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Competition and Cooperation in Network Games*

Alexander Konovalov[†]

Abstract

We consider games where agents are embedded in a network of bilateral relationships and have multivariate strategy sets. Some components of their strategies correspond to individual activities, while the other strategic components are related to joint activities and interaction with the partners. We introduce several new equilibrium concepts that account for the possibility that players act competitively in individual components of their strategy but cooperate on the components corresponding to joint activity or collaboration. We apply these concepts to the R&D collaboration networks model where firms engage in bilateral joint projects with other firms. The analysis shows that investments are highest under bilateral cooperation and lowest under full cooperation because the spillovers associated to bilateral collaboration are bound to the partnership. This leads to welfare being maximized under bilateral collaboration when there are a few firms in the market and under non-cooperation in markets with many firms; full cooperation is never social welfare maximizing. Investigating the issue of endogenous network formation, we find that bilateral cooperation increases (lowers) the profits of more (less) connected firms. However, this does not always lead to a denser stable network of R&D collaboration under bilateral cooperation.

JEL classification: L13, L14, L22, O31, O32

Key words: network games, bilateral cooperation, hybrid equilibrium, R&D collaboration networks

1 Introduction

For long strategic interactions have been studied without taking into account the importance of the structure of relationships between players. It was simply assumed that every player

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interacts with every other player. The introduction of network games (see Goyal (2007), Jackson (2008), Jackson and Zenou (2013) for reviews of the literature) has made the structure of social relationships within the reach of formal game-theoretical analysis. This is an important but relatively recent development and the analyses are sometimes still based on somewhat tenuous assumptions. One is the rather arbitrary assumption about the behavioral mode of the players in different stages of the interaction, which is assumed to be either cooperative or non-cooperative. In particular, in the *network formation literature* it is usually assumed that players cooperate on forming links (long-run decisions) and compete in subsequent stages of the game, e.g., Goyal and Moraga-González (2001), Deroian and Gannon (2006), and Goyal, Konovalov, and Moraga-González (2008) among others. The opposite situation in which long-run decisions are made competitively but with the understanding that collusion will follow in subsequent stages of the game is typically studied in the *literature on partial collusion (semi-collusion)*, see Steen and Sorgard (2009) for an overview of models and results. In this literature the long-run variables which are chosen non-cooperatively by firms before collusion are the locations in the product space in Friedman and Thisse (1993), capacity in Osborne and Pitchik (1987) and Davidson and Deneckere (1990), R&D expenditures in Fershtman and Gandal (1994) and Brod and Shivakumar (1999), and pricing policy in Colombo (2011). Both assumptions, non-cooperative behavior followed by fully cooperative behavior and the inverse sequence of behavioral modes, seem questionable for many important applications. On the other hand, assuming a cooperative mode or a non-cooperative mode of behavior throughout a multistage game is unrealistic in many environments as well.

But even in simultaneous move games, assuming the same behavioral mode across different tasks may be inappropriate. Consider, for instance, a multi-product firm which produces a standardized vitamin (and competes with other suppliers of the vitamin in the corresponding product market) and (simultaneously) decides upon a research joint venture on a new beta blocker with one of its competitors in the vitamin market. Arguably, antitrust legislation will force the firms into non-cooperative behavior in the vitamin market, but the corresponding R&D units are expected to collaborate for the new beta blocker. For instance, the two firms could mutually agree upon a contract that fixes the task and profit division and is on the Pareto frontier, i.e. no other contract between the firms can make one of the contractual partners better off without making the other one worse off. In this sense, firms exhibit cooperative behavior in the design of the research joint venture.

In the present paper we introduce a framework that allows the behavioral mode (cooperative or non-cooperative) to depend on the type of activity. For that purpose it is assumed that each player engages in multiple activities. Some activities are employed in (bilateral) partnerships and these are decided on cooperatively; other activities are employed in isolation and made in non-cooperative fashion. Broadly speaking, one can say that in our approach the behavioral mode depends on the network structure in which that activity is embedded. We argue that this approach is relevant for a number of applications (see Sections 3 and 5 for examples) and that it opens up several intriguing theoretical and experimental questions and hypotheses.

Taking the literature on R&D collaboration networks as an example, Goyal, Konovalov, and Moraga-González (2008; in what follows - GKM) is one of the first attempts to study a network game with multi-dimensional link specific strategies. In their model individual firms carry out in-house research on core activities and undertake bilateral joint projects on non-core activities with other firms. In this context, a link between firms i and j corresponds to a joint research project between these firms. If $N_i(g)$ is the set of firm i's neighbors in the R&D collaboration network g, then firm i's strategy is a vector of investment levels $x_i = (x_{ii}, (x_{ij})_{j \in N_i(g)})$, where x_{ii} is the firm's investment in its in-house project, and x_{ij} is its investment in the joint project with firm j. These investments together with firm i's collaborators' efforts x_{ji}, $j \in N_i(g)$ reduce firm i's operating costs (in another interpretation, raise the demand for i's product due to its increased quality). Given the costs, firms earn profits competing in prices or quantities in the market with differentiated goods. In our R&D application, we take the GKM model as our working horse. The focus of our study is on *bilateral cooperation*, a novel form of cooperative decision-making where a pair of firms jointly chooses investment levels for their common project to maximize their joint profit taking as given investments in other projects. Another important question we address is the issue of endogenous network formation: how the process of network formation changes if agents foresee higher (or lower) degrees of cooperation in subsequent stages and how playing hybrid equilibrium in later stages affects the architecture and welfare properties of stable networks.

Other works studying network games with multi-dimensional link specific strategies include the papers by Bloch and Dutta (2009) and Rogers (2005) on network formation with endogenous link quality, Brueckner (2006) and and Jackson, Rodriguez-Barraquer, and Tan (2012) on networks of friendship and favor exchange, Goyal, van der Leij, and Moraga-González (2006) on co-authorship networks, Bloch and Jackson (2007) on network formation with transfers, Franke and Öztürk (2009) on conflict networks, and Elliot, Golub, and Jackson (2013) on financial networks. Also related are the recent papers on bilateral bargaining in trading networks, eg. Elliot (2013) and Konovalov and Stennek (2013). An earlier paper, where agents can at an additional cost acquire the ability to choose different actions with different partners, is Goyal and Janssen (1997). Our intention is to contribute to this strand of research, which is still at its early stage.

The rest of the paper is organized as follows. In the next section we present our basic framework and our new notions of hybrid equilibria. In Section 3, an R&D collaboration network model is introduced and our results for this case are presented. Section 4 discusses the issue of endogenous network formation. Section 5 contains the discussion of other network games and possible applications of our approach. Section 6 briefly concludes.

2 General Model

In this section we propose a context-free framework and several new equilibrium concepts that allow for *simultaneous* cooperation and competition at the same stage of a game.[1] As mentioned in the introduction, this is in contrast to earlier literature which typically has studied sequential cooperation and competition at different stages of multi-stage games. Two assumptions about players' strategy sets are crucial for our analysis. First, we assume

[1]The idea that cooperation and competition can coexist is not new. The buzz word "co-opetition" was introduced into the literature by Brandenburger and Nalebuff (1996), who wrote in their book: "Business is cooperation when it comes to creating a pie and competition when it comes to dividing it up. In other words, business is War *and* Peace. But it's not Tolstoy - endless cycles of war followed by peace followed by war. It's simultaneously war and peace".

that strategies are multi-dimensional. Budget decisions, distribution of time across different projects, and pricing decisions by multi-product firms are examples of multivariate choice situations. Second, we assume that agents are embedded in a network of bilateral relationships, and that some components of their strategies correspond to individual activities, while others are related to joint activities and interaction with partners. Our main idea is that cooperation in the latter is more likely. For instance, due to the fact that individual activities are not directly observable by the neighbors, they may face difficulties assessing "how nice" the player is towards them in the "individual" component of her strategy set. On the other hand, the level of collaborative activity may be directly observable by the partner, being too selfish here seems less likely since such a course of action may result in punishment. Experimental evidence indicates that the kind and intensity of the externality associated with an activity is another factor (beyond observability) that is important for cooperation. For instance, Suetens (2005) found that non-binding cheap talk between two players engaged in joint R&D activity is sufficient to move levels of investments towards R&D cooperation. By contrast, if players conduct only in-house research and there are no spillovers, binding contracts are needed for the decisions to deviate from the Nash level towards the cooperative level.

To introduce our formal framework let $N = \{1, \ldots, n\}$ be the set of players. Consider a network (an undirected graph) g with n nodes, where each node i corresponds to a player. As before, whenever $ij \in g$, this means that there exists a link between players i and j. Let $N_i(g) = \{j \in N | ij \in g\}$ be the neighborhood of i, that is, the set of agents with which player i is linked in network g. A game in normal form $G(g)$ is called a *network game*, if the strategy set X_i of player i can be represented as a product of $|N_i(g)| + 1$ components:

$$X_i = X_{ii} \times \prod_{j \in N_i(g)} X_{ij},$$

where elements of X_{ii} are called *internal strategies* of i, and elements of $\prod_{j \in N_i(g)} X_{ij}$ are called *external* or *collaborative strategies* of i.

Let x_i be an element of X_i. By $x \in X = \prod_{i \in N} X_i$ we denote a strategy profile $x = (x_1, \ldots, x_n)$ as well as a corresponding outcome of the game. Furthermore, let x_{-i}, x_{-ii}, and $x_{-ij,ji}$ stand for vectors consisting of all components of x except those of x_i, x_{ii}, as well as x_{ij} and x_{ji}, respectively. Let $\pi_i : X \to I\!R$ be the payoff function of player i. We do not make the assumption that π_i depends only on the actions of i and i's neighbors, since it is often violated in applications, as, for instance, in the R&D application studied in this paper (where investments of non-neighbor firms have an indirect effect on the profit of a given firm via their impact on the product-market behavior of firms that are directly affected by the investments).[2]

Definition 1 *A strategy profile x is called a* hybrid (or co-opetitive) equilibrium *(HE) of the network game $G(g)$ if the following two conditions are satisfied:*

(i) *For any $i \in N$ there is no x'_{ii} such that $\pi_i(x'_{ii}, x_{-ii}) > \pi_i(x)$,*

(ii) *For any $ij \in g$ there is no x'_{ij} and x'_{ji} such that $\pi_i(x'_{ij}, x'_{ji}, x_{-ij,ji}) \geq \pi_i(x)$ and $\pi_j(x'_{ij}, x'_{ji}, x_{-ij,ji}) \geq \pi_j(x)$ with at least one inequality strict.*

If condition (ii) is replaced by $(ii)'$:

$$\pi_i(x) + \pi_j(x) = \max_{x'_{ij}, x'_{ji}} \pi_i(x'_{ij}, x'_{ji}, x_{-ij,ji}) + \pi_j(x'_{ij}, x'_{ji}, x_{-ij,ji}),$$

we talk about a *hybrid equilibrium with transfers* (HET). Obviously, any HET is HE. Since it is likely that in a continuous setting there exist multiple hybrid equilibria, we regard hybrid equilibrium with transfers as an important refinement of the proposed concept.

Despite the fact that networks are ubiquitous, other forms of collaborative structures can easily be observed in real life. Joint research ventures that involve more than two firms and scientific papers with more than two coauthors are among the obvious examples.[3] The concept of hybrid equilibrium can easily be generalized to account for such situations. The main idea is the same as in the generalization of the classic pairwise stability concept introduced by Jackson and van der Nouweland (2005). Let us call a subset $\mathcal{S}$ of 2^N a *collaboration structure (or generalized network)* if any $s \in \mathcal{S}$ has at leat two elements. The sets in a

[2]Our definition of a network game is thus different from the one given in Galeotti *et al.* (2010).

[3]For example, among the 63 papers published in 2009 in *Econometrica*, 23 (or 37%) are single-authored, 28 (44%) have two coauthors, and 12 (19%) have three or more coauthors.

collaboration structure are not necessarily disjoint, thus the property of non-exclusiveness still holds. If any set from $\mathcal{S}$ has exactly two elements, then $\mathcal{S}$ is a network. A game $G(\mathcal{S})$ is a *game on collaboration structure* $\mathcal{S}$ if strategy set X_i of player i is the product

$$X_i = X_{ii} \times \prod_{s \in \mathcal{S}, i \in s} X_{is}.$$

Definition 2 *A strategy profile x is called a* generalized hybrid equilibrium *(GHE) of the game on collaboration structure $G(g)$ if the following two conditions are satisfied:*

(i) For any $i \in N$ there is no x'_{ii} such that $\pi_i(x'_{ii}, x_{-ii}) > \pi_i(x)$,

(ii) For any $s \in \mathcal{S}$ there is no $x'_s = (x'_{is})_{i \in s}$, such that $\pi_i(x'_s, x_{-s}) \geq \pi_i(x)$ for any $i \in s$ with at least one of these inequalities strict.

In this definition members of coalition s are allowed to change external components of their strategies x_{is} in order to improve upon a non-cooperative outcome.

Finally, consider numbers $\alpha_{ij} \in [0, 1]$, $ij \in g$. Define a *trust equilibrium* as an outcome x satisfying the conditions

(i) For any $i \in N$ there is no x'_{ii} such that $\pi_i(x'_{ii}, x_{-ii}) > \pi_i(x)$,

$(ii)''$ $\pi_i(x) + \alpha_{ij}\pi_j(x) = \max_{x'_{ij}} \pi_i(x'_{ij}, x_{-ij}) + \alpha_{ij}\pi_j(x'_{ij}, x_{-ij})$.

The coefficient α_{ij} can be interpreted as a level of trust of agent i towards agent j. Obviously, the case when all coefficients α_{ij} are zero corresponds to a Nash equilibrium; while the case when all α_{ij} are equal to one corresponds to a hybrid equilibrium with transfers.

To motivate our new concepts and to show that they yield interesting results in important applications we now turn to the example of R&D networks.

3 R&D Networks

In the last three decades there has been a significant increase in the number of R&D collaboration agreements between firms. This number rose sharply in the 1990's and has remained high in recent years.[4] There are three features of these relationships worth to highlight: collaborations are often (i) *bilateral* (Delapierre and Mytelka, 1998; Mody, 1993), so investments are *link-specific*; (ii) *non-exclusive* (Milgrom and Roberts, 1992) so a firm i may have a collaboration with firm j, but also a collaboration with firm ℓ; and (iii) *non-transitive*, that is, firms i and j may have a collaboration, as well as j and ℓ, but this does not imply that firms i and ℓ also collaborate. These features imply that firms nowadays are embedded in networks of bilateral collaborations.

Since a typical firm carries out several joint research projects with distinct partners and since these projects are of diverse content (see for example the research profile of leading firms in the pharmaceutical sector such as Bayer, Bristol-Myers-Squibb, and Glaxo-Smith-Kline), several authors in the business strategy literature have argued that firms are moving towards a novel organizational form in which divisions of different firms have very close ties, sometimes closer than the ties they have with other divisions within their own firms (see e.g., Delapierre and Mytelka, 1998; and Podolny and Page, 1998). In this paper we provide a natural framework to model this emergent organizational structure.

In our model, prior to competing in the market, firms can engage in bilateral research projects with other firms to lower their costs of production. The bilateral nature of the projects gives raise to a novel form of cooperative decision making which we refer to as *bilateral cooperation*: every pair of firms involved in a collaborative research project chooses the aggregate level of R&D investment of the common project so as to maximize the joint profit of the two research partners, while taking as given investments in other projects. The levels of in-house R&D investments are chosen non-cooperatively. In other words, bilateral cooperation results in firms playing a *hybrid equilibrium with transfers* in the R&D stage of the game.

A comparison of R&D investments under different decision making regimes yields the following ranking: *all R&D investments under bilateral cooperation are higher than investments with non-cooperation, which are in turn higher than under full cooperation.* This result arises due to two reasons: one, because bilateral cooperation allows for an internalization of the

[4] See Hagedoorn (2002) for an analysis of general trends and patterns in inter-firm R&D partnering since the early 1960s.

pairwise externalities associated with collaborative research, research efforts are greater under bilateral cooperation than under non-cooperation; and two, because bilateral collaboration limits the spillovers to the partnership level, research investments under full cooperation are lower than under non-cooperation. We also find that, when there are just a few firms in the market, welfare is maximized by firms which engage in bilateral cooperation while if there are many non-cooperative decision making yields the highest welfare level.

Our results contribute to the literature on cooperative R&D in oligopoly. Research activity creates spillover externalities which non-cooperative firms do not take into account. The scope of cooperative decision making among firms in resolving the resulting incentive problem has been explored in great detail by an extensive literature (see e.g., d'Aspremont and Jacquemin, 1988; De Bondt and Veugelers, 1991; Kamien, Muller and Zang, 1992; Leahy and Neary, 1997; Suzumura, 1992; Amir et al., 2003; and Zikos, 2010). In this literature models start with the assumption that either firms work independently or all together. Thus the main contribution of our paper to this strand of work is a model where bilateral collaboration can be explicitly accounted for. Our analysis suggests that the modeling innovation has serious implications for firm behavior and appropriate policies. An important result in the earlier work is that R&D investments by non-cooperative firms are lower than investments by cooperative firms. By contrast, we find the reverse ranking of R&D investments. The reason for this difference lies in the fact that collaboration is bilateral in our model, as opposed to the earlier work where, typically, firms engage in an industry-wide collaborative agreement. This also has serious implications for social welfare. In earlier work welfare is maximized under cooperation, while we find that non-cooperative firms attain higher welfare if there are many firms. In our view, this reversal of welfare ranking is important from a policy perspective as it suggests that cooperation should be disallowed when collaboration is bilateral.

We envisage a market where firms produce complex products involving a range of technologies and skills that are difficult to master all individually. By pooling skills/assets with partners, firms open avenues of research to reduce their costs of production. Our model is inspired by the GKM paper. While the focus in that paper is on the interplay between in-house independent research and collaborative research and features non-cooperative firms, here we focus on the advantages and disadvantages of cooperation.

Let $N = \{1, 2, \ldots, n\}$ be the set of ex-ante identical firms. We shall assume that $n \geq 2$. Firms play a two-stage game. In the first stage, each firm allocates resources to its private R&D project and to every joint R&D project available to the firm. These decisions determine the *effective* costs of production of every firm. In stage two, firms engage in Cournot competition[5]. The set of potential research projects a firm can engage in with other firms is represented by an (undirected) network g with n nodes. Whenever $ij \in g$, this means that there exists a potential *collaborative research project* between firms i and j;[6] this project is activated if firms i and j pool resources in the research trajectory ij. Let $N_i(g) = \{j \in N | ij \in g\}$ be the set of firms with which firm i has a joint project in network g. Our work will focus on symmetric structures, i.e., $|N_i(g)| = |N_j(g)| = k$; we shall refer to k as the *degree of collaborative activity.*

Given the set of potential collaboration projects g, a firm i chooses the amount of money $x_{ij} \in I\!R_+$ to be spent on the joint project with firm j, $j \in N_i(g)$, as well as the level of the R&D expenditure in its own core (in-house) project $x_{ii} \in I\!R_+$. Let $x_i = (x_{ii}, (x_{ij})_{j \in N_i(g)})$ be the research strategy of firm i and $x = (x_i)_{i \in N}$ denote a strategy profile for the n firms.

We assume that the returns to R&D investment in in-house research are given by the function $f : I\!R_+ \to I\!R_+$, while every collaborative project yields returns given by the function $h : I\!R_+^2 \to I\!R_+$. Let f and h be strictly increasing, strictly concave, and twice differentiable on $I\!R_+^2$. In addition, we impose some boundary conditions on f and h to guarantee the existence of an interior symmetric equilibrium: $f(0) = 0$, $h(0, 0) = 0$, the derivative of f and the partial derivatives of h at zero are sufficiently large and, finally, functions f and h_d are bounded for large investment levels.

Given the research strategy x_i of a firm i, this firm obtains a research output $f(x_{ii})$ from

[5]There is evidence that R&D cooperation facilitates product market collusion, see Suetens (2008). In the current paper we assume that cooperation in the market stage is disallowed, for example, due to legal restrictions. In the subsequent work, we plan to relax this assumption.

[6]We normalize the number of projects any pair of firms can undertake as well as the number of in-house projects per firm to one.

its in-house project, while it obtains a research outcome $h(x_{ij}, x_{ji})$ from the research project with firm j, $j \in N_i(g)$. Given the outcomes of all research projects of a firm i, its unit cost of production is given as follows:

$$c_i(g, x) = \bar{c} - f(x_{ii}) - \sum_{j \in N_i(g)} h(x_{ij}, x_{ji}). \tag{1}$$

The additive structure of this cost-reduction formulation reflects the fact that distinct joint projects go along different research trajectories and there are no technological spillovers across them. The absence of technological spillovers is motivated by the growing complexity of many products and the distinct technologies that are involved in different aspects of the product.[7]

Moreover, since f and h are concave, the additive structure provides an incentive for spreading efforts across different projects and therefore an increase in the number of projects embodies efficiency gains. The role of this additive structure is clarified in GKM by considering the case of a single research trajectory. In such a case, all efforts of a firm enter the same R&D production function and a firm has an incentive to engage in independent research only.

Given the costs $c_i(g, x)$, firms operate in the market by choosing quantities $(q_i(g, x))_{i \in N}$. The inverse demand function is given by $p = A - \sum_{i \in N} q_i(g, x)$. Then, the equilibrium quantity of firm i is

$$q_i(g, x) = \frac{A - nc_i(g, x) + \sum_{j \neq i} c_j(g, x)}{n + 1}, \tag{2}$$

and the profits of the Cournot competitors are given by

$$\pi_i(g, x) = (q_i(g, x))^2 - \sum_{j \in N_i(g)} x_{ij} - x_{ii}. \tag{3}$$

[7]For example, in the automobile industry investments in improving lamp illumination or in reducing lamp costs should have no influence on the cost of producing engine starting procedures, nor on the cost of producing collision safety devices like airbags. GKM shows that the main insights from this model are robust to the inclusion of moderate spillovers across projects.

We shall compare social desirability of equilibria under different decision making regimes. For this comparison we shall employ the usual measure of social welfare:

$$W(g,x) = \sum_{i \in N} \pi_i(g,x) + \frac{1}{2} Q(g,x)^2, \tag{4}$$

where $Q(g,x) = \sum_{i \in N} q_i(g,x)$.

Given the network g and other firms' R&D investments, firm i chooses $x_{ii} \geq 0$, $x_{ij} \geq 0$, $j \in N_i(g)$ to maximize its profits:

$$\begin{aligned} \max \quad & \pi_i(g,x) \\ \text{s.t.} \quad & x_{ii} \geq 0, \\ & x_{ij} \geq 0, \quad j \in N_i(g). \end{aligned} \tag{5}$$

The set of first-order conditions for the interior solution of this problem is

$$\frac{\partial \pi_i(g,x)}{\partial x_{ii}} = \frac{2q_i(g,x)n}{(n+1)} \frac{\partial f(x_{ii})}{\partial x_{ii}} - 1 = 0, \tag{6}$$

and

$$\frac{\partial \pi_i(g,x)}{\partial x_{ij}} = \frac{2q_i(g,x)(n-1)}{(n+1)} \frac{\partial h(x_{ij}, x_{ji})}{\partial x_{ij}} - 1 = 0, \quad j \in N_i(g). \tag{7}$$

These equations say that a firm should continue to invest in a research project up to the point where the value of the marginal cost reduction obtained from the project equals the marginal cost of R&D investment. Note that the cross partial-derivative of the profit function with respect to x_{ii} and x_{ij}, for all $j \in N_i(g)$, and with respect to x_{ij} and x_{ik}, for all $j \neq k$ are positive. This implies that distinct research projects are *complementary* activities. This result, which is analyzed more deeply in GKM, is fairly general and holds also under price competition with differentiated products.

Collaboration generates efficiency gains but it also creates perfect technological spillovers within a joint project. This raises a classical question: are there institutional mechanisms which can be used to alleviate this negative effect? In the economics literature a great deal of attention has focussed on the role of R&D cooperation and we now examine the implications of cooperative decision making by firms in our model.

In our context, every joint project engages two firms and so it is natural to consider cooperation at a bilateral level. However, when a firm has two or more partners bilateral cooperation needs to take into account the investments of the firms in projects with other firms. This leads us to propose a novel mode of cooperation: *bilateral cooperation*. This is a mode of cooperation where each pair of firms involved in a collaborative research project chooses the aggregate level of R&D investment on the common project so as to maximize the joint profit of the two research partners, taking as given the decisions on other joint projects. As we mentioned in the introduction, bilateral cooperation offers a formal model of an organizational form in which divisions of different firms have very close ties, sometimes being closer than ties within the firm, an idea that has been discussed in the business strategy literature (see e.g., Delapierre and Mytelka, 1998; and Podolny and Page, 1998).

Formally, the investments in a joint project $ij \in g$ solve the following problem

$$\begin{aligned} &\max \quad \pi_i(x; g^k) + \pi_j(x; g^k) \\ &\text{s.t. } x_{ij} \geq 0, \; x_{ji} \geq 0 \end{aligned}$$

given the investments of firms i and j in independent research and in all other joint projects they maintain. Firm i investment in in-house research is decided unilaterally and thus solves the problem

$$\begin{aligned} &\max \quad \pi_i(x; g^k) \\ &\text{s.t. } x_{ii} \geq 0 \end{aligned}$$

given firm i investments in all joint projects.

The first order conditions with respect to R&D variables are

$$\frac{\partial \pi_i(x; g^k)}{\partial x_{ii}}(x) = \frac{2q_i(g, x)n}{(n+1)} \frac{\partial f}{\partial x_{ii}}(x) - 1 = 0, \tag{8}$$

and

$$\frac{\partial(\pi_i(x; g^k) + \pi_j(x; g^k))}{\partial x_{ij}}(x) = \frac{4q_i(g, x)(n-1)}{(n+1)} \frac{\partial h}{\partial x_{ij}}(x) - 1 = 0, \quad j \in N_i(g). \tag{9}$$

Let us focus on symmetric equilibria, i.e., where all firms have the same number $k = 1, 2, ..., n-1$ of potential partners and every firm puts in the same amount of effort in every joint project it maintains. Let x_{ii}^{BC} and x_{ij}^{BC} denote symmetric non-cooperative research efforts of a firm in the in-house project and in each joint project, correspondingly, and let x_{ii}^{BC}, x_{ij}^{BC} be research efforts corresponding to a symmetric hybrid equilibrium with transfers.

To prove existence and uniqueness of symmetric equilibria, let us parameterize functions f and h as follows: $f(x_{ii}) = \frac{1}{\gamma}\tilde{f}(x_{ii})$, $h(x_{ij}, x_{ji}) = \frac{1}{\gamma}\tilde{h}(x_{ij}, x_{ji})$. Parameter $\gamma \in (0, +\infty)$ is simply a shift-scalar that reduces the returns from R&D. The following result shows that when γ is large enough, the system of first order conditions has a unique symmetric solution.

Proposition 3 *Let g^k be a symmetric network of degree k. Let $f(x) = \frac{1}{\gamma}\tilde{f}(x_{ii})$, $h(x_{ij}, x_{ji}) = \frac{1}{\gamma}\tilde{h}(x_{ij}, x_{ji})$. Then if*

$$\min\{f'(0), h_d'(0)\} > \frac{(n+1)^2}{(n-1)(A-\bar{c})}$$

there exists γ_0 such that for all $\gamma \geq \gamma_0$ there exist a unique symmetric Nash equilibrium $x^{NC} \in \mathbb{R}_{++}^{(k+1)n}$ and a unique hybrid equilibrium with transfers $x^{BC} \in \mathbb{R}_{++}^{(k+1)n}$.

Proof. See the Appendix.

We now compare the private incentives to invest in research with those under bilateral cooperation. We note that the first order condition (8) is exactly identical to (6). As a result, the implicit function defined by the first order condition (8), denoted $x_{ii}^{BC}(x_{ij})$, is equal to $x_{ii}^{NC}(x_{ij})$. Denoting the implicit function defined by equation (9) as $x_{ij}^{BC}(x_{ii})$, it is easy to see that it lies to the right of $x_{ij}^{NC}(x_{ii})$. These remarks are illustrated in Figure 1. Inspection of this graph reveals that non-cooperative firms invest less in in-house less than firms that cooperate bilaterally.

This observation is easily explained. Under bilateral cooperation, each R&D unit internalizes the positive externality it confers to the collaborator so investments in joint R&D projects are larger than under non-cooperative decision making. Complementarity between different research projects implies that non-cooperative in-house R&D is also lower than in-house investments under bilateral cooperation.

In the existing literature firms either conduct research independently or all together. This

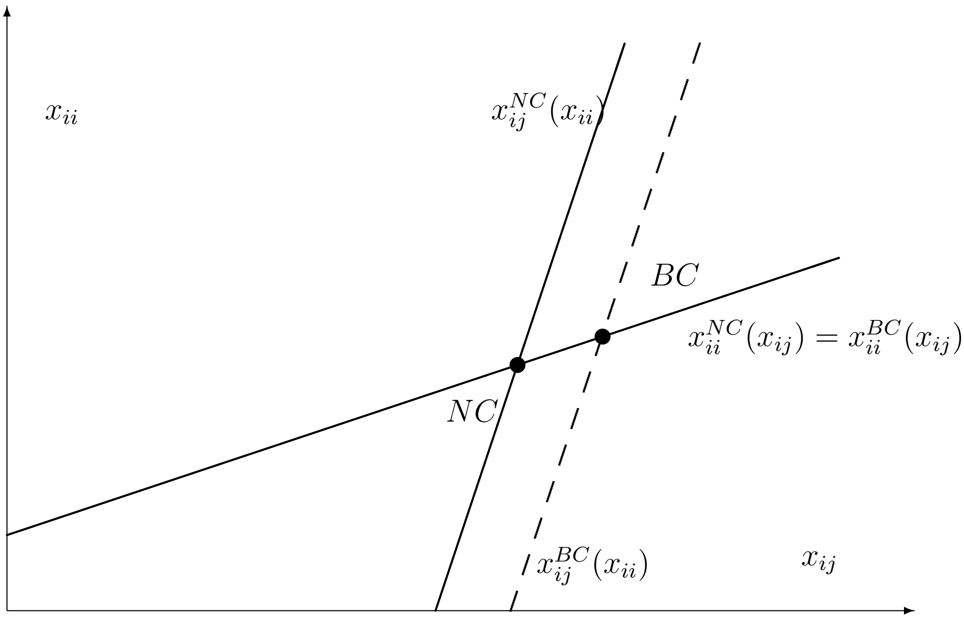

Figure 1: Non-Cooperative (NC) and Bilateral Cooperative (BC) R&D Investments

has led to the study of a model of *full cooperation* where firms decide on investments to maximize the joint profits of all the firms operating in the industry. We discuss this form of cooperation as a benchmark case. Given a symmetric network g^k, R&D investments that maximize industry profits solve the following problem:

$$\begin{aligned} &\max \sum_{i=1}^{N} \pi_i(x; g^k) \\ \text{s.t. } & x_{ii} \geq 0, \ i \in N \\ & x_{ij} \geq 0, \ i \in N, \ j \in N_i(g^k) \end{aligned}$$

The first order conditions with respect to R&D variables are

$$\frac{\partial \sum_{i=1}^{N} \pi_i(x; g^k)}{\partial x_{ii}}(x) = \frac{2q_i(g,x)}{(n+1)} \frac{\partial f}{\partial x_{ii}}(x) - 1 = 0, \tag{10}$$

and

$$\frac{\partial \sum_{i=1}^{N} \pi_i(x; g^k)}{\partial x_{ij}}(x) = \frac{4q_i(g,x)}{(n+1)} \frac{\partial h}{\partial x_{ij}}(x) - 1 = 0, \quad j \in N_i(g). \tag{11}$$

Again, existence and uniqueness of an industry profit maximizing profile of R&D investments follow from similar arguments as those behind Proposition 3. Let $(x_{ii}^{FC}, x_{ij}^{FC})$ denote the symmetric solution of this problem. As above, let us denote the implicit function defined by the first order condition (10) as $x_{ii}^{FC}(x_{ij})$. A comparison of equations (6) and (10) reveals that the function $x_{ii}^{FC}(x_{ij})$ lies beneath $x_{ii}^{BC}(x_{ij})$. Likewise, equation (11) implicitly defines a function $x_{ij}^{FC}(x_{ii})$ that lies to the left of $x_{ij}^{NC}(x_{ii})$, provided that $n \geq 3$. These two observations lead to the finding that investments in R&D are greater under non-cooperative decision making as compared to what is industry profit maximizing.

The following proposition summarizes our findings on investment levels under different regimes of decision make.

Proposition 4 *Suppose $n \geq 3$. Then, bilateral cooperation yields the highest level of R&D investments, while full cooperation yields the lowest:*

$$(x_{ii}^{BC}, x_{ij}^{BC}) \geq (x_{ii}^{NC}, x_{ij}^{NC}) \geq (x_{ii}^{FC}, x_{ij}^{FC}) \quad k > 0, \quad i \in N.^{8} \tag{12}$$

In the pure in-house research case ($k = 0$), independent R&D under non-cooperative decision making is excessive as compared to what firms would like collectively.[9]

Proof: Let $T = \{NC, FC, BC\}$. Then for every $t \in T$ a pair (x_{ii}^t, x_{ij}^t) is a fixed point of the correspondence $F^t(x_{ii}, x_{ij})$, where

$$F_1^t(x_{ii}, x_{ij}) = \frac{2\alpha_1^t}{(n+1)^2}(A - \bar{c} + f(x_{ii}) + kh_d(x_{ij}))f'(x_{ii}) - 1 + x_{ii},$$

$$F_2^t(x_{ii}, x_{ij}) = \frac{\alpha_2^t}{(n+1)^2}(A - \bar{c} + f(x_{ii}) + kh_d(x_{ij}))h_d'(x_{ij}) - 1 + x_{ij}.$$

Denoting $\alpha^t = (\alpha_1^t, \alpha_2^t)$ we observe that $\alpha^{NC} = (n, n-1)$, $\alpha^{FC} = (1, 2)$, and $\alpha^{BC} = (n, 2(n-1))$. If $n \geq 3$ then $\alpha^{BC} \geq \alpha^{NC} \geq \alpha^{FC}$. Moreover, F_1^t is increasing in x_{ij} and F_2^t is increasing in x_{ii} for every t. Theorem 4 in Milgrom and Roberts (1994) implies that $(x_{ii}^{BC}, x_{ij}^{BC}) \geq (x_{ii}^{NC}, x_{ij}^{NC}) \geq (x_{ii}^{FC}, xij^{FC})$. The last statement of the proposition is obvious. □

[8] We note that $x_{ii}^{NC} > x_{ii}^{FC}$, $i \in N$, for $k = 0$ also. In addition, we can state that $(x_{ii}^{BC}, x_{ij}^{BC}) \geq (x_{ii}^{FC}, x_{ij}^{FC})$ and $(x_{ii}^{BC}, x_{ij}^{BC}) \geq (x_{ii}^{NC}, x_{ij}^{NC})$ for $n = 2$.

[9] This last result is in line with the finding of Leahy and Neary (1997), who also obtain an excessive individual incentives result for low global spillovers. The complementary nature of investments in independent and joint R&D reinforces this result for any level of collaboration.

This ranking of research intensities brings out the role of two forces created by cooperation: internalization of bilateral spillovers and internalization of the combined-profits externality. The former force affects R&D efforts positively while the latter force affects them negatively, relative to the non-cooperative case. Our analysis has clarified that bilateral cooperation basically works on the former while full cooperation works on both effects. This leads to research efforts being highest under bilateral cooperation, and lowest under full cooperation, while non-cooperative decision making yields intermediate effort levels.

Our result that non-cooperative investments are lower than joint profit-maximizing effort levels contrasts the results in Kamien *et al.* (1992). This difference in result is important for policy purposes and we explain the reasons for it. In our model of bilateral collaborative projects, the investment put in a joint research project is shared only with a single collaborator. This implies that the combined-profits externality is overall negative since just a single competitor benefits from the investment of a given firm in collaborative R&D. In the model of Kamien, Muller and Zang (1992), by contrast, a firms investment confers a cost advantage to all the firms in the collaboration structure and thus the combined-profits externality is positive.

Social Welfare

Our analysis so far has ranked investment levels under different regimes of decision making but the social desirability of cooperation remains an open question. This section addresses this issue. Given the network g^k, R&D investments that maximize social welfare solve the following problem:

$$\begin{aligned} &\max W(x; g^k) \\ \text{s.t. } & x_{ii} \geq 0,\ i \in N \\ & x_{ij} \geq 0,\ i \in N,\ j \in N_i(g^k) \end{aligned}$$

The first order conditions with respect to R&D variables are

$$\frac{\partial W(x;g^k)}{\partial x_{ii}}(x) = \frac{(n+2)q_i(g,x)}{(n+1)}\frac{\partial f}{\partial x_{ii}}(x) - 1 = 0, \tag{13}$$

and

$$\frac{\partial W(x;g^k)}{\partial x_{ij}}(x) = \frac{2(n+2)q_i(g,x))}{(n+1)}\frac{\partial h}{\partial x_{ij}}(x) - 1 = 0, \quad j \in N_i(g^k). \tag{14}$$

Our assumptions imply that there exists a unique symmetric solution to this system of equations. Let x_{ii}^W, x_{ij}^W denote the socially desirable investment levels.

How do social incentives compare with private incentives? The first remark we make here is that the comparison between the non-cooperative investment levels and the welfare maximizing ones does not yield clear-cut results. To see this, note that the first order condition (13) implicitly defines a function $x_{ii}^W(x_{ij})$ that lies beneath the function $x_{ii}^{NC}(x_{ij})$ defined by equation (6). Further, the first order condition (14) implicitly defines a relationship $x_{ij}^W(x_{ii})$ that lies to the right of $x_{ij}^{NC}(x_{ii})$, which follows from equation (7). As a result, without further specifications of the return to investment functions f and h, the private incentives to invest in R&D of the Cournot sellers cannot be compared with the social incentives. What is clear is that these remarks point towards excessive incentives to invest in own research and insufficient incentives to invest in joint R&D.

The second observation we make here is about the relation between the effort levels of cooperating firms and the effort levels that maximize social welfare.

Proposition 5 *If $n \geq 4$ then bilateral cooperation yields too much R&D while full cooperation leads to too little investments, that is,*

$$(x_{ii}^{BC}, x_{ij}^{BC}) \geq (x_{ii}^{W}, x_{ij}^{W}) \geq (x_{ii}^{FC}, x_{ij}^{FC}) \quad k > 0, \quad i \in N.^{10} \tag{15}$$

Proof: The proof builds on the arguments in Proposition 4 and will be kept brief. Set $T = \{BC, FC, W\}$ in that proposition and note that $\alpha^W = (\frac{n+2}{2}, n+2)$. Moreover, F_1^w is increasing in x_{ij} and F_2^w is increasing in x_{ii}. Theorem 4 in Milgrom and Roberts (1994) and Proposition 4 implies that if $n \geq 4$ then $\alpha^{BC} \geq \alpha^W \geq \alpha^{FC}$, and $(x_{ii}^{BC}, x_{ij}^{BC}) \geq (x_{ii}^{W}, x_{ij}^{W}) \geq (x_{ii}^{FC}, x_{ij}^{FC})$. $\square$

[10] In fact, $(x_{ii}^{W}, x_{ij}^{W}) \geq (x_{ii}^{FC}, x_{ij}^{FC})$ for any n. If $n = 2$ then $(x_{ii}^{W}, x_{ij}^{W}) \geq (x_{ii}^{BC}, x_{ij}^{BC})$.

We have seen that, from the point of view of the society as a whole, cooperation may lead to too large or to too small incentives to invest in R&D; this depends on the extent to which firms can coordinate their R&D activities. We would like to compare welfare levels for different modes of decision making. However, this cannot be done for a general specification of the return to investment function h since the different systems of first order conditions that determine R&D investments under distinct modes of decision making cannot be solved.

To conduct such welfare analysis we assume the following quadratic specification of the return to R&D functions: $f(x_{ii}) = 1/\gamma\sqrt{x_{ii}}$ and $h(x_{ij}, x_{ji}) = 1/\gamma(\sqrt{x_{ij}} + \sqrt{x_{ji}})$.[11] This function satisfies all the conditions mentioned above except for differentiability at zero. We note, however, that all results presented above are also true for this specification. To guarantee existence of equilibrium in investment levels, we simply require that γ is sufficiently large ($\gamma > 2\sqrt{n}$ suffices).

The welfare levels under non-cooperation, bilateral cooperation and full cooperation can be computed easily; the details are given in the appendix. A comparison of the welfare levels under different modes of decision making yields the following result.

Proposition 6 *Let $h(x_{ij}, x_{ji}) = 1/\gamma(\sqrt{x_{ij}} + \sqrt{x_{ji}})$, and γ sufficiently large. If $3 \le n \le 6$, then for any degree of collaboration k*

$$W(x^{BC}, g^k) > W(x^{NC}, g^k) > W(x^{FC}, g^k),$$

while if $n \ge 7$ then

$$W(x^{NC}, g^k) > W(x^{BC}, g^k) > W(x^{FC}, g^k).$$

Proof. See the Appendix.

A noteworthy result is that welfare under full cooperation is always lower than under non-cooperative behavior. This result, which is in contrast to the finding in Kamien *et al.* (1992), highlights again the detrimental effects of fully coordinated behavior to restrict R&D investments when collaboration spillovers are bilateral. It is also important to notice that

[11] It is possible to generalize the results of the rest of the paper to the family of functions $h(x_{ij}, x_{ji}) = \sqrt{x_{ij} + x_{ji} + \beta\sqrt{x_{ij}x_{ji}}}$, where $\beta > 0$ is a technological complementarity parameter.

welfare under bilateral cooperation is always greater than under full cooperation. This highlights the potential welfare enhancing effects of a novel form or R&D organization.

4 Endogenous Network Formation

To understand how playing hybrid equilibria with different behavioral modes across tasks affects endogenous networks, we consider the following network formation process preceding the network game $G(g)$ introduced in the previous section. In stage 1, agents observe link formation costs $l \in [\underline{l}, \bar{l}]$ where $\underline{l}$ and $\bar{l}$ are lower and upper bounds of profits an agent can enjoy in $G(g)$. Subsequently, agents simultaneously announce a set of intended links with other agents. At stage 2, the game is played on the endogenously generated network structure as described in the previous section for a fixed network.

A link between two agents is formed if and only if both agents intend to make it. If a link is formed, each of the two agents incurs costs l. Let $g - ij$ denote a network that can be obtained from g by deleting a link between players i and j. Correspondingly, $g + ij$ is a network that is obtained from g by adding ij. A network g is *pairwise stable*[12] if

(i) If $ij \in g$, then $\pi_i(g) \geq \pi_i(g - ij) + l$ and $\pi_j(g) \geq \pi_j(g - ij) + l$.

(ii) If $ij \notin g$, then $\pi_i(g + ij) - l > \pi_i(g) \Rightarrow \pi_j(g + ij) - l < \pi_j(g)$.

Here, $\pi_i(g)$ denotes agent i's continuation profit for network g in game $G(g)$. The first condition says that every link is profitable for the two players involved in a link. The second condition requires that for each link not present in the network it must be the case that at least one of the involved players strictly loses from forming it. Let

$$\alpha_{ij}(g) = \min_{i,j} \left(\pi_i(g + ij) - \pi_i(g), \pi_j(g + ij) - \pi_j(g)\right)$$

[12] We use the standard notion of pairwise stability due to Jackson and Wolinsky (1996). It should be noted, however, that the measure of stability introduced below can easily be adopted for other network stability concepts such as strong stability (Jackson and van den Nouweland, 2005), pairwise equilibrium (Belleflamme and Bloch, 2004, and Goyal and Joshi, 2006), etc.

be the lower of the two extra payoffs players i and j gain in game $G(g+ij)$ compared to game $G(g)$. With $\bar{\alpha}(g) = \max_{ij \notin g} \alpha_{ij}(g)$, condition (ii) of the definition above is equivalent to saying that $\bar{\alpha}(g) < l$. Furthermore, let

$$\beta_{ij}^{i} = \pi_i(g - ij) - \pi_i(g)$$

be the extra profit that player i gets in game $G(g-ij)$, and $\bar{\beta}(g) = \max_{i,\, ij \in g} \beta_{ij}^{i}$. Condition (i) of the definition of pairwise stability is equivalent to saying that $l \leq -\bar{\beta}(g)$. For determinacy, put $\bar{\beta}(g^e) = -\infty$ for the empty network g^e and $\bar{\alpha}(g^c) = 0$ for the complete network g^c. As pairwise stability implies that $\bar{\beta}(g) \leq 0$, the condition of pairwise stability is equivalent to $-\bar{\beta}(g) \geq \bar{\alpha}(g)$ and $l \in (\bar{\alpha}(g), -\bar{\beta}(g)]$.

So for a given hybrid equilibrium at stage 2, we expect a pairwise stable network to form at stage 1. Generically, there will be multiple pairwise stable networks and stage 1 thereby represents a coordination problem. Depending on the application, different selection criteria such as Pareto optimality, core or setwise stability, or risk dominance could be employed.[13] Without specifying $G(g)$, however, we may compare the stability of networks as follows. If costs of link formation l are distributed on $[\underline{l}, \bar{l}]$ or are subject to an occasional shock, the interval $(\bar{\alpha}(g), -\bar{\beta}(g)]$ can be used to construct a measure of stability of a network g. From an ex-ante perspective, networks that are stable for a larger set of link formation costs (according to set inclusion) are more likely to be stable if link formation costs are drawn from a distribution with full support. Likewise, networks that are stable for a larger set of link formation costs are more likely to remain stable ex-post if link formation costs are subject to ideosyncratic shocks. To be specific, suppose that profits and, hence, values of $\bar{\alpha}$ and $\bar{\beta}$ depend on the parameter a, so $\bar{\alpha}(g) = \bar{\alpha}_a(g)$ and $\bar{\beta}(g) = \bar{\beta}_a(g)$. Let $\mu_a(g) = (\bar{\alpha}_a(g), -\bar{\beta}_a(g)]$ if $-\bar{\beta}_a(g) \geq 0$, $-\bar{\beta}_a(g) \geq \bar{\alpha}_a(g)$, and $\mu_a(g) = \emptyset$ otherwise. We say that a network g is *more*

[13] Jackson and Watts (2002) formulated a dynamic network generation process and suggest stochastically stable networks as a solution concept to the coordination problem – analogously to equilibrium selection in normal form games by learning dynamics as introduced in Kandori et al. (1993) or Young (1993). However, these processes are based on boundedly rational agents which is at odds with our assumptions on hybrid equilibrium at the final stage of the game. We regard a boundedly rational analysis of games with multidimensional strategy spaces an interesting topic for future research which, however, appears to be beyond the purpose of our paper. As a result we refrain from an explicit modeling of the dynamic network formation process and work with the following ad hoc stability measure that only implicitly incorporates dynamic considerations.

stable under the value $a = a'$ than under the value $a = a''$ if

$$\mu_{a'}(g) \supset \mu_{a''}(g),$$

that is, under $a = a'$ g is stable for a larger range of parameter values l.

Hence, introducing a cost of link formation allows us to analyze the (pairwise) stability of different network architectures. In addition, the set of link formation costs which supports the stability of a given network can be used to construct a measure of stability and introduces a partial order that ranks networks according to their (relative) stability. As we will illustrate in the context of R&D networks, stable network architectures are rather sensitive to the (anticipated) behavioral mode and the payoff characteristics of the game.

Endogenous Formation of R&D Networks

To illustrate how the anticipation of different behavioral modes influence network formation, we study a simple three firms example to answer two questions: 1. How does switching from NC to BC affect profits of the firms in different network topologies? 2. Which networks are more stable under BC and which under NC?

To keep computations simple we focus on a world with three symmetric firms, 1, 2, and, 3, and assume quadratic cost reduction functions. Without loss of generality, we assume that firm 1 is the most connected, and firm 3 is the least connected firm in the network. There are only four possible network architectures: the empty network g^e (in such a network the outcomes under the two regimes coincide), the partially connected network $g^p = (12)$, the star $g^s = (12, 13)$, and the complete network $g^c = (12, 13, 23)$. The answer to the first question is that BC increases the profit of more connected firms, and decreases the profit of less connected ones compared to NC. In g^p, for example, the profit of the connected firms is higher under BC than under NC, and the profit of the isolated firm is lower under BC. In g^s the profit of the hub firm is higher under BC, and the profit of the spoke firms is lower under BC. In the symmetric network g^c the profit of all firms is lower under BC. The last result is connected with the fact that the overall investment level in the NC mode is already excessive from the industry-wide perspective, and this problem is further exacerbated when

BC drives investment levels further upwards.

To answer the second question, we compare stability intervals $\mu(g)$ of the four networks under the two decision modes. The empty network g^e is less stable under BC — establishing a link between two firms brings them more profit when they cooperate on a joint project. It is interesting that the complete network is more stable under BC than under NC despite the decrease in profits of all firms. The reason is that the profits of the spoke firms in the star decrease even more dramatically, which makes deleting a link less attractive. The partially connected network g^p can be stable ($-\bar{\beta}(g^p) \geq \bar{\alpha}(g^p)$) under NC only for low values of the research cost parameter γ ($\gamma < 1.56$), that is, only in R&D intensive industries. BC increases stability of this network, and even makes stability of g^p possible for less R&D intensive industries ($\gamma < 2.82$). There are two reasons for this. First, the profit of firms in the empty network is too low compared to high profits of connected firms in g^p under BC, so deleting a link is less attractive. Second, creating a new link is less attractive for the isolated firm under BC since this confers too much market power to the hub in the star which is thus created.[14] Finally, stability of the star g^s decreases under BC. In particular, g^c can be stable under NC for high γ ($\gamma > 1.84$), but it is never stable under BC ($-\bar{\beta}(g^s)$ is always less than $\bar{\alpha}(g^s)$)! Again, the reason for that is that the spokes are in a relatively worse position under BC than under NC and gain more from linking.

To summarize, all four network architectures can be stable under NC, and only three under BC. For γ sufficiently high, only two stable network architectures are possible under BC: the empty and the complete. So, for instance, if the costs of linking l decline, the emergence of a stable network follows a bang-bang pattern as appears to be intuitive given that internal and external strategies are complements. Note however that it is *not* always the case that BC leads to the formation of a more connected network. For instance, a partially connected network can be stable under BC, while under NC the star or the complete network would have been formed.

Finally, we have a preliminary result for the general case of n firms. We have found that the

[14] Interestingly enough, in Goyal and Moraga-Gonzalez (2001) in the setting describing different way of creating and sharing knowledge g^p can also be stable, but there it is a connected firm finds linking with the isolated firm unattractive.

profit of all firms decreases under BC (for $n > 2$) but in spite of that the complete network is more stable under BC. This generalizes the result obtained for three firms.[15]

5 Further Examples of Network Games

We conclude our discussion of applications with a list of other research questions that may benefit from the tools proposed by the paper.

Product Innovation Networks in Multi-Product Oligopolies. Despite the fact that most of the theoretical literature on R&D focuses on process innovation, the largest part of R&D investments by firms (for example, about three-fourth of R&D investments by firms in the United States in the 80s, Scherer and Ross, 1990) is devoted to product innovation. This fact motivates studying product innovation networks (and other forms of collaborative structures) in industries where firms produce multiple (horizontally or vertically) differentiated products. Suppose that a firm can develop and produce goods either on its own or in collaboration with other firms, in the latter case collusion in the price of the jointly developed product(s) occurs. Different assumptions can be used about the demand structure. One could consider, for example, a discrete choice model or related to it the CES representative consumer model, see Konovalov and Sandor (2010) and references cited there. Another possibility is to have a look at a model with spacial competition. In the first case, the internal strategy x_{ii} of firm i is the price vector for goods produced exclusively by firm i. The external strategy x_{ij} is the price of a product jointly developed by firms i and j. In the case of spacial competition, the decision on the location of products also enters a firm's internal and collaborative strategies. The condition of symmetry, $x_{ij} = x_{ji}$, should be imposed in both cases. The possibility for two or more firms to collaborate on product innovation reduces fixed costs incurred by the firms, increases the variety of products on the market, boosts competition and reduces equilibrium prices. Partial collusion on prices however drives equilibrium prices upward. Which effect prevails may depend on the product innovation network structure.

[15] The results are available form the author upon request.

International Trade Agreements. Consider a network of bilateral preferential trade agreements between countries as in Furusawa and Konishi (2007) or Goyal and Joshi (2006). The external strategy x_{ij} is country i's tariff (or quota) on the production of country j with which i has a bilateral agreement, while x_{ii} stands for the profile of tariffs imposed on the production of countries with which i has no agreement. The lowering of tariff x_{ij} benefits country j's firms, hurts profits of other firms active inside country i and also benefits country i's consumers due to reduction of prices on the domestic market. Hybrid equilibria arise naturally in this setting — in fact, the meaning of a bilateral trade agreement is nothing else but to increase the welfare of both participating sides. Goyal and Joshi (2006) argued that in the symmetric case joint social welfare of participating countries is maximized under zero tariffs, $x_{ij} = x_{ji} = 0$. Thus, a hybrid equilibrium with transfers underlies any symmetric network of *free trade agreements*. An interesting research question is the characterization of hybrid equilibrium in the case of *ex post* (the network is asymmetric) or *ex ante* (countries differ in size and efficiency of domestic firms) asymmetry. The latter assumption also allows to study the *trade diversion effect* (a potentially welfare decreasing diversion of trade to a higher cost producer when a trade agreement with a less-efficient country is made).

Cross-border Pollution Games. Cross-border pollution is the pollution that originates in one country but potentially causes damage in another country's environment by crossing borders through pathways like water or air. Pollution can be transported across hundreds and even thousands of kilometers. One of the problems with cross-border pollution is that pollutants can be carried from a heavy emitter to a nation whose emissions are relatively low. Consider a network g, where $ij \in g$ if there is a flow of pollution between countries i and j. Let x_{ii} be the activity of country i aiming at the reduction of its own pollution stock, while x_{ij} is its undertaken abatement activity to reduce the flow of pollution to country j. Two aspects of this model make it very different, for example, from the R&D collaboration network model considered earlier. First, the presence of technological spillovers: activity x_{ii} reduces to some extent the need in activity x_{ij} and vice versa, making them strategic substitutes as in the public good examples. Second, due to asymmetry of pollution flows caused by winds, oceanic and river flows, etc., activity x_{ij} might increase the utility of country j, but not so much the utility of country i itself – an asymmetry not present in our public good analysis with undirected networks. Hybrid equilibrium with transfers thus reflects the "Polluter Pays Principle" adopted in particular by OECD and EC countries. A number of issues can be studied in this setup, for example, the question whether a system of bilateral pollution abatement agreements between the countries can lead to a globally desirable outcome in the absence of a supranational authority – as suggested by our discussion of efficient networks for public good provision in the previous subsection.

Communication Games. The following sketch of a model combines elements of the models by Bloch and Dutta (2009) and by Galeotti and Goyal (2010). Players can invest either in collecting information individually or establishing bilateral links with other players which will allow them to get access to the information collected by others. The individual effort of player i is x_{ii}. Investment x_{ij} of player i into the link with player j improves its quality and increases the magnitude of information accessible via the network. Information can flow between any two directly or indirectly connected players with decay determined by the endogenous strength of the links. The payoff of an agent is the sum of the value of individually obtained information and the value of information received from others minus the cost of effort. A result that can be expected here is that agents invest too little into bilateral relations since they do not take into consideration the positive externality on indirectly connected agents. Bilateral cooperation may alleviate this problem. A realistic feature not modelled in Bloch and Dutta (2009) is the budget constraint $x_{ii} + \sum_{j \in N_i(g)} x_{ij} = B$, where B can be the amount of time with which each player is endowed. Increasing the level of activity x_{ij} must then result in the decrease of the level of some other activity x_{ik} and increase of its marginal return. In Galeotti and Goyal (2010), x_{ij} is assumed to be either 1 or 0. The model of Bloch and Dutta (2009) is more general in this respect but it does not consider internal strategies of the players. Other related papers that feature communication games on networks are Brueckner (2006) and Rogers (2005).

Co-Authorship Games. In Goyal, van der Leij, and Moraga-González (2006)[16] the following model of co-authorship is considered. Scientists can spend a costly effort on writing

[16]This paper does not use Nash equilibrium as solution concept but one of its refinements: coalitions of two players are also allowed to deviate. Note that this concept is different from HE — we allow two players to deviate by coordinating the corresponding collaborative components of their strategies only.

either single-authored papers or papers in collaboration with other researchers. The internal strategy x_{ii} is scientist i's effort on writing papers alone. The external strategy x_{ij} is (g_{ij}, e_{ij}), where $g_{ij} \in \{0, 1\}$ is a yes/no decision by i whether to write a paper with j, and e_{ij} is the effort spent on the joint project. The profit of player i depends on the number of published papers which in its turn depends on her exogenously given talent level, her internal and external efforts, and on the talent levels and external efforts x_{ji} of her co-authors. Another assumption that can be made here is that the utility of an author depends not on the number of her papers but on her status in the scientific community defined by the share of her publications in the total research output. This creates a negative externality and may result in an overworking problem that could be exacerbated by bilateral collaboration.

6 Concluding remarks

In recent years, the typical firm is engaged in joint bilateral research projects with various other firms. This paper presents a model to study this novel form of organization of R&D activities. Specifically, the paper makes explicit an essential feature: the idea of investments being link-specific gives raise to a novel form of cooperation, namely, bilateral cooperation.

We find that R&D investments under bilateral cooperation are larger than under non-cooperation and this is because bilateral cooperation allows for the internalization of bilateral externalities within a collaboration. In contrast to standard results in the literature full cooperation yields the lowest level or research activity and this is because spillovers are bilateral here rather than industry-wide. When there are just a few firms in the industry welfare is maximized under bilateral cooperation. However, when there are many firms in the market the excessive incentives for R&D under bilateral cooperation render non-cooperative decision making preferable from a welfare point of view.

Investigating the issue of endogenous network formation, we find that bilateral cooperation increases (lowers) the profits of more (less) connected firms. However, this does not always lead to a denser stable network of R&D collaboration under bilateral cooperation. a partially connected network can be stable under bilateral cooperation, while under non-cooperative investment strategies the star or the complete network would have been formed.

7 Appendix

Proof of Proposition 3: A symmetric equilibrium must satisfy the first order conditions (6) and (7), which can be written as follows:

$$\frac{2n}{(n+1)^2}(A-\bar{c}+f(x_{ii})+kh_d(x_{ij}))f'(x_{ii})-1=0, \tag{16}$$

$$\frac{\alpha(n-1)}{(n+1)^2}(A-\bar{c}+f(x_{ii})+kh_d(x_{ij}))h'_d(x_{ij})-1=0, \tag{17}$$

where $\alpha \in \{1,2\}$. The case $\alpha = 1$ corresponds to the Nash equilibrium, while the other case corresponds to the hybrid equilibrium with transfers.

Show that the assumptions imposed on functions $f(\cdot)$ and $h(\cdot,\cdot)$ guarantee the existence of a symmetric solution to the system of the first order conditions (16)-(17). Consider a function $R(y) = (h'_d)^{-1}(\frac{2n}{\alpha(n-1)}f'(y))$, and put $R(y)$ equal to 0 if $(h'_d)^{-1}(\frac{2n}{n-1}f'(y))$ is not determined. By the concavity of the functions f and h (and, therefore, $h_d(y)$), the function $R(y)$ is continuous, non-negative and monotonically non-decreasing for all $y \geq 0$. Moreover, if (x_{ii}, x_{ij}) is an interior solution of (16)-(17), then $x_{ij} = R(x_{ii})$. A function $\frac{2n}{(n+1)^2}(A-\bar{c}+f(y)+kf(R(y)))f'(y)-1$ is continuous and strictly positive at zero by the boundary conditions. It is strictly negative if y is sufficiently large. Therefore, there is a point $y = x^*_{ii} > 0$ such that

$$\frac{2n}{(n+1)^2}(A-\bar{c}+f(x^*_{ii})+kf(R(x^*_{ii})))f'(x^*_{ii})-1=0.$$

Let $x^*_{ij} = R(x^*_{ii})$. By construction, (x^*_{ii}, x^*_{ij}) is a solution of the system (16)-(17). Since $h'_d(0) > \frac{(n+1)^2}{(n-1)(A-\bar{c})}$, x^*_{ij} is strictly positive. Show now that every function $\pi_i(x_i, x^*_{-i})$, $i \in N$ is strictly concave if γ sufficiently large. Without loss of generality, assume that x_i belongs to a compact convex set $K_i \subset I\!R^{k+1}_+$. It is sufficient to verify that $D^2_{x_i}\pi_i(x_i, x^*_{-i})$ is negative definite for every $x_i \in K_i$. The matrix $D^2_{x_i}\pi_i(x_i, x^*_{-i})$ can be represented as a linear combination

$$D^2_{x_i}\pi_i(x_i, x^*_{-i}) = \frac{1}{\gamma}B_i(x_i, x^*_{-i}) + \frac{1}{\gamma^2}C_i(x_i, x^*_{-i}),$$

where matrices $B(.)$ and $C(.)$ do not depend on γ, and $B(.)$ is diagonal and strictly negative definite for all $x_i \in K_i$. The functions $u \cdot B_i(x_i, x^*_{-i})u$ and $u \cdot C_i(x_i, x^*_{-i})u$ are continuous and bounded if $x_i \in K_i$ and $u \in \{v \in I\!R^{k+1} \mid ||v|| = 1\}$. Therefore it is possible to find a sufficiently large γ such that for all such x_i and u the inequality $\frac{1}{\gamma}|u \cdot B_i(x_i, x^*_{-i})u| > \frac{1}{\gamma^2} u \cdot C_i(x_i, x^*_{-i})u$ is satisfied. This implies that $D^2_{x_i}\pi_i(x_i, x^*_{-i})$ is also negative definite for every x_i in K_i and all sufficiently large γ. Hence, a profile of investment levels $x^* = (x^*_{ii}, x^*_{ij})_{i \in N, ij \in g}$ solving (16)-(17) for $\alpha = 1$ is an interior symmetric Nash equilibrium. In a similar way, it can be shown that for γ sufficiently large $\pi_i + \pi_j$ is concave in (x_{ij}, x_{ji}), so the solution x^* for $\alpha = 2$ is an interior symmetric hybrid equilibrium with transfers.

To prove the uniqueness of a symmetric equilibrium, it is sufficient to show that the function $\tilde{F} : I\!R^2 \to I\!R^2$ defined by the right-hand side of the system (16)-(17) is strictly monotonic on some compact set containing all (u^*, v^*) such that $x_{ii} = u^*$, $i \in N$, $x_{ij} = v^*$, $ij \in g^k$, is a symmetric equilibrium. This can be done by analogy with the first part of the proof.

The condition that γ should be sufficiently large is essential. As numerical examples show, if returns from R&D activity are too high, no equilibrium exists in the model. □

Proof of Proposition 6: Non-cooperative equilibrium investment levels are given by the solution to (6)-(7). In this case we obtain:

$$x^*_{ii} = \left(\frac{(A - \bar{c})n\gamma}{\gamma^2(n+1)^2 - n - 2k(n-1)} \right)^2, \tag{18}$$

$$x^*_{ij} = \left(\frac{(A - \bar{c})(n-1)\gamma}{\gamma^2(n+1)^2 - n - 2k(n-1)} \right)^2, \tag{19}$$

Plugging these R&D efforts in (4), we can calculate the level of welfare under non-cooperative decision making:

$$W(x^*, g^k) = \frac{(A - \bar{c})^2\gamma^2[n\gamma^2(n+2)(n+1)^2 - 2kn(n-1)^2 - 2n^3]}{2\left(\gamma^2(n+1)^2 - n - 2k(n-1)\right)^2} \tag{20}$$

Equilibrium welfare under non-cooperative decision making is given by (20). The welfare level attained under bilateral cooperation is

$$W(x^{BC}, g^k) = \frac{(A-\bar{c})^2\gamma^2(-2n^3 - 8k(-1+n)^2 n + n(1+n)^2(2+n)\gamma^2)}{2(n+4k(-1+n)-(1+n)^2\gamma^2)^2},$$

and the welfare level under full cooperation is

$$W(x^{FC}, g^k) = \frac{(A-\bar{c})^2 n\gamma^2(-2-8k+(1+n)^2(2+n)\gamma^2)}{2(1+4k-(1+n)^2\gamma^2)^2}.$$

First, notice that

$$\lim_{\gamma\to\infty} \gamma^2(W(x^*, g^k) - W(x^{BC}, g^k)) = \frac{(A-\bar{c})^2 k(n-7)(n-1)n}{(n+1)^4}.$$

Therefore, for a sufficiently large γ, welfare under non-cooperative decision making is higher than under bilateral cooperation if $n > 7$ and lower if $n < 7$. If $n = 7$ then

$$\lim_{\gamma\to\infty} \gamma^4(W(x^*, g^k) - W(x^{BC}, g^k)) = \frac{21(A-\bar{c})^2 k(35+18k)}{65536},$$

so the equilibrium welfare is higher than that under bilateral cooperation. Furthermore,

$$\lim_{\gamma\to\infty} \gamma^2(W(x^*, g^k) - W(x^{FC}, g^k)) = \frac{(A-\bar{c})^2 n(n-1+k(n^2-9))}{(n+1)^4},$$

which means that welfare under non-cooperative behavior is higher than under full cooperation if $n \geq 3$ and γ is large enough. □

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