Topics on Harmonic analysis and Multilinear Algebra

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Thesis for the degree of Doctor of Philosophy to be defended in public on **Thursday 22th of October 2015, at 13:15 in room Pascal**, Department of Mathematical Sciences, Chalmers Tvärgata 3, Göteborg.
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Abstract

The present thesis consists of six different papers. Indeed, they treat three different research areas: function spaces, singular integrals and multilinear algebra. In paper I, a characterization of continuity of the p-Λ-variation function is given and Helly's selection principle for $\Lambda BV^{(p)}$ functions is established. A characterization of the inclusion of Waterman-Shiba classes into classes of functions with given integral modulus of continuity is given. A useful estimate on the modulus of variation of functions of class $\Lambda BV^{(p)}$ is found. In paper II, a characterization of the inclusion of Waterman-Shiba classes into H^q_ω is given. This corrects and extends an earlier result of a paper from 2005. In paper III, the characterization of the inclusion of Waterman-Shiba spaces $\Lambda BV^{(p)}$ into generalized Wiener classes of functions $BV(q;\,\delta)$ is given. It uses a new and shorter proof and extends an earlier result of U. Goginava. In paper IV, we discuss the existence of an orthogonal basis consisting of decomposable vectors for all symmetry classes of tensors associated with Semi-dihedral groups SD_{8n}. In paper V, we discuss o-bases of symmetry classes of tensors associated with the irreducible Brauer characters of the Dicyclic and Semi-dihedral groups. As in the case of Dihedral groups [46], it is possible that $V_\phi(G)$ has no o-basis when ϕ is a linear Brauer character. Let $\vec{P} = (p_1, \ldots, p_m)$ with $1 < p_1, \ldots, p_m < \infty$, $1/p_1 + \cdots + 1/p_m = 1/p$ and $\vec{w} = (w_1, \ldots, w_m) \in A_{\vec{P}}$. In paper VI, we investigate the weighted bounds with dependence on aperture α for multilinear square functions $S_{\alpha,\psi}(\vec{f})$. We show that

$$\|S_{\alpha,\psi}(\vec{f})\|_{L^p(\nu_{\vec{w}})} \leq C_{n,m,\psi,\vec{P}}\ \alpha^{mn}[\vec{w}]_{A_{\vec{P}}}^{\max(\frac{1}{2},\frac{p_1'}{p},\ldots,\frac{p_m'}{p})} \prod_{i=1}^{m} \|f_i\|_{L^{p_i}(w_i)}.$$

This result extends the result in the linear case which was obtained by Lerner in 2014. Our proof is based on the local mean oscillation technique presented firstly to find the sharp weighted bounds for Calderón–Zygmund operators. This method helps us avoiding intrinsic square functions in the proof of our main result.

Keywords: Generalized bounded variation, Helly's theorem, Modulus of variation, Generalized Wiener classes, Symmetry classes of tensors, Orthogonal basis, Brauer symmetry classes of tensors, Multilinear singular integrals, weighted norm inequalities, weighted bounds, local mean oscillation, Lerner's formula