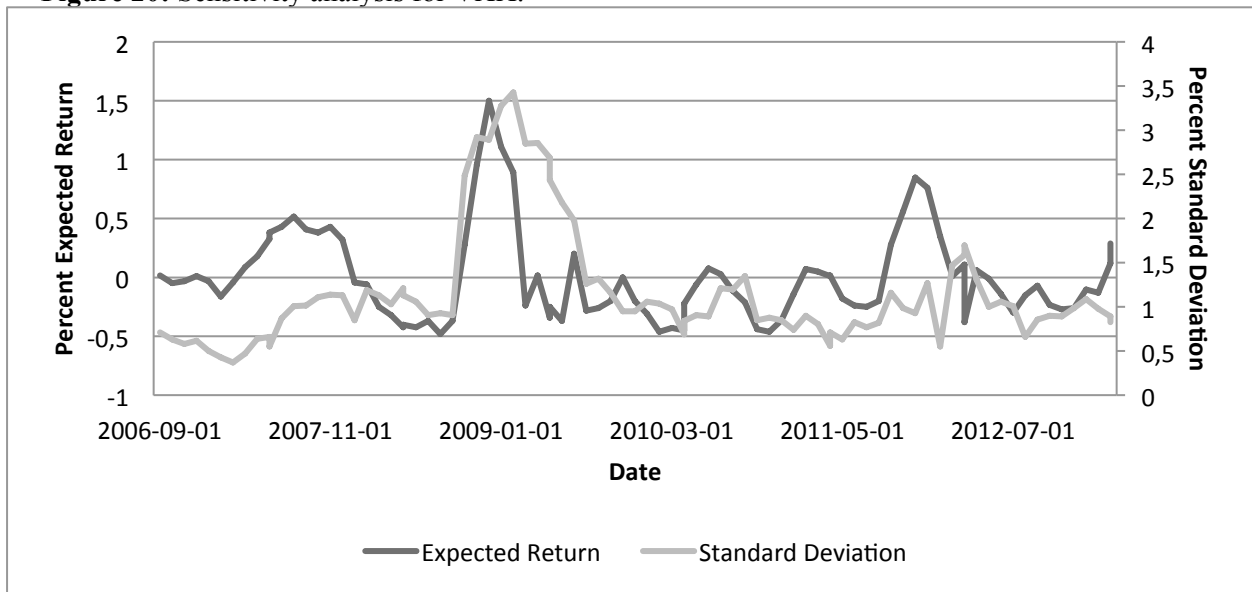
VXZ, graphed in Figure 21, shows a similar pattern as VXX. The correlation between standard deviation and portfolio returns seems to be positive, meaning that they move in the same

direction. An increased expected return also means an increased standard deviation. Compared to VIX and VXX, VXZ shows a relative stability after the financial crisis. This means that the calculated minimum variance portfolio will show somewhat similar properties regardless of time period chosen, which is not true when considering Figure 19 and Figure 20.

Figure 21: Sensitivity analysis for VXZ.
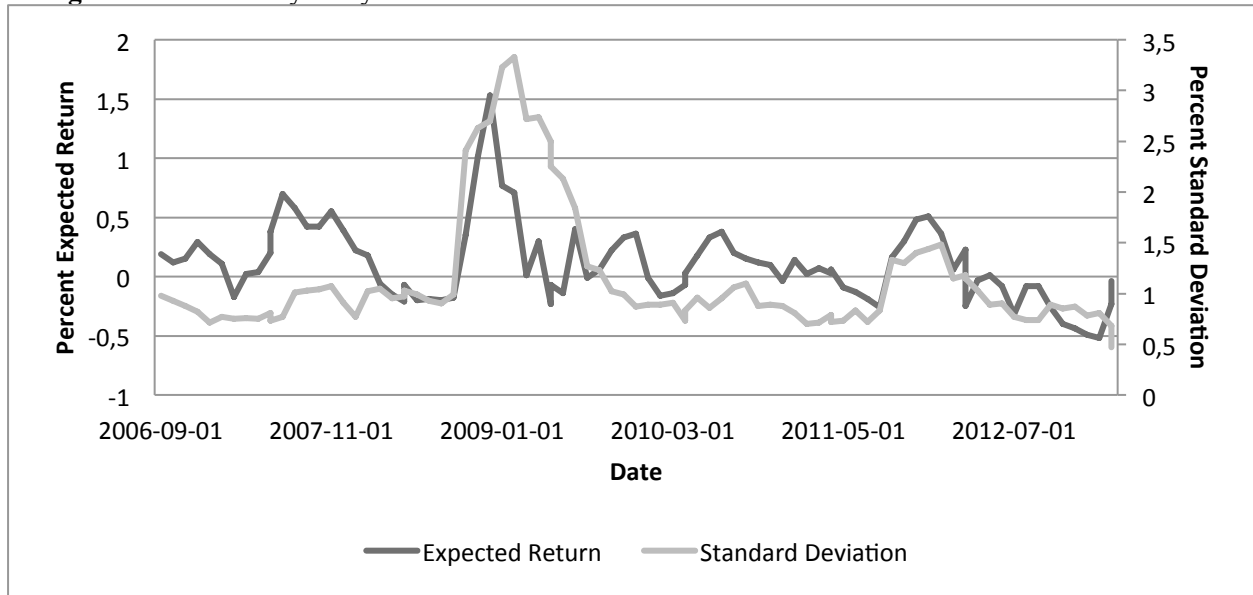
Gold is also showing this instability, according to Figure 22. The standard deviation of the minimum variance portfolio is ranging from 1 percent to 4.5 percent depending on the time period chosen. Even though the expected return is relatively consistent after the crisis, the standard deviation fluctuates during this period. Therefore, gold can also be considered to be sensitive with respect to the chosen data, with a performance largely depending on the time period.

Figure 22: *Sensitivity analysis for gold.*
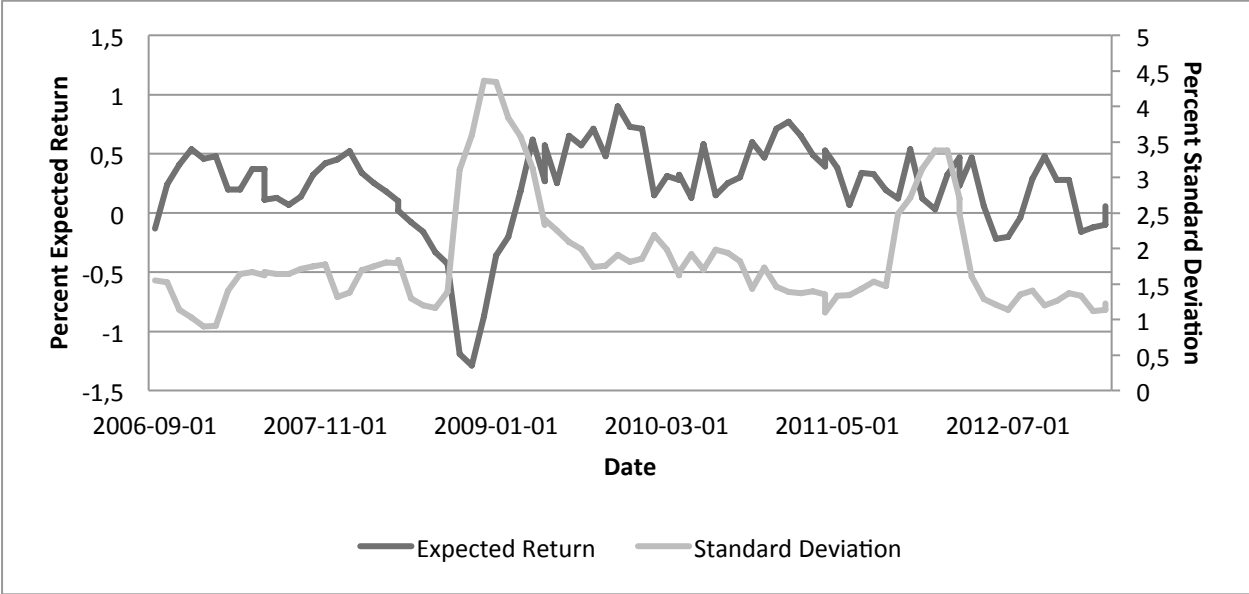
5. Conclusion

All three volatility-based instruments (VIX, VXX and VXZ) show statistical properties that make them theoretically suitable to use in order to reduce the risk in a stock portfolio. Our calculated portfolios show that all three volatility-based instruments, during all time periods, in practice would have reduced portfolio risk. Thus, it is possible to diversify and reduce risk in a stock portfolio using volatility.

VIX is the superior instrument to use, especially during times of financial market stability. The addition of VIX is, along with reducing portfolio risk, more often than not also increasing portfolio returns. Here, VXX's and VXZ's poor abilities of tracking VIX are evident.

The statistical properties of gold suggest that its risk reducing abilities is worse than those of the volatility-based instruments. This is confirmed when looking at the constructed portfolios: during all time periods, the risk reducing effect of adding gold to a stock portfolio is worse than the risk reducing effect of adding any volatility-based instrument.

All four examined instruments show a volatility suggesting that the results and expectations in terms of risk and return is sensitive with respect to the data used, and therefore largely dependent upon the chosen time period. Hence, a small change in the time period can possibly have a big impact on portfolio expected return and portfolio risk.

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7.   Appendix

APPENDIX 1: Black-Scholes

The formula for Black-Scholes is found in a press release dated 14 October 1997 by the Royal Swedish Academy of Sciences.

The Black-Scholes formula is, for a European call option, defined as

$$C = SN(d) - Le^{-rt}N(d - \sigma\sqrt{t})$$

Where the variable d is defined as

$$d = \frac{\ln\left(\frac{S}{L}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$C = $ The value of the call option

$S = $ Share price today

$\sigma = $ Volatility of share price

$r = $ Risk-free interest rate

$t = $ Time to maturity

$L = $ Strike price

$N = $ A normal distribution function measuring the probability of option exercising

APPENDIX 2: Maximum Drawdown

Maximum drawdown was calculated using Microsoft Excel and a spreadsheet supplied by Turnkey Analyst Wesley R. Gray. The source code for maximum drawdown is written below, strictly taken from Turnkey Analyst (Gray, 2013):

```
Function drawdown(port_series As Range)
Application.EnableCancelKey = xlDisabled
' Find the biggest cumulative drawdown for a performance series
```

```
‘ dates = dates typically alongside the returns
‘ port_series = port_series returns in % * 100 format

Dim counter As Integer
Dim cume As Double
Dim max As Double

counter = 1
cume = 1
max = 1
For counter = 1 To port_series.Count
If port_series(counter).Value <> “–” Then
cume = (cume * (port_series(counter).Value + 1))
End If

If cume >= 1 Then
cume = 1
End If

If cume < max Then
max = cume
End If

Next counter

‘If there never was a drawdown
If max = 1 Then
drawdown = “N/A”
Else
drawdown = (max – 1)
End If

End Function
```