THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Tracking mathematical giftedness in an egalitarian context

LINDA MATTSSON

Division of Mathematics
Department of Mathematical Sciences
CHALMERS UNIVERSITY OF TECHNOLOGY
AND UNIVERSITY OF GOTHENBURG
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Department of Mathematical Sciences
Chalmers University of Technology
and University of Gothenburg
SE-412 96 GÖTEBORG, Sweden
Phone: +46 (0)31-77 210 00

Author e-mail: linda.mattsson@bth.se

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To KRISTIAN,
EMINA and IDA

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Abstract

In three different studies upper secondary school head teachers' characterization and identification of mathematical giftedness was investigated. A survey study (Paper II) explored the conceptions held by 36 randomly selected upper secondary school head teachers in mathematics. An interview study (Paper III) investigated the conceptions held by three purposively selected head teachers working at the longest running gifted programs in mathematics in Swedish upper secondary schools. A third study (Paper IV) looked for creativity, the characteristic head teachers' most frequently associated with giftedness, in the admission tests used at the cutting-edge programs in mathematics in upper secondary school.

As compared to theoretical models (Krutetskii, 1976; Renzulli, 1978; 2005), results showed that the head teachers collectively expressed nuanced characterizations of mathematical giftedness and the identification thereof. This was especially demonstrated by the head teachers at the gifted mathematics programs. Still, for individual head teachers, there is a need to further their knowledge about the different abilities contributing to manifestations of mathematical giftedness. This would increase the possibility to identify and develop the mathematical abilities of an even greater number of mathematically promising students. Krutetskii's (1976) structure of mathematical abilities manifested by capable mathematics students was used as a framework for the content analysis in the first two studies, and Lithner's (2008) framework for creative and imitative reasoning was used in the third study.

In a fourth study (Paper V) the representation of different student groups at five purposively selected gifted programs in upper secondary school was investigated. Findings from this comparative study of demographical factors – gender, geographical origin, and highest education of parents – were complemented by findings from the interview study where the cognitive, as well as personal and social, characteristics of students participating at three gifted programs were expressed.

Results from the interview study indicated that students participating at the gifted programs showed signs of mathematical giftedness that are not necessarily connected to schoolhouse giftedness (Renzulli, 2005). Both mathematically gifted students who were individualists and reluctant to participate in traditional school mathematics, and those who were hardworking and ambitious, were recognized. Participating students had special needs, such as to approach mathematical tasks in their own way and learn how to communicate mathematics in written solutions, connected to their giftedness that call for special education. The demographical study also showed that it is mostly males with highly educated parents who have found their way to gifted programs in mathematics. In sum, results indicate that the head teachers at the gifted programs acknowledge the special educational needs of gifted mathematics students. There is also a call for the development of complementary educational activities to reach a greater number of gifted students.

List of Papers

Paper I. Mattsson, L., & Bengmark, S. (2011). On track to gifted education in mathematics in Sweden. In B. Sriraman & K.H. Lee (Eds.), *The elements of creativity and giftedness in mathematics* (pp. 81-101). Rotterdam: Sense Publishers.

Paper II. Mattsson, L. (2010). Head teachers' conception of gifted students in mathematics in Swedish upper secondary school. *Nordic Studies in Mathematics Education*, *15*(3), 3-22.

Paper III. Mattsson, L. (to be submitted). Conceptions of mathematical giftedness according to head teachers' at gifted programs in mathematics in Swedish upper secondary school.

Paper IV. Mattsson, L. (in review). Requirements for mathematically founded reasoning in the identification of mathematical giftedness in Sweden. *Journal of Advanced Academics.*

Paper V. Mattsson, L. (2012). Demographics of self-selected participants in mathematical tracks in Swedish upper secondary school. *The Mathematics Enthusiast*, *9*(3), 271-302.

Content

1. Introduction

For centuries there have been individuals who have amazed us with their extraordinary and innovative performances or products. And there continue to be. Recently, many of us were impressed by the 100 meter run of Shelly-Ann Fryser-Pryse and Usain Bolt in the World Championships in Moscow; tomorrow others will be overwhelmed by the soloist's creative interpretation of a piano concert of Mozart; and in July 2014 many mathematicians will be fascinated by the achievement of the participants in the International Mathematical Olympiad in South Africa. The interest in these gifted individuals, however, is not only in order to enjoy the beauty of their achievements. The reasons for attending to them are a function of who you ask and when you ask. For example, the launch of Sputnik 1 by the Soviet Union led the United States federal government to initiate education that was to support the gifted mathematics and science students in the U.S. The reason for stimulating students to choose advanced studies in mathematics and science was to make sure that the Soviet Union did not get a head start in the space race (Sheffield, 2006). From a global perspective there are currently three driving forces behind our interest in gifted individuals: the potential contribution of the gifted to (1) the growth of global economy,(2) the development of welfare and economic success of individual nations; and (3) the individual needs and wants of the gifted themselves (Persson, 2013). The former two perspectives are connected with individuals' extreme ability to contribute to meeting the *production needs* of society, whereas the latter perspective attends to the *social and emotional needs* of the individual (Persson, in press). The different views decide who the gifted ones are (Persson, in press.).

According to the UNESCO Salamanca Statement and Framework for Action on Special Needs (United Nation Educational, Scientific, and Cultural Organization, 1994) school education should attend to the individuals' needs:

> "Every child has a fundamental right to education, and must be given the opportunity to achieve and maintain an acceptable level of learning. Every child has unique characteristics, interests and abilities and learning needs. Education systems should be designed and educational programs implemented to take into account the wide diversity of these characteristics and needs". (p.viii)

This also includes gifted individuals. The obligation to attend to gifted students is also shown in the Swedish Education Act:

> "All children and students should be given the support and stimulation they need in their learning and personal development in order to, from their own conditions, develop as far as possible according to the educational goals.

> Students who easily reach the minimum knowledge requirements which shall be reached should be given guidance and stimulation to come further in their knowledge development" (2010:800, chapter 3, 3§).

Possibly due to the strongly rooted egalitarian heritage of our country (Persson, Joswig, & Balogh, 2000), there is, however, little focus on gifted education within the national education system. Recent research showed serious shortcomings in the current school education of gifted students (Pettersson, 2011; Persson, 2010, 1998). In case studies it was shown that gifted students in inclusive settings were opposed as they solved mathematical tasks in advanced ways (Pettersson, 2011). The lack of appropriate education and appreciation from teachers in some cases led to students' being depressed and formed a frustrating situation for the gifted mathematics students (Pettersson, 2011). Thus, there is a need to create a foundation on which development of a successful gifted education in mathematics in our egalitarian society can be built. This thesis is meant to contribute to such a foundation. However, what are the important components in such a development? What issues could research in gifted education in mathematics address?

1.1 Identifying areas for development in gifted education

In the search for proper aims to govern this thesis a literature review of the current status of gifted education in mathematics in Sweden was done (Paper I). A central issue was to identify areas of research and development work that need to be considered for the improvement of Sweden's national gifted education. In order to enable a broad but comprehensive review that identified topics that are considered as essential in the development of gifted education, the review focused on four components: a) national polices, b) advocacy groups, c) research, teacher education and curriculum development, and d) implementation in schools (Phillipson, Kaur, & Phillipson, 2003). (For a description of the structured work on this review see the Methodology section.)

The review showed that Sweden has recently taken important initial steps towards the development of gifted education. Previously, although both the earlier Educational Act (Swedish statute-book, 1985) and the former curricula commented that school education should be adapted to "students in need of special support", gifted students have not been given special attention in school practice. However, new national policies such as the new Education Act (Swedish statute-book, 2010) explicitly attend to students at the higher end of the ability scale when emphasising the right to get guidance and stimulation for students who can easily reach the knowledge requirements. Advocacy groups such as The National Center for Mathematics Education (NCM), which is an influential organisation that promotes development in mathematics at all school levels in Sweden, has been giving gifted education in mathematics some support. Their support has included articles about gifted students or gifted education in literature addressed to mathematics teachers. NCM also singled out gifted education as one of their topics at a national conference for Mathematical Developers, enabling participants from more than 250 out of 290 Swedish municipalities to get a half-day lecture on the topic. This support complements advocacy from other actors such as Mensa Sweden and promoters for different mathematical competitions. The implementation of gifted education in schools has, to some extent, been sanctioned by the State. There are now pilot activities of gifted programs in specific subjects, including mathematics, running both in lower and upper secondary school. There is, however, only a small group of people that does research within gifted education in Sweden. Further, in teacher education, schooling in gifted education is a very uncommon element. (For more details about the current state of gifted education in mathematics see Paper I). Thus, there are still areas that need to be dealt with in order to improve the possibility for the successful development of gifted education. Six of these areas are listed below.

1.1.1 Six areas for development

During the literature review my research partner and I identified six developmental areas that we believe need to be addressed in order to develop a sustainable gifted education practice in mathematics. Since development of gifted education in the subject of mathematics is at the forefront of the development of the field of gifted education in academic subjects in Sweden in general, most of the identified challenges are important to enable further evolvement of gifted education in any specific subject.

First, the *gifted students need to get legal recognition in the national policies.* Based on experiences from other countries, the odds for the development of a variety of gifted education practices increase as gifted students are recognized in legal documents. Moreover, it is a matter of recognizing that there are other gifted students, in addition to the ones who are high achieving in traditional school education, who also are in need of a special and challenging mathematics education. An acceptance by formal educational policies has been described as "paramount to the psychological well-being of the highly able" (Persson, in press, p. 5).

Second, in order to get access to a proper guidance and stimulation, *the gifted mathematics students need to be identified*. This requires that teachers know about and recognize different expressions of mathematical giftedness. At the same time, in order for students to show their mathematical giftedness and thus be identified, they need to be offered challenging tasks. This relationship between the identification of and provision for giftedness is essential for educators. As giftedness will manifest in students at different ages and in regular classrooms there is a need for teachers to be knowledgeable about giftedness.

The third area that needs to be considered is the *introduction of gifted education during Swedish teacher education*. Gifted education is still not included in mandatory courses in the Swedish teacher education.

Fourth, there is *a need to strengthen the connection between research and implementation of gifted education.* Gifted programs in mathematics have been running for many years in Sweden. Still, there has been little research interest in these educational practices. By studying different aspects of the already existing gifted programs, research and practice in gifted education could be developed side by side. Important issues concern, among others, evaluation of the learning effectiveness of the gifted programs, how to develop learning opportunities aimed at developing certain mathematical abilities and access to gifted education. Research need not focus only on the gifted programs. It might just as well focus on gifted educational practices in other forms and settings, such as on enrichment practices in inclusive settings.

The fifth area identified in our review concerned the *need for coordinating measures for development of gifted students.* It would be valuable to have a national agency that could coordinate support activities and transfer information about developments in gifted education between schools, parents and others. It could also give access to platforms where educators, parents and gifted individuals could share their views and discuss issues of interest from a personal perspective. The availability of the contributions of others would allow for a faster and more structured development, as one could build on earlier work. During recent years there have been more and more actors who have shown a concern for gifted students and their diverse educational needs. Psychologists, educators and agencies such as the Swedish Association of Local Authorities and Regions are among the advocates. This has led to an increasing amount of local websites or social media groups that informs users about giftedness as well as about measures for giftedness development[1].

Finally, there is a need to complement the recognition of the domain-specific intellectual needs, and also *attend to the social and emotional needs of gifted students.* There are witnesses from the Swedish context that gifted students often are perceived as socially different as compared to other age peers (Kullander, 2012; Persson, 2009, 2010; Pettersson, 2011). Since gifted students might not conform to behaviors which

[1] For examples of websites and social media groups see contributions of Roland Persson, http://hj.se/hlk/en/research/faculty-and-staff/child/roland-s-persson.html; Linnaeus University, http://giftedmath.se/; Mensa Sweden, www.mensa.se; Anita Kullander, https://www.facebook.com/filurum; and Staffan Harling, http://www.begåvadebarn.se/

are considered as the norm in school and society, they are sometimes judged as socially different or disturbed. There is obviously a need to further parents', teachers', counselors', school welfare officers', etc. understanding of gifted students' different needs for social and emotional stimulation, as compared to those of their age peers. Influenced by measures taken in other countries (Alencar, Fleith, & Arancibia, 2009; Phillipson, Shi, Zhang, Tsai, Quek, Matsumura & Cho, 2009), my research partner and I believe there is also a need for the gifted student to understand his/her role in society better. We believe that experience with extracurricular activities that focus on the appreciation of individual differences and the understanding of the gifted students themselves in relation to peers would be valuable to the development of the gifted students' self-esteem.

1.2 Aims of this thesis

1.2.1 Exploring conceptions of mathematical giftedness

The first aim of this thesis is to explore conceptions of giftedness held within the Swedish school practice. Understanding these conceptions will shed light on what characterizes the students that might be identified as manifesting mathematical giftedness. Thus, this aim relates to the identification of gifted students.

Most recent models of giftedness consider giftedness as evolvable (Davidson, 2009; Gagné, 2005; Mönks, 1992; Renzulli, 1986; 2005; Sternberg, 2005; Subotnik, Olszewski-Kubilius, & Worell, 2012). The development of giftedness does not happen spontaneously, but is dependent on proper stimulation and activities (Bloom, 1985; Krutetskii, 1976; Mönks, 1992; Renzulli, 1986, 2005; Subotnik et al., 2012; Winner, 1996). The importance of getting the opportunity to develop is crucial in the realization of giftedness (Bloom, 1985; Renzulli, 1986, 2005; Subotnik et al., 2012). Access to gifted education in school is considered as one such important opportunity (Subotnik et al., 2012). In order to enable a beneficial support for the development of giftedness, there is a need for teachers and other school administrators to recognize signs of giftedness. The conception of what constitutes giftedness is thus a central question for the identification of (potential) mathematical giftedness and the selection of support activities in school.

There has been little attention paid to this issue within national research as well as within the teacher education. For example, Sweden has had special classes for gifted students in mathematics at upper secondary school for a quarter of a century. Yet these activities have, from an educational policy perspective, gone almost unnoticed. During the last four years there have been evaluations done on a few aspects of some of the gifted programs, the so-called cutting edge programs, in mathematics, natural sciences, humanities and social studies respectively (Swedish National Agency for Higher Education, 2010, 2011, 2012, 2013). However, nowhere in these evaluations are the “most important conceptual foundations of a gifted program” (Moon & Roselli, 2000, p. 500) discussed; that is, “the theory of giftedness that undergirds the program” (Moon & Roselli, 2000). The interdependency between the theory of giftedness to which one subscribes, the particular gifted program goals, delivery format and instructional strategies, as well as the student identification process (see e.g. Feldhusen & Jarwan, 2000; Moon & Roselli, 2000; Renzulli, 2004), has thus not yet been focused upon.

Several models of giftedness (e.g. Kiesswetter, 1985; Krutstskii, 1976; Mönks, 1992; Renzulli, 1986, 2005; Sternberg, 2005; Subotnik et al., 2012) emphasize the character traits or attributes that need to be synthesized in order for a person to be considered gifted or to show gifted behavior. Thus, the characteristics of giftedness will be a central focus in this thesis. Following Mönks (1992), I consider the development of giftedness to be dependent on the interaction between, among others, the personal characteristics and school. The teachers are thus seen as main actors in facilitating the development of character traits vital to the expression of giftedness. Further, teachers’ conceptions of what characterizes giftedness are considered as essential influences on the character traits students are supported to develop. Teachers’ ways of detecting giftedness is seen as an important component in order to enable the development of the character traits and also to provide further information on what might be signs of potential giftedness.

Acknowledging the domain-specific expressions and developmental paths of giftedness (Mayer, 2005; Subotnik et al., 2012), this thesis focuses on teachers’ conceptions of mathematical giftedness. I specifically concentrate on upper secondary school mathematics head teachers’ conceptions of giftedness. Hence, the first two general research questions are:

- How do head teachers in upper secondary school characterize mathematical giftedness?
- How is mathematical giftedness detected by head teachers in upper secondary school?

1.2.2 Investigating representations of different students group in gifted educational practices

The second aim of this thesis is to investigate representations of different students group in gifted educational practices in mathematics. These representations are fundamental in discussions of educational equity in gifted education.

The gifted education developed in a culture is greatly influenced by its' educational philosophy (Moon & Roselli, 2000). In an *equal opportunity philosophy* the emphasis is on meeting the individual needs of different students. Regardless of background the students should have equal access to opportunities to develop their abilities and interests. Thus, in an equal opportunity culture, the difference between students might very well increase (Moon & Roselli, 2000). Within an *egalitarian philosophy* on the other hand, education aims at creating similar outcomes for all students by providing all students the same educational experience. Equal opportunity cultures, such as the U.S. for example, usually encourage the development of a variety of special provisions for gifted students. By contrast, egalitarian cultures, such as Sweden for example, have been described as being hostile to gifted education (Persson, Joswig & Balogh, 2000) or supportive of gifted education only in inclusive settings (Moon & Roselli, 2000).

By introducing nationally established gifted programs in Sweden, the strongly rooted egalitarian view of how to achieve equity in education is being challenged. The aim of education changes and the size of the educational target group decreases. This awakens a concern among the opposition that gifted education might be socially stratifying (Wistedt & Raman, 2011). If an educational practice is found to be socially stratifying, history has shown that there is a risk that it will be withdrawn. This was the case with earlier differentiated curriculum in mathematics for secondary school (Wistedt & Raman, 2011)

The question of educational equity in gifted education is not a new one, but has been a main issue since the rise of gifted education. This issue can be discussed from a variety of perspectives, but continually addresses the need of different student groups to get access to educational practices aimed at developing giftedness (see e.g. Bianco, Harris, Garrison-Wade & Leech, 2011; Borland & Wright, 2000; Donovan & CRos, 2002; Feldhusen & Jarwan, 2000; Ford, 2003; Freeman, 2003; McBee, 2010; Nissen, 2013; Persson, 2010; Preckel, Goetz, Pekrun & Kleine, 2008; Rimm, 2004; Sarouphim, 2002; Swedish National Agency for Education, 2011, 2012, 2013;

Torrance, 2004; Wallace & Eriksson, 2006; Yoon & Gentry, 2009). In the egalitarian context of Sweden there are at least two different perspectives of special concern: the need to address students diversity as regards gender, origin, and socioeconomic status (Swedish National Agency for Education, 2011, 2012, 2013), as well as the need to address the highly intellectually gifted children (Persson, 2010). This thesis aims at contributing to further our knowledge about student representations in gifted educational practices in mathematics by relating to both of these perspectives. Thus, the focus of the third general research question of this thesis is:

- What is the representation of different student groups in gifted educational practices in mathematics in the Swedish upper secondary school?

1.3 Structure of thesis

In order to give the reader an idea of the variation of and complex relationship behind the construct conception of giftedness at focus in this thesis the following chapter presents different *General models of giftedness.* The next chapter focuses on the specific conceptions found in *Research on mathematical giftedness*. Special attention is given to characteristics of and identification of mathematical giftedness, as well as the relationship between mathematical giftedness and mathematical creativity. These aspects of mathematical giftedness are central to the work in this thesis. Then, in the *Methodology* chapter, the design and method behind each of the studies that are to contribute to furthering our understanding of the general research questions are presented. This is follow by a detailed description of the reflections of the scientific quality of the work in this thesis. The chapter ends with a discussion on the ethical considerations applied in this work. The fifth chapter is a *Summary of* the five *Papers* found in the Appendices. Here the research questions, methods, analysis procedures and the main findings from each of my studies are briefly presented. These studies lay the foundation for the *Conclusions* drawn about the general research questions in the following chapter. The thesis ends with a *Discussion* on how the findings in this thesis may affect the educational practice, and act as a springboard for future research.

1.4 Thesis structure

In order to give the reader an idea of the variation of and complex relationship behind the construct “conception of giftedness” in focus in this thesis the following chapter presents different *General models of giftedness.* The next chapter focuses on the specific conceptions found in *Research on mathematical giftedness*. Special attention is given to characteristics of and identification of mathematical giftedness, as well as the relationship between mathematical giftedness and mathematical creativity. These aspects of mathematical giftedness are central to the work in this thesis. Then, in the

Methodology chapter, the design and methods behind each of the studies that are to contribute to furthering our understanding of the general research questions are presented. This is followed by a detailed description of the reflections on the scientific quality of the work in this thesis. The chapter ends with a discussion on the ethical considerations applied in this work. The fifth chapter is a *Summary of* the five *Papers* found in the Appendices. Here, the research questions, methods, analysis procedures and the main findings from each of my studies are briefly presented. These studies lay the foundation for the *Conclusions* drawn about the general research questions in the following chapter. The thesis ends with a *Discussion* on how the findings in this thesis may affect the educational practice, and act as a springboard for future research.

2. General models of giftedness

This chapter begins with an orientation of various models of giftedness. It is meant to provide the reader with a reference to the consensus and controversies between different conceptions of giftedness. It is also intended to provide information on concepts such as, e.g., dynamic assessment and school house giftedness, which are used in the Discussion chapter of this thesis. Next, there is a brief presentation of factors that in different models of giftedness are found to contribute to giftedness. The chapter ends with a section on the identification of giftedness. While this chapter presents research on general aspects of giftedness the succeeding chapter will deal with research on giftedness in mathematics specifically.

2.1 Definitions of giftedness

In research in gifted education there is no consensus in the use of terminology. The most commonly used terms are "giftedness" and "talent". However, there are no agreements in the use of these terms (Gagné, 2004, 2005). Sometimes the terms are used as synonyms (e.g., Csikszentmihaliyi & Robinson, 1986). Other times, talent is considered as a sub-category of giftedness (Haensly, Reynolds, & Nash, 1986). Yet at still other times, talent is considered a potential for giftedness and gifted achievements (Feldhusen, 1986). On other occasions, giftedness stands for the stage of development that precedes the outstanding mastery called talent (Gangé, 2004; Tannenbaum, 1986). In the presentation of different theories here I have tried to stay true to the terminology adopted by the scholars.

The more systematic search for an understanding of giftedness started in the late nineteenth century. Since then scholars have presented an increasing number of definitions of giftedness (Subotnik et al., 2012). Some claim more than hundreds of definitions now exist (Hany, 1987). Starting with the work of Galton, giftedness was equated with high intelligence (Ziegler, 2009). According to Galton, giftedness was normally distributed among humans and the intelligence corresponding to giftedness could be measured by tests. This was the idea behind the famous studies done by Terman (1922) in 1920s. IQ tests were the supreme measure of giftedness until the 1980s. From the perspective of this genetic oriented definition of giftedness, intelligence was seen as purely innate and thus the IQ test score permitted testers to predict future achievements (Mönks & Katzko, 2005). The only controversy within this conception of giftedness was how high to set the cut-off score for the identification of giftedness (Renzulli, 1999.) Terman argued that giftedness meant scoring in the top 1% of the IQ tests, while others argued that top 3% or top 5% should be the criterion. By time it was shown that IQ tests did not correctly account for talents such as creativity and domain-specific talent, and that those tests told little about students' ability and thought processes (Wertheimer, 1999). This resulted in scholars advancing their definitions of giftedness by identifying different domains of giftedness. Important work includes Gardner's (1983) theory of multiple intelligences (MI). The eight intelligences identified by Gardner (2003) are linguistic, musical, logical-mathematical, spatial, bodily-kinesthetic, intra and interpersonal, naturalistic and existentialistic intelligence. The logical-mathematical intelligence includes the ability to detect patterns, reason deductively and think logically. Gardner's definition of intelligence has been described as "a biopsychological potential to design culturally valued products and solutions" (Davidson, 2009, p. 83). Often different intelligences interact in the creation of such a product or solution. Unlike the earlier model of giftedness, the MI theory stresses the developmental nature of intelligence.

In order to allow for quality aspects in the identification of gifted individuals, cognitive models of giftedness were developed (Mönks & Mason, 2000). In these models, the focus is on what distinguishes highly gifted individuals from average individuals in their way of receiving and processing information. The research done by Krutetskii (1976) on the structure of the mathematical mind uses this approach (see further descriptions of Krutetskii's model in the next chapter). A general model of giftedness within this orientation is given by Sternberg (1985). According to his triarchic theory of successful intelligence, successful intelligence is "(1) the ability to achieve one's goals in life, given one's sociocultural context, (2) by capitalizing on strengths and correcting or compensating for weaknesses (3) in order to adapt to, shape, and select environments (4) through a combination of analytical, creative, practical abilities." (Sternberg, 2005, p. 328). Individuals do not necessarily have to be equally strong in all aspects of the abilities. However, all of the three aspects of the abilities are seen as evolvable. Since the first presentation of the triarchic theory of successful intelligence, Sternberg has presented several models that contribute to the understanding of giftedness development (see, e,g., Sternberg, 2000, 2003, 2005). Sternberg's model (2000) of developing expertise is influenced by contemporary research on expertise (see below). In this model, giftedness is not a feature of a fixed group of individuals, since what counts as expertise, and thus giftedness, changes with time (Davidson, 2009). A person might be considered as gifted at one stage of life but not in another. In his latest model of giftedness, called wisdom, intelligence, and creativity synthesized (WICS), wisdom plays a prominent role (Sternberg, 2005).

The influence of the non-cognitive traits on the realization of giftedness is accounted for in the achievement oriented models. These models typically make a distinction between potential for giftedness and realized giftedness. One influential model of this type is Renzulli's three-ring conception of giftedness (1999, 2005). Renzulli believed that the long-term purpose of special programs for gifted individuals was to increase "society's supply of potentially and creative and productive adults" (1999, p 8). He proposed two types of giftedness – "schoolhouse giftedness" and “creative-productive giftedness”. School house gifted students are more easily found as they are good test-takers and lesson-learners. They also cover regular curricular material at advanced rates and levels of understanding (Renzulli, 2005). Further, the abilities that make gifted students perform highly in traditional school settings (for example: to learn deductively, structure thinking processes, and to acquire, store and retrieve information) are rather stable over time. This group of students is one type of gifted students that might be called high performers. Within creative-productive giftedness the emphasis is on the development of “original material and products that are purposefully designed to have an impact on one or more target audiences” (Renzulli, 2003, p. 185). Creative-productive gifted students act as committed inquirers and work on self-selected problems. Creative-productive giftedness is defined as an interaction between three clusters of character traits: above average ability, high levels of creativity and high task commitment (Renzulli, 1978, 1986). In this three-ring conception of giftedness, “Gifted and talented children are those possessing or capable of developing this composite set of traits and applying them to any potentially valuable area of human performance” (Renzulli, 1978, p. 261). Renzulli emphasizes the developmental nature of giftedness, and later (see, e.g., Renzulli, 1998) prefers to talk about the development of gifted behaviors rather than gifted students.

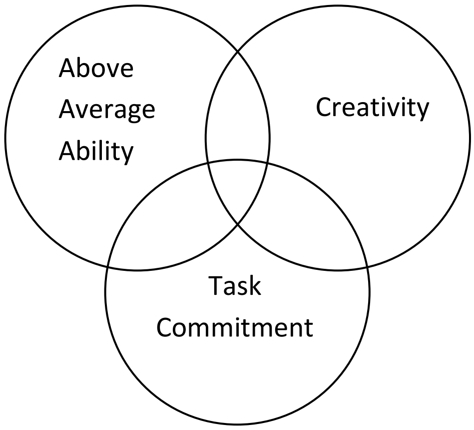

Figure 1. The-three ring conception of giftedness.

During recent years Renzulli (2002, 2003, 2005) has expanded his conception of giftedness to also include co-cognitive traits (optimism, courage, romance with a discipline, sensitivity to human concerns, physical/mental energy and vision/sense of destiny) and to promote social capital. This work is ongoing under the name of Operation Hundstooth.

In the work by Sternberg as well as Renzulli, the gifted individual can be seen as acting on the environment (Davidson, 2009). Other models recognize the environment acting on the gifted individual (Davidson, 2009). Examples include Mönk’s (1992) multifactor model of giftedness, Gagné’s (2005) developmental model of giftedness and talent (DMGT), and Tannenbaum’s (1983) star definition of giftedness, presented below.

In order to attend to the importance of the individual’s social interaction in the emergence and development of gifted behavior, the three-ring conception of giftedness was modified by Mönks (1992). Mönks’ model, now called the multifactor model, recognizes both personality and environmental aspects of giftedness.

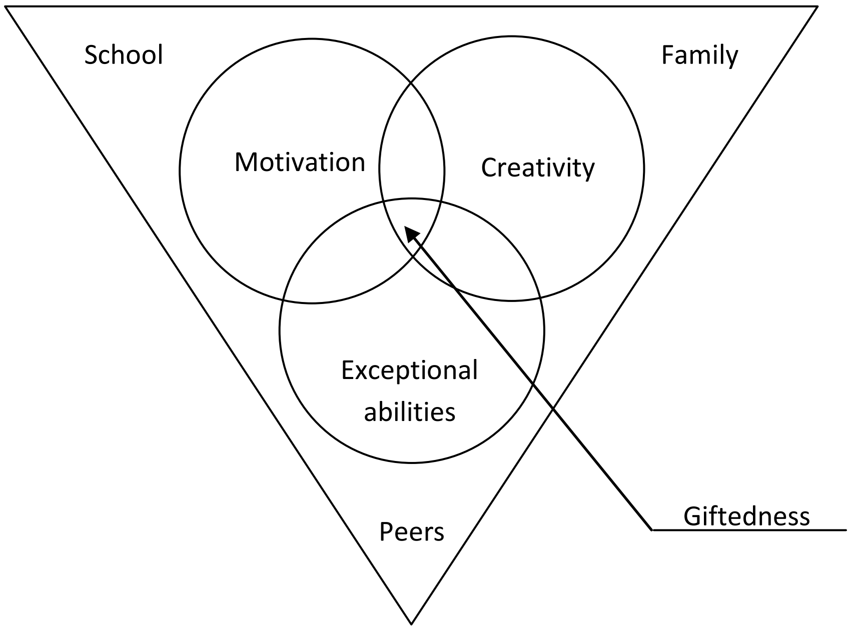

Figure 2. Mönk’s Multifactor Model of giftedness.

The environmental factors that were found to be most important are: family, school and peer group. The personal traits of this model are: outstanding/exceptional abilities, creativity and motivation. Mönks sharpened the requirements of giftedness by replacing above average ability with exceptional abilities. Renzulli does not give any numerical values to clarify the level of abilities as compared to peers. However, when referring to well-above average ability he means individuals who perform or have the potential for performing at the top 15-20 percent of any given area of human endeavor (Renzulli, 1998). By outstanding abilities Mönks refers to abilities in specific domains that exceed 5-10 percent of performance (Mönks & Mason, 2000). In Mönks’ model motivation includes task commitment. Mönks’ model has also been extended to comprise risk taking, future time perspective, anticipation, planning and emotional factors (Mönks & Mason, 2000). Giftedness has been described as "an individual potential for exceptional or outstanding achievements in one or more domains" (Mönks & Katzko, 2005, p. 191), and can only be developed in the fruitful interaction among the personal and environmental components.

Also Gagné's (2005) developmental model of giftedness and talent (DMGT) is achievement oriented. In his DMGT, he makes a clear distinction between potential for and realization of eminent achievements. Giftedness is associated with the outstanding natural or innate abilities. Gifted individuals should have natural abilities at least to the degree of the top 10 percent of age peers. The talent development process is about transforming the genetic endowments into expertise in a given field. This expertise, called talent, is the product of the interaction of the innate abilities, intrapersonal factors (such as self-management) and environmental factors (such as school provisions and home) surrounding the individual, and chance. A talented individual should show mastery in at least one specific domain area, called fields of human activity, to a degree of at least the top 10 percent of age peers (Gagné, 2005). Mathematics is one out of more than 40 fields of human activities. The model, however, does not include descriptions of the signs of talent in each field of activity. Gagné proposed a five-level system of cutoffs to differentiate five levels of giftedness or talent within each specific domain area: mildly (1:10), moderately (1:100), highly (1:1,000), exceptionally (1:10,000), and extremely (1:100,000).

From a systemic perspective there is a need to attend to the influences of different systems of society, such as the economic situation, political orientation, and the culturally dominant values and beliefs, in order to understand what counts as an outstanding achievement and thus decide who will be considered as gifted (Mönks & Mason, 2000; Persson, 2013). This approach is taken by Tannenbaum (1983, 2003). He defines giftedness in the following way: *"Keeping in mind that developed talent exist primarily in adults, I propose a definition of giftedness in children to denote their potential for becoming critically acclaimed performers or exemplary producers of ideas in spheres of activity that enhance the moral, physical, emotional, social, intellectual, or aesthetic life of humanity."*(Tannenbaum, 2003, p. 45). Like Gagné, Tannenbaum links the promise of giftedness with the fulfillment of giftedness. According to his star model of giftedness there are five psychological and social elements that contribute to high performance: superior general intellect (high IQ), distinctive special abilities (e.g., a mathematical cast of mind (Krutetskii, 1976)), non-intellective requisites (e.g., interpersonal skills, motivation, and perseverance), environmental support (from, e.g., peer groups), and chance (e.g., good luck with genes and family background). These five components must all come together for the fulfillment of gifted behavior. The minimum level for each element depends on the talent area. Tannenbaum stresses that chance factors should never be trivialized in the study of giftedness. He states “without some good fortune, no amount of potential can be truly realized” (Tannenbaum, 2003, p 56).

Other systemic models of giftedness are based on research on expert performance (e.g., Ericsson, Nandagopal, & Roring, 2009). While previous models of giftedness often focus on early identification of individuals that have the potential for high achievements in the future, research on expert performance tries to clarify the conditions that enable individuals to become excellent performers (Ziegler, 2009). From an expert performance perspective, giftedness is equated with exceptional performance in specific domains which is a result of extended and deliberate practice. The deliberate practice is a structured activity in which the individual persistently strives to improve his or her performance. In order to become an expert individuals need to practice deliberately for about 10 000 intensive hours (Ericsson, Krampe, & Heizmann, 1993). The development of expertise has three phases (Ericsson et al., 1993; Ericsson, Krampe & Tesch-Römer, 1993). The first phase is a joyful and playful phase where the individual gets to know the field of interest. In the second phase, the individual engages in deliberate practice. The third phase is a stage where the individual commits full time to the field of interest in order to reach a professional level of performance.

Ziegler’s (2005) actiotope model of giftedness is an example of a systemic approach to eminent achievement. In response to earlier models of giftedness Ziegler tried to clarify the relationship between gifts and excellent achievements (Phillipson & Callingham, 2009). Ziegler rejects the idea that talents and gifts are personal attributes (Ziegler, 2005). Instead focus is on the actions and their determinants of an individual. Developing expertise means being able to do more and more complex actions within a specific domain. The actiotope model is based on the idea that the next learning step is always within reach for a healthy individual (Ziegler, 2009). An individual is seen as talented if it is possible to identify a path of learning from the present action repertoire to the repertoire of excellence. In the identification of this path there is a need to include the living situation for the individual, including family situation, access to teachers, effects on different aspects of life due to time consuming practice, etc. In addition to this environmental factor, the goals of an individual and the action opportunities that an individual believes he/she has are important components in the development of his/her action repertoire (Ziegler, 2005).

2.2 Factors contributing to realization of giftedness

Sociocultural factors influence what is considered a domain of giftedness and the definition of giftedness (Csikszentmihaliyi, 1988; Freeman, 2005; Persson, 2013, in press; Sternberg, 2004, 2005). Although there are multiple definitions of giftedness there are several aspects of giftedness on which most scholars agree. Based on a review of conceptions of giftedness presented in the literature in psychological science Subotnik, Olszewski-Kubulus, and Worrell (2012) conclude that giftedness is domain specific, developmental and that biological, cognitive, psychosocial and pedagogical factors influence the realization of giftedness. The scholars specially stress that the recent models underlines that, "*Both the cognitive and psychosocial variables are malleable and need to be deliberately cultivated*" . (p.3)

The factors that were found to contribute to giftedness were general as well as domain-specific ability, creativity, motivation, personality, interest, passion, opportunity, chance, parents and emotional trauma (Subotnik et al., 2012). The first six factors are personal character traits that can be seen as contributing to giftedness. These traits can be affected in the interaction with the environment. The last four factors have a closer relation to the environment than to the character of the individual. In the following the later factors will be briefly described. In the next chapter, the other six factors will be treated in relation to expressions of mathematical giftedness.

Getting, and taking, the opportunity to nurture giftedness is vital for the realization thereof (Barnett & Durden, 1993; Noble, Subotnik, & Arnold, 1996; Tannenbaum, 1983). This can be exemplified in the study by Bloom (1985). Bloom's work is based on a retrospective study of individuals who had reached world-class level in their fields. Twenty of these individuals were eminent mathematicians. By interviewing the gifted individuals, their parents, their teachers and others, Bloom attempted to understand the processes that enabled the development of their mastery in a specific field. The essential result of this study can be captured by the following citation: "no matter what the initial characteristics (or gifts) of the individuals, unless there is a long and intensive process of encouragement, nurturance, education, and training, the individuals will not attain extreme levels of capability in these particular fields" (Bloom, 1985, p 3). Thus, getting the support and opportunity to develop is crucial in the realization of giftedness. This underlines the importance of educational practices such as, for example, gifted programs, in order to allow for talent development.

Some opportunities seem to appear by chance. However, according to Austin, (1978), successful individuals have often prepared themselves and learned to take advantage of developmental opportunities that they happen to come acRos. They could, for example, grab the chance to participate in a gifted group or to pick up comments and remarks that will make their work evolve further (Subotnik et al., 2012). Thus, chance factors are not only considered as good or bad luck.

Parents have been found to play a vital role in encouraging and intellectually stimulating their children who have turned into eminent contributors in their respective field in adulthood. This was shown in Bloom's study (1985), as well as in other retrospective studies (e.g., Goertzel & Goertzel, 1962). Parents' appreciation of intellectual excellence was also a common factor.

In order to facilitate creative productivity individuals need characteristics such as "an ability to cope with high level of stress, resiliency, emotional strength, a tolerance for ambiguity, intellectual risk taking, and a preference for challenge" (Subotnik et al., 2012, p. 19). Some scholars argue that such characteristics might be developed in some individuals as they experience challenging family situations and emotional traumas (Subotnik et al., 2012). However, emotional trauma is neither a pre-requisite for giftedness nor a guarantee for development of giftedness.

2.3 Identification of giftedness

Most scholars agree that abilities are derived from both genetic endowments and learning experiences (Mayer, 2005; Subotnik et al., 2012). However, scholars argue whether the main emphasis is on the genetic (see, e.g., Gagné, 2005; Persson, 2013; Sternberg, 2001) or the environmental component (Ericsson et al., 2009). Depending on the model of giftedness the identification process is meant to identify gifted individuals or gifted achievements/behaviors.

The procedure for how to identify giftedness is, or should be, a function of what is considered as giftedness. For example, according to a psychometric definition, giftedness means high scores on a psychological test like intelligence tests or creativity tests (Feldhusen & Jarwan, 1993). On the other hand, from a performance perspective, giftedness means extraordinary achievements. From this perspective, giftedness might mean top performance in a class or at work (Feldhusen & Jarwan, 1993). Research has shown that only about 15 percent of the individuals considered as gifted based on their performance reach the cut of scores for giftedness in psychometric tests (Ziegler, 2009). At the same time, less than 50 percent of the individuals labeled gifted from a psychometric definition show evidence of performance in line with their intelligence (Ziegler, 2009). An individual, product or performance might thus be considered as gifted from one perspective but not from another.

The majority of current models of giftedness are multi-factorial, which means that giftedness is seen as integral to and dependent on several cognitive as well as non-cognitive components. Just as scholars recommend (see, e.g., Feldhusen & Jarwan, 2000; Renzulli, 2004; Sheffield, Bennett, Berriozabal, DeArmond, & Wertheimer, 1999), we might thus expect that giftedness must be identified by multiple criteria. However, extended analysis of criteria used for identification of giftedness, in scientific studies and for practical purposes in educational practices, showed that about two thirds of the identification procedures built on only one criterion (Ziegler, 2009; Ziegler & Raul, 2000). The criteria found were achievement and intelligence (32,3%), achievement (31,0%), intelligence (21,2%), teacher or parent nominations (8,6%), other multiple criteria (4,4%), behavior (1,3%) and creativity (1,3%) (Ziegler, 2009). The intelligence criterion included various intelligence and aptitude tests such as the Scholastic Aptitude Test and Stanford-Binet L-M (Ziegler & Raul, 2000). High achievement test performance, scholastic awards and recognitions by colleagues are examples of indicators that were included in the achievement criterion. Measures of creativity included creative ability tests, group inventories for creative talent and demonstrated creative products or performances as judged by teachers or parents. The criterion on behavior referred to, for example, observations or analysis of gifted behavior checklists and the Scale for Rating Behavioral Characteristics of Superior Students. The criteria for identification of giftedness were highly age-dependent. In the younger years the intelligence tests dominated, whereas the achievement based criteria increased with age (Ziegler & Raul, 2000). General ability, which is often measured by IQ-tests, is often seen as necessary but not sufficient prediction of optimal performance or gifted behavior (Mönks, 1992; Renzulli, 2005; Subotnik et al., 2012). Thus, the different ways of identifying giftedness at different ages are consistent with a current comprehensive definition of giftedness in which "*giftedness can be viewed as developmental, in that in the beginning stages, potential is the key variable; in later stages, achievement is the measure of giftedness; and in fully developed talents, eminence is the basis on which this label is granted*". (Subotnik et al., 2012, p. 3)

As indicated above there are many ways to assess giftedness. Some of the measures at hand, for example the intelligence tests, are characterized as static, normative and standardized (Kanevsky, 2000). Scores from these measures and other types of traditional tests, as well as previous school marks, reflect what an individual knows as compared to age-peers and as a result of earlier experience. This type of information is sometimes called status information and is considered valid information for identification of school house giftedness (Renzulli, 2004). Dynamic assessments, on the other hand, relate to assessments of an individual's processes in what Vygotsky (1978) called the zone of proximal development. Thus, in dynamic assessments the individual's learning progress, as the individual interacts in a teaching/learning situation, is in focus (Kanevsky, 2000). This type of assessment has, in some sense, a close relationship to the dynamic interaction, characterized by high level of investigative or creative activity, central for gathering action information. However, whereas dynamic assessment is meant to evaluate the individual's learning development, action information is meant to give the teacher information on how to proceed in order to create the most beneficial conditions for a dynamic interaction to appear (Renzulli, 2004). This latter form of information is crucial for enabling the development of creative-productive giftedness (Renzulli, 2004).

No matter which definition of giftedness and aligning identification criterion we use, giftedness is always a function of the choice of cut off scores for giftedness on tests or the operationalization of the term extraordinary achievement. Recent scholars suggest different numbers of prevalence of giftedness among a cohort like age peers: 1-3% (Robinson, 2005), 3% (Brody & Stanley, 2005), 5% (Mayer, 2005), 5-10% (Freeman, 2005), 10% (Gagné, 2005), 15% (Gordon & Bridgall, 2005) and 15-20% (Renzulli, 2005). It is to be noted that that all judgments of exceptionality depends on the norm group used (Lohman, 2009). As previous opportunities to develop or manifest giftedness may vary a lot between individuals in different schools or from different socio-economic places, there should be a concern about comparing students score's or performances to a proper norm group (Lohman, 2009).

There is always a risk that identification procedures are biased in some way. For example, IQ-tests have been criticized as they have been found to underestimate the potential of, for example, culturally and socio-economically different children (Frasier & Passow, 1994; Skuy, Kaniel, & Tzuriel, 1998). This means that students in certain groups risk being underrepresented in gifted educational services. It is to be noted that, due to their underrepresentation in mathematically intensive programs and professions, gifted mathematics females are also considered as a group which needs special attention in the identification process and in the gifted education practice (Feldhusen & Jarwan, 2000).

3. Research on Mathematical Giftedness

This chapter focuses on conceptions of mathematical giftedness as described by scholars who have studied gifted mathematics students. Mathematical giftedness is often described in terms of ability in mathematical problem solving processes (Sriraman, 2005). Most often, recent studies of mathematical giftedness (e.g., Bicknell, 2009; Chyriwsky & Kennard, 1997; Gavin, Casa, Adelson, Carroll & Sheffield, 2009; Koshy, Ernest & Casey, 2009; and studies presented in Leikin, Berman & Koichu, 2009; Pettersson, 2011; Sriraman, 2008) relate to the mathematical abilities identified by Krutetskii's (1976) in his famous study of the components that distinguish capable mathematics problem solving students from less capable ones. Krutetskii's (1976) structure of mathematical abilities also serves as the basis for my studies on conceptions of giftedness. Therefore, the following presentations of mathematical abilities and mathematical creativity will proceed from his study findings. Other descriptions and definitions of mathematical giftedness and mathematical creativity treated below give complementary information on mathematical giftedness, and will be dealt with in the discussion on head teachers' conceptions of mathematical abilities. The chapter begins, however, with a discussion on the relationship between mathematical giftedness and mathematical creativity.

3.1 Mathematical Giftedness and Mathematical Creativity

In general models of giftedness there is sometimes a distinction made between giftedness, ability and creativity (see, e.g., Mönks, 1992; Renzulli, 1978). Although mathematical giftedness has been described in many ways (for an overview see, e.g., Phillipson & Callingham, 2009; Sriraman, 2005; Wiecerkowski, Cropley & Prado, 2000), there is often no clear division made between mathematical giftedness, mathematical abilities and mathematical creativity. Instead creativity has in many cases been connected with the highest level of mathematical ability (Leikin, 2009). In Sheffield's (1999) presentation of the continuum of mathematical proficiency, which mathematics students need to move along, creators of mathematics are at the highest level (see Figure 3).

$$\longleftrightarrow$$

innumerates	doers	computers	consumers	problem solvers	problem posers	creators

Figure 3. Sheffield's (1999) continuum of mathematical proficiency.

When describing the developmental steps, from (mathematical) ability in childhood to (mathematical) eminence in adulthood, Subotnik (Subotnik, Pillmeier & Jarvin, 2009; Subotnik et al., 2012) and her colleagues emphasize the importance of different types of creativity. At the highest level, eminent achievements are shown by creative products that move the field (of mathematics) forward (Subotnik et al., 2009; Subotnik et al., 2012). Creative productivity is thus considered as a stage beyond expertise (Subotnik et al., 2012). In Usiskin's (2000) eight-tiered classification of mathematical talent, the professional mathematician is at level 5. The highest levels – levels 6 and 7 – are reserved for the creative mathematicians. For Sriraman (2008) this meant that "at this level mathematical giftedness does not necessarily imply creativity, but the converse is certainly true" (2008, p 85). This is partly in line with Krutetskii's assertion about gifted children: "many mathematically gifted students demonstrate a kind of creative mathematical thought" (1976, p.69). Building on Usiskins's work, Sriraman (2005) presented a model of how to work in a classroom in order to enable gifted and/or creative mathematics students to move from levels 3 and 4 to level 5. The aim of this model was to increase the level of creativity in gifted students.

These examples show that mathematical creativity is highly valued in the development of mathematical eminence. Still, the relationship between mathematical giftedness and creativity is not yet fully understood. Issues of concern include whether creativity and ability are distinct phenomena, whether creativity is a general trait or a domain-specific component and whether it is a component that allows us to identify gifted achievements (Subotnik et.al., 2012). Questions also include whether mathematical creativity is a subset of mathematical giftedness, and whether mathematical creativity is a prerequisite for mathematical giftedness in youths (Sriraman, 2005).

One of very few (Leikin, 2009) studies that bridges general models of giftedness with studies of mathematical giftedness and creativity is done by Leikin, Koichu and Berman (2009). In this study, mathematical giftedness is associated with the ability to solve and pose mathematical problems. The study is built on Renzulli's (1978) three-ring definition of giftedness, and there is a distinction made between above-average ability, creativity and task commitment. The specific descriptions of these characteristics of gifted and/or creative mathematics students are related to earlier studies thereof. In the following I will present more detailed interpretations of these and other characteristics.

3.2 Characteristics of Mathematical Giftedness and Creativity

Although gifted mathematics individuals are not a homogeneous group (Majoram, 1992) they still share some common characteristics (for an overview see, e.g., Phillipson & Callingham, 2009; Sriraman, 2005; Wiecerkowski, Cropley & Prado, 2000). In the following sections I will present mathematical abilities, non-cognitive traits and components of mathematical creativity that research finds characterize giftedness in mathematics.

3.2.1 Mathematical abilities

Several models of giftedness consider general abilities and domain-specific abilities as important for optimal achievement in a specific field (Mayer, 2005; Subotnik et al., 2012). General abilities in academic domains are often defined by abilities measured by intelligence tests. As noted by the term, domain-specific abilities need to be described in relation to the specific field of giftedness.

According to Krutetskii (1976), “mathematical giftedness is characterized by generalized, curtailed and flexible thinking in the realm of mathematical relationships and number and letter symbols, and by mathematical cast of mind” (p. 352). Krutetskii states that mathematical abilities are not innate, but admits that the inclination for developing mathematical giftedness differs between individuals. He argues that “anyone can become an ordinary mathematician: one must be born an outstanding mathematician” (p. 361). Mathematical abilities are dynamic and they only exist, manifest and develop in a mathematical activity. Thus, a systematic and purposeful mathematical activity is needed for abilities to develop also in individuals with excellent inclination for the domain. Under certain beneficial conditions any individual may make significant progress in mathematics, but s/he needs to put much more time and energy into the activity than individuals with high inclination for mathematics.

Krutetskii distinguishes between abilities and skills. Whereas abilities are character traits of an individual, skills are characteristics of (the content of) an activity. For example, students can have skills in formulating an equation out of the information in a given problem. This skill may be due to an underlying ability to grasp the formal structure of the problem. Mathematical abilities are central in Krutetskii’s work. The concise expression of giftedness above is a summary of his general outline of mathematical abilities that characterize mathematical giftedness in the problem-solving activity. Following the basic stages of this process – obtaining, processing and retaining mathematical information – he identified abilities including the ability to a) grasp a formal structure of a problem, b) think logically in spatial, numeric and symbolic relationships, c) rapidly and comprehensively generalize mathematical material, d) curtail mathematical reasoning processes, e) be flexible with mental processes, f) strive for clarity, simplicity and rationality of solutions, g) to reverse and reconstruct mental processes, and h) memorize schemes, methods, principles and relationships. These abilities are interrelated and form a general synthetic component, “a distinctive syndrome of mathematical giftedness, the mathematical cast of mind” (Krutetskii, 1976, p 351.). This might be interpreted as a tendency to view the world through a mathematical eye. It includes the individuals’ high level of concentration and interest. Krutetskii argued that the abilities above open up the possibility of creative mastery of mathematics.

Krutetskii also explicitly mentions components that might be useful but not obligatory for mathematical giftedness. However, the degree of development of these components determines the type of mathematical mind. The components that are found not to have a decisive value for the manifestation of mathematical giftedness include the swiftness of mental processes; the ability to rapidly compute in the head; the memory for symbols, numbers and formulas; the ability for spatial concepts; and the ability to visualize mathematical relationships and dependencies. It is also to be noted that Krutetskii does not equate mathematical giftedness with high achievements in school mathematics. This is also true for other later researchers (see, e.g., Diezmann & Watters, 2002; Greenes, 1981; House, 1999; Pettersson, 2011).

Numerous studies confirm the existence of the abilities identified by Krutetskii (Wertheimer, 1999). Other mathematical abilities or skills that have been identified as important in earlier studies involve, for example, managing data (Greenes, 1981; Yakimanskaya, 1970), having an intuitive awareness of mathematical proof (Sriraman, 2004c), transferring ideas to new situations (Greenes, 1981), constructing related problems (Kiesswetter, 1985), advanced analytical or spatial reasoning (Diezmann & Watters, 1996; 2000), proper decision-making when solving problems (Schoenfeld, 1985; Sriraman, 2003) and distinguishing between empirical and theoretical principles (Davydov, 1988, 1990 and Vygotsky, 1962, 1978 in Sriraman, 2005).

Some researchers believe that ability for learning at a faster pace is a sign of mathematical giftedness (e.g. Heid, 1983). The ability for learning mathematics is also regarded as manifestation of mathematical ability according to Krutetskii.

3.2.2 Non-cognitive characteristics

Current models of mathematical giftedness recognize the role of non-cognitive personality traits in the manifestation of mathematical giftedness (Subotnik et al., 2012; Wiecerkowski, et al., 2000). Such features are not explicitly stated in the structure of mathematical abilities by Krutetskii (1976). Nevertheless, along with mathematical abilities Krutetskii does mention general psychological conditions that a person needs in order to perform successfully in an activity. These conditions include some character traits (e.g., diligence and finding pleasure in intensive work), a positive attitude toward the mathematical activity (e.g., interest, inclination and passion to study mathematics), and a presence of mental state (e.g., incentive and concentration). (Krutetskii also add knowledge and skills in mathematics to these psychological conditions.)

Krutetskii emphasizes that capable students show perseverance, capacity for work and diligence during extended and intensive mathematical activity. Those traits are found crucial in order to master mathematics. This indefatigability of mathematically gifted students has been interpreted (Wiecerkowski, et al., 2000) as included in his general component “mathematical cast of mind”. Gifted mathematics individuals’ tendency not to tire when engaging in mathematics has been noted by other researchers as well (e.g., Bloom, 1985; Greenes, 1981; Leikin et al., 2009). A keen drive to engage in work in the domain of giftedness is a striking character trait of gifted individuals (Csikszentmihayili, 1985; Winner, 1996). In general models of giftedness this trait is sometimes connected to task commitment and is often described as “perseverance, endurance, hard work, dedicated practice, self-confidence and belief in one’s ability to carry out important work” (Renzulli, 1986, p. 96). Along with, for example, curiosity, task commitment is often considered as a component in motivation.

Motivation is considered as an essential component for eminent levels of achievement (e.g., Duckworth, Kirby, Tsukayama, Bernstein & Ericsson, 2010; Gagné, 2005, 2010). Researchers often talk about two different types of motivation, intrinsic and extrinsic motivation. Intrinsic motivation refers to the individuals’ engagement in a task for the sake of exploration, learning or enjoyment. This is the type of motivation that Krutetskii refers to. On the other hand, extrinsic motivation relates to the individuals engagement in tasks driven by external rewards. Gifted individuals have generally been associated with intrinsic motivation (Subotnik et al., 2012). However, recent studies have shown that many high achievers are motivated also by extrinsic factors (Subotnik et al., 2012). One might ask whether this shift in the motivation of gifted individuals is at least partly due to the change in society’s view of the reasons for focusing on the gifted ones, and thus the very definition of giftedness.

The difference between motivation and interest is not clear cut. Still, interest has been described as a different component that plays a vital role in the future development of outstanding performance (Subotnik et al., 2012). For example, a person who is high achieving in mathematics as a youth, but who prefers a non-natural science or non-mathematics career, is less likely to get a degree in mathematics-related domains than a person who is an average mathematics achiever as a youth, but who has science-related career interests (Subotnik et al., 2012). Research on reasons for female under-representation in fields that are mathematically intensive has shown that females do not have a desire to pursue a mathematic-intensive career to the same extent as male (Ceci & Williams, 2010). Gifted students’ interests are thus governing the choice to further or not to further the development of giftedness in the domain. Krutetskii stressed the importance of a positive attitude which required an interest in studying mathematics. He described the highest level of positive attitude as a “passionate enthusiasm”. Current researchers portray passion as a call, directed towards a specific domain, which drives an individual’s exploration of that domain (Subotnik et al., 2012). However, recent research has mostly linked passion with work in nonacademic domains (Subotnik et al., 2012).

Several researchers agree that personality traits influence the realization of giftedness (Subotnik et.al., 2012). However, researchers have not yet found a complete answer to the question of how personality contributes to development of giftedness. In a summary of the personality characteristics of intellectually gifted individuals, the individuals were described as independent and creative, as mentioned earlier. But they were also described as non-conformist, reflective, curious, imaginative, dominant and individual (Janos & Robinson, 1985). When portraying how the highly-able intellectual individuals are perceived by others Persson (in press) uses words such as difficult and oppositional, as the highly able individuals shake things up when questioning authorities and norm systems. Gifted students have also been described by their unwillingness to accept statements without critical examination thereof, and their reluctance to practice skills already mastered (Wolfle, 1986).

3.2.3 Mathematical creativity

Krutetskii (1976) mentions mathematical creativity several times in his work, but he never gives a precise meaning of it. Scholars still have no clear-cut and broadly accepted definition of mathematical creativity. Instead there exist many descriptions and definitions of the construct (Mann, 2006; Sriraman, 2005). However, like most models of giftedness (Davidson, 2009; Subotnik et al., 2012), Krutetskii emphasizes originality or novelty in the manifestation of creativity. He acknowledges that there is a difference between the creativity shown by a professional mathematician and a student, but argues that this difference is not absolute. The creativity of professional mathematicians is associated with groundbreaking achievements which change the direction of and move the field of mathematics forward. The creativity of mathematics students is instead demonstrated as s/he rediscovers, invents, and independently achieves something subjectively new in mathematics. Thus, like several current researchers (e.g., Leikin, 2009; Liljedahl & Sriraman, 2006; Lithner, 2008; Sheffield, 2009; Sriraman, 2005; Subotnik et al., 2012), Krutetskii takes the subjective – or relative – nature of creativity into consideration when defining or describing mathematical creativity in school mathematics.

In school settings, mathematical creativity is usually associated with original approaches and unusual solutions to problem solving or problem posing (e.g., Greenes, 1981; Kiesswetter, 1985; Leikin et al., 2009; Sheffield, 2009; Silver, 1997; Sriraman, 2005). Although, Krutetskii does not clearly state which mathematical abilities are most closely connected to the manifestation of mathematical creativity, other researchers mention gifted persons' strive for elegance and clarity in explaining reasoning, flexibility, and reversibility (see, e.g., Leikin et al., 2009; Sheffield, 2009) in this regard. Fluency is also considered an important component (Greenes, 1981; Leikin et al, 2009).

Diezmann and Watters (2000) argue that mathematically creative breakthroughs are more often based on a high spatial ability rather than on a high analytical ability. This distinction between abilities calls to mind the two types of structure of mathematical minds identified by Krutetskii, that is the geometrical and the analytical cast of mind. Most gifted mathematics individuals in Krutetskii's study had a mix of these types of mind, called a harmonic type of mind. Krutetskii stated that the extent of giftedness had nothing to do with the presence or non-presence of one or the other type of mind. However, he did express that leaning towards a pure analytical or a pure geometrical type of mind may put constraints on the possibility to pursue some types of mathematical activities.

Krutetskii does, however, emphasize the role of independence in the demonstration of creativity. In his explanation of the abilities, characterizing the mathematically capable children, he describes that they are "related to an independent creative mastery of mathematics under the conditions of school instruction, to the independent formulation of uncomplicated mathematics problems, to finding ways and means of solving these problems, to the invention of theorems, to the independent deduction of formulas, and to finding original methods of solving nonstandard problems" (Krutetskii, 1976, p. 68). Sriraman (2005, p. 24) underlines the creative students "fiercely independent" thinking and independent discovery of mathematical principles (Sriraman, 2004a; 2004b). The mathematically gifted students in Bloom's (1985) study were also characterized by their independence, as they had a predilection during their school years for doing mathematics as they wished.

In some general models of giftedness, creative work is not only original but also useful (Renzulli, 2005) and adaptive (Sternberg & Lumbart, 2000). It has to be noted, though, that creative mathematical work does not necessarily fulfill the requirement of usefulness as applicability in the real world as intended by these general models (Sriraman, 2008).

3.3 Identification of Mathematical Giftedness

3.3.1 Goals for identification

The choice of how to define and how to identify mathematically gifted children and youth depends on the educational ideology to which one subscribes (Wertheimer, 1999). Often, different ideologies exist in parallel and influence existing educational practices. One goal for identifying mathematical giftedness might be to find a small group of excellent mathematicians in order to develop the field of mathematics further. The content of *mathematics* is central in this perspective. A second, *social*, perspective emphasizes that all individuals are mathematically promising and should get the opportunity to develop. The goal in this case is to reduce the stratification of society. Another goal may be to meet the need from the *job market* with the intention to ensure the well-being of a region or nation. From still another ideology, the focus is on the individuals needs and on developing the *individuals*' unique potential for mathematics.

Identification procedures focusing on finding the individuals who are seen to have the best potential to be creative mathematicians include competitive-selections. The best performing students in national as well as international competitions are thus selected for furthering their mathematical giftedness (Wertheimer, 1999). Although not explicitly stated, this perspective may be enacted in Sweden in the special mathematical course given to the finalist students in the national selection for the Mathematical Olympiad.

The second ideology, representing the socialist perspective, stresses that all students are mathematically promising and are able to achieve as long as they are motivated, believe in themselves and are given the right opportunities to develop (Wertheimer, 1999). From this perspective there is no need for special identification procedures as all students are supported or self-selected. This view of education is very much present in the egalitarian school development in the 20th century in Sweden (see Paper I).

Ideologies that attempt to meet the mathematical needs from the job market seem to imply an identification procedure that is neutral with regard to giving all gifted students equal opportunities to be identified. However, this approach has been accused of maintaining hierarchical structures in society (Ernest, 1991, in Wertheimer, 1999). This may be due to factors such as provisions which are given in later years of schools and that many students with less supportive environments may have lost interest or motivation before the opportunity turns up. Although a variety of indicators such as diagnostic tests and tests of intellectual curiosity may be used to identify mathematical giftedness, still motivation and interest are needed.

An ideology underpinning the establishment of gifted education in our country is the individual centered approach. The needs and wants of the students are in focus in this fourth ideology. This is an approach often taken by educators, including me in my choice to focus on the topics treated in this thesis. A variety of tools have been used in attempts to identify mathematical giftedness in this perspective. Examples include general intelligence tests; specific aptitude tests;

achievement tests; teacher, parent and peer nominations; self-selection; and observations during problem solving. The Swedish nationally established gifted programs in upper secondary school use written enrollment tests in mathematics to test the basic knowledge and skills, as well as the problem solving ability of their applicants. Some programs also require a nomination letter from the students' previous school. In identification procedures for gifted programs in other countries, cognitive tests are sometimes complemented by interviews or questionnaires in order to find out about the non-cognitive attributes of the applicants (Wertheimer, 1999). There are gifted programs in mathematics in the Swedish upper secondary school that includes interviews in their identification procedures. The purpose is to ensure that there is a strong personal interest influencing the student to participate in the program, instead of expectations from parents and other influential people close to the students.

Continuing the focus on this individual centered perspective on identification of giftedness, we turn our attention to tools and models used in various identification procedures from this approach.

3.3.2 Tools and models of identification of mathematical giftedness and mathematical creativity

There are several models for identification of mathematical giftedness (e.g., Brody & Stanley, 2005; Kiesswetter, 1985; Krutetskii, 1976; Leikin et al., 2009; Vilkomir & O´Donoghue, 2009; various models presented in Wertheimer, 1999). Some of them stem from general models of giftedness, whereas others are developed from studies of certain cognitive mathematical components. Like Krutetskii (1976), other scholars often stress the dependency between provision for and the manifestation of giftedness (Kanevsky, 2000; Renzulli, 2004; Sheffield, 1999; Vilkomir & O´Donoghue, 2009). Researchers (Diezmann and Watters, 2002; Krutetskii, 1976) also draw attention to the fact that students may have different types of mathematical minds that need different kinds of mathematical activities in order for the mathematical ability to be demonstrated. Moreover, researchers often emphasize the use of multiple criterion the identification procedures (e.g., Renzulli, 2004; Sheffield, 1999; Subotnik et al., 2012). Some suggest that students should collect their best work, including projects or creative work, in portfolios as an evidence of their mathematical development (Sheffield, 1999; 2003).

Practice does not always follow recommendations from research. For example, due to their time efficiency, often high scores on mathematics achievement tests or IQ scores above 130 are used as signs of giftedness (Silverman, 2009). This is especially true for identification procedures for gifted programs. However, considering that most gifted students work within inclusive settings, we turn our attention to also other types of identification of mathematical giftedness.

In some cases teachers or parents use checklists of typical characteristics of gifted students as a reference for identification. Such a list may, for example, include:

- "an early ability to speak in sentences and to sustain a conversation with an adult;
- a wide vocabulary and excellent reading ability;
- an inveterate curiosity and questioning attitude;
- an ability for sustained concentration particularly when the subject really interests them;
- demonstrating a preference for complex thought;
- able to work with abstract thought, often using higher level thinking skills
- showing a preference for socializing with older pupils, or with adults – they sometimes have difficulty making friends within their own peer group". (Bates & Munday, 2005, p. 6)

Given a definition of giftedness not based on manifested performance, there is a possibility for a student to be considered an underachiever. There is, thus, a gap between the potential and the performance of an underachiever. Also these students are sometimes identified based on checklists of their typical characteristics. Characteristics might comprise:

- "orally good, demonstrating quick thinking, the ability to build on other people's ideas and to apply learning in different situations;
- able to argue and justify with ease;
- poor written work; unfinished, untidy, completing the minimum to get by;
- may seem bored, lethargic and uninterested and anxious to finish the school day;
- may be restless and tentative;
- able to manipulate other people and situations;
- able to ask searching and perhaps provocative and challenging questions". (Bates & Munday, 2005, p. 13)

In order to include the potentially highly intelligent but in school lower performing and/or non-conformist young child in the identification procedure, student checklists have been developed (Nissen, 2013). An example of such a checklist includes items such as:

- "I often hide what I know"
- "School does not do enough for me"
- "I am a fast learner"

- “I am good at understanding complicated relationships”
- “I use another language than my age peers”
- “I am bored in school because it is too easy”
- “I am good at technology, e.g., Lego”. (Nissen, 2013, p.2, my translation)

The above mentioned checklists present features of gifted students in general and thus give little information about how to identify mathematical giftedness specifically. However, Table 1 gives an overview of five different models for identification of mathematical giftedness and/or creativity. These are chosen in order to show a variety of approaches used to identify mathematical giftedness and/or mathematical creativity in practice or for research purposes in different places. The models may be used in other countries and other settings than stated. The geographical reference is only an indication of where a model was initially developed. Although the models below include identification procedures that some may use in order to identify giftedness at a specific time of life of the student, many scholars suggest that the stimulation and identification of giftedness must continue over long periods of time in order to enable a proper nurturing as well as identification of the giftedness (e.g., Diezmann and Watters, 2002; Lohman, 2009; Renzulli, 2004; Sheffield, 1999).

3.3.2.1 The SMPY model

Much research done on mathematical giftedness stems from the work by Stanley and his colleagues (for an overview see Brody, 2009). In 1971, Julian Stanley developed the Study of Mathematically Precocious Youth (SMPY) at John Hopkins University (Brody & Stanley, 2005). The purpose of this program was to help students who exhibit exceptional reasoning ability in mathematics to achieve their full potential. Precocious youths are identified by their score on the mathematics part of the Scholastic Aptitude Test (SAT-M), a test that is used to determine success in college. The test consists of 60 multiple choice tasks that are to be covered in one hour. By using above-grade-level tests, the tests make it possible to discriminate among students who all would score high on in-grade-level tests. Stanley was, thus, one of the first to use domain-specific aptitude tests, and also a forerunner in the use of out-of-grade-level tests in the identification process of talent or giftedness (Wertheimer, 1999). These tests are given in annual talent searches. Mathematically gifted or, as put by the scholars, precocious students are those who reason exceptionally well in mathematics (Brody & Stanley, 2005). This has been interpreted as meaning the students who excel in school and university mathematics (Wertheimer, 1999).

Model	Central to identification	Definition	Identification procedure
Study of Mathematically Precocious Youth (SMPY) in the U.S. and other countries (Stanley, 1976)	Content specific knowledge	Students who reason exceptionally well in mathematics as measured by an aptitude test	High scores on the SAT-M test at an early age
Hamburger model of giftedness (Kiesswetter, 1985)	System of complex mathematical activities	Students who are capable of complex thinking processes in mathematics as described by Kiesswetter (1985)	High scores on Hamburger Test for Mathematical Giftedness
Mathematically promising students in Ireland (Vilkomir & O´Donoghue, 2009)	Cognitive abilities in mathematics	Mathematically gifted students are those who have a mathematical cast of mind and show signs of ability to generalize, and ability for flexibility and reversibility of mental processes as defined by Krutetskii (1976)	Observation of levels of abilities demonstrated in a problem solving situation
Mathematical giftedness as a model of problem solving acts in Israel (Leikin, Koichu & Berman, 2009)	Creativity, efficiency and task commitment	Students who manifest creativity, effectiveness and commitment in the acts of solving a mathematical problems in different ways and posing related problems.	Observation of high quality in problem solving acts
Framework for creative and imitative reasoning in Sweden (Lithner, 2008)	Creative mathematically founded reasoning	Students giving evidence of novelty, plausibility and mathematical foundation in their mathematical reasoning when solving mathematical tasks	Observation of novelty, plausibility and mathematical foundation of reasoning in task solving acts

Table 1. Five well used models of identification of mathematical giftedness and/or mathematical creativity.

In order for the students to succeed in the acceleration practice offered by SMPY, students need to be motivated and interested (Brody & Stanley, 2005). However, Brody & Stanley find it "unwise" to include affective traits as defining characteristics of giftedness as many of these traits can be altered by interventions. Instead, these traits like motivation and perseverance are facilitated and engendered by the appropriate learning opportunities that are matched to each student.

At SMPY, creativity is considered to be embedded in content areas. Students need to master deep and broad content knowledge in order to produce true creative achievements. By giving students the opportunity to learn more content knowledge in less time, the students are allowed to pursue creative work at a younger age. Thus, at SMPY, creativity is not seen as separate trait from precociousness, nor as a key component for the identification of gifted students (Brody & Stanley, 2005).

Research has shown that the domain-specific identification system developed by SMPY has great predictive validity (Subotnik et al., 2012). The mathematics and science programs built on domain-specific identification systems have also shown efficiency in the development of high talent in the specific domain (Subotnik et al., 2012).

The model of SMPY is, however, criticized for issues usually associated with the use of IQ tests in the identification procedures. It does not meet the requirements of equity, as there is an overrepresentation of white and Asian males. Nor does it allow for an identification of students who exhibit high motivation, creative or other mathematical abilities that are not measured through performance on aptitude tests (Wertheimer, 1999).

3.3.2.2 Hamburger model of giftedness

The talent search that built on the Hamburger model of giftedness started out in 1983 in the University of Hamburg in Western Germany (http://www.williamsterngesellschaft.de/). From 1985 it has been run by the William-Stern-Gesellschaft. The purpose of this talent search is to find mathematically gifted, 12-13 year old students and give them the opportunity to join an enrichment program outside school. The program is three hours long, once a week. The selection procedure has two tests. The first test is the same kind of SAT-M test used by SMPY. The second test is the Hamburger test for mathematical giftedness (HTMB). This test is based on Kiesswetter's (1985) study of complex mathematical thinking which includes six activities: "(1) Organizing materials", "(2) Recognizing patterns or rules", "(3) Changing the representation of the problem and recognizing patterns and rules in this new area", "(4) Comprehending and working with highly complex structures", "(5) Reversing and inverting processes", and "(6) Finding (constructing) related problems" (Kiesswetter, 1985, in Wiecerkowski, et al., 2000). The HTMB consists of seven openended problems which should be worked on in two hours. In the evaluation of students' solutions, correct answers as well as applications of the six components of Kiesswetter are counted for.

3.3.2.3 Mathematically promising students

According to Krutetskii (1976) mathematical abilities can only manifest in a mathematical activity. Thus, there is a need to observe the students engaging in mathematical problem solving activities in order to identify signs of mathematical giftedness. There have been several attempts to transfer the ideas of Krutetskii into the identification procedure of mathematical giftedness (Wertheimer, 1999). One of these is the model of mathematically promising students presented by Vilkomir and O´Donoghue (2009). The researchers' overall purpose is to identify and properly nurture mathematically gifted students in general classrooms. In order to find these students they suggest a model in which mathematically promising (a term adopted by the NCTM task force, see Sheffield et al., 1999) students are initially developed and, if they are seen as having a mathematical cast of mind (Krutetskii, 1976), thereafter identified as mathematically gifted. In line with the method used in Krutetskii's study, all students are given a set of specially designed problems which are meant to trigger the manifestation of certain abilities. Students are then observed as they work on these problems. By using a checklist of observables for different levels of mathematical abilities, teachers are expected to be able to evaluate the level of development of each mathematical ability for each student (Vilkomir & O´Donoghue, 2009). According to this model the abilities most closely linked to the demonstration of giftedness are the ability to generalize, and the abilities of flexibility and reversibility of mental processes.

3.3.2.4 Mathematical giftedness as a model of problem solving acts

Leikin, Koichu and Berman (2009) developed their model in order to evaluate how various types of ability grouping contributed to students' mathematical giftedness and creativity. The model is meant to characterize the mathematical creativity and giftedness of students. They base their model on Renzulli's (1978) three ring model. Above average general ability is measured by effectiveness in solving mathematical problems (Leikin et al., 2009). Effective problem solving means finding one or more solutions to a given problem and posing new solvable related problems. Mathematical creativity is defined by the components fluency, flexibility and originality (Leikin et al., 2009.). Whereas fluency is associated with the number of answers found to a problem, flexibility refers to the different approaches used in a problem. Originality is connected to finding an unusual idea as compared to many other individuals on the same task. The last component, task commitment, is related to completions of tasks, the capability to carry on work through failure, and the ability to perform highly, independently of outside monitoring. The presence or absence of different qualities of the components effectiveness, creativity and commitment in the problem solving act form the base for evaluation of an individual's mathematical giftedness. Leikin, Koichu and Berman excluded the co-cognitive traits, introduced in Renzulii's (2002) definition of giftedness, in their model as they were seen as impossible to consider when observing students solve problems.

3.3.2.5 Framework for creative and imitative reasoning

The framework for creative and imitative reasoning by Lithner (2008) was initially developed in order to understand the reasoning types required by text books tasks (Lithner, 2000a, 2000b, 2003, 2004). In his studies of students' reasoning of different tasks, Lithner distinguishes between imitative and creative reasoning. Whereas the former type is characterized by a complete recalling of an answer or a solution algorithm, the latter is associated with requirements of mathematically well founded and novel reasoning (Lithner, 2008; for further details about the framework see Paper IV). It should be noted that the model has never been used as a tool to identify mathematically creative students for certain provisions. It has been used to evaluate task requirements directly from studies of test tasks in upper secondary school (Palm, Boesen & Lithner, 2011) as well as at the university (Bergqvist, 2007). The main feature separating this framework from other structures of task analysis is that it focuses on the minimum requirements of solving a task. This means it may be used as a tool to examine requirements of tasks that are used for identification of creative reasoning. This is done in a study of the extent of creativity requirements in the identification of gifted students in the Swedish upper secondary school (Paper IV).

4. Methodology

Methodological considerations are about the choice, reflection, evaluation and justification of the methods used in research studies (Wellington, 2000). This chapter attends to these activities. In order to understand the underlying ideas behind the choice of the studies carried out in this thesis, read the section entitled Study Design and Methods. The sections following the Study Design and Methods section deal with the scientific quality of and the ethical consideration applied in this work. These issues have, to some extent, been discussed in Papers I-V, but are dealt with here in more detail. A reader interested in thoroughly looking over these matters is advised to read these sections as well. Other readers, after reading the Study Design and Methods, might skip to chapter 5.

4.1 Study Design and Methods

The following sections describe how the different studies presented in Papers II-V are meant to contribute to further our knowledge about the general research question presented in the introduction. Details about the methods of each of the studies are left out as these are presented in each of the papers. However, I begin by describing the method underpinning the literature review (Paper I) as this was not included in the report of that study.

4.1.1 Literature review

The purpose of the literature study was to give a broad, yet comprehensive, review of the Swedish status of topics found to be essential for the development of gifted education (Phillipson, Kaur, & Phillipson, 2003). In order to enable a thorough search for relevant information, the information gathering process was open but still very structured. Since one aim of the study was to enhance our understanding of the topic at issue, I followed the interpretative paradigm and did a narrative review (Bryman, 2012). This means that the information gathering process did not follow rigorous rules, but was more of an open process in which data from sources other than research were sometimes essential contributors. The initial literature search procedure was based on combining key terms in online databases. The key terms used were both in English and Swedish and read gift*, begåv*, talent*, cutting edge, spetsutbildning, high ability, creative, creativity, kreativ*, elit* and synonyms. Since I searched for information of various kinds I needed to use different databases or online resources. For literature on national polices, implementation in schools, teacher education and curriculum development I used the online databases at the Government Offices of Sweden, The Swedish National Agency for Education, and Ministry of Education and Research; for advocacy groups I used Mediearkivet (Archive of media); for national research I used the databases Diva, ERIC (ESSCO), Academic Search Elite, but also lists of publications of licentiate and doctoral thesis in mathematics education on the web pages of National Center for Mathematics Education (NCM). I also searched the national database LIBRIS which provides information on titles held by Swedish universities and research libraries.

The second aim of this literature review was to accumulate knowledge about current national research in gifted education in mathematics. Influenced by the systematic literature review practice (Bryman, 2012), I therefore added a quality requirement for the literature that was to be included. The requirement was that the research should at least be on the level of a licentiate thesis, or published in peer-reviewed journals. Other types of documents were included if they were considered to contribute to a more credible description of the four components (Phillipson, Kaur, & Phillipson, 2003) central to the literature review study.

In order to find additional information on teacher education, implementation in schools and, not least, about advocacy groups, I contacted different actors who I knew worked with the intention to serve gifted mat hematics students. These actors were, for example, head teachers at gifted programs in mathematics, mathematics professors at different Swedish universities working with mathematics competitions, math circles and special courses for non-university school students, as well as researchers who worked in this area. Like a snowball effect, this search for information made it possible to get to know more actors and gather more information.

The analysis process of the identification of specific issues that we need to develop further in Sweden was influenced by my, and my co-writer's, reading of developments done in other countries. The identified issues are not to be considered as including all possible areas to which we need to attend. Rather, they are a product of our evaluation of what needs to be done, given the development and outcomes presented in the review articles from other countries.

4.1.2 Studies of conceptions of mathematical giftedness

As earlier research has shown shortcomings in the teaching of gifted mathematics students (Persson, 2010; Pettersson, 2011) in the egalitarian context of Sweden, there is reason to believe that there are many teachers who have a limited conception of mathematical giftedness in our country. In the studies of teachers' conceptions of mathematical giftedness I therefore chose to concentrate on conceptions held by head teachers who specialize in teaching mathematics at higher grades. Results from such studies may have the potential to serve as grounds for further discussions on the possible need for and development of conceptions of mathematical giftedness within the egalitarian Swedish school system.

Head teachers' characterizations of mathematical giftedness and ways of detecting mathematical giftedness were investigated in three different studies – a survey study, an interview study and a study of mathematical requirements in admission test tasks used in the enrollment process for gifted programs in mathematics. The survey study (see Paper II) provided information about conceptions held within the larger community of head teachers in mathematics at upper secondary school. The target group taught students in inclusive settings. The interview study (see Paper III) complemented these findings by providing data on how mathematical giftedness was conceived by head teachers forming the educational practices at gifted programs in mathematics in upper secondary school. This study allowed for discussions on the potential for also developing characteristics of mathematical giftedness. The study of the admission test tasks allowed for consideration of conceptions of mathematical giftedness expressed in the mathematical requirements of the test tasks. This study focused on furthering our understanding of how the relationship between mathematical giftedness and mathematical creativity is conceived by upper secondary school teachers. More specifically, the question of whether mathematical creativity is a prerequisite for mathematical giftedness was the central issue. The findings from such a study would not only further the research field of gifted education, but they would also provide very relevant information for the question of how to detect mathematical giftedness in Swedish schools.

In all of these studies I follow Pehkonen and Hannula's (2004) interpretation of "conception". Conception is thus seen as the subset of belief, emphasizing the cognitive component of beliefs. Also, I consider only the conscious components of conceptions. This means that conceptions are principally viewed as "professed beliefs" (Speer, 2005), which can be expressed in writings and oral utterances.

4.1.3 Study of student representations in gifted education

Most researchers agree that giftedness is not based on factors such as race, ethnicity, gender or socioeconomic background (Mayer, 2005). Thus, the representation or enrollment of students with different demographic backgrounds in gifted education practices continues to be an important issue to consider (see e.g. Bianco et al., 2011; Borland & Wright, 2000; Donovan & CRos, 2002; Feldhusen & Jarwan, 2000; Ford, 1998; Freeman, 2003; McBee, 2010; Preckel et al., 2008; Sarouphim, 2002; Wallace & Eriksson, 2006; Yoon & Gentry, 2009). This was also the topic for the study presented in Paper V. By designing a comparison study, it was possible to evaluate the representation of students at gifted programs in mathematics in the Swedish upper secondary school regarding their gender, geographical origin and parents' educational levels. (For further descriptions of selection of students and choice of comparison groups see Paper V.)

The representation of students in gifted educational practices can, however, also be studied in terms of whether the participating students fulfill the requirements of the target group of the specific gifted programs, and whether participating students show signs of specific needs in the development of their mathematical giftedness. Therefore findings from Paper III will be included in the conclusion of and discussion on the second aim of this thesis.

4.2 Scientific quality

In conducting research we should always be aware of the quality aspects of our work. Each step of the research process is preceded by a decision about which path to take, or not to take. In each decision-making step we thus need to evaluate how the quality of our work is affected by our choice of path. In the following I will present how I have reflected upon the scientific quality of my work throughout my research, using the scientific quality criteria – relevance; coherence and validity; originality; competence, rigor and precision; predictability; openness, reproducibility and objectivity; and relatedness (Kilpatrick, 1993; Lester & Lambdin, 1998; Sierpinska, 1993).

4.2.1 Relevance

Given the recent introduction of national gifted programs in Sweden, and the growing concern among educators that we need to specifically attend to gifted students at the higher end of the ability scale (see Paper I; Persson, 1998, 2010; Pettersson, 2011) the work of this thesis was right on time – an important aspect of the relevance criteria (or the criteria of worthwhileness as it is called by Lester & Lambdin, 1998). Moreover, the background study of this thesis (Paper I) directly addresses one of the key indicators of relevance, as the very purpose of the literature review study was to inform the mathematics education practice about the current status of gifted education in mathematics in our country (Lester & Lambdin, 1998). The purpose of identifying areas of research that need to be considered in the development of the field also addresses this key indicator (Lester & Lambdin, 1998). This latter purpose relates to a second key indicator as well, as it informs the reader about areas which might generate good future research questions (Lester & Lambdin, 1998; Sierpinska, 1993).

In the studies attending to the first aim of this thesis, I wanted to emphasize the importance of teachers' involvement in the development of gifted education. The teachers are seen as vital participants in the advancement of the field. I believe that holding this view also might enhance the practical usefulness of this research, as stressed by Kilpartick (1993).

4.2.2 Coherence and validity

The criteria of coherence can be seen as encompassing the specific question of whether the research method has allowed us to investigate what we intended to investigate. This is sometimes referred to as the question of measurement validity (Bryman, 2012). However, the concept of validity also includes questions about the conclusions drawn from the study. These questions concern whether a claim that includes a causal relationship between variables holds (internal validity) and whether the result can be generalized beyond the context of the research study (external validity) (Bryman, 2012).

In hermeneutic research there is a trend to talk about credibility instead of validity (e.g., Lester & Lambdin, 1998). The credibility criterion has to do with the extent to which the readers of a report find the conclusions drawn in the report of the study believable. In order for the reader to judge whether a research finding is credible or not, the research report should include a clear description of the evidence at hand and the interpretations drawn. It should be possible for the reader to verify or refute the conclusions drawn. Hence, the criterion of openness (see below) is an integral aspect of the criterion of credibility (Lester & Lambdin, 1998).

When I attend to this latter criterion of scientific quality I choose to use the term validity, although this thesis includes hermeneutic as well as positivistic research.

4.2.2.1 Validity in the literature review

The literature review gave me information on the current status of gifted education in Sweden. Naturally, in an overview study there is a need to find a good balance between giving a detailed and in-depth description, and an account that is manageable to review. The final report of the study (Paper I) is thus a result of my, and my co-writer's, judgment in this aspect. Although the areas of development that are identified in this study are subjectively selected, they are all followed by a discussion on why they are considered as important. This makes it possible for the reader to evaluate for him/herself if the arguments hold. The substance of the review part of this study is most likely possible to repeat, if the study is done in future (see arguments in the section Reproducibility of findings in the literature review).

4.2.2.2 Validity in the survey study

The purpose of the study was to find out how head teachers in the Swedish upper secondary school characterized and detected mathematical giftedness. In order to attend to the possibility to generalize findings to the whole target group, that is, to allow for external validity, I decided to do a random sampling of head teachers. The crucial question was which method to use to get access to data from all over the country and, at the same time, get as high a measurement validity as possible.

The decision to collect data by using a free description questionnaire meant several conveniences for both me and the respondents. However, it also placed some constraints that might have affected the measurement validity of the study. For example, the request for written statements, instead of answers on a multiple choice questionnaire, may have affected the exhaustiveness of the head teachers' expressions. Though I cannot say for sure, it is, however, reasonable to believe that the head teachers included their most highly valued gifted characteristics and most common detection indicators in their statements. In such a case, the choice of method would not lower the validity of the most strongly held conceptions. Still, the method may affect the validity of results concerning the possible complexity of the head teachers' conceptions of giftedness.

Other limitations of this method relates to the precision in meaning of the survey questions as well as the answers. In the section Competence, rigor and precision I will describe how the latter issue was reflected upon in my study. The former issue, however, needs some attention. The very questions posed in the survey might have been interpreted differently by different respondents. However, one might say that that was partly the very purpose of the formulation of the question. The question was meant to allow head teachers to think of "their" type of "gifted" mathematics student. Therefore, in the research question I used a quite neutral formulation which excluded the labels or wordings that I thought were the most common ones used in school practice. This means that the results from this study are based on descriptions from head teachers who may be thinking of quite different students. Still, the compilation of the study results does give us information on the character traits and the detection indicators that are most valued in by head teachers in mathematics in upper secondary school.

In this study I investigated head teachers' professed beliefs. This meant that I only analyzed and drew conclusions from the utterances that the teachers actually made in the survey. Still, I need to acknowledge that during the process of categorization the beliefs expressed by the head teachers were, to some extent, attributed by me. This is further affected by the fact that I did not contact each of the head teachers' afterwards to find out if we had a shared understanding of the head teacher's written expressions. By interviewing the head teachers I might have gotten additional information that would have made the interpretations drawn from the study more valid. But this latter method also contains arbitrary elements and does not exclude the risk of not reaching a shared understanding (Speer, 2005). Instead I focused on achieving precision in meaning (see the section Competence, rigor and precision for further details) as I included a detailed description of the analysis framework and many examples of units of data in the report of the study. This makes it possible for the reader to evaluate the validity of this study. Since respondents were allowed to answer by free description they included character traits of different conceptual "levels". For example, the ability to solve hard mathematical problems might very well include a strong logical ability or a high ability to generalize.

4.2.2.3 Validity in the interview study

In the pilot study the interview guide was found to be helpful in the information gathering process. Thus, the same guide was used in the other interview studies. The flexibility of the interview structure made it possible for the interviewees to ask for clarification if they were uncertain of the meaning of any question posed. In the pilot study I only made one classroom observation. In order to get a better understanding of what head teachers meant in the interviews considering, for example, what students actually did in the classroom, the classroom atmosphere, the students' characteristics, the teacher's role in the classroom, etc., I found it necessary to do more classroom observations in the other studies. This also increased the likelihood of having shared meaning in the interviews held after the observations as we could relate to shared situations. For information on the actions taken to ensure precision in meaning, which affect the analysis validity, see the section Competence, rigor and precision.

In order to enhance confidence in the findings from the interview study I used complementary methods for gathering data. Findings from head teachers' statements were cRos-checked with findings from other data sources, including classroom observations, written documents and conversations with students and other teachers.

The results of the interview study are not meant to be generalized to the target group of head teachers at gifted programs in mathematics in Sweden. Rather they are viewed as examples of how conceptions of giftedness are conceived and enacted in different gifted programs in mathematics.

4.2.2.4 Validity in the study of admission tests

The framework (Lithner, 2008) used in the study of admission tests was chosen as it focused on the minimum requirements of creative mathematically founded reasoning (CR). As such, it met the demand for a tool to measure prerequisites of CR. The range of the findings of this study was, however, affected by the use of the mathematics textbook as the reference for the head teachers' reference for students' earlier experience. Although the Swedish mathematics education is very much governed by work in the textbook, this is a simplification of the measurement that needs to be acknowledged.

This study meant to capture one aspect of mathematical giftedness currently enacted in the Swedish school practice. Findings showed that the requirement for CR in the admission test tasks has increased over the years. Still this result is not necessarily to be generalized over time. The mutual development of research and practice in gifted education in Sweden has just begun, and discussions on how to identify potential for mathematical giftedness will continue. Future studies will show whether this process leads to a change regarding the prerequisite of CR for the identification of mathematical giftedness at the Swedish nationally established gifted programs, i.e. the cutting-edge programs, in mathematics.

4.2.2.5 Validity in the demography study

The validity measurement for this study is to be considered high as the measures of this study – gender, parent, origin and educational level – were closely aligned with both the research hypothesis and the definitions of the corresponding measures at Statistic Sweden.

The findings of this study give a snapshot of the current representations of gender, students with foreign background and students with highly educated parents in gifted programs in mathematics. As the study was not built on random sampling, the study findings cannot simply be generalized to all the gifted programs in mathematics in Sweden. Since economy did not allow for a longitudinal study, the results are not generalizable over time for each of the selected mathematical schools. To generate more valid knowledge of the demography of students in gifted programs in mathematics we need to repeat the study and compare results over time. However, this study confirms the low representation of women and students whose parents have low education levels in mathematically intensive education, as seen in several other investigations. Moreover, for the year the study was done, it also confirms head teachers common belief about high representations of foreign students (also see the section Reproducibility and objectivity of the demography study).

4.2.2.6 Validity of findings in this thesis

The conclusions drawn about the general research questions attending to the first aim of this thesis are built on three different studies. Together these studies give a rich picture of conceptions of mathematical giftedness held by head teachers working at mathematically intensive programs in upper secondary school. With one exception, there were different head teachers contributing to the data analyzed in these studies. Assuming that the validity of each single study holds, this triangulation of studies strengthen the validity of the fusion of the findings of this thesis.

The findings of the demography study which contribute to the conclusions drawn on the topic in the second aim of this thesis should be carefully interpreted as the study methods chosen do not allow for generalization over time (see above). The validity of results from the interview study that contributed to the conclusion for the second aim was strengthened by the triangulation of data. The value of the conclusions posed in this thesis might, however, not lay in the question of the generalizability of the single studies, but rather on the different perspectives raised in the discussion of student representation in gifted programs in mathematics.

4.2.3 Originality

Kilpatrick (1993) states that “Powerful research is research that is original” (p. 24) and gives examples of landmark studies in mathematics education. At the same time, we have learned that “thesis is awarded for original contributions to knowledge” (Phillips and Pugh, 2000, p. 63-64). Although, or perhaps due to the fact that, mathematics education research is a field that is very hard to review I can say that my thesis is not pioneering work in the astonishing way described by Kilpatrick.

However, Sekiguchi (1998), stresses that mathematical education research is socially and culturally situated, and that this nature of the field needs to be taken into account when evaluating the quality of research. Further, regardless of whether the specific research issues was paid international attention years ago, it can still be an important contribution to the domestic educational research field. This criterion of originality of research is presented by Phillips and Pugh (2000). Considering the situated nature of my research, and the more indulgent interpretation of what can be considered as original contributions in research presented by Phillips and Pugh (2000), I believe this thesis holds work that is original.

Moreover, this thesis includes results that, in some sense, are original. For example, although not surprising to the head teachers at the gifted programs studied, the result from the study of demography that there was no under-representation of foreign students was somehow unexpected for actors such as The Swedish National Agency of Education (2011). This result also goes against results found in similar studies abroad. Another example is taken from the study of the admission test tasks. In this study I argued, and provided clear evidence, that there is a need to develop the analysis framework in order to make more valid comparisons between different study results when using this framework to classify tasks that require creative mathematically founded reasoning in several ways.

Another area of originality of this thesis is the use of various research methods. The work includes five studies with different methods. There are both quantitative and qualitative studies. The sampling of informants or data is done in a variety of ways: purposive sampling, random sampling, snowball sampling and sampling of the whole target group. I also use different kinds of analysis techniques. Examples include content analysis, with and without a given framework, and tests of hypothesis. This also means attending to the scientific quality of studies in several ways.

4.2.4 Competence, rigor and precision

In order to conduct high quality research the investigator must have the skills to carry out data collection, analysis and interpretation techniques in a competent way (Lester & Lambdin, 1998). This requirement is closely related to the scientific quality criterion of rigor. Depending on the nature of the research, this criterion might mean precision of measurement and/or precision of meaning (Kilpatrick, 1993). The former relates to the rigorous research techniques including, for example, random selection, reliable measurement and use of statistics for estimating error (Kilpatrick, 1993). The latter meaning concerns the researcher's sensitivity to and care taken in the details of observations and the consideration of various interpretations of meaning in the analysis process (Kilpatrick, 1993).

This thesis is built on studies with five quite different ways of collecting, analyzing, and interpreting data. In the following sections I present how I addressed the criterion of competence, rigor and precision in each of my studies.

4.2.4.1 Competence, rigor and precision in the literature study

Although the literature review did not follow precise rules as in a systematic review, this study can still fulfill requirements of precision in the meaning. Rigor and precision have to do with the repeatability of study findings, and in the section Reproducibility and objectivity of the literature study I argue that this is the case for the presentation of the overview of the current status of gifted education in Sweden. Considering the personal judgments underpinning the selection of issues that need to be dealt with in order to develop gifted education in Sweden, there is, in some sense, less rigor in these study findings.

4.2.4.2 Competence, rigor and precision in the survey study

In the sampling process I addressed the criterion for rigor and precision in a traditional way. First, all sampling units, and hence all the head teachers in the target group, were included in the sampling frame. Second, simple random sampling was used to enable generalization of the results to the entire target population (Bryman, 2012). Third, the non-response was very low (15%) (see also the section Reproducibility of the survey study).

In the analysis, however, I strove for precision in meaning. This is shown in the decision to let the collected data form the basis of the categorization. For the purpose of our study, and since I did not find a suitable framework for categorization in the literature, I developed my own scheme of categories (see Paper II for details). In order to comply with both the breadth and the depth of the teachers' expressions, I decided to add four categories that might partially overlap or include some other category. This decision meant that the rigor and precision in the scheme of categorization decreased. On the other hand, it increased the possibility of precision in meaning. This includes avoiding ascribing meaning to statements that was not evident from the statement, and instead focusing on possible interpretations thereof. In the process of categorization, data from each head teacher was split into coherent units, and sorted into one category. This process was conducted by two researchers independently. As all data were similarly categorized by the two researchers, the inter-coder reliability (Bryman, 2012; Krippendorff, 2004) of this study is high. Finally, since there are several examples of units of data presented in the report of the study, it is possible for the reader to evaluate the precision of some of the work him/herself.

4.2.4.3 Competence, rigor and precision in the interview studies

In order to increase my competence in carrying out interviews and analyzing the information gathered, I did a pilot study at another gifted program in mathematics in Sweden. The set up was the same as in the interview studies. For two days I visited the head teacher at the gifted program and pursued interviews, did classroom observations and collected additional data such as documentation of class room exercises and tests. I transcribed the interview and did an analysis of the transcription as well.

In order to allow for the method of control of the analysis process as stressed by Kvale (1997), I have presented an example of how a part of the interview transcription has been analyzed. In order to increase the precision in meaning, and thus the credibility of my study results, I sent the descriptive dictum of the interviews to the head teachers and asked them to evaluate if I had described their thoughts in an accurate way. In this process head teachers had the opportunity to add information in order to make their ideas more clear in the resulting presentation.

4.2.4.4 Competence, rigor and precision in the study of admission tests

In the study of admission test tasks there were no sampling errors as all tests that had been used were collected. In the analysis process, I used a framework that has been successfully used by earlier researchers to discern the reasoning requirements in test tasks (Boesen, 2006). In order to learn how to use the framework, and how to document the findings for different task variables, I repeatedly had discussions with one of the researchers, PhD Jesper Boesen, who has the most experience of using it for classifying test tasks. As I was able to access parts of the documented analysis from earlier research, I could practice the work of categorization by comparing my analysis with the one performed by the earlier research group. My experience from this practice was then discussed with another researcher who was to categorize my data. The categorization of test tasks in my study was thus done by myself and another researcher independently. Nine months after the first categorization I did a second categorization of data. In order to test the reliability of the distribution of tasks that required CR, a Krippendorff's α-test was made (Krippendorff, 2004). This test is dependent on the variation of data. Since no admission tasks were expected to be categorized as MIR, which the results also showed, I excluded this variable when calculating the α- coefficient. Thus, I calculated a $_{\text{binary}}\alpha$-coefficient. The inter-coder reliability test score was .847 and the intra-coder reliability test score was .887. These scores are higher than the cutoff score of .800, indicating that the coding meets the required level of reliability in research (Krippendorff, 2004).

4.2.4.5 Competence, rigor and precision in the study of demography

In the study of demographics of students in gifted programs, I conducted a secondary analysis, that is, an analysis of data which I have not collected myself (Bryman, 2012). Instead, data was collected and compiled by Statistic Sweden. The reliability of the data in this national register is considered to be high. There are of course aspects that do influence the reliability of data, e.g. concerning the educational background of parents, in a negative way (see Paper V). However, there is no reason to believe that I would have been able to gather more reliable data myself. Since the personal identification numbers were deleted when I got the data from Statistic Sweden, it was also impossible for me to collect information about the parents whose educational background was unknown in the national register. Thus, the reliability of data is considered as high as it can be under the given circumstances.

Some readers may think that it might have been more interesting to use a more complex measure of socio-economic status (SES) in this study. However, after consulting Statistic Sweden I chose not to use this data as the national SES register had not been updated in several years. This would have made the data less reliable. As a result, by using the specific measure of “parents’ educational level”, it was possible to get more precise information on the demography of students in the gifted programs studied.

The analysis was conducted using statistical hypothesis testing. Since I considered data as randomly selected in time, it was possible to do a one-sided test of significance (see, e.g., Milton & Arnold, 2003). It is to be noted that I chose only to test three hypotheses, as multiple tests made on the same data affect the total level of significance of the study.

4.2.5 Predictability

Predictability is a scientific quality that stems from the positivistic field of research. Traditionally it has been a test of scientific theory (Kilpatrick, 1993). In hermeneutic research the criterion of predictability of research can be dealt with in another sense by asking “how well the inferences we draw from a body of research informed by a covering theory allows us to anticipate what will happen in a given [teaching or learning] situation” (Kilpatrick, 1993 p. 28). The work in this thesis is not the type of research that has as a goal to predict certain outcomes. Rather, it is a work where I try to interpret contexts and concepts in order to further our understanding of them. The results from my thesis might very well lead to changes in the studied contexts and concepts. This coincides with the overall purpose of this work as the development of gifted education in Sweden is dependent on such changes. However, the scope of this thesis does not encompass predicting what will happen.

4.2.6 Openness, reproducibility and objectivity

In order to attend to one facet of the criteria of openness I repeatedly have used the expression “I believe…” throughout this thesis. By making my personal assumptions clear the reader will know which glasses I looked through when conducting my investigations (Lester & Lambdin, 1998). Further, in the sections on reproducibility and objectivity of findings for below, I discuss how my biases and assumptions might have affected each study. This also implies attending to the criterion of objectivity (Kilpatrick, 1993). Even if hermeneutic research cannot reach absolute objectivity, I endeavored to rule out obvious bias in my work.

As I have carefully described the research methods and analysis techniques used in order to allow for the reader to scrutinize them, I have also considered the other facet of the criteria of openness (Lester & Lambdin, 1998). I have allowed for replication of the study design, which is a necessity to make it possible to reproduce the research (Kilpatrick, 1993). However, reproducibility is not only about making clear what has been done, but also about making clear what part of a study that others should reproduce in order to allow for the same study outcome (Sierpinska, 1993). Thus the criterion of reproducibility is an aspect of generalization (Kilpatrick, 1993). In the following I will comment this criterion, as well as the criterion on objectivity, in relation to each of my studies.

4.2.6.1 Reproducibility and objectivity of the literature review

In this thesis I have gotten the opportunity to elaborate on the methods used in the literature study. As the data sources are available to other researchers it is possible to reproduce and check the data underpinning the analysis. The interpretation of the data, however, might not necessarily be the same if done by other researchers. It could be that the “story” reported on would be presented in another tone than my report was. For example, if one values gifted education mainly because it benefits the individuals could influence the interpretation of the data differently from if one values gifted education because it contributes to society. In this thesis I have tried to display my standpoint and in that way fulfill the criterion of reproducibility and objectivity, as the interpretations of a specific finding are allowed to differ (Kilpatrick, 1993).

4.2.6.2 Reproducibility and objectivity of the survey study

In the survey study, data was collected according to the process of random selection. This means that the selected group of head teachers can be seen as a representative sample of the population of head teachers at the Natural Science program and the Technology program. This also means that the term “generalizability” is applicable for this study (Bryman, 2012). The response rate for the third specific research question was 85% and for the fourth question the corresponding proportion was 75%. It is to be noted that these response rates are rather high compared to many other survey studies. For example, a study of response rates of questionnaire-based articles in five journals in the area of management studies showed that the average (mean) response rate was 64.4% in 1975, 55.7% in 1985 and 52.2% in 1995 (Bryman, 2012).

In the report of this study (Paper II), other steps of the research process are described in detail as well. First, the evolvement of the framework for the categorization of data is presented. Next, the process of categorization is described and exemplified. Finally, the results are presented in frequency tables of the different categories, along with several examples of data. In order to strengthen the objectivity, each of two researchers first split the data into units and then coded it independently. Considering the high response rate and the careful description of the whole research process, including examples, it is likely that the data, as well as the main pattern of results, of this study are reproducible. It might be the case that in a replicate study there may be a need to introduce new categories into the framework. However, there is no reason to believe that the pattern of the most common answers will change in such a study in the near future. Pursuing the same study in ten years may not yield the same results though, as concepts might change and be affected by the increasing discussions on the issue. In such a replicated study the purpose might rather be to show the differences in responses to the same survey.

4.2.6.3 Reproducibility and objectivity of the interview study

It is not possible to replicate my interview study, not even in the first sense of the reproducibility criterion. Even if I gather the same head teachers, and talk about the same main issues again, the circumstances would be different than the first time. Accordingly, there is no generalization, in the positivistic sense, to be drawn from single interviews with head teachers of gifted programs in mathematics to all gifted programs in mathematics. The purpose of these interviews was to gather information of *current* aspects of conceptions of giftedness in the Swedish school system. Thus, the lack of reproducibility must not be a lack of scientific quality in this study.

4.2.6.4 Reproducibility and objectivity of the study of admission test tasks

The study of reasoning requirements of admission test tasks was an overall study. This means that I can say for sure (assuming that the study was done in line with other scientific criteria) that the results pertain to the body of all tasks that such tests used before 2013; no generalization is needed. The criterion of objectivity of the study was attended to as two researchers coded the work. Yet I cannot say that future studies will generate the same result. I can, however, identify some aspects that make this study reproducible. If, for example, the head teachers running the gifted programs continue to work in “the same way” as before, if they do not advance their knowledge about the textbook tasks that most of the Swedish students are taught in their mathematics studies in ninth grade, and if they do not consider the work presented in my study, there is a great possibility that the result will be generalizable also over time. On the other hand, since I am interested in the development of gifted education in our country, I make sure my research results are spread to all actors who have contributed to my work. This means that the creators of the admission test tasks are informed about how they might work in order to preserve or change the results of a replicated study. Thus, this study is considered as reproducible, but not necessarily generalizable over time.

4.2.6.5 Reproducibility and objectivity of the study of demography

The study of students' demography was a snapshot of the backgrounds of the current students at selected gifted programs. This means that the results cannot be generalized to all other gifted programs in mathematics in Sweden during that time, or for other times. Nevertheless, the results might not change much between different years in a specific gifted program. The reason I argue this is that the study was conducted at gifted programs, three of which had been running for over 15 years. During these years certain cultures have developed at each school which affects which students choose to apply to these programs. The general findings of my study also agreed with the preliminary results predicted by the head teachers running these three programs. This also holds for the result, which from an international perspective may perhaps be surprising, that native Swedes might be under represented at gifted programs in mathematics. In interviews head teachers said, for example:

> "It is more multicultural in the mathematics class than in the ordinary NV. I think it is due to the fact that there are immigrants with good mathematical background that come here and enroll to such education. I think that the parents of these students are highly educated. Some of the students with foreign background are probably born here and others have moved here."

> "Several of the students that have studied in other countries have come and been very good at mathematics."

The under representation of females, as well as of students with low parental educational levels, confirms many earlier studies and reports about their participation in various mathematically intensive education in Sweden (Paper V). This does not, of course, ensure that my study findings are reproducible. There is, however, a strong possibility that future study results will tend to point at the same pattern.

The objectivity, interpreted in the traditional sense, of this study is to be regarded as high. The research design fits within the positivistic research field, as data needs no interpretation and the analysis procedure is almost purely statistical. There are, of course, facets of the formulation of the hypotheses to be tested that may have been influenced by my personal choices. However, Paper V explains why the different hypotheses are chosen. Considering the information about gifted students' origin that was given to me by the head teachers at the gifted programs before the study, the carful reader might ask why I did not test the hypothesis: "Students with foreign backgrounds are over-represented". The main reason, which has dropped out of the report, is that the Swedish National Agency of Education had made a comment on the under-representation of students with foreign background in the cutting-edge programs (2011). I wanted to see if that was true for my selected programs. Another aspect that reveals my intention for objectivity is that the categories for different measures follow external, nationally stated, definitions.

4.2.7 Relatedness

This thesis does not have any focus on the very praxis of teaching. Moreover, it does not exclusively focus on mathematics. A reader might thus conclude that this work does not fulfill the criterion of relatedness to mathematics education as stated by Sierpinska (1993). However, Kilpatrick (1993) gives a somewhat extended definition of relatedness and underlines that we should be careful to reject research that does not make use of mathematics in an "integral and enlightened fashion" (p. 30) as unhelpful or uninformative. I argue that this thesis includes research that tells us something helpful about the teaching practice in gifted education, specifically within mathematics, both directly and indirectly.

The literature review study (Paper I) indirectly tells us something about the teaching practice for gifted mathematics students. It informs us, for example, of the organizational set-ups of current gifted mathematics programs, the gifted mathematics programs vulnerability regarding teachers' workload and knowledge transfer, the use of admission tests to gifted programs, and the lack of teacher education in gifted education. It also informs about the collected research on the lack of requirements of creative mathematical reasoning in textbook tasks, national tests and teacher made tests, as well as on how teachers support gifted students in compulsory school. In addition, it also enlightens us about what could be done, for example, formulating specific programs goals in order to develop our practice. Thus, the literature review gives us an important gathered picture of conditions of the teaching practice in gifted education in mathematics. Considering that mathematics is the only domain in which there have been gifted programs running for several years, and that the research in domain-specific areas within gifted education is mostly pursued in relation to the subject of mathematics, this literature review study might be useful also in the discussion on development of gifted practices in other domains. However, that does not exclude its importance for the insight into and development of the teaching practice in gifted education in mathematics.

Studies relating to the first aim of this thesis focus on the detection and characterization of mathematical giftedness. These concrete aspects of giftedness are closely related to the teaching practice. The findings are also discussed in relation to theories of giftedness in the specific domain of mathematics. Also, the very use of an admission test is specific for the gifted programs in mathematics as opposed to other national gifted programs. By focusing on these tests we thus focus on something that distinguishes these mathematics programs from the other gifted programs. In the study of the reasoning requirements of admission test tasks (Paper IV) there is also direct evidence of the educational practice that gifted students encounter in gifted programs in mathematics, as there are some examples of tasks discussed in detail in the report of the study. The studies pertaining to the second aim of this thesis thus fulfill the criterion of relatedness, as both Sierpinska (1993) and Kilpatrick suggested (1993).

I must admit, however, that the study of the demography among participants has no close relatedness to the teaching praxis in mathematics. However, it informs us about a situation that is specific to the gifted programs in mathematics as opposed to national gifted programs in other subjects – a situation which might affect the social outcomes of gifted education in mathematics. In relation the definition of educational research as defined by the Australian College of Education, this study should also be counted as research in (mathematics) education.

> "Educational research is the 'systematic and in-depth inquiry concerning the purposes of education; processes of teaching, learning and personal development; the work of educators; the recourses and organizational arrangements to support educational work; the policies and strategies to achieve educational objectives; and the social, political, cultural and economic outcomes of education'" (Strategic Review of Research in Education, 1992, p.8, in Leder, 1998).

4.3 Ethics

Research in mathematics education is never done in isolation from the surrounding world. This means that the research is influenced by the work of other researchers and, most often, is dependent on contributions from many other people in order to, for example, have financial support, get access to data, supervise the research, etc. To acknowledge the research contributions from different actors is but an obvious good ethical practice (Lester & Lambdin, 1998). In the extended Acknowledgements in the beginning of this thesis I credit the different people who, through interactions with me or my work in various ways, enabled me to do the work presented in this thesis. In this thesis I also continuously recognize the work of earlier researchers or contributors by referencing their work and discussing how it has influenced my research. This includes always trying to stay true to the meaning of earlier work. That was the main ethical concern for the literature study.

In social research we also need to consider ethical issues in order not to harm participants or the group these participants represent. In the following sections I present how the ethical principles – claim of information, informed consent, confidentiality and consequences of use of data (Bryman, 2012; Kvale, 1997) – are reflected upon in my empirical studies.

4.3.1 Ethics in the survey study of head teachers' conceptions of giftedness

Through e-mail, teachers in the survey study were informed about who I was and the purpose of the study. It was made clear that the participants responded out of free will. Teachers were welcomed to make contact with us if they had any further questions about the study. In order to be able to remind teachers who had not responded to the survey within a certain time, I kept track of which teachers had responded. This was done by ascribing a number to each teacher's school and noting which schools had responded. Thus, although teachers did not answer anonymously, their information was treated with confidentiality. In the final report of the study there was no possibility to identify either the participating teachers or schools.

4.3.2 Ethics in the interview study

In the interview study I tried to clearly present the aims of the study to the teachers. Their participation was by free will and there was an informed consent. In order to hinder the possibility to identify single teachers, I decided not to reveal the teachers' school or gender. In order to minimize the risk of identifying a person from their way of expressing themselves, I did not use direct quotes from teachers but only brief descriptive dictum (Kvale, 1997). I found this to be especially important in the case where one teacher describes personal characteristics, which might be interpreted as negative characteristics, of students participating in the gifted program. Considering the confidentiality of this study, I also asked the teacher for permission to publish this information. The teacher agreed to publish the text as presented in this thesis. For the record, I would like to add that the teacher in question never spoke about the students in a negative tone. Rather, s/he was very concerned about the students, acknowledged their giftedness and was very happy to teach them. S/he also emphasized the importance and value of these students having each other.

4.3.3 Ethics in the study of admission tests

The study of reasoning requirements of admission test tasks would not have been possible without the willingness of the head teachers of the cutting-edge programs. These head teachers let us take part in all of their admission tests, although most tests are kept secret to outsiders as some tasks are used every year. I thus found it important to deal with the tasks in such a way that we did not reveal task items that schools were to use the coming years. This secrecy of the admission test is the reason why no basic sample of the admission tests is presented in this thesis. For each of the task items shown in the study I have asked for teachers' permission to use them. As results showed, there was a gifted program in one school for which admission tests required a rather low proportion of global creative mathematically founded reasoning. Since this could be seen as a critique against this particular gifted program I chose not to specify the number of task items from each school as this could have revealed the identity of the specific program.

4.3.4 Ethics in the study of demography of students in gifted programs

In the study of students' demography I needed information about what is considered to be sensitive personal data. By Swedish law I had to apply to a regional ethical review board in order to get permission to get access to data. The application was approved. Following the commissions I presented in the application I sent written information about the study to the head teachers in mathematics at each selected gifted program. The text presented the purpose of the study, what personal data we wanted to access, where the students could find out about the data that was gathered about them and that study results were to be presented for larger groups of students. In addition, there was explicit information for how to contact us in order to be excluded from the study. If a student did not contact us it was interpreted as an informed consent. Students were also welcomed to contact us if they had further questions about the study. The head teachers informed their students orally about the study. The written information was posted on the ordinary billboard as well as the school billboard on the web. To ensure confidentiality, data was coded, kept and handled in line with the Personal Data Act and constitutions from the Data Inspection Board. Only two people have access to data and the coding system. In the final study report no data was presented at the school level. All of these measures minimized the possibility to identify single participants.

There was, however, another aspect of this study that needed to be reflected on from an ethical point of view. It was possible that study results were to reveal overrepresentation among groups who are considered as already privileged in some sense. Given our egalitarian culture, such a result might underpin discussions about gifted programs as being elitist. However, our overarching purpose with this study was to understand which groups take the opportunity to participate in the present form of gifted programs, and to open up for a discussion on how we could develop our gifted programs and complement them with other gifted education practices in order to serve as many gifted students as possible. Therefore, we decided to pursue this study although there might be a risk that other people might use the result in a sense that might affect the development of gifted education in our country in a negative way. Interestingly enough, results later showed that the biased recruitment of students with foreign background, which the Swedish National Agency for Education underlined in one of their comments on the evaluation of all the cutting-edge programs in Sweden

(National Agency of Education, 2011), was not valid when looking at cutting edge programs in mathematics.

It needs to be noted that this study would not have been possible without the help from mathematics teachers at gifted programs. Not only did they inform the students about the study, they also gathered personal identification numbers of all participating students.

5. Summary of Papers

This chapter briefly presents the findings from each paper.

5.1 Paper I

On track to gifted education in mathematics in Sweden

The aim of Paper I was to twofold. First, it was meant to further our knowledge about the encompassing current status of gifted education in mathematics in Sweden. Second, it was intended to identify areas of research and developmental work that need to be considered for the improvement of Sweden's national gifted education. The study was carried out by doing a literature review that focused on four components: a) national polices, b) advocacy groups, c) research, teacher education and curriculum development, and d) implementation in schools.

Results showed that Sweden has taken important initial steps in the development of gifted education. Six developmental areas, which may govern a further evolvement of the field, were identified and presented: (1) a lack of recognition of gifted students in the national policies, (2) a need to introduce structured ways to identify mathematically gifted students in order to properly provide for them, (3) a lack of gifted education within the Swedish teacher education, (4) a need to develop and strengthen the connection between research and implementation of gifted education, (5) a call for coordinating measures for and information on/to gifted students, and (6) a need to attend to the social and emotional needs of gifted students.

5.2 Paper II

Head teachers' conception of gifted students in mathematics in Swedish upper secondary school

The aim of this study was to explore teachers' conception of giftedness in mathematics. These conceptions were seen as fundamental to our understanding of who might be identified as gifted in mathematics. This study explored the variation and commonality of how Swedish head teachers in mathematics in upper secondary school characterize and detect mathematically gifted students using a survey. The randomly selected head teachers worked at the most mathematically intensive programs in the Swedish upper secondary school – the Natural Science program (NV) and the Technology program (TE). The analysis was done using content analysis (Berg, 2007) and sorting units of data into schemes of categories. These schemes were developed from the data in order to capture both the width and breadth of teachers' responses (see Paper II). Krutetskii's (1976) structure of mathematical abilities of the gifted mind was used as a base framework for the development of the schemes.

Results provided evidence of a variety of conceptions of mathematical giftedness held by the head teachers in this study. Findings showed that the head teachers most often characterized mathematically gifted students by their creativity, strong logical ability and keen motivation for mathematics. The head teachers detected the mathematically gifted students primarily by the students' own initiative for engaging in mathematics, their inclination to reason orally about mathematics and their successfulness on tests. The conclusion was that, as a group, head teachers have an awareness that there are different, interacting traits that need to be present in order for giftedness manifest. The emphasis on students' own initiative and motivation for doing mathematics does, however, raise a concern for the possibility of overlooking gifted students who are reluctant to do standardized school mathematics.

5.3 Paper III

Conceptions of mathematical giftedness according to head teachers at gifted programs in mathematics in Swedish upper secondary school

The purpose of this study was to further explore teachers' conceptions of mathematical giftedness in an egalitarian setting. These conceptions were seen as governing which character traits of mathematical giftedness teachers identify and seek to develop within their mathematics students. An earlier study (Paper II) investigated conceptions of giftedness held by head teachers in mathematics working at ordinary upper secondary schools. This study complemented the previous study by focusing on how mathematical giftedness was conceived by actors forming the gifted programs in mathematics in upper secondary school. Head teachers at the three longest running gifted programs in mathematics in Swedish upper secondary schools were purposively selected for a semi-structured interview study. The analysis was done by concentration of meaning (Kvale, 1997), where the central themes of the interview were distinguished.

Results showed that all head teachers in this study expressed a developed way of looking at realization of mathematical giftedness, when compared to theoretical models of giftedness (Lohman, 2009; Renzulli, 2005). Interest in and pleasure for engaging in mathematics, along with fluency in basic mathematical knowledge and skills were considered as signs of mathematical promise by all head teachers in this study. The characteristics most closely connected to mathematical giftedness were: problem solving ability, mathematical creativity and swiftness of thought. Most cognitive characteristics of mathematically gifted students were similarly described by the head teachers. However, there were diverse characterizations of students' personal and social character traits. Whereas some students were described as schoolhouse gifted and were, to a higher extent, driven by external rewards, others were described as lazy, resistant to traditional school mathematics and having a stronger internal motivation for engaging in mathematics they found interesting. This contradicts the result that students identified as gifted conform to, or have the responsibility to support, the positive social behavior in the class room, as was found in an earlier study done in the egalitarian context of Sweden (Persson, 1998). Findings suggest that in the identification of mathematical giftedness in all kinds of educational settings, teachers need to attend to the students' personal motive for engaging in mathematics.

5.4 Paper IV

Requirements for mathematically founded reasoning in the identification of mathematical giftedness in Sweden

The aim of this study was to investigate to what extent mathematical creativity is considered as a prerequisite for mathematical giftedness in school practice. The identification of mathematical giftedness is considered to be a vital step in order to give gifted students a proper provision for further development. In the findings presented in Paper II and Paper III head teachers in mathematics in upper secondary school emphasized the ability to be mathematically creative among the characteristics of their mathematically gifted students. From these studies it is not clear whether mathematical creativity is considered to be a prerequisite for mathematical giftedness. This is a question that also foils researchers seeking to understand the relationship between mathematical giftedness and mathematical creativity (e.g., Sriraman, 2005).

This study was carried out by examining the requirements for creative mathematically founded reasoning, CR, (Lithner, 2008) in the admission tests for the Swedish gifted programs, the so-called cutting-edge programs, in mathematics. All tests (n=16) used until 2012 were examined. The reasoning skills required to solve a test task were determined by considering the similarity between test tasks, reasoning types (Lithner, 2008) and the knowledge that students taking the admission tests can be expected to have.

Results showed that 64% of the admission test tasks that focused on problem solving required Local CR (4%) or Global CR (60%). Often these tasks required GCR in several aspects. The reasoning requirement differed widely between gifted programs. The percentage of tasks requiring CR for the different gifted programs was 82%, 75%, 60% and 26%. All the other tasks required algorithmic reasoning (AR). Results also showed that all admission tests in this study included tasks that required CR. A detailed examination of the tests from each gifted program indicated that head teachers at three out of four gifted programs are likely to consider mathematical creativity as a prerequisite for mathematical giftedness, as most of their tasks required CR. The conclusion was that mathematical creativity is treated as a prerequisite for giftedness in the cutting-edge programs in mathematics in the Swedish upper secondary school practice.

5.5 Paper V

Demographics of self-selected participants in mathematical tracks in Swedish upper secondary school

The second aim of this thesis was to understand the representation of different student groups in gifted educational practices in the Swedish upper secondary school. This study investigated the demography of students (n=147) participating in five purposively selected gifted programs in mathematics. The factors studied were gender, geographical origin and parents' educational level. Comparisons were made, through statistical tests of significance, with corresponding student distributions in the closely related Natural Science program (n = 1528).

Results confirmed that females are underrepresented in the gifted mathematics programs as compared to the comparison group ($p<.01$). Results also showed that parents of students in the gifted programs in mathematics are more highly educated than the parents of students in the comparison group ($p<.05$). However, findings do not show that students with foreign backgrounds are underrepresented in gifted programs. Instead, the results question whether Swedish students are underrepresented in these programs. The underrepresentation of females as well as the possible underrepresentation of Swedish students are results that are specific to the gifted programs *in mathematics*.

6. Conclusions

6.1 Conceptions of mathematical giftedness in an egalitarian context

In spite of the egalitarian context, head teachers in mathematics in the studies included in this thesis (Paper II and Paper III) described mathematically gifted students using several of the characteristics that are used by researchers in mathematical giftedness. Head teachers primarily mentioned cognitive attributes such as, for example, creative ability, problem solving ability and an ease for learning mathematics, and personal characteristics such as a keen motivation for engaging in mathematical work. All head teachers at the gifted programs in mathematics stressed the same three character traits – high interest and pleasure in mathematics, mathematical creativity and fluency in basic mathematical knowledge and skills – that need to be present in an individual in order for mathematical giftedness to be manifested. Although head teachers were not educated in gifted education research, this result showed that these experienced head teachers individually had an awareness of a conception of giftedness closely related to Renzulli's three-ring conception of giftedness (1986, 2005). Taken as a group, this also holds for the head teachers who worked in ordinary mathematically-intensive programs where mathematically gifted students appeared in inclusive settings.

Still, findings from Paper II and Paper III showed that most single head teachers, and also head teachers who have worked with mathematically gifted students for years, had a hard time describing the cognitive characteristics of mathematical giftedness in detail. In order to allow for the development of mathematical giftedness it is important to attend to a variety of mathematical abilities. By specifying the specific abilities we want our students to develop, the greater the chances are to create learning activities with a clear goal to further evolve these abilities. More important, by clearly expressing the characteristics of mathematical giftedness we may increase the chance to identify different aspects of mathematical giftedness in our students. In the studies of this thesis, there was a huge emphasis on mathematical creativity when head teachers described non-cognitive characteristics of mathematical giftedness. This suggests that mathematical creativity is a highly valued manifestation of mathematical giftedness. However, if we consider mathematical creativity as the highest level of mathematical proficiency (Sheffield, 2009), we also need to identify and properly nurture students who are mathematically promising but do not yet show signs of mathematical creativity. Further, according to the study presented in Paper IV there was a significant requirement for creative mathematically founded reasoning in the admission test tasks for the four nationally established gifted programs in mathematics. This was interpreted as a sign for head teachers' expression of mathematical creativity as necessary for mathematical giftedness. But, it is possible to challenge this interpretation by posing the question: Could it be that mathematical creativity, considered as the highest level of giftedness, is a rather easy characteristic to detect and is therefore used as a sign that there is no doubt of mathematical giftedness? If so, how does this affect the identification, and thus nurturing, of other high-ability mathematics students who may not yet have had the chance to develop this character trait? These students may also be considered as mathematically gifted (Sriraman, 2005). Thus, a more detailed description of various mathematical abilities, and their possible observable expressions when students are engaging in mathematical activities (see e.g., Leikin et al., 2009; Vilkomir & O'Donoghue, 2009), seems to be important knowledge for teachers to be aware of. Such knowledge may also inspire educators to offer new types of learning activities that attend to mathematical abilities not explicitly mentioned, and perhaps not yet acknowledged, by some teachers. Learning activities may, for example, attend to developing the memorization of generalized structures of mathematical problems. The mathematically gifted students' specific memorization has been emphasized in other research (Krutetskii, 1976) but was not explicitly mentioned by any teachers in my studies.

The mathematically gifted students' ability to easily and rapidly learn mathematics was only mentioned by a few head teachers working in the inclusive settings, but was stressed by all head teachers at the gifted programs in mathematics. This characteristic that takes the dynamic and developmental aspect of giftedness into consideration is emphasized by researchers of mathematical giftedness as well (ref; Krutetskii, 1976). In order to identify this characteristic students need to be invited to dynamic assessments (Kanevsky, 2000) where it is possible to observe the students' response to teaching. It is, however, to be noted that one of the head teachers at the gifted programs in mathematics attempted to attend to the possibility of identifying test writers' learning potential by introducing new definitions followed by a series of problems dealing with this definition. This is one way to test learning abilities in written tests. This type of tasks was recognized in the study of requirements of the admission tests (Paper IV). Still, in inclusive settings, and within gifted programs, research suggests (Kanevsky, 2000) it would be beneficial for students if their learning ability is continually assessed when students interact with teachers.

Written achievement tests and diagnostic tests are not considered as dynamic tests (Kanevsky, 2000). Still, high scores on such tests are used to identify mathematical giftedness by one third of the head teacher in the survey study (Paper II). In general diagnostic tests are used to test what students remember from earlier education. As such they may reflect the quality of the students' previous learning environment, and thus tell us little about the students' potential for profiting from future teaching. However, looking at students' solutions of mathematical problems, as suggested by one-fifth of the head teachers in the survey study, may give extra information on students' mathematical ability. This assumes that the problems are carefully chosen and that there is a list of highly valued competences, abilities or actions that might be looked for in the solutions. This is the procedure used to identify gifted mathematics students according to the Hamburger model of giftedness (http://www.williamsterngesellschaft.de/).

Findings from Paper III indicate that mathematically gifted students often lack the ability to communicate in written presentations. This result suggests actions to be taken. First, there is a need for compulsory school teachers to make all students practice mathematically written communication. Second, there is a need for complementary means to identify mathematical giftedness. Given some mathematically gifted students' predilection for oral communication due to their speed in switching over from one train of thought to another (Greenes, 1981; Krutetskii, 1976) oral assessments of mathematical reasoning should also be used. This was also a quite common way for head teachers in inclusive settings as well as for head teachers at gifted programs to detect mathematical giftedness (see Paper II and Paper III).

Contrary to earlier research of teachers' conceptions of high ability in the egalitarian context of Sweden (Persson, 1998), findings in Paper III showed that mathematically gifted students were not overly described by their positive classroom functioning and compliance. Instead, one head teacher at a gifted program described most of the mathematically gifted students' as lazy and individualistic. These characteristics were not seen as a prerequisite for mathematical giftedness, but as a character trait that needed to be attended to in the adjustment of teaching these students. There was a concern to engage students both in the course work stipulated by the national curricula but also in mathematical work attending to the students' personal interests. Some of the head teachers working in the inclusive setting also acknowledged gifted mathematics students reluctance to standardize teaching (Paper II). Students were described as hard to catch.

Almost half of the head teachers in the survey study detected mathematically gifted students by a student's own initiative for engaging in mathematics (Paper II). Although, a keen drive to pursue mathematics is a typical characteristic for mathematical giftedness (Krutetskii, 1976; Winner, 1996) there is a need to be careful in order not to overlook identification of some gifted mathematical students. Considering that there are (potentially) mathematically gifted students who are reluctant to participate in standardized teaching (see Paper II), as well as students with low self-esteem or low profile, there is a need to invite students to a variety of mathematical activities in order to allow for mathematical giftedness to be demonstrated by all potentially gifted students. This also holds for students who tend to have an inclination for creative-productive giftedness rather than schoolhouse giftedness, as defined by Renzulli (2005).

In sum, the identification of and characterization of mathematical giftedness are two topics that all mathematics teachers need to be educated about in order to properly nurture their students (Greenes & Mode, 1999). Taken together, as compared to theoretical models (Krutetskii, 1976; Renzulli, 1978; 2005), results showed that the head teachers collectively expressed nuanced characterizations of mathematical giftedness and the identification thereof. This was especially demonstrated by the head teachers at the gifted mathematics programs. Still, for individual head teachers, there is a need to further their knowledge about the different abilities contributing to manifestations of mathematical giftedness. The conceptions collectively expressed by the head teachers in this thesis could act as a foundation for further development of the individual teachers' knowledge. This would increase the possibility to identify and develop the mathematical abilities of an even greater number of mathematically promising students.

6.2 Representation of different groups of students in gifted programs in mathematics in the Swedish upper secondary school

Results from this thesis confirm that there are under representations of certain groups in gifted programs in mathematics in the Swedish upper secondary school. Based on the results from the single study presented in Paper V it might be tempting to conclude that gifted programs in mathematics preserve hierarchical structures in our society, as parents of students in these programs have higher educational levels than parents of students in comparison groups. Findings may also indicate that these gifted practices maintain the stereotypical image of mathematics as a male domain, since females are underrepresented. As such, gifted programs can be seen as a stratifying arena. However, considering that the purpose of gifted education is to attend to the individual needs and wants of the gifted themselves we have to include other aspects than only demographical factors in the equity discussion of representation of different student groups in gifted programs.

Including the results from the studies in Paper III it is clear that the students who participate in the gifted programs in mathematics show signs of (exceptional) mathematical giftedness. Some of the students are hard working, ambitious and high achieving in school. Most notably, however, are the findings that the Swedish gifted programs in mathematics also attract mathematically able students who are reluctant to participate in standardized teaching and care more about learning mathematics they find fascinating than focusing on getting the highest grade. In gifted programs in mathematics, students who are described as individualists find a place where they are accepted and find strength in each other. Thus, students who sometimes are overlooked in the ordinary school practice in egalitarian settings, due to their non-conformist and sometimes oppositional personalities (Persson, 1998), are encouraged to develop their individual mathematical giftedness in gifted programs in mathematics. From a social perspective of gifted education (Wertheimer, 1999) there could be a concern that we give to the ones who are already privileged. Such an advantage might be the socioeconomic status of the gifted student's family. However, I argue that many students in gifted programs in mathematics do not have it all. The study of students' characteristics in gifted mathematics programs (Paper III) clearly point out that gifted mathematical students have special needs, such as to learn how to communicate mathematics. These needs have not been met by their earlier education or their highly educated parents.

7. Discussion

7.1 Implications for the educational practice

A committed and well-prepared teaching staff is one of the most important factors in the successful implementation of gifted education (Dai, 2010). Since most education for gifted students in egalitarian nations such as Sweden is within inclusive settings, there is a need for every teacher to be knowledgeable about gifted education. As giftedness is a social construction (Persson, in press), the interpretation of its meaning needs to be understood in the context of the educational ideology to which one subscribes. Historically, Sweden has had an ideology, with a social activist's agenda, to reduce the hierarchic structure in our society. The main goal for defining and identifying gifted mathematics students in this ideology is to help all students to gain access to mathematical knowledge and career opportunities (Wertheimer, 1999). However, there is now an emphasis on the interest of the gifted students as well as on the development of the abilities of the gifted students in Swedish schools. Thus, the goal for defining and identifying mathematical giftedness is to help individual students to reach their unique potential. This implies that teachers need to also consider other aspects of giftedness than schoolhouse giftedness (Renzulli, 2005).

This thesis connects empirical findings of conceptions of mathematical giftedness held by teachers, with theoretical models of giftedness. Collectively, head teachers in this study noticed many of the character traits that are described in research. Hence, this thesis contributes to the credibility of these common character traits, both from the perspective of teachers and of researchers. The insights can therefore be used for educational purposes and pave the way for a deepened discussion about mathematical giftedness among different groups involved in the nurturing of giftedness and development of gifted education.

In the development of the practices of gifted education the concern for equity in the representation of students with different demographical factors should always be present. However, in doing so, we must allow for a challenge to our own preconceptions before drawing conclusions that might seem correct from a political and egalitarian point of view. This thesis contributes with aspects that can be used in discussions about what equity in gifted education means. Equity may be founded in demographical factors, as well as in the different cognitive and psychological needs of students developing giftedness. Based on the findings from this thesis, instead of asking the question of whether to have or not to have educational practices for the development of giftedness, I suggest we pose the question of how to develop and complement the current provisions in order to give more students access to a variety of activities. As for now, only

0.15% of the Swedish students in each peer group have access to nationally supported gifted provisions in mathematics (Paper I).

7.2 Suggestions for future research

In this work I assume that teachers' conceptions of mathematical giftedness affect their way of teaching. However, it is not clear how different conceptions are enacted in the class room. Knowledge about this relationship can be used to realize which aspects of their conceptions are most important in order to affect the classroom practice.

In Sweden there have been gifted programs in mathematics for a long time and there is little attention paid to them in research. The explicit definition of giftedness has been described as a keystone for the development of gifted programs (Feldhusen & Jarwan, 2000). The identification of giftedness, program goals and curriculum offerings should have a close relationship with the definition of giftedness. Research focusing on revealing, developing and evaluating the relationships between these elements would be valuable. Such research could, for example, start from the conceptions of giftedness expressed by head teachers in this thesis.

Moreover, the work in this thesis could serve as a basis for studies on whether and how different mathematical activities attend to the evolvement of certain desirable mathematical abilities.

One question raised in this thesis concerns the role of creativity in the identification if mathematical giftedness. Findings in this thesis suggest that mathematical creativity is used as a sign of mathematical giftedness. But the relationship between giftedness and creativity is not evident, neither in research nor for head teachers. Should mathematical creativity be seen as equivalent to giftedness? Or is creativity just a subset of giftedness, and hence the logic behind the extensive use of creativity is that it is a safe and easy way to detect gifted students? A way to investigate this relationship between mathematical creativity and mathematical giftedness could be to look at more examples of how teachers in practice identify mathematically gifted students. A systematic analysis of teachers' descriptions of character traits of promise in individuals who are not yet considered creative but in later stages are considered as gifted would be useful in the discussion about creativity and giftedness. This knowledge could also be used in the development of identification procedures of mathematical giftedness.

Researchers within gifted education emphasize the interrelationship between research and practice. The overarching common purpose is to give individuals the best opportunities to show and develop their (mathematical) promise or giftedness.

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