Tight maps, a classification

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Abstract

This thesis concerns the classification of tight totally geodesic maps between Hermitian symmetric spaces of noncompact type.

In Paper I we classify holomorphic tight maps. We introduce a new criterion for tightness of Hermitian regular subalgebras. Following the classification of holomorphic maps by Ihara and Satake we go through the lists of (H2)-homomorphisms and Hermitian regular subalgebras and determine which are tight.

In Paper II we show that there are no nonholomorphic tight maps into classical codomains (except the known ones from the Poincaré disc). As the proof relies heavily on composition arguments we investigate in detail when a composition of tight maps is tight. We develop a new criterion for nontightness in terms of how complex representations of Hermitian Lie algebras branches when restricted to certain subalgebras. Using this we prove the result for a few low rank cases which then extends to the full result by composition arguments.

The branching method in Paper II fails to encompass exceptional codomains. We treat one exceptional case using weighted Dynkin diagrams and the other by showing that there exists an unexpected decomposition of homomorphisms in Paper III. Together these three papers yield a full classification of tight maps from irreducible domains.