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PARENTAL INVESTMENTS IN CHILD HEALTH – the importance of paternalistic altruism, child egoism and short-sightedness

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ABSTRACT

Parent and child interaction is an important determinant of child health. Typically, parents are more forward-looking than their children and, hence, care about investments in human capital to a larger extent. In this paper we consider the parent-child health-related interaction, when the parent is altruistic and forward-looking and the child is egoistic and short-sighted. The child receives a monetary transfer, from the parent, which is used to finance either health-unrelated consumption or unhealthy behaviour. We apply a simple differential-game approach, assuming linear-state preferences, and study equilibrium time-paths of (a) the parental transfer, (b) the unhealthy behaviour, and (c) the stock of child health capital. We distinguish between the case in which the child is perfectly myopic and the case in which he or she is forward looking.

Keywords: Health capital; parent-child interaction; myopic behaviour; differential game.

JEL classification: I12

INTRODUCTION

Family decision making and human-capital related decisions are intimately connected, and the resulting intertemporal allocation of resources is the fundament for most economies. Gary Becker (1972a, b; 1974, 1976), who is the most renowned researcher in this field of economics, was one of the pioneers of the development of the economic theories of the family and of human capital. His contributions have inspired several currently thriving directions of research. Individual health-related decision making is but one example. Investments in child health capital are decided and executed in a family context. There are several reasons why – for instance, the frequently mentioned demographical transition that most Western societies are going through – investments in child health may be among the most important issues for economists to study. Theoretical efforts to integrate human-capital theory with economic theories of family decision making have not produced tools for the analysis of investments in child human capital, in a family context, when actions cannot be governed by enforceable contracts. In other words, human-capital theory needs to be integrated with methods for handling dynamic and strategic family-based decision making.

The human-capital theory developed by Becker (1964) was extended by Michael Grossman (1972a, b) in order to include health as a separate component of the individual's stock of human capital. Grossman's model has since then been extended towards a family setting (Jacobson, 2000; Bolin et al., 2001, 2002a, b). The importance of the parent-child interaction for the investments in child health in a theoretical health-capital context has received little or no attention, though. However, strategic parent-child interaction has been recognised in the economic theoretical literature as a fundamental aspect of family decision-making, at least since Becker published his treatise on the family (1981).

Several authors have argued that, since actions within the family cannot be regulated by enforceable contracts, strategic behaviour needs to be taken into account when analysing intra-family decision making (see, for instance, King, 1982; Konrad and Lommerud, 1995; Dufwenberg, 2002; Oosterbeek et al., 2003). The survey of economic theories of the family that was published by Bergström (1997) focused on consensus models of the family, and strategic considerations were only briefly discussed. No doubt, this owes to the fact that, at the time, non-cooperative game theory was largely unexplored as a means of studying the family. More recent contributions to this field include the parent-child interaction, but from the perspectives of children competing for parental resources (Chang, 2009; and Chang et al., 2005).

Typically, theoretical approaches to the parent-child interaction involves altruism, on account of the parent, and egoism, on account of the child(-ren). In general, incentives for strategic behaviour arise whenever enforceable contracts cannot be used to govern individual actions. Few, if any, domestic actions taken by individual members of a family can be controlled by legally enforceable contracts. Thus, the family scene is exposed to strategic behaviour, and the parent-child relationship is no exception. However, parental decisions taken with the objective of investing in their childrens' health may be less vulnerable to strategic behaviour since (a) children make no intentional investments on their own and, hence, cannot strategically offset parental investment by lowering their own investments, and (b) parents are able to monitor and enforce – at least to some degree – healthy habits. On the other hand, children that have reached the age where *some* autonomy becomes an essential part of eventually becoming an independent adult, but are still dependent on the family for survival, may be able to behave strategically in relation to the "freedom" they are offered by their parents as an essential part of becoming an adult. This may have important consequences for the development of human capital of children and adolescents. In this paper, we will develop an economic theoretical model of parent-child strategic and dynamic interaction as a determinant of unhealthy child behaviour. Adolescent behaviour that is not

approved of by parents may, nevertheless, be paid for, unintentionally, by parents who transfer monetary resources to their children.

In spite of the large stock of theoretical health-economic research that has been published during the last three decades, no attention has been paid to parental investments in child health, explicitly taking into account offsetting unhealthy child behaviour. Becker (1981) argued that parents have become less confident that they are able to govern the behaviour of their children. Even though Becker did not explicitly mention health-related behaviour, there can be no doubt that some adolescent behaviours include actions that have, potentially, considerable adverse health effects. Although parents' preferences typically are altruistic with respect to their children, it seems natural to assume that this altruism is somewhat mixed with paternalism. More specifically, parents take the wellbeing of their children into account when allocating time and resources between different competing uses, but they do so by making decisions that differ from the decisions that the child him- or herself had made had he or she been given perfect authority over the resources. In a widened perspective, parents may also hold altruistic preferences towards grand-children and subsequent generations. Therefore, parental resources may not only be devoted to the making of healthy and well-educated children, but also take grandchildren into account.

In contrast to parental far-sighted and altruistic preferences that may embrace at least two succeeding generations, the preferences of children and adolescents are in general thought of as being more or less egoistic and relatively short-sighted. Thus, any resources controlled by adolescents in a family will be put to use in a way which is perceived, by the parents, possible to improve. For example, parents and children may disagree concerning the optimal level of certain child activities and consumption, for instance, hazardous sports, alcohol consumption, pre-marital sex, schoolwork habits and unwholesome eating.

In a world of perfect information and perfect ability of enforcing rules, parents would “only” be faced with the planners problem of deciding the optimal allocation of resources. However, since children can only be governed imperfectly, parents have to decide how resources should be allocated *given* strategic, selfish and short-sighted child behaviour. This is reminiscent of Becker’s famous Rotten-kid theorem, which implies that a selfish child will cooperate in maximising family welfare if financial incentives are properly tuned. Thus, a parent who faces a related, but more complex, situation of making optimal investments in child health during the child’s upbringing may use monetary transfers not only as a means of fulfilling altruistic objectives, but also as a tool for influencing child behaviour. Thus, the size of monetary transfers is rationally determined as a balance between altruism and paternalistic parental preferences.

In this paper, we will focus on the importance of the parent-child interaction in determining child health-related behaviour and future health. We will analyse the significance of myopic, as well as of more farsighted, child preferences. For this purpose, we need to apply the techniques developed for analysing equilibrium strategies in differential games (Dockner et al., 2000). For simplicity, a particularly tractable class of games will be applied – linear state games (Dockner et al., 2000). This class of games is both (relatively) tractable and has the advantageous property that an open-loop Nash equilibrium is Markov perfect (Dockner et al., 2000). Differential games have been applied to various areas in economics, for example, resource economics, industrial organisation and different political economics issues (for an application to terrorism, see, Novak et al., 2010). Some applications have also been published in the theoretical human-capital based macroeconomic literature (see, for instance, Hori and Shibata, 2010). The theoretical modelling of dynamic and strategic interactions within the family has also been subject to efforts from mathematicians and biologists (see, for instance, Ewald et al., 2007).

The natural point of departure for an economic analysis of the parent-child interaction in the determination of child health is the Grossman model. Grossman's demand-for-health model is the dominating economic-theoretical model of individual health behaviour (Grossman, 1972a, b).¹ In this paper, we will adopt the basic notion of the demand for health model – that individuals produce their own health. The parent-child interaction in the process that determines investments in child health capital has not yet been analysed within a demand-for-health framework (to the best of our knowledge). The focus of our interest is the effect of child farsightedness when it comes to the allocation of parental monetary transfers between different uses that are health-related or independent on health. A more farsighted child would have stronger preferences for health than a myopic child, *ceteris paribus*, which means that a given transfer would be used in order to exert a smaller detrimental health effect. On the other hand, parental incentives for making the transfer would be less curbed if the child is more farsighted.

The structure of the rest of the paper is as follows. Next, we will present the model, which we will solve for both the case of a myopic child and the case of a more farsighted child. After that, we will discuss the optimality conditions and compare parental and child behaviour in these two cases. The final section contains discussion and conclusions.

THE MODEL

General structure

The general structure of the demand-for-human-capital-type model developed below is as follows: one parent with preferences that are altruistic and dynastic, and one egoistic child, interact in a

¹ Since its introduction, the demand-for-health model has been extended in various ways. For instance, in order to incorporate (a) uncertainty (Dardanoni and Wagstaff, 1987, 1990, Selden, 1993, Chang, 1996, and Liljas, 1998, 2000); (b) the family as producer of health (Jacobson, 2000, Bolin et al., 2001, 2002a); (c) and the employer as producer of health (Bolin et al., 2002b), and (d) healthy and unhealthy consumption (Forster, 2001). The basic features of the demand-for-health model are (1) that the individual demands health (a) for its utility enhancing effects (the consumption motive), and (b) for its effect on the amount of healthy time (the investments motive), (2) that the demand for investments in health is derived from the more fundamental demand for health, (3) that the investments in health is produced by the individual, and (4) that the stock of health depreciates at each point in time.

dynamic setting which determines the time-path of child human capital, $H(t)$. The parent cares about the child's welfare beyond the current finite planning horizon ($t = T$). More specifically, the value that the parent puts on child health at the end of the planning horizon is strictly larger than zero, reflecting the parent's dynastic preferences. The child allocates a parental monetary transfer, $M(t)$, between an activity, which is detrimental to human capital, $\theta(t)$, and consumption, $c(t)$, that does not influence the stock of human capital, while the parent makes choices about the size of the transfer, based on preferences that balances paternalism and altruism. We assume that the parent derives instantaneous utility from current child human capital and the monetary transfer, but not from the child's activities. Thus, the parent is altruistic in a paternalistic sense.²

Budget constraint

For simplicity, we assume that there are no financial markets and, hence, that total parental spending at time t equals income at time t . Total parental income is comprised of a guaranteed income, \bar{y} , and a variable income determined by the stock of child human capital.³ The marginal productivity of child health, with respect to parental income, is constant, and denoted h . Hence, the parent's budget constraint is, with the constant marginal cost of gross investments in child human capital, $I(t)$, denoted π :

$$M(t) = y(t) - \pi \cdot I(t) = \bar{y} + h \cdot H(t) - \pi \cdot I(t). \quad (1)$$

The child performs the human-capital detrimental activity at a constant marginal cost, p_θ , and buys the the human-capital unrelated consumption good at the constant marginal cost 1. Thus, the child's budget constraint is:

² Of course, this setting constitutes a harsh simplification of the true interactions between parent and child. For instance, parental altruism and paternalism may reach beyond caring for child health and being indifferent to child consumption choices, and there may be several other arenas for strategic behaviour other than child human capital. For our purpose, however, it suffices to model one asset stock – child human capital – and two choice variables that make the allocation strategic – the parental transfer and the level of the child's unhealthy activities. Essentially, this means that we assume that the decisions pertaining to child human capital can be made separately from other decisions in the family. This is also consistent with, or even implied by, the linear-state assumption.

³ Child health is part of the stock of child human capital, and child health influence the parent's labour market performance.

$$M(t) = c(t) + p_\theta \cdot \theta(t). \quad (2)$$

Preferences

Parental preferences

We employ a linear-state framework, and assume that the parent's instantaneous marginal utility of making the transfer, $M(t)$, is strictly increasing and concave in $M(t)$.⁴ Further, for simplicity we assume that the instantaneous marginal utility of the transfer is quadratic function of $M(t)$, with marginal utility equal to $U_0 > 0$ when $M(t) = 0$. Thus, parental preferences are represented by the following instantaneous quasi-linear utility function:

$$U^p(H(t), M(t)) = H(t) + U_0 \cdot M(t) - \frac{1}{2} \cdot M(t)^2, \quad (3)$$

And the value that the parent associates with end-state child health, $a \cdot H(T)$ (a is a strictly positive constant).

Child preferences

The child receives utility from own health, unhealthy behaviour, $\theta(t)$, and from health-unrelated consumption, $c(t)$. The behaviour is unhealthy in the sense that the specific level of behaviour at t , $\theta(t) = \hat{\theta}$, adds negatively to the health stock at that point in time. Due to the linear-state framework adopted, child preferences are represented by a quasi-linear Cobb-Douglas-type utility function:

$$U^c(\theta(t), C(t)) = H(t) + \theta(t) \cdot c(t). \quad (4)$$

Equation of motion of the child health stock

The stock of child health evolves through time as a consequence of (1) parental investments, $I(t)$, (2) the child's unhealthy behaviour, $\theta(t)$, and (3) natural depreciation of the existing stock of child

⁴ Linear-state games have applied to different areas of economics. For an overview see, for instance, Van Long (2010).

health capital at rate δ . We assume that the rate of depreciation is constant over the planning horizon, δ ($0 < \delta < 1$).^{5,6} Thus, the equation of motion for the stock of health capital is:

$$\dot{H}(t) = I(t) - \varphi \cdot \theta(t) - \delta \cdot H(t), \quad (5)$$

where φ reflects the severity of the adverse impact that each unit of θ imposes on health. We assume that $\varphi > 0$.

The control problems

We will analyse two types of situations. In the first, the child is perfectly myopic and, hence, cares only about instantaneous utility. In the second case, the child is less short-sighted and assigns a strictly positive value to his or her health at $t = T$. The planning horizon is fixed at $t = T$ in both cases (for the parent only, in the first case).

The myopic child

In this case, the child's allocation at t can be incorporated as part of the parent's optimization problem, making this a one-individual problem. The parent discounts future utility at rate ρ_p and, hence, acts as if solving the following vertical terminal line problem:⁷

$$\max_{M, H(T)} \int_0^T e^{-\rho \cdot t} \cdot [H(t) + (U_0 \cdot M(t) - \frac{1}{2} \cdot M(t)^2)] dt + a \cdot H(T) \cdot e^{-\rho_p \cdot T}, \quad (6)$$

subject to (4) and the transversality condition $H(T)$ free $\Rightarrow \lambda(T) = a$, and $H(0) = H_0 \geq 0$.

⁵ This may seem as a restrictive assumption. However, the focus of our analysis is not the effects of ageing and, hence, a rate of depreciation that increases over time is not essential for our purposes.

⁶ Grossman (1972), Muurinen (1982), Wagstaff (1986), Liljas (1998), Jacobson (2000), and Bolin et al. (2001a), examined models with time-dependent rates of depreciation.

⁷ A vertical terminal line problem is to be distinguished from a horizontal terminal line problem. The optimality conditions resulting from these two types of problems only differ regarding the transversality conditions. Endogenous length of life is possible to handle using the horizontal terminal line problem. The individual's life-time optimisation problem was formulated as a vertical time line problem by, for instance, Bolin et al (2001; 2002b, c).

At each point in time the child allocates the parental transfer, $M(t)$, in order to maximize (4). This gives the child's optimal behaviour as $\theta(t) = \frac{M(t)}{2 \cdot p_\theta}$. Applying optimal control theory to solve the parent's optimization problem, (6), leaves us with the following current-value Hamilton function:⁸

$$\mathcal{H}(t) = H(t) + (U_0 \cdot M(t) - M(t)^2) + \lambda(t) \cdot \left(\frac{y(H(t)) - M(t)}{\pi} - \varphi \cdot \frac{M(t)}{2 \cdot p_\theta} - \delta \cdot H(t) \right),$$

where $M(t) = y(H(t)) - \pi \cdot I(t)$ has been used to substitute for $I(t)$. The maximum principle yields the following equation of motion (arguments left out for brevity):

$$\dot{\lambda} = -\mathcal{H}_H + \rho \cdot \lambda = -1 - \lambda \cdot \left(\frac{h}{\pi} - \delta \right) + \rho \cdot \lambda. \quad (7)$$

Then, we have the first order conditions for the control variable:

$$\mathcal{H}_M = (U_0 - 2 \cdot M^*) + \lambda^* \cdot \left(-\frac{1}{\pi} - \frac{\varphi}{2 \cdot p_\theta} \right) \leq 0. \quad (8)$$

From these first-order conditions the following conclusion can be drawn:

Claim 1: assume that the marginal productivity of (child) health capital, h , is larger than the marginal cost of (child) health capital, $(\delta + \rho_p) \cdot \pi$, then the parental transfer, M , and the amount of the health-adverse behaviour, θ , are increasing over time.

Proof: the solution to the differential equation (7) is: $\lambda(t)^* = e^{\left(\frac{h}{\pi} - \delta - \rho_p\right) \cdot (T-t)} \cdot \left(a + \frac{1}{\frac{h}{\pi} - \delta - \rho_p} \right) - \frac{1}{\frac{h}{\pi} - \delta - \rho_p}$. Since, by assumption, $\frac{h}{\pi} - \delta - \rho_p > 0$, it follows that $\dot{\lambda} < 0$. Taking the time derivative of

(8) establishes that $\dot{M}(t) \geq 0$ for $t \geq 0$. The second part of the Claim follows from $\dot{\theta}(t) = \frac{\dot{M}(t)}{2 \cdot p_\theta}$.

⁸ The maximum principle gives necessary and sufficient conditions for the optimal control of $M(t)$, given the time path of the state variable $H(t)$, provided that the Hamilton function is strictly concave. The Hamilton function is concave (H, M) , as can be seen by the following: *First*, notice that $\mathcal{H}_{HM} = \mathcal{H}_{MH} = 0$. Second, the diagonal terms are: $\mathcal{H}_{HH} = 0$, and $\mathcal{H}_{MM} = -2$. Thus, the Hamiltonian is jointly concave in (H, M) .

Notice, that Claim 1 does not rule out that the transfer equals 0 for an initial time-period. More precisely, if $U_0 < \lambda(0) \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta}\right)$, the transfer will be 0 at $t = 0$ (since $\partial_H[H(t) + (U_0 \cdot M(t) - M(t)^2)] = 1$, and since $\lambda(T) = a$, we are guaranteed that $\lambda(t) > 0 \forall t \in [0, T]$ (Caputo, 2005; p. 56). If this is the case, the transfer will become strictly positive at a specific point in time, $t = \hat{t}$, for which $U_0 = \lambda(\hat{t}) \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta}\right)$. Further, it follows from the above that if $U_0 < a \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta}\right)$, the parent will never make a strictly positive transfer. The optimal time-path of the parental transfer, $M(t)^*$, can be constructed by solving the differential equation (7) and substitute the solution in (8). This gives: $M(t)^* = \frac{U_0}{2} - \frac{\lambda(t)^*}{2} \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta}\right)$. Given the level of inherited health (H_0), a closed-form expression of the optimal time-path of child health capital can be found by solving the differential equation obtained from substituting for $M(t)^*$ in (5) (see the appendix).

The following conclusions can be drawn regarding the optimal time path of health capital and gross health investments.

Claim 2: Assume that $h > \delta \cdot \pi$, then $\frac{\bar{y}}{\pi} > \frac{U_0}{2} \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta}\right)$ is a sufficient condition for child health capital to be increasing over the entire planning period.

Proof: the claim about the health stock follows from, *first*, formulating the general solution of equation (5), and, *second*, taking the time derivative of this expression (using Leibnitz rule). For details, see the appendix.

The result stated in Claim 2 assures that for a high enough guaranteed income both the parental transfer and the stock of child health will increase over time. The time-path of parental

investments in child health does not mirror the optimal time-path of health capital. This is so since the child influences his or her health by exerting the health-related behaviour, θ . The parental investment in child health over time can be obtained from the parent's budget condition: $\dot{I}(t) = \frac{1}{\pi}(h\dot{H}(t) - \dot{M}(t)\varphi)$, the sign of which may be positive or negative, depending on the relation between the time paths of health capital and the transfer. Under the assumption made in Claim 1, it is clear that gross health investments are on a decreasing trajectory if the health stock is stationary or at a decreasing trajectory. However, that clear-cut result presupposes that the assumption made in Claim 2 is violated. In cases when Claim 2 apply, however, the direction of the optimal time path of gross health investments is ambiguous. However, adding the requirement that $\left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_{\theta}}\right) > \frac{h}{\pi} - \delta - \rho_p$ guarantees that gross health investments are on an increasing trajectory.

The results when the child is myopic can be summarized as follows: Due to the Cobb-Douglas preferences assumed for the child he or she chooses $\theta(t)^* = \frac{M(t)^*}{2 \cdot p_{\theta}}$, i.e., a fixed share of the transfer is used for financing the unhealthy behaviour. The parent optimally transfers $M(t)^* = \frac{U_0}{2} - \frac{\lambda(t)^*}{2} \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_{\theta}}\right)$, which means that the parent transfers more and more over time; at $t = T$ the transfers has grown (maintaining the assumption that if the marginal productivity of health capital, h , is larger than the marginal cost of health capital, $(\delta + \rho_p) \cdot \pi$) to $\frac{U_0}{2} - \frac{a}{2} \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_{\theta}}\right)$. Moreover, the parent transfers more money at each point in time, ceteris paribus, the higher the marginal cost of θ , the higher is the marginal cost of gross health investments, and the smaller is the marginal health damage of θ , φ . Further, the higher (marginal) value that the parent puts on child end-state health, reflected by a , the smaller amount is transferred at each t . There is a level of guaranteed income for which the health stock will increase over the entire planning horizon.

The forward-looking child

When the child begins to take future consequences of current behaviour into account, the parent-child interaction becomes strategic in a non-trivial sense. When deciding about the transfer to the child, the parent can no longer ignore intertemporal strategic effects. Similarly, the child starts to take parental responses into account when deciding about how to divide a given transfer between unhealthy behaviour and health-unrelated consumption. Formally, the parent acts as if solving (6), in which $\theta(t) = \frac{M(t)}{2 \cdot p_\theta}$ has been replaced by the child's strategic response, $\theta(t)$. The child acts as if solving the following vertical terminal line problem, discounting future utility at rate ρ_c , and :

$$\max_{\theta, H(T)} \int_0^T e^{-\rho_c t} \cdot [H(t) + \theta(t) \cdot C(t)] dt + b \cdot H(T) \cdot e^{-\rho_c T}, \quad (9)$$

subject to (4) and the transversality condition $H(T)$ free $\Rightarrow \lambda^c(T) = b$.

Thus, the choices that result from this strategic parent-child interaction are assumed to be arrived at as if both parties simultaneously solve their respective dynamic optimization problem, each taking the strategic response by the other into account. Formally, we seek time-paths for $M(t)$, $\theta(t)$, and $H(t)$ so that both parties optimality conditions and transversality conditions are simultaneously satisfied. This leaves us with the following system of Hamilton functions:

$$\begin{aligned} \mathcal{H}^p(t) &= H(t) + (U_0 \cdot M(t) - M(t)^2) + \lambda^p(t) \cdot \left(\frac{y^{(H(t)) - M(t)}}{\pi} - \varphi \cdot \theta(t) - \delta \cdot H(t) \right), \\ \mathcal{H}^c(t) &= H(t) + \theta(t) \cdot C(t) + \lambda^c(t) \cdot \left(\frac{y^{(H(t)) - M(t)}}{\pi} - \varphi \cdot \theta(t) - \delta \cdot H(t) \right). \end{aligned} \quad (10)$$

Further, the maximum principle yields the following equations of motion (again, arguments left out for brevity), which must be fulfilled by the solution $(\lambda^{p*}, \lambda^{c*})$:

$$\begin{aligned} \dot{\lambda}^{p*} &= -\mathcal{H}_H^p + \rho \cdot \lambda^{p*} = -1 - \lambda^{p*} \cdot \left(\frac{h}{\pi} - \delta \right) + \rho_p \cdot \lambda^{p*}, \\ \dot{\lambda}^{c*} &= -\mathcal{H}_H^c + \rho \cdot \lambda^{c*} = -1 - \lambda^{c*} \cdot \left(\frac{h}{\pi} - \delta \right) + \rho_c \cdot \lambda^{c*}. \end{aligned} \quad (11)$$

In both cases, closed-form solutions can be obtained:

$$\lambda^p(t)^* = e^{\left(\frac{h}{\pi} - \delta - \rho_p\right) \cdot (T-t)} \cdot \left(a + \frac{1}{\frac{h}{\pi} - \delta - \rho_p} \right) - \frac{1}{\frac{h}{\pi} - \delta - \rho_p}, \text{ and}$$

$$\lambda^c(t)^* = e^{\left(\frac{h}{\pi} - \delta - \rho_c\right) \cdot (T-t)} \cdot \left(b + \frac{1}{\frac{h}{\pi} - \delta - \rho_c}\right) - \frac{1}{\frac{h}{\pi} - \delta - \rho_c}. \quad (12)$$

Further, the first-order conditions for optimal choices of the transfer and the unhealthy behaviour, $M(t)^*$ and $\theta(t)^*$, are:

$$\begin{aligned} \mathcal{H}_M^p &= (U_0 - 2 \cdot M^*) + \lambda^{p*} \cdot \left(-\frac{1}{\pi}\right) = 0, \\ \mathcal{H}_\theta^c &= M^* - 2 \cdot p_\theta \cdot \theta^* + \lambda^{c*} \cdot (-\varphi) = 0. \end{aligned} \quad (13)$$

Solving this system for M^* and θ^* yields: $M^* = \frac{U_0}{2} - \frac{\lambda^{p*}}{2\pi}$ and $\theta^* = \frac{U_0}{4p_\theta} - \frac{\lambda^{p*}}{4p_\theta\pi} - \frac{\varphi\lambda^{c*}}{2p_\theta}$. It is immediately clear that the parental transfer is lower (1) the lower the marginal cost of health investments, (2) the higher the value that the parent puts on end-point child health (a), and (3) the more forward looking the parent is (lower ρ_p). Similarly, less of the health-adverse behaviour will be exerted (1) the more forward looking the parent is (lower ρ_p), (2) the more forward looking the child is (lower ρ_c), and (3) the larger the marginal adverse health effect (φ) of the behaviour. The corresponding changes over time of the parental transfer, the unhealthy behaviour exerted, and the amount of child health capital fulfils the following statement:

Claim 3: (i) the parental transfer, M , and the amount of the health-adverse behaviour, θ , are increasing over time; and

(ii) assume that $h > \delta \cdot \pi$, then $\frac{\bar{y}}{\pi} > \frac{U_0}{2} \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta}\right)$ is a sufficient condition for the stock of child health capital to be increasing over the entire planning period.

Proof: part (i) follows from the time derivatives of M^* and θ^* . Part (ii) is shown in the appendix.

One observation seems particularly important: the conclusions that the parent makes strictly larger transfers in the forward-looking case than in the myopic case, and that the level of child health is

increasing over the entire planning horizon, do not rely on any specific assumption concerning how the level of child health is valued at the end of the planning horizon or the rate of time preferences.

In the previous section, we showed that under specific assumptions the health stock increases over time for the myopic child. The same condition applies also in the forward-looking case. So far, we have not said anything about the size of the health stock when the child is forward looking as compared to when he or she is perfectly myopic. The results in Claim 3 do not in tell us whether or not the health stock is smaller or larger when the child is forward looking rather than myopic, *ceteris paribus*, since the parental transfer lowers the investments in health, while a reduction in the unhealthy behaviour, due to more child farsightedness, lowers the *disinvestment* in health. However, in cases in which the forward looking child exerts more than the myopic child in the unhealthy behaviour, health must necessary be lower for the former type.

In order to compare child health levels in the two cases we solve the equation of motion of health capital in the forward-looking case and compare the solution to the corresponding solution in the myopic case. Equation system (13), together with the solutions (12), provides the optimal time paths for the parental transfer and child behaviour. The relation between health stocks on the one hand, and the price of health investment, the price of health-related behaviour, and the rate at which the behaviour influences, on the other, is summarized in Claim 4:

Claim 4: assume that two children are identical, except that one is perfectly myopic (m) and the other is forward looking. Let $M^m(t)^*$, $\theta^m(t)^*$, $H^m(t)^*$ and $M(t)^*$, $\theta(t)^*$, $H(t)^*$ be the optimal time-paths of the parental transfer, the health-related behaviour and the stock of health capital in

the myopic and forward looking cases, respectively. Then, (i) $M^m(t)^* < M(t)^*$; (ii) $\theta^m(t)^* \leq \theta(t)^*$ if, and only if, $p_\theta \leq \frac{1}{4}$; and, (iii) $H(t)^* \leq H^m(t)^*$ if, and only if, $\varphi \leq \frac{2p_\theta}{\pi(4p_\theta-1)}$ for $\forall t$.

Proof: since $\lambda^{p^*} = \lambda^*$ (shadow price in the myopic case) $\forall t$, part (i) follows from comparing (8) with the first equation of (13). Parts (ii) and (iii) are shown in the appendix.

The relation between parental investments in child health in the two cases can be inferred from these results. There are two cases in which the investments are unambiguously larger or smaller in the forward-looking case: *First*, since the parental transfer is always larger in the forward-looking case, parental investments in child health will be smaller when the child is forward looking if the amount of health capital is smaller in this case, i.e., if $\varphi < \frac{2p_\theta}{\pi(4p_\theta-1)}$. *Second*, if health capital is larger and the child exerts more of the health-related behaviour, it follows that also parental investments in child health must necessarily be larger. This situation occurs in the forward-looking case if $p_\theta < \frac{1}{4}$ (since $\varphi > 0$).

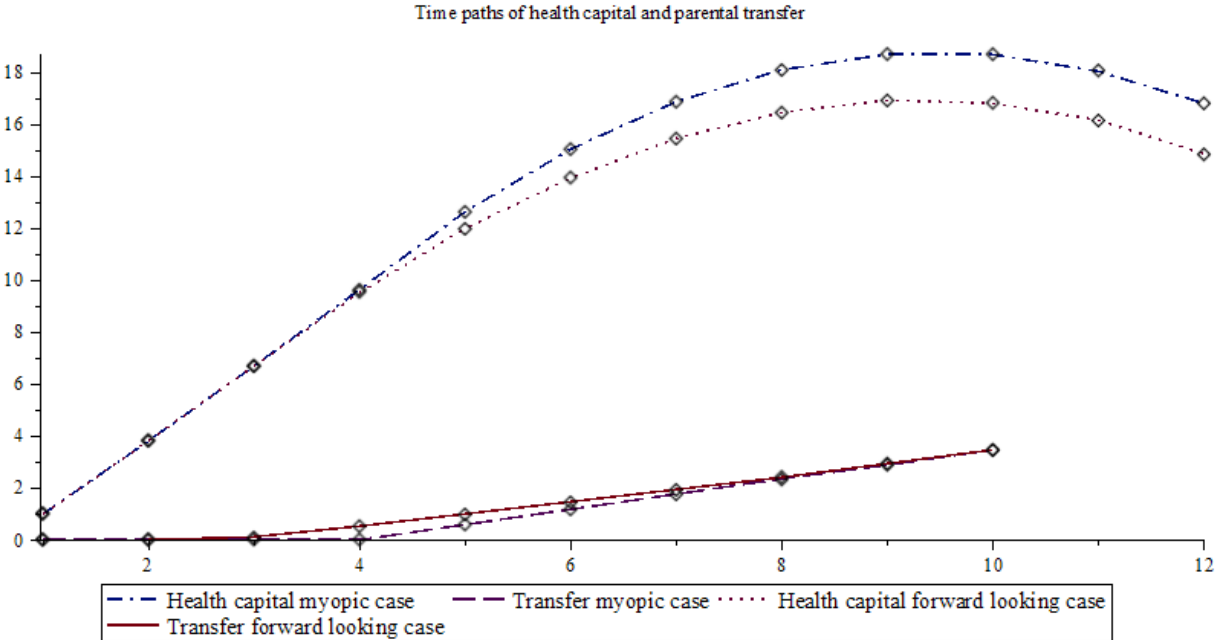
Further, these results imply that for a high enough price, p_θ , the forward-looking child will be more healthy than the myopic child, ceteris paribus. Similarly, for a given price, p_θ , there is a level of the adverse health impact caused by a unit of θ , φ , above (below) which the forward-looking child is more (less) healthy than the myopic child. As was the case with the optimal time paths of the transfer and the health stock (Claim 3).

The impact of less short-sightedness on health-related behaviours and health capital can be summarized as follows: when the child starts to take the future into account, the incentives for behaving unhealthy are weakened, since he or she begins to value not only current health and resources, but also future health and the stream of parental transfers. In particular, the child starts

to value the amount of health at the end of the planning horizon. The net effects on health-related behaviour, health investments and health capital of less short short-sightedness depend on the marginal cost and the marginal health impact of the health behaviour.

Numerical illustration

We utilize different parametrizations of the closed-form solutions $M^m(t)^*, \theta^m(t)^*, H^m(t)^*, I^m(t)^*$ and $M(t)^*, \theta(t)^*, H(t)^*, I(t)^*$ in order to illustrate the solutions. Thus, subsequent values of the solutions were calculated and plotted in diagrams with time on the horizontal axis. The following parameter values were used throughout: rization was used: $U_0 = 8; H_0 = 1; a = 0.005; b = 0.001; h = 0.02; \rho_p = 0.05355; \rho_c = 0.3; \delta = 0.00058555; T = 10; p_\theta = 4.9; \varphi = 0.002$. At each step, i , of the recursion, we checked that $M(t)^* \geq 0$, and if it was not, we adjusted the recursion equation so that $M(t)^*$ was assigned the value 0. Figure 1 illustrates the time-paths in the myopic and forward-looking cases, for this parametrization.



4. DISCUSSION

In this paper we have developed a dynamic and strategic model of parental investments in child health, when the child enjoys certain behaviours that are detrimental to health, and the parent is altruistic in the sense that he or she cares about child health and child consumption that is unrelated to health. The model includes a minimum required amount of child health capital at the end of the planning horizon. Our model adds to the small but growing literature that is concerned with strategic incentives when deciding about health-related efforts in a family context. The optimal behaviour responses were calculated applying a simple dynamic-game structure.

The following predictions were derived *ceteris paribus*:

- (1) Parent-child transfers are larger when the child is forward-looking than when he or she is myopic;
- (2) A myopic child exerts more (less) of the health-adverse behaviour than a forward-looking child if the price of the behaviour is high (low); and
- (3) Child health is larger in the forward-looking case than in the myopic case, for behaviours that have a large adverse health impact, and lower for behaviours that have a small health impact.

Comparative dynamics with respect to the exogenous parameters of the model is relatively straightforward, since we are able to obtain closed-form solutions for all endogenous variables. So, for instance,

The required amount of child health capital at the time when the parent's planning horizon ends also plays a crucial role.

- The examples used in the text are assuming equal time preferences and equal values put on end-point health. – maybe add something about what happens to the results in Claim 4 when these parameters change.

APPENDIX

Proof of Claim 2

The equation (5) is an ordinary linear and non-homogenous differential equation which has the general solution: $H(t)^* = H_0 \cdot e^{-(\delta - \frac{h}{\pi})t} + e^{-(\delta - \frac{h}{\pi})t} \cdot \int_0^t \left(\frac{\bar{y} - \frac{1}{\pi} M(\tau)^* - \varphi \cdot \theta(\tau)}{e^{-(\delta - \frac{h}{\pi})\tau}} \right) d\tau$. Let $\frac{\bar{y} - M(\tau)^* \cdot (\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta})}{e^{-(\delta - \frac{h}{\pi})\tau}} =$

$f(\tau)$. Then, by Leibnitz rule the time derivate of $H(t)^*$ is: $\left(\frac{h}{\pi} - \delta \right) \cdot e^{-(\delta - \frac{h}{\pi})t} \cdot \left[H_0 + \int_0^t f(\tau) d\tau \right] + e^{-(\delta - \frac{h}{\pi})t} \cdot \int_0^t f_t(\tau) d\tau + e^{-(\delta - \frac{h}{\pi})t} \cdot e^{(\delta - \frac{h}{\pi})t} \cdot \left[\frac{\bar{y}}{\pi} - M(t)^* \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta} \right) \right]$. $f_t(\tau) = 0$

and, hence, it suffices to examine the first and third terms. Since $M(t)^* = \frac{U_0}{2} - \frac{\lambda(t)^*}{2} \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta} \right)$

they are both positive if $\frac{\bar{y}}{\pi} > \frac{U_0}{2} \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta} \right)$.

Proof of Claim 3

Part (ii): clearly, based on the proof of Claim 2, we only need to examine the third term of the time

derivative of $H(t)^*$: In the forward-looking case we have: $H(t)^* = H_0 \cdot e^{-(\delta - \frac{h}{\pi})t} + e^{-(\delta - \frac{h}{\pi})t} \cdot$

$\int_0^t \left(\frac{\bar{y} - \frac{1}{\pi} M(\tau)^* + \varphi \cdot \theta(\tau)^*}{e^{-(\delta - \frac{h}{\pi})\tau}} \right)$. Again, applying Leibnitz rule, gives $\dot{H}(t)^* = \left(\frac{h}{\pi} - \delta \right) \cdot e^{-(\delta - \frac{h}{\pi})t} \cdot$

$\left[H_0 + \int_0^t f(\tau) d\tau \right] + e^{-(\delta - \frac{h}{\pi})t} \cdot \int_0^t f_t(\tau) d\tau + e^{-(\delta - \frac{h}{\pi})t} \cdot e^{(\delta - \frac{h}{\pi})t} \cdot \left[\frac{\bar{y}}{\pi} - \frac{1}{\pi} M(t)^* - \varphi \cdot \theta(t)^* \right]$.

$f_t(\tau) = 0$ and, hence, it suffices to examine the first and third terms. Using the expressions for

$M(t)^*$ and $\theta(t)^*$ it is clear that $\dot{H}(t)^* > 0$ if $\frac{\bar{y}}{\pi} > \frac{U_0}{2} \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta} \right)$.

Proof of Claim 4

The time-path of health capital in the myopic (m) case is: $H^m(t)^* = H_0 \cdot e^{-(\delta - \frac{h}{\pi})t} + e^{-(\delta - \frac{h}{\pi})t} \cdot$

$\int_0^t \left(\frac{\bar{y} - M(\tau)^* \cdot (\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta})}{e^{-(\delta - \frac{h}{\pi})\tau}} \right) d\tau$. In the forward-looking case the time-path is: $H(t)^* = H_0 \cdot e^{-(\delta - \frac{h}{\pi})t} +$

$e^{-(\delta-\frac{h}{\pi})t} \cdot \int_0^t \left(\frac{\bar{y} - \frac{1}{\pi} M(\tau)^* + \varphi \cdot \theta(\tau)^*}{e^{-(\delta-\frac{h}{\pi})\tau}} \right)$. Comparing the integrands, it is clear that $H(t)^* < (>) H^m(t)^*$ if,

and only if, $\bar{y} - M^m(\tau)^* \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta} \right) > (<) \bar{y} - \frac{1}{\pi} \cdot M(\tau)^* + \varphi \cdot \theta(\tau)^*$. Expanding the left- and

right hand sides gives (equation system (13) gives the expressions for M^* and θ^*):

$$\bar{y} - \left(\frac{U_0}{2} - \frac{\lambda^*}{2} \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta} \right) \right) \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta} \right) \quad \text{and} \quad \bar{y} - \frac{1}{\pi} \cdot M(\tau)^* + \varphi \cdot \theta(\tau)^* = \bar{y} - \frac{1}{\pi} \left(2p_\theta \left(\frac{U_0}{4p_\theta} - \frac{\lambda^{p^*}}{4p_\theta\pi} - \frac{\lambda^{c^*}\varphi}{2p_\theta} \right) + \lambda^{c^*}\varphi \right) + \varphi \left(\frac{U_0}{4p_\theta} - \frac{\lambda^{p^*}}{4p_\theta\pi} - \frac{\lambda^{c^*}\varphi}{2p_\theta} \right).$$

From $a = b$ and $\rho_p = \rho_c$ it follows that $\lambda^{p^*} = \lambda^{c^*} = \lambda^*$. Thus, the second left-hand side term is (leaving out \bar{y}): $\frac{U_0}{2\pi} + \frac{U_0\varphi}{4 \cdot p_\theta} - \frac{\lambda^*}{2}$.

$$\left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta} \right)^2 = \frac{U_0}{2\pi} + \frac{U_0\varphi}{4 \cdot p_\theta} - \frac{\lambda^*}{2\pi^2} - \frac{\lambda^*\varphi}{2p_\theta\pi} - \frac{\lambda^*\varphi^2}{8p_\theta^2}.$$

Terms two and three on the right hand side are: $\frac{U_0}{2\pi} - \frac{\lambda^*}{2\pi^2} - \frac{\lambda^*\varphi}{\pi} + \frac{\lambda^*\varphi}{\pi} + \frac{U_0\varphi}{4p_\theta} - \frac{\lambda^*\varphi}{4p_\theta\pi} - \frac{\lambda^*\varphi^2}{2p_\theta}$. Comparing terms gives the following condition for

$$H(t)^* < (>) H^m(t)^*: -\frac{\lambda^*\varphi}{2p_\theta\pi} - \frac{\lambda^*\varphi^2}{8p_\theta^2} < (>) -\frac{\lambda^*\varphi}{4p_\theta\pi} - \frac{\lambda^*\varphi^2}{2p_\theta}, \text{ which is equivalent to: } \frac{1}{2p_\theta\pi} + \frac{\varphi}{8p_\theta^2} <$$

$(>) \frac{1}{4p_\theta\pi} + \frac{\varphi}{2p_\theta}$. Gathering terms and rearranging yields: $H(t)^* \leq H^m(t)^*$ if, and only if, $\varphi \leq$

$$\frac{2p_\theta}{\pi(4p_\theta-1)}.$$

For behavioural differences we have $\theta^m(t)^* - \theta(t)^* = \frac{1}{2p_\theta} \left(\frac{U_0}{2} - \frac{\lambda^*}{2} \cdot \left(\frac{1}{\pi} + \frac{\varphi}{2 \cdot p_\theta} \right) \right) -$

$\frac{1}{2p_\theta} \left(\frac{U_0}{2} - \frac{\lambda^{p^*}}{2\pi} - \varphi\lambda^{c^*} \right)$. Since $\lambda^* = \lambda^{p^*} = \lambda^{c^*}$, $\theta^m(t)^* - \theta(t)^* = -\frac{\varphi\lambda^*}{8p_\theta^2} + \frac{\varphi\lambda^*}{2p_\theta}$, and, hence, ,

$$\theta^m(t)^* \geq \theta(t)^* \text{ when } p_\theta \geq \frac{1}{4}.$$

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