UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

Master Degree Project in Finance

The Low Risk Anomaly
Evidence from Sweden

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Abstract

This paper finds that the low risk anomaly is present on NASDAQ OMX Stockholm during January 2005 until December 2014. The result has been produced with a survivorship bias-free sample, consisting of 25 108 firm-month observations in total. We sort stocks into quintile portfolios based on both rolling total volatility and rolling beta with a one-month holding period strategy. Both value-weighted and equal-weighted portfolios are used to obtain Jensen's alpha and Sharpe Ratio, leading to the same conclusion. The low risk anomaly is found in all market stages except for the bear market in 2007-2008. Benchmarking is one of the variables that explain the presence of the low risk anomaly in the Swedish market. A potential investment opportunity is thus to invest in low risk stocks and leverage the portfolio to increase expected risk-adjusted returns.

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1. Introduction

"The long-term outperformance of low-risk portfolios is perhaps the greatest anomaly in finance "

Baker, Bradley and Wurgler (2011)

1.1 Background

One of the most well-known relationships in finance is the one between risk and return, higher risk is rewarded with higher expected return. The capital asset pricing model (CAPM) measures the risk of a stock as the covariance with the market divided with the variance of the market. A fundamental principle of CAPM is that all investors invest in the portfolio with the highest expected excess return per unit of risk, and leverage or deleverage his portfolio to adjust their risk with respect to risk preferences. However, many investors, regardless if the investor is an individual, a pension fund or a mutual fund etc., are constrained in the leverage that they can take and may therefore overweight riskier assets instead of using leverage to suit their risk preferences Black, Jensen & Scholes (1972) showed that the security market line (SML) is flatter than predicted in CAPM and is better explained by CAPM with restricted borrowing (Black, 1972). In many recent studies, there is evidence, both on American and International level, that there is a negative relationship between risk and returns on a risk-adjusted basis (Ang et al. 2006; Ang et al. 2009; Clarke, de Silva and Thorley, 2006; Blitz and van Vliet, 2007; Baker, Bradley and Wurgler, 2011; Baker, Bradley and Taliaferro, 2013; Blitz and van Vliet, 2011; and Frazzini and Pedersen, 2011). Their findings violate one of the most known and used financial theories, higher risk leads to higher returns, confirming a low risk anomaly. However, a recent study on the Norwegian stock market could not find any evidence of a risk anomaly (Hafskör & Östenäs 2013)[1].

1.2 Contribution

As most research up to this date has been on either U.S. or broad international data, our main contribution is to test if the anomaly is present in the Swedish stock market as well. The contradicting results from Norway, a market similar to Sweden, emphasize further that analyses need to be done on each market. We use data on Swedish stocks from 2000 until 2014, with the first 5 years used as sampling period. Following Baker, Bradley and Wurgler (2011); and Baker, Bradley and Taliaferro (2013), we test the anomaly using both beta and total volatility as risk measurements. Further on, we test the anomaly in different market stages as well as for different market capitalization for robustness as well as to find explanatory factors of the anomaly. The goal is to either confirm the anomaly in the Swedish stock market as well, and thereby help investors boost their risk-adjusted returns, or to confirm that the Swedish stock market is efficient regarding risk-adjusted returns.

1.3 Research Question

The research question is formulated to answer the question whether NASDAQ OMX Stockholm is an effective market or not. If the null-hypothesis can be rejected this implies that investing in a leveraged low risk portfolio could increase returns.

H_0 : NASDAQ OMX Stockholm is efficient. There is no low risk anomaly present on the market

The Swedish market efficiency is determined by evaluating the respective portfolios' performance with Jensen's alpha, Sharpe Ratio and Treynor Ratio.

[1] Note that this is a master thesis. Not a peer-reviewed publication.

1.4 Results

In this paper, we find that the low risk anomaly is present in the Swedish stock market during 2005-2014. This is in line with previous literature (see for example Ang. et al., 2006; Baker, Bradley and Wurgler, 2011), confirming that low risk stocks yields higher risk-adjusted returns. We look at both equal-weighted and value-weighted portfolios sorted on risk in order to see if there are any discrepancies in results between the two methods; both leading to the same conclusion. Benchmarking as well as short-selling constraint are two possible explanations of the phenomenon. Inconsistent with our overall conclusion, findings are reversed when looking at the bear market in isolation, the financial crisis of 2007-2008. This contradicts the study of Ang. et al. (2006), who found the low risk anomaly to be present also in bear markets.

1.5 Delimitations

Many comparable studies (see for example Ang. et al, 2006; Baker, Bradley and Wurgler, 2011; Blitz & Vliet, 2007) use returns over a longer time period. It could be argued that using a longer time-period than our 15 year period could give more robust results. Yet, we choose a shorter time period as we want to investigate if the anomaly exists in present time and a shorter time period is more relevant when drawing conclusions regarding the current state of the financial market.

We have limited our results to only include return data on a monthly level as this is the most frequently used method. Using daily or weekly return data could improve our result as a check of robustness, but has been found to easily be disturbed by issues such as micro-noise data and thus be less reliable (see the critics from Bali and Cakici, 2008).

2. Literature Review

The low risk anomaly stands in direct contrast to financial theory and the subject has drawn a large amount of attention, in modern days especially since the study of Ang et al. (2006). The literature review is divided into three parts. In 2.1, we will discuss the different approaches used to measure risk. We look at the provided evidence for and against the existence of a low risk anomaly in 2.2, and in section 2.3 we examine proposed explanations for this phenomenon.

2.1 Proxy for risk

There are three definitions of risk used by researchers in this field. These are CAPM's beta, total volatility and idiosyncratic volatility. According to Baker, Bradley and Wurgler (2011), and also Trainor (2012), beta and total volatility have high correlation so which measurement you use will matter only to a lesser extent. This has also been shown by Blitz & Vliet (2007). Baker, Bradley and Wurgler (2011) argue that total volatility will play a part only if the portfolios are not diversified enough to remove idiosyncratic risk. Riley (2014) provides evidence that idiosyncratic volatility and total volatility are hard to separate and high (low) total volatility follows high (low) idiosyncratic volatility.

2.2 The Low Risk Anomaly

Ang et al. (2006) looks at U.S. data between 1963 and 2000, and find that firms with high idiosyncratic volatility have abysmally low average returns. These results were robust, controlling for size, book-to-market, momentum and liquidity. It was also persistent in bull and bear market as well as during both volatile and stable market conditions. Furthermore, they found that it could not be explained by higher exposure to aggregate volatility.

Empirical evidence of the existence of the anomaly on a global level has been presented by Ang et al. (2009), Blitz and Vliet (2007), Baker, Bradley and Taliaferro (2013), and Frazzini and Pedersen (2011), see Table 8 in the appendix for a summary. Blitz and Vliet (2007) use data from 86-06, and find that the annual spread in alpha, CAPM's measurement of risk-adjusted performance, between the global low and global high volatility docile portfolio amounts to 12%. They also provide evidence for the low risk anomaly in the European, American and Japanese market independently, the Japanese findings confirmed in a study by Iwasawa and Uchiyama (2014).

Baker, Bradley and Wurgler (2011) sorted portfolios according to risk, using beta and total volatility instead of idiosyncratic volatility as proxy for risk. They found that a 1 dollar investment into the lowest risk portfolio in 1968 would increase to 59.55 dollar at the end of 2008. The same investment in the riskiest portfolio would have been reduced to 58 cent. The monthly alpha for the low risk portfolio was significant and economically large, between 2 and 3 percent depending on sorting method.

Bali and Cakici (2008) present some critical evidence against a low risk anomaly. They find a positive correlation between risk and return and argue that difference in methodology applied is the reason behind this conflicting evidence. Three issues skewing the results were daily instead of monthly data collection frequency used to capture volatility, faulty weighting schemes and break points for portfolios, and filtering rules. To prove this they replicated the study by Ang et al. (2006), using daily volatility frequency sampling and found the same negative correlation between volatility and returns. Conversely, when they instead sample volatility on a monthly basis, they could not find evidence of a negative correlation. Instead the expected returns in correlation to volatility were flat or very weak. They also provide statistical evidence that monthly sampling of volatility works as a better proxy for future expected volatility than daily sampling.

Fu (2009) rejected the low risk anomaly. He argued that past idiosyncratic volatility is a poor measure for future idiosyncratic volatility and instead uses an EGARCH model to investigate the connection between volatility and returns. With his model, the results are reversed and he finds a positive relation between conditional volatility and expected returns. On the other hand, Guo, Kassa and Ferguson (2014) find that Fu (2009) the model has a look-ahead bias and provide evidence that in-sample EGARCH creates a significantly large bias that drives the results. The authors do not claim that the result of a positive correlation between volatility and returns is untrue but simply that the look-ahead bias in the EGARCH model is large enough to make the results of Fu (2009) unreliable.

A recent study by Hafsskär and Östnenes (2013) found no evidence of a low (idiosyncratic) volatility anomaly when investigating the Norwegian market from 1981 to 2012. Contrary to the conclusions drawn from U.S. and international data, they find a positive correlation between volatility and returns. Their results are robust for different methodologies, using different subsamples and industry exposure.

2.3 Explanations To The Low Risk Anomaly

Several explanations to the low risk anomaly have been proposed. Roughly put, these can be divided into mathematical compounding, behavioural and rational explanations. A short summary of the proposed explanations can be found in Table 9 in the appendix.

Trainor (2012) shows how the mathematical compounding of calculating beta and cumulative returns' attributes to the conclusion of a low risk anomaly. He finds that the conclusion of Ang et al. (2006; 2009) and Baker, Bradley and Wurgler (2011) have their roots in this compounding problem. Trainor (2012) found that for periods of low volatility in the market, high beta outperforms low beta. However, high beta was found to be doomed in the long run and consistently produce worse results due to the market being excessively volatile over time. However, Trainor

(2012) also notes that the compounding problem cannot explain why the monthly average returns of higher beta portfolios are not associated with higher returns and that his tests ignore transaction costs, and may also be troubled by survivorship bias.

Behavioural rationale that may explain this issue are lottery behaviour and overconfidence. Shefrin and Statman (2000) argue that investors will pay too much for stocks that would give large earnings quickly if successful, and only bear a small loss if unsuccessful. Bali Cakici and Whitelaw (2011) provide evidence for this behaviour to over-valuate lottery-like assets with high volatility. They found a strong negative relationship between the high performance in the previous month and performance in the current. Cornell (2009) argues that overconfidence, i.e., overestimation of private information, makes investors want to invest in stocks with high volatility as this market is where such skills are rewarded the most. Baker, Bradley and Wurgler (2011) expands on this subject and argues that when investors disagree on stock valuation they tend to stick to their own valuation, which causes wide differences in expected future stock returns. This issue becomes more present the more volatile the stock is, i.e., stocks with high uncertainty.

Even if such behaviour exists on the market, should not the rational, risk-neutral investor use this opportunity to his/her advantage? Baker, Bradley and Wurgler (2011) find that even though the institutional investors' share of the U.S. stock market has increased from 30 to 60% in the time period 1968-2008, the low risk anomaly has only become more apparent. Some suggested rational reasons to why this arbitrage has not been capitalized on are leverage constraints, shorting constraints and/or benchmarking. Frazzini and Pedersen (2011) argue that many investors such as individuals, pension funds and mutual funds, are constrained from taking leverage. Instead, to take on risk, they will overweight risky securities. Consistent with this, Baker, Bradley and Wurgler (2011) found that none of the five largest American mutual funds took leverage. Stambaugh, Yu and Yuan (2012) argue that short-sellers face more constraints than purchasers which make it difficult to exploit overpricing. Similarly, Bali, Cakici and Whitelaw (2011) suggest that it may be too costly to short-sell and that this is why the anomaly has not been capitalized on.

Baker, Bradley and Wurgler (2011) suggest that the anomaly can be explained partly by benchmarking. Benchmarking is that a manager needs to beat a benchmark, for example SP500 for the U.S market, without taking on too much, if any, leverage and are thus incentivised to pick stocks with higher volatility. They exemplify this by showing that for large cap, as fund managers tend to pay more attention to the larger stocks, a relatively high Sharpe Ratio (0.46) for their low volatility portfolio is insufficient to incentivize a fund manager to invest in the portfolio when the Information Ratio, which captures portfolio returns minus the benchmark return, is low (0.08). The benchmarking-argument is supported by their discovery that the average mutual fund had a beta of 1.10 at the time. Iwasawa and Uchiyama (2014) find that foreign institutional investors' behaviour is the main reason to why the low risk anomaly is present on the Japanese market. This is because institutional investors are aiming to beat a benchmark, which causes them to over invest in high-beta stock.

Loh and Hou (2014) explore many of the suggested variables that might explain the puzzle. They find that the behavioural explanations rather than rational explanations answer a larger portion of the anomaly. Especially the suggestion by Bali, Cakici and Whitelaw (2011) that there is an over investment in stocks that achieved the highest daily maximum returns the previous month, called lottery preference, seems to be able to explain a large portion (29-61%), of the anomaly (Loh and Hou, 2014).

3. Theoretical Framework

In this section we will cover the theoretical foundation used to examine our research question. We provide a detailed version of the development of CAPM in 3.1, a summary of its assumptions in 3.2, and in 3.3 we present some of the most frequently used measurements of performance in the finance sector.

3.1 CAPM

The Capital Asset Pricing Model, CAPM, was developed by Sharpe (1964), Lintner (1965) and Mossin (1966) as an extension from Makrowitz's (1952) portfolio model. It is the workhorse model in pricing theory and can be expressed as:

$$R_{it} - R_{ft} = \alpha_i + \beta_i\left(R_{mt} - R_{ft}\right) + \varepsilon_{it}$$

The model and how it is calculated will be discussed more extensively in the methodology section. A β (beta) for firm i is the sensitivity for the individual firm towards the market. If a firm or portfolio has a beta higher than 1, it is riskier than the average. The α (alpha) is the intercept and is used as a measure of skill as alpha will capture the excess return. If the alpha is positive, this is interpreted as an over performance of the portfolio compared to the market and the opposite conclusion is drawn if the portfolio has a negative alpha. How to calculate the so called Jensen's alpha is provided in the methodology section. The variance of ε_{it} is used as to measure idiosyncratic risk and have been conducted by among others Ang et al. (2006), Ang et al. (2009), and Chen et al. (2010). See Hafskör and Östenäs (2013) for a detailed version how this methodology can be applied.

3.2 Assumptions of CAPM

The assumptions underlying CAPM are individual behaviour and market structure. Individual behavior assumptions are that investors are rational, with the same expectations and the same planning horizon. Assumptions regarding market structure are that all assets are traded publicly, short-selling is possible and that investor can borrow to the risk-free rate. In addition, all information is publicly available and there are neither taxes nor transaction costs (Bodie et al. 2014).

Some of these assumptions are problematic and hard to satisfy in reality. As discussed in the literature review, leverage and internal short-selling constraints are one of the proposed reasons to why the anomaly appears. It also might not be possible to short-sell, both due to availability and restricting laws. It has also been proven by Black, Jensen & Scholes (1972) that the security market line is flatter than predicted by CAPM.

3.3 Performance Measurement

The Sharpe Ratio was developed by William Sharpe (1966) and is now one of the most used measurements of risk-adjusted performance of security portfolios. It is calculated as the portfolio's return minus the risk-free rate divided by the standard deviation of the returns. It is widely used because of its simplicity in both calculation and interpretation. The ratio describes how much extra excess return you get by adding one unit extra of volatility (risk).

In difference to the Sharpe Ratio, Treynor Ratio (1966) uses the Security Market Line as a base instead of the Capital Market Line. This means that the denominator in formula is changed from volatility to the beta from CAPM. The interpretation is the same as in the Sharpe Ratio, i.e., how much extra excess return you get by adding one unit extra of beta (risk). Sharpe Ratio is the more used of these to measurements, but which is preferable to the other is of small importance.

Information Ratio is the excess return of the portfolio compared to a benchmark divided by the tracking error. Tracking error is defined as the standard deviation of active returns. The ratio measures the excess return per unit of risk which in theory has the potential to be diversified away by holding the index portfolio of the market (Bodie et al. 2014).

The Information Ratio is important as benchmarking is widely used among fund managers, Sensoy (2009) found that 91.8% of all U.S mutual funds were benchmarked during 1994-2004. It is also common practice to show results compared to a benchmark when advertising a fund, even though the rational investor should be more concerned about performance in terms of Sharpe Ratio. Thus, the Information Ratio in comparison to Sharpe Ratio or alpha can be revealing in whether benchmarking is part of the reason for a low risk anomaly (Baker, Bradley and Wurgler, 2011)

4. Data

We use Swedish data on large-, medium- and small cap stocks listed on the Stockholm Stock Exchange from 2000-01-01 until 2014-12-31. We collect the data using the Bloomberg terminal. Relevant data is monthly prices for all stocks. We use last close price as the price, if it is not available, we take the mean between bid- and ask-price. The data is adjusted for dividends, stocks split, etc. In total, we have 373 stocks over our time period. However, the data set is not without survivorship bias; this has been adjusted for by manually adding stocks that have been delisted during the selected timeframe. Also, list changes have been adjusted for such as deleting observations for stocks that was listed on a smaller stock exchange (as for example First North) but are later listed on the main list. Also, Swedish companies often issue stocks with different voting rights such as a-, b-, and c-stocks. These stocks' valuations are based on the same fundamental information, i.e., future cash flow. To deal with this issue of duplicate stocks from the same firm, which could possibly skew our results, we remove the most illiquid stock, i.e., the A/B/C-stock with the lowest turnover. The average amount of stocks included per month totalled 208, with a maximum of 226, and a minimum of 192. We use the SIXRX-index, obtained from SIX Financial information, as the market return and the Swedish 3-month Stibor as the risk-free rate. The risk-free rate is downloaded from the Swedish Riksbank's homepage. Also, in order to be able to sort stocks into the three different categories large, medium and small cap stocks we collect the EUR/SEK rate from the Bloomberg terminal as well.

5. Methodology

We test the low risk anomaly in the Swedish market by using both beta and total volatility as risk measures, following Baker, Bradley and Wurgler (2011) as our main source of inspiration. We estimate the beta and the volatility for all stocks by using a 5-year trailing technique. Therefore, we use the first 5 years as an estimation period and divide stocks into quintile-portfolios starting 2005 based on the estimated risk.

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)} \qquad (1)$$

The beta of each stock, *i*, is calculated each month using the past 60 month's return of the stock as well as the return of the market (SIXRX), r_m. The same method is used for calculating the volatility of the stocks, where $\bar{R}$ is the average return for the period and n equals 60.

$$\sigma_i = \sqrt{\frac{\sum_{i=1}^{n}(R_i - \bar{R})^2}{n-1}} * \sqrt{12} \qquad (2)$$

At each month, we divide stocks into equal-weighted quintile portfolios based on risk. The portfolio is rebalanced every month based on the new estimated 5-year trailing risk. Excess monthly returns of each quintile portfolio are calculated as the monthly return minus the return of the risk-free rate on a monthly basis, measured as the return from 3-month STIBOR in one month. The performance of each portfolio is then tested to see if any statistical and economical differences can be found in performance between the portfolios.

We look at Jensen's alpha, calculated each month as:

$$\alpha_p = r_p - \left[r_f + \beta_p\left(r_m - r_f\right)\right] \qquad (3)$$

Where r_p is the return of the portfolio, r_f is the return of the risk-free rate and β_p is the beta of the portfolio p calculated as the sum of each stock's weight in the portfolio multiplied with the beta of the stock.

In addition, Sharpe Ratio is used to evaluate each quintile portfolio.

$$SR_p = \frac{r_p - r_f}{\sigma_p} \tag{4}$$

Where $r_p - r_f$ is the mean excess return and σ_p is the volatility of the portfolio p.

Following Baker, Bradley and Wurgler we also look at the Information Ratio and tracking error calculated as follows:

$$TE = \omega = \sqrt{Var(r_p - r_b)} \tag{5}$$

Where r_b is the return of the benchmarking, in our case the same as r_m.

$$IR = \frac{r_p - r_b}{\omega} \tag{6}$$

Where including variables has the same interpretation as earlier.

To further help evaluate and analyze the performance of our portfolios we include an additional measure of performance, the Treynor Ratio:

$$TR = \frac{r_p - r_f}{\beta_p} \tag{7}$$

Again, above variables have the same interpretations as previously described.

To broaden our research we expand the base from Baker, Bradley and Wurgler to see how other methods affect the results, we value-weight our quintile portfolios to follow the more updated research from Baker, Bradley and Taliaferro (2013). As short-selling constraint might be an explanation of the low risk anomaly, we look at the large cap stocks separately, which are assumed to be easier to short-sell for large institutions. We thereby want to see if the anomaly disappears when the short-selling constraint reduces. In addition, we also check if the anomaly is present for small and mid cap stocks as well. Another interesting topic in this field of research is to see if the anomaly is present during all market stages. We therefore perform a test in the bull market from January 2005 to December 2014, excluding the financial crisis in 2007-2008, and test the bear market that occurred in July 2007 to November 2008.

6. Results & Analysis

In our result and analysis sector, we refer to the portfolio with the lowest risk as "portfolio 1" and the portfolio with the highest risk as "portfolio 5". The other portfolios; 2, 3, 4, are on an increasing scale of risk in terms of either total volatility or beta.

The chapter is divided into four parts. First, we discuss our main findings from equal-weighted quintile portfolios, mirroring the study of Baker, Bradley and Wurgler (2011). We then discuss our results from value-weighted portfolios, for increased inference. Lastly, we divide our sample into different subsamples to further examine the presence of the anomaly and find possible causes to its appearance

6.1 Equal-Weighted Portfolios

This section looks at our main result, which can be seen in Table 1. The period consists of 25 108 firm-month observations divided into quintile equal-weighted portfolios selected on volatility and beta respectively as in Baker, Bradley and Wurgler (2011).

Looking at the results when sorting portfolios on volatility, we see that all measurement reported indicates above average performance of the stocks with lower volatility. The Sharpe Ratio, which accounts for and penalizes the riskiness of each portfolio, is substantially better for portfolio 1 and 2 (0.86 and 0.69 respectively) compared to their peers. Our monthly Jensen's alpha for each portfolio gives further evidence of a low risk anomaly with both economically (0.64% per month) and statistically (at the 1% level) significant alpha for portfolio 1. The high risk portfolio also has a significant alpha at the 1% level, but negative and large.

When sorting by beta, the spread between the different portfolios declines considerably. Now, portfolio 2 is the top performer in terms of Sharpe Ratio while portfolio 1 is to be preferred in terms of the Treynor Ratio. Portfolio 1 and 2 have a positive alpha at 0.37% and 0.47%, but neither is statistically different from zero. Again, the alpha for portfolio 5 is negative (-1.13%) and significant at the 1% level.

Even though the alpha is not statistically significant, the overall tendency also in the beta sorting is that low beta stocks outperform the market in risk-adjusted terms. Thus, our main findings are in line with Baker, Bradley and Wurgler (2011), the low risk anomaly is present also at the Swedish market.

Table 1: Returns by Volatility and Beta Quintile, January 2005-December 2014

	Equal-Weighted				
	1	2	3	4	5
A. Volatility sorts					
Average R_p-R_f	12.91%	12.38%	9.81%	11.43%	0.47%
Standard Deviation	15.09%	17.82%	20.29%	22.34%	23.54%
Sharpe Ratio	0.86	0.69	0.48	0.51	0.02
Treynor Ratio	0.21	0.16	0.10	0.10	0.00
Average R_p-R_m	2.36%	1.83%	-0.74%	0.88%	-10.08%
Tracking Error	6.45%	8.10%	9.25%	12.33%	16.09%
Information Ratio	0.37	0.23	-0.08	0.07	-0.63
Beta	0.61	0.79	0.96	1.19	1.43
Monthly alpha	0.64%	0.39%	0.00%	-0.18%	-1.41%
p-value (alpha)	0.0002***	0.0717*	0.9923	0.5869	0.0022***
B. Beta sorts					
Average R_p-R_f	8.01%	11.98%	8.79%	10.83%	7.43%
Standard Deviation	15.10%	17.60%	18.89%	22.06%	24.42%
Sharpe Ratio	0.53	0.68	0.47	0.49	0.30
Treynor Ratio	0.20	0.18	0.10	0.09	0.04
Average R_p-R_m	-2.54%	1.44%	-1.76%	0.29%	-3.12%
Tracking Error	10.01%	11.30%	8.57%	10.89%	12.18%
Information Ratio	-0.25	0.13	-0.21	0.03	-0.26
Beta	0.40	0.68	0.92	1.21	1.77
Monthly alpha	0.37%	0.47%	-0.05%	-0.22%	-1.13%
p-value (alpha)	0.1552	0.1054	0.8311	0.4431	0.0031***

Notes: 25 108 observations. For each month, we form portfolios by sorting all stocks into five equal-weighted quintile portfolios according to trailing volatility (standard deviation) for Panel A and trailing beta for Panel B. We estimate volatility and beta by using up to 60 months of trailing returns (i.e., return data starting as of January 2000). The Information Ratio uses the market return of SIXRX. Average returns are monthly averages multiplied by 12. Standard deviation and tracking error are monthly standard deviations multiplied by the square root of 12.

*** Significant at the 1% level

** Significant at the 5% level

* Significant at the 10% level

6.2 Value-Weighted Portfolios

This section looks at our value-weighted result and is presented in Table 2. The data is value-weighted portfolios selected on volatility and beta respectively as in Baker, Bradley and Taliaferro (2013). The difference between equal-weighted and value-weighted portfolios is that value-weighted adjust for market capitalization instead of investing an equal amount of stocks from each firm. This methodology gives more relative importance to the stocks with high market capitalization. From this section and forward, if not otherwise mentioned, all presented results are value-weighted. In unreported results we have also looked at equal-weighted portfolios to confirm our findings.

Table 2: Returns by Volatility and Beta Quintile, January 2005-December 2014

	Value-Weighted				
	1	2	3	4	5
A. Volatility sorts					
Sharpe Ratio	0.64	0.52	0.54	0.67	0.46
Treynor Ratio	0.14	0.11	0.09	0.12	0.06
Monthly alpha	0.28%	0.13%	-0.15%	0.16%	-1.15%
p-value (alpha)	0.0773*	0.53	0.62	0.67	0.0189***
B. Beta sorts					
Sharpe Ratio	0.69	0.67	0.47	0.49	0.55
Treynor Ratio	0.26	0.15	0.10	0.09	0.07
Monthly alpha	0.54%	0.33%	-0.01%	-0.20%	-0.81%
p-value (alpha)	0.0492**	0.17	0.96	0.34	0.0271**

Note: Full version of the Table is found as Table 2 in the Appendix.

Portfolio 1 is, as in the equal-weighted sorting case, outperforming the average as indicated by the Sharpe and Treynor Ratio. Looking further at the Sharpe Ratio, there is in this case no clear trend going from the high to low risk as the best portfolio in terms of Sharpe Ratio is the 4th portfolio. This is different from the findings in the equal-weighted sorting. Still, portfolio 1 has a 0.28% monthly alpha which is significantly different from zero at the 10% level, and the portfolio 5 has a monthly alpha at -1.15% significant at the 5% level.

Sorting by beta conveys a similar story but with a few discrepancies. In risk-adjusted terms, portfolio 1 and 2 outperforms their more risky peers. Yet again, only portfolio 1 (+0.54%) and 5 (-0.81%) have monthly alphas that are significantly different from zero.

Changing methodology from equal-weighted to value-weighted portfolios consolidates the results from our main approach and of the existence of a low risk anomaly on the Swedish market.

6.3 Large, Mid and Small Cap

We divided our sample into three subsamples; Large, Mid and Small cap, to control for whether the anomaly was present in all sub-markets. Additionally, short-selling restriction (see Stambaugh, Yu and Yuan, 2012) and benchmarking (see Baker, Bradley Wurgler (2011); Iwasawa and Uchiyama (2014)) are two of the proposed reasons for the low risk anomaly. We are especially interested in the large cap sample for two reasons. For short-selling restriction, we expect that large cap should show no or at least less evidence of an anomaly as these stocks are expected to be the easiest to short-sell. For benchmarking, fund managers commonly put more weight in the larger stocks and thus large cap is where any signs of benchmark should become most apparent.

Our large cap subsample is tested with value-weighted portfolios based on beta and volatility respectively. The subsample contains a total of 7 036 firm-month observations over the time period. From the results, we can still see a significant alpha on portfolio 1 in both beta and volatility sorting. However, even though the sign is still negative, portfolio 5 is in this sample not significantly different from zero. The alpha from portfolio 5 is also not as economically large as previously, in the volatility sorting it even outperforms portfolio 3.

Table 3: Returns by Volatility and Beta Quintile, January 2005-December 2014

	Large Cap				
	1	2	3	4	5
A. Volatility sorts					
Sharpe Ratio	0.69	0.66	0.23	0.53	0.70
Treynor Ratio	0.17	0.14	0.04	0.10	0.10
Information Ratio	-0.16	0.26	-0.65	0.16	0.51
Monthly alpha	0.35%	0.28%	-0.53%	-0.12%	-0.27%
p-value (alpha)	0.0752*	0.2385	0.0314**	0.6328	0.4944
B. Beta sorts					
Sharpe Ratio	0.98	0.32	0.45	0.42	0.68
Treynor Ratio	0.33	0.06	0.09	0.08	0.09
Information Ratio	0.22	-0.59	-0.18	-0.13	0.59
Monthly alpha	0.83%	-0.16%	-0.02%	-0.30%	-0.40%
p-value (alpha)	0.0017***	0.4696	0.9237	0.2266	0.1847

Note: Full version of the Table is found as Table 3 in the Appendix.

A look at Table 3 could give an indication of why the anomaly exists on the Swedish market. For large cap, the Sharpe Ratio is about equal at 0.69 for portfolio 1 and 0.70 for portfolio 5 in our volatility sorting. However, the Information Ratio shows a much more attractive 0.51 for portfolio 5 compared to -0.16 for portfolio 1. Sorting on beta reveals similar results with better alpha for portfolio 1 but with the difference that the Sharpe Ratio for portfolio 1 is now considerably better (0.98 versus 0.68) than portfolio 5. Again, however, portfolio 1 has a considerably lower Information Ratio. This highlights why a fund manager that needs to beat a benchmark would prefer the riskier portfolio 5, even though portfolio 1 has equal or better Sharpe Ratio and significantly higher alpha. The difference between Sharpe Ratio and Information Ratio is also more apparent in our large cap result where fund managers are presumed to be the most active, compared to results from the overall sample. These results confirm the finding of Baker, Bradley and Wurgler (2011) and thus a likely explanation to why this arbitrary opportunity has not been capitalized on is that fund managers are evaluated not by total return in relation to total risk, but on active return relative to active risk. This discourages investing in low beta stocks, leading to an anomaly.

The mid cap subsample contains a total of 6 629 firm-month observations (Table 5 in the appendix). Sorting on volatility gives some unexpected results as portfolio 1, portfolio 2 and portfolio 3 all have positive and significant alphas. The high portfolio has a negative and significant alpha, which differs from our large cap subsample where the alpha was insignificant. Sorting on beta gives results similar to the main results with significant portfolio 1 (positive alpha) and 5 (negative alpha) respectively.

The small cap is our largest subsample with a total of 11 415 firm-month observations (found in Table 6 in the appendix). Here we find no significant evidence of a low risk anomaly when sorting on beta, with a monthly alpha of 0.35 for portfolio 1. The anomaly is significant on the 10 per cent level when sorting on volatility. The Treynor Ratio shows a downward sloping trend from portfolio 1 to 5 independent of sorting method.

Comparing our different subsample, we can see that the low risk anomaly is more apparent in the case of large- and mid-cap. The result from small cap is still indicating a positive alpha for portfolio 1, however not as economically or statistically significant. Another interesting notice is that large cap and mid cap outperforms small cap in returns by quite the margin during our time period. This is an indication that size may affect value, at least on the Swedish market.

Though in this subsample we do not explicitly test the short-selling constraint, our large cap sample is the only sample when we have insignificant alpha for portfolio 5. This validates our hypothesis that portfolio 5, which have been economically (negatively) large in our main results, should be less significant in the large cap subsample as this is where short-sellers can exploited overvalued stocks to a larger extent. We interpreted this as an indication that these stocks are indeed easier to short-sell and that this has, to some extent, been exploited by the market

6.4 Bull and Bear market

From Table 4, we see the results for the Bear market of the financial crisis. Starting point is July 2007 and the bear market ends in November 2008, which gives a total of 3635 firm-month observations. The results are now practically reversed from our previous findings. When using the volatility sort there are no significant differences in alphas from zero, however, it seems like the trend is towards the better performance going from 1 to 4. Even though Sharpe Ratio is a less reliable performance measure during bear markets, the interpretation is that the more negative Sharpe Ratio, the worse the portfolio performs. In this case, the trend is the same as with the alphas, performance increase going from 1 to 4. Portfolio 5 is performing about average when looking at both measurements. The same analysis can be drawn when looking at the Treynor Ratio as well.

Table 4: Returns by Volatility and Beta Quintile

	Bear Market (July 2007-November 2008)				
	1	2	3	4	5
A. Volatility sorts					
Sharpe Ratio	-2.10	-1.94	-1.45	-1.21	-1.89
Treynor Ratio	-0.53	-0.52	-0.42	-0.31	-0.37
Monthly alpha	-0.32%	-0.14%	0.86%	2.20%	0.93%
p-value (alpha)	0.46	0.79	0.37	0.14	0.49
B. Beta sorts					
Sharpe Ratio	-1.83	-2.63	-2.27	-2.06	-1.39
Treynor Ratio	-0.66	-0.60	-0.60	-0.52	-0.27
Monthly alpha	-0.65%	-0.59%	-0.78%	-0.24%	2.75%
p-value (alpha)	0.47	0.17	0.12	0.66	0.0005***

Note: Full version of the Table is found as Table 4 in the Appendix.

Looking at the beta sorting, the results are now somewhat easier to interpret. Portfolio 5 is now showing a strong significant positive alpha. Looking at the Treynor Ratio we see that portfolios with higher beta outperform portfolios with lower beta on a risk-adjusted basis. This goes against all previous results earlier in this study, and also against the findings of Ang et al. (2006), who had significant low risk anomaly in all market stages.

The results of the bear market are quite remarkable as results are reversed. In order to find possible explanations to our bear market results, we need to dig deeper into the specific market investigated, i.e., the financial crisis of 2007-2008, as well as the characteristics of the Swedish market. The financial crisis hit the banks hard, and holding bank stocks before the financial crisis were considered to be safer than the stock market in general. This can be confirmed by looking at the four big banks in Sweden; Nordea, Swedbank, Handelsbanken & SEB which had an average 5-year trailing beta of 0.87, 0.74, 0.54 and 0.78 respectively prior to the financial crisis. Further on, the banking sector is a big part of the Swedish Stock market, representing almost a third of the OMXS30 index. The aforementioned stocks lost 58%, 90%, 50% & 84% respectively of their market value during the financial crisis, which has an immense impact on the results. Because of their low beta value and extremely bad returns during the financial crisis as well as their significant weights in their portfolios, we believe that this heavily affects the results in the reversed direction compared to our overall results. This also relates to Fu (2009) saying that risk based on past data is a poor measure of risk which is the case here since these stocks had much higher risk than the market during the financial crisis. An interesting field for further research would be to expand the time period to include the dot-com bubble burst to compare the results of that bear market to the financial crisis to validate or reject our hypothesis.

Table 7 in the appendix shows the results during a bull market. We defined the bull market as the whole time-period less the bear market in the former paragraph. The expected results of the bull market are easy to predict since the main results confirm the low risk anomaly on the Swedish market, and the bear market shows reversed results. Therefore, it is straight forward to expect that the low risk anomaly should be even stronger than in the main results when the bear market is deducted from the sample. Both volatility and beta sorting confirm these results, i.e., portfolio 1 has a significant positive alpha whereas portfolio 5 has a significant negative alpha in both cases. The anomaly is thereby highly present in the bull stages of the stock market. When looking at both Sharpe Ratio and Treynor Ratio, there is a downward trend in performance with increasing risk. This is consistent with the main results and the results of previous studies.

7. Conclusion

The main purpose of this study is to investigate whether the low risk anomaly, i.e., that portfolios consisting of low risk stocks outperform portfolios of stock with higher risk on a risk-adjusted basis, is present on the Swedish market. This is conducted by using CAPM's beta and volatility as proxy for risk, and Jensen's alpha and Sharpe Ratio as performance measures. The data is stocks traded at NASDAQ OMX Stockholm from 2000-01-01 to 2014-12-31, with the first 5 years used as sampling period. Quintile portfolios on an increasing scale of risk are created and rebalanced each month with a one-month holding period strategy. We find that the low risk anomaly is present in the Swedish market during our investigated time span. Our low risk portfolio has a high, positive alpha in all but one of our results, even though it fluctuates slightly in economical and statistical significance. In addition, the low risk portfolios constantly outperform the market in terms of Sharpe Ratio and Treynor Ratio. The low risk anomaly is found independently of the methodological approach regarding equal-weighted or value-weighted portfolios, and is present in large, mid and small capitalization separately.

The exception where portfolio 1 does not show a positive alpha, is found in the bear market of 2007-2008, where results are reversed and the low risk portfolio performs worse in risk-adjusted terms. We conclude that one of the reasons for this peculiarity is found in the characteristics of the Swedish market. Banks suffered heavy losses during the crisis and the four major banks, with low trailing-risk all sorted into the lower portfolios, composed a large part of the total market capitalization traded on NASDAQ OMX Stockholm, skewing our results. The short time period with a fairly small number of observations, 3565 firm-month, may also trouble our findings. To conclude whether the low risk anomaly is consistently not present (or reversed) during bear market in Sweden, further investigation is needed.

Factors that drive the anomaly on the Swedish market include benchmarking and short-selling restriction. Benchmarking becomes apparent when we compare the Information Ratio with the Sharpe Ratio and alpha between our quintile portfolios. Even though our low risk portfolio outperforms the high risk portfolio in terms of Sharpe Ratio and alpha, the high risk portfolio still has, for a fund manager, a more attractive Information Ratio. Investors that are judged by benchmarking are thereby discouraged from investing in low beta stocks. Ample evidence of short-selling constriction is found in our large cap sample where we found that our high risk portfolio did not perform as poorly compared to the overall results. An indication that short-selling has been used to exploit overvaluation of high risk stocks. These findings do not exclude other explanatory factors, and is a potential topic for future research.

The anomaly presents a potential investment opportunity for fund managers that are not bound by borrowing constraints. Higher expected risk-adjusted returns could be obtained by investing in low risk stocks and leverage their portfolio.

Lastly, our results in this thesis have been made by using CAPM, and produced during a limited time period. Thus, the flaws of CAPM, and the limitation in robustness checks should be taken into consideration when evaluating these results.

8. References

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Appendix

Table 2: Returns by Volatility and Beta Quintile, January 2005-December 2014

	Value-Weighted				
	1	2	3	4	5
A. Volatility sorts					
Average R_p-R_f	9.08%	10.97%	11.80%	17.52%	11.56%
Standard Deviation	14.12%	21.17%	21.92%	26.08%	25.06%
Sharpe Ratio	0.64	0.52	0.54	0.67	0.46
Treynor Ratio	0.14	0.11	0.09	0.12	0.06
Average R_p-R_m	-1.40%	0.60%	1.12%	6.90%	0.97%
Tracking Error	0.68%	0.76%	0.99%	1.30%	1.31%
Information Ratio	-2.07	0.79	1.13	5.32	0.74
Beta	0.63	1.01	1.25	1.50	1.87
Monthly alpha	0.28%	0.13%	-0.15%	0.16%	-1.15%
p-value (alpha)	0.0773*	0.53	0.62	0.67	0.0189***
B. Beta sorts					
Average R_p-R_f	9.19%	10.44%	9.04%	10.48%	13.07%
Standard Deviation	13.35%	15.53%	19.22%	21.31%	23.65%
Sharpe Ratio	0.69	0.67	0.47	0.49	0.55
Treynor Ratio	0.26	0.15	0.10	0.09	0.07
Average R_p-R_m	-1.41%	-0.18%	-1.56%	-0.14%	2.43%
Tracking Error	13.35%	10.24%	6.63%	8.20%	9.46%
Information Ratio	-0.11	-0.02	-0.24	-0.02	0.26
Beta	0.36	0.68	0.91	1.19	1.83
Monthly alpha	0.54%	0.33%	-0.01%	-0.20%	-0.81%
p-value (alpha)	0.0492**	0.17	0.96	0.34	0.0271**

Table 3: Returns by Volatility and Beta Quintile, January 2005-December 2014

	Large Cap				
	1	2	3	4	5
A. Volatility sorts					
Average R_p-R_f	8.91%	12.78%	4.58%	12.17%	16.90%
Standard Deviation	13.00%	19.30%	19.54%	22.77%	24.23%
Sharpe Ratio	0.69	0.66	0.23	0.53	0.70
Treynor Ratio	0.17	0.14	0.04	0.10	0.10
Average R_p-R_m	-1.64%	2.23%	-5.97%	1.62%	6.35%
Tracking Error	10.20%	8.67%	9.14%	10.04%	12.50%
Information Ratio	-0.16	0.26	-0.65	0.16	0.51
Beta	0.52	0.89	1.06	1.25	1.72
Monthly alpha	0.35%	0.28%	-0.53%	-0.12%	-0.27%
p-value (alpha)	0.0752*	0.2385	0.0314**	0.6328	0.4944
B. Beta sorts					
Average R_p-R_f	13.29%	4.79%	9.22%	9.35%	15.95%
Standard Deviation	13.50%	14.97%	20.54%	22.23%	23.39%
Sharpe Ratio	0.98	0.32	0.45	0.42	0.68
Treynor Ratio	0.33	0.06	0.09	0.08	0.09
Average R_p-R_m	2.75%	-5.76%	-1.33%	-1.19%	5.40%
Tracking Error	12.75%	9.79%	7.59%	9.49%	9.14%
Information Ratio	0.22	-0.59	-0.18	-0.13	0.59
Beta	0.40	0.77	0.99	1.20	1.75
Monthly alpha	0.83%	-0.16%	-0.02%	-0.30%	-0.40%
p-value (alpha)	0.0017***	0.4696	0.9237	0.2266	0.1847

Table 4: Returns by Volatility and Beta Quintile, January 2005-December 2014

	Bear Market (July 2007-November 2008)				
	1	2	3	4	5
A. Volatility sorts					
Average R_p-R_f	-38.59%	-56.09%	-48.91%	-46.14%	-57.60%
Standard Deviation	18.34%	28.91%	33.78%	38.11%	30.44%
Sharpe Ratio	-2.10	-1.94	-1.45	-1.21	-1.89
Treynor Ratio	-0.53	-0.52	-0.42	-0.31	-0.37
Average R_p-R_m	10.60%	-6.89%	0.28%	3.05%	-8.41%
Tracking Error	8.22%	9.41%	16.11%	22.76%	17.79%
Information Ratio	1.29	-0.73	0.02	0.13	-0.47
Beta	0.73	1.08	1.17	1.50	1.55
Monthly alpha	-0.32%	-0.14%	0.86%	2.20%	0.93%
p-value (alpha)	0.4605	0.7969	0.3762	0.1459	0.4940
B. Beta sorts					
Average R_p-R_f	-29.97%	-43.54%	-56.85%	-60.26%	-44.30%
Standard Deviation	16.38%	16.56%	25.07%	29.29%	31.82%
Sharpe Ratio	-1.83	-2.63	-2.27	-2.06	-1.39
Treynor Ratio	-0.66	-0.60	-0.60	-0.52	-0.27
Average R_p-R_m	19.23%	5.66%	-7.65%	-11.07%	4.89%
Tracking Error	18.37%	8.81%	6.97%	9.33%	11.47%
Information Ratio	1.05	0.64	-1.10	-1.19	0.43
Beta	0.45	0.73	0.95	1.16	1.66
Monthly alpha	-0.65%	-0.59%	-0.78%	-0.24%	2.75%
p-value (alpha)	0.4744	0.1706	0.1195	0.6629	0.0005***

Table 5: Returns by Volatility and Beta Quintile, January 2005-December 2014

	Mid Cap				
	1	2	3	4	5
A. Volatility sorts					
Average R_p-R_f	11.55%	16.29%	19.61%	17.06%	2.28%
Standard Deviation	17.44%	19.65%	25.57%	26.86%	27.07%
Sharpe Ratio	0.66	0.83	0.77	0.64	0.08
Treynor Ratio	0.20	0.22	0.21	0.14	0.01
Average R_p-R_m	1.00%	5.74%	9.06%	6.51%	-8.27%
Tracking Error	10.31%	11.80%	14.80%	16.78%	17.18%
Information Ratio	0.10	0.49	0.61	0.39	-0.48
Beta	0.58	0.74	0.92	1.25	1.67
Monthly alpha	0.50%	0.73%	0.85%	0.21%	-1.55%
p-value (alpha)	0.0757*	0.0225**	0.0301**	0.6281	0.0013***
B. Beta sorts					
Average R_p-R_f	13.88%	12.89%	14.94%	13.51%	12.08%
Standard Deviation	17.74%	20.01%	23.39%	23.69%	30.81%
Sharpe Ratio	0.78	0.64	0.64	0.57	0.39
Treynor Ratio	0.30	0.18	0.16	0.11	0.07
Average R_p-R_m	3.33%	2.34%	4.39%	2.96%	1.53%
Tracking Error	12.39%	11.24%	14.25%	13.63%	19.57%
Information Ratio	0.27	0.21	0.31	0.22	0.08
Beta	0.46	0.71	0.92	1.23	1.86
Monthly alpha	0.77%	0.51%	0.45%	-0.02%	-0.95%
p-value (alpha)	0.0234**	0.1024	0.2420	0.9650	0.0587*

Table 6: Returns by Volatility and Beta Quintile, January 2005-December 2014

	Small Cap				
	1	2	3	4	5
A. Volatility sorts					
Average R_p-R_f	11.79%	4.63%	4.60%	13.65%	-9.20%
Standard Deviation	17.17%	20.12%	26.88%	24.41%	27.01%
Sharpe Ratio	0.69	0.23	0.17	0.56	-0.34
Treynor Ratio	0.20	0.06	0.04	0.11	-0.06
Average R_p-R_m	1.25%	-5.92%	-5.95%	3.10%	-19.75%
Tracking Error	12.70%	11.85%	17.82%	17.75%	22.22%
Information Ratio	0.10	-0.50	-0.33	0.17	-0.89
Beta	0.59	0.80	1.08	1.29	1.42
Monthly alpha	0.58%	-0.19%	-0.52%	-0.12%	-2.21%
p-value (alpha)	0.0671*	0.5483	0.2821	0.8012	0.0005***
B. Beta sorts					
Average R_p-R_f	7.15%	5.83%	7.62%	3.45%	4.77%
Standard Deviation	17.60%	18.50%	22.10%	26.33%	26.57%
Sharpe Ratio	0.41	0.32	0.34	0.13	0.18
Treynor Ratio	0.19	0.09	0.08	0.03	0.03
Average R_p-R_m	-3.40%	-4.72%	-2.93%	-7.10%	-5.77%
Tracking Error	14.78%	14.60%	14.59%	17.77%	18.29%
Information Ratio	-0.23	-0.32	-0.20	-0.40	-0.32
Beta	0.37	0.64	0.90	1.17	1.72
Monthly alpha	0.35%	0.01%	-0.11%	-0.77%	-1.23%
p-value (alpha)	0.3440	0.9806	0.7826	0.1053	0.0272**

Table 7: Returns by Volatility and Beta Quintile, January 2005-December 2014

	Bull Market (excl. July 2007-November 2008)				
	1	2	3	4	5
A. Volatility sorts					
Average R_p-R_f	17.01%	21.63%	21.08%	27.79%	23.35%
Standard Deviation	11.93%	17.60%	17.92%	22.38%	22.54%
Sharpe Ratio	1.43	1.23	1.18	1.24	1.04
Treynor Ratio	0.28	0.22	0.17	0.19	0.12
Average R_p-R_m	-3.39%	1.22%	0.67%	7.38%	2.95%
Tracking Error	7.36%	8.03%	10.05%	12.55%	13.78%
Information Ratio	-0.46	0.15	0.07	0.59	0.21
Beta	0.61	0.99	1.27	1.50	1.93
Monthly alpha	0.37%	0.11%	-0.37%	-0.21%	-1.47%
p-value (alpha)	0.0314**	0.6059	0.2271	0.5625	0.0053***
B. Beta sorts					
Average R_p-R_f	15.57%	18.89%	19.60%	21.47%	21.82%
Standard Deviation	11.89%	13.65%	16.19%	17.64%	20.88%
Sharpe Ratio	1.31	1.38	1.21	1.22	1.04
Treynor Ratio	0.46	0.28	0.22	0.18	0.12
Average R_p-R_m	-4.84%	-1.52%	-0.81%	1.07%	1.41%
Tracking Error	12.23%	10.53%	6.72%	8.00%	9.19%
Information Ratio	-0.40	-0.14	-0.12	0.13	0.15
Beta	0.34	0.67	0.90	1.19	1.86
Monthly alpha	0.73%	0.44%	0.09%	-0.25%	-1.46%
p-value (alpha)	0.0099***	0.1011	0.6398	0.2807	0.0002***

Table 8: Articles investigating the low risk anomaly

Year	Author	Data & Year	Low Risk Anomaly	Proxy for risk	Method
2006	Ang. et al	USA 63-00	True	Ivol	FF
2009	Ang. et al	Int (G7) 80-03	True	Ivol	FF
2007	Blitz & Vliet	International 86-06	True	Ivol	FF
2011	Baker, Bradley & Wurgler	USA 68-08	True	Beta, Tvol	CAPM
2011	Frazzini & Pedersen	International 63-12	True	Beta	CAPM
2012	Dutt & Humphery-Jenner	International 90-10	True	Tvol	FF
2008	Bali & Cakici	USA 63-04	False	Ivol	FF
2011	Bali, Cakici & Whitelaw	USA 62-05	False	Ivol	FF
2014	Riley	USA 90-12	True	Tvol, Ivol	FF
2010	Chen, Jiang, Xu and Yao	USA 63-10	True	Ivol	FF
2006	Clarke, De Silva & Thorley	USA 70-05	True	Tvol	FF
2013	Baker, Bradley & Taliaferro	USA 68 -12, Int 89-12	True	Beta	CAPM
2014	Iwasawa & Uchiyama	Japan 85-13	True	Beta	FF
2009	Fu	USA 63-06	False	Ivol	EGarch
2014	Guo, Kassa & Ferguson	USA 63-06	Reject Fu(2009)	Ivol	-
2013	Hafskör & Östenäs	Norway 81-12	False	Ivol	FF

Note : Ivol = Idiosyncratic volatililty. Tvol = Total volatility. FF = Fama-French

Table 9: Suggested explanations to the low risk anomaly

Year	Author	Explanation
2009	Cornell	Overconfidence
2011	Frazzini & Pedersen	Leverage Constrain
2012	Stambaugh, Yu & Yuan	Short-selling Constrain
2011	Baker, Bradley and Wurgler	Benchmarking
2000	Shefrin & Statman	Lottery
2011	Bali Cakici & Whitelaw	Lottery & Short-sell
2008	Barberis and Huang	Asymmetric Payoffs
2012	Hou and Loh	Lottery
2012	Trainor	Mathematical compounding
2014	Iwasawa & Uchiyama	Benchmarking