



**GÖTEBORGS UNIVERSITET  
HANDELSHÖGSKOLAN**

**A study in the effectiveness of predicting default using the  
Merton model during financial distress**

*Bachelor degree thesis*

*Authors: Martin Gholami and Andreas Hjelm*

*Supervisor: Evert Carlsson Ph.D.*

*Gothenburg University*

*School of Business, Economics and Law*

*Department of Finance*

*2 June 2014*

## **Abstract**

### **Bachelor thesis in financial economics**

Applied financial pricing theory

Department of finance

**School of Business, Economics and Law**

**Gothenburg University**

2 June 2014.

**Title:** A study in the effectiveness of predicting default using the Merton model during financial distress

**Author:** Martin Gholami and Andreas Hjelm

**Supervisor:** Evert Carlsson, Ph.D.

**Background:** There are many approaches for calculating the default probability for a corporate bond, but none so important and widely used as the Merton model. The Merton model is a firm value model for pricing risky corporate bonds, from 1974 by Robert Merton.

**Purpose:** The purpose of this paper is to show how well the Merton model predicts corporate default during a period of financial distress.

**Motivation:** We intend to give the reader a step-by-step introduction to the Merton model and how to apply the model on real corporate data.

**Methodology:** First, we extract all the necessary data from the balance sheet and market quotes for equity. Second, a risk-free discount rate is constructed from two generic US government bonds. Then, we discount all future cash flows of the bond to be able to solve for the volatility. Finally, default probabilities are calculated.

**Conclusion and Discussion:** In our study we analyzed the outcome of the results and tried to find shortcomings and advantages in the Merton model. However in a period of financial distress it is hard to say if the model predicts default better or worse than more complex models.

**Further research:** There are many more developed and complex firm value models. An interesting approach would be to convey a comparative study of different models to conclude each model's advantages and limitations.

**Key words:** Balance sheet, Bloomberg, Default probability, Excel, Ford Motor Company, Volatility

## Acknowledgments

First, we would like to express our deepest gratitude to our supervisor Evert Carlsson for his patience during the process of writing this thesis. Thanks also to family and friends for their support. Finally, thanks to our loving girlfriends Jonna and Fanny.

*“The way that the calculations are performed instead gives us an intuitive explanation for this contra-intuitive result...”*

*-Written one early morning, or late night, during hard work, Andreas Hjelm*

## Table of content

<b>Abstract .....</b>	<b>i</b>
<b>Acknowledgments.....</b>	<b>ii</b>
<b>Introduction .....</b>	<b>1</b>
<b>1.1 Background .....</b>	<b>1</b>
<b>1.2 Objective and outline.....</b>	<b>5</b>
<b>Theoretical framework .....</b>	<b>7</b>
<b>2.1 The Merton model .....</b>	<b>7</b>
<b>Data .....</b>	<b>11</b>
<b>3.1 Fundamental data .....</b>	<b>11</b>
<b>3.2 Bond .....</b>	<b>13</b>
<b>3.3 Interest rate .....</b>	<b>14</b>
<b>Methodology.....</b>	<b>15</b>
<b>4.1 Software .....</b>	<b>15</b>
<b>4.2 Balance sheet .....</b>	<b>15</b>
4.2.1 Balance sheet - Theoretical .....	16
4.2.2 Balance sheet- Real data .....	18
<b>4.3 Interest Rate .....</b>	<b>20</b>
<b>4.4 Applied Merton model.....</b>	<b>21</b>
<b>Results.....</b>	<b>22</b>
<b>5.1 Volatility .....</b>	<b>22</b>
<b>5.2 Default probability.....</b>	<b>24</b>
<b>Conclusion and Discussion.....</b>	<b>26</b>
<b>Bibliography.....</b>	<b>27</b>
<b>Appendix .....</b>	<b>29</b>
<b>A Stochastic analysis and pricing theory .....</b>	<b>29</b>
<b>B Interpolated risk free rate .....</b>	<b>34</b>
<b>C Balance sheet .....</b>	<b>35</b>
C1 Asset .....	35
C2 Equity .....	36
<b>D Bond Price- Last price.....</b>	<b>37</b>
<b>E Historical market capitalization .....</b>	<b>38</b>

# Chapter 1

## Introduction

In this paper we study the Merton model, or more specifically, use the Merton model to estimate the default risk of Ford Motor Company during the financial crisis, 2007 to 2009. The purpose of this paper is to show how well the Merton model predicts corporate default for the period 2008-2009. The reason for choosing Ford is that the American automotive industry suffered from a dramatic drop in car-sales and many companies, among them Ford, needed to apply for emergency loans. Ford is one of the major corporates in the American automotive industry and therefore it seemed as a good choice of a firm in financial distress.

We conduct this study by firstly extracting key values on the solidity of Ford from the accounting notes. Secondly, these numbers is adjusted to fit the Merton model in such a way that the default probability can be estimated. Finally, the predicted default probabilities are analyzed. The motivation for this thesis is the lack of detailed documentation on how to implement the Merton model on real data for the purpose of predicting corporate default. Many papers has been written on the theory of the Merton and many of these papers also study the performance of the model; however, we have found very little documentation on how to actually apply the Merton model on real data.

## 1.1 Background

It is essential for an economy that firms and households can choose an appropriate level of risk for their financial transactions, since individuals have different risk attitudes. The price of a bond is closely related to the risk an investor is exposed to when purchasing the bond and the risk of the bond is related to the default risk of the firm emitting the bond. In 1974 Robert C. Merton published a paper providing a closed form solution for the price of risky corporate bonds, for this work he was awarded the Nobel Prize in economics in 1997. There are many different types of risk connected to investing in a corporate bond. In this thesis we are, however, only concerned with the risk of corporate default.

We will now give a brief presentation of the three big rating companies: Standard and Poor's, Moody's and Fitch Ratings (Pichler, Hornik, Leitner, Grün & Hofmarcher, 2013). It seems relevant to mention these firms since they pioneered the theory of credit rating and default risk estimation and they have an important role in doing unbiased estimations of the

credibility of a company. Further more, the rating of a bond is essential when trying to apply the Merton model, since the choice of bond must be such that the price of the bond is not mainly determined on the liquidity of the firm. The focus in the following section will be on the rating firms historical contribution to the subject of credit rating.

The history of Standard and Poor's starts in 1860, when Henry Varnum Poor published the book: *History of Railroads and Canals in the United States* (Poor, 1860). This book attempts to compile operational and financial information about U.S. railroad companies. In 1906, Luther Lee Blake founded the Standard Statistics Bureau (Standard & Poor's Financial Services LLC, 2014). Blake intended to provide investors with financial information on companies on a frequent basis. In 1941 the two firms merged and became Standard and Poor's (Standard & Poor's Financial Services LLC, 2014).

In 1900 John Moody started to publish his *Moody's Manual of Industrial and Corporation Securities* (Moody's Corporation, 2014). In 1909 Moody introduced a system for rating bonds as part of *Moody's Analyses of Railroad Investments*. The rating system was based on assigning symbols from Aaa through C for highest quality bonds and lowest quality bonds respectively; the mercantile credit-reporting firms inspired him when he constructed this rating system. Moody gathered and analyzed data from US railroad corporates in order to provide independent rating opinions to his readers. This was intended to help investors with their credit risk managing (Moody's Corporation, 2014).

John Knowles Fitch founded the Fitch Publishing Company in 1913 (Fitch Ratings, Inc., 2014). Fitch published financial statistics for the investment industry. In 1924 Fitch introduced the AAA through D rating system, which has now become standard.

In the following section we will give a brief historical presentation of different methods for estimating the default risk of a firm. Four important classes of models will be presented, these are:

- Models based of fundamental analysis
- Empirical models
- Reduced form models
- Structural models

The intention is to shed light on the evolution of bond pricing and risk estimation and these four classes, and combinations of them, constitute the majority of the existing theory on the subject.

Prior to the Merton model, a model using stochastic calculus and risk neutral pricing theory, methods for determining the default probability of corporate bonds relied on fundamental analysis, where the financial statement of a company was used to evaluate the risk an investor was exposed to, when buying bonds. One example of a fundamental analysis based model is Hickman (1958), which describes how specific characteristics of bond issues directly or indirectly are related to the bond quality i.e. the risk of the bond. Key values such as: the margin of safety (ratio of net income to gross income of the companies offering the bond), the times-charges-earned ratio (ratio of income before fixed charges to charges), the lien position, the size of issues and the asset size of obligor are typical such characteristics. Models such as Hickman was built upon and extended in order to standardize and amend the method for default risk estimation; these extended models are examples of what is known as empirical models, which are based on the assumption that it is difficult to mathematically assign the risk of debt in an adequate way. The idea is to calculate a set of values from different company's financial statement. These values are assumed to indicate different aspects of the company's financial health. The values can then be compared to the values of companies that have defaulted on their debt and to the values of companies that have not defaulted on their debt. One example of an empirical model is the well-known Altman model (1968), which uses five simple financial ratios that are used to estimate the risk of default. In his paper Altman reports bankruptcy prediction accuracy between 70% and 90%.

Even though Altman claim a high level of accuracy in bankruptcy predictions, empirical models has been given much critique for their generally poor performance. The unsatisfactory performance resulted in that the methods evolved and the models for evaluating the risk of a corporate bond grew in two main branches: the reduced form models and the structural models, also called firm value models. Reduced form models do not use the financial reports of a company when trying to predict the likelihood of default. Instead, these models take use of existing financial securities on the market to price bonds; assuming that the efficient market hypothesis holds, they argue that the market prices contain all information about the risk of debt.

According to these models, default occurs unexpected and cannot be predicted, since an accurate prediction of a default would imply that a company could act to avoid bankruptcy.

One approach for calculating the probability of a default is to use mathematical simulations with the use of a Poisson process. The Poisson process is a stochastic process and is often used to model rare event, such storms, earthquakes or company default. One of the most well known reduced form models is the Jarrow-Turnbull model (1995). The problem faced when using these models is to find the right intensity of the default process; this, however, is a hard task and the models display modest empirical evidence of actually price bonds successfully.

The difficult procedure and the unsatisfying performance of the reduced form models led researchers to develop a different approach for bond pricing, based on the idea of the efficient market hypothesis and risk neutral pricing theory, often used for option pricing. The first documentation of options is from medieval times, where the contract was written on agricultural products. These contracts most likely was priced using experience and common sense and it took a long time to find a theory that could rather satisfyingly price options.

In 1900 Louis Bachelier presented a model to find the price of a call option contract (Bachelier, 1900). This model assumed that the price-process of a stock could be modeled with a Brownian motion-process. The big break-thru in option pricing came in 1973 when Fisher Black and Myron Scholes presented their celebrated model (Black & Schoels, 1973). This model assumed that a stock price could be modeled with a geometrical Brownian motion. Independently from Black and Scholes, worked Robert C. Merton (1974) on a bond-pricing model. Merton found a model to price risky bonds using the exact same model that Black and Scholes used to price options. In comparison to Black and Scholes the Merton model suffers from the fact that asset values cannot be observed in the market in the same way that a stock price can. Further, the Merton model is developed for zero-coupon bonds that are rather unusual in the exchange market. This is why the model has been further developed by many authors to better match a real market. Zhou (1997) has developed an alternative to the Merton using jump-diffusion processes, furthermore Hilberink and Rogers (2002) and Mason and Bhattacharya (1981) contributed with some of most well known models using jump-diffusions processes. Also, in the real world flat interest rate curves are seldom found, a stochastic interest rate is more likely and in “Some Empirical Estimates of the Risk Structure of Interest Rates” (Shimko, David, Naohiko, & Van Deventer, 1993) the authors proposed an extended model to handle that.



## 1.2 Objective and outline

We have used the Merton model to estimate the default probability of Ford during the period of financial distress, 2008 and 2009. The estimates have been calculated once a week for a year (52 weeks), with data from Ford's quarterly financial reports. The data is collected from Bloomberg and has been processed (*chapter 4*) for the application of the model.

The method we use, to apply the Merton model, is to assume that our bond is the only issued bond by the firm. This means that we assume that the bond represents the value of all bonds issued by the firm. We use historical market prices of the bond and key values from the financial reports to calculate the price of the bond, assuming it represents the total debt of Ford. The Merton model is developed to find the price of a bond. In this thesis we use the price of the bond to find the volatility of the assets, which is the only unknown parameter we need to finally find the default probability. This can be accomplished by inverting the Merton model.

We conduct this study with the belief that a model with few input parameters is more effective than a model with many input parameters, when trying to predict default during a financial crisis. We base this belief on the fact that models tend to be more sensitive, the more input parameters they have. Hence, for a period of financial distress, the models with fewer input parameters would be more robust.

The reason for choosing the Merton model is that it is the most fundamental of the firm value models, with the smallest set of input parameters. We choose the Merton model prior to e.g. a fundamental empirical model because the Merton model performs better than models such as Altman (Hillegeist, Keating, Cram & Lundstedt, 2004).

With this thesis we intend to do a qualitative study of the effectiveness of predicting default during a period of financial distress, by applying the Merton model on real data from Ford.

This thesis is outlined in the following way:

*Chapter 2:* Presents the Merton model.

*Chapter 3:* Presents the data.

*Chapter 4:* Presents how we processed and adapted our data to fit the Merton model.

*Chapter 5:* Presents the results.

*Chapter 6:* Presents our conclusions.

# Chapter 2

## Theoretical framework

In this chapter we will present the Merton model. The intention is to give the reader an intuitive understanding of the model and those properties of the model that are important for understanding this thesis. The first part of this chapter covers the underlying assumptions of the Merton model and the second part of the chapter deals with the pricing formulas and the formula for finding the default probability. Depending on the reader's prior knowledge in mathematics, this chapter can appear rather technical. For this reason, we provide *Appendix A*, as a sort of crib.

### 2.1 The Merton model

In 1974 Merton proposed a model that provides a closed form solution for calculating the price of default-able bonds. The model relies on a number of assumptions made by Merton, which will now be stated and explained.

The first assumption concerns the capital structure of the bond-issuing firm. It is assumed that a company can only raise money by issuing two types of securities: equity and debt. The total asset value of the firm is thus the sum of the equity and the debt. Hence, mathematically it must hold that:

$$\text{Assets} = \text{Equity} + \text{Debt} \qquad \text{Equation 1.a}$$

which in symbols will be written:

$$V_t = S_t + B_t \qquad \text{Equation 1.b}$$

where the index  $t$  indicates that this equality must hold for each given time. Furthermore, the equity is assumed to not pay dividend during the life of the bond.

Next, the assumptions concerning the market efficiency are the same as those in the setting of the standard Black-Scholes model. The Black-Scholes model assumes a market with continuous trading which is competitive and frictionless in the sense that:

- Trading in assets does not affect the asset prices (agents are price takers).
- There are no transaction costs.
- You have unlimited access to short selling.

- You can borrow and lend money in the market at the fixed risk-free interest rate  $r$ .

Given this last assumption we can conclude that in the risk-free money market the evolution of price is deterministically given by:

$$\beta_t = e^{rt} \quad \text{Equation 2.a}$$

Equivalently, discounting in the risk-free money market is given by:

$$\beta_t^{-1} = e^{-rt} \quad \text{Equation 2.b}$$

We will now present the assumption made on the underlying stochastic price-process. This means that we determine the type of random process we wish to use for modeling the firms asset value. Merton assumed that the asset value follows a geometrical Brownian motion. This is a stochastic differential equation of the form:

$$dV_t = \mu V_t dt + \sigma V_t dW_t^{\mathbb{P}} \quad \text{Equation 3}$$

where  $W_t^{\mathbb{P}}$  is the standard Brownian motion defined on the probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . Consult *Appendix A* for the details regarding Brownian motion and probability space. The solution of this stochastic differential equation can be found using Ito's lemma. The solution is given by:

$$V_t = V_0 e^{\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t^{\mathbb{P}}\right)} \quad \text{Equation 4}$$

Consider a contingent claim on the asset with payoff  $H(V_T)$  at the maturity date  $T$ . We seek a fair price  $h(t)$  of that contract at the time  $t$ . The conditional expectation of a random variable  $Y$  with respect to the filtration  $\mathcal{F}_t$  (the information generated by  $W_t$  up to time  $t$ ), can be expressed as:

$$E[Y|\mathcal{F}_t] \quad \text{Equation 5}$$

Since we are in a Black-Scholes setting it can be shown that there exists a risk-neutral probability measure  $\mathbb{Q}$ , such that:

$$h(t) = E^{\mathbb{Q}}[e^{-r(T-t)}H(V_t)|\mathcal{F}_t] \quad \text{Equation 6}$$

and  $h(t)e^{-rt}$  is a martingale (a process for which the expected future value equals the present value) under  $\mathbb{Q}$  with regards to the filtration  $\mathcal{F}_t$ . This is a very important result since it implies that there are no arbitrage opportunities in the market. The “no arbitrage“ result is an

assumption of the model and must not be interpreted as an absence of arbitrage in the real world.

The asset process  $V_t$  under the risk-neutral probability measure  $\mathbb{Q}$  is given by:

$$V_t = V_0 e^{\left( \left( r - \frac{1}{2} \sigma^2 \right) t + \sigma W_t^{\mathbb{Q}} \right)} \quad \text{Equation 7}$$

where  $W_t^{\mathbb{Q}}$  is a Brownian motion under the risk-neutral probability measure  $\mathbb{Q}$ . Note that under the risk-neutral measure we have successfully eliminated  $\mu$  from the equation and we can instead use the risk-free interest rate  $r$ . This achievement is of great importance when one should apply the model since the risk-neutral interest rate can easily be observed in the market, whereas the  $\mu$  parameter is very complicated to evaluate. The change in parameters is due to mathematical analysis and will be of no further interest in this thesis.

The payoff-function for a risky bond can be expressed as  $B_T = \min(D, V_T) = D - \max(D - V_T, 0)$  and the payoff-function for equity as  $S_T = \max(V_T - D, 0)$ . Thus, the bond is a portfolio containing a short position in a put-option contract written on the asset value with strike-price equal to the debt. Similarly, equity is a call-option contract written on the asset value with strike-price equal to the debt.

The present value of the bond and the equity can now be expressed as:

$$B_t = D e^{-r(T-t)} - P^{BS}(V_t, D, \sigma, r, T - t) \quad \text{Equation 8}$$

$$S_t = C^{BS}(V_t, D, \sigma, r, T - t) \quad \text{Equation 9}$$

where  $P^{BS}$  notates the present value of the put-option formula and  $C^{BS}$  notates the present value of the call option.

The Black-Scholes model provides a closed form solution for the call-option  $C^{BS}$ :

$$C^{BS}(V, D, T, \sigma, r) = V_t N(d_1) - D e^{-r(T-t)} N(d_2) \quad \text{Equation 10}$$

where  $N$  is a standard normal cumulative distribution function. We can now use the put-call parity:

$$P^{BS}(V_t, D, \sigma, r, T - t) = C^{BS}(V, D, T, \sigma, r) + D e^{-r(T-t)} - V_t \quad \text{Equation 11}$$

to find the price on the bond as:

$$B_t = V_t - C^{BS}(V, D, T, \sigma, r) \quad \text{Equation 12}$$

or, directly as:

$$B_t = V_t(1 - N(d_1)) - De^{-r(T-t)}N(d_2) \quad \text{Equation 13}$$

where  $d_1$  and  $d_2$  are given by:

$$d_1 = \frac{\log\left(\frac{V_t}{D}\right) + rT + \frac{1}{2}\sigma^2T}{\sigma\sqrt{T}} \quad \text{Equation 14}$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad \text{Equation 15}$$

The default probability under the risk-neutral probability measure  $\mathbb{Q}$  is given by:

$$\mathbb{Q}[V_T < D] = N\left(\frac{\ln\left(\frac{D}{V_0}\right) - \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) \quad \text{Equation 16}$$

The price of a bond under the Merton model has the following dependence on its parameter.

- It is increasing in the asset value  $V_t$
- It is increasing in the face value  $D$
- It is decreasing in the risk free interest rate  $r$
- It is decreasing in time-to-maturity  $T - t$
- It is decreasing in the asset volatility  $\sigma$

The inverted relation between volatility and bond-price is due to the short position in the call-option. Thus, a long position in a call-option increase in value as volatility increase.

# Chapter 3

## Data

In the following chapter we are going to present the data that we use in our calculations. The data has been collected from Bloomberg, which is a market-leading corporation in providing financial software tools. It is important to keep in mind that although some of the data is collected on a quarterly basis the calculations are done on a weekly basis, for 52 weeks. The intention of this chapter is not to list all the gathered data, but to explain in what way the data has been used in the application of the Merton model. Chapter 4 covers the details on the calculations.

*For a complete list of the data the reader is advised to consult appendix B-D.*

Section 3.1 will cover the data from the balance sheet of Ford. Section 3.2 will cover the data of the chosen bond. In Section 3.3 the data for the generic US government bond, used as risk-free interest rate, is presented.

### 3.1 Fundamental data

This section covers the data collected from the financial reports of Ford. The data consists of the four quarterly financial published from 2008-06-30 to 2009-03-31. The publishing dates for all financial reports are listed in *Table 1*. The items from the financial reports that are used in this thesis are listed in *Table 2*. The details on how these items have been used can be found in Section 4.2.

*Table 1 Publishing dates of financial reports*

Date	Quarter
<b>2008-06-30</b>	CQ2
<b>2008-09-30</b>	CQ3
<b>2008-12-31</b>	CQ4
<b>2009-03-31</b>	CQ1

*Publishing dates of the four quarterly financial reports. CQ indicates that the reports refers to the passed calendar quarters.*

The historical market-cap item in *Table 2* is not given in the financial report but is calculated by multiplying historical stock prices with the total number of outstanding stocks once every quarter (Appendix E). The total equity item is itself composed of three items as shown in *Table 2*.

*Table 2 List of key value identifiers from the balance sheet used in the calculations*

<b>Description</b>	<b>Bloomberg identifier</b>
Short term Cash items	BS_CASH_NEAR_CASH_ITEM
Market securities	BS_MKT_SEC_OTHER_ST_INVEST
Account notes	BS_ACCT_NOTE_RCV
Total Equity	TOTAL_EQUITY
Accounts payable	BS_ACCT_PAYABLE
Total assets	BS_TOT_ASSET
Total liability and equity	TOT_LIAB_AND_EQY
Historical market Cap	HISTORICAL_MARKET_CAP

*These are the Bloomberg identifier codes we have used in order to find our key values.*

*These values are used in Chapter 4.2.2.*

The data presented in this section is used for three parts of the application of the Merton model:

- For calculating the asset value
- For calculating the equity value
- For calculating the value of the debt

These three parts constitute all the terms in *Equation 1.b*.



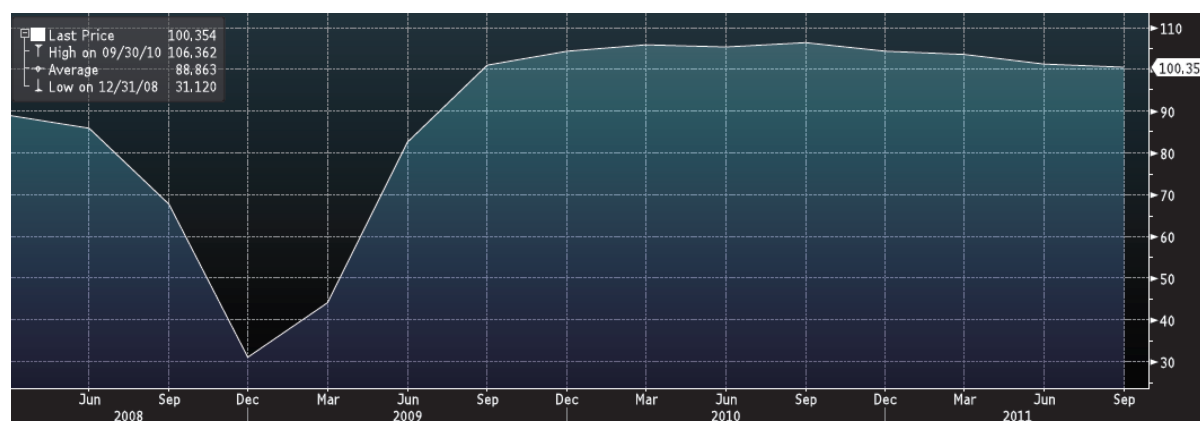
### 3.2 Bond

To apply the Merton model we must choose a corporate bond such that the value of the bond not only depends on the interest rate and the value of the firm but also on the markets expected risk. We also require that the bond should not have a long time to maturity, in order to have a principal value that dominates the value of the bond.

The name of the bond we use is F 9 ½ 09/15/11 with code 345370AZ Corporate bond.

The F in the beginning of the name tells us that it is a Ford US Motor bond. The 9 ½ means that it is a 9,5% coupon rate. 09/15/11 indicates the maturity date of the bond, in our case 15 September 2011. The bond has a semi-annually coupon payment and is therefore paid out once every 6 month, with a rate of 4,75%.

*Figure 1 – Market bond price, Bloomberg chart*



*The data is collected from Bloomberg. Although the bond has matured we managed to use Bloomberg's SRCH command to find historical values of the bond. The data was collected as weekly data from 2 July 2008 to 1 July 2009, 52 weeks. The values are in USD/bond.*

The data we have gathered on the bond displays that it is frequently traded (*see Appendix D*); it is important for the accuracy of the results to have a frequently traded bond. *Figure 1* shows the evolution of the price from May 2008 to September 2011.

The bond data is used in *Equation 13* and *Equation 14* for parameter D. The bond price is converted into a multiplier and multiplied to the total asset and liability so we can have the market value, which can fulfill the assumption that the bond is the only bond issued on the firm's debt.

### 3.3 Interest rate

For the risk-free interest rate we use a two- and a five-year generic US government bond. The reason for using a generic bond and not a treasury bill is that the rates are comprised of Generic United States on-the-run government bill/note/bond indices. This will give us a more accurate risk-free interest rate.

The risk-free interest will be used for two parts of the calculations:

- For discounting the coupons of the Ford bond in order to make the bond better match the requirement of a zero-coupon bond in the Merton model.
- Directly in the application of the Merton model e.g. the default probability formula (*Equation 16*).

The data is collected quarterly from 30 June 2008 and 4 quarters forward, ending 31 Mars 2009. The values of the risk-free interest rates are given in *Table 3*.

*Table 3-Risk-free interest rates*

Date	USGG2YR	USGG5YR
2009-03-31	1,6551	0,7960
2008-12-31	1,5489	0,7643
2008-09-30	2,9793	1,9599
2008-06-30	3,3271	2,6164

*Interest rates, given in %, that are used as risk-free interest rates in the application of the Merton model.*

In chapter 4 we explain how linear interpolation of the values in *Table 3* is used to approximate the risk-free interest rate for each week we calculate the default probability.

# Chapter 4

## Methodology

When pricing an option with the Black-Scholes model, the underlying stock-value can easily be observed on the market. This is, however, not the case when pricing bonds with the Merton model. Here, the underlying price-process is the value of the firm assets. Thus, the asset value must be estimated. Further, since the Merton model relies on assumptions of a very simple capital structure of the firm, we must make some approximations before the model can be applied.

This chapter outlines the method we have used to evaluate the default probability of Ford. The calculations have been done for 52 weeks. The first week is indexed 0 and the last week is indexed 51. Section 4.1 covers the Software that we have used for our calculations. Section 4.2 presents how the data from the balance sheet is adapted to fit the application of the Merton model. To make the presentation clear we give a theoretical example of the procedure before we tackle the real data. Section 4.3 presents the procedure for finding risk-free interest rate for each period. Section 4.4 covers the application of the Merton model.

### 4.1 Software

For our calculations we have used Excel together with the solver-add in by Frontline Systems, Inc. The solver uses a variety of methods, from linear programming and nonlinear optimization to genetic and evolutionary algorithms, for finding the solution of a problem.

### 4.2 Balance sheet

In order to use the Merton model, we must make the assumption that all debt is identical. This implies that we consider all debt of the company to be written on one single bond. We also make the assumption that no dividend payments are allowed during the life of the bond.

The main subject in this section is a step-by-step explanation of how to find the appropriate data from the balance sheet. We start this chapter by doing a theoretical case; this is outlined with made up numbers to clearly display what is going on. In reality the identification and categorization of numbers is more complex. Chapter 4.2.2 will guide you, in the same manner, how this is done on a real data.

### 4.2.1 Balance sheet - Theoretical

First we start with a theoretical example. These are made up numbers just to give the reader some prerequisites for our real data scenario.

First off we setup a balance sheet with assets on one side and liabilities on the other as see on in *Table 4*. Cash-like items are every item on the firm that is liquid or near liquid, in contrast to other assets that is assets whom can be convertible into cash within one business cycle.

*Table 4 Balance sheet, step 1*

<b>Balance Sheet – Book Value</b>			
<i>Cash-like items</i>	100	200	<i>Short-term</i>
		500	<i>Other debt</i>
<i>Other assets</i>	900	300	<i>Equity</i>
<b>Assets</b>	<b>1000</b>	<b>1000</b>	<b>Liabilities</b>

*Balance sheet with made up numbers, these numbers are found to give us rounded numbers.*

On the liability side there is 3 items, short-term debt, other debt and equity. Short-term debt is all of a company’s debt that is due within one year. Other debt is long term debt, meaning everything more than one year. The equity item is all of the outstanding shares times the market value on the shares.

We now explain the method of finding asset value and bond value from the balance sheet. The first step of this procedure is to aggregate all short-term assets e.g. cash, receivables and payables.

The calculations of the next step as seen on *Table 5*, is to net all cash-like items from the balance sheet. This is done since we do not intend to price the bond on any short-term assets.

*Table 5 Balance sheet, step 2*

<b>Balance Sheet – Book Value – “No Cash items”</b>			
<i>Cash like items</i>	0	100	<i>Short-term</i>
		500	<i>Other debt</i>
<i>Other assets</i>	900	300	<i>Equity</i>
<b>Assets</b>	<b>900</b>	<b>900</b>	<b>Liabilities</b>

*Cash like items is settled against short-term items.*

The last step is to calculate the market value of the assets from the observed market value of equity, as seen in *Table 3*.

*Table 6 Balance sheet, step 3*

<b>Balance Sheet – Market Value</b>			
<i>Cash like items</i>	0	0	<i>Short-term</i>
		600	<i>Other debt</i>
<i>Other assets</i>	1200	600	<i>Equity</i>
<b>Assets</b>	<b>1200</b>	<b>1200</b>	<b>Liabilities</b>

*As one can see the entire cash like items are settled so that asset are equal to liabilities.*

## 4.2.2 Balance sheet- Real data

This section contains a detailed guide of how we have extracted the values for assets, debt and equity from the balance sheet.

Step 1: All short term items are identified from the balance sheet. See *Table 2* for a complete list of identifiers.

*Table 7 Sample data from Q2, 2008*

Identifier	Value
BS_CASH_NEAR_CASH_ITEM	30066
BS_MKT_SEC_OTHER_ST_INVEST	19872
BS_ACCT_NOTE_RCV	5116
TOTAL_EQUITY	-3229
BS_ACCT_PAYABLE	24216
BS_TOT_ASSET	270450
TOT_LIAB_AND_EQY	270450
HISTORICAL_MARKET_CAP	10913,89

*Sample data from quarter 2, 2008. This is the data used for this section. Values are in Million USD.*

In table 8 we start by defining short-term assets and short-term debt. The values we find from the balance sheet are then added. This is done to simplify further computations.

*Table 8 Short-term posts*

Short term			
<i>Cash, near cash item</i>	30066	24216	<i>Accounts and payables</i>
<i>Market securities and other investments</i>	19872		
<i>Account notes and receivables</i>	5116		
<b>Short-term assets</b>	<b>55054</b>	<b>24216</b>	<b>Short-term debt</b>

*Short-term items are extracted from balance sheet and added to two posts either short-term assets or short term debt. This is done to simplify further calculations. . Values are in Million USD.*

Step 2: We use the added values of the short-term items from *Table 8* and categorize it as assets or liabilities and identify the value of account and payables and categorize it under liability as seen in *Table 9*.

*Table 9 Real data, book value*

<b>Balance Sheet – Book-value</b>			
<i>Cash-like items</i>	55054	24216	<i>Short-term</i>
		249463	<i>Other debt</i>
<i>Other assets</i>	215396	-3229	<i>Equity</i>
<b>Assets</b>	<b>270450</b>	<b>270450</b>	<b>Liabilities</b>

*Book values are categorized and summed up to total assets and liabilities.*

*Values are in Million USD.*

Step 3: We add the account and payables seen in *Table 8* with short-term items within assets, which can be seen in *Table 10*. Also we add the total equity to the item other debt.

*Table 10 Balance sheet, no cash items*

<b>Balance Sheet – Book Value – “No Cash items”</b>			
<i>Cash like items</i>	30838	0	<i>Short-term</i>
		243005	<i>Other debt</i>
<i>Other assets</i>	212167	0	<i>Equity</i>
<b>Assets</b>	<b>243005</b>	<b>243005</b>	<b>Liabilities</b>

*Equity is added to other debt and short-term assets are adjusted against cash like items.*

*This step is conducted to make it possible to apply the Merton model, because bonds are long-term securities.*

Step 4: Last step is to find the bond price for the corresponding week by using Bloomberg. In this particular week (*week 0 for our calculations*) the closing price is: 0,78971. This value is then multiplied to all the items to acquire the market value of debt as seen in *Table 11*.

*Table 11 Market values*

<b>Balance Sheet – Market Value</b>			
		191903,4	<i>Market bond value</i>
<i>Other assets</i>	202817,4	10914	<i>Market equity value</i>
<b>Assets</b>	<b>202817,4</b>	<b>202817,4</b>	<b>Liabilities</b>

*This table gives us the market values; this is accomplished by multiplying the bond closing price to the values.*

### 4.3 Interest Rate

The Merton model assumes that there are no coupon payments; bonds of this character are called zero-coupon bonds and are rarely observed in the corporate bond market. The interest rate presented in section 3.3 is used to discount all the coupon payments of the Ford bond in order to be able to regard the bond as a zero-coupon bond. The calculation procedure is as follows:

1. Calculate the time to each coupon payment
2. Discount each coupon with the coupon rate and interest rate from section 3.2 and 3.3 respectively
3. Step forward one week and start over from step 1

This is done for 52 weeks and the result is vector of bond prices.

The interest rate is a parameter in the default probability formula (Equation 16). To calculate the default probability for each of the 52 weeks, we follow the same basic principle as above:

1. Calculate the time to maturity of the bond
2. Calculate the default probability
3. Step forward one week and start over from step 1

The result is a vector of default probabilities.

Since the interest rate differs for different maturities, we must find the interest rate that corresponds to each of these time variables. This is done using linear interpolation of the values in table 3, section 3.3. For every week we get a set of interest rates that correspond to each of the coupon payments and the maturity of the bond.

By letting the interest rate take different values, we violate the assumption of constant interest rate in the Merton model. We motivate this by the fact that the bond is not really a zero-coupon bond and to discount the value of the coupons with the same interest rate would be to simplify the problem drastically. Also, since we study the default probability for an entire year, it would be naive to consider the interest rate to be constant during that period.



#### 4.4 Applied Merton model

In this section we explain how the default probability is calculated. We will follow the procedure in section 4.3 to do the calculations for each week. The first step is to calculate the discounted value of all future cash flows. Thus, all future coupon payments and the face value of the bond are discounted using the formula:

$$B = c_k e^{-rt} \left[ N(d_2) + \frac{N(-d_1)}{a} \right] \quad \text{Equation 17}$$

where  $c_k$  is the value of the coupon payment or the face. The debt-to-asset ratio  $a$  is given by:

$$a = \frac{X e^{-rt}}{A} \quad \text{Equation 18}$$

where  $A$  is the asset value and  $X$  is the value of the debt. In contrast to equation 16, we here use  $A$  and  $X$  instead of  $V_0$  and  $D$  to emphasize that these are the approximated values. The parameters in the cumulative normal distribution function are given by:

$$d_1 = \frac{\ln\left(\frac{A}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \quad \text{Equation 19}$$

$$d_2 = d_1 - \sigma\sqrt{t} \quad \text{Equation 20}$$

The only unknown parameter is the volatility  $\sigma^2$ . To find the value of the volatility the observed bond price is set equal to the sum of all discounted cash flows. By setting observed bond price equal to the discounted sum, we can then use the solver from section 4.1 to force the volatility to take the value that makes Equation 21 hold.

$$P_t = \sum_{k=1}^K B(R_k, t_k) c_k = \sum_{k=1}^K B(r_k, \sigma, t_k, A, X, c_k) \quad \text{Equation 21}$$

This is done for every week and the result is a vector of 52 volatility values. When the volatility is known there is no problem calculating the default probability. The default probability formula given by the Merton model is:

$$\mathbb{Q}[A < X] = N\left(\frac{\ln\left(\frac{X}{A}\right) - \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) \quad \text{Equation 22}$$

The result of these calculations is a vector of 52 default probability values.

# Chapter 5

## Results

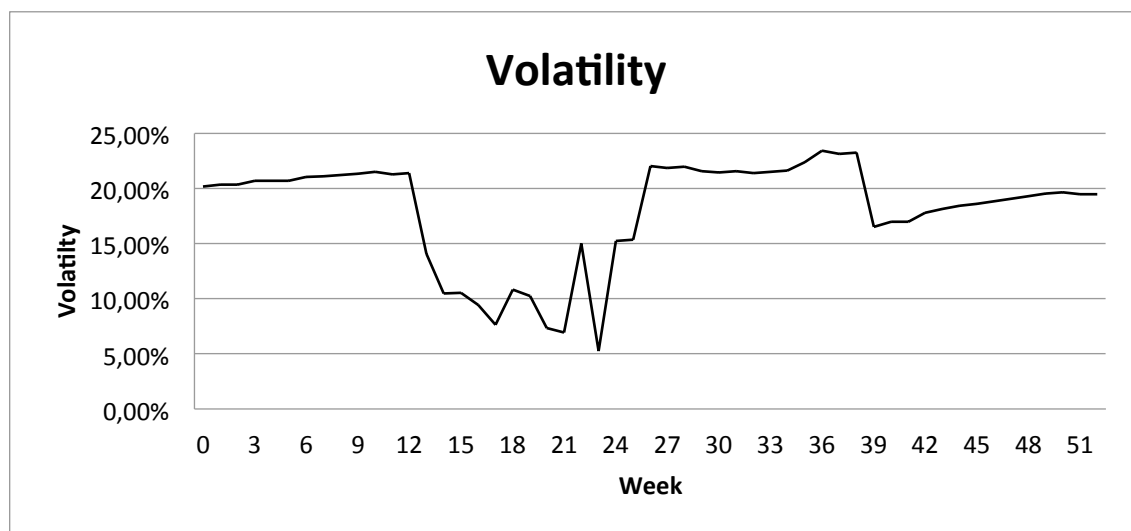
In this chapter the results of our calculations will be presented. In section 5.1 we present and analyze the graph showing our calculated values of the asset volatility. In section 5.2 the graph of estimated default probability values is presented and analyzed. We will try to analyze these results with the premise that we later wish to draw conclusions on the efficiency of the Merton model for periods of financial distress.

### 5.1 Volatility

In this section we will present the result of the asset volatility that is calculated by solving for the  $\sigma$ -parameter in *Equation 21* of Section 4.4.

By studying the graph of asset volatility values (*Figure 2*) we observe the large changes on a quarterly basis. The reason for these drastic changes is due to the publishing of new quarterly financial reports. Based on this periodicity, we will try to analyze the results for each quarter separately.

*Figure 2 – Asset volatility graph*



*Here one can see how the yearly asset volatility changes over time, starting week 0 to 51.*

For the first period (week 0 to week 12), the estimated total asset volatility is over 20%, which to us seems high since we expect such values for the equity volatility, but not for the total asset volatility of seventh biggest company in the USA. However, when studying the

implied equity volatility for the same period of time, we observe that the equity volatility during this period was remarkably high and peaked at a value of about 300%. Even though the equity only constitutes a small part of the total asset value, this gives us an indication of what condition the US market is in.

In the beginning of the second period (week 13 to week 25), the bond price starts to decline from 74.5 at week 12 to 31.12 at week 25 (*Figure 1 and Appendix D*); that kind of decline in the bond price leads us to think that the insecurity on the market and the market volatility should increase, but this idea conflicts with the result in *Figure 2*. The explanation to this outcome is that when the bond price falls, so does the value of the debt. Since we have assumed that all the debt is issued on one single bond these changes in value are equally large. Also, when the debt value decrease, so does the asset value. These changes are, however, not equally large. As the market equity value, in our calculations, is constant during every quarter the debt to asset ratio (*Equation 18*) decrease. Thus, the asset value becomes larger in relation to the value of the debt, which implies that the risk to default decreases.

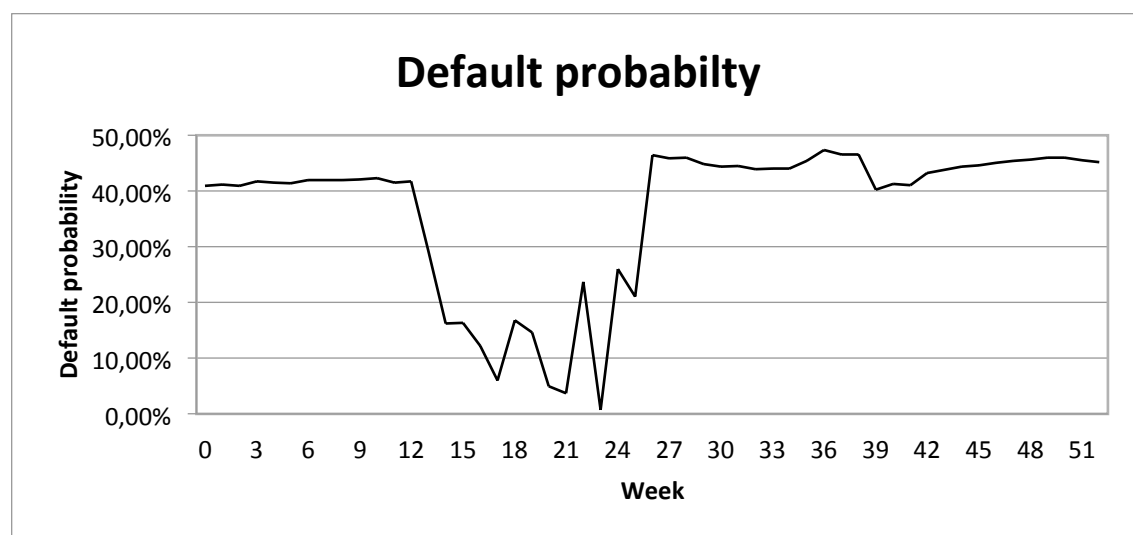
In the third period (week 26 to week 38) we observe a decline in the market equity value (*Appendix E*) of around 50% that make the debt-to-asset ratio increase. At the same time the market price on the bond increases, resulting in an additional increase of the debt-to-asset ratio. These combined events lead to that the asset volatility takes values in the span: 20% to 25% as seen in *Figure 2*.

For the fourth and last period (week 39 to week 51), Ford is not as exposed to drastic changes as for the first three periods. The market value on equity increases slightly, which can possibly explain the initial drop in volatility. The upward sloping tail in the right end can be explained by a successive increase in the bond price.

## 5.2 Default probability

The principal structure of the default probability curve follows that of the volatility curve. This is natural since a higher value of the volatility in the Merton model yields a larger default probability.

*Figure 3- Default probability graph*



*This graph shows the calculated risk of defaulting, within one year, for each of the weeks 0 to 51.*

For the first period (week 0 to week 12) we can observe a default probability that is around 40%, *Figure 3*. Since the default probability is closely correlated to the asset volatility, we note a similar shape of the graph.

For the second period (week 13 to week 25) the default probability is taking values of 0.7% to 26% and displays an erratic behavior. This behavior is hard to explain since we cannot find any major changes in the input parameters.

In the third period (week 26 to week 38), the default probability is around 45% and, as with the case of the asset volatility, this is explained by a change in the market equity value.

For the last period (week 39 to week 51) the Merton model yields default probability values in the span 40% to 45%.

It is hard to comment on if the magnitude of the default probability is high or low. Other authors find that the Merton model tend to underestimate the asset volatility on safe investment and overestimate the asset volatility on risky investments (Ho Eom, Helwege & Huang, 2004). Since we conduct this study during a period of financial distress it seems reasonable to assume that an investment in Ford would be risky. This means that that the asset volatility is overestimated and therefore also the default probability.

# Chapter 6

## Conclusion and Discussion

In this thesis we have tried to make a qualitative study on the effectiveness of predicting default with the Merton model in a period of financial distress. We argued that a model with few input parameters would be more robust than a model with many input parameter in predicting default for a period of high market insecurity. To be able to answer if this truly is the case, one would have to conduct a comparative study on this matter. There are many structural models, extending the Merton model, that take into account more market parameters than the Merton model does e.g. Longstaff-Schwarz (1995). In this thesis we refer to the results in Ho Eom, Helwege & Huang (2004), to be able to get an indication on the authenticity of our initial assumption.

Since we have only conducted this study on Ford, it is hard to say if the Merton model overestimates the default probability or not. It would have been interesting to compare the estimated default probability of Ford with the estimated default probability of a smaller company in order to get an indication of the accuracy and the effectiveness of prediction.

## Bibliography

- Altman, E. I. (1968). Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *Journal of Finance* , 23 (4), 589-609.
- Bachelier, L. (1900). Théorie de la spéculation. *Annales scientifiques de l'Ecole normale supérieure* 17 , 21 (8).
- Black, F., & Schoels, M. (1973). The pricing of options and corporate liabilities. *The Journal of Political Economy* , 81 (3), 637-654.
- Braddock Hickman, W. (1958). *Corporate Bond Quality and Investor Experience*. Massachusetts: National Bureau of Economics Research, Inc.
- Chacko, G. C., Sjöman, A., Motohashi, H., & Dessain, V. (2006). *Credit Derivatives: A Primer on Credit Risk, Modeling and Instruments*. Upper Saddle River, New Jersey, USA: Wharton School Publishing.
- Fitch Ratings, Inc. (2014, 01 29). *Fitch Ratings, About us*. Retrieved 01 29, 2014, from Fitch Ratings: <https://www.fitchratings.com/web/en/dynamic/about-us/about-us.jsp>
- Herwig, L., & Patricia, L. (2008). *The Rating Agencies and Their Credit Ratings: What They Are, How They Work, and Why They are Relevant*. England: Wiley Finance.
- Hilberink, B., & Rogers, L. (2002). *Optimal capital structure and endogenous default*. Bath, UK: Springer-Verlag.
- Hillegeist, S. A., Keating, E. K., Cram, D. P., & Lundstedt, K. G. (2004). Assessing the probability of Bankruptcy. *Review of Accounting Studies* , 9 (1), 5-34.
- Ho Eom, Y., Helwege, J., & Huang, J.-Z. (2004). Structural Models of Corporate Bond Pricing: An Empirical Analysis. *Review of Financial Studies* , 17 (2), 499-544.
- Jarrow, A. R., & Turnbull, M. S. (1995). Pricing derivatives on financial securities subject to Credit risk. *Journal of finance* , 1 (1), 53-85.
- Jarrow, R. A., & Protter, P. (2004). Structural versus reduced form models: A new information based perspective. *Journal of Investment Management* , 10 (2), 1-10.
- Lando, D. (2004). *Credit risk modeling, Theory and application*. Princeton and Oxford, UK: Princeton University Press.
- Longstaff, F. A., & Schwartz, E. S. (1995). A simple approach to valuing risky fixed and floating rate debt. *The Journal of Finance* , 50 (3), 789-819.
- Mark, M. (2012). *Bonds, an introduction to the core concepts*. Singapore: John Wiley & Sons.
- Mason, S. P., & Bhattacharya, S. (1981). Risky debt, jump processes, and safety covenants. *Journal of financial Economics* , 9 (3), 281-307.
- Merton, R. C. (1974). On the pricing of corporate rate: The risk structure of interest rates. *The Journal of Finance* , 29 (2), 449-470.

Moody's Corporation. (2014, 01 29). *Moody's History*. Retrieved 01 29, 2014, from Moody's Corporation: <https://www.moodys.com/Pages/atc001.aspx>

Pichler, S., Hornik, K., Leitner, C., Grün, B., & Hofmarcher, P. (2013). Deriving consensus ratings of the big three rating agencies. *Journal of Credit Risk* , 9 (1), 75-98.

Poor, H. V. (1860). *History of the Railroads and Canals of the United States America*. New York: J.H. Schultz & Company.

Shimko, David, Naohiko, T., & Van Deventer, D. (1993). Some Empirical Estimates of the Risk Structure of Interest Rates. *Journal of Finance* , 44 (5), 1351-1360.

Standard & Poor's Financial Services LLC. (2014, 01 29). *A history of Standard & Poor's*. Retrieved 01 29, 2014, from Standard & Poor's: <http://www.standardandpoors.com/about-sp/timeline/en/us/>

Zhou, C. (1997). *A Jump-Diffusion Approach to Modeling Credit Risk and Valuing Defaultable Securities*. Peking: Peking University - Guanghua School of Management - Finance.



## Appendix

### A Stochastic analysis and pricing theory

This section shortly reviews the basics of stochastic analysis and pricing theory, necessary to fully understand the rest of the text. The starting point is the concept of stochastic processes. In order to give this presentation a fairly adequate mathematical meaning, we must however set up the space in which the processes are defined.

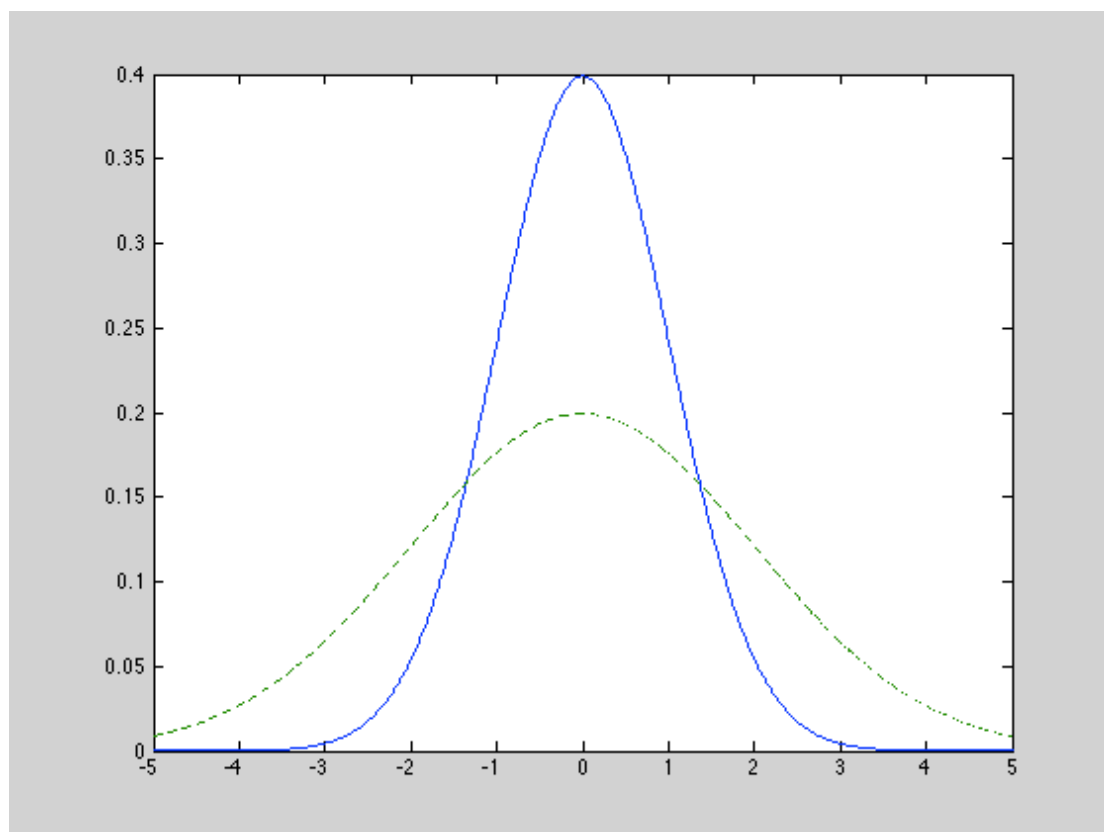
We always consider the time space  $T$  to be finite i.e.  $T < \infty$ . The space of all possible random outcomes is denoted by  $\Omega$ . On this space  $\Omega$  we must now define a sigma-algebra. A sigma-algebra is a family of subsets of  $\Omega$ , with the property that it is closed under countable infinite unions and intersection. In addition if a set  $A$  is a subset of  $\Omega$  and in the sigma-algebra, so is its complement  $A^c$ . The sigma-algebra is denoted by  $\mathcal{F}$ . The probability of an event in  $\mathcal{F}$  is given by the probability  $\mathbb{P}$ . We are now almost done defining our space  $(\Omega, \mathcal{F}, \mathbb{P})$  called a probability space. What is left is to equip the space with a filtration. A filtration is a family of sub-sigma-algebras that are non-decreasing, meaning that  $\mathcal{F}_s \subset \mathcal{F}_t$  for  $s < t$ . The filtration  $\mathcal{F}$  is assumed to be  $\mathbb{P}$ -complete, a condition meaning that if  $B \in \mathcal{F}$ ,  $\mathbb{P}(B) = 0$  and  $A \subset B$  then  $A \in \mathcal{F}$ .

A stochastic process  $X = \{X_t, 0 \leq t \leq T\}$  is a collection of random variables defined on a complete probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . When a stochastic process has the property that for each  $t$ ,  $X_t$  is  $\mathcal{F}_t$ -measurable, we call it adapted to the filtration. This means that the information in  $\mathcal{F}_t$  is enough to resolve  $X_t$ . The two most commonly used stochastic processes are the Brownian motion and the Poisson process. They represent the simplest form of continuous process and jump process respectively. In order to understand the concept of Brownian motion it is fundamental that one is familiar with the Gaussian distribution. The Gaussian distribution, sometimes called Normal distribution, is a bell-shaped distribution. It is symmetric around its mean and has always kurtosis equal to 3. The shape of and position of the Gaussian distribution is solely determined by two parameters.

The mean  $\mu$  determines the horizontal position of the density and the variance  $\sigma^2$  determines how spread out the curve is. The density of the Gaussian distribution is given by

$$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

*Figure 4 Gaussian distribution*



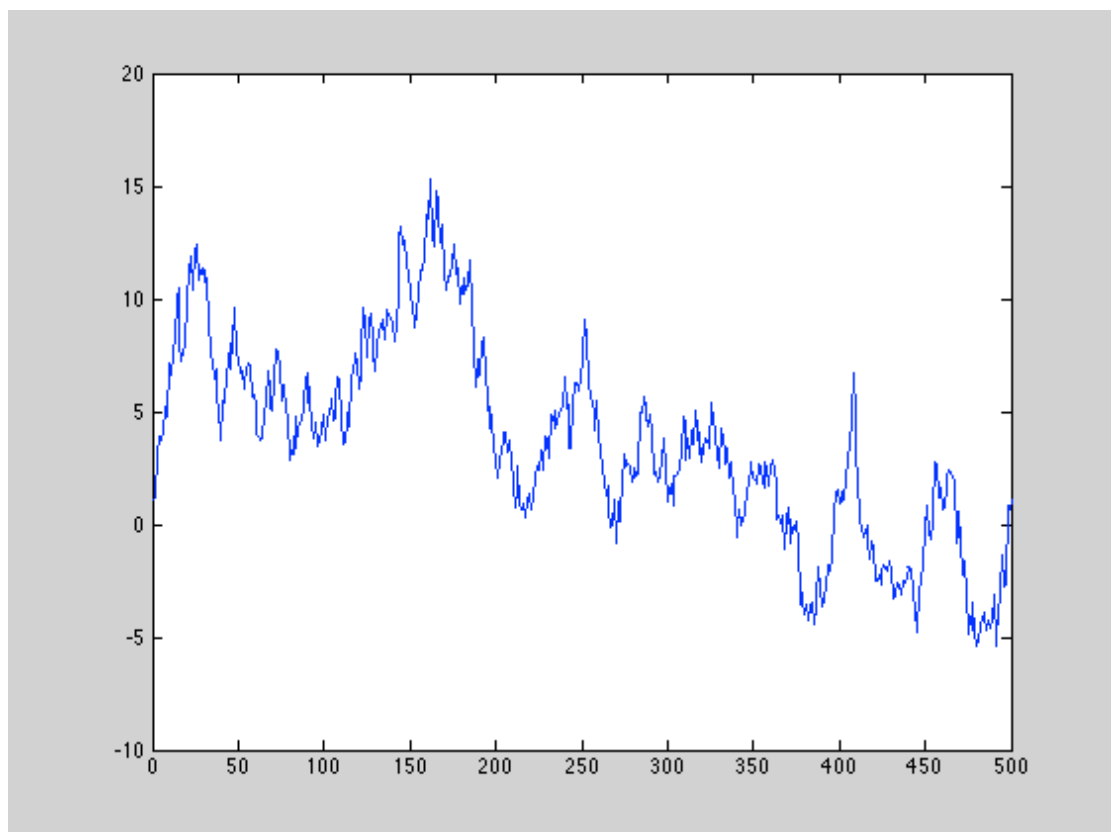
The Gaussian distribution is now given by

$$\Phi(x) = \int_{-\infty}^x f(v; \mu, \sigma^2) dv$$

**Definition 1.1** (Brownian Motion) A stochastic process  $W = \{W_t, t \geq 0\}$  is a Brownian motion if the following conditions hold:

1.  $W_0 = 0$ .
2. The process has stationary increments. This means that the distribution of the increments  $W_{t+s} - W_t$  does not depend on  $t$ , but only on  $s$ .
3. The process has independent increments. This means that for non-overlapping intervals  $l < u$  and  $s < t$  and  $u \leq s$ , the stochastic variables  $W_t - W_u$  and  $W_s - W_l$  are independently distributed.
4. For  $0 \leq s < t$  the stochastic variable  $W_s - W_t$  follows a Gaussian distribution with mean 0 and variance  $t - s$ .

*Figure 5 Brownian Motion*



**Definition 1.2** (Risk-Free Asset) The price process  $B = \{B_t, 0 \leq t \leq T\}$  of a risk-free bond is determined by the differential equation

$$dB_t = r_t B_t dt$$

where  $r = \{r_t, 0 \leq t \leq T\}$  is the short rate. The short rate can either be constant, a deterministic function of time or stochastic process. If the short rate is stochastic we require that it is an adapted process.

**Definition 1.3** (Poisson Process) A stochastic process  $N = \{N_t, t \geq 0\}$  with intensity parameter  $\lambda > 0$  is a Poisson process if the following conditions hold:

1.  $N_0 = 0$ .
2. The process has independent increments
3. The process has stationary increments.
4.  $s < t$  the stochastic variable  $N_s - N_t$  has a Poisson distribution with parameter  $\lambda(t - s)$ :

$$P(N_t - N_s = n) = \frac{\lambda^n (t - s)^n}{n!} e^{-\lambda(t-s)}$$

**Definition 1.3** (Discount factor) We indicate by  $D(t, T)$  the discount factor, determined by

$$D(t, T) = \mathbb{E} \left[ \frac{B_t}{B_T} \right]$$

Where the right hand side is the expected value of the fraction between the bond price at time  $t$  and the bond price at time  $T$ . The discount factor can be thought of as the price of a risk-free zero-coupon bond at time  $t$ , with face value 1 and maturity  $T$ . Solving the differential

equation (1.1) for the three different types of short rate dynamics generates the discount factor for each of the cases respectively.

In the case where the interest  $r$  is constant, the discount factor becomes

$$D(t, T) = \exp(-r(T - t))$$

For the case where  $r$  is a deterministic function of time, there is no stochastic dynamics driving the process of the bond price. The value of  $r_t$  is deterministically known for each time  $t$ . Solving equation (1.1) now yields the discount factor

$$D(t, T) = \exp\left(-\int_t^T r_s ds\right)$$

When the bond price is driven by a stochastic short rate process the discounting factor is found to be

$$D(t, T) = \mathbb{E}\left[\exp\left(-\int_t^T r_s ds\right)\right]$$

## B Interpolated risk free rate

	CQ2 2008	CQ3 2008	CQ4 2008	CQ1 2009					
<b>Time to maturity</b>	0,25	0,75	1,25	1,75	2,25	2,75	3,25		
<b>Risk free rate</b>	0,42%	1,25%	2,08%	2,91%	3,27%	3,15%	2,91%		
<b>Time to maturity</b>		0,5	1	1,5	2	2,5	3		
<b>Risk free rate</b>		0,74%	1,49%	2,23%	2,98%	2,81%	2,64%		
<b>Time to maturity</b>			0,25	0,75	1,25	1,75	2,25	2,75	
<b>Risk free rate</b>			0,19%	0,58%	0,97%	1,36%	1,48%	1,48%	
<b>Time to maturity</b>				0,5	1	1,5	2	2,5	3
<b>Risk free rate</b>				0,41%	0,83%	1,24%	1,66%	1,51%	1,37%

## C Balance sheet

### C1 Asset

Ticker F US Equity  
 Currency USD  
 Name FORD MOTOR CO  
 Start Date 2008-04-01  
 End Date 2013-09-17  
 Periodicity Calendar Quarterly  
 Display Order Chronological  
 Data Type Bloomberg Fundamentals  
 Reported Status Most Recent  
 Consolidation Level Default

Date:		<b>CQ2 2008</b>	<b>CQ3 2008</b>	<b>CQ4 2008</b>	<b>CQ1 2009</b>
Status:		Restated	Restated	Restated	Restated
<b>Cash &amp; Near Cash</b>	<i>BS_CASH_NEAR_CASH_ITEM</i>	30066	24894	22049	21093
<b>Mrktable Sec &amp; Oth ST Invts</b>	<i>BS_MKT_SEC_OTHER_ST_INVEST</i>	19872	17212	17903	20720
<b>Accounts &amp; Notes Rec</b>	<i>BS_ACCT_NOTE_RCV</i>	5116	4623	3065	2694
<b>Inventories</b>	<i>BS_INVENTORIES</i>	12987	12048	6988	6575
<b>Other Current Assets</b>	<i>BS_OTHER_CUR_ASSET</i>	8472	6266	5787	5276
<b>Current Asst Reported</b>	<i>BS_CUR_ASSET_REPORT</i>	76513	65043	55792	56358
<b>Net fixed assets</b>	<i>BS_NET_FIX_ASSET</i>	31909	30027	23930	23590
<b>LT Investments &amp; LT Receivables</b>	<i>BS_LT_INVEST</i>	110776	129634	119221	86713
<b>Oth Asst/Def Chgs&amp;Oth</b>	<i>BS_OTHER_ASSETS_DEF_CHRG_OTHER</i>	51252	21845	24004	40609
<b>Total Assets</b>	<i>BS_TOT_ASSET</i>	270450	246549	222947	207270

## C2 Equity

Ticker F US Equity  
 Currency USD  
 Name FORD MOTOR CO  
 Start Date 2008-04-01  
 End Date 2013-09-17  
 Periodicity Calendar Quarterly  
 Display Order Chronological  
 Data Type Bloomberg Fundamentals  
 Reported Status Most Recent  
 Consolidation Level Default

Date:		<b>CQ2 2008</b>	<b>CQ3 2008</b>	<b>CQ4 2008</b>	<b>CQ1 2009</b>
Status:		Restated	Restated	Restated	Restated
<b>Accounts Payable</b>	<i>BS_ACCT_PAYABLE</i>	24216	20358	13145	12882
<b>ST borrowings</b>	<i>BS_ST_BORROW</i>	62019	59247	43404	55013
<b>Other ST liab</b>	<i>BS_OTHER_ST_LIAB</i>	31219	27764	32374	29417
<b>Current Liabilities</b>	<i>BS_CUR_LIAB</i>	117454	107369	88923	97312
<b>LT Debt</b>	<i>BS_LT_BORROW</i>	104006	97851	109665	90930
<b>Other LT Liabilities</b>	<i>BS_OTHER_LT_LIABILITIES</i>	52219	45869	38886	35505
<b>Total Liabilities</b>	<i>BS_TOT_LIAB2</i>	273679	251089	237474	223747
<b>Minority Interest</b>	<i>MINORITY_NONCONTROLLING_INTEREST</i>	1459	1458	1195	1100
<b>Total common equity</b>	<i>TOT_COMMON_EQY</i>	-4688	-5998	-15722	-17577
<b>Tot Shareldr Eqty</b>	<i>TOTAL_EQUITY</i>	-3229	-4540	-14527	-16477
<b>Tot liab &amp; equity</b>	<i>TOT_LIAB_AND_EQY</i>	270450	246549	222947	207270
<b>Shares Outstanding</b>	<i>BS_SH_OUT</i>	2269	2375	2412	2421



## D Bond Price- Last price

<b>Week</b>	<b>Date</b>	<b>Last Price</b>	<b>Week</b>	<b>Date</b>
0	2008-07-02	78,971	26	2009-01-07
1	2008-07-09	79,424	27	2009-01-14
2	2008-07-16	77,437	28	2009-01-21
3	2008-07-23	82,193	29	2009-01-28
4	2008-07-30	79,85	30	2009-02-04
5	2008-08-06	77,5	31	2009-02-11
6	2008-08-13	80,906	32	2009-02-18
7	2008-08-20	79,751	33	2009-02-25
8	2008-08-27	78,944	34	2009-03-04
9	2008-09-03	78,626	35	2009-03-11
10	2008-09-10	80,045	36	2009-03-18
11	2008-09-17	73,272	37	2009-03-25
12	2008-09-24	73,5	38	2009-04-01
13	2008-10-01	64	39	2009-04-08
14	2008-10-08	45,188	40	2009-04-15
15	2008-10-15	45,076	41	2009-04-22
16	2008-10-22	41,938	42	2009-04-29
17	2008-10-29	38,511	43	2009-05-06
18	2008-11-05	44,875	44	2009-05-13
19	2008-11-12	43	45	2009-05-20
20	2008-11-18	37,623	46	2009-05-27
21	2008-11-29	36,99	47	2009-06-03
22	2008-12-03	37,69	48	2009-06-10
23	2008-12-10	35,52	49	2009-06-17
24	2008-12-17	42,9	50	2009-06-24
25	2008-12-24	31,12	51	2009-07-01

## **E Historical market capitalization**

Date	Period	Period (Weeks)	Historical market capitalization
2008-06-30	CQ2 2008	0-12	10914
2008-09-30	CQ3 2008	13-25	12350
2008-12-31	CQ4 2008	26-38	5523
2009-03-31	CQ1 2009	39-51	6367