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Incentive Schemes Under New Regulations

Master's Thesis

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Abstract

Due to the financial crisis of 2007, regulatory authorities engaged in a fundamental reconsideration of how they approach financial regulation and supervision. Therefore, that financial crisis caused the need of a significant change of the regulations of remuneration in the financial sector in the EU. It is worth to notice that the EU led the way internationally as the EU' s approach has exerted externally. The European Securities and Markets Authority published guidelines on the managers' remuneration policies of investment funds. The guidelines introduced new standards for investment managers.

In this thesis, we study the managerial behaviour due to changes to incentive schemes. We consider a manager who is offered an awarded fee by the investor based on a specific model, which we will subsequently analyse. Moreover, the manager withholds a percentage of the earned bonuses in an account for some time period. We study how the management fee ranges accordingly to how small or big is the amount which is saved in the account. Finally, we observe that the existence of the account evolves more risk taking but it gives higher total fees.

Keywords–Regulations, Bellman equation, bonus, dynamic programming, incentives, portfolio choice, value function, manager, account

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Contents

1 Introduction **1**

2 Model **4**
- 2.1 Fund Value Evolution 4
- 2.2 Remuneration Scheme 5

3 Theoretical Analysis **6**

4 Numerical Solution **8**
- 4.1 Case I ($\theta = 100\%$) 8
- 4.2 Cases II ($\theta = 50\%$) and III ($\theta = 75\%$) 9
- 4.3 Parameter Choices 9

5 Results **11**
- 5.1 Case I - Reference Scheme, $\theta = 100\%$ 11
- 5.2 Case II, $\theta = 50\%$ 12
- 5.3 Case III, $\theta = 75\%$ 16
- 5.4 Comparison Of Surfaces 20
  - 5.4.1 $\theta = 50\%$ 20
  - 5.4.2 $\theta = 75\%$ 20
- 5.5 Comparison of Optimal Choices 20
- 5.6 Comparison of Managerial Fees And Investor Returns in The Different Cases 21

6 Conclusions and Future Directions **24**
- 6.1 Conclusions 24
- 6.2 Future Work 24

Bibliography **27**

A Appendices **28**
A.1 Constant Relative Risk Aversion Utility function 28
A.2 Bellman Equation 29

1

Introduction

Before we start analysing the aim of this thesis, let us mention that the last years new stricter regulations on financial sector remuneration have been introduced. According to the Financial Stability Board (FSB) [1]:

> Compensation practices at large financial institutions are one factor among many that contributed to the financial crisis that began in 2007. High short-term profits led to generous bonus payments to employees without adequate regard to the longer-term risks they imposed on their firms. These perverse incentives amplified the excessive risk-taking that severely threatened the global financial system and left firms with fewer resources to absorb losses as risks materialised. The lack of attention to risk also contributed to the large, in some cases extreme absolute level of compensation in the industry.

Consequently, during 2010 the EU passed legislation imposing new restrictions on financial sector remuneration [2]. The EU Institutions claimed that EU 's new requirements were some of "the strictest rules in the world on bankers' bonuses". Member States were required to implement the new requirements into their domestic law by 1 January 2011, including applying them to bonuses for 2010. To assist in the application of the new requirements, the Committee of European Banking Supervisors (CEBS, but from 2011 replaced by the European Banking Authority (EBA)), issued guidelines on remuneration policies and practices.

The new EU remuneration restrictions define the target in terms of the remuneration incentives in order to limit the financial sector's desire for high-risk, short term financial strategies. Since the solution should be compatible with the problem, the success of regulatory reform of financial sector pay should properly be judged by reference to its capacity to restrain excessive risk-taking, not wider redistributive goals [3] [4].

This thesis report studies the managerial behaviour, due to changes in incentive schemes.

But do managers really attempt to maximize fund value? It is easy to see how they might be tempted to engage in activities not in the best interest of shareholders. Their goal is not necessarily to outperform a particular competitor or benchmark, but to perform well enough to keep and, hopefully, grow their assets under management. There are a number of ways to overcome this incentive problem, such as managers can be given salary incentive schemes to get their interests to move toward those of shareholders. Therefore, variable incentive schemes are an increasingly fundamental part of total remuneration of managers for investors. In the context of asymmetry of information between investors and their managers, we investigate the use of incentive contracts as strategic variables. Usually investors split their assets between index-managers and specialists. Specialists are the ones who take on large risks and their remuneration is often performance based, therefore they seem to be the ones with incentive schemes.

Management fee structures vary from fund to fund as they can manage mutual funds, private equity, units trust, hedge funds and offshore companies on behalf of investors/owners. However, they are typically based on a percentage of assets under management. Most of the times it is a fixed fee and a bonus fee based on the excess return related to some index to the order of which, fees are proportional.

Our work is motivated by the paper ”On Incentives for Sustainable Investments” [5] which studies the effect of high water mark based bonus schemes and compares the different types of those schemes. Specifically, four different managerial remuneration schemes are considered, one reference setting and three different water mark settings of different height. Our work aims to study such incentive schemes under the new regulations. We do not examine the effect of high water mark but we add a new regulation and we consider only one remuneration scheme, the reference setting. Moreover, the thesis proposes incentive structures for investments in environmentally friendly assets, i.e., sustainable investments. Remuneration schemes for those types of investments mainly aim to yield higher managerial income if the ”green” (sustainable) asset outperforms the ”black” (conventional) asset. We consider an account where the manager withholds a proportion of earned bonuses for four time periods and it is managed exactly like the fund. More analytically, the investor awards the manager a bonus after the end each time point and the manager saves a percentage of this bonus in an account, keeping the rest of the bonuses in her ”pocket”. At the end of the terminal point, the manager empties the account at a discount rate of $1 + \delta + \delta^2$. We need to use a discount factor because it considers not just the time value of money, but also the risk or uncertainty of future cash flows. The greater the uncertainty of future cash flows, the higher the discount rate.

However, we do not keep track of when the bonuses were earned. Furthermore, by changing the percentage of the amount which the manager saves in the account, we observe how the awarded fee ranges. Also, we compare these results with the result of the reference scheme where there does not exist any account.

The rest of this thesis is organised as follows:

In Chapter 2, we introduce the model that we use in order to achieve our results, followed by the theoretical analysis of this model in Chapter 3. Chapter 4, consists of the numerical solution, followed by the results that we got in Chapter 5. You find our conclusions and the presumptive future work that we could continue with in Chapter 6.

2

Model

2.1 Fund Value Evolution

In this section we will specify the model that governs the development of the fund value.

Similarly with the paper ”On Incentives for Sustainable Investments” [5], we assume that the capital to be handled by the manager is initially equal to 1. We also assume that her mandate runs over the time interval [0,T]. In our case, we set that the total time periods are four. As we have already mentioned the manager is expected to allocate capital in two different investments, which may be considered ”green” and ”black”, respectively. So at discrete time points t, for $t = 0, 1, 2, ..., T-1$, the manager allocates capital according to

$$\lambda_t e^{X_{t+1}} + (1-\lambda_t)e^{Z_{t+1}}, \tag{2.1}$$

where $e^{X_{t+1}}$ represents the green investment and $e^{Z_{t+1}}$ represents the black investment. The index $t+1$ is used to indicate that the returns on choices made at time t are not realized until time $t+1$. The choice λ_t is a real number between 0 and 1. At time $t+1$ while the investor observes the fund value, she offers the manager an awarded fee which is denoted as Φ_{t+1}. The manager deducts this fee from the fund value before she makes her next choice. Letting $f(\lambda_t) = \lambda_t e^{X_{t+1}} + (1-\lambda_t)e^{Z_{t+1}}$, the value of the fund before rewarding the manager, is assumed to be evolving according to

$$Y_{t+1} = f(\lambda_t)(Y_t - \Phi_t), \tag{2.2}$$

where Φ_t is the manager's awarded fee at time t. Hence,

$$\tilde{Y} = Y_t - \Phi_t \tag{2.3}$$

where $\tilde{Y}$ denotes the fund value after rewarding the manager.

2.2 Remuneration Scheme

At each time point $t+1$ the manager receives a fraction α of the fund value, and if she performs better than the "conventional" or the black asset she receives a fraction β of the percentage of her bonuses that she does not put on the account of the excess return and also receives that percentage of the fund value of the account value. We also note that the bonus, i.e., the manager's extra reward of the excess return, is linear in the choice λ_t.

Thus the incentive scheme is given by

$$\Phi_{t+1}(\lambda_t) = [\alpha f(\lambda_t) + \theta\beta\lambda_t \max\{e^{X_{t+1}} - e^{Z_{t+1}}, 0\}]\tilde{Y_t} + f(\lambda_t)\theta A_t \tag{2.4}$$

for the time points $t \in \{0, ..., T-2\}$, where:

α and β are fixed prices, $1-\theta$ indicates the percentage of the manager's bonus that is put on her account and A_t is the account value at the time t and is given by:

$$A_t = (1-\theta)[f(\lambda_t)A_{t-1} + \beta\lambda_t \max\{e^{X_{t+1}} - e^{Z_{t+1}}, 0\}\tilde{Y_t}], \forall t \geq 2 \tag{2.5}$$

and

$$A_t = f(\lambda_t)A_{t-1} + \beta\lambda_t \max\{e^{X_{t+1}} - e^{Z_{t+1}}, 0\}\tilde{Y_t}, \forall t \leq 1 \tag{2.6}$$

Moreover for $t = T-1$ the awarded fee is given by:

$$\Phi_T(\lambda_{T-1}) = f(\lambda_t)[\alpha\tilde{Y_t} + \frac{1}{3}A_t(1+\delta+\delta^2)] + \lambda_t\tilde{Y_t}[\theta + \delta^3(1-\theta)]\beta \max\{e^{X_{t+1}} - e^{Z_{t+1}}, 0\}\tilde{Y_t} \tag{2.7}$$

where δ is the discount factor.

The discount factor δ is needed because it takes into account not just the time value of money, but also the risk or uncertainty of future cash flows. The greater the uncertainty of future cash flows, the higher the discount rate. At this point it is worth to notice that we do not keep track of the amount that is on the account when the deposits are made.

In order to see if the total fees are affected by the percentage of the bonuses that are kept in the account, we test the model for different values of this percentage. First, we consider as case I, the hypothesis that there does not exist any account. In that case, θ equals to 100%. Afterwards, when θ equals to 50% we meet case II and then case III is the one when θ is equal to 75%. In the first case where $\theta = 100\%$, the manager does not save any money at the account, fact that means the new regulations are not applied here. In the second case when $\theta = 50\%$, the percentage of the manager's bonus that is put on her account is 50%, i.e. she saves the half of her bonuses. Thirdly, while $\theta = 75\%$, the manager keeps 25% of her bonuses on an account.

3

Theoretical Analysis

In order to briefly outline the methodology for solving the optimization problems, we need to use the constant relative risk aversion utility function. The Constant Relative Risk Aversion utility function [6] is the most straightforward implication of relative risk aversion that occurs in the context of forming a portfolio with one risky asset and one risk-free asset, therefore it consists the most appropriate utility function for our aim.

The Constant Relative Risk Aversion utility function is

$$U(x) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 0, \gamma \neq 1, \\ ln\gamma & \text{if } \gamma = 1. \end{cases} \tag{3.1}$$

The parameter γ measures the degree of relative risk aversion that is implicit in the utility function. Before we continue to the methodology, it is worth to notice that when total utility is a sum of CRRA functions, the ratios between the consumption of different goods depend only on relative prices, not on the income level. This means that if there is an increase in real income level, the consumption of all goods will go up in the same proportion as income. In other words, all income elasticities are equal to one. In the section of Appendices we refer more thoroughly to the properties of the function.

We continue by assuming that given the CRRA utility function and letting δ be the discount factor or time preference, the manager's aim is to maximize the expected sum of utilities of her fees. Thus, the latter, similar to Foufas, Sundén and Carlsson's[5], is given by:

$$E[\sum_{t=0}^{T-1} \delta^{t+1} U(\Phi_{t+1}(\lambda_t))] = \frac{1}{1-\gamma} E[\sum_{t=0}^{T-1} \delta^{t+1} \Phi_{t+1}^{1-\gamma}(\lambda_t)], \tag{3.2}$$

with respect to $\lambda_0, \lambda_1, ..., \lambda_{T-1}$. The maximization will be done using dynamic programming methods. We define the value function

$$V_t(s) = \sup_{\lambda_t,\ldots,\lambda_{T-1}} \frac{1}{1-\gamma} E[\sum_{i=t}^{T-1} \delta^{i-t+1} \Phi_{t+1}^{1-\gamma}(\lambda_i)|S_t = s], \tag{3.3}$$

where S_t is the state of the system at time t and especially it is two-dimensional with $S_t = (\tilde{Y_t}, A_t)$.

According to the dynamic programming literature, e.g. Stokey, Lucas and Prescott [7], the results give that the value function V_t satisfies the Bellman equation

$$V_t(s) = \sup_{\lambda} \delta E[\frac{\Phi_t^{1-\gamma}(\lambda)}{1-\gamma} + V_{t+1}(S_t(\lambda))|S_{t-1} = s], \tag{3.4}$$

for $t \in \{1, ..., T\}$, and where $V_{T+1} = 0$ as the manager-investor relation is terminated at time T.

Moreover, while the state of the system at time $T-1$ is indicated as S_{T-1} we are to find the optimal choices $\hat{\lambda}_{T-1}(s)$, such that

$$\hat{\lambda}_{T-1}(s) = \operatorname*{argmax}_{\lambda} E[\frac{\Phi_T^{1-\gamma}(\lambda)}{1-\gamma}|S_{T-1} = s]. \tag{3.5}$$

as

$$\operatorname*{argmax}_{x} f(x) := \{x|\forall y : f(y) \leq f(x)\} \tag{3.6}$$

The corresponding value function is the right hand side of 3.5 with the argmax_λ substituted by the $\sup_\lambda$.

$$V_T(s) = \sup_{\lambda} E[\frac{\Phi_T^{1-\gamma}(\lambda)}{1-\gamma}|S_{T-1} = s]. \tag{3.7}$$

At time points $t = T-2, ..., 0$ the problem is to find the numbers $\hat{\lambda}_t(s)$ such that

$$\hat{\lambda}_t(s) = \operatorname*{argmax}_{\lambda} E[\frac{\Phi_{t+1}^{1-\gamma}(\lambda)}{1-\gamma} + V_{t+2}(S_{t+1}(\lambda))|S_t = s]. \tag{3.8}$$

with corresponding value functions given by

$$V_{t+1}(s) = \sup_{\lambda} E[\frac{\Phi_{t+1}^{1-\gamma}(\lambda)}{1-\gamma} + V_{t+2}(S_{t+1}(\lambda))|S_t = s]. \tag{3.9}$$

Thus, we find the array of optimal choices

$$\{\hat{\lambda}_0, \hat{\lambda}_1(s_1), ..., \hat{\lambda}_{T-1}(s_{T-1})\}. \tag{3.10}$$

4

Numerical Solution

To numerically solve the Bellman equations for the optimal choices, a tree of state space going into each time period is spanned. By using the optimization and interpolation routines in MATLAB, we compute the fund value function and the optimal choices at each node of the tree. The optimal choice $\hat{\lambda}_{t-1}$ is found using the "fminbound" routine in MATLAB.

However to numerically solve the Bellman equations for the optimal choices at hand, the state space is as discretized.

4.1 Case I ($\theta = 100\%$)

This is the simplest setting considering that there are no new regulations and the manager does not put any amount of the bonuses on the account. In mathematical terms, the fee Φ_t at time t can be written

$$\Phi_{t+1}(\lambda_t) = [\alpha f(\lambda_t) + \beta \lambda_t \max\{e^{X_{t+1}} - e^{Z_{t+1}}, 0\}]\tilde{Y}_t \tag{4.1}$$

In this point of view we should backdate in equation 2.4 where if we set

$$M_t = \max\{e^{X_{t+1}} - e^{Z_{t+1}}, 0\}, \tag{4.2}$$

for notational convenience, we get

$$g(\lambda_t) = \alpha f(\lambda_t) + \beta \lambda_t M_t. \tag{4.3}$$

Then at time $t = T-1$ the optimal choice is

$$\hat{\lambda}_{T-1}(s) = \underset{\lambda}{\operatorname{argmax}} \frac{1}{1-\gamma} E[(g(\lambda)\tilde{Y}_{T-1})^{1-\gamma} | \tilde{Y}_{T-1} = y] \tag{4.4}$$

$$= \underset{\lambda}{\operatorname{argmax}} \frac{y^{1-\gamma}}{1-\gamma} E[g^{1-\gamma}(\lambda)] \tag{4.5}$$

where y is the fund value instantly after the manager is rewarded the amount Φ_{T-1}. But $g(\lambda)$ and $\tilde{Y}_{T-1}$ are independent therefore the optimal choice is independent of the fund value y. We find the optimal choice $\hat{\lambda}_{T-1}$ in MATLAB by using the "fminboud" routine.

The corresponding value function V_T is given by

$$V_T = \sup_{\lambda} \frac{1}{1-\gamma} E[(g(\lambda)\tilde{Y}_{T-1})^{1-\gamma} | \tilde{Y}_{T-1} = y] \tag{4.6}$$

$$= \sup_{\lambda} \frac{y^{1-\gamma}}{1-\gamma} E[g^{1-\gamma}(\lambda)] \tag{4.7}$$

as $g(\lambda)$ and $\tilde{Y}_{T-1}$ are independent.

While the account is empty, we set $A_t = 0$, henceforth, the state S_t becomes $\tilde{Y}_t$. Thus, similarly as previously, by using the equation 3.8 we can obtain that all the optimal choices are independent of the current fund value. Our numerical procedure generates a sequence of real numbers $\lambda_0, \lambda_1, ..., \lambda_{T-1}$.

4.2 Cases II ($\theta = 50\%$) and III ($\theta = 75\%$)

In these two remaining settings the manager saves an amount of the bonuses on the account and therefore the state S_t is equal to $(\tilde{Y}_t, A_t)$. With this state and with Φ_t given by 2.4 and 2.6, the Bellman equation gives

$$\lambda_t(y,a) = \operatorname*{argmax}_{\lambda} E[\frac{\Phi_{t+1}^{1-\gamma}(\lambda)}{1-\gamma} + V_{t+1}(\tilde{Y}_t(\lambda), A_t) | \tilde{Y}_t = y, A_t = a] \tag{4.8}$$

The corresponding value function similarly is

$$V_t(y,a) = \sup_{\lambda} \delta E[\frac{\Phi_{t+1}^{1-\gamma}(\lambda)}{1-\gamma} + V_{t+1}(\tilde{Y}_t(\lambda), A_t) | \tilde{Y}_t = y, A_t = a] \tag{4.9}$$

Thus for these two cases, there is no independence between the optimal choices and the fund value and therefore we get surfaces (and not a sequence of real numbers) of optimal choices. In all cases the simulation procedure starts from the optimal starting choice λ_0.

4.3 Parameter Choices

The manager is assumed composing a portfolio with higher yield than the point of reference and also her portfolio is assumed to be more risky than the point of reference. The green asset also presents higher volatility than the black asset. Moreover, as we noticed before, the X_t and Z_t are Gaussian random variables with correlation parameter given by:

$$Corr(Z_t, X_{t+h}) = \begin{cases} \rho & \text{if } h = 0, \\ 0 & \text{if } h \neq 0. \end{cases} \tag{4.10}$$

as

$$Cov(X_t, X_{t+h}) = 0, h \neq 0 \tag{4.11}$$

and

$$Cov(Z_t, Z_{t+h}) = 0, h \neq 0 \tag{4.12}$$

This choice is done for transparency and it is not obligatory to work with this distribution exclusively.

Below (tables 4.1 and 4.2) we appose the parameters that we chose for the distribution of the stochastic variables and for the remuneration scheme.

Table 4.1: Parameters for the distribution

Green asset expected return	$\mu_{gr.}$	0.08
Black asset expected return	μ_{bl}	0.03
Green asset volatility	σ_{gr}	0.3
Black asset volatility	σ_{bl}	0.2
Correlation between two assets	ρ	0.7

Table 4.2: Parameters for the remuneration scheme

Fixed fee fraction	Bonus fraction	Risk aversion	Discount factor	Interest rate
α	β	γ	δ	r
0.02	0.2	2	0.96	0.15

5

Results

In this chapter we will see the prices of the optimal starting choices, the graphs of the remaining three optimal choices and the histograms of the optimal choices from 100000 simulations for each of the cases that we face.

5.1 Case I - Reference Scheme, $\theta = 100\%$

Running the optimization for this reference scheme we find the optimal choices as

Table 5.1: Optimal choices

$\hat{\lambda}_0$	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$
0.690	0.745	0.825	0.962

5.2 Case II, $\theta = 50\%$

Below are the optimal choices in Case II where the manager puts the 50% of her bonuses on the account for four periods. The optimal starting choice is

$$\hat{\lambda}_0 = 1.0000 \tag{5.1}$$

The second, the third and the fourth optimal choices as functions of the fund value and the account value are found in the graphs that follow, Figures (5.1), (5.2) and (5.3) We should mention that there are some numerical "glitches" which make the surfaces non-smooth.

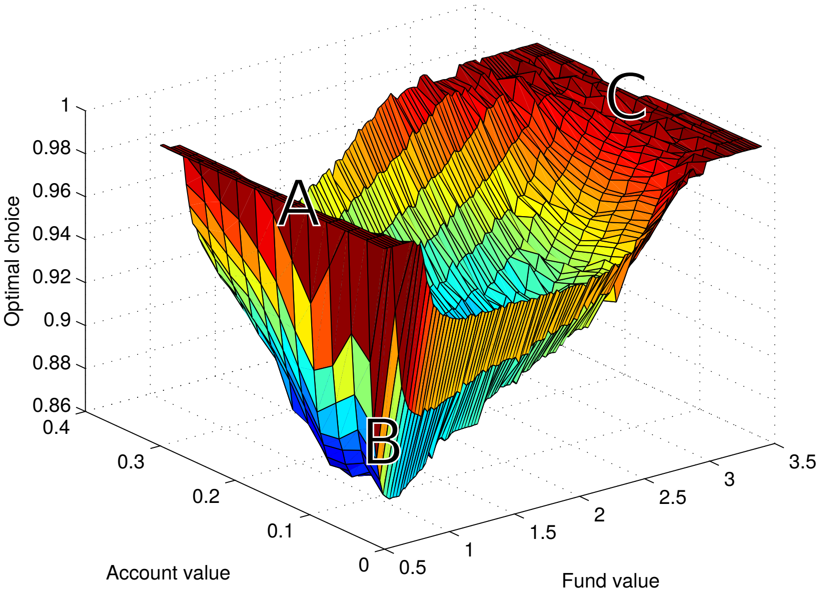

Figure 5.1: The optimal choice $\hat{\lambda}_1$ after the first period as a function of the fund value and the value of the account when $\theta = 50\%$

Using the terminology of Hodder and Jackwerth [8], "Gambler's ridge" in area A of figure (5.1) where the fund value is rather low is not surprising in this case. Here manager is in a situation that could be described as "heads: I win, tails: I don't lose very much". As we see in this $\hat{\lambda}_1$ plot for low fund values, the manager optimally becomes risky and "goes all in" willing to gamble with a very large λ. Even more interesting is the "Hill of Anticipation" toward the area B of figure 5.1. This area is a novel area of managerial behaviour. She has more to lose and more time left to manage the fund than on the Gambler's Ridge area, and this moderates her behaviour regarding λ. As the fund value increases, it is worth more, therefore the bets become higher and the manager keeps being risky, as we see in area C of the same figure.

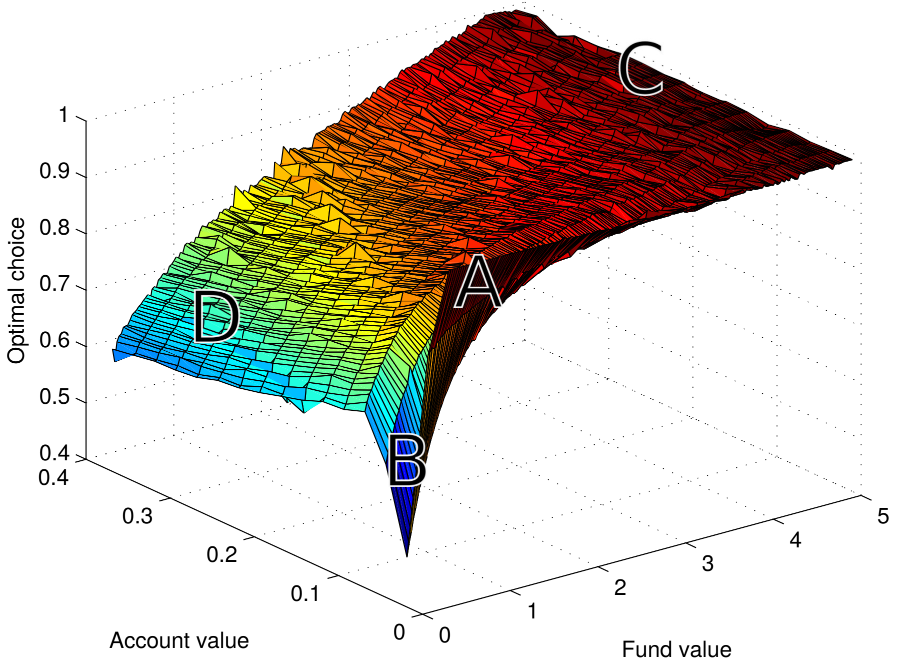

Figure 5.2: The optimal choice $\hat{\lambda}_2$ after the second period as a function of the fund value and the value of the account when $\theta = 50\%$

In the $\hat{\lambda}_2$ plot (5.2), we can clearly see that while the account is empty as we can notice at area A, the manager behaves too risky. As the account value starts increasing and the fund value is still low, we observe an "option ridge" in the plot which is seen at area B. Now the manager has more to lose and more time left to manage the fund and this moderates her behaviour regarding λ. She decreases the risk-taking and moves safer. However, above option ridge, the manager's optimal λ increases consecutively and also ramps up faster towards an upper "Merton Flats" region, area C, which exists at very high fund values.

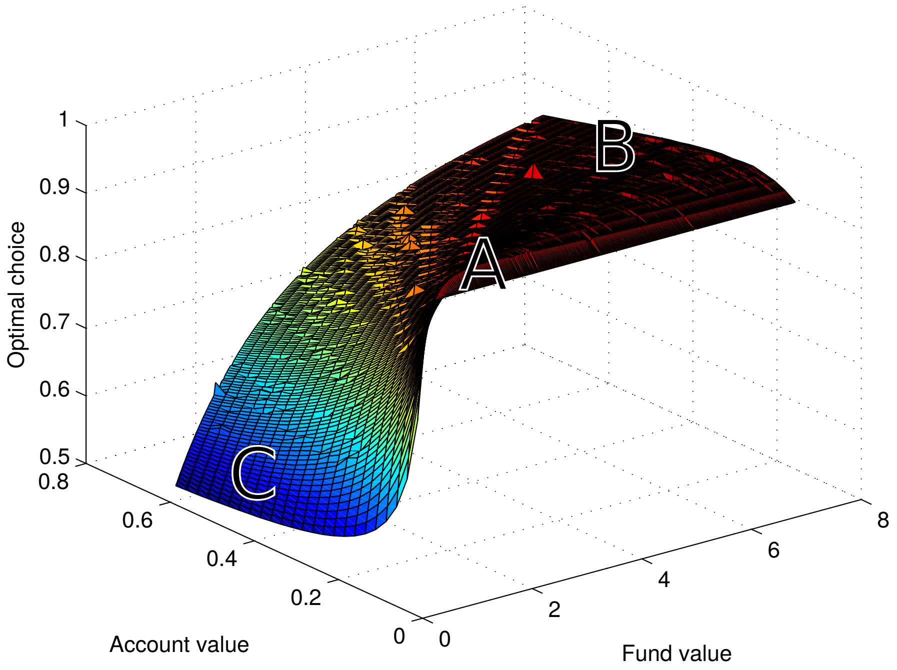

Figure 5.3: The optimal choice $\hat{\lambda}_3$ after the third period as a function of the fund value and the value of the account when $\theta = 50\%$

The $\hat{\lambda}_3$ plot (5.3) bears similarities with the $\hat{\lambda}_2$ plot (5.2) as while the account is empty, area A, the risk is too high, but here we also see lowered risk taking when the fund value is very low independently of account value, area C. As the fund value reaches its highest values, area B, the manager continues being risky enough.

In figure (5.4) we can see the histograms of the optimal choices from 100000 simulations. From left to right, we see $\hat{\lambda}_1$, $\hat{\lambda}_2$ and $\hat{\lambda}_3$. Going chronologically through time periods, we see that the choices are increasing on average. This means that the manager optimally takes on more risk as she approaches termination of her contract. We note that even though the Merton flat is large in figure 5.1, trajectories only end up in one percent of the runs is whereas the optimal choice in figure 5.4 is one in a 10 percent of the runs.

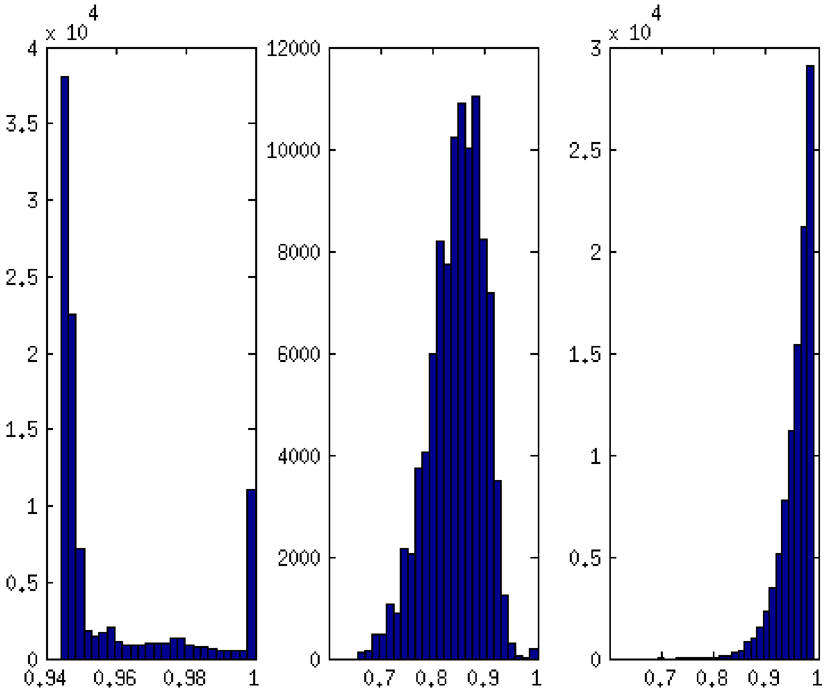

Figure 5.4: The optimal choices from 100000 simulations when $\theta = 50\%$

5.3 Case III, $\theta = 75\%$

In this section the percentage that the manager saves in the account is about 25% and the optimal starting choice in this case is

$$\hat{\lambda}_0 = 1.0000 \tag{5.2}$$

The remaining optimal choices as functions of the fund value and the account value can be found in the graphs 5.5, 5.6 and 5.7 As we mentioned before, there are some numerical "glitches" which make the surfaces non-smooth.

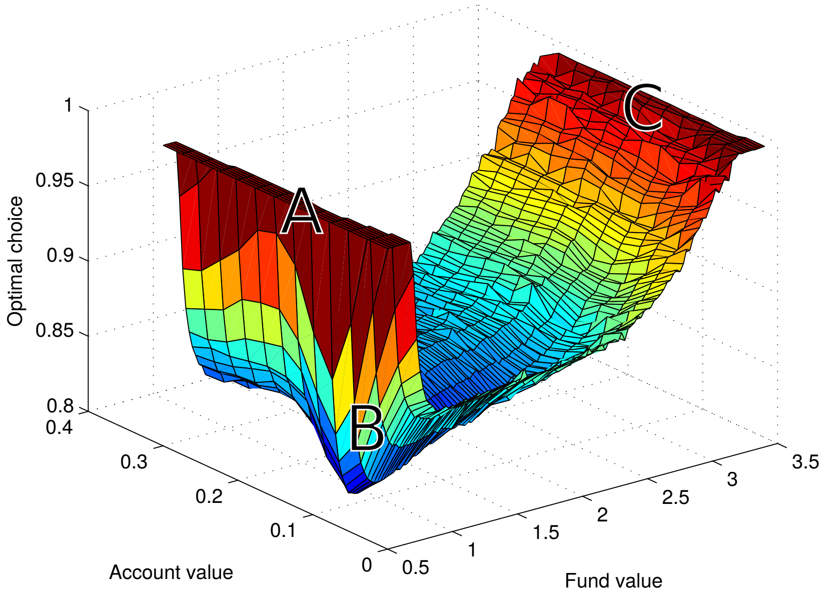

Figure 5.5: The optimal choice $\hat{\lambda}_1$ after the first period as a function of the fund value and the value of the account when $\theta = 75\%$

Here the $\hat{\lambda}_1$ plot (Figure 5.5) bears many similarities with the $\hat{\lambda}_1$ plot in case II where the amount that is kept in the account is 50%. (5.1) Again the manager optimally becomes too risky for low fund values and she "goes all in" as we can observe at area A. While the fund value starts increasing and the account value also gets a bit higher, as we see at area B, the manager decelerates and gets less risky. As we get closer to the terminal point of this period and the fund value peaks its highest value (area C) the bets get really high and the manager "goes all in" again as fund worth more.

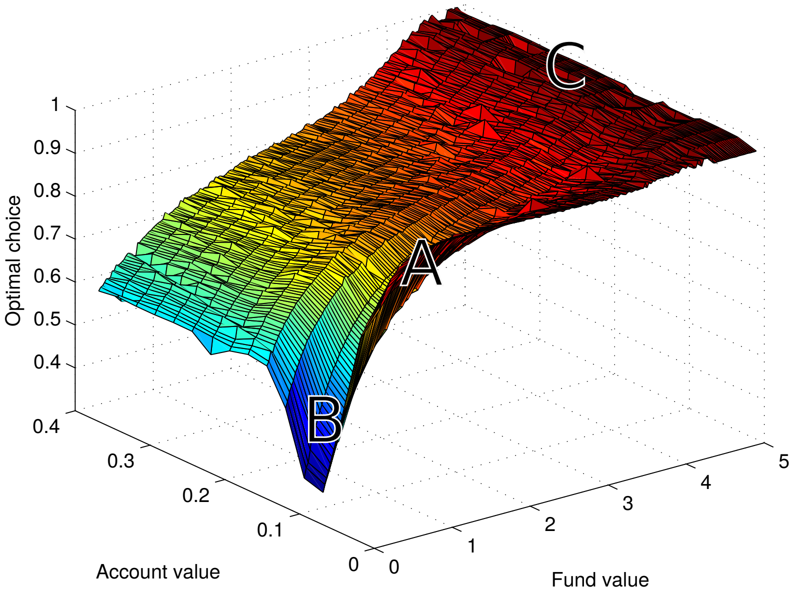

Figure 5.6: The optimal choice $\hat{\lambda}_2$ after the second period as a function of the fund value and the value of the account when $\theta = 75\%$

In the $\hat{\lambda}_2$ plot (5.6) the manager behaves in the same way as when $\theta = 75\%$. Again at area A of this plot we notice that for too low account value and low fund value, the manager takes high risk. As the account value gets a little higher and the fund value remains low (area B), there is an option ridge in the plot where the manager "slows down" and becomes less risky. Afterwards, as being observed at area C while the fund value reach its highest level, we notice Merton flats in the plot and the that is the time where the manager "goes all in" again.

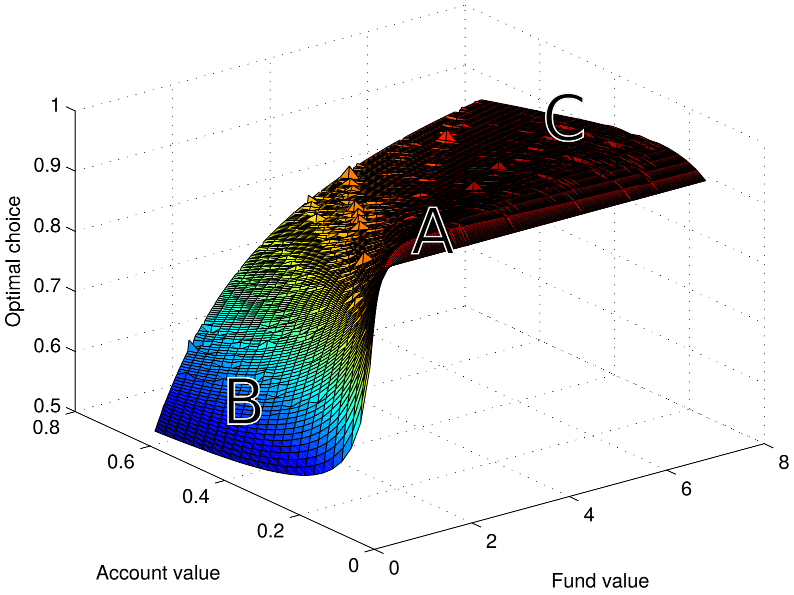

Figure 5.7: The optimal choice $\hat{\lambda}_3$ after the third period as a function of the fund value and the value of the account when $\theta = 75\%$

In the $\hat{\lambda}_3$ plot (5.7) we clearly see many similarities with the $\hat{\lambda}_3$ plot (5.3). Again the risk is too high (area A) when there is no money in the account and too low when the fund value is low (area B). Moreover, as the fund value reaches its highest level (area C), the risk-taking rockets.

The histograms of the optimal choices for 100000 runs follow in figure (5.8). From left to right, we see $\hat{\lambda}_1$, $\hat{\lambda}_2$ and $\hat{\lambda}_3$.

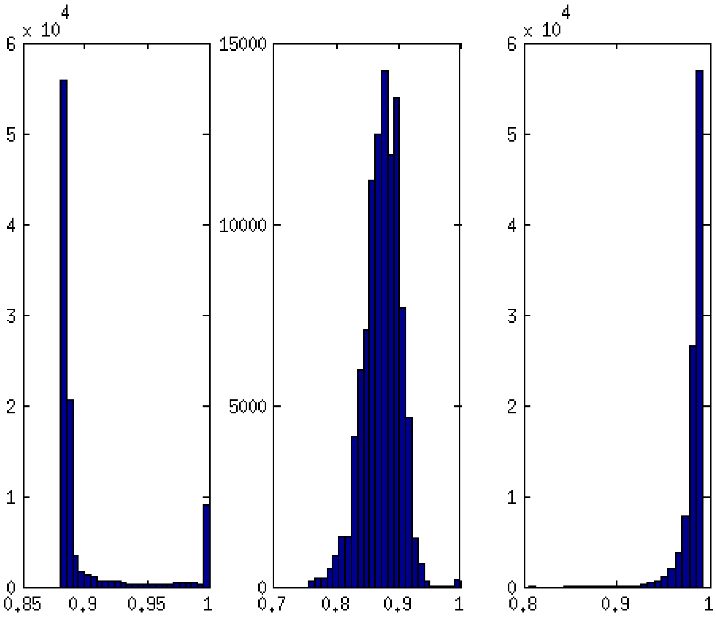

Figure 5.8: The optimal choices from 100000 simulations when $\theta = 75\%$

Again in Figure (5.8) we see that the optimal choices are increasing on average and comparing to the previous case, we note that the optimal choice levels on average are similarly concentrated in both cases. In addition, similarly with previous case, we note again that even though the Merton flat in figure 5.5 is large enough, trajectories only end up in almost one percent of the runs whereas the optimal choice one as we can see in figure 5.8 in a 10 percent of the runs.

5.4 Comparison Of Surfaces

5.4.1 $\theta = 50\%$

We note that while the fund value is low enough, the risk taking is really high in the $\hat{\lambda}_1$ plot as we can see at area A, but it is lowered in the $\hat{\lambda}_2$ plot (area D) and rather more lowered in the $\hat{\lambda}_3$ plot (area C). Also, we can notice that there is an "option ridge" present in the plots for $\hat{\lambda}_1$ (area B) and $\hat{\lambda}_2$ (area B) but not for $\hat{\lambda}_3$.

5.4.2 $\theta = 75\%$

In this case we note the same similarities and differences in the plots of optimal choices as in case II. Similarly, the risk taking is getting considerably reduced from $\hat{\lambda}_1$ plot to $\hat{\lambda}_3$ plot while the fund value is rather low. Again, there is an "option ridge" present in the plots for $\hat{\lambda}_1$ (area B) and $\hat{\lambda}_2$ (area B) but not for $\hat{\lambda}_3$.

5.5 Comparison of Optimal Choices

In the table (5.2) below we compare the mean of the optimal choices when $\theta = 50\%$ and $\theta = 75\%$ with the optimal choices when $\theta = 100\%$. As we can see at the table, the means of the optimal choices when there exists an account at the time points, 0, 1 and 2 are apparently higher than the corresponding optimal choices when such an account does not exist. However, it is worth noticing that at the last time point the results are reversed as the optimal choice while there is not an account is higher than the means of the optimal choices when there is account. That fact is reasonable as at the end of the time periods when there are savings, the manager withdraws the money and the account empties. Overall, we can obtain that the manager takes much more risk when she saves a percentage of the bonuses in an account.

Table 5.2: Optimal choices

$\theta = 50\%$	$\theta = 75\%$	$\theta = 100\%$
$\hat{\lambda}_0 = 1.000$	$\hat{\lambda}_0 = 1.000$	$\hat{\lambda}_0 = 0.690$
$\bar{\hat{\lambda}}_1 = 0.9467$	$\bar{\hat{\lambda}}_1 = 0.8946$	$\hat{\lambda}_1 = 0.745$
$\bar{\hat{\lambda}}_2 = 0.8886$	$\bar{\hat{\lambda}}_2 = 0.8530$	$\hat{\lambda}_2 = 0.825$
$\bar{\hat{\lambda}}_3 = 0.8878$	$\bar{\hat{\lambda}}_3 = 0.8906$	$\hat{\lambda}_3 = 0.962$

5.6 Comparison of Managerial Fees And Investor Returns in The Different Cases

In this section we try to compare the performance of the manager between the three different cases while considering the differences in terminal fund values.

Below in figures (5.9), (5.10) and (5.11) we can notice the fixed, bonus and total fees for each of the cases respectively. Regarding the case where there is no account (5.9), we can clearly see that the overall performance of the total fees shows a continued increase and they reach their highest level of approximately 0.058. On the other hand, at the cases where there exists an account, things are a bit different. (5.10) and (5.11) We notice that the total fees begin from a starting point much higher than the fees in case I and after some fluctuations, they end up to the same value which they started from. More specifically, when the manager puts half of her bonuses on an account, the high initial value of total fees is followed by a notable decrease at the third time point. Then we notice a rapid rise to the same peak as the initial value. In case III where the manager saves 25% of the bonuses at the account, the total fees behave similarly with case II. However it is observed a slight difference as we can clearly see that the fees remain steady between the second and third time points. After this the total fees shoot up to the highest level of approximately 0.062.

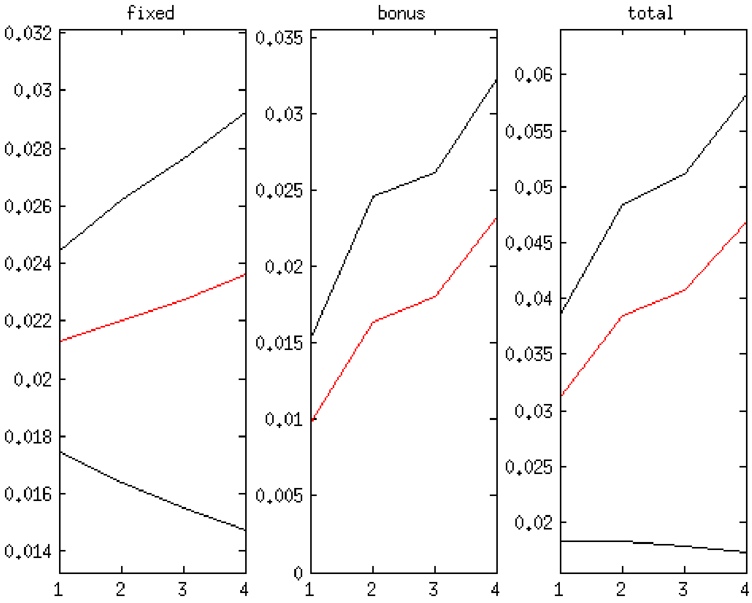

Figure 5.9: Fees when $\theta = 100\%$

Overall we conclude that the total fees are slightly higher when the manager puts on the account 25% of her bonuses.

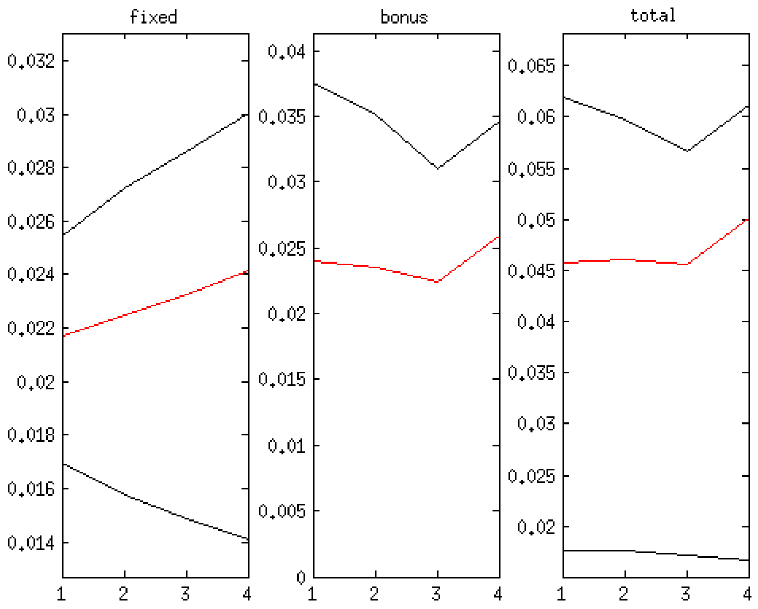

Figure 5.10: Fees when $\theta = 50\%$

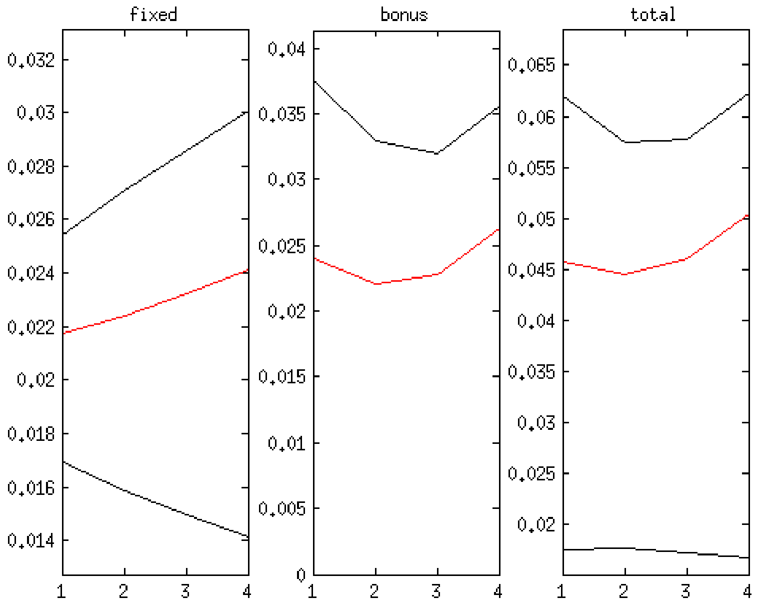

Figure 5.11: Fees when $\theta = 75\%$

Moreover, it is quite of interest to take a look at the fund value from the investor's point of view (5.3) As seen in Table 5.3 the spread between upper and lower bounds for case I when $\theta = 100\%$ are narrower than those for the other two cases where there exists an account. Furthermore, the cases where the manager saves money from the bonuses, have higher terminal fund value and also case I where there does not exist any account, gives a lower mean of the terminal value than the other two cases. We see that the differences between the cases where there exists an account are really small.

Table 5.3: Mean, 25% and 75%-quantiles for the fund value at termination

Cases	$\theta = 50\%$	$\theta = 75\%$	$\theta = 100\%$
upper bound	1.4385	1.4394	1.4017
mean	1.1574	1.1567	1.1352
lower bound	0.6834	0.6851	0.7146

Additionally at (5.4) we observe the values of the account at termination. Of course, we do not consider the case of no new regulations because it is obvious that there is no money at the account yet.

Table 5.4: Mean, 25% and 75%-quantiles for the account value at termination

Cases	$\theta = 50\%$	$\theta = 75\%$
upper bound	0.0686	0.0342
mean	0.0540	0.0268
lower bound	0.0131	0.0066

6

Conclusions and Future Directions

6.1 Conclusions

All the tests that we run lead us to the conclusion that those different features in portfolio managers' remuneration schemes show some differences in optimal choices between two risky assets. Especially, we have noted that the introduction of an account with bonus savings increases risk and also gives more opportunity for the manager and the investor to maximize their profits.

Looking at simulation results we see that the cases where the manager puts a percentage of her bonuses in an account, naturally give choices that are increasing in time approaching the termination of the contract. Also it is worth to notice that in both those settings, the choices are similarly concentrated.

Furthermore, by comparing all the surface plots we see that they are all similar to each other. Thus we obtain the information that if the manager saves a percentage of her bonuses in an account, it is significant how great is that percentage. We saw that the differences between $\theta = 50\%$ and $\theta = 75\%$ are quite small but we should mention that the total fees are slightly higher when the manager saves the smallest amount of the bonuses. Hence, we can obtain that the case III with $\theta = 75\%$ shows a very small advantage comparatively to case II. However, it does not matter how big or small will this percentage be but it surely matters if there exists or not a percentage!

6.2 Future Work

The approach that has been studied in this thesis, can generate new questions for future work. It would be really interesting to research these incentive schemes under an improved version of this regulation.

In our research, we briefly examine what are the consequences while the manager puts a percentage of her bonuses in an account and keeps in her ”pocket” the remaining amount. Nevertheless, we could address another question on that approach. What if she reinvests the remaining amount of her bonuses with investor’s fund? Addressing this question will allow us to reconsider the whole model under new regulations. The manager should be more careful with the investment choices that she would make as she will be directly affected.

Thus we answer the question that we addressed in the beginning of this report. Do the managers really attempt to maximize firm/fund value? The answer is yes if they have such incentives!

Bibliography

[1] F. S. Board, Fsf principles for sound compensation practices, Financial Stability Forum, 2009.

[2] E. Directive, Directive 2010/75/eu of the european parliament and of the council of 24 november 2010 on industrial emissions (integrated pollution prevention and control), Official Journal of the European Union L 334 (2010) 17–119.

[3] E. Ferran, New regulation of remuneration in the financial sector in the eu, European Company and Financial Law Review 9 (1) (2012) 1–34.

[4] M. Goldstein, P. Turner, Banking crises in emerging economies: origins and policy options, in: BIS Economic Paper 46. Bank for International Settlements, Monetary and Economic Department, Basle, Citeseer, 1996.

[5] G. Foufas, M. Sundén, E. Carlsson, On Incentives for Sustainable Investments, Sustainable Investment and Corporate Governance Working Paper 2010/1, Sustainable Investment Research Platform (2010).
URL `https://ideas.repec.org/p/hhb/sicgwp/2010_001.html`

[6] J. Geweke, A note on some limitations of crra utility, Economics letters 71 (3) (2001) 341–345.

[7] N. L. Stokey, Recursive Methods in Economic Dynamics, Harvard University Press, 1989.

[8] J. E. Hodder, J. C. Jackwerth, Incentive contracts and hedge fund management, Journal of Financial and Quantitative Analysis 42 (2007) 811–826.
URL `http://journals.cambridge.org/article_S0022109000003409`

[9] L. P. Hansen, K. J. Singleton, Stochastic consumption, risk aversion, and the temporal behavior of asset returns, The Journal of Political Economy (1983) 249–265.

[10] Z. Bodie, A. J. Marcus, A. Kane, Investements, Irwin, 1996.

[11] S. Garrett, An introduction to the mathematics of finance: a deterministic approach, Butterworth-Heinemann, 2013.

[12] M. Bardi, I. Capuzzo-Dolcetta, Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equations, Springer Science & Business Media, 2008.

A

Appendices

A.1 Constant Relative Risk Aversion Utility function

As we already noticed in Chapter 3 the Constant Relative Risk Aversion (CRRA) [9] [10] utility function is defined as:

$$U(x) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 0, \gamma \neq 1, \\ ln\gamma & \text{if } \gamma = 1. \end{cases} \tag{A.1}$$

where the parameter γ measures the degree of relative risk aversion that is implicit in the utility function.

We suppose that we have two goods and that

$$U = u(x_1) + u(x_2) \tag{A.2}$$

Since the first derivative of the CRRA utility function is

$$u'(x) = x^{-\gamma}, \tag{A.3}$$

the marginal rate of substitution is

$$\frac{u'(x_1)}{u'(x_2)} = \frac{x_1^{-\gamma}}{x_2^{-\gamma}} = (\frac{x_2}{x_1})^{\gamma} \tag{A.4}$$

$$\frac{x_2}{x_1} = (\frac{u'(x_1)}{u'(x_2)})^{\frac{1}{\gamma}} \tag{A.5}$$

Henceforth, $\frac{1}{\gamma}$ is the elasticity of the ratio of the consumed quantities of the two goods with respect to the marginal rate of substitution. By definition $\sigma = \frac{1}{\gamma}$ is then the elasticity of substitution and is a measure of the strength of the substitution effect that a change in relative prices urges. This is clearly seen as the first order condition for the consumer makes the marginal rate of substitution equal to the price ratio [11]. Therefore,

$$\frac{c_2}{c_1} = (\frac{p_1}{p_2})^{\sigma} \tag{A.6}$$

A.2 Bellman Equation

A Bellman equation is a dynamic programming equation which writes the value of a decision problem at a certain point in time in terms of the payoff from some initial choices and the value of the remaining decision problem that results from those initial choices. The solution of this equation is the "value function" which gives the minimum cost for a given dynamical system with an associated cost function. For a general optimization problem, we consider x_t a state at time t and x_0 the initial state. When action a_t represents one or more variables and the current state is represented by $T(x,a)$, the current payoff is $F(x,a)$. We assume "impatience" (rate of time preference) is represented by a discount factor $0 < \delta < 1$. Under these assumptions, an infinite-horizon decision problem becomes [12]

$$V(x_0) = \max_{\{a_t\}} \sum_{t=0}^{\infty} \delta^t F(x_t,a_t), \tag{A.7}$$

subject to the constraints a_t, $x_{t+1} = T(x_t,a_t)$ Moreover, we should mention that $V(x_0)$ is the value function. Henceforth,

$$V(x_0) = \max_{a_0}\{F(x_0,a_0) + \delta V(x_1)\} \tag{A.8}$$

subject to the constraints a_0, $x_1 = T(x_0,a_0)$. Therefore the Bellman equation can be written as

$$V(x) = \max_{a}\{F(x,a) + \delta V(T(x,a))\}. \tag{A.9}$$

In this thesis, the manager chooses a sequence $\{\lambda_t\}$, $t \in [0,T]$ with state of the system at time t, $S_t = (\tilde{Y}, A_t)$ in such a way that the expected sum of utilities of her fees are maximized:

$$\max_{\lambda_t} E[\sum_{t=0}^{T-1} \delta^t U(\Phi(\lambda_t))] \tag{A.10}$$

as $F(x_t,a_t)$ now is equal to $U(\Phi(\lambda_t))$.

The Bellman equation takes a very similar form:

$$V_t = \max_{\lambda_t}\{U(\Phi(\lambda_t)) + \delta V(S(\lambda_t))\} \tag{A.11}$$