



MASTER THESIS (30 hp) IN FINANCIAL ECONOMICS  
AT THE DEPARTEMENT OF ECONOMICS

---

# Carry trade optimization

---

*Using a covariance matrix estimated with the  
intrinsic currency framework*

Peter Mannerbjörk  
06/01/2015

Supervisor: Charles Nadeau

## Abstract

This thesis investigates various possible improvements of implementing the carry trade. For this purpose a number of benchmark carry trade strategies are formed to which the results of the modified strategies are compared. The modified carry strategies take the correlations and/or volatilities of the currencies into account, aiming for a more efficient portfolio in the return to risk sense. Specifically, the effects of optimizing the carry strategy using a covariance matrix estimated with the intrinsic currency valuation framework is investigated. This is compared both with the benchmark carry strategies and the carry strategy optimized using an ordinary covariance matrix. I find that the carry strategy can be improved with portfolio optimization techniques. Both the information ratio of the strategy, as well as the skewness and kurtosis benefits from diversifying the trade across several currencies. However, the choice of which covariance matrix to use in the optimization is not important.

**JEL classification:** G11

**Key Words:** carry trade, intrinsic currency value, portfolio optimization

---

### Acknowledgements

I would like to thank Senior Lecturer Charles Nadeau for valuable comments and supervision of the thesis. I would also like to thank Christina Varkki who helped me with all practical issues required for me to start writing this thesis.

### Author Contact Details

Peter Mannerbjörk, E-mail: [peter.mannerbjork@gmail.com](mailto:peter.mannerbjork@gmail.com)

## Contents

Introduction.....	4
<i>Purpose and contribution</i> .....	4
<i>Background</i> .....	4
<i>Research questions</i> .....	5
<i>Results</i> .....	5
<i>Delimitations</i> .....	6
Literature review.....	7
Data and Methodology.....	10
<i>Summary overview</i> .....	10
<i>Data</i> .....	11
<i>Methodology</i> .....	11
<i>Portfolio construction methodology</i> .....	14
<i>Evaluated portfolios</i> .....	15
Results.....	17
<i>Benchmark portfolios</i> .....	17
<i>Intrinsic currency covariance estimation</i> .....	20
<i>Portfolio construction using the intrinsic covariance matrix</i> .....	21
Conclusions.....	25
References.....	26
Appendix.....	27

# Introduction

## *Purpose and contribution*

This thesis investigates the benefits of carry trade diversification using an intrinsic currency valuation (ICV) framework and mean-variance optimization techniques. To do so, I use the interest rates for the currencies as expected return and the covariance matrix for the portfolio currencies is estimated by the ICV-estimation process described in Doust et al. (2008, 2012). Even though the carry trade is well documented in the literature, little work has been done on how to construct optimal carry portfolios. The results of this thesis may serve as a guide for investors that want to implement the carry trade strategy how to create portfolios that are efficient in a mean-variance sense.

## *Background*

The carry trade is a simple and well documented investment strategy in which the investor sells a currency with low interest rate and invests the proceeds in a currency with high interest rate. The investor is aiming for earning the interest rate difference between the currencies. This is the return that the investor of a carry trade expects, at least in average. The carry trade is often executed in the FX market through FX-forward contracts. Since the strategy is a net zero investment where the investor is long and short equal amounts in both currencies, there are possibilities for leverage and high returns. This seemingly simple strategy has historically shown a high return to risk ratio, information ratio (IR), Burnside et al. (2011).

There has been some work done on the predictability of the carry trade returns, and with this information the investor can benefit from reversing the carry trade at appropriate times, Laborda et al. (2014). Opposed to this strategy, this thesis takes the expected return for the carry trade as given by the interest rates, and aims for constructing an optimal portfolio in an IR sense.

The purpose of the ICV framework is to estimate the value and risk of currencies in their own right, contrary to the instruments that are quoted in the FX market. In the FX-market, only

relative values and risks can be observed since currencies are quoted in pairs as values of one currency in terms of another currency. Using currency pairs in a mean-variance optimization procedure forces the investor to make arbitrary choices regarding which currency pairs to use in the optimization. ICV regards each currency as a separate asset which means that the investor avoids that problem.

### *Research questions*

The research question of the thesis is to examine if a number of benchmark carry strategies can be improved with statistical techniques and portfolio optimization. The benchmark strategy goes long a number of high yielding currencies by the same amount and short the same number of low yielding currencies. This is the carry strategy that the modified carry strategies are compared with. The statistical measure to evaluate improvement is information ratio (IR) but other measures as skewness and kurtosis are also documented. Specifically, the thesis investigates if a mean-variance optimization with a covariance matrix estimated following the ICV framework improves the carry strategy.

There are several different ways that the covariance matrix can be used to enhance the IR of the strategy. The most obvious way is to involve the risk of the individual currencies and choose the currency with the highest interest rate to risk ratio, rather than just the highest interest rate currency. Another way to use the covariance matrix is to let an optimizer use not only the variances but also the correlations among the currencies to form a portfolio with as high IR as possible. A third way is to use the covariance matrix to diversify through time, that is to create a carry strategy that has equal risk at each point in time, so that the risk of having particular bad performance when the risk in the strategy is high is minimised.

### *Results*

Results in this thesis show that variances and covariances as well as portfolio optimization using the ICV covariance matrix enhances the carry strategy. It raises the IR and makes the return distribution less negatively skewed and less fat tailed. However, results also show that the same improvements can be achieved with other covariance matrices as well, for example a covariance matrix estimated with returns from ordinary FX crosses where all currencies are denominated against the USD.

### *Delimitations*

This thesis involves only developed markets currencies. The reason for this is that developed market currencies are thought to have less risk for jumps in their valuation than emerging market currencies. These jumps are hard to estimate in the ICV framework used in this thesis. Including currencies like this would lead to non-accurate estimation of their risk and the results of the mean-variance optimization procedure less reliable. The expected return of each currency in the thesis is assumed to be the nominal interest rate in that currency. One could think of other variables than the nominal interest rate to serve as expected returns. For example the real interest rate could be used, but this is something that is beyond the scope of this thesis.

## Literature review

The first study on the concept of ICV framework was made by Doust et al. (2008). That study introduces the concept of ICV and presents a procedure how to estimate the currency values in their own right as well as the risk and covariances between them. Their paper uses two different approaches for estimation purposes, historical FX-quotes only or FX-option quotes to estimate the risks in the currencies. This thesis uses the historical FX-quotes method to perform all estimations. The reason for this is that the carry trade can be executed in a number of currencies on which there are no options traded, or at least the FX-options are very illiquid, which means that the use of such prices would be dubious.

One initial study of the topic of portfolio optimization was made by Markowitz (1952). In his framework each asset has an expected return and the assets has individual variances and covariances with each other. His work states that for a given portfolio expected return, one should invest in the portfolio with lowest risk. All optimal portfolios end up on the efficient frontier and the actual choice of portfolio is a result of the preferences of expected return and risk of the investor. This thesis creates optimal portfolios in the same sense as Markowitz (1952).

Since the carry trade is executed through FX futures, the possibilities to profit from this strategy depends on if the uncovered interest rate parity (UIP) holds or not. UIP states that the high interest rate currency on average should depreciate by a percentage equal to the interest rate differential, so that the carry trade on average should give the investor zero return.

The plausibility of the UIP is related to the efficiency of the FX-markets. If UIP holds, it is not possible to form profitable systematic FX-strategies. One initial study on the UIP is made by Fama (1984) where he splits the observed forward exchange rate into an expected future spot rate and a premium. By running a regression, he investigates if the difference between the current forward and spot exchange rate has power to predict a future change in the spot rate. According to the UIP, the regression that Fama constructed should produce a regression coefficient close to 1. However, what Fama finds is that the regression coefficient is negative for all nine currencies against the U.S. Dollar. This means that if the UIP predicts a depreciation of one currency against another, that currency actually on average appreciates

against the other currency. This is what is called the "Forward premium puzzle" in the literature.

One could also investigate the predictability of the spot exchange rates by forming different forecasting models and comparing them to a simple random walk model. This is done by Meese et al. (1983) for the U.S. dollar spot exchange rate to the yen, mark and pound and they find that a random walk model performs as well as the other forecasting models.

A study where systematic FX strategies are documented is Burnside et al. (2011), where the carry strategy and the momentum strategies are investigated. They find that the carry trade for 20 different currencies against the U.S. dollar has an average Sharpe-ratio of 0.42. By creating a portfolio of equally weighted carry trades against the U.S. dollar the Sharpe-ratio is increased to 0.89. The increase in Sharpe-ratio points in the direction that diversification and/or portfolio optimization can be useful for creating efficient carry portfolios.

The rather high IR for such a simple strategy as the carry trade raises questions if there are risks that has not been thought of or has not occurred in the sample. One possible explanation that has been discussed in the literature is that the carry trade has an unattractive return profile, i.e. negative skewness. It could be the case that the carry trade has a small probability of giving large negative returns, but this has not occurred during the sample. If that is the case, the carry trade has an expected return that is not as high as documented or it could even have a zero expected return. Brunnermeier et al. (2009) studies individual carry trades and portfolios of two and three equally weighted carry trades and find that the carry trade has negative skewness and excess positive kurtosis. They mean that the carry trade is exposed to crash risk, and will once in a while experience large sudden losses. They also find that these unwanted characteristics do not get diversified away in a portfolio of carry trades, at least not with two or three equally weighted carry trades. Furthermore, they find that currency crashes are correlated with common risk factors such as implied stock market volatility and funding illiquidity.

Since carry trades seem to be exposed to crash risk, it is a legitimate question to ask if the high returns for the strategy is just a compensation for bearing risk. With help of FX options, Jurek (2014) constructs crash-hedged carry trades of all G10 currency pairs. From comparing unhedged and hedged carry trades he concludes that a crash risk-premia accounts for one third of the excess return of the carry trade in his sample.

For an investor who wants to implement a carry strategy, questions regarding portfolio construction arises. When it comes to optimal carry implementation there has been some work done. Often the literature makes use of a naïve carry trade that goes long 1-3 high interest currencies and short the same number of low interest currencies. Then the investor has the option to divide his investment between a risk-free asset that yields the risk-free rate of interest and the naïve carry trade. How much to allocate to each strategy is decided by some other variables that are believed to have predictive power of the return of the carry trade. For example, Laborda et al. (2014) investigates the predictiveness of the U.S. average forward discount, lagged returns of the carry trade, the VIX index, U.S. TED spread, the CRB Industrial return and a global monetary policy indicator to predict future carry trade returns. From these variables they construct a model that can actually go short the carry trade, if the variables indicate bad carry returns. This model successfully identifies one of the worst carry trade periods in history, late 2008, as a period to go short the carry trade.

There is a statistical problem of fitting a model that allows the investor to go short the carry trade to a sample with one or two carry trade crashes. If you are able to find a variable that reverses the carry trade during 2008, the backtest will look good in-sample. However, it is hard to verify that the model performs well the next time that the carry trade crashes. This is unavoidable since any sample data contains relatively few crashes.

This thesis focuses on another type of optimization, namely mean-variance optimization. There is one difficulty to be able to construct an optimal FX-forward portfolio compared to equities or bonds. The FX-forwards, which are used to implement the carry trade are not individual currencies, but ratios of currencies where the only available information in the market are the relative values between currencies. Because of that, it is not clear what conclusion regarding the risk of the individual currencies one can draw from the volatilities of the currency pairs. Instead of using expected returns and risks in the FX-pairs, this thesis uses the interest rates and risks in the individual currencies estimated in the ICV framework to form mean-variance optimal currency carry portfolios. A topic that to my knowledge has not been documented in the literature.

## Data and Methodology

### *Summary overview*

The concept of intrinsic currency values aims for estimating values and risks of each currency in its own right and still be consistent with the observed FX-market quotes, where each currency is always valued in relation to another currency. The challenge to do so is that if you observe  $n$  number of FX-quotes, you must estimate  $n+1$  number of currency values and risks. The logic behind the last condition to fix the remaining degree of freedom is that if each currency is to be regarded as an asset of its own, the correlations between the currencies should be small. This translates into an optimization problem that seeks the intrinsic currency values that minimizes the correlations among the intrinsic currencies subject to the constraint that the ratios of the intrinsic currency values must be equal to the observed FX-market quotes. After estimating these intrinsic currency values, the intrinsic currency covariance matrix is easy to compute with ordinary statistical methods.

Interest rates in the different currencies are reflected in the FX-forward quotes. For arbitrage reasons, the FX-forward price has to be the FX-spot price adjusted for the interest rate differential between the two currencies. Because of that, FX-forward contracts can be used to calculate the implied interest rate differential between all currencies, which is used as input as expected return in the carry trade.

With interest rates as the expected returns and an estimated covariance matrix it is possible to perform portfolio optimizations for creating efficient carry portfolios. To conclude if the portfolio optimization and covariance matrix adds value to the carry strategy, three benchmark carry portfolios are formed. These portfolios go long the one, two and three highest interest rate currencies and short the same number of the lowest interest rate currencies. These are naïve carry strategies that only involves the level of interest rates but no risks. If the ICV covariance matrix can be used to enhance the IR of these benchmark carry strategies, the ICV concept has added value for a carry investor. The criteria to evaluate the portfolios is their IR.

### *Data*

The data used for this thesis is collected from Thomson Reuters. The FX data that is used are spot exchange rates for nine currencies against the US dollar (AUD, CAD, CHF, EUR, GBP, JPY, NOK, NZD and SEK) and one month forward exchange rates for the same currencies. These currencies are all very liquid and traded in large amounts, which makes the reliability of the quotes high. From the forward and spot exchange rate it is possible to derive the interest rate differential between the two involved currencies. Since the carry trade is executed through forward contracts in the FX markets, this is the interest rate that is used in our model. The data ranges from 1999-01-01 to 2014-10-07, which gives us about 15 years of history. Exchange rates are snapped at 4 pm CET.

### *Methodology*

The process to estimate intrinsic currency values and returns follows Doust (2008).

Denote  $X_i^t$  to be the intrinsic currency value of currency  $i$  at time  $t$ . The quantity that can be observed in the FX markets is

$$S_{ij}^t = \frac{X_i^t}{X_j^t} \quad (1)$$

which is the spot exchange rate between currency  $i$  and  $j$  at time  $t$ . That means that the quantity  $S_{ij}^t$  shows how many units of the currency  $j$  that corresponds to one of currency  $i$ .

The purpose of intrinsic currency analysis is to estimate all the intrinsic currency values  $X_i^t$ , at every point in time in the data sample and at the same time satisfy Equation (1).

With  $N$  currencies in the data sample, there are  $(N-1)$  independent spot exchange rates at each date. To be able to estimate the intrinsic currency values, we must find a way to fix the last degree of freedom.

To be able to write the constraints given by Equation (1) in a linear way, we transform the problem into log scale.

Define:

$$Z_i^t = \ln(X_i^t)$$

so the constraints are

$$\ln(S_{ij}^t) = Z_i^t - Z_j^t \quad (2)$$

and from that:

$$\Delta \ln(S_{ij}^t) = \Delta Z_i^t - \Delta Z_j^t \quad (3)$$

where  $\Delta$  denotes a difference in time.

Equation (3) states that an observed percentage difference in a spot exchange rate between currency  $i$  and  $j$  corresponds to an equal percentage difference between the intrinsic currency values of currency  $i$  and  $j$ . You know the relative value difference between the currencies, but it is impossible to know the absolute percentage value difference of the individual currencies.

If the currencies are to be regarded as separate assets, they should have limited influence on each other. That means that the correlation among the currencies should be close to zero. To be able to fix the remaining degree of freedom, this is the condition the ICV framework uses. In other words, we would like to find intrinsic currency values that minimizes the squared correlation sum, subject to Equation (3).

$$\min \sum_{i,j} \lambda_{ij} (\hat{\rho}_{ij})^2 \quad (4)$$

where  $\lambda_{ij}$  is a weight parameter between 0 and 1. This parameter can be used for adjusting the weight in case of two currencies that actually are expected to have a correlation different from 0.  $\hat{\rho}_{ij}$  is the sample correlation coefficient between the returns of intrinsic currency  $i$  and  $j$ . In this thesis,  $\lambda$  is set to 1 for all currencies, that is no correlations are excluded from the objective function in the minimization process. To solve Equation (4) I use the NLPQN procedure in SAS IML which is a nonlinear optimization routine, for description and syntax see internet address in the references section.

Furthermore, the equation for the return correlation between intrinsic currencies  $i$  and  $j$  is given by:

$$\hat{\rho}_{ij} = \frac{n \sum_{t=0}^{n-1} \Delta Z_i^t \Delta Z_j^t - \left( \sum_{t=0}^{n-1} \Delta Z_i^t \right) \left( \sum_{t=0}^{n-1} \Delta Z_j^t \right)}{\sqrt{n \sum_{t=0}^{n-1} (\Delta Z_i^t)^2 - \left( \sum_{t=0}^{n-1} \Delta Z_i^t \right)^2} \sqrt{n \sum_{t=0}^{n-1} (\Delta Z_j^t)^2 - \left( \sum_{t=0}^{n-1} \Delta Z_j^t \right)^2}} \quad (5)$$

In Equation (5) one can see that the optimization problem involves all the  $\Delta Z_i^t$ . That means that all intrinsic currency values at each point in time are involved in the optimization. Because of the many degrees of freedom, this problem is very computationally expensive. To reduce computation time one can use the constraints in Equation (3) directly in the optimization. This means that only one of the intrinsic currency values has to be decided, and then all the other currencies intrinsic values follow from the constraints. This makes sense, since we only have one degree of freedom left as stated above.

The aim for using the intrinsic currency value framework is to construct a covariance matrix for the individual currencies and not the FX-pairs to see if the carry trade can be improved through portfolio optimization with individual currencies as assets. When deciding how long history to use for this covariance matrix estimation one has to make a trade-off between long history and many data points in the estimation process and the ability to estimate changes in the correlations and variances in the currencies by not including too much historical data. When solving Equation (4) you get time series of intrinsic currency value returns and their correlations. This can be used for constructing a covariance matrix of intrinsic currency values:

$$Cov(\Delta Z_i, \Delta Z_j) = std(\Delta Z_i) Corr(\Delta Z_i, \Delta Z_j) std(\Delta Z_j) \quad (6)$$

where  $Corr(\Delta Z_i, \Delta Z_j)$  is given by (5) and  $std(\Delta Z_i)$  is:

$$std(\Delta Z_i) = \sqrt{\frac{1}{T} \sum_{t=1}^T (\Delta Z_i^t - \mu)^2}$$

$$\mu = \frac{1}{T} \sum_{t=1}^T \Delta Z_i^t$$

One could study how to choose the length of history as well as other models for the variance of the  $\Delta Z_i$  such as garch models, but this is beyond the scope of this thesis. I choose  $T=365$  days, i.e. one year.

The interest rates in the different currencies are derived from the forward and spot exchange rates. Interest rate parity states the following condition:

$$\ln\left(\frac{F_{ij}}{S_{ij}}\right) = \frac{R_i - R_j}{12} \Leftrightarrow (R_i - R_j) = 12 \log\left(\frac{F_{ij}}{S_{ij}}\right) \quad (7)$$

where  $F_{ij}$  is the one month forward rate and  $S_{ij}$  is the spot rate between currency  $i$  and  $j$ .  $R_i$  and  $R_j$  are the interest rates in currency  $i$  and  $j$ . This means that the interest rate differential between currency  $i$  and  $j$  can be determined by studying the forward and spot exchange rates between the currencies.

The interest rate differences between the currencies given by Equation (7) are used as carry signals to construct carry portfolios.

To determine if the use of the intrinsic currency covariance matrix adds value to the carry strategy I form a number of different carry portfolios each month, and evaluate them by their IR.

### *Portfolio construction methodology*

The process to implement the carry strategy is as follows: By the seventh of each month in the backtest period is used as a rebalance date. If the seventh of the month is a non-bank day, the first bank day prior to the seventh is used. At each rebalance date, the currencies are evaluated and selected by a carry criteria. This criteria could be the highest carry against the lowest carry or the highest carry-to-risk to the lowest carry-to-risk currency or something

more sophisticated such as a portfolio optimization. The carry is determined by Equation (7) above and if the risk is used in the selection process, the time period used to estimate the risk is one year of data prior to the rebalance date. The reason for not including the whole data sample in the estimation process is that only including data known at that point in time makes the backtest as realistic as possible. After the rebalance date, the positions are held until the next rebalance date one month later, when the process is repeated. One month after each rebalance date, it is possible to calculate the monthly return for the carry strategy. These monthly returns are then added together to a return series for the strategy spanning the whole backtest.

### *Evaluated portfolios*

The most basic carry strategy is going long an equal amount of the 1-3 highest interest rate currencies and short the 1-3 lowest interest rate currencies. This is the strategy that I compare the modified carry strategies with, the base strategy. Going from one to three long positions lowers the carry of the portfolio, but may also lower the risk of the portfolio which means that the IR potentially increases. This shows if diversification is beneficial for the carry strategy. These basic carry strategies are called S1, S2 and S3 and serves as naïve benchmark strategies which the modified carry strategies can be compared with.

Instead of ranking the currencies by their interest rates, one could rank the interest rate difference to risk ratio for each currency pair and use that as selection criteria. Risk is estimated return volatility for each individual currency pair. The purpose is to take not only the interest rates but also the risk into account, to hopefully construct a strategy that has better IR. Three strategies are constructed that invest in 1, 2 and 3 currency pairs with the best IR. For diversification reasons, each currency is allowed only once. These strategies are called S1\_riskadj, S2\_riskadj, S3\_riskadj.

Note that all of the strategies above all involve currency pairs, not individual currencies. The first carry portfolio construction technique using the intrinsic covariance matrix is as follows. At each backtest date, the risk for each of the FX crosses is calculated using the intrinsic covariance matrix after which the FX crosses are ranked by their interest rate difference to intrinsic risk ratio. Then I construct three portfolios which invest in the 1, 2 and 3 FX crosses with the best IR. These strategies are called S1\_i, S2\_i and S3\_i.

I then turn to using the intrinsic covariance matrix in a portfolio optimization. For the portfolio optimization, the NLPQN procedure in SAS IML is used. The objective is to create a portfolio with minimal variance that has a carry equal to 0.1%. This is the optimal portfolio according to Markowitz (1952) since no other portfolio with the same expected return (carry) has lower risk, or to put another way, no other portfolio with the same amount of risk has higher expected return. The optimal portfolio is constructed at each backtest date, only using data that is known at that point in time. This strategy is called Opt1. This is a portfolio that takes both standard deviations of the currencies and the correlations of the currencies into account. Information about the correlations gives the optimizer the possibility of diversification.

There is another possibility for diversification except from diversifying at a specific point in time through the optimization process. An updated covariance matrix gives the investor the possibility to construct a carry portfolio with constant estimated risk over time. This serves as a diversification through time, so that the investor is less exposed to the risk of having particularly low portfolio returns at times when the realized risk is particularly high. To investigate this, the positions in Opt1 are scaled so that the resulting portfolio has equal estimated risk at each rebalance date. This strategy is called Opt2.

To conclude if the ICV framework adds value compared to an ordinary portfolio optimization, the counterparties to Opt1 and Opt2 are constructed but using a covariance matrix estimated from FX cross returns. USD is the base currency and the implied interest rates are used as expected returns for each currency against the USD. In this case there are nine assets, all currency crosses to the USD. Apart from having nine assets instead of ten, the exact same backtest procedure as before is followed. These two strategies are called Opt\_FX1 and Opt\_FX2. The results for these two strategies can be compared with the results for the Opt1 and Opt2 strategies to see what the ICV optimization adds to an ordinary optimization.

To conclude if the improved carry strategies are statistically significant better than the naïve strategies, I perform a t-test for the difference in monthly returns for the best naïve strategy and the best improved strategy. In this test,  $H_0$  is that there is no monthly return difference between the strategies and  $H_1$  is that there is monthly return difference between the strategies.

## Results

### *Benchmark portfolios*

This section describes the results of the basic carry strategy of going long 1-3 currencies and short the same number of currencies. Table 1 describes how often the different currencies are used as funding and investment currencies during the 167 backtest months in the sample. S1, S2, S3 are the strategies going long and short 1, 2, 3 currencies respectively.

Table 1: Number of occurrences as funding and investment currency for the three basic strategies

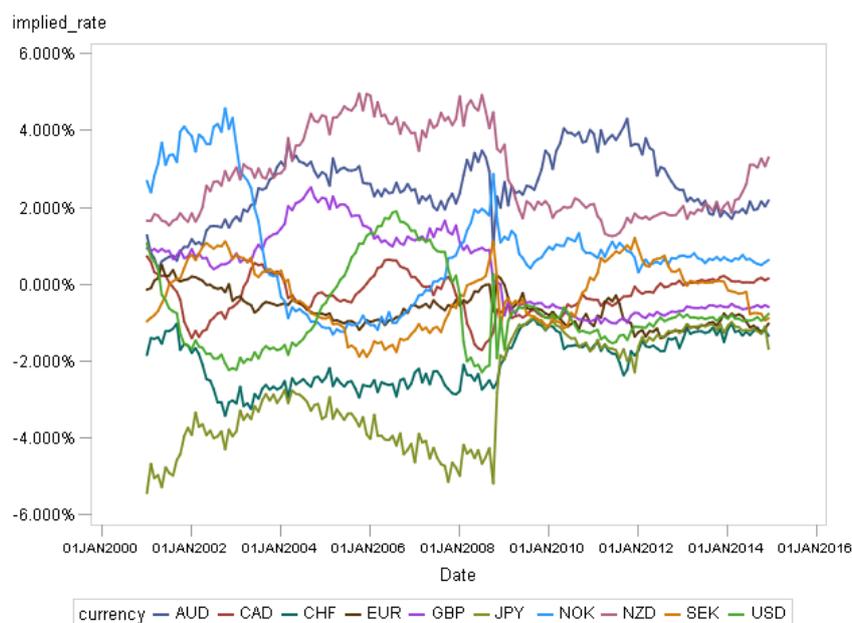
Currency	Strategy					
	S1		S2		S3	
	Funding	Investment	Funding	Investment	Funding	Investment
AUD	0	58	0	137	0	158
CAD	0	0	0	0	6	0
CHF	61	0	165	0	167	0
EUR	0	0	12	0	34	0
GBP	0	0	0	0	0	48
JPY	105	0	155	0	167	0
NOK	0	26	0	30	8	105
NZD	0	83	0	167	0	167
SEK	1	0	1	0	45	10
USD	0	0	1	0	74	13

We can see that CHF and JPY are dominant as funding currencies and AUD and NZD are dominant as investment currencies.

This can also be illustrated by the interest rates of the different currencies. That is what is shown in Graph 1 below. The interest rates are the implied rates from the FX-futures markets and they are normalized so that the rate shown is the deviation from the average interest rate. In other words, the interest rates are centered on zero in the graph. Another thing that is clear from Graph 1 is that the interest rates have become more homogenous over time. The

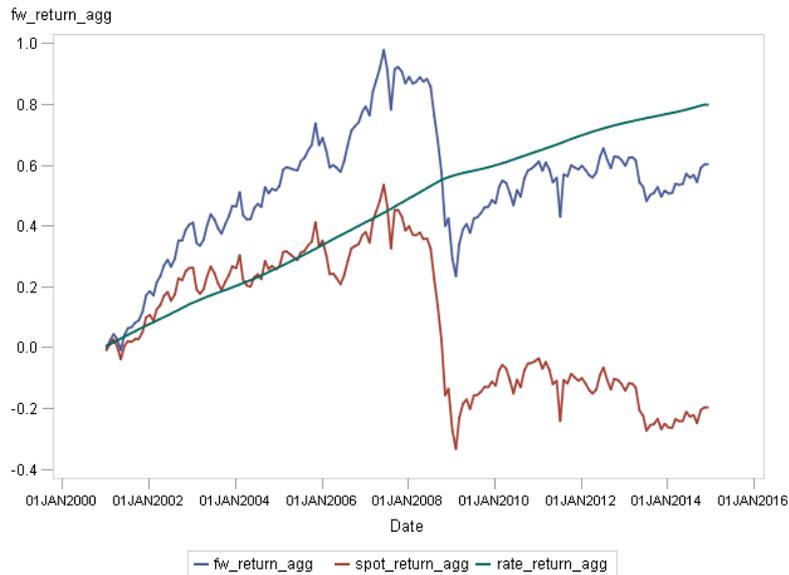
difference between the highest and the lowest interest rates is smaller now than in the beginning of the backtest period, something that may decrease the returns, or at least the IR, of the carry strategy.

Graph 1: The evolution of the interest rates over time



The aggregated return of strategy S1 is shown in Graph 2. The total return of a FX-forward contract can be divided into two parts, the return from the interest rate differences and the return from the spot exchange rates. That makes it possible to divide the carry strategy return into three components, the aggregated carry strategy return (`fw_return_agg`), the return from the interest rate differentials (`rate_return_agg`) and the return from spot exchange rate changes (`spot_return_agg`). The UIP states that the positive return from the interest rate differentials in average should be offset by an equal amount of negative return from spot exchange rates. This is something that cannot be seen in this backtest, where the spot exchange rate changes contribute negatively, but this negative return is by far compensated by the positive return from interest rate differentials. This result shows why the carry trade has been a successful strategy the last years. Note that this is not a formal test of the UIP as in Fama (1984) or if the expected spot returns follow a random walk as in Meese and Rogoff (1983), since no significance tests are performed.

Graph 2: Carry strategy return divided into total return, rate return and spot return



Another thing that is clear from Graph 2 is that the carry trade had severe losses during the financial crisis. Statistics for the carry trade that are interesting are of course the average return and risk, but also the skewness and kurtosis that describe at which extent the return distribution is deviating from a normal distribution. To be exact, the skewness measures if the distribution has more large returns in the positive or negative direction and the kurtosis is telling if the distribution is more fat tailed than the normal distribution. These statistics are shown in Table 2 for strategies S1, S2, S3.

Table 2: Descriptive statistics for the three basic carry strategies

Statistics	Strategy		
	S1	S2	S3
Average return	4.27%	5.94%	4.74%
Standard dev	15.03%	14.00%	12.85%
IR	0.284	0.425	0.369
Skewness	-0.78	-0.13	-0.58
Kurtosis	3.10	1.79	1.85

All three strategies show excess kurtosis and negative skewness, which means that extreme returns are more likely than if the underlying return distribution was a normal distribution and extreme negative returns are more likely than extreme positive returns. The positive effect of diversification can be seen from that the standard deviation of the returns decrease if more currencies are used in the implementation. However, the IR is not increasing with more instruments added in the strategy since the average return is also affected. The IR here are smaller than what is documented by Burnside et al. (2011) which finds an average IR of 0.42 for the carry strategy. However, in that paper they use a much longer backtest period which makes the bad carry period during the financial crisis less dominant.

Now I investigate if the estimated currency pair risk adds value to the carry investment process. The descriptive statistics for the strategies that rank currencies regarding their expected IR, S1\_riskadj, S2\_riskadj, S3\_riskadj are shown in Table 3.

Table 3: Descriptive statistics for the three carry strategies that ranks by IR

	Strategy		
Statistics	S1_riskadj	S2_riskadj	S3_riskadj
Average return	2.41%	2.99%	3.82%
Standard dev	10.36%	8.40%	7.31%
IR	0.233	0.356	0.523
Skewness	-0.39	-0.05	-0.65
Kurtosis	0.99	0.72	0.93

The most striking difference between the descriptive statistics for the strategies in Table 2 and Table 3 is that the strategies that rank the currency pairs by their estimated IR has much lower kurtosis, that is less fat tails. Skewness and IR are not affected uniformly, the standard deviation is decreased and so is the return.

#### *Intrinsic currency covariance estimation*

Turning to the ICV correlations, the correlation matrix and the standard deviations when using the whole backtest period of data is shown in Table 4 and Table 5 in the Appendix.

As one expects, the AUD-NZD and CHF-EUR correlations are high, since the AUD-NZD are closely linked economies and the CHF is managed to the EUR. There are some other quite high correlations as well, CAD-USD and JPY-USD and SEK-NOK.

The same estimation is done with FX-crosses, the nine currencies against the dollar. The results are shown in Table 6 and Table 7 in the Appendix.

It is clear that the correlations in Table 6 is generally higher than in Table 4. This is expected because all the FX crosses in Table 6 share one currency, the USD. This common currency may also explain why the standard deviations in Table 7 are more homogenous than the standard deviations in Table 5.

If not using the whole backtest period for estimating the intrinsic currency covariance matrix, but instead estimating a new covariance matrix each month, only using one year of data prior to that date one can investigate how the covariance matrix has changed through time. Since there are 10 different currencies we have 10 unique standard deviations and 45 unique correlations, I plot only the standard deviation for the USD and the CAD-USD correlation in Graph 3 and Graph 4 in the Appendix.

The standard deviation of the return of the intrinsic USD has varied during the backtest period, with a peak during the financial crisis. The correlation between the intrinsic CAD and USD has varied a lot, it has ranged from above 0.8 in the beginning of the period to around 0.1 in the beginning of 2010. These two graphs indicate that the risk of a portfolio of the same currencies can have very different risk profiles depending on the current risk and correlation structure.

### *Portfolio construction using the intrinsic covariance matrix*

The results for the first set of strategies that use the intrinsic covariance matrix is shown in Table 8. These portfolios rank the currency pairs with respect to their carry to risk ratio when the risk is estimated using the intrinsic covariance matrix.

Table 8: Statistics for strategy that ranks by interest rate to intrinsic risk

Statistic	Strategy		
	S1_i	S2_i	S3_i
Average return	2.41%	2.71%	4.07%
Standard dev	10.36%	8.65%	7.37%
IR	0.233	0.313	0.552
Skewness	-0.325	-0.097	-0.678
Kurtosis	1.18	0.622	0.871

If one compare these statistics with the statistics in Table 3, where the same selection criteria is used but the risk is the ordinary FX cross standard deviation, there are no large differences. Using the standard deviation estimated from the intrinsic covariance matrix seems to result in approximately the same carry portfolios as using the ordinary FX cross standard deviation.

Before using portfolio optimization techniques for constructing the carry trade portfolios, I want to highlight Graph 3 and Graph 4. These show that the risk and correlations in the currencies are changing substantially over time. This indicates that an updated covariance matrix could be valuable for the investor. Opt1 chooses the portfolio with best estimated IR and the Opt2 strategy scales the Opt1 bets so that all portfolios have equal estimated total variance. Statistics for these two strategies are shown in Table 9. The accumulated return for the Opt1 and Opt2 strategies are shown in Graph 5 and Graph 6. The accumulated return for the Opt2 strategy together with the S3\_riskadj strategy which is the best naïve carry strategy is shown in Graph 7 in the Appendix.

Table 9: Statistics for optimized portfolio using intrinsic covariance matrix

Statistics	Strategy	
	Opt1	Opt2
Average return	4.49%	4.67%
Standard dev	7.47%	6.30%
IR	0.601	0.741
Skewness	-0.028	-0.25
Kurtosis	1.414	0.135

The statistics in Table 9 shows the benefits of portfolio optimization during the backtest period. If we look at Opt1, we see an IR that is higher and the skewness is closer to zero than any of the previous strategies. The kurtosis is smaller than all of the benchmark strategies that

are not adjusted for risk but larger than the kurtosis for the strategies that ranks currencies or currency pairs by their IR. These numbers can be compared with Jurek (2014) who tries to improve the carry trade by constructing crash-neutral portfolios by using FX-options. He documents a skewness of -0.19 and kurtosis of 3.5 when implementing a crash-neutral portfolio with 9 equally weighted currencies against the dollar using 10 $\delta$  options between 1999-2007, that is before the credit crisis. Using ATM or 25 $\delta$  options raises the skewness to a positive number.

Looking at Opt2, we see that diversification through time raises the IR to 0.741, it also lowers the kurtosis substantially and increases the skewness somewhat. This shows that keeping a constant estimated risk in the carry portfolio is an easy way to improve the IR. The fact that the kurtosis is decreased can be explained with this constant risk profile. The other strategies that do not keep a constant estimated risk will have large risk in turbulent times, for example during the financial crisis. This will produce strategy returns of large magnitude during those times compared with calmer periods in the backtest. Brunnermeier et al (2009) documented that the skewness and kurtosis cannot be diversified by investing in more than one currency cross for carry trade implementation. However, here we can see that portfolio optimization can be used to decrease the skewness and a constant estimated risk portfolio decreases the kurtosis. This is a complete opposite approach to enhance the carry trade compared with Laborda et al. (2014). They try to predict the return of the carry trade and change the exposure to the carry risk and possibly reversing the exposure. The results above show that it is possible to improve the carry trade by instead having a constant exposure to carry risk.

Graph 5 in the Appendix shows the accumulated returns for the Opt1 strategy. The green line is the rate return which is a straight line because of the constraint in the optimization that keeps the rate return exactly at 0.1%.

Graph 6 in the Appendix shows the accumulated returns for the Opt2 strategy. The rate return is not constant for Opt2 as for Opt1. Since the positions are scaled to give the Opt2 portfolio constant risk over time, the rate return is no longer constant. Another interesting result is that both Opt1 and Opt2 actually profits from spot exchange rates.

When comparing the Opt2 and S3\_riskadj strategies the most striking difference is that the large losses during the financial crises is smaller in the Opt2 strategy. This is because of the constant risk property of the Opt2 strategy. This can be seen in Graph 7 in the Appendix. I have also performed a significance test for the monthly return differences between the Opt2

and S3\_riskadj strategies.  $H_0$  is that there is no monthly return difference between the strategies and  $H_1$  is that there is monthly return difference between the strategies. This test gives a monthly average difference of 0.023 and a standard deviation of 0.262, which gives us a t-statistic of 0.089.

To conclude if the ICV framework adds value compared to an ordinary portfolio optimization, we turn to the Opt\_FX1 and Opt\_FX2 strategies. The statistics for a constant carry portfolio and a constant risk portfolio is shown in Table 10 and should be compared with the Opt1 and Opt2 strategies above.

Table 10: Statistics for optimized portfolios using the FX cross covariance matrix

	Strategy	
	Opt_FX1	Opt_FX2
Average return	5.0%	4.68%
Standard dev	8.33%	6.3%
IR	0.601	0.741
Skewness	-0.0137	-0.25
Kurtosis	1.449	0.14

One can see that the statistics for the Opt\_FX1 and Opt\_FX2 strategies are pretty much the same as the statistics for the Opt1 and Opt2 strategies. This means that optimization is adding value to the carry strategy portfolio construction, but the choice of covariance matrix is not important.

## Conclusions

The main objective of this thesis is to investigate to what extent optimization is useful for currency carry portfolio. Specifically, the benefits of a covariance matrix estimated by the ICV framework are investigated. This is done through constructing optimized portfolios using the ICV covariance matrix and comparing the return distribution with the return distribution of benchmark strategies that do not involve any covariance matrix information and with return distribution of carry portfolios optimized using a covariance matrix estimated from ordinary FX cross returns with the USD as base currency.

The benchmark strategies show positive IR, which means that they are profitable. They also show negative skewness and excess kurtosis, which means that the return distribution is fat-tailed and has more frequent large losses than large profits.

When using portfolio optimization techniques all these statistics are improved. The IR is increased and the skewness is closer to 0 and the kurtosis is reduced. However, it seems like it is not important which covariance matrix that is used in the optimization process. The statistics are practically the same regardless if an intrinsic covariance matrix or an ordinary covariance matrix is used. Furthermore, the improvement of the IR compared to the best benchmark strategy is not statistically significant.

The benefits of using a covariance matrix in an optimization is twofold. Firstly, the optimizer is able to diversify risk using the correlations between the currencies. This lowers both the risk and carry of the portfolio, but given that the optimizer does its job, the risk is lowered more than the carry which gives the resulting portfolio higher IR. Secondly, the use of a covariance matrix gives the investor the possibility to create a carry strategy with constant risk. This is a diversification through time.

These results are limited to the backtest period used in the thesis and for this specific strategy and currencies. It would be interesting to increase the number of currencies and/or to extend the backtest period to see if the results are the same. It would also be interesting to see if portfolio optimization is useful for other currency investment strategies than the carry strategy as well.

## References

Brunnermeier, M.K., Nagel, S. and Pedersen, L.H. (2009). Carry trades and currency crashes. *NBER Macroeconomics Annual*, 23, pp. 313-387.

Burnside, C., Eichenbaum, M. and Rebelo, S. (2011). Carry trade and momentum in currency markets. *Annual review of Financial Economics*, 3, pp. 511-535.

Doust, P., Chen J. (2008). Estimating intrinsic currency values. *Risk Magazine*, 19, pp. 381-445.

Doust, P. (2012). The stochastic intrinsic currency volatility model: A consistent framework for multiple FX rates and their volatilities. *Applied Mathematical Finance*, 19, pp. 381-445.

Fama, E.F. (1984). Forward and spot exchange rates. *Journal of Monetary Economics*, 14, pp. 319-338.

Jurek, J.W. (2014). Crash-neutral currency carry trades. *Journal of Financial Economics*, 113, pp. 325-347.

Laborda J., Laborda R., Olmo J. (2014). Optimal currency carry trade strategies. *International Review of Economics and Finance*, 33, pp. 52-66

Markowitz, H. (1952). Portfolio selection. *Journal of Finance*, 7, pp. 77-91.

Meese, R.A. and Rogoff, K. (1983). Empirical exchange rate models of the seventies. *Journal of International Economics*, 14, pp. 3-24.

SAS IML documentation

<http://support.sas.com/documentation/onlinedoc/iml/>

SAS IML NLPQN Call documentation:

[http://support.sas.com/documentation/cdl/en/imlug/59656/HTML/default/viewer.htm#langref\\_sect191.htm](http://support.sas.com/documentation/cdl/en/imlug/59656/HTML/default/viewer.htm#langref_sect191.htm)

## Appendix

Table 4: Intrinsic correlation matrix

Currency	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
AUD	1.00	0.35	-0.03	-0.16	0.11	-0.16	0.07	0.71	0.07	0.07
CAD	0.35	1.00	0.03	-0.26	0.16	0.09	-0.08	0.26	-0.09	0.46
CHF	-0.03	0.03	1.00	0.77	0.17	0.37	0.17	-0.02	0.17	0.28
EUR	-0.16	-0.26	0.77	1.00	-0.13	0.01	0.02	-0.18	0.13	0.08
GBP	0.11	0.16	0.17	-0.13	1.00	0.12	-0.18	0.11	-0.21	0.39
JPY	-0.16	0.09	0.37	0.01	0.12	1.00	-0.24	-0.14	-0.27	0.56
NOK	0.07	-0.08	0.17	0.02	-0.18	-0.24	1.00	0.02	0.34	-0.16
NZD	0.71	0.26	-0.02	-0.18	0.11	-0.14	0.02	1.00	0.02	0.06
SEK	0.07	-0.09	0.17	0.13	-0.21	-0.27	0.34	0.02	1.00	-0.18
USD	0.07	0.46	0.28	0.08	0.39	0.56	-0.16	0.06	-0.18	1.00

Correlations for the intrinsic currencies. Correlations are generally positive but there are a couple of negative observations. Closely linked currencies such as AUD-NZD show positive correlations.

Table 5: Intrinsic currency annualized standard deviations

Currency	Std yearly
AUD	10.02%
CAD	8.37%
CHF	8.34%
EUR	3.65%
GBP	6.44%
JPY	11.83%
NOK	6.18%
NZD	10.38%
SEK	6.16%
USD	9.54%

Annualized intrinsic currency standard deviations. These quantities are hard to have an a priori opinion about since they cannot be observed in the market.

Table 6: FX cross return correlations

FX cross	AUD/USD	CAD/USD	CHF/USD	EUR/USD	GBP/USD	JPY/USD	NOK/USD	NZD/USD	SEK/USD
AUD/USD	1.00	0.61	0.39	0.55	0.53	0.02	0.59	0.83	0.60
CAD/USD	0.61	1.00	0.32	0.45	0.44	0.00	0.51	0.55	0.51
CHF/USD	0.39	0.32	1.00	0.84	0.55	0.37	0.66	0.40	0.67
EUR/USD	0.55	0.45	0.84	1.00	0.67	0.25	0.81	0.54	0.83
GBP/USD	0.53	0.44	0.55	0.67	1.00	0.14	0.61	0.53	0.61
JPY/USD	0.02	0.00	0.37	0.25	0.14	1.00	0.16	0.04	0.15
NOK/USD	0.59	0.51	0.66	0.81	0.61	0.16	1.00	0.57	0.83
NZD/USD	0.83	0.55	0.40	0.54	0.53	0.04	0.57	1.00	0.56
SEK/USD	0.60	0.51	0.67	0.83	0.61	0.15	0.83	0.56	1.00

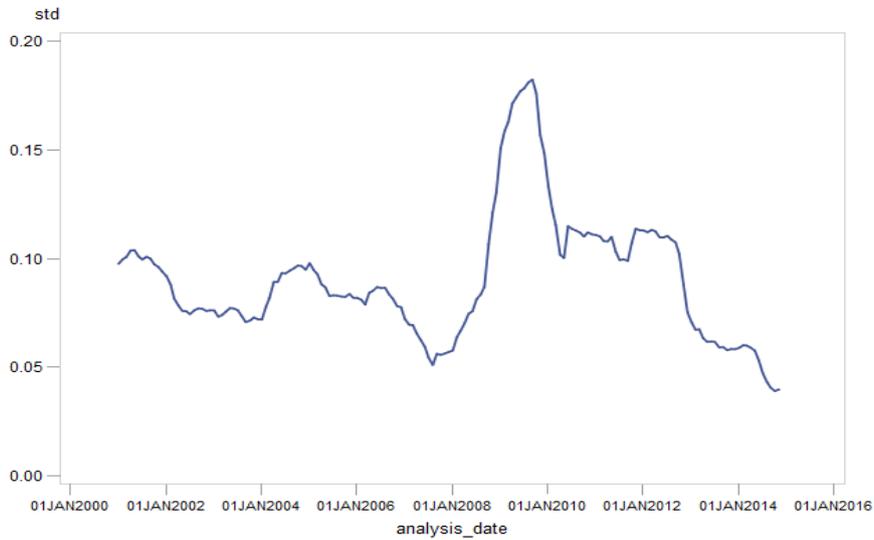
FX cross return correlations. These numbers can be compared with the numbers in Table 4. However, the numbers in Table 6 are expected to be higher than those in Table 4 since the USD is included in all pairs in Table 6.

Table 7: FX cross annualized standard deviations

FX cross	Std yearly
AUD/USD	13.17%
CAD/USD	9.13%
CHF/USD	10.75%
EUR/USD	9.87%
GBP/USD	9.06%
JPY/USD	10.38%
NOK/USD	11.91%
NZD/USD	13.54%
SEK/USD	12.01%

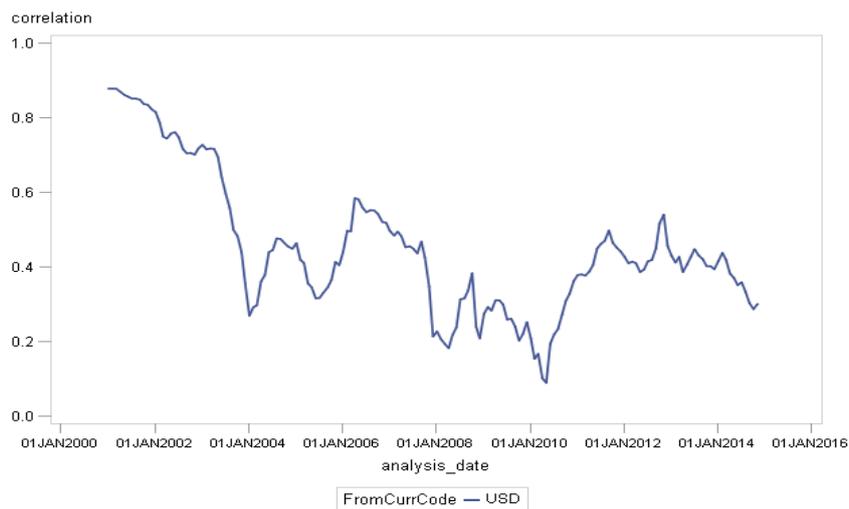
The FX cross return volatilities for all currencies against the USD estimated from daily returns.

Graph 3: Intrinsic USD return annualized standard deviation



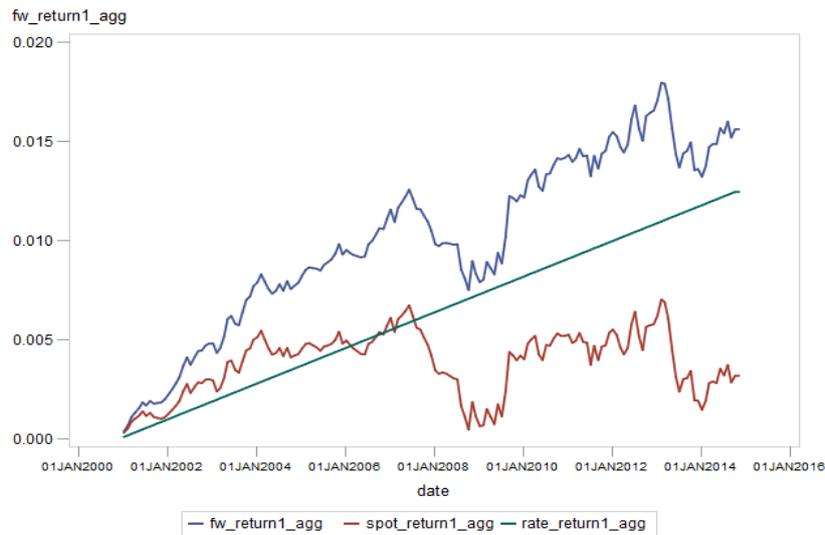
The intrinsic USD standard deviation shows a peak during the financial crisis and has after that declined.

Graph 4: Intrinsic CAD – USD return correlation



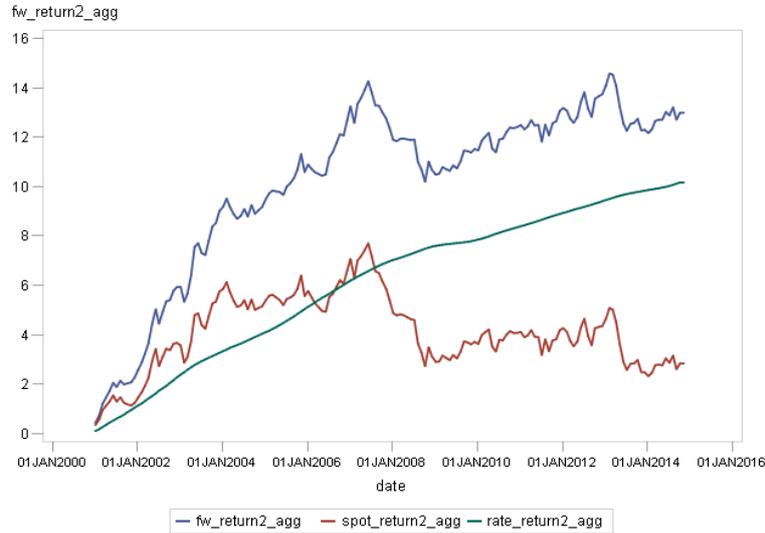
The CAD-USD intrinsic return correlation shows that the correlation between currencies can vary substantially through time.

Graph 5: Accumulated return for the Opt1 strategy



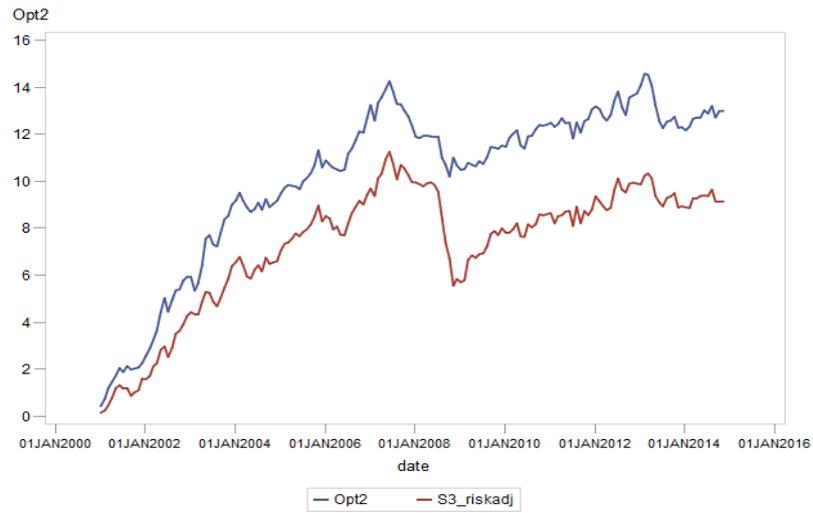
Accumulated return for the Opt1 strategy. This strategy has a constant carry pickup by construction, which makes the accumulated carry pickup a straight line.

Graph 6: Accumulated return for the Opt2 strategy



Accumulated return for the Opt2 strategy. This strategy has a constant estimated risk at each rebalance date. Because of that, the carry pickup is no longer constant, which can be seen from that the accumulated carry pickup is not a straight line.

Graph 7: Accumulated returns for the Opt2 and S3\_riskadj strategies



Comparison of the accumulated total returns of the Opt2 and S3\_riskadj strategies scaled to the same total risk for comparison purposes.