UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

WORKING PAPERS IN ECONOMICS

No 662

On-the-job search and city structure

Aico van Vuuren

June 2016

ISSN 1403-2473 (print)
ISSN 1403-2465 (online)

Department of Economics
School of Business, Economics and Law at University of Gothenburg
Vasagatan 1, PO Box 640, SE 405 30 Göteborg, Sweden
+46 31 786 0000, +46 31 786 1326 (fax)
www.handels.gu.se   info@handels.gu.se

EFMD
EQUIS
ACCREDITED

On-the-job search and city structure*

Aico van Vuuren †

Version: February 2016

Abstract

This paper investigates an equilibrium search model in which search frictions are increasing with the distance to a city's central business district, allowing for on-the-job search and endogenous wage formation and land allocation. The findings suggest that the decentralized market results in a more segregated outcome than may be socially desirable. The externality comes from the misguided incentives for the low-paid workers, who have a high preference for central locations in order to climb up the job ladder. Policies reducing the rental costs of unemployed workers for locations close to the central business district may potentially increase welfare.

Keywords: Search, city structure, urban economics

JEL-codes: J00, J64, R14

*We would like to thank Yves Zenou, Florian Sniekers, Jos van Ommeren as well as participants of the Search and Matching Conference in Edinburgh and the workshop on labor mobility in Louvain-la-Neuve in particular Paul Beaudry and Etienne Wasmer for useful comments.

†University of Gothenburg.

1 Introduction

This paper investigates the structure of cities, extending the mono-centric city model of Wasmer and Zenou (2002) by including on-the-job search and endogenous wage formation. Wasmer and Zenou (2002) found that there are two mutually exclusive equilibrium city structures in the absence of on-the-job search and assuming ex-ante homogeneous workers. In the first city structure, unemployed workers live as close as possible to the central business district (CBD), while employed workers live farther away. This equilibrium, labeled as the "integrated equilibrium", exists whenever the level of search frictions is low. Another city structure, labeled as the "segregated equilibrium", occurs in the case of high search frictions. In that case, the unemployed live as far away as possible from the CBD (and hence pay low rents), while the employed workers are centrally located. This paper finds that these two equilibria are special cases of a large range of equilibria characterized by the location of the unemployed. Our findings suggest that neither an integrated nor a segregated city is formed when the distance-dependent search efficiency of employed workers is identical to that of unemployed workers (while their commuting costs are low). Instead, we obtain an internal solution of the decentralized market in which low-paid workers are located the closest to the CBD, while high-paid workers are located the farthest away. The unemployed are located in between these two groups of workers.

The decentralized market outcome is found to be inefficient under very general circumstances. The externality comes from the misguided incentives given to the low-paid workers. These workers prefer to locate themselves close to the CBD in order to obtain a job in which they receive a higher wage. However, since our basic analysis assumes homogeneity in worker productivity and firm matches, social welfare is not affected by a job-to-job transition, but only by a transition of a worker from unemployment into a paid job. The social planner thus prefers a situation in which unemployed workers are located at a closer distance to the CBD than the market would allocate. In terms of the location decision, worker incentives are aligned with the maximization of social welfare only when wages are equal to worker productivity. In this case, the monopsony power of the firms is the single determinant for the externality. However, since this monopsony power is also necessary in the first place for firms to open vacancies and hire workers, the market can never result in an efficient outcome. Our model features a robustness check whereby the assumption of homogeneneity in the productivity of job matches is shown not to be essential to obtain this result.

We close our model by assuming that firms post wages as in Burdett and Mortensen (1998). Most of our results are not dependent on this assumption, while the remaining results can be adapted to other wage-setting mechanisms, conditional on the assumption that monopsony wages are paid. Gautier et al. (2010) identified one good reason to focus on wage-posting models. Abstracting from endogenous land allocation, they look at many different wage-setting mechanisms (such as wage-posting and wage-bargaining models), and conclude that wage posting is the only framework in which the market outcome can be constrained efficient. As we show in this paper that even wage posting can never result in an efficient allocation, our inefficiencies must be due to endogenous land allocation.

Our paper makes a case for the presence of subsidized housing for the unemployed workers. Although rewarding unemployed workers for living closer to the CBD may increase social welfare, such subsidies should be based on employment status and not on earned income, and should terminate directly after the acceptance of a job. We also find that the potential gains for such a policy are limited: our calibration exercises indicate that the inefficiencies that result from endogenous land allocation are only around 1 percent of the total loss of production due to search frictions. Nevertheless, the potential regional consequences of such a policy are enormous. We find, in some cases, that the decentralized market chooses a city structure that is almost completely segregated, while the optimal city structure turns out to be an integrated city structure.

Like the original paper by Wasmer and Zenou (2002), our paper is highly dependent on the assumption that workers face lower access to jobs when residing at a greater distance from those jobs. There is ample evidence that even during the age of the internet, workers are reluctant to apply for jobs that are farther away from them. For example, Marinescu and Rathelot (2015) use data from CareerBuilder.com to show that the vast majority of job seekers send applications for jobs that are no farther than ten miles from their present residence. In their study of the United Kingdom, Manning and Petrongolo (2015) find an even higher bias towards the present residence.

Our paper is closely related to Kawata and Sato (2012), who also extend the model of Wasmer and Zenou (2002) by including on-the-job search. There are two important differences. First, we use other assumptions on how wages are offered to workers.[1] Second, they assume search effort to be independent of distance to the CBD, which implies that unemployed workers are always located the farthest away from the CBD (due to the higher commuting costs of the employed workers). This also implies that our welfare results concerning the location of the workers do not exist in their model. Our paper is also related to the work of Smith and Zenou (2003), who extend the model of Wasmer and Zenou (2002) by endogenizing search effort. They show that this extension can result in an equilibrium in which the unemployed live in both central areas as well as areas close to the city border. However, their analysis does not include on-the-job search,

[1] In particular, they use the competitive search framework of Garibaldi and Moen (2010) and the directed search framework of Menzio and Shi (2011).

and features exogenous wage formation.

The remainder of this paper is as follows. Section 2 sets up the model and Section 3 looks at the partial equilibria of the labor and housing market. Section 4 analyzes the general equilibrium. Section 5 looks at the social planner, while Section 6 calibrates the model. Section 7 discusses our results and looks at potential extensions; Section 8 concludes.

2 The Model

2.1 General notation and assumptions

We assume that the total number of workers equals unity and that workers are uniformly located along a linear, closed and mono-centric city.[2] Time is continuous and land is owned by absentee landlords. We define μ as the total number of matches per unit of labor supply; u is the unemployment rate and v is the total number of vacancies. Moreover, we define d, with $0 \leq d \leq 1$ as the distance to the CBD. Let $s(d)$ be the search efficiency of an unemployed worker at distance d. As in Wasmer and Zenou (2002), we assume the following function:

$$s(d) = s_0 - ad, \tag{1}$$

where s_0 and a are relative efficiency parameters and $s_0 \geq a$, since otherwise some workers would have negative search efficiency. Moreover, we assume that the search efficiency of the employed workers at distance d equals $\psi s(d)$, $0 \leq \psi \leq 1$. The parameter ψ is relative search efficiency of employed workers. Workers and jobs arrive according to a Poisson process.

We assume a general expression for the contact technology between job seekers and vacancies (Gautier et al., 2010):

$$\mu = \alpha \left[s(\overline{d}_u)u + (1-u)\psi s(\overline{d}_e) \right]^{\nu} v^{\xi}, \tag{2}$$

[2]see Zenou (2009) for an explanation of these terms.

where $\overline{d}_u$ and $\overline{d}_e$ are the average distance to the CBD of, respectively, the unemployed and employed workers. The parameters ν with $0 \leq \nu \leq 1$, and ξ with $0 \leq \xi \leq 1$, measure the relative contributions of job seekers and vacancies to the contact technology. The parameter α measures the overall efficiency of the matching process. Let $\lambda := \mu / \left[s\left(\overline{d}_u\right)u + (1-u)\psi s\left(\overline{d}_e\right)\right]$. The implied contact rates for unemployed and employed workers are $\lambda s(d)$ and $\lambda \psi s(d)$.

Vacancies can be opened by firms with a per-period cost equal to γ. We assume $\tau \geq 0$ to be the commuting costs at distance d from the CBD for employed individuals. Without loss of generality, we set the transportation costs of the unemployed equal to zero. In addition, we denote the land rent at distance d as $R(d)$ and denote $R(1) = R_A$ as the exogenous rental costs of agricultural land. We assume that workers produce b in case of unemployment and p in case of being matched to an employer. We assume that $p - b > \tau$, which allows us to obtain sensible equilibria in which unemployed workers accept jobs. Wages are denoted by w. The wage-offer distribution of firms is denoted by F. We denote the distribution of wages among employed workers by G and we assume that matches are destroyed with an exogenous job destruction rate equal to δ.

2.2 Workers

Define $V_u(d)$ as the lifetime discounted value of a worker who is unemployed and living at distance d from the CBD and denote $V_e(d, w)$ as the lifetime discounted value of an employed worker also living at distance d from the CBD and working at a wage equal to w. We obtain the following Bellman equations:

$$\rho V_u(d) = b - R(d) + \lambda s(d) \int_{\varphi}^{w^+} \left(\max_{d'} V_e(d', w) - V_u(d) \right) dF(w), \tag{3}$$

and

$$\begin{aligned}\rho V_e(d,w) =& w - \tau d - R(d) + \lambda\psi s(d)\int_w^{w^+}\left(\max_{d'} V_e(d',x) - V_e(d,w)\right) dF(x) \\ &+ \delta(\max_{d'} V_u(d') - V_e(d,w)),\end{aligned} \tag{4}$$

where φ is the reservation wage of the unemployed and w^+ is the upper bound of the support of the wage-offer distribution. Every period, unemployed workers receive their home production minus their rental costs. In addition, they have the possibility of receiving a job offer (with a rate equal to $\lambda s(d)$); that job offer is accepted whenever the value of accepting that job offer is larger than the value of rejecting it. Employed workers receive their wages minus the sum of the transport and rental costs. They also have the possibility of receiving a job offer and – if a job offer is received, they accept it when the value is higher. Finally, they have the possibility of losing their jobs; in that case, they receive the value of an unemployed worker.

2.3 Firms

We define $V_v(w)$ as the value of a firm that opens a vacancy that pays w in the case of a match; $V_j(w)$ is the value of a match that pays wage w. We obtain

$$\rho V_v(w) = \frac{\lambda}{v}\left(us\left(\overline{d}_u\right) + (1-u)\psi\int_{\varphi}^{w} s(\overline{d}_e(x))dG(x)\right) V_j(w) - \gamma, \tag{5}$$

where $\overline{d}_e(w)$ is defined as the average distance to the CBD of an employed worker that earns w. The right-hand side of equation (5) can be explained as follows: the value of a vacancy equals the rate at which a match is formed multiplied by the value of a match, *i.e.* $V_j(w)$. In addition, we have to subtract the costs of keeping a vacancy open, *i.e.* γ. A match is formed when the firm connects with either an unemployed worker or an employed worker who earns less than w. Hence, the rate at which a match is formed is the sum of the rates of these two events. Taking into account that matching is random, these rates can be shown to equal $\lambda us(\overline{d}_u)/v$ and

$\lambda(1-u)\int_{\varphi}^{w}\overline{d}_e(x)dG(x)/v$ for unemployed and employed workers. The value of a match can be calculated by its Bellman equation; after rewriting, we obtain

$$V_j(w)=\frac{p-w}{\rho+\delta+\lambda\psi s(\overline{d}_e(w)))(1-F(w))}.$$

Hence, the value of a match equals the properly discounted flow of profits, taking into account that a match can be ended by either exogenous reasons (occurring at a rate equal to δ), or because the worker finds a better match (occurring at a rate equal to $\lambda\psi s(\overline{d}_e(w))(1-F(w)))$.

2.4 Wage-setting mechanism

In line with Burdett and Mortensen (1998), we assume that firms post wages to workers, implying that in the case of homogeneous firms, all firms should have the same expected pay-off at the moment that they open a vacancy (*i.e.* $V^j(w)$ should be independent from the wage in equilibrium). This setup implies that the wage-offer distribution has three important properties: (i) it has no mass points, (ii) the lowest offered wage equals the reservation wage of the unemployed workers, (iii) $w^+ < p$. The proofs of these claims are standard and can be found in other papers (Burdett and Mortensen, 1998; Gautier et al., 2010, among others).

3 Partial equilibrium

3.1 Partial equilibrium at the housing market

As in Wasmer and Zenou (2002), we can use the condition that, in the absence of relocation costs, unemployed workers should have the same value independent of the distance to the CBD. Otherwise, some unemployed workers might be able to relocate themselves and gain in terms of utility – and this situation cannot be an equilibrium. Hence, we stipulate that $V_u(d)=\overline{V}_u$ for the value of the unemployed. Likewise, we stipulate that employed workers with a wage w should receive the same value as well (*i.e.* $V_e(d,w) = \overline{V}_e(w)$). Moreover, define the bid-rent function, $\Psi_u(d)$, as the maximum rent that an unemployed worker is able to pay for residing at distance d from the CBD in order to obtain the value $\overline{V}_u$. Likewise, we define the bid-rent function $\Psi_e(d,w)$ for the employed workers.[3] As an equilibrium condition for the housing market, we stipulate that $R(d) = \max_{w\in[\varphi,w^+]}\{\Psi_u(d), \Psi_e(d,w)\}$, that is, the rent equals the maximum bid of the workers. Based on equation (3),

$$\Psi_u(d) = b + \lambda s(d) \int_{\varphi}^{w^+} \left(\overline{V}_e(w) - \overline{V}_u\right) dF(w) - \rho\overline{V}_u.$$

Likewise, define the bid-rent function $\Psi_e(d,w)$. Based on equation (4), we obtain

$$\begin{aligned} \Psi_e(d,w) =& w - \tau d + \lambda\psi s(d) \int_{w}^{w^+} \left(\overline{V}_e(x) - \overline{V}_e(w)\right) dF(x) \\ &+ \delta(\overline{V}_u - \overline{V}_e(w)) - \rho\overline{V}_e(w). \end{aligned} \tag{6}$$

Note that the bid-rent functions are not all that interesting on their own, because the maximum amount a person is willing to pay depends on the amount that she has to pay for alternative locations. In addition to that, the bid rents depend on the unknown equilibrium values $\overline{V}_u$ and $\overline{V}_e(w)$. Therefore, the derivatives of the bid rents with respect to d are more valuable for the analysis. First, we find that the higher the absolute values of these derivatives, the more centrally located are the individuals. Second, it turns out that we are able to write these derivatives of the bid-rent functions as independent of the unknown values of $\overline{V}_u$ and $\overline{V}_e(w)$. Therefore, we concentrate on the derivatives of the bid-rent functions rather than on the bid-rent functions themselves. Note that the bid-rent function can be derived from its derivative as the solution to a differential equation.

[3]Since the concept of bid rents is standard in urban economics, we do not elaborate further on it. See Fujita (1989) and Zenou (2009). Also note that we use somewhat abusive notation, since Ψ_u also depends on $\overline{V}_u$ and $\overline{V}_e(w); \varphi \leq w \leq w^+$.

The (partial) derivatives of the bid rents, Ψ_u and Ψ_e with respect to d, can be obtained by taking derivatives of these values, *i.e.*

$$\Psi_u'(d) = -a\lambda \int_{\varphi}^{w^+} \left(\overline{V}_e(w) - \overline{V}_u\right) dF(w), \tag{7}$$

which is negative. It is easier to interpret this first-order derivative by looking at its absolute value, which equals the amount that an unemployed worker is willing to pay for a more central location. Looking at the absolute value of the right-hand side, this willingness equals the increase in the rate at which a worker receives additional job offers when moving to a more central location (*i.e.* $a\lambda$) times the average gain received from such an offer. Likewise, we can construct the first-order derivative of $\Psi_e(d, w)$

$$\frac{\partial \Psi_e(d, w)}{\partial d} = -\tau - a\lambda\psi \int_{w}^{w^+} \left(\overline{V}_e(x) - \overline{V}_e(w)\right) dF(x), \tag{8}$$

which is also negative. The absolute value of the right-hand side of this equation equals the reduction in commuting costs plus the increase in the rate at which a worker receives additional job offers (*i.e.* $a\lambda\psi$) times the average gain received from such an offer. As in Wasmer and Zenou (2002), the second-order derivatives with respect to d equal zero. The cross-partial derivative equals

$$\frac{\partial^2 \Psi_e(d, w)}{\partial d \partial w} = a\lambda\psi \frac{\partial \overline{V}_e(w)}{\partial w}(1 - F(w)), \tag{9}$$

which is positive for all $w < w^+$. Workers with higher wages have less to gain by moving closer to the CBD, since the likelihood of obtaining even better job offers declines with the wage level. The distribution of land is now determined by the workers who have the highest bid rent. Let $\mathcal{W}(d)$ be the set of wages paid to workers with distance d, or

$$\mathcal{W}(d) = \left\{ w | \forall w' \in [\varphi, w^+] : \Psi_e(d, w) \geq \Psi_e(d, w') \wedge \Psi_e(d, w) \geq \Psi_u(d) \right\}.$$

Note that this set can be empty in the case that the willingness to pay for the unemployed is strictly larger. In addition, $\mathcal{W}(d)$ should have measure zero, since d has measure zero. Hence, $\mathcal{W}(d)$ cannot be an interval. The following lemma states that the sets must be strictly increasing in d.

Lemma 1 *Suppose that we have $d_1, d_2 \in [0,1]$ and $d_1 < d_2$. Then, for any $w_1 \in \mathcal{W}(d_1)$ and $w_2 \in \mathcal{W}(d_2)$ we have $w_1 < w_2$.*

Proof: See Appendix A. ■

A direct result of Lemma 1, together with the non-interval restriction of $\mathcal{W}(d)$, is that $\mathcal{W}(d)$ is single valued. We therefore denote $w(d) = \mathcal{W}(d)$, using $d(w) = \overline{d}_e(w)$ for its inverse. After partially integrating (7) and (8) and taking the derivatives of the left- and right-hand sides of equation (4), we obtain the following first-order derivative of the bid-rent function for unemployed and employed workers:[4]

$$\Psi'_u(d) = -a\lambda \int_{\varphi}^{w^+} \left[\frac{1-F(w)}{\rho+\delta+\lambda\psi s(d(w))(1-F(w))}\right] dw, \tag{10}$$

and

$$\frac{\partial \Psi_e(d,w)}{\partial d} = -\tau - a\lambda\psi \int_{w}^{w^+} \left[\frac{1-F(x)}{\rho+\delta+\lambda\psi s(d(x))(1-F(x))}\right] dx. \tag{11}$$

We define the set of distances at which the unemployed workers live from the CBD by $\mathcal{D}_u$. The introduction of $\mathcal{D}_u$ has an implication for $w(d)$, because it is not defined for any $d \in \mathcal{D}_u$. Therefore, we define $w(d) = \mathrm{supp}_{d'<d; d' \notin \mathcal{D}_u} w(d')$ for any $d \in \mathcal{D}_u$. Another result of Lemma 1 is that we can write $R(d) = \Psi_e(d, w(d))$; hence, the first-order derivative of the rents equals

$$R'(d) = \frac{\partial \Psi_e(d,w(d))}{\partial d} + w'(d)\frac{\partial \Psi_e(d,w(d))}{\partial w} = \frac{\partial \Psi_e(d,w(d))}{\partial d} \tag{12}$$

[4]Note that $\overline{V}'_e(w) = \frac{\partial V_e(w,d(w))}{\partial w} + \frac{\partial V_e(w,d(w))}{\partial d} d'(w) = \frac{\partial V_e(w,d(w))}{\partial w}$, where the last equality holds due to the first-order condition of V_e with respect to d.

for any $d \notin \mathcal{D}_u$. The second equality is a direct result of the first-order condition of $\max_w \Psi_e(d, w)$. Again, it is easier to interpret $R'(d)$ when using its absolute value, which equals the additional rent that needs to be paid when moving to more central locations. Hence, equation (12) states that this increase in rents should equal the additional amount that employed workers are willing to pay for being more centrally located on any location where employed workers are living. The following lemma makes a statement about $\mathcal{D}_u$.

Lemma 2 *$\mathcal{D}_u$ is convex, with lower bound $\underline{d}_u$ and upper bound $\underline{d}_u + u$.*

Proof: See Appendix B. ■

The intuition behind Lemma 2 is based on the fact that $\rho V_u'(d)$ can be shown to equal $\Psi_u'(d) - R'(d)$, implying that when an unemployed worker relocates towards a more central location, her gains equal the additional amount that she is willing to pay for that location in comparison to her present location, minus the increase in rents that she has to pay for being more centrally located. The increase in rents equals exactly this additional amount for any distance at which the unemployed are living (since $V_u(d)$ must be constant there). Likewise, it equals $|\partial \Psi_e(d, w(d))/\partial d|$ on any other point. Based on the linearity of $s(d)$, we know that the willingness to pay for more central locations is constant. This willingness to pay for employed workers is strictly decreasing for any location where the employed workers are living and it is constant at the locations of the unemployed workers.[5] It implies that $V_u(d)$ is concave and even strictly concave for any distance where the employed workers are living. This concavity is a sufficient condition for the set at which $V_u(d)$ is maximized (*i.e.* $\mathcal{D}_u$) to be convex.

To explain the implications of this lemma, Figure 1 draws the absolute values of the first-order

[5] The derivative of $|\partial \Psi_e(d, w(d))/\partial d|$ equals $-w'(d)\partial^2 \Psi_e(d, w(d))/(\partial d \partial w)$, which is strictly negative due to (9) (unless $w'(d)$ is zero).

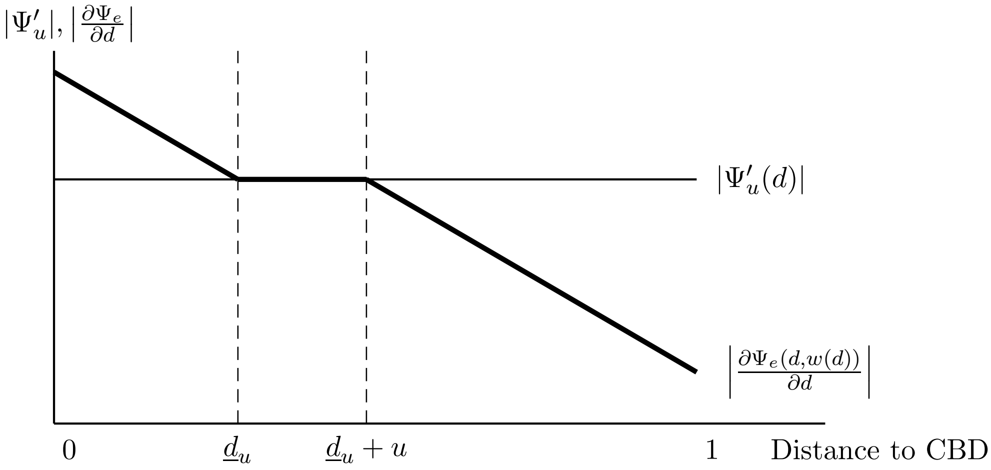

Figure 1: Illustration of the willingness to pay for more central locations

derivatives of the bid-rent functions. As stated above, $|\Psi'_u(d)|$ is constant, while $|\partial\Psi_e(d, w(d))/\partial d|$ decreases with d. Unemployed workers are never located to the right of $\underline{d}_u + u$, since for those distances they are always willing to pay more for more central locations than the employed workers are. In other words, the rent increase for more central locations is always lower than what unemployed workers are willing to pay for those locations for any point to the right of $\underline{d}_u + u$, and hence they are able to increase $V_u(d)$ by reducing d (*i.e.* $V'_u(d)$ is negative). The opposite can be said about the points to the left of $\underline{d}_u$. Hence, there are low-paid employed workers to the left of $\underline{d}_u$, high-paid employed workers to the right of $\underline{d}_u + u$, with the unemployed workers located in between these workers.

Lemma 2 implies that there is a one-to-one mapping between wages and distance. We can write $d(w)$ as

$$d(w) = \begin{cases} (1-u)G(w) & \text{if } w < \widetilde{w} \\ \\ u + (1-u)G(w) & \text{if } w \geq \widetilde{w} \end{cases}, \tag{13}$$

where $\widetilde{w} = G^{-1}\left(\underline{d}_u/(1-u)\right)$. Moreover, the following condition applies for any point $d \in \mathcal{D}_u$ (see

Figure 1):

$$\Psi_u'(d) = \frac{\partial \Psi_e(d, w(d))}{\partial d} = \frac{\partial \Psi_e(d, \widetilde{w})}{\partial d}, \tag{14}$$

where the last equality follows from the fact that $\widetilde{w} = w(d)$ for all $d \in \mathcal{D}_u$. Equation (14) states that employed and unemployed workers must have equal incentives to live closer to the CBD at any point in $\mathcal{D}_u$. Substitution of (10) and (11) into (14) results in

$$\begin{aligned}\tau = a\lambda(1-\psi) \int_{\varphi}^{w^+} &\left[\frac{(1-F(x))}{\rho+\delta+\lambda\psi s(d(x))(1-F(x))}\right] dx \\ + a\lambda\psi \int_{\varphi}^{\widetilde{w}} &\left[\frac{(1-F(x))}{\rho+\delta+\lambda\psi s(d(x))(1-F(x))}\right] dx,\end{aligned} \tag{15}$$

which gives a restriction for $\underline{d}_u$ since $\widetilde{w}$ depends on $\underline{d}_u$. Employed workers have two reasons to locate themselves closer to the CBD: a reduction in commuting time and an improvement in labor market opportunities. For unemployed workers, only the latter reason is relevant – but their improvement in labor market opportunities is larger than for employed workers. Hence, the reduction in commuting costs of employed workers who locate themselves closer to the CBD should be exactly equal to the difference in additional labor market opportunities for the unemployed, minus the labor market opportunities for the employed. This last difference can be subdivided into two components represented by the terms on the right-hand side of (15). The first term equals the difference due to the fact that employed workers have a lower probability of receiving job offers for any distance. The second term comes from the fact that not all job offers are acceptable for employed workers. Consider the possibility of

$$\tau < a\lambda(1-\psi) \int_{\varphi}^{w^+} \left[\frac{1-F(x)}{\rho+\delta+\lambda\psi s(d(x))(1-F(x))}\right] dx. \tag{16}$$

In that case, all unemployed workers allocate themselves closer to the CBD than all employed workers, and $\underline{d}_u = 0$. This is the extreme case that was called the "integrated city" by Wasmer and Zenou (2002). Note that this situation is impossible to obtain when $\psi = 1$ and τ positive.

This result is easily explained, since in such a case workers who receive the lowest wage have a higher willingness to pay for living at distance zero than do the unemployed workers. The intuition is that these workers have the same opportunities on the labor market as unemployed workers, but have higher commuting costs. Since $\partial\Psi_e/\partial w$ is continuous in w at $\underline{d}_u$, it implies that $\underline{d}_u$ must be positive. When $\tau = 0$, then the only possible outcome is the integrated city equilibrium. It is also possible that[6]

$$\tau > a\lambda \int_{\varphi}^{w^+} \left[\frac{1 - F(x)}{\rho + \delta + \lambda\psi s(d(x))(1 - F(x))}\right] dx. \tag{17}$$

In that case, all unemployed workers allocate themselves farther away from the CBD than all employed workers, and $\underline{d}_u = 1 - u$. This case was called the "segregated city" by Wasmer and Zenou (2002).

The final step is to determine the equilibrium land rents. We can use that $R'(d) = \partial\Psi_e(d, w(d)) /\partial d$ for all $d \notin \mathcal{D}_u$. Hence, the derivation of $R(d)$ between $\underline{d}_u + u$ and 1 results from solving the differential equation with initial condition $R(1) = R_A$. The solution of $R(d)$ between $\underline{d}_u$ and $\underline{d}_u + u$ is in line with Wasmer and Zenou and simply equals $R(\underline{d}_u + u) + \Psi_u'(\cdot)(d - \underline{d}_u - u)$. The differential equation for values between 0 and $\underline{d}_u$ is identical to the one for values between $\underline{d}_u + u$ and 1, but now with an initial condition for $R(\underline{d}_u)$. Further details of the derivation of the equilibrium land rents can be found in Appendix C.

Unlike the situation of Wasmer and Zenou (2002), our rent function is not linear for all $d \notin \mathcal{D}_u$, since a decrease in distance implies not only an improvement of search efficiency of the workers and a decrease in commuting costs but it also implies that the wage of the worker is lower when located closer to the CBD. Hence, such a worker has a higher likelihood of obtaining wage offers higher than the present wage than does the worker who lives a little farther away

[6]This inequality can be derived after equating (10) and (11) and using $w = \widetilde{w}$ and $\widetilde{w} > \varphi$ and $\psi < 1$.

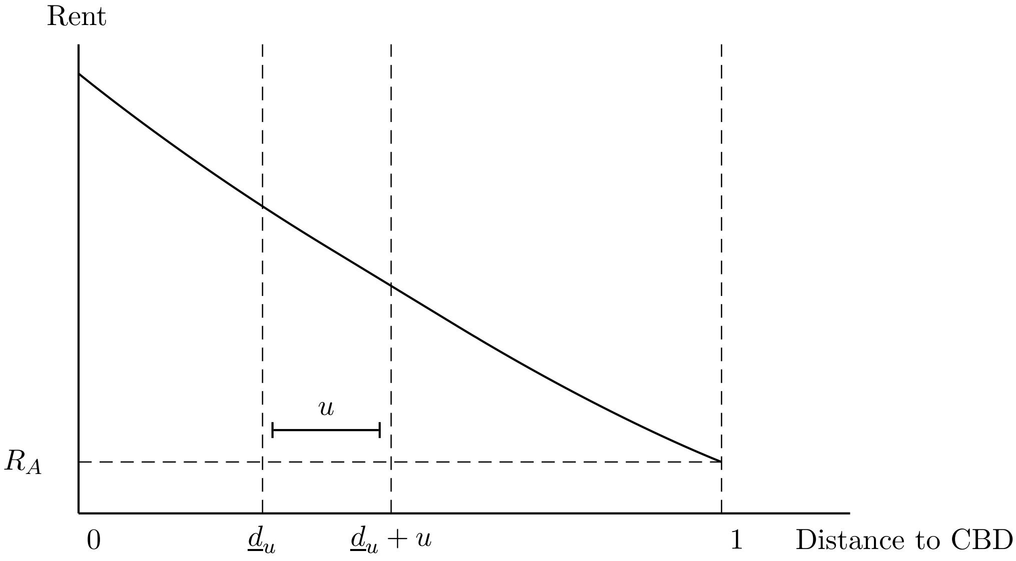

Figure 2: Illustration of the partial equilibrium at the housing market

from the CBD, even if both workers had been living at equal distance.

Figure 2 illustrates the partial equilibrium at the housing market. Rent equals R_A at distance 1 from the CBD and gradually increases when moving closer to the CBD. There is an increasing rate at which the rent increases when moving left in the figure, because $\partial\Psi_e(d, w(d))/\partial d$ increases with distance. The unemployed live in between $\underline{d}_u$ and $\underline{d}_u + u$ from the CBD. Rents are linear in this interval. Workers with low wages live somewhere between 0 and $\underline{d}_u$ and these workers pay the highest rents.

Comparing our results with those of Wasmer and Zenou (2002), we conclude that the introduction of on-the-job search makes it possible to explain many more equilibrium city structures than only the segregated and integrated city structures in a model without on-the-job search and exogenous search effort. Obviously, many cities, especially in the mainland of Europe, cannot be classified as either segregated or integrated and are more in line with the donut-shaped city outcome that we described in this section with employed workers at a short and high distance from the CBD and the unemployed in between these employed workers.[7] Of course, the cities in

[7]There is some empirical evidence for this statement: Gobillon et al. (2010) investigate the inner city of Paris

mainland Europe are also those with a high level of (historical) amenities, while their city structures are also highly influenced by social housing policies. Still, our paper makes one important prediction, and that is that when ψ is high, then it is more likely to obtain a donut-shaped city than when ψ is very low (in which case, only a segregated or integrated city can occur). We are aware of only one paper, Ridder and Van den Berg (2003), that tries to estimate ψ for a number of different countries using macro-data. They find ψ to be substantially lower in the United States and the United Kingdom, than in the Netherlands, Germany and France.

3.2 Partial equilibrium at the labor market

Equalizing in- and outflow of the unemployed results in the following steady-state level of unemployment:

$$(1-u)\delta = u\lambda s\left(\underline{d}_u + \frac{u}{2}\right). \tag{18}$$

Similarly, equalizing the in- and outflow of workers with a wage at least paying w results in

$$\delta(1-G(w))(1-u) = \lambda(1-F(w))\left\{us\left(\underline{d}_u + \frac{u}{2}\right) + (1-u)\psi\int_{\varphi}^{w} s(d(x))dG(x)\right\}, \tag{19}$$

where the left-hand side is the number of workers dismissed from a job that pays at least w. The right-hand side is the number of unemployed workers who find a job paying at least w plus the number of employed workers paid less than w who find such a job. Appendix D shows that conditional on the wage-offer distribution F, the function G can be solved as a root of a second-order polynomial.

and conclude that the highest unemployment rates are located close to the CBD. However, unemployment rates in the CBD itself are lower than the metropolitan average. Finally, Åslund et al. (2010) find that for Stockholm the highest job density can be found in the CBD and that unemployment rates are especially high in the suburban areas around that district.

The reservation wage is determined by

$$\rho V_u(\underline{d}_u) = \rho V_e(d(\varphi), \varphi). \tag{20}$$

Lemma 1 states that $d(w)$ is decreasing. From the second assumption of Subsection 2.4 we know that $d(\varphi)$ can only take two values: either it equals zero when $\underline{d}_u > 0$ or it equals u when $\underline{d}_u = 0$. Appendix E calculates these two cases.

3.3 Wage posting

Using Lemma 1 and equations (18) and (19), we see that the value of a vacancy that posts a wage w equals

$$\rho V_v(w) = \frac{1-u}{v}\delta\frac{1-G(w)}{1-F(w)}\frac{p-w}{\rho+\delta+\lambda\psi s(d(w))(1-F(w))} - \gamma. \tag{21}$$

Since all firms are homogeneous, it is necessary that $V_v(\varphi) = V_v(w)$, for all wages in between φ and $w^1 < \widetilde{w}$. Note that it can never be optimal to post a wage just below $\widetilde{w}$. This is because there is a discontinuity in the quit rate of workers at this wage level. Workers who are paid marginally below this wage live closer to the CBD than do unemployed workers, while workers who are paid exactly this wage or more live farther away from the CBD than unemployed workers do. This implies that the number of workers who quit their jobs because they find a better wage offer drops sharply at this wage level. This does not occur because of a mass point in the wage-offer distribution, but merely because the workers have a lower search intensity. It also implies that the value of a vacancy (*i.e.* $V_j(w)$) jumps upward at the wage level of $\widetilde{w}$ and that no firms will pay slightly below this wage. Therefore, a firm only considers paying below $\widetilde{w}$ when its profit margin per worker is sufficiently higher than the firms that pay above the threshold. The condition to be satisfied is $V_j(w^1) = V_j(\widetilde{w})$. The wage distribution above $\widetilde{w}$ can be obtained by using $V_j(\widetilde{w}) = V_j(w)$ for every w between $\widetilde{w}$ and w^+.

4 General equilibrium

We close our model by assuming that opened vacancies must have an expected profit equal to zero. Using this assumption and substituting $w = \varphi$ into equation (21), we obtain

$$\frac{1-u}{v}\delta\frac{p-\varphi}{\rho+\delta+\lambda\psi s(d(\varphi))} = \gamma. \tag{22}$$

For the general equilibrium, we need to determine the variables φ, u, $\underline{d}_u$ and v, the distributions F and G and the rent function R. The restrictions for this equilibrium are equations (15), (18), (19), (20) and (22), as well as the solutions for the land rent derived in Appendix C.

Unless otherwise stated, we assume for this section and the next that $\mu = \xi = \psi = 1$. Our main reason is that general results are extremely hard to obtain. There is also another reason to focus on this case: Gautier et al. (2010) show in a model without endogenous land allocation that among a large set of wage-setting mechanisms, only wage posting can result in an efficient outcome, as long as $\nu = \xi = \psi = 1$. The issue here is whether this result still holds for our extended model. Note that the literature on job search models without on-the-job search has not found a lot of evidence for an increasing-returns-to-scale matching technology (Petrongolo and Pissarides, 2001). Therefore, we also look at the constant-returns-to-scale matching technology in the calibration section.

Theorem 3 *An equilibrium with a positive number of vacancies exists whenever*

$$p - b > \frac{2\gamma(\rho+\delta)}{\alpha}\frac{1}{2s_0 - a}. \tag{23}$$

Proof: See Appendix F.■

Condition (23) is standard and states that we can expect a positive number of vacancies if there is a sufficient difference between production at the workplace and at home. Unfortunately, a general proof of uniqueness is difficult for this model, but we can prove uniqueness under the conditions of Lemma 4.

Lemma 4 *Suppose that $\rho/\delta \downarrow 0$ and (23) applies. Then, the equilibrium is unique.*

Proof: See Appendix G. ■

The condition that $\rho/\delta \downarrow 0$ is standard in job search models, but obviously debatable. It is possible to obtain the following very conservative sufficient condition

$$\frac{\rho}{\delta} < \frac{a\underline{d}_u}{s_0 - a\underline{d}_u}.$$

Note that the right-hand side of this restriction is the percentile loss in the search efficiency of the most efficient unemployed worker in comparison to her search efficiency at distance zero from the CBD. To check whether this restriction is satisfied for reasonable values of ρ, s_0, a and δ, we look at estimates obtained from Ridder and Van den Berg (2003). They estimate δ to be in between 5 percent for Germany and 42 percent for the United States. Recent estimates of the discount rates indicate figures as low as 2 percent (see Laibson et al., 2007, and Gautier and Van Vuuren, 2015). It implies that in the United States, workers located in the CBD should be at least about 2 percent more efficient in their search as the unemployed for the condition to be satisfied. In Germany, unemployed workers should have at least a 40 percent lower search efficiency than the workers located in the CBD. Note that the restriction is conservative, since it is unlikely to hold whenever a is very small – whereas our model with $a = 0$ has been proven to have a unique equilibrium under very general conditions.

5 Welfare analysis

In line with Zenou (2009), we define the social welfare function as

$$\begin{aligned}\Omega(v,\underline{d}_u) &= \int_0^\infty \left((1-u(t))p + u(t)b - \gamma v - (1-u(t))\tau\overline{d}_e\right) e^{-\rho t}dt \\ &\equiv \int_0^\infty \omega_t(v,\underline{d}_u)e^{-\rho t}dt.\end{aligned} \tag{24}$$

The first term between brackets is production of the employed workers, the second term is production of the unemployed homeworkers, the third term is total costs for vacancies and the last term is the average commuting costs of the employed workers. We consider the situation in which the social planner faces the same search frictions as the decentralized market, and investigate the outcome when the social planer is able to maximize welfare by choosing v and $\underline{d}_u$. This implies that the social planner has as state variable u with

$$\frac{du}{dt} = \delta(1-u(t)) - \lambda u(t)s\left(\underline{d}_u + \frac{u(t)}{2}\right). \tag{25}$$

We show in Appendix H that the first-order derivative of Ω with respect to $\underline{d}_u$ is strictly positive whenever

$$\tau > \lambda a \frac{p-b}{\rho+\delta+\lambda s_0}, \tag{26}$$

assuming that either ν in (2) or ψ equals one, while it is strictly negative in the case when the >-sign is replaced by a <-sign. This implies that, with the exception of equality of the left- and the right-hand sides, the social planner prefers either $\underline{d}_u = 0$ or $\underline{d}_u = 1-u$. This can be explained as follows: suppose that the social planner is able to move the unemployed workers somewhat closer to the CBD – say, by distance Δ. The social planner benefits from this move, because of the reduction in the equilibrium unemployment rate (*i.e.* $u'(\overline{d}_u)\Delta$) times the difference in production between employed and unemployed workers ($p-b$). Second, these newly employed workers increase total commuting costs (*i.e.* $u'(\overline{d}_u)\tau\overline{t}_e\Delta$). Third, the commuting costs further increase by $\tau u\Delta$, because the existing employed workers have to commute a greater distance (since there are u unemployed workers that move a distance Δ closer to the CBD and hence also u employed workers should move the same distance from the CBD). It can be shown that the first term is proportional to $(p-b)a\lambda$, while the last two terms are proportional to $(\rho+\delta+\lambda s_0)\tau$, resulting in (26).

From the discussion following equation (16), we know that when $\psi = 1$, the decentralized market always implies that $\underline{d}_u > 0$, even when the employed have very low commuting costs. This result automatically implies the inefficiency of the decentralized market outcome, including wage posting. Still, the question remains whether the social planner would prefer a more integrated city structure than that obtained by the market. We arrive at the following result.

Lemma 5 *Suppose that* $\rho < (a(1-u)/(s_0 - a(1-u)))\delta$. *Suppose that the social planner prefers the segregated city to be the preferred outcome. Then the segregated city is also the outcome of the decentralized market.*

Proof: see Appendix I.■

The condition in Lemma 5 is conservative. Our simulation results indicate that the results presented in this lemma are true even when the condition is not satisfied. The proof behind the lemma is based on the fact that for a fixed level of λ (and hence the number of vacancies), the location incentives of the workers are only in line with the policy maker when workers are paid their marginal productivity. In contrast, unemployed workers are less willing to pay for closer locations if the wages are lower than their marginal productivity, while the willingness of employed workers to pay for these locations is higher. Hence, in the market equilibrium, employed workers live too close to the CBD. This result is intuitive and implies that workers do not make socially optimal location decisions when not fully compensated for their search activities. It also implies that any model in which workers are paid monopsony wages can be expected to make this prediction. Among these are wage-bargaining models and models in which firms are allowed to make counteroffers (such as in Postel-Vinay and Robin, 2002). The only reason why we still need the assumption of wage posting is to allow for endogenous vacancy creation as well. We can show in our case of wage posting that the unemployment rate is higher under the market than under the social planner.[8] We show in Appendix I that this difference further enlarges the gap between the outcomes of the social planner and the market.

The fact that unemployment is higher under the market outcome is in line with previous results. Gautier et al. (2010) find that the unemployment rate under wage posting and $\xi = \nu = \psi = 1$ is equal for the market and the planner. This result implies that the wages have exactly the level under wage posting to obtain the right amount of vacancies. In our case, however, wages are higher and hence unemployment is higher, since firms have an additional reason to offer higher wages. In the standard wage-posting framework, firms offering higher wages reduce the likelihood for their workers to receive better outside offers. In our case, firms that offer higher wages also reduce the number of these outside offers, since better-paid workers live farther away from the CBD.

A couple of instruments may be used to restore the housing decisions of the decentralized market. We propose here an instrument that reduces the costs of unemployed workers to live closer to the CBD. The introduction of such an instrument implies that condition (15) becomes

$$\tau - \chi = a\lambda \int_{\varphi}^{\widetilde{w}} \left[\frac{1 - F(x)}{\rho + \delta + \lambda s(d(x))(1 - F(x))} \right] dx$$

for the case that $\psi = 1$, where χ is the rent reduction received by the unemployed worker when

[8] This statement is formalized in Lemma 6.

she lives one unit closer to the CBD. It implies that she receives $(1-d)\chi$ when she lives at distance d from the CBD. This policy instrument can restore the location decisions made by the decentralized market. Note that this policy instrument is exactly the same instrument as the single effective instrument considered by Smith and Zenou (2003).

6 Simulations of the model

We simulate our model using the same set of parameter values as Wasmer and Zenou (2002), who use a bargaining model with the bargaining power equal to 0.5 and assume that there is no on-the-job search (*i.e.* $\psi = 0$). Hence, $\delta = 0.1$, $\rho = 0.05$, $\tau = b = 0.3$, $\alpha = p = 1$, $\xi = \nu = 0.5$ and $\gamma = 0.3$. In addition, we set $\psi = 1$ for the model with on-the-job search. The simulation results are listed in panel A of Table 1. For the sake of completeness, we replicate the results of Wasmer and Zenou and report the optimal welfare for both models.[9] Although welfare in the Wasmer and Zenou (2002) model is optimal, given both the city structure and model parameters, a suboptimal city structure can be chosen by the market. In fact, equation (26) is also the condition for the social planner to choose the segregated city structure, while Wasmer and Zenou (2002) show that the market chooses this city structure whenever

$$\tau > \frac{\gamma a v}{us(\underline{d}_u + 1/2)}.$$

Our simulations show that the market chooses a suboptimal city structure only when $a = 0.5$ and the loss in welfare is small.

[9]See also page 531 of Wasmer and Zenou, 2002. The only difference in our table is welfare, since we assume absent commuting costs for the unemployed.

	Wasmer and Zenou						On-the-job search					
a	u	v	$\underline{d}_u$	$s(\overline{d}_u)$	ω_t	$\omega_t^{*\,a}$	u	v	$\underline{d}_u$	$s(\overline{d}_u)$	ω_t	$\omega_t^{*\,a}$
A. $\nu = \xi = 0.5, \alpha = 1$												
1	0.0686	0.131	0	0.966	0.763	0.763	0.139	0.246	0.049	0.881	0.684	0.693
0.75	0.0685	0.130	0	0.974	0.764	0.764	0.153	0.258	0.106	0.864	0.674	0.684
0.6	0.0685	0.129	0	0.979	0.764	0.764	0.230	0.247	0.612	0.564	0.665	0.680
0.55	0.0685	0.129	0	0.981	0.764	0.764	0.252	0.238	0.748	0.519	0.668	0.679
0.5	0.121	0.120	0.879	0.530	0.763	0.764	0.236	0.299	0.759	0.560	0.674	**0.680**
0.25	0.100	0.106	0.894	0.763	0.776	**0.776**	0.202	0.226	0.798	0.775	0.695	**0.702**
0.1	0.0915	0.100	0.900	0.905	0.782	**0.782**	0.186	0.219	0.813	0.909	0.704	**0.712**
B. $\nu = \xi = 1, \alpha = 8$												
1	0.0806	0.149	0	0.959	0.750	0.750	0.0786	0.157	0.027	0.934	0.749	0.750
0.75	0.0805	0.147	0	0.970	0.750	0.751	0.0786	0.157	0.047	0.935	0.750	0.751
0.6	0.0804	0.146	0	0.976	0.750	0.751	0.0799	0.156	0.091	0.922	0.750	0.751
0.55	0.0804	0.146	0	0.977	0.751	0.751	0.0820	0.156	0.142	0.899	0.750	0.751
0.5	0.1458	0.137	0.854	0.536	0.710	0.751	0.1057	0.137	0.543	0.702	0.749	0.751
0.25	0.1188	0.121	0.881	0.765	0.733	**0.764**	0.1098	0.133	0.890	0.763	0.764	**0.764**
0.1	0.1082	0.114	0.891	0.905	0.742	**0.771**	0.0996	0.125	0.900	0.905	0.771	**0.771**

[a]Numbers in bold imply that the planner chooses a segregated city.

Table 1: Calibration results. The variable ω_t^* is the per period optimal welfare.

Comparing both models, it becomes immediately clear that the unemployment rates are higher in our model with on-the-job search. This result is not surprising, given the fact that we use a constant-returns-to-scale contact technology, which implies that the search efficiency of the unemployed is frustrated by the search activities of the employed workers. Since the vacancy rate is higher under on-the-job search, it also implies that welfare should be lower. More surprising is the result that welfare increases for values of a between 0.5 and 1. This result is also related to the choice of a constant-returns-to-scale contact technology – since even the social planner is not able to obtain higher welfare levels when a becomes smaller. Thus, a lower level of a also makes search more efficient for the employed workers, thereby reducing the opportunities of the unemployed in such a way that it is even possible that more vacancies are needed in order to obtain the same level of unemployment.

Note that for the model of on-the-job search only the simulations with $a < 0.6$ result in a completely segregated labor market (where $\underline{d}_u = 1 - u$), while all other cases result in a mixed city in which the unemployed are living in between the low-paid and high-paid employed workers.

Comparison between the two models is complicated by the constant-returns-to-scale matching function, since employed workers congest the labor market for unemployed workers, thereby considerably reducing their contact rate. Therefore, we also look at a model with an increasing-returns-to-scale matching function, where $\nu = \xi = 1$. All other parameters are the same – apart from α, which is set to 8 here in order to obtain reasonable unemployment rates. The results are presented in panel B of Table 1. The calibration results of the two models are very similar for a up to 0.5. For smaller values, the unemployment rate for the model with on-the-job search is somewhat lower and welfare is somewhat higher than under the model without on-the-job search,[10] implying that the high unemployment rates in Panel A were due completely to

[10]Note that the Hosios condition of the model without on-the-job search no longer holds, implying that welfare

congestion. This result is also true for the difference between the optimal and market welfare. Welfare can be interpreted here as the realized percentage of total potential output in the case without any search frictions and commuting costs. This interpretation implies that the market loses about 25 percent of that potential production when $a = 1$ and that the social planner is able to reduce that loss only by less than half a percent (*i.e.* 0.001 / 0.25). The welfare loss due to suboptimal location choices of the workers is therefore quite small. Hence, a policy to reduce the loss of efficiency due to suboptimal locations (such as the rent subsidy for the unemployed discussed in the previous section) cannot have a large impact.

7 Discussion and extensions

The main contribution of this paper is the observation that a social planner (weakly) prefers a more integrated city than the city structure that is formed by the decentralized market. This result is easily explained in a model in which there are homogeneous jobs, since in such a model on-the-job search does not add any value to social welfare. The social planner is therefore only interested in the commuting costs of employed workers and the job-finding probabilities of unemployed workers. Hence, one of the necessary restrictions to obtain this outcome in the market is that employed workers also only consider their commuting costs when making decisions about their location. However, since there are differences in wages between identical jobs, employed workers also look at the potential to receive a higher wage offer – and this externality explains our inefficiency result.

The question arises whether this result is robust when we loosen the assumption of homogeneity in jobs. We therefore look in this subsection at a stochastic job-matching model in the tradition of Pissarides (2000, chapter 6). In order to ease notation, we assume that p follows a

within a city structure may no longer be optimal.

standard uniform distribution (although our results can easily be extended to any distribution). It has been shown in earlier work that wages are ranked according to their productivity; hence, workers can only have different wages if they also have different levels of productivity (Gautier et al. (2010) provide a formal proof).

We denote the productivity distribution of employed workers by H. In addition, we denote $K(p)$ for the wage that is paid for a match that yields productivity level p. Note that this function is strictly increasing, since wages are ordered based on their productivity. Hence, $F(K(p)) = p$ and $G(K(p)) = H(p)$, where F and G still follow the definitions of the earlier sections. Using the same techniques as in the standard version of our model, we can use bid rents to determine the location of unemployed workers. We thus rewrite equation (15) to obtain

$$\begin{aligned} \tau = a\lambda(1-\psi) \int_{\underline{p}}^{1} K'(x) \left[\frac{1-x}{\rho + \delta + \lambda\psi s(d(x))(1-x)} \right] dx \\ + a\lambda \int_{\underline{p}}^{\widetilde{p}} K'(x) \left[\frac{1-x}{\rho + \delta + \lambda\psi s(d(x))(1-x)} \right] dx, \end{aligned} \tag{27}$$

where $\widetilde{p} = H^{-1}(\underline{d}_u/(1-u))$ and $\underline{p}$ is the lowest productivity level accepted by unemployed workers. The welfare function of the social planner becomes

$$\Omega(v, \underline{d}_u, \underline{p}) = \int_0^\infty \left\{ (1-u(t))\mathrm{E}_H p + u(t)b - v(t)\gamma - \overline{d}_e\tau \right\} e^{-\rho t} dt.$$

Solving the planner's problem is complicated by its dependence on the equilibrium distribution H. Therefore, we follow an alternative: in Appendix J we show that for any given value of λ, we have

$$\Omega(v, \underline{d}_u, \underline{p}) = \rho V_u^*(\underline{d}_u + u/2) = b + \lambda s(\underline{d}_u + u/2) \int_{\underline{p}}^{1} \frac{1-p}{\rho + \delta + \lambda\psi s(d(p))(1-p)} dp, \tag{28}$$

whenever $\rho/\delta \downarrow 0$ and where V_u^* is the value of the unemployed workers when everybody is paid according to their productivity level. In addition, we prove in Appendix K that

$$\Omega(v, \underline{d}_u, \underline{p}) = \rho V_e^*(p, d(p)) = p - \tau d(p) + \lambda\psi s(d(p)) \int_{p}^{1} \frac{1-p}{\rho + \delta + \lambda\psi s(d(p))(1-p)} dp, \tag{29}$$

whenever $\rho/\delta \downarrow 0$ and where V_e^* is the value of the employed workers when everybody is paid according to their productivity level. The combination of the last two equations implies that when everybody is paid according to their productivity, then the lifetime expected utility is independent of the labor market state and equal to the welfare function of the planner. This result comes from the fact that when ρ is small, then the social planner is interested only in the steady-state distribution of the workers and not in the state of the worker at the particular moment that the social planner has to make her decision. This result also implies that by placing an unemployed worker closer to the CBD, the gains must outweigh the costs of placing employed workers farther away from the CBD. Hence, the condition for Ω to be maximized with respect to $\underline{d}_u$ is identical to the condition

$$\frac{\partial V_e^*(p, d(p))}{\partial d(p)} = \frac{\partial V_u^*(\underline{d}_u)}{\partial \underline{d}_u}.$$

Moreover, by equating the derivatives of (28) and (29), we obtain

$$\begin{aligned}\tau = a\lambda(1-\psi) \int_{\underline{p}}^{1} \left[\frac{1-x}{\rho + \delta + \lambda\psi s(d(x))(1-x)} \right] dx \\ + a\lambda \int_{\underline{p}}^{\widetilde{p}} \left[\frac{1-x}{\rho + \delta + \lambda\psi s(d(x))(1-x)} \right] dx.\end{aligned} \tag{30}$$

This differs from (27) only with regard to the disappearance of $K'(\cdot)$. Since a monopsony model in general implies that $K'(\cdot) \neq 1$, the outcome of the social planner differs from the market equilibrium.

Table 2 reports the results of a calibration exercise based on the model with heterogeneous firms. We use exactly the same calibrated parameters as in Panel B of Table 1 except for the commuting costs, which are set to 0.15. Setting these to their original level would have resulted in a segregated market equilibrium for any level of a (which is not such an interesting case to investigate). In line with the results of this section, we find a difference between the market and

a	Planner u	v	$\underline{d}_u$	$s(\overline{d}_u)$	ω_t	$\underline{p}$	Market u	v	$\underline{d}_u$	$s(\overline{d}_u)$	ω_t	$\underline{p}$
$\nu = \xi = 1, \alpha = 8$												
1	0.057	0.270	0	0.971	0.7063	0.290	0.060	0.291	0	0.970	0.706	0.300
0.75	0.056	0.257	0	0.979	0.708	0.290	0.070	0.286	0.152	0.859	0.702	0.324
0.6	0.055	0.249	0	0.983	0.708	0.290	0.074	0.281	0.240	0.857	0.702	0.358
0.55	0.053	0.214	0.060	0.952	0.707	0.290	0.076	0.280	0.274	0.829	0.702	0.343
0.5	0.055	0.213	0.065	0.954	0.708	0.293	0.079	0.278	0.329	0.816	0.702	0.353
0.25	0.056	0.247	0.102	0.967	0.710	0.288	0.098	0.261	0.856	0.773	0.707	0.430
0.1	0.070	0.329	0.390	0.903	0.716	0.276	0.084	0.261	0.915	0.904	0.715	0.431

Table 2: Calibration results for the model with heterogeneous firms.

the planner; surprisingly, the differences in welfare are somewhat higher than those reported in Table 1.

Note that our model is also restrictive in the linearity of the search efficiency function. Using a slight variation of the derivations in Appendix H, we obtain the following first-order condition of $\underline{d}_u$ for an interior solution of the social planner:

$$\tau = \frac{\lambda s'(\underline{d}_u + u/2)}{\rho + \delta + \lambda s(\underline{d}_u + u) + \lambda s'(\underline{d}_u + u/2)}.$$

Hence, interior solutions are possible even in the case of homogeneous workers, but with a non-linear search efficiency function. However, this is not likely to change the conclusion that the market yields a more segregated outcome than the social planner does.

8 Concluding remarks

This paper investigated the impact of the location decision of workers when they are allowed to search on the job. Two conclusions from our model differ markedly from the present state of the literature (*i.e.* Wasmer and Zenou, 2002). First, it is possible to explain a large set of city structures rather than merely the extreme types (*i.e.* completely integrated or segregated). We realize that we are not the first to obtain such a result, but we show that this result can be derived even without assuming house consumption to be a normal good for any income level (as in, for example Fujita, 1989). Second, it is possible to show that the city structure that results from the market is inefficient under very general conditions. Hence, although there is a role for governmental intervention, we have also shown that the benefits from such a policy are potentially small.

Even though some of our results are based on wage posting, many results can be shown to translate automatically to other models, such as wage bargaining models. Still, our model does not allow for all types of wage-setting mechanisms – such as models in which firms make counteroffers (as in Postel-Vinay and Robin, 2002). For this model, the only potential equilibrium results in a completely segregated city structure. This result is obtained from the fact that unemployed workers are offered their value of unemployment when accepting a job; their values therefore do not depend on distance other than the rents. This also implies that their bid-rent function is always equal to zero. Hence, location decisions are obviously inefficient in this wage-setting mechanism as well.

In future work, we can extend our analysis taking account of endogenous search effort such as in Smith and Zenou (2003).

References

Åslund, O., J. Östh and Y. Zenou (2010), "How important is access to jobs? Old question improved answer", *Journal of Economic Geography*, **10**, 389–422.

Burdet, K. and D.T. Mortensen (1998), "Wage differentials, employer size and unemployment", *International Economic Review*, **39**, 257–73.

Fujita, M. (1989), "*Urban economic theory: land use and city size*", Cambridge University Press, Cambridge.

Garibaldi, P. and E.R. Moen (2010), "Job to job movements in a simple search model", *American Economic Review*, **100**, 343–47.

Gautier, P.A. and A.P. van Vuuren (2015), "The estimation of present bias and time preferences using land-lease contracts ", working paper, VU Amsterdam.

Gautier, P.A. C.N. Teulings and A.P. van Vuuren (2010), "On-the-job search, mismatch and efficiency", *Review of Economic Studies*, **77**, 245–72.

Gobillon, L. T. Magnac and H. Selod (2010), "The effect of location and finding a job in the Paris region", *Journal of Applied Econometrics*, **26**, 1079–1112.

Kawata, K. and Y. Sato (2012), "On-the-job search in urban areas", *Regional Science and Urban Economics*, **42**, 715–26.

Laibson, D. A. Repetto and J. Tobacman (2007), "Estimating discount functions with consumption choices over the lifecycle", NBER Working Paper No. 13314, Cambridge (MA).

Manning, A. and B. Petrongolo (2015), "How local are labor markets? Evidence from a spatial job search mode", working paper, London School of Economics.

Marinescu, I. and R. Rathelot (2015), "Mismatch unemployment and the geography of job search", working paper, University of Chicago.

Menzio, G. and S. Shi (2011), "Efficient search on the job and the business cycle", *Journal of Political Economy*, **119**, 468–510.

Petrongolo, B. and C.A. Pissarides (2001), "Looking into the black box: a survey of the matching function", *Journal of Economic Literature*, **39**, 390–431.

Pissarides, C.A. (2000), "*Equilibrium unemployment theory* ", The MIT Press, Cambridge (MA).

Postel-Vinay, F. and J.M. Robin (2002), "Equilibrium wage dispersion with worker and employer heterogeneity", *Econometrica*, **70**, 2295–2350.

Ridder, G. and G.J. van den Berg (2003), "Measuring labor market frictions: a cross-country comparison", *Journal of the European Economic Association*, **1**, 224–44.

Smith, T.E. and Y. Zenou (2003), "Spatial mismatch, search effort, and urban spatial structure", *Journal of Urban Economics*, 129–56.

Wasmer, E. and Y. Zenou (2002), "Does city structure affect job search and welfare", *Journal of Urban Economics*, **51**, 515–41.

Zenou, Y. (2009), "*Urban labor economics*", Cambridge University Press, Cambridge.

Appendix

A Proof of Lemma 1

We have that $\Psi_e(d_1, w_1) \geq \Psi_e(d_1, w_2)$ and $\Psi_e(d_2, w_2) \geq \Psi_e(d_2, w_1)$. Hence for any pair of d_1 and d_2 we have

$$\begin{aligned}\Psi_e(d_1, w_1) - \Psi_e(d_2, w_1) =& \Psi_e(d_1, w_1) - \Psi_e(d_1, w_2)\\ &+ \Psi_e(d_1, w_2) - \Psi_e(d_2, w_2)\\ &+ \Psi_e(d_2, w_2) - \Psi_e(d_2, w_1)\\ &\geq \Psi_e(d_1, w_2) - \Psi_e(d_2, w_2).\end{aligned}$$

Since the second-order derivatives equal zero, we have

$$\frac{\partial \Psi_e(d, w_1)}{\partial d} = \frac{\Psi_e(d_2, w_1) - \Psi_e(d_1, w_1)}{d_2 - d_1},$$

and

$$\frac{\partial \Psi_e(d, w_2)}{\partial d} = \frac{\Psi_e(d_2, w_2) - \Psi_e(d_1, w_2)}{d_2 - d_1}.$$

Using the inequality above and $d_2 > d_1$, we obtain

$$\frac{\partial \Psi_e(d, w_1)}{\partial d} \leq \frac{\partial \Psi_e(d, w_2)}{\partial d}.$$

But then we obtain that $w_1 \leq w_2$ since we know that the cross-partial derivative of Ψ is positive. Now suppose that we have equality. This implies that also the values in between d_1 and d_2 should be occupied by workers with wage w_1.[11] Hence there is a mass point at w_1. This is ruled out by the wage posting, *i.e.* see Section 2.4. ■

B Proof of Lemma 2

Suppose that $\mathcal{D}_u$ is not convex. It implies that there is an interval (d_1, d_2) that is not occupied by unemployed workers while $d_1, d_2 \in \mathcal{D}_u$. Since the value of unemployed workers should be constant on $\mathcal{D}_u$, we have $V_u(d_1) = V_u(d_2)$. From the main text (proven below lemma 12), we also know that $R'(d) = \partial\Psi_e(d, w(d)/\partial d$ for all $d \in (d_1, d_2)$. Taking derivatives of (3) and using the same steps to obtain equation (10), we obtain

$$\rho V_u'(d) = \Psi_u'(d) - R'(d) = \Psi_u'(d) - \frac{\partial \Psi_e(d, w(d))}{\partial d}$$

for all $d \in (d_1, d_2)$. Note that since by assumption d_1 is a local maximum of $V_u(d)$ we must have that $\rho V_u'(d_1) < 0$ and hence $\Psi_u'(d) < \partial\Psi_e(d, w(d))/\partial d$. Since we know that for all $d \in (d_1, d_2)$, the second-order derivative of $R(d)$ equals

$$\frac{\partial \Psi_e(d, w(d))}{\partial d \partial w} w'(d) > 0$$

where the inequality comes from Lemma 1 and (9). Since we know that $\Psi_u'(d)$ is constant, it implies that $\rho V'(d) < \rho V'(d_1) < 0$ for all $d \in (d_1, d_2)$. But then $V_u(d_2) < V_u(d_1)$ which is a contradiction to the assumption that both should be equal. ■

C Derivation of the land rents

As stated in the main text, we have that

$$R'(d) = \frac{\partial \Psi_e(d, w(d))}{\partial d}. \tag{31}$$

Using the fact that $R(1) = R_A$ and substitution of (31) results in

$$R(d) = R_A - \int_d^1 \frac{\partial \Psi_e(d, w(x))}{\partial d} dx. \tag{32}$$

Using (11) and changing the order of the integrals, the second term at the right-hand side of (32) equals

$$\int_d^1 \frac{\partial \Psi_e(d, w(x))}{\partial d} dx = \tau(d-1) - a\lambda\psi \int_{w(d)}^{w^+} \frac{(1-F(w))}{\rho + \delta + \lambda\psi s(d(w))(1-F(w))} \int_d^{d(w)} dx dw.$$

Solving for the integrals and substitution of the result into (32) gives

$$R(d) = R_A - \tau(d-1) + a\lambda\psi \int_{G^{-1}\left(\frac{d-u}{1-u}\right)}^{w^+} \frac{(1-F(x))(d(x)-d)}{\rho + \delta + \lambda\psi s(d(x))(1-F(x))} dx.$$

For $\underline{d}_u < d \leq \underline{d}_u + u$, we have

$$R(d) = R\left(\underline{d}_u + u\right) - \int_d^{\underline{d}_u + u} R'(x) dx, \tag{33}$$

[11] Suppose that $w_1 = w_2$ and suppose that there is a $c \in (0,1)$ for which there is a $w^* \neq w_1$ with $\Psi(d(c), w_1) < \Psi(d(c), w); d(c) = cd_1 + (1-c)d_2$. Suppose that $w^* > w_1$, then since $d(c) < d_2$, we have from the above that $w^* \leq w_2 = w_1$ which contradicts that $w^* > w_1$. Suppose that $w^* < w_1$, then since $d(c) > d_1$ we must have $w^* \geq w_1$.

and

$$R(d) = \Psi_u(d).$$

Hence

$$R'(d) = \Psi_u'(d) = -a\lambda \int_{\varphi}^{w^+} \left[\frac{(1 - F(w))}{\rho + \delta + \lambda\psi s(d(w))(1 - F(w))} \right] dw. \tag{34}$$

Substitution of (34) into (33) results in

$$R(d) = R(\underline{d}_u + u) - a\lambda \int_{\varphi}^{w^+} \frac{(1 - F(x))}{\rho + \delta + \lambda\psi s(d(x))(1 - F(x))} dx(d - \underline{d}_u - u).$$

Finally, when $d < \underline{d}_u$, we can use (31) to obtain

$$R(d) = R(\underline{d}_u) - \int_{d}^{\underline{d}_u} \frac{\partial \Psi_e(d, w(x))}{\partial d} dx. \tag{35}$$

Using (11) and changing the order of the integrals, we obtain

$$\begin{aligned}\int_{d}^{\underline{d}_u} \frac{\partial \Psi_e(d, w(x))}{\partial d} dx = -\tau(\underline{d}_u - d) - a\lambda\psi \int_{w(\underline{d}_u)}^{w^+} \frac{(1 - F(w))}{\rho + \delta + \lambda\psi s(d(w))(1 - F(w))} \int_{d}^{\underline{d}_u} dx dw \\ + \int_{w(d)}^{w(\underline{d}_u)} \frac{(1 - F(w))}{\rho + \delta + \lambda\psi s(d(w))(1 - F(w))} \int_{d}^{d(w)} dx dw.\end{aligned}$$

Solving for the integrals and substitution of the result into (35) results in

$$\begin{aligned}R(d) = R(\underline{d}_u) - \left(\tau + a\lambda\psi \int_{\widetilde{w}}^{w^+} \frac{(1 - F(x))}{\rho + \delta + \lambda\psi s(d(x))(1 - F(x))} dx \right) (d - \underline{d}_u) \\ + a\lambda\psi \int_{G^{-1}\left(\frac{d}{1-u}\right)}^{\widetilde{w}} \frac{(1 - F(x))(d(x) - d)}{\rho + \delta + \lambda\psi s(d(x))(1 - F(x))} dx.\end{aligned} \tag{36}$$

D Derivation of $G(w)$

Based on the definition of $s(\cdot)$ from equation (1) and the definition of $d(\cdot)$ from equation (13), we obtain

$$\int_{\varphi}^{w} s(d(x))dG(x) = \begin{cases} G(w)s_0 - (1-u)a\frac{1}{2}G(w)^2 & \text{if } w \leq \widetilde{w} \\ G(w)(s_0 - au) - (1-u)a\frac{1}{2}G(w)^2 + \frac{\underline{d}_u}{1-u}au & \text{otherwise} \end{cases}.$$

It implies that $G(w)$ is the root of a second-order polynomial.

E Derivation of the reservation wages

Using equation (20) and the values of V_u and V_e in equations (3) and (4) and then using equation (36), we obtain that

$$\begin{aligned}\varphi = b + \tau\underline{d}_u + \lambda[1 - \psi] s(\underline{d}_u) \int_{\varphi}^{w^+} \frac{(1 - F(w))}{\rho + \delta + \lambda\psi s(d(w))(1 - F(w))} dw \\ - a\lambda\psi \int_{\varphi}^{\widetilde{w}} \frac{(1 - F(w))(\underline{d}_u - d(w))}{\rho + \delta + \lambda\psi s(d(w))(1 - F(w))} dw.\end{aligned} \tag{37}$$

in the case that $\underline{d}_u > 0$. For the case that $\underline{d}_u = 0$, we obtain

$$\varphi = b + \tau u + \lambda s(u)[1 - \psi] \int_{\varphi}^{w^+} \frac{(1 - F(w))}{\rho + \delta + \lambda\psi s(d(x))(1 - F(w))} dw. \tag{38}$$

F Proof of Theorem 3

For the moment suppose that $\underline{d}_u > 0$. Using $\nu = \xi = 1$, the zero-profit condition (22) can be written as

$$u^2 s^2 \left(\frac{u}{2} + \underline{d}_u\right) \frac{p - \varphi}{us\left(\frac{u}{2} + \underline{d}_u\right)\frac{\rho+\delta}{\delta} + (1-u)\psi s_0} = \frac{\gamma\delta}{\alpha}. \tag{39}$$

For $u = 0$, the left-hand side equals zero. For $u = 1$, the left-hand side equals

$$s\left(\frac{1}{2} + \underline{d}_u\right)(p - \varphi)\frac{\delta}{\rho + \delta}.$$

Since $u = 1$, we have that $\varphi = b + t\underline{d}_u$. From equation (17), we obtain that $\underline{d}_u = 1 - u = 0$ and hence $\varphi = b$. Substitution of this result into the left-hand side of (39) and using the definition of $s(.)$, we obtain that the left-hand side of (39) at $u = 1$ equals

$$\left(s_0 - \frac{a}{2}\right)(p - b)\frac{\delta}{\rho + \delta}.$$

Hence equation (39) has a root under the assumptions made in the proposition. The reservation wage equation (37) has a root, because if $\varphi = 0$, then the left-hand side of that equation equals zero, while the right-hand side is positive. When $\varphi = p$, then the left-hand side equals p, while the right-hand side equals $b + t\underline{d}_u < b + t$. Since $p - b$ is assumed to be larger than τ, it immediately implies that this equation has a root. In the case that $\underline{d}_u = 0$, equation (39) must be replaced by

$$u^2 s^2 \left(\frac{u}{2} + \underline{d}_u\right) \frac{p - \varphi}{us\left(\frac{u}{2} + \underline{d}_u\right)\frac{\rho+\delta}{\delta} + (1-u)\psi s(u)} = \frac{\gamma\delta}{\alpha}.$$

The arguments above still apply for this case. ■

G Proof of Lemma 4

For the situation of $\psi = 1$ and $\rho/\delta \downarrow 0$ we can rewrite equation (39) as

$$u^2 s^2 \left(\frac{u}{2} + \underline{d}_u\right) \frac{p - \varphi}{us\left(\frac{u}{2} + \underline{d}_u\right) + (1-u)s_0} = \frac{\gamma\delta}{\alpha}. \tag{40}$$

The derivative of the numerator with respect to u is obviously positive. The denominator can be rewritten as

$$s_0 - au\left(\frac{u}{2} + \underline{d}_u\right).$$

This has a first-order derivative equal to

$$-a\left(u + \underline{d}_u + u\frac{\partial \underline{d}_u}{\partial u}\right). \tag{41}$$

When $\underline{d}_u < 1 - u$, we obtain from the flow condition (18) that

$$\frac{\partial \underline{d}_u}{\partial u} = \frac{au\lambda}{\delta + \lambda s\left(\underline{d}_u + u\right)} > 0.$$

In the case that $\underline{d}_u = 1 - u$ it equals one and hence $\partial \underline{d}_u / \partial u$ in (41) is larger than -1. Hence the first-order derivative of the denominator of (40) is smaller than $-a\underline{d}_u$ which is strictly negative in the case that $\psi = 1$. Hence the left-hand side of (40) is strictly increasing and hence has a single root. ■

H Derivation of equation (26)

Using the definition of $\overline{d}_e$ and (13) and solving for the integrals results in

$$\overline{d}_e = \int_{\underline{w}}^{w^+} d(x)dG(x) = (1-u)\int_{\underline{w}}^{w^+} G(x)dG(x) + u\int_{\widetilde{w}}^{w^+} dG(x) = \frac{1}{2}(1+u) - \frac{\underline{d}_u}{1-u}u. \tag{42}$$

After substitution of (42), the Hamiltonian for the maximization of Ω with respect to equation (25) equals

$$\mathcal{H} = e^{-\rho t}\left[(1-u)p + ub - \gamma v - \frac{1}{2}(1-u)(1+u)\tau + \underline{d}_u u\tau\right] + \zeta\left[\delta(1-u) - \lambda us\left(\underline{d}_u + \frac{u}{2}\right)\right],$$

where ζ is the costate variable. Note that we have

$$\lambda = \left(s_0 - \frac{a}{2} - (1-\psi)\left[s_0(1-u) + au(\underline{d}_u + u/2)\right]\right)^{\nu-1} v^{\xi}.$$

The partial derivatives of λ with respect to u and $\underline{d}_u$ equal 0 whenever either $\nu = 1$ or $\psi = 1$. Based on this, the Euler conditions with respect to v and u equal

$$\frac{\partial\mathcal{H}}{\partial v} = \gamma e^{-\rho t} - \zeta\frac{(\xi-1)\lambda us\left(\underline{d}_u + \frac{u}{2}\right)}{v} = 0, \tag{43}$$

and

$$\frac{\partial\mathcal{H}}{\partial u} = -e^{-\rho t}\left[p - b - (u + \underline{d}_u)\tau\right] - \zeta\left[\delta + \lambda s\left(\underline{d}_u + u\right)\right] = -\dot{\zeta}, \tag{44}$$

while the first-order derivative with respect to $\underline{d}_u$ equals

$$\frac{\partial\mathcal{H}}{\partial\underline{d}_u} = e^{-\rho t}u\tau + \zeta u\lambda a. \tag{45}$$

Taking derivatives of equation (43) with respect to t, we can derive that $\dot{\zeta} = -\rho\zeta$ and substitution of this result into (44), we obtain

$$\frac{\partial\mathcal{H}}{\partial u} = -e^{-\rho t}\left[p - b - (u + \underline{d}_u)\tau\right] - \zeta\left[\rho + \delta + \lambda s\left(\underline{d}_u + u\right)\right] = 0. \tag{46}$$

Solving for ζ from this equation and substitution into (45) and rewriting results in

$$\frac{\partial\mathcal{H}}{\partial\underline{d}_u} = \frac{e^{-\rho t}u}{\rho + \delta + \lambda s\left(\underline{d}_u + u\right)}\left[(\rho + \delta + \lambda s_0)\tau - a\lambda(p-b)\right],$$

which is larger than zero, because the term between brackets was assumed to be positive.

I Proof of Lemma 5

Define τ_M and τ_P as the levels of τ that equalize the left- and the right-hand side of (17) respectively (26) and define $u_M(\tau_M)$ and $u_P(\tau_P)$ as their corresponding unemployment rates. Moreover, we denote $\lambda_P = \lambda(u_P)$ and $\lambda_M = \lambda(u_M)$. We have to prove that $\tau_P > \tau_M$. We use an intuitive proof for this. Take into account that the condition that we have a completely segregated market equilibrium can also be written as

$$\Psi_u(1-u) - \Psi_u(1) < \Psi_e(1-u) - \Psi_e(1), \tag{47}$$

i.e. the worker who is at the city edge would benefit less from moving to the location of the highest paid worker (*i.e.* 1-u) then the best paid worker loses from moving to the city edge. Now, fix for the moment λ_M and suppose that we have a situation in which all workers are paid their marginal productivity p.

Then, using standard Bellman techniques it is possible to obtain that the value of an unemployed worker V^{U^*} equals

$$\rho V^{U^*}(d) = b - R(d) + \lambda s(d) \left(\overline{V}^{E^*}(p) - V^{U^*}(d) \right), \tag{48}$$

where $\overline{V}^{E^*}(p) = \max_d V^{E^*}(p, d)$ and where $V^{E^*}(p, d)$ is the value of an unemployed worker eanring p at distance d. In addition, we have

$$V^{E^*}(p, d) = \frac{p - \tau d - R(d) + \delta \overline{V}^{U^*}}{\rho + \delta}. \tag{49}$$

Substitution of (49) into (48) and solving for V^{U^*} results in

$$\rho V^{U^*}(d) = b - R(d) + \frac{\lambda s(d)}{\rho + \delta + \lambda s(d)}(p - b - \tau + R(d)).$$

The bid rents of the unemployed workers can now be found by solving for $R(d)$ in the equality $V^{U^*}(d) = V^{U^*}(1)$. It equals

$$\Psi_u^*(d) = R_A + \lambda(p - b - \tau)\frac{a(1 - d)}{\rho + \delta + \lambda s(1)}.$$

The bid-rent of the employed workers can be found likewise. It equals

$$\Psi_e(d) = \tau(1 - d) + R_A.$$

We have

$$\Psi_u(1 - u) - \Psi_u(1) < \Psi_u^*(1 - u) - \Psi_u^*(1),$$

and

$$\Psi_e(1 - u) - \Psi_e(1) > \Psi_e^*(1 - u) - \Psi_e^*(1).$$

Moreover

$$\Psi_u^*(1 - u) - \Psi_u^*(1) = \lambda(p - b - \tau)\frac{au}{\rho + \delta + \lambda s(1)},$$

and

$$\Psi_e^*(1 - u) - \Psi_e^*(1) = u\tau.$$

Hence a sufficient condition for (47) is

$$(p - b - \tau)\frac{\lambda a}{\rho + \delta + \lambda_M s(1)} < \tau.$$

Solving for τ this results in

$$\tau > \frac{(p - b)\lambda_M a}{\rho + \delta + \lambda_M s_0},$$

and hence τ_M should be smaller than the left-hand side of this equality. Now, we still have to prove that this also holds for endogenous λ. In Lemma 6 we prove that $u_M(\tau_M) > u_P(\tau_P)$ and hence $\lambda_M \equiv \lambda(u_M(\tau_M)) < \lambda(u_P(\tau_P)) \equiv \lambda_P$. Hence

$$\tau_P = a\lambda_P \frac{p - b}{\rho + \delta + \lambda_P s_0} > a\lambda_M \frac{p - b}{\rho + \delta + \lambda_M s_0} = \tau_M.$$

■

Lemma 6 *Suppose that $\psi = \xi = \nu = 1$. We have that $u_M(\tau_M) > u_P(\tau_P)$.*

Proof: Using the first-order condition of v of the planner in (43) and (46) to solve for ζ results in

$$\frac{\partial \mathcal{H}}{\partial v} = \gamma - \frac{p - b - (u_P + \underline{d}_u)\,\tau}{\rho + \delta + \lambda s\,(\underline{d}_u + u_P)} \alpha u_P s \left(\underline{d}_u + \frac{u_P}{2}\right) = 0.$$

Substitution of $\underline{d}_u = 1 - u_P$, we obtain:

$$\frac{\partial \mathcal{H}}{\partial v} = \gamma - \frac{p - b - \tau}{\rho + \delta + \lambda s\,(1)} \alpha u_P s \left(1 - \frac{u_P}{2}\right) = 0, \tag{50}$$

and substitution of τ_p from (26) into (50) and rewriting results in

$$u_P s \left(1 - \frac{u_P}{2}\right) \frac{p - b}{\rho + \delta + \lambda s_0} = \frac{\gamma}{\alpha}.$$

Substitution of the flow condition for unemployment (18) results in

$$Q(u_P(\tau_P)) \equiv u_P^2 \frac{\left(s_0 - a\left(1 - \frac{u_P}{2}\right)\right)^2}{(\delta + \rho u_P)s_0 - u_P a\left(1 - \frac{u_P}{2}\right)(\rho + \delta)} = \frac{\gamma}{\alpha(p - b)}.$$

For the decentralized market we assume that $\tau = \tau_M$ and therefore substitute $\underline{d}_u = 1 - u$ into equation (39) to obtain

$$Q(u_M(\tau_M))(p - \varphi) = \frac{\gamma\delta}{\alpha},$$

Since $\varphi > b$, we obtain that

$$Q(u_M(\tau_M)) > \frac{\gamma\delta}{\alpha(p - b)},$$

which implies that $Q(u_M(\tau_M)) > Q(u_P(\tau_P))$. Note that the numerator of Q is increasing in u, while the denominator is decreasing in u. Hence Q is an increasing function and therefore $u_P(\tau_P) < u_m(\tau_m)$ ■

J Derivation of equation (28)

Define $\widetilde{V}_e^*(p, d)$ as

$$\widetilde{V}_e^*(p, d) = V_e^*(p, d) - \frac{\delta}{\rho + \delta} V_u^*(\overline{d}_u). \tag{51}$$

The standard Bellman equation of $V_u^*(\overline{d}_u)$ equals

$$\rho V_u^*(\overline{d}_u) = b + \lambda s(\overline{d}_u) \int_{\underline{p}}^{1} \left(V_e^*(x, d(x)) - V_u^*(\overline{d}_u)\right) dx. \tag{52}$$

Substitution of (51) into (52) and rewriting, we obtain

$$\rho V_u^*(\overline{d}_u) = ub + \lambda u s(\overline{d}_u) \int_{\underline{p}}^{1} \widetilde{V}_e^*(p, d(p)) dp, \tag{53}$$

while using the same techniques we can also derive that

$$\left[\rho + \delta + \lambda\psi s(d(p))(1 - p)\right] \widetilde{V}_e^*(p, d(p)) = p - d(p)\tau + \lambda\psi s(d(p)) \int_{p}^{1} \widetilde{V}_e^*(x, d(x)) dx. \tag{54}$$

We prove (28) by using the intermediate step that if we prove

$$\begin{aligned} \rho V_u^*(\overline{d}_u) &= ub + (1 - u) \int_{\underline{p}}^{p} (x - d(x)\tau)\, dH(x) \\ &\quad + \left\{(\rho + \delta + \lambda\psi(1 - p)s(d(p)))\,(1 - u)H'(p)\right\} \int_{p}^{1} \widetilde{V}_e^*(x, d(x)) dx. \end{aligned} \tag{55}$$

then we are done by substitution of $p = 1$ in (55). We prove this by induction. The base is easily obtained by substitution of $p = \underline{p}$ in equation (55) and substitute the steady-state condition $(\delta + \lambda\psi(1 - \underline{p})s(d(\underline{p})))(1-u)H'(\underline{p}) = \lambda s(\overline{d}_u)u$ in (53). For the induction step we need to prove

$$\begin{aligned} \rho V_u^*(\overline{d}_u) = ub + (1-u)\int_{\underline{p}}^{p+\Delta} (x - d(x)\tau)\, dH(x) + (1-u)H'(p+\Delta)\times \\ \{(\rho + \delta + \lambda\psi(1-(p+\Delta))s(d(p+\Delta)))\}\int_{p+\Delta}^{1} \widetilde{V}_e^*(x, d(x))dx. \end{aligned} \tag{56}$$

Based on the assumption that (55) is correct. From (55), we have

$$\begin{aligned} \rho V_u^*(\overline{d}_u) = ub + (1-u)\int_{\underline{p}}^{p} (x - d(x)\tau)\, dH(x) + \\ (\rho + \delta + \lambda\psi(1-p)s(d(p)))\,(1-u)H'(p)\widetilde{V}_e^*(p, d(p))\Delta + \\ (\rho + \delta + \lambda\psi(1-p)s(d(p)))\,(1-u)H'(p)\int_{p+\Delta}^{1} \widetilde{V}_e^*(x, d(x))dx. \end{aligned} \tag{57}$$

Substitution of (54) into (57) and rewriting results in

$$\begin{aligned} \rho V_u^*(\overline{d}_u) = ub + (1-u)\int_{\underline{p}}^{p+\Delta} (x - d(x)\tau)\, dH(x) + (1-u)H'(p)\times \\ \{\lambda\psi s(d(p))\Delta + (\rho + \delta + \lambda\psi(1-p)s(d(p)))\}\int_{p+\Delta}^{1} \widetilde{V}_e^*(x, d(x))dx. \end{aligned} \tag{58}$$

Since the in- and outflow of workers with a productivity level exactly equal to p should be equal to each other in steady-state, we have

$$(\delta + \lambda\psi(1-p)s(d(p)))\,(1-u)H'(p) = \lambda u s(\overline{d}_u) + \lambda\psi(1-u)\int_{\underline{p}}^{p} s(d(x))dH(x), \tag{59}$$

while also the in- and outflow of workers with a productivity level exactly equal to $p + \Delta$ should be equal to each other in steady-state or

$$(\delta + \lambda\psi(1-(p+\Delta))s(d(p+\Delta)))\,(1-u)H'(p+\Delta) = \lambda u s(\overline{d}_u) + \lambda\psi(1-u)\int_{\underline{p}}^{p+\Delta} s(d(x))dH(x). \tag{60}$$

Combining equations (59) and (60), we obtain

$$\begin{aligned} (\delta + \lambda\psi(1-(p+\Delta))s(d(p+\Delta)))\,(1-u)H'(p+\Delta) \\ = (1-u)H'(p)\,\{\lambda\psi s(d(p))\Delta + (\rho + \delta + \lambda\psi(1-p)s(d(p)))\} \end{aligned} \tag{61}$$

Substitution of (61) into (58) results in (56). ■

K Derivation of equation (29)

This is a direct result of (54) and the definition of $\widetilde{V}_e^*$ in (51)

$$\begin{aligned} \rho\widetilde{V_e}^*(p, d(p)) = \frac{\rho/\delta}{\rho/\delta + \frac{\delta + \lambda s(d(p))(1-p)}{\delta}} \left\{ p - d(p)\tau + \lambda s(d(p))\int_{p}^{1} \widetilde{V}_e^*(x, d(x))dx \right\} \\ + \frac{1}{1+\rho/\delta} V_u^*(\overline{d}_u). \end{aligned}$$

Now, using $\rho/\delta \downarrow 0$ gives the desired result. ■