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Crack spacing and crack widths due to normal force or bending moment

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National Swedish Building Research

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by Åke Holmberg & Sten Lindgren

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Foreword

Studies concerning crack spacing and crack widths due to an external normal force or a bending moment were carried out by Centerlöf & Holmberg AB, Consulting Engineers, on a grant from the National Council for Building Research and the National Committee on Concrete.

The Report is based on a large amount of information from literature as shown in a separate list, on tests carried out by A-Betong and on a number of complementary tests which were kindly incorporated by Professor Anders Losberg of Chalmers Institute of Technology, Gothenburg, into the work at the Institute. A test series concerning cracks in reinforced concrete pipes was made available by Alfa-Rör AB.

Several authors, quoted in the Report, kindly answered questions put to them and in this way extended their own reports. Fruitful discussions, which had the effect of providing guidance for the investigations, were held in the course of the work with Dr. Júlio Ferry Borges, Lisbon, Dr. Eivind Hognestad, Skokie, and Professor Hideo Yokomichi, Sapporo, in addition to the members of the National Committee on Concrete.

The Report as a whole is substantially the product of Sten Lindgren's work. Åke Holmberg took part in discussions and contributed to writing the Report.

Introduction

This Report deals with stable /67 Kr/ cracks in reinforced concrete structures subjected to tension and bending moment. The effect of shearing force is dealt with briefly in connection with crack width. No original work was carried out on the crack-promoting effect of crossing reinforcement or on crack shape. Reference is to be made in connection with crack shape to some studies /66 Ba, 65 Br, 65 Bro and 66 Br/ concerning the variation of width on the concrete surface and to some /35 Em, 65 Br, 65 Bro and 66 Br/ concerning its variation inside the concrete. It is probable that these must be augmented and extended before questions concerning the effect of the cracks on durability and working can be answered. All the observations recounted and the hypotheses put forward relate to cracks on the surface of the concrete right over a reinforcing bar. All types of reinforcement, however, are dealt with with regard to the age and loading history of the structure and to the stress in the reinforcement. The influence of prestressing to varying degrees is also taken into account.

The Report is based substantially on tests, our own and others', and references are made to a list of literature; far too much material that has been written to date has had to be omitted, however, one of the reasons being incomplete documentation. Apparent omissions in the list of references are due to this.

The main features of the arrangement reflect the ideas and hypotheses on which the work has been based, which may be summarized as follows:

1. It is possible to attain final values of crack spacing in a short time using test techniques.
2. In addition to deformations due to loads and restraints on the structure, crack widths also depend on the repetition and duration of these. They cannot therefore be reproduced in a short time.
3. Crack widths in models, in order to compare with crack widths in structures, are to be studied during decreasing load, since practically every structure has at one time carried a load greater than the one being considered.

Symbols

A	cross-sectional area of the principle reinforcement
B_o	the maximum concrete area whose centre of gravity coincides with that of the principle reinforcement
B_1	total concrete area subjected to tension
E_a	strain modulus of the steel
E_b	strain modulus of the concrete
N	No. of
c	cover, measured at right angles from the surface of the reinforcing bar or tendon to the nearest concrete surface
c_h	cover at side of beam. For slabs, this is one half of the clear distance between two bars
c_v	cover at the surface subjected to the greatest tension (the bottom)
c_1	appropriate value of c used, c_h or c_v
c_2	mean value of c_h and c_v
c_3	c_1 plus $\varnothing/2$
c_4	c_2 plus $\varnothing/2$
Δl	crack spacing
Δl_{av}	mean crack spacing
Δl_{max}	maximum crack spacing
Δl_o	observed value of crack spacing
Δl_c	calculated value of crack spacing
S_1	standard deviation
S_2	standard deviation as a percentage of l_c
S_3	standard deviation as a percentage of l_o
n	exponent
x	depth of concrete area in compression
w	crack width
w_{av}	mean crack width
w_{max}	maximum crack width
w_1	mean crack width at the level of the reinforcement
w_2	mean crack width at the maximum distance from the neutral plane
a, β	constants
ε	strain
ε_a	strain of the reinforcement, without regard to the restraint due to the surrounding concrete ($\varepsilon_a = \sigma_a/E_a$)
ε_{av}	mean strain at side of beam (slab etc.) at the level of the reinforcement (including cracks)
ξ, η	general expressions for parameters determining crack spacing
$\tilde{\omega}$	percentage of reinforcement
$\tilde{\omega}_o$	100 A/B_o
$\tilde{\omega}_1$	100 A/B_1
σ_a	tensile stress in the reinforcement
σ_b	tensile stress in the concrete
$\varnothing$	diameter of reinforcing bar or tendon. For strand or bundled reinforcement and for bars, wires or strand in grouted ducts, $\varnothing = \sqrt{\frac{4}{\pi} \times \text{the cross sectional area}}$

All lengths in cm, areas in cm^2 and forces in kg.

1 Crack spacing due to normal force and moment

During loading of short duration, crack spacing is a function of the stress in the reinforcement in such a way that a low stress gives rise to large crack spacing /59 Ef/ as shown in FIG. 1. The phenomenon has been described extensively, and for this reason our studies have been concentrated to values of $\sigma_a \geqq 3000$ kg/cm². This only resulted in errors in exceptional circumstances and never in any major ones. The ratio of Δl at $\sigma_a = 1000$ kg/cm² (Δl_{1000}) to that at $\sigma_a = 4000$ kg/cm² (Δl_{4000}) is shown in FIG. 1 at a value $\frac{B_o}{\Sigma \varnothing} = 17.8$. The ratio is 2.8.

FIG. 2 illustrates the increase in the number of cracks from the year of construction, 1954, until 1956 and 1962 respectively /65 Ku/ for the bridge over Sagån at Östanbro /51 Ho/. The stress in the reinforcement at midspan due to permanent load was on an average 1000 kg/cm² with a gradually increasing addition due to an observed support displacement. The value of $\frac{B_o}{\Sigma \varnothing}$ is 17.8 and the increase 280 %. It is probable that cracks which were very small originally were not discovered until later. The hypothesis that a high stress has the same effect as duration therefore receives some support, which is also the case in other investigations /68 Ab/.

The work on which the analysis of what determines final mean crack spacing in structures subjected to tension and bending is based, comprises 239 No observations on beams and 81 No on slabs reinforced with deformed bars (not special types such as Sonderstahl /63 Rü/ and Square Twisted /66 Ba/. Of the material available /56 Cl, 63 Ka, 63 Rü, 66 Ba, 66 Te and 68 Lo/, all that which had not obviously been affected by very large bond stresses /63 Rü/ has been taken into consideration. Individual beam sides and bottoms have been regarded as individuals as long as published data permitted this. Where the mean value of the cover, $0.5(c_h + c_v)$, has been used, the exception is made in some cases of estimating one of the c values, which halves the error. For slabs c_h was considered to be equal to half the clear distance between two bars.

After some attempts, the expression

$$\Delta l = \alpha\, \xi + \beta \eta$$

was chosen for the crack spacing, with α and β to be determined by means of a regression analysis and ξ and η according to the following alternatives.

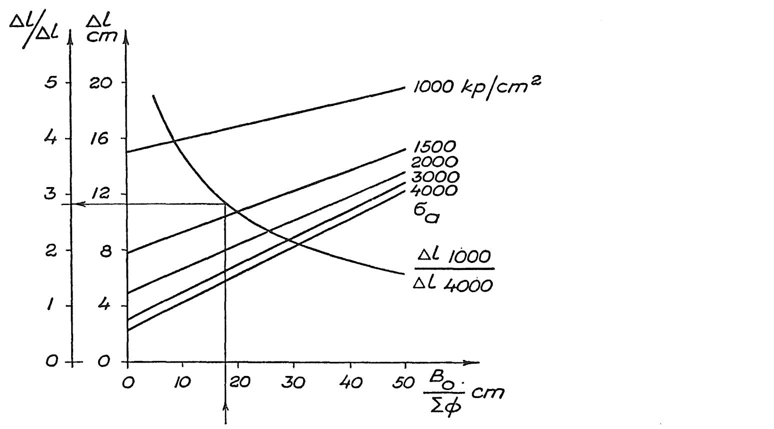

FIG. 1. Relationship of crack spacing and σ_a and $\frac{B_o}{\Sigma \varnothing}$ according to /59 Ef/.

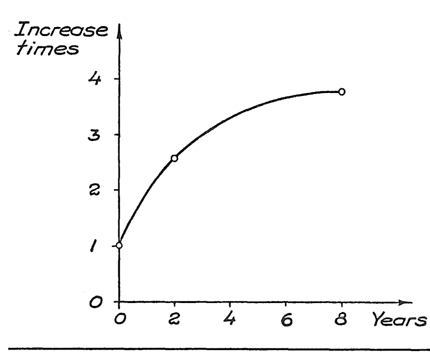

FIG. 2. Increase in the number of cracks in the bridge over Sagån at Östanbro. Stress in reinforcement at midspan due to permanent load approximately 1000 kg/cm².

$\xi = c_1 =$ appropriate value of c_h or c_v (cm)

$c_2 = 0.5(c_h + c_v) \approx 0.5\sqrt{c_h^2 + c_v^2}$ /10 Gr and 66 Yo/

$$c_3 = c_1 + \frac{\varnothing}{2} \text{ /65 Bro/}$$

$$c_4 = c_2 + \frac{\varnothing}{2}$$

and at the same time

$$\eta = \varnothing\left(\frac{B_o}{\Sigma\varnothing^2}\right)^n;\ n = 0,\ 1,\ \frac{1}{2},\ \frac{1}{3}$$

$$\varnothing\left(\frac{c}{\varnothing}\ \frac{B_o}{\Sigma\varnothing^2}\right)^n;\ n = 1,\ \frac{1}{2},\ \frac{1}{3}$$

$$\varnothing\left(\frac{c^2}{\varnothing^2}\ \frac{B_o}{\Sigma\varnothing^2}\right)^n;\ n = \frac{1}{2},\ \frac{1}{3}$$

$$c\left(\frac{B_o}{\Sigma\varnothing^2}\right)^n;\ n = \frac{1}{3}$$

or $\xi = 1$ (cm)

and at the same time

$$\eta = \varnothing\left(\frac{c}{\varnothing}\ \frac{B_o}{\Sigma\varnothing^2}\right)^n;\ n = 1,\ \frac{1}{2},\ \frac{1}{3}$$

$$\varnothing\left(\frac{c^2}{\varnothing^2}\ \frac{B_o}{\Sigma\varnothing^2}\right)^n;\ n = \frac{1}{2},\ \frac{1}{3}$$

$$c\left(\frac{B_o}{\Sigma\varnothing^2}\right)^n;\ n = 0,\ \frac{1}{3}$$

Absolute and relative deviations were calculated as follows:

$$S_1 = \sqrt{\frac{\Sigma(\Delta l_o - \Delta l_c)^2}{N-1}}$$

$$S_2 = \sqrt{\frac{\sum\left(\frac{\Delta l_o - \Delta l_c}{\Delta l_c}\right)^2}{N-1}}$$

$$S_3 = \sqrt{\frac{\sum\left(\frac{\Delta l_o - \Delta l_c}{\Delta l_o}\right)^2}{N-1}}$$

Δl_o is the observed value
Δl_c is the calculated value
N is the number of samples

Eight calculations, among them seven with the least deviation, are reproduced in TAB. 1.

TAB. 1 contains elements from the bulk of the recent literature on this subject. Detailed analysis of the results shows that 3 No beams /68 Lo/ representing 12 No samples assumed a dominance which their extreme nature did not warrant. The covers were about 8 and 13 cm respectively. Recalculation without these gave results as in TAB. 2.

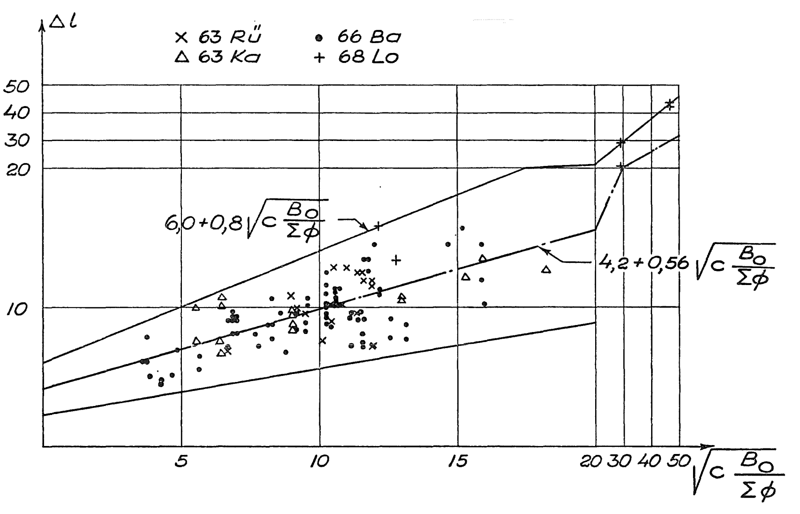

FIG. 3. Measured crack spacing (cm) on sides of beams with deformed bars, compared to spacing calculated from formulae (1) and (2).

TAB. 1. Eight calculations, among them the seven best for all observations on test specimens with deformed bars.

Δl (cm)	S_1 (cm)	S_2 (%)	S_3 (%)
$1.3 + 0.76\sqrt{c_3 \frac{B_o}{\Sigma\varnothing}}$	2.12	24.2	24.7
$2.2 + 0.79\sqrt{c_1 \frac{B_o}{\Sigma\varnothing}}$	2.14	23.7	25.1
$8.6 + 1.15\sqrt[3]{c_3^2 \frac{B_o}{\Sigma\varnothing}}$	2.16	25.1	23.7
$3.2 + 0.65c_3\sqrt[3]{\frac{B_o}{\Sigma\varnothing^2}}$	2.18	23.6	24.2
$1.05 + 1.21\sqrt[3]{c_2^2 \frac{B_o}{\Sigma\varnothing}}$	2.49	25.3	29.9
$-0.8 + 1.94\sqrt[3]{c_1 \frac{\varnothing^2 B_o}{\Sigma\varnothing^2}}$	2.50	28.4	25.7
$2.4 + 0.64c_4\sqrt[3]{\frac{B_o}{\Sigma\varnothing^2}}$	2.51	24.6	30.6
$-1.2 + 1.83\sqrt[3]{c_3 \frac{\varnothing^2 B_o}{\Sigma\varnothing^2}}$	2.65	29.1	26.5

Arrangement strictly in accordance with the results would presumably show a preference for

$$\Delta l = a \cdot 1 + \beta \sqrt{c_3 \frac{B_o}{\Sigma\varnothing}}$$

which is best in the first case and second best in the second case. In order however to attain some simplification in the practical case, the value chosen was nevertheless

$$\Delta l = a \cdot 1 + \beta \sqrt{c_1 \frac{B_o}{\Sigma\varnothing}}$$

which gives insignificantly greater deviations. When applied only to beams, with the three mentioned above excluded, we have

$\Delta l = 4.6 + 0.54 \sqrt{c_1 \frac{B_o}{\Sigma\varnothing}}$ with $S_1 = 1.72$ cm, $S_2 = 17.6$ %, $S_3 = 19.8$ %

and when applied to slabs,

$\Delta l = 3.9 + 0.49 \sqrt{c_1 \frac{B_o}{\Sigma\varnothing}}$ with $S_1 = 2.18$ cm, $S_2 = 28.8$ %, $S_3 = 30.3$ %.

The procedure is that the whole of the material, divided into beam sides, FIG. 3, beam bottoms, FIG. 4, and slabs, FIG. 5, is compared with the calculated mean value without the extreme beams

$$\Delta l = 4.2 + 0.56 \sqrt{c_1 \frac{B_o}{\Sigma\varnothing}} \qquad (1)$$

and is augmented by $2S_2 = 2 \times 21.4 = 42.8$ % to give

$$\Delta l(1 + 2S_2) = 1.428\left(4.2 + 0.56 \sqrt{c_1 \frac{B_o}{\Sigma\varnothing}}\right) = 6.0 + 0.8 \sqrt{c_1 \frac{B_o}{\Sigma\varnothing}}$$

Everything seems then to be in order, apart from a dissimilarity between two test series on slabs /56 Cl and 66 Te/ for which no explanation can be found. Transverse reinforcement in one /66 Te/ is too light to give any appreciable effect.

It is therefore possible to stipulate that the mean values of the final crack spacing for structures subject to tension and bending, reinforced with deformed bars, should be

$$\Delta l = 6.0 + 0.8 \sqrt{c \frac{B_o}{\Sigma\varnothing}} \text{ cm} \qquad (2)$$

As regards the maximum crack spacing, the material is not so complete. Most of the test beams are too short to be reasonably expected to contain the maximum crack. There is however reasonable justification for the maximum value to be set at 70 % above the mean value. There are indications of higher values /66 Br and others/ but these are not taken into account since the

TAB. 2. The eight best calculations for all observations on test specimens with deformed bars, with the exception of three beams /68 Lo/ with extremely high values of $\sqrt{c \frac{B_o}{\Sigma\varnothing}}$.

Δl (cm)	S_1 (cm)	S_2 (%)	S_3 (%)
$3.4 + 0.80 \sqrt[3]{c_3^2 \frac{B_o}{\Sigma\varnothing}}$	1.84	21.0	23.1
$3.6 + 0.54 \sqrt{c_3 \frac{B_o}{\Sigma\varnothing}}$	1.88	21.2	24.7
$3.0 + 1.20 \sqrt[3]{c_1 \frac{\varnothing^2 B_o}{\Sigma\varnothing^2}}$	1.88	21.4	23.8
$4.2 + 0.56 \sqrt{c_1 \frac{B_o}{\Sigma\varnothing}}$	1.92	21.4	25.1
$3.0 + 1.09 \sqrt[3]{c_3 \frac{\varnothing^2 B_o}{\Sigma\varnothing^2}}$	1.92	21.8	24.3
$4.7 + 0.47 c_3 \sqrt[3]{\frac{B_o}{\Sigma\varnothing^2}}$	1.96	21.9	24.4
$3.4 + 0.72 \sqrt[3]{c_4^2 \frac{B_o}{\Sigma\varnothing}}$	2.04	22.2	27.6
$4.1 + 0.75 \sqrt[3]{c_2^2 \frac{B_o}{\Sigma\varnothing}}$	2.10	22.5	28.7

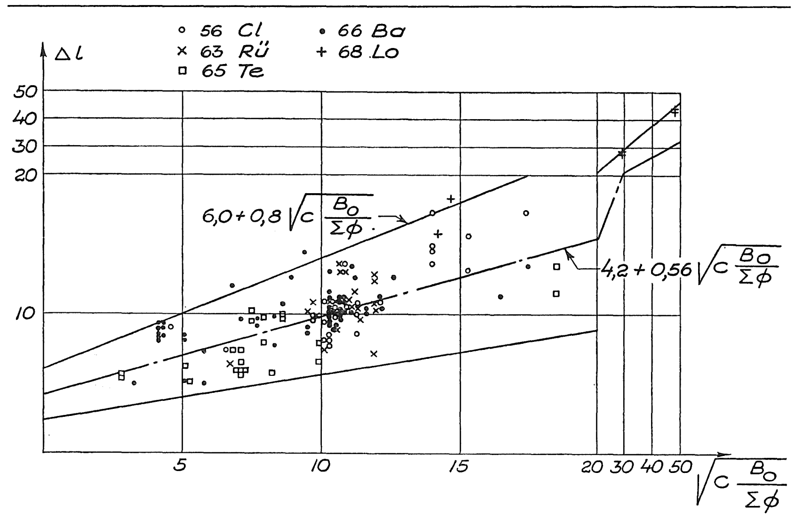

FIG. 4. Measured crack spacing (cm) with deformed bars for the beam surfaces subjected to the greatest tension (bottoms), compared to spacing calculated from formulae (1) and (2).

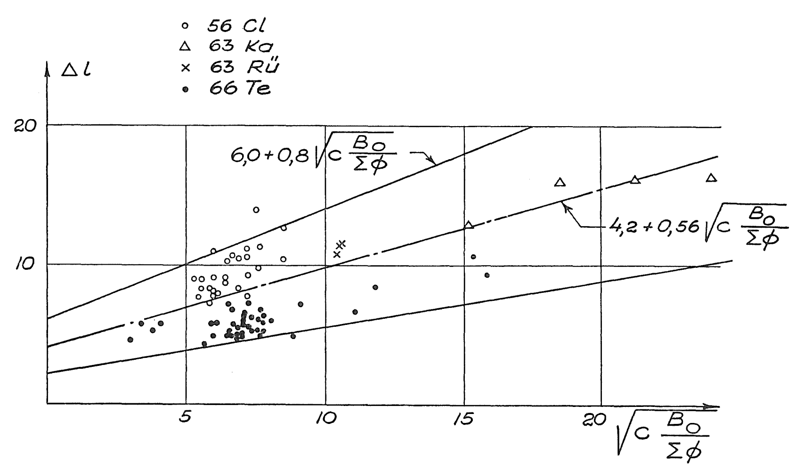

FIG. 5. Measured crack spacing (cm) on slabs with deformed bars, compared with spacing calculated from formulae (1) and (2).

mean value itself has already been corrected by $2S_2$. The expression is therefore

$$\Delta l_{\max} = 1.7\left(6.0 + 0.8\sqrt{c\frac{B_o}{\Sigma \varnothing}}\right) \tag{3}$$

An analysis, similar to the previous one, of crack spacing based on 67 No observations on beams reinforced with plain bars /66 Ba and 63 Rü/ gives the results shown in TAB. 3.

The slight dispersion is presumed to be due to the limited number of tests. This circumstance is purposely made use of to bring about agreement with Formula (1), the following being obtained

$$\Delta l = 4.4 + 0.72\sqrt{c_1\frac{B_o}{\Sigma \varnothing}} \tag{4}$$

This is corrected for plain bars by $2.5S_2$, to result in

$$\Delta l = 6.0 + 1.0\sqrt{c\frac{B_o}{\Sigma \varnothing}} \tag{5}$$

with

$$\Delta l_{\max} = 1.7\left(6.0 + 1.0\sqrt{c\frac{B_o}{\Sigma \varnothing}}\right) \tag{6}$$

Considered against the common belief of the dominant significance of the diameter on crack spacing, results (2) and (4) are surprising, until account is taken of the fact that regulations concerning cover often relate this to the diameter. The slight difference between deformed and plain bars is also verified elsewhere /57 Ho/, but it disagrees nevertheless with what is often considered to be an experience. The explanation probably lies in the fact that structures reinforced with plain bars attain the full crack width more quickly than do those reinforced with deformed bars. The results for plain bars, divided over beam sides and beam bottoms, are shown in FIG. 6 and 7.

The crack spacing according to (1)—(6) will be compared with further test results, which have not been taken into account before because they were too specialised or showed too little variation of the term

$$\sqrt{c\frac{B_o}{\Sigma \varnothing}}.$$

For beams reinforced with deformed bars with variable diameter within the beam /63 Rü and 68 Lo/, FIG. 8 shows a comparison between test results and (1) which it is found can be used even if the results are scant and the dispersion large.

Some test results on bundled reinforcement /62 Te, 65 Te, 65 Tep and 66 Tep/ are compared in FIG. 9 with (1) and show good agreement. A comparison between 12 No beams with deformed bars and 12 No with plain bars /66 Ba/ has not been included before, since the beams are quite similar and would have too great a significance.

TAB. 3. Nine calculations, among them the seven best for all observations on test specimens with plain bars.

Δl (cm)	S_1 (cm)	S_2 (%)	S_3 (%)
$3.4 + 0.74c_4\sqrt[3]{\frac{B_o}{\Sigma \varnothing^2}}$	1.45	13.7	14.1
$4.7 + 0.76c_2\sqrt[3]{\frac{B_o}{\Sigma \varnothing^2}}$	1.51	14.0	14.6
$3.1 + 1.24\sqrt[3]{c_2^2\frac{B_o}{\Sigma \varnothing}}$	1.51	14.1	14.5
$3.6 + 0.82\sqrt{c_2\frac{B_o}{\Sigma \varnothing}}$	1.62	15.0	15.7
$4.4 + 0.72\sqrt{c_1\frac{B_o}{\Sigma \varnothing}}$	1.65	15.4	15.5
$2.9 + 1.07\sqrt[3]{c_4^2\frac{B_o}{\Sigma \varnothing}}$	1.69	15.8	16.3
$4.2 + 0.68\sqrt{c_3\frac{B_o}{\Sigma \varnothing}}$	1.75	16.3	17.0
$4.9 + 1.18\sqrt[3]{c_1\frac{\varnothing^2 B_o}{\Sigma \varnothing^2}}$	2.01	19.1	19.6
$5.5 + 0.97\sqrt[3]{c_3\frac{\varnothing^2 B_o}{\Sigma \varnothing^2}}$	2.14	20.3	21.7

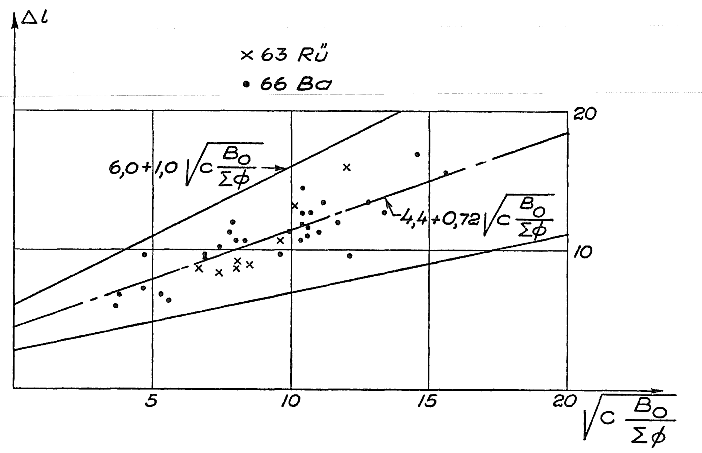

FIG. 6. Measured crack spacing (cm) on sides of beams with plain bars, compared with spacing calculated from formulae (4) and (5).

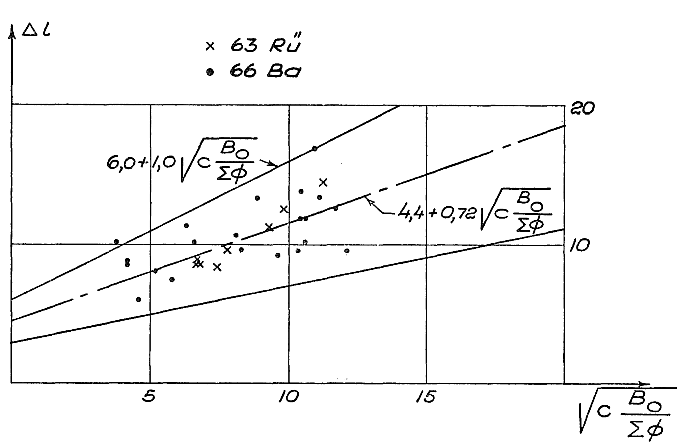

FIG. 7. Measured crack spacing (cm) with plain bars for the beam surfaces subjected to the greatest tension (bottoms), compared with spacing calculated from formulae (4) and (5).

The ratios between observed values of mean crack spacing and those calculated according to (1) and (4) are

0.96: 1; (1.07: 1 − 0.84: 1) for deformed bars and 0.92: 1; (1.04: 1 − 0.83: 1) for plain bars.

Another comparison between 5 No beams with deformed bars and 3 No with plain bars /65 Mu/ gives the ratio 1.10: 1 (1.19 − 0.90) for deformed bars and 0.96: 1 (1.06 − 0.77) for plain bars, between measured values and those calculated according to (1) and (4).

A study of 12 No beams with plain bars and 2 No with deformed bars only states the maximum crack spacing /47 Wä/. For the 12 No beams, the measured crack spacing in relation to 1.7× the value according to (4) was on an average 1.13 (1.32 − 0.83) and for the 2 No beams the ratios of the measured spacing to 1.7× the value according to (1) were 1.03 and 0.95 respectively. For a beam with plain bars and stirrups spaced at 12.5 cm, the ratio was 1.04.

There is another study on 15 No beams which only states the maximum spacing /66 Yo/. For 13 No of these, without stirrups, the measured crack spacing was 1.7× that according to (1) with a standard deviation of 8 %. For one beam with stirrups spaced at 15 cm, the crack spacing was 0.97 of that according to (1)×1.7 and for another one with a stirrup spacing of 10 cm, it was 0.76, which illustrates the crack-promoting effect of the stirrups.

The supposed effect of the direction in which the concrete is cast, or perhaps rather more of the thickness of the concrete layer below the reinforcement, is studied in a number of test series /63 Rü, 66 Ba and 66 Tep/, as shown in FIG. 10. No such effect is found.

Our own tests set out to show the effect of prestress (none, half, full) and of the surface condition. The results as shown in FIG. 11 indicate that the prestress has no effect, and also that even such an insignificant treatment of the surface as indentation or crimping converts the reinforcement or prestressing bars, from the point of view of the behaviour being investigated, into deformed bars. (After preliminary studies, all deformed bars have been regarded in this study as one type.) It is also shown that for values of $\tilde{\omega}_o$ less than 1 %, reinforcement ceases to have any crack-controlling effect. Attempts to find the influence of concrete quality from the test results which are available have been unsuccessful. It would be reasonable to suppose otherwise that it may have some effect at least on the above limit for $\tilde{\omega}_o$.

The constants α and β have been made equal to 4.2 and 0.56 respectively in accordance with (1) for strand and indented and crimped wire, 4.4 and 0.72 respectively in accordance with (4) for plain wire and 4.4 and 1.1 respectively for bars in sheaths. Calculations for bars in sheaths in combination with deformed bars were carried out

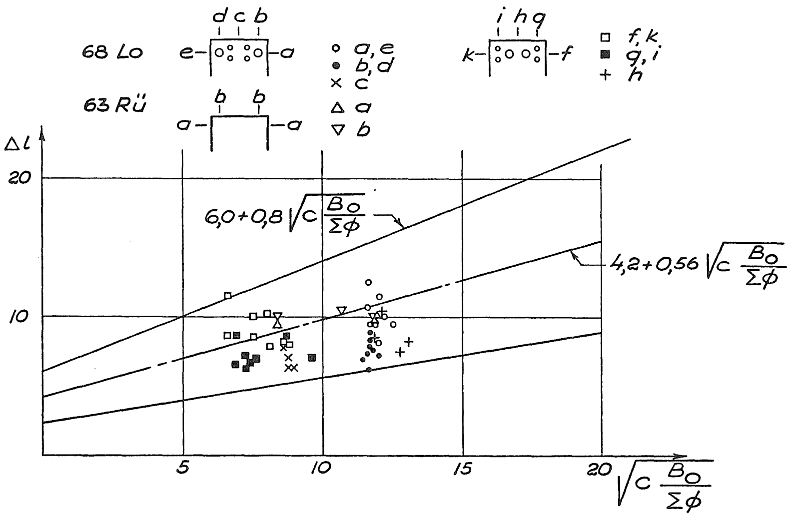

FIG. 8. Measured crack spacing (cm) for beams with deformed bars of different diameters and with variable cover, compared with spacing calculated from formulae (1) and (2).

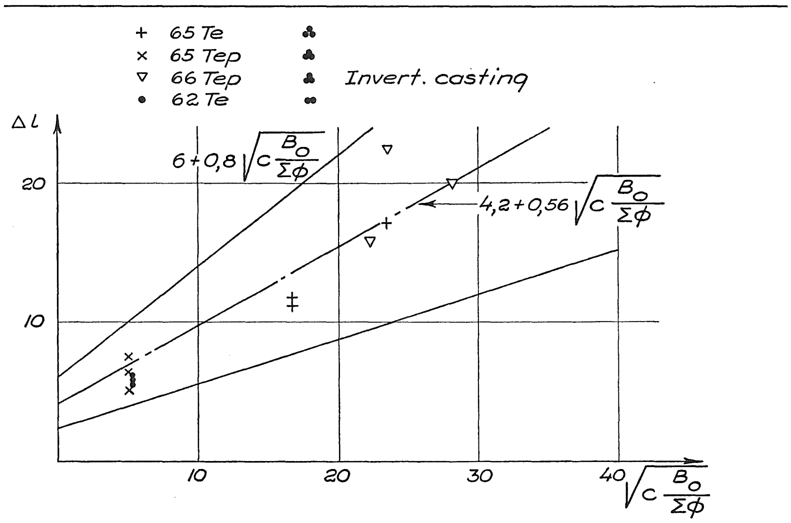

FIG. 9. Measured crack spacing (cm) for bundled deformed reinforcement, compared with spacing calculated from formulae (1) and (2).

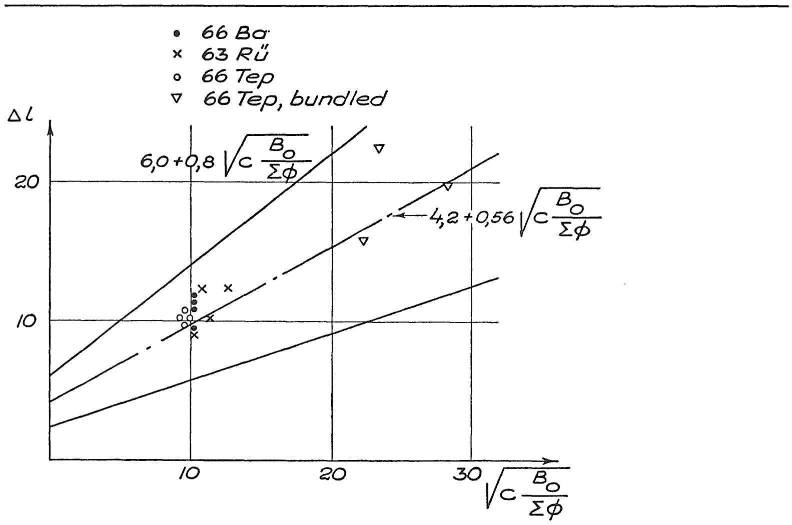

FIG. 10. Measured crack spacing (cm) for beams with deformed bars which were at the top of the beam on casting.

according to (1), in which connection the diameter of the bar, uncorrected, was included in the calculation in $\Sigma\varnothing$.

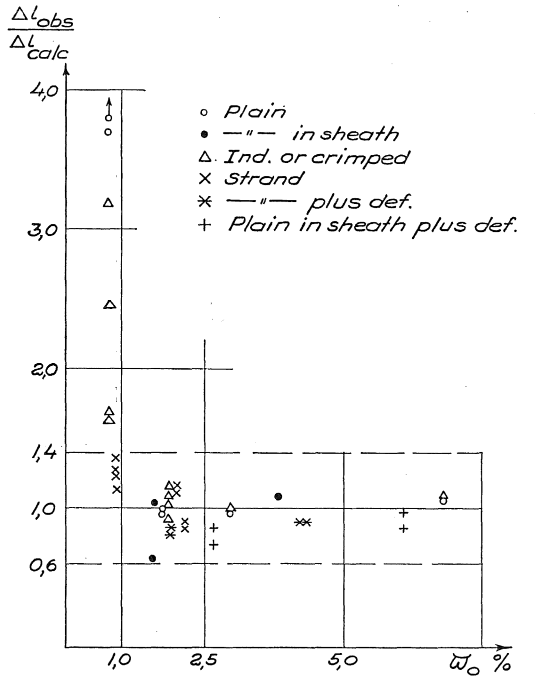

FIG. 11. Relationship between observed and calculated crack spacing for different types of steel in present tests. Each mark refers to mean values for two sides of beam. For strand alone or in combination with deformed bars, for indented and crimped wire and for plain bars in sheaths, in combination with deformed bars, Δl has been calculated from formula (1). For plain wire, Δl has been calculated from formula (4). For only plain bars in a sheath,

$$\Delta l_{\text{calc}} = 4.4 + 1.1\sqrt{c\frac{B_0}{\Sigma\varnothing}}.$$

The worst result with a percentage of reinforcement less than 1 was obtained for beams with 50 No plain wire of 2.5 mm diameter.

2 Value of strain to determine crack width

The width of cracks is given by crack spacing multiplied by a certain value of the strain. It has been shown earlier /66 Ba/ that this strain, with the application of some correction factor, is the mean strain in the concrete (including the cracks) at the level of the reinforcement or prestressing tendon. This state of affairs is illustrated in FIG. 12 and 13 which refer to beam No XIIB in our own tests. FIG. 12 shows the observation length, with deflection assumed to follow a circular arc, and illustrates the relationship between the deflection f and the mean crack width w_{av} as a function of the bending moment M. FIG. 13 shows for the same beam the mean strain ε_{av} calculated from f in relation to the maximum crack width w_{max} and the mean crack width w_{av}. Both of these are represented by straight lines through the origin.

As regards the correction factor, there is at present no more known than that it would appear /66 Ba/ to be less than 1 for small values of c and to be nearly 1 for large values of c. Experimental verification of this, based on short-term tests, will be discussed in Section 3.

The mean strain seems /63 Rü and our own tests/ to approach ε_a in the crack after some loading and unloading cycles. This is illustrated in FIG. 14 which refers to the same beam as FIG. 12 and 13. For the sake of comparison, the line

$$\varepsilon_a = \varepsilon_{av} + \frac{7.5}{E_a \, \tilde{\omega}_1}$$

has been drawn in /66 Fe/. This formula is obviously derived from first loading tests.

The figure should not be regarded as presenting an accurate record, since stresses and strains must be calculated on the basis of assumed values for residual prestress and for the strain moduli of the concrete and steel respectively. Both of these were affected above the limit of proportionality.

It is accepted that $\varepsilon_{av} = \varepsilon_a$ (7)

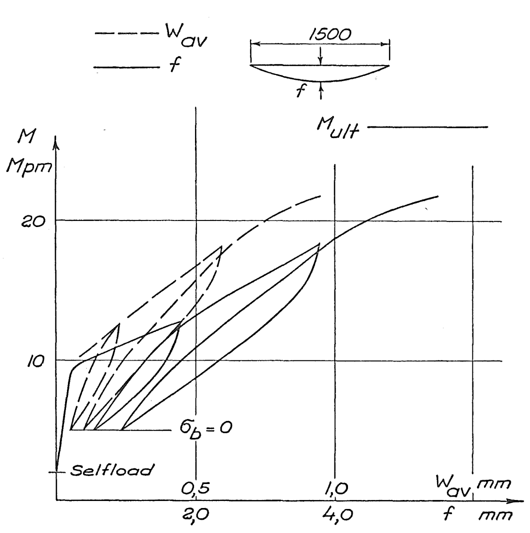

FIG. 12. Deflection over a 1.5 m section at midspan and mean crack width for beam XIIB in present tests.

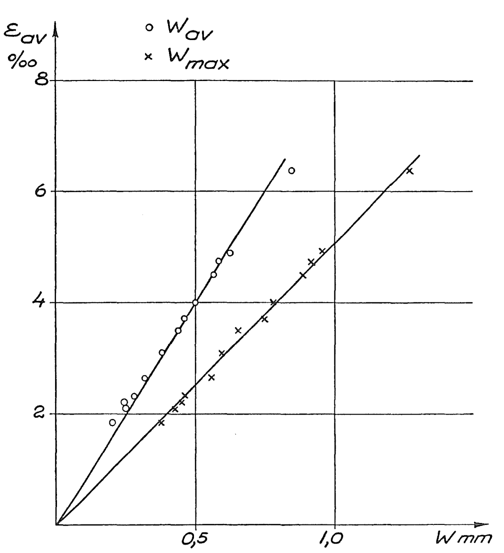

FIG. 13. Mean and maximum crack widths for beam XIIB in the present tests, compared with mean strain at the level of the steel calculated from the curvature.

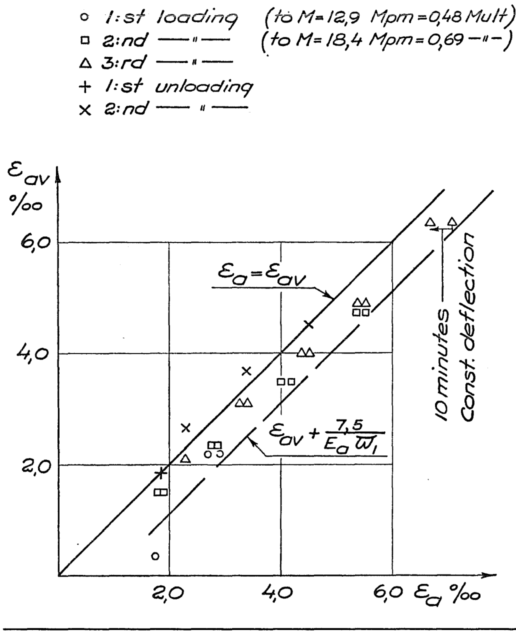

FIG. 14. Strain in steel calculated without regard to surrounding concrete, compared with mean strain at the level of the steel calculated from curvature, for beam XIIB in present tests.

3 Influence of repetition and duration

The correction factor referred to in Section 2 is less than 1 in most tests. This phenomenon is probably due to the fact that the concrete is deformed in the direction of strain in the steel without obstructing this strain apart from very early stages of loading. See (7). This is explained, as far as the concrete subjected to tension nearest the steel is concerned, by the occurrence of internal cracks /65 Br/. As regards the concrete that may be subjected to compression between the cracks on the surface, the explanation is probably to be found in plastic extension prior to cracking.

A lot remains to be investigated. The problems are the strain in the concrete and its internal rupture before and after external cracking, as well as the shape of the crack surfaces.

It is shown in FIG. 15 that the correction factor is not the same for all beams. This figure reproduces results of our own tests, arranged according to the arbitrarily chosen parameter

$$\sqrt{c\frac{B_o}{\Sigma\varnothing}}.$$

For beams that are otherwise similar, the correction factor, expressed as

$$\frac{w_{av}}{\varepsilon_a \Delta l_{av}},$$

is obviously larger for large

$$\sqrt{c\frac{B_o}{\Sigma\varnothing}}.$$

FIG. 16 and 17 give an indication of the connection between these phenomena. These figures show how repetition and duration gradually increase the value of the correction factor towards 1, in which connection

$$w = \varepsilon_a \Delta l \tag{8}$$

The sequence of events may be assumed to be as follows: compressive stresses in the concrete surface gradually induce compressive strain, the bond between the concrete and the steel yields, and the inner cracks are closed.

It is not claimed that the above is a complete explanation.

No attempt is made to seek a relationship between the magnitude of the correction factor and its alteration and the factor

$$\sqrt{c\frac{B_o}{\Sigma\varnothing}},$$

or any other parameter.

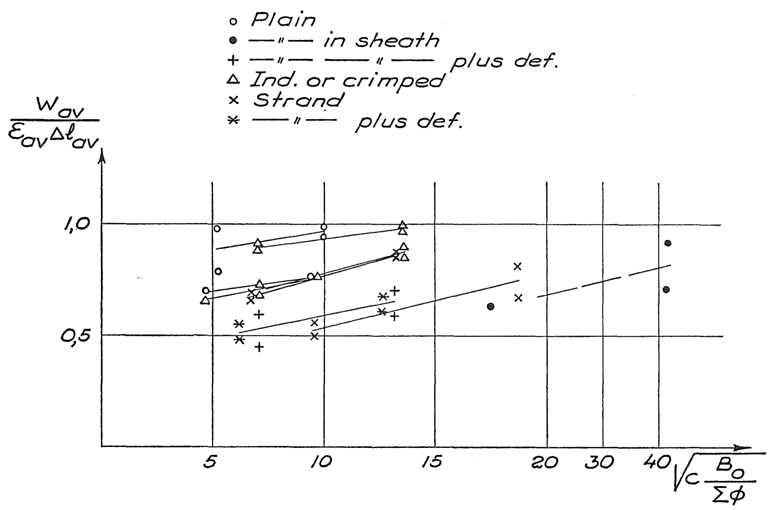

FIG. 15. Values of $w_{av}/\varepsilon\Delta l_{av}$ in present tests. Five beams have values near unity. Four of these, with indented or crimped plain wire, had a steel percentage less than 1 (see FIG. 11) and one beam with 2.5 mm diameter plain wire failed in bond.

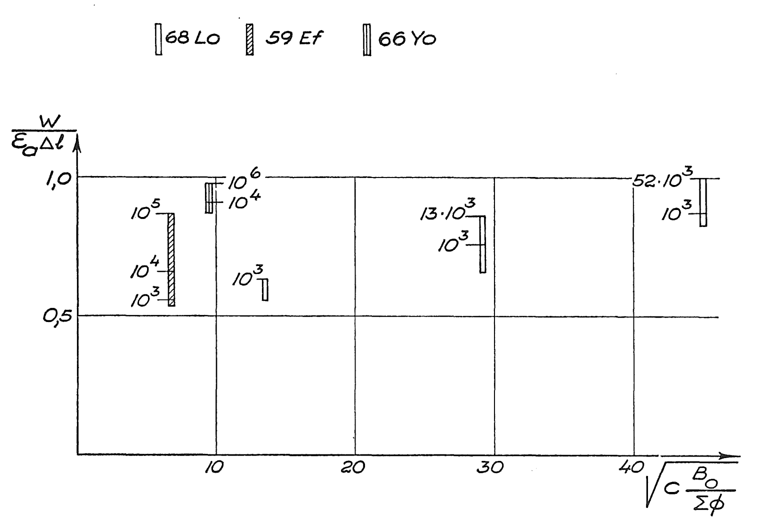

FIG. 16. Increase of $w/\varepsilon_a\Delta l$ on repeated loading for beams with deformed bars. Numbers at staples denote number of load repetitions.

Another observation which will be described refers to a number of beams with deformed bar reinforcement /57 Bj/ with $\sigma_a = 4000$ kg/cm^2 and the value

$$\sqrt{c\frac{B_o}{\Sigma\varnothing}} = 5.$$

With the correction factor = 1, the mean value of the crack width after a long time should be 0.13 mm and its maximum value equal to $1.7 \times \frac{6.0}{4.2} \times 0.13 = 0.32$ mm, in accordance with (1), (7) and (8). Over a period of 2.25 years with constant load, the maximum crack width for a total of 4 No beams increased from 0.15 mm to 0.30 mm. These values do not, however, refer to the same crack.

The objection could be made that this investigation takes no account of the shrinkage of concrete, which at least over a long period should have some significance. The counterargument is that the whole of this Section has rather an uncertain foundation, and since the correction is seldom greater than that corresponding to $\sigma_a =$ 800 kg/cm^2, the objection would not be justified.

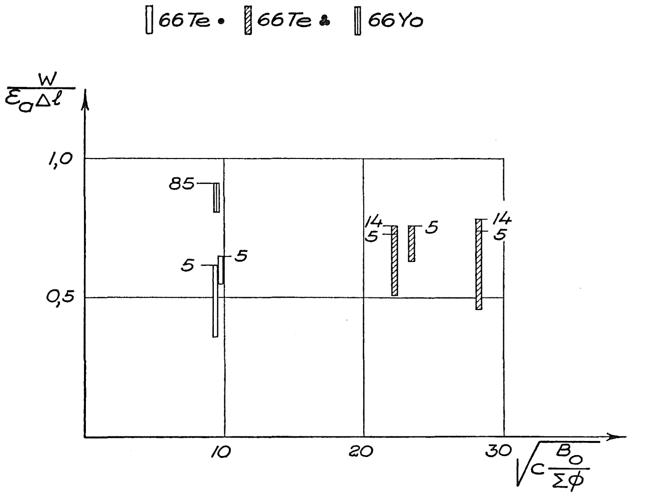

FIG. 17. Increase of $w/\varepsilon_a \Delta l$ on loading of long duration for beams with deformed bars. Number at staples denote the time of loading in days.

4 Influence of shear force

Little is known of the influence of shear force on crack spacing and crack widths. In these tests, the greatest crack under the loading points was observed outside the actual area of observation. In FIG. 18 and 19, both this crack and the greatest crack within the area observed are compared to those calculated in accordance with 1.7 × (1) and 1.7 × (4). The only observations that show appreciable deviation are those on a beam with 2.5 mm diameter plain reinforcement (No XVIIIA), in which the reinforcement slipped and the beam finally failed in bond. It is found that the shear force has no effect on the crack width, which is also borne out by other investigations /63 Rü/, as long as the bond stress is not too high.

An unpublished investigation on 13 No concrete pipes, with an internal diameter of 60 cm, wall thickness of 8 cm and reinforcement varying in four groups, loaded on two opposite generatrices, showed the mean value of the maximum crack width to be 0.6 × that calculated in accordance with (3) and (9), with the standard deviation approximately 40 %. The calculation was carried out for $B_0 = 0.75 \times$ the cross section around the reinforcement (central reinforcement) and with the purely formal assumption that it was possible for the full crack spacing to be developed.

The observed cracks are thus small, which may be a result of their being measured on first loading. Unintentional variations in c show, however, that the ratio w_{obs}/w_{calc} becomes less as the value of c increases. This is a possible consequence of the special loading case with rapidly variable stress. Here is an open field of research.

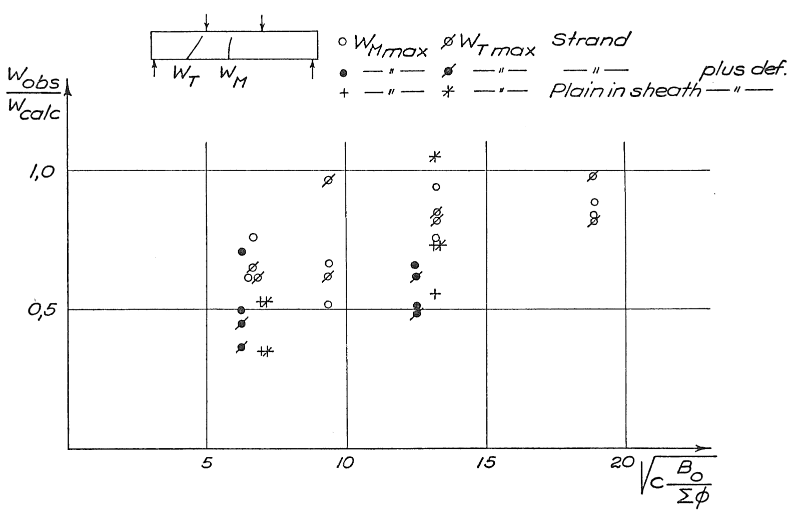

FIG. 18. Observed maximum crack widths inside and outside the region with constant moment respectively, compared with calculated maximum crack widths in present tests. Beams with strand or deformed bars combined with other steel. Beams with a percentage of reinforcement less than 1 not included.

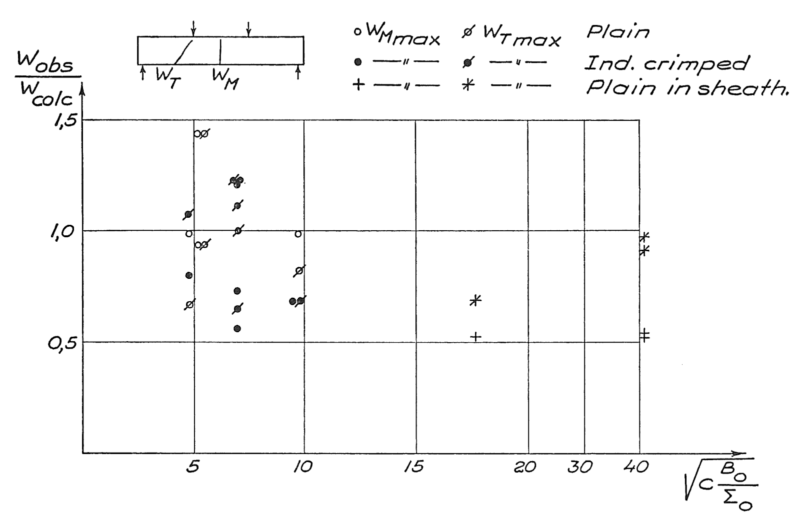

FIG. 19. Observed maximum crack widths inside and outside the region with constant moment respectively, compared with calculated maximum crack widths in present tests. Beams with plain indented or crimped wires and with plain bars in sheaths. The beam with the w_{obs}/w_{calc} ratio of 1.43, which was reinforced with 50 No 2.5 mm wires, failed in bond. Beams with a percentage of reinforcement less than 1 not included.

5 Recommendations for regulations

For a constant moment and normal force, which for a constant inner lever arm corresponds to a constant force in the reinforcement, and thus for a constant steel area to constant strain in the steel, and with this state of affairs being also approximately true over a fairly small multiple of the calculated crack spacing, it is probable that the following expression, with a reasonable repetition of the construction element will hold for the crack spacing governing design

$$\Delta l = 6 + \beta \sqrt{c \frac{B_o}{\Sigma \varnothing}} \text{ cm}$$

where $\beta = 1.0$ for plain bars or wire
$= 0.8$ for indented bars or wire
$= 0.8$ for crimped bars
$= 0.8$ for deformed bars
$= 0.8$ for strand
$= 1.5$ for reinforcement in sheath

Where transverse reinforcement firmly connected to the main reinforcement does not have a crack-promoting effect, these values should be used as the basis for prediction of the crack spacing.

It is to be assumed that the mean crack width, opposite the reinforcement and at the same distance from the neutral layer as this, will be given by the following expression after about 2500 hours or 10^6 load applications

$w_1 = \varepsilon_a \times \Delta l$
with ε_a measured from $\sigma_b = 0$.

The maximum crack width should be assumed to be $1.7 \times$ the mean crack width.

Under the conditions specified, the crack width at the maximum distance from the neutral layer is to be assumed /66 Ba/ to be equal to

$$w_2 = w_1 \frac{h_t - x}{h - x} \tag{9}$$

The crack width determined by a unit of reinforcement should be assumed to grow in size, measured along the concrete surface, as the distance from the reinforcement is increased, until it reaches a value of $2.5 \times$ the calculated value at a distance of 5—$6 \times c$ /66 Ba/.

Experimental determination of Δl should be carried out at a value of σ_a at least $1.5 \times$ that being considered, and on so many similar models that the results will provide a reliable basis for the mean value and twice the standard deviation.

Where the strain in the steel shows a substantial variation within a small multiple of the calculated crack spacing and the bond of the steel is at the same time fully satisfactory, it is to be supposed that the maximum crack width will be less than that according to the above. For an experimental determination of this, the specification should be that the basis of comparison is to be the mean value + twice the standard deviation determined during decreasing load from 1.5 × the value of σ_a considered.

Strain due to enforced deformations is not considered here. No attempt has been made to correlate the widths of cracks to the possible corrosion of the steel in corrosive environments or to the function of the structure.

It was not considered that there was any reason to take into account any possible effect due to the steel being situated at the top of the structure while the concrete is being cast.

The recommendations are limited in their scope by the requirement that $\tilde{\omega} \geqslant 1$ % and by the requirement as to reliable bonding of the steel.

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Appendix 1. Material used. Reinforcement and prestressing tendons

TAB. 4. Main steel used in the various beams.

Beam No.	Type	Dia. mm	σ_u kg/mm²	$\sigma_{0\cdot 2}$ kg/mm²	No. of	Area cm²	Effective prestress kg/mm²	Figure
IA, IB	Indented[1]	10	66	54	12	9.45	—	20
IIA, IIB	Plain[2]	10	63	52	12	9.45	—	20
IIIA	Plain[3]	26	109	89	1	5.3	56	21
IIIB	Plain[3]	26	109	89	1	5.3	57	21
IVA	Plain[3]	26	109	89	1	5.3	27.5	21
IVB	Plain[3]	26	109	89	1	5.3	28	21
VA	Plain[3]	26	109	89	1	5.3	54	21
	Deformed[4]	10	67	49	4	3.1	—	20
VB	Plain[3]	26	109	89	1	5.3	56	21
	Deformed[4]	10	67	49	4	3.1	—	20
VIA	Plain[3]	26	109	89	1	5.3	27	21
	Deformed[4]	10	67	49	4	3.1	—	20
VIB	Plain[3]	26	109	89	1	5.3	28	21
	Deformed[4]	10	67	49	4	3.1	—	20
VIIA	Strand	12.7	188	172	3	2.85	100	22
VIIB	Strand	12.7	188	172	3	2.85	102	22
VIIIA	Strand	12.7	188	172	3	2.85	50	22
VIIIB	Strand	12.7	188	172	3	2.85	51	22
IXA	Strand	12.7	188	172	3	2.85	100	22
	Deformed[4]	10	67	49	4	3.1	—	20
IXB	Strand	12.7	188	172	3	2.85	102	22
	Deformed[4]	10	67	49	4	3.1	—	20
XA	Strand	12.7	188	172	3	2.85	50	22
	Deformed[4]	10	67	49	4	3.1	—	20
XB	Strand	12.7	188	172	3	2.85	51	22
	Deformed[4]	10	67	49	4	3.1	—	20
XIA	Strand	6.3	200	174	12	3.0	93	22
XIB	Strand	6.3	200	174	12	3.0	96	22
XIIA	Strand	6.3	200	174	12	3.0	47	22
XIIB	Strand	6.3	200	174	12	3.0	48	22
XIIIA	Indented	5.0	190	170	14	2.75	102	21
XIIIB	Indented	5.0	190	170	14	2.75	105	21
XIVA	Indented	5.0	190	170	14	2.75	52	21
XIVB	Indented	5.0	190	170	14	2.75	53	21
XVA	Crimped	5.0	184	142	14	2.75	102	21
XVB	Crimped	5.0	184	142	14	2.75	105	21
XVIA	Crimped	5.0	184	142	14	2.75	52	21
XVIB	Crimped	5.0	184	142	14	2.75	53	21
XVIIA	Plain	2.5	220	207	50	2.45	115	22
XVIIB	Plain	2.5	220	207	50	2.45	119	22
XVIIIA	Plain	2.5	220	207	50	2.45	58	22
XVIIIB	Plain	2.5	220	207	50	2.45	60	22

[1] Ps 50 Swedish Standard 21 25 19
[2] Ss 50 Swedish Standard 21 25 18
[3] In 30 mm diameter sheath. Grouted
[4] Ks Swedish Standard 21 25 13

The steel was washed in carbon tetrachloride before being placed.

Concrete

Cement (Gullhögen, rapid-hardening)	354 kg/m³
C/W	1.7
Maximum aggregate size	10 mm

Appendix 2. Shape of test specimens and test procedure

The test specimens, which were T-beams or I-beams, are shown in FIG. 23 and 24. Details of the principle reinforcement are given in TAB. 4. The tops of the beams were reinforced with 2 No ⌀12 deformed bars (Ks 40). The beams were provided with stirrups, ⌀8 at 200 mm spacing (Ks 40), between the supports and the point loads. The placing of the main steel is shown in FIG. 25 a—h.

Pretensioned reinforcement was released slowly and post-tensioned reinforcement was stressed when the concrete had reached a strength of at least 300 kg/cm².

The method of loading of the beams is shown in FIG. 23, while the load cycle is shown in FIG. 26. The failing moment specified in the figure has been calculated under the assumption of parabolic distribution of compressive stress in the concrete and with nominal values for the strengths of the concrete and the steel. The cracking moment is the moment at which the first crack was observed. The deflection was maintained constant during stops to take readings. The load had to be decreased during such stops when the load was being increased, and increased slightly when it was being reduced. All crack widths were measured at every loading stage within the observation distance (1500 mm).

TAB. 5. Results of crack measurements. Each observed value is the mean for the two sides of a beam. For strand alone or in combination with deformed bars, for indented and crimped wires and for plain bars in sheaths in combination with deformed bars, $\Delta l_{av} = w_{av}/\varepsilon_{av}$ has been calculated from formula (1). For plain embedded steel, Δl_{av} has been calculated from formula (4) and for plain bars in grouted sheaths from the expression $\Delta l_{av} = 4.4 + 1.1 \sqrt{cB_o/\Sigma\varnothing}$. Maximum crack widths, both inside and outside the region with constant moment, have been calculated from the expression $w_{max}/\varepsilon_{av} = 1.7\ \Delta l_{av}$.

Beam	B_o	A	$\tilde{\omega}_o$	c_h	$\Sigma\varnothing$	$\sqrt{c_h \frac{B_o}{\Sigma\varnothing}}$	Δl_{av}			w_{av}/ε_{av}			$w_{av}/\varepsilon_{av}\Delta l_{av}$	w_{max}/ε_{av}				
							Calc	Obs	Obs/Calc	Calc	Obs	Obs/Calc	Obs	Calc	Obs[2] (M)	Obs[3] (T)	Obs/Calc (M)	Obs/Calc (T)
No.	cm²	cm²	%	cm	cm	cm	cm	cm		cm	cm		cm	cm	cm	cm		
IA	140	9.45	6.8	2.0	12.0	4.8	6.9	7.5	1.09	6.9	5.0	0.73	0.67	11.7	9.4	12.6	0.80	1.08
IB	320	9.45	3.0	3.5	12.0	9.7	9.6	9.7	1.01	9.6	7.4	0.77	0.76	16.3	11.2	11.2	0.69	0.69
IIA	140	9.45	6.8	2.0	12.0	4.8	7.9	8.3	1.05	7.9	5.8	0.74	0.70	13.4	13.4	9.0	0.99	0.67
IIB	320	9.45	3.0	3.5	12.0	9.7	11.4	11.1	0.97	11.4	8.4	0.74	0.76	19.4	19.2	15.8	0.99	0.82
IIIA	140	5.30	3.8	5.7	2.6	17.6	23.7	25.5	1.08	23.7	16.0	0.68	0.63	40.3	20.5	27.4	0.51	0.68
IIIB	320	5.30	1.7	14.7	2.6	42.6	51.3	52.0	1.01	51.3	36.8	0.72	0.71	87.3	45.5	84.0	0.52	0.96
IVA[1]	140	5.30	3.8	5.7	2.6	17.6	23.7	33.5	1.41	23.7								
IVB	320	5.30	1.7	14.7	2.6	42.6	51.3	32.0	0.63	51.3	29.4	0.57	0.92	87.3	45.5	79.0	0.52	0.91
VA	140	8.45	6.0	2.4	6.6	7.1	8.2	7.0	0.86	8.2	3.1	0.38	0.44	13.9	4.8	4.8	0.35	0.35
VB	320	8.45	2.6	3.7	6.6	13.4	11.7	10.0	0.86	11.7	5.9	0.51	0.59	19.8	14.5	14.5	0.73	0.73
VIA	140	8.45	6.0	2.4	6.6	7.1	8.2	7.9	0.97	8.2	4.6	0.56	0.58	13.9	7.4	7.4	0.53	0.53
VIB	320	8.45	2.6	3.7	6.6	13.4	11.7	8.4	0.72	11.7	5.9	0.51	0.70	19.8	11.1	20.7	0.56	1.05
VIIA	140	2.85	2.0	2.1	3.3	9.4	9.5	11.1	1.17	9.5	6.3	0.66	0.57	16.2	10.9	15.7	0.67	0.97
VIIB	320	2.85	0.9	3.7	3.3	18.9	14.8	18.8	1.27	14.8	15.2	1.03	0.81	25.2	21.0	24.8	0.84	0.99
VIIIA	140	2.85	2.0	2.1	3.3	9.4	9.5	10.7	1.12	9.5	5.3	0.56	0.50	16.2	8.4	10.1	0.52	0.62
VIIIB	320	2.85	0.9	3.7	3.3	18.9	14.8	20.0	1.35	14.8	13.3	0.91	0.67	25.2	22.2	20.4	0.89	0.82
IXA	140	6.00	4.3	2.1	7.3	6.4	7.8	6.9	0.89	7.8	3.8	0.49	0.55	13.3	9.3	5.9	0.71	0.45
IXB	320	6.00	1.9	3.7	7.3	12.7	11.3	9.7	0.86	11.3	5.9	0.52	0.61	19.2	10.0	12.2	0.51	0.62
XA	140	6.00	4.3	2.1	7.3	6.4	7.8	7.0	0.90	7.8	3.4	0.44	0.49	13.3	6.5	4.8	0.50	0.37
XB	320	6.00	1.9	3.7	7.3	12.7	11.3	9.4	0.83	11.3	6.3	0.56	0.67	19.2	13.0	9.6	0.66	0.49
XIA	140	3.00	2.1	2.2	6.8	6.7	8.0	7.3	0.91	8.0	5.1	0.64	0.70	13.6	10.4	8.8	0.76	0.65
XIB	320	3.00	0.9	3.7	6.8	13.2	11.6	13.1	1.13	11.6	11.1	0.96	0.85	19.7	14.8	16.3	0.75	0.83
XIIA	140	3.00	2.1	2.2	6.8	6.7	8.0	6.9	0.86	8.0	4.6	0.58	0.67	13.6	8.4	8.4	0.62	0.62
XIIB	320	3.00	0.9	3.7	6.8	13.2	11.6	14.3	1.23	11.6	12.5	1.08	0.87	19.7	18.5	16.7	0.94	0.85
XIIIA	145	2.75	1.9	2.2	7.0	6.9	8.1	9.4	1.16	8.1	8.3	1.03	0.88	13.8	16.6	13.8	1.20	1.00
XIIIB	335	2.75	0.8	3.7	7.0	13.5	11.8	29.0	2.45	11.8	29.0	2.46	1.00	20.0	33.8	25.8	1.70	1.30
XIVA	145	2.75	1.9	2.3	7.0	6.9	8.1	8.8	1.09	8.1	6.0	0.74	0.68	13.8	9.5	10.6	0.69	0.77
XIVB	335	2.75	0.8	3.8	7.0	13.5	11.8	19.5	1.67	11.8	16.7	1.42	0.86	20.0	22.8	19.0	1.14	0.96
XVA	145	2.75	1.9	2.3	7.0	6.9	8.1	8.3	1.02	8.1	7.6	0.94	0.92	13.8	16.8	16.8	1.22	1.22
XVB	335	2.75	0.8	3.8	7.0	13.5	11.8	38.0	3.20	11.8	36.8	3.12	0.97	20.0	41.8	39.0	2.09	1.96
XVIA	145	2.75	1.9	2.3	7.0	6.9	8.1	7.5	0.93	8.1	5.5	0.68	0.73	13.8	10.1	15.2	0.73	1.10
XVIB	335	2.75	0.8	3.8	7.0	13.5	11.8	19.0	1.62	11.8	17.1	1.45	0.90	20.0	32.3	32.3	1.62	1.62
XVIIA	140	2.45	1.8	2.4	12.5	5.2	8.1	7.9	0.98	8.1	6.3	0.78	0.80	13.8	13.0	13.0	0.94	0.94
XVIIB	320	2.45	0.8	3.9	12.5	10.0	11.6	43.5	3.75	11.6	40.6	3.50	0.95	19.7	58.5	38.6	2.97	1.96
XVIIIA	140	2.45	1.8	2.4	12.5	5.2	8.1	8.1	1.00	8.1	8.0	0.99	0.99	13.8	20.0	20.0	1.43	1.43
XVIIIB	320	2.45	0.8	3.9	12.5	10.0	11.6	63.5	5.50	11.6	62.5	5.40	0.99	19.7	70.5	58.5	3.60	2.97

[1] Beam IVA had been damaged before the test, so that there was a crack at one end of the observation distance at the beginning of the test.
[2] Largest crack within the observation distance. [3] Largest crack in the vicinity of the point loads.

In addition to this, the largest cracks in the vicinity of the point loads were also measured, since these cracks could be expected to be larger than those within the area of constant moment. Cracks were observed on both sides of the beams at the level of the centre of gravity of the steel. Deflections were measured at the centre of the observation distance and at both its ends.

The mean strain at the level of the steel (ε_{av}) was calculated from the curvature and the measured crack widths were related to this mean strain. The reason for this is illustrated by a comparison between FIG. 12 and 13.

Test results are shown in TAB. 5 and TAB. 6. The observed crack spacing and crack widths quoted are means for the two sides of the beams. The calculated failing moments shown in TAB. 6 have been calculated on the assumption that the distribution of compressive stress in the concrete is parabolic, with the measured value of the cube strength as the maximum value. Maximum concrete strain has been assumed to be 0.45 %. Stress-strain curves according to FIG. 20—22 have been used for the steel.

TAB. 6. Calculated and observed failing moments.

Beam	Cube strength	Failing moment, ton metre			
No.	kg/cm²	Calc	Obs	Obs/Calc	Cause of failure
IA	538	27.2	27.7	> 1.02	Large deflection
IB	555	27.2	28.4	1.04	Concrete failure
IIA	423	25.8	23.6	0.92	Concrete failure
IIB	437	25.8	24.1	0.93	Concrete failure
IIIA	526	24.5	25.2	1.03	Concrete failure
IIIB	500	24.4	24.9	1.02	Concrete failure
IVA	472	24.2	24.4	1.01	Concrete failure
IVB	467	24.2	24.7	1.02	Concrete failure
VA	436	30.3	31.6	1.04	Concrete failure
VB	458	30.3	30.7	1.01	Concrete failure
VIA	494	30.5	30.8	1.01	Concrete failure
VIB	487	30.5	30.4	1.00	Concrete failure
VIIA	511	23.3	23.8	> 1.02	Large deflection
VIIB	491	23.3	24.7	1.06	Concrete failure
VIIIA	475	23.3	22.7	0.98	Concrete failure
VIIIB	508	23.3	23.4	> 1.00	Large deflection
IXA	440	29.5	29.7	1.01	Concrete failure
IXB	486	29.7	30.1	1.01	Concrete failure
XA	415	29.2	30.1	1.03	Concrete failure
XB	427	29.2	29.7	1.02	Concrete failure
XIA	482	26.2	26.6	1.02	Concrete failure
XIB	496	26.2	27.0	1.03	Concrete failure
XIIA	496	26.2	26.2	1.00	Concrete failure
XIIB	500	26.2	26.6	> 1.02	Large deflection
XIIIA	507	22.6	23.7	1.05	Concrete failure
XIIIB	503	22.6	23.4	1.04	Fracture of wire
XIVA	465	22.5	23.4	1.04	Fracture of wire
XIVB	427	22.5	23.5	1.04	Fracture of wire
XVA	451	21.7	22.4	1.03	Fracture of wire
XVB	431	21.7	22.1	1.02	Fracture of wire
XVIA	458	21.7	22.3	> 1.03	Large deflection
XVIB	400	21.7	22.4	1.01	Fracture of wire
XVIIA	408	23.4	23.7	1.01	Fracture of wire
XVIIB	456	23.5	22.4	0.95	Fracture of wire
XVIIIA	432	23.5	20.9	0.89	Bond failure
XVIIIB	410	23.4	19.5	0.83	Bond failure

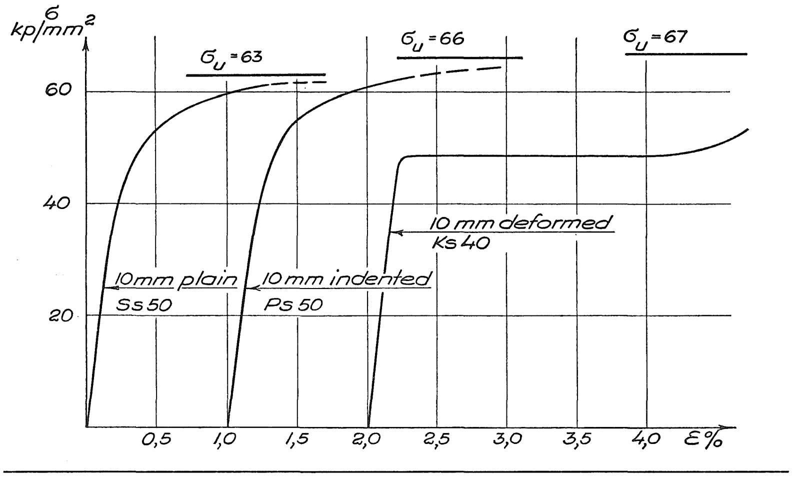


FIG. 20. Stress-strain diagrams for plain and indented reinforcing wire (Ss 50 and Ps 50) and for deformed bars (Ks 40).

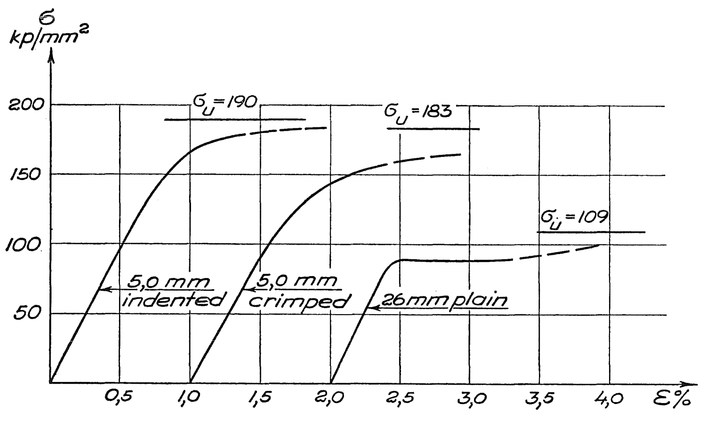


FIG. 21. Stress-strain diagram for 5 mm indented and crimped prestressing wire and for 26 mm plain prestressing bars.

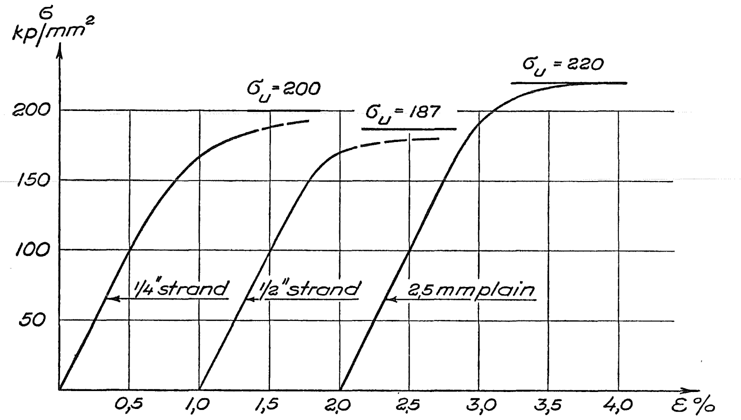


FIG. 22. Stress-strain diagram for 1/4″ and 1/2″ strand and for 2.5 mm plain prestressing wire.

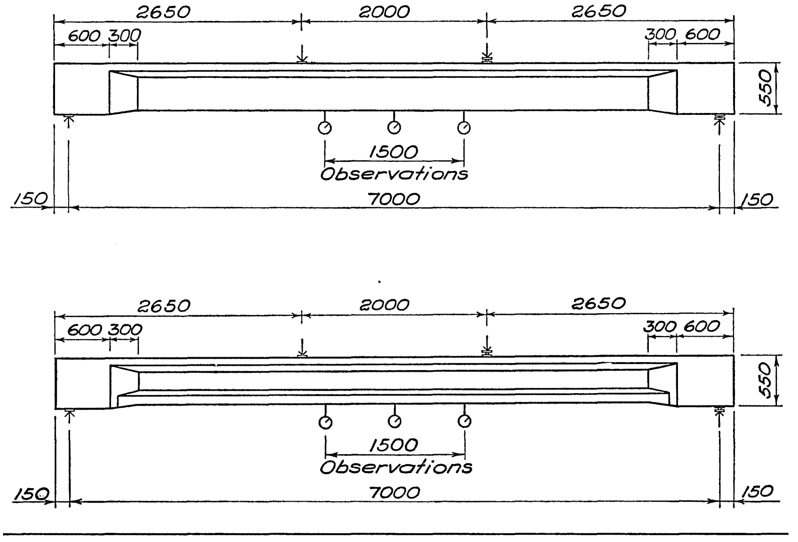


FIG. 23. Test beams. A = T-beam; B = I-beam.

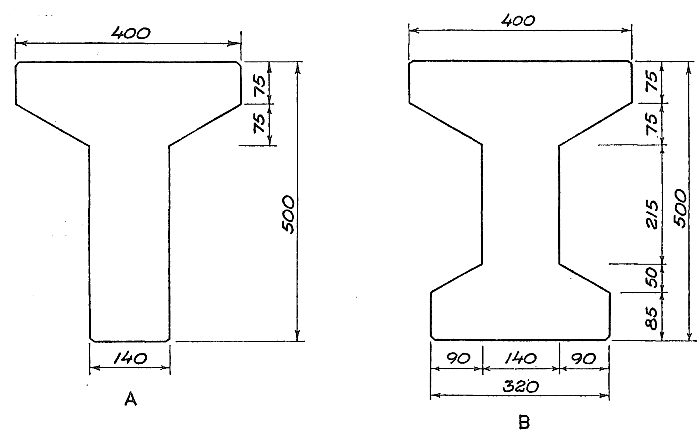


FIG. 24. Test beams. Cross sections.

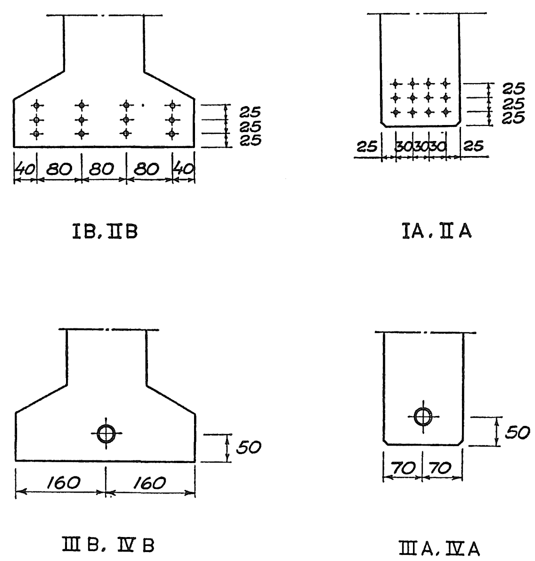
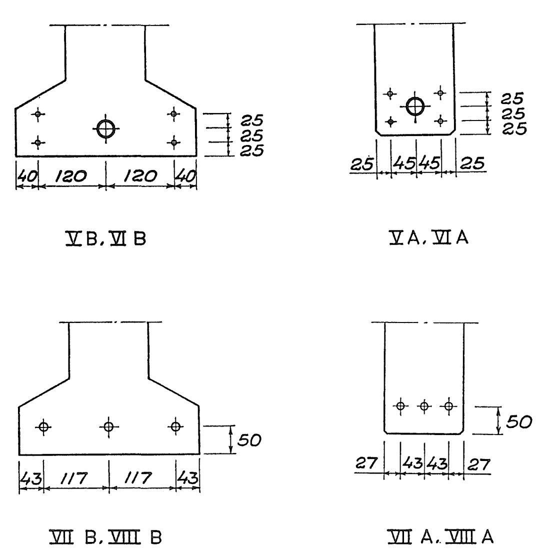
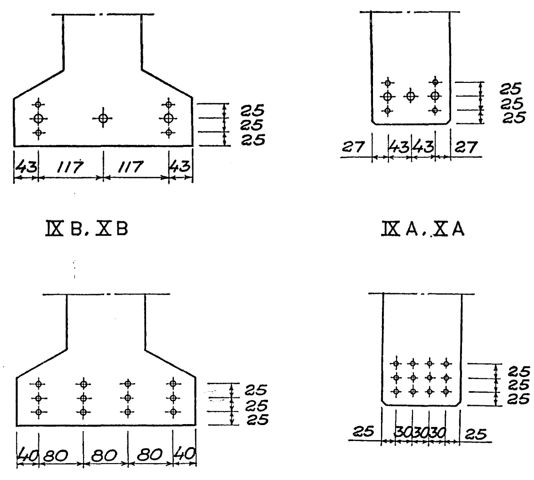


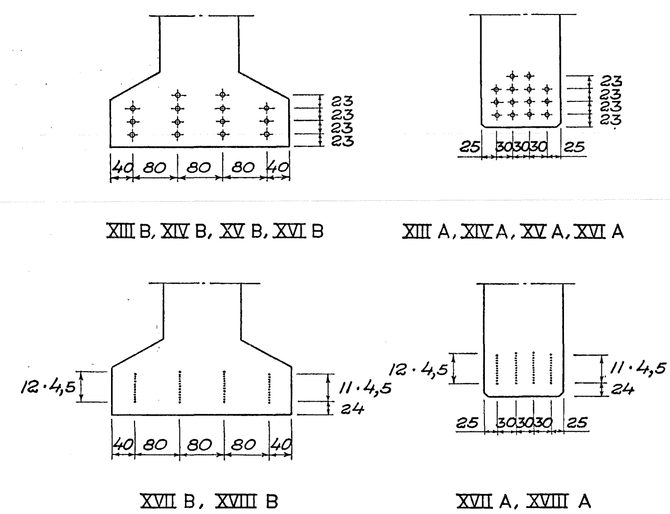
FIG. 25. Arrangement of steel. See also TAB. 4.
a) Unstressed reinforcement ∅ 10 mm.
b) Post-tensioned ∅ 26 mm bar in sheath.



c) Post-tensioned ∅ 26 mm bar in sheath in combination with ∅ 10 mm deformed bars.
d) Pretensioned 1/2″ strand.



e) Pretensioned 1/2″ strand in combination with ∅ 10 mm deformed bars.
f) Pretensioned 1/4″ strand.



g) Pretensioned indented and crimped ∅ 5 mm wires.
h) Pretensioned ∅ 2.5 mm plain wires.

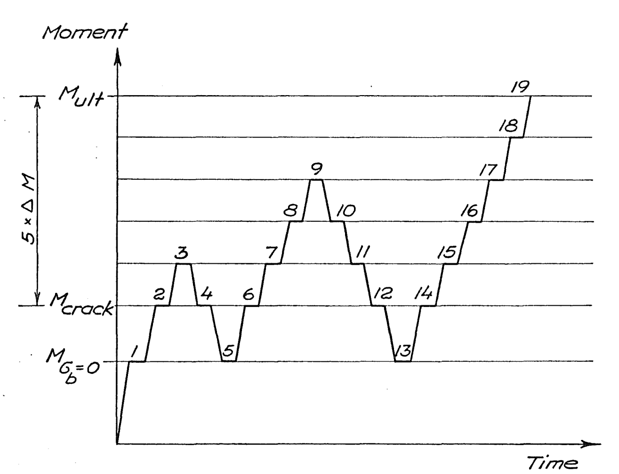


FIG. 26. Loading cycle diagram. $M_{\sigma_b=0}$ is the moment at which the stress in the concrete at the level of the steel is calculated to be nil. M_{crack} is the moment at which the first crack is observed. M_{ult} is the failing moment calculated from the nominal concrete and steel strengths. Observations were made at each step right up to No. 17. Observations could also be made in most cases at step No. 18.

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