UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

Bachelor Thesis in Finance

The Effect of Bond Convexity in Abnormal Volatility

An Empirical Study on Returns and Convexity in the U.S. Treasury Market 2007-2012

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Abstract

According to earlier empirical studies, convexity in the U.S. treasury market is arbitrage-free priced. This paper study whether the arbitrage-free pricing of convexity held even in the financial crisis and the volatile period that followed.

A fixed-income investor must consider interest rate exposure since it is the factor affecting the value of the security the most. This direct relationship can be shown in the negative correlation between interest rates and price; if rates decline prices increase. The shape of this price-yield relationship is more or less convex for individual bonds. Theory suggests that this convex property is desirable for investors experiencing yield shifts in the market. However, no return enhancing aspect of convexity has been found in empirical studies before, which proposes convexity to be arbitrage-free priced. By analyzing U.S. treasury bonds in the volatile interest rate period of 2007-2012, we extend earlier research of convexity's impact on bond returns.

Our regression results show no significant effects of convexity generating excess returns. The tests are based on yearly data from 2007-2012 and monthly data from 2008-2009, respectively. Results for entire periods show no positive significant results, neither for the yearly or monthly datasets. When analyzed period-by-period significant effects of convexity are found, but with just as many results showing negative effects on returns as positive. Implications of these results are that convexity is priced by the market, sometimes even over-priced, and higher convexity is thereby not consistent with excess returns for longer periods. Hence, our findings strengthen earlier research and conclude that convexity does not generate excess returns, even in the most volatile periods.

Keywords: Convexity, Duration, Fixed-Income, Volatility, U.S. Treasury Bonds, Bond Returns

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1. Introduction

1.1 Background

The global financial crisis that erupted in 2008 caused the most severe recession since the Great Depression in the 1930s (FRED, 2016), and different markets as well as different sectors suffered due to the scale of the crisis. Even the most liquid market of them all, the U.S. Treasury Bond Market, exhibited volatility well above average (Engle et al., 2012). The financial crisis led to decreased U.S. treasury yields, due to ‘flight-to-safety’ where investors moved from riskier positions to U.S. treasury securities (Noeth & Sengupta, 2010). In 2009 however, the stability of the entire financial system was questioned and U.S. treasury yields rose (FRED, 2016). On the 3rd November 2010, the Fed announced “QE2”, a purchasing program of $600 billion in U.S. treasuries (Federal Reserve, 2010). Overall, these market changes resulted in large movements in bond yields completely unforeseen in 2007.

Bonds have a reverse relationship between yield, or interest rate if you like, and bond prices. The direct effect of decreased rates is price appreciation, and vice versa. This is a well-known behavior of bonds. Since interest rates are the single factor affecting bond prices most, managers must consider interest rate risk and quantify the exposure. Such measures of interest rate exposure are duration and convexity. Duration measures the linear effect on bond prices of one percent change in interest rates. Convexity on the other hand, describes the non-linear price-yield relationship (Fabozzi, 1999). Thus, duration underestimates the new price after an interest rate change if convexity is not considered. Consequently, the property of bond convexity should be favorable to an investor when yields shift either way.

The financial crisis and the bond price mechanism together constituted problematic preconditions for bond investors with implications for their investment strategies. The increased volatility in the crisis period forced investors to evaluate bond yield risk more precisely, where convexity play an important role. However, the crisis period is not relevant for its categorization as a ‘financial crisis’ but for the particular aspect of large shifts in yields that the period involved. These circumstances suggest, at least theoretically, excess returns attributable to increased convexity exposure.

1.2 Problem Discussion

The convexity discussion which developed in the late 20th century could be seen as a dilemma between financial mathematics and the arbitrage-free aspect of efficient market theory, where all available information is reflected in the pricing of a security (Fama, 1970). As will be explained in the theoretical framework, the concept of convexity is doubtless – in response to a yield change, higher convexity leads to higher bond price all other equal. The question is how higher convexity is priced in the first place. Numerous empirical studies, e.g. Kahn & Lochoff (1990), and Lieu & Shyy (1994), have found no excess returns associated with convexity. These studies are consistent with an efficient market where returns on high-convexity bonds diminish due to higher pricing in the first place. If this aspect of the efficient market hypothesis holds in periods of abnormal volatility, when convexity in theory is as most valuable, has never been studied before.

To further expand the convexity dilemma, there is an ongoing discussion whether convexity can be used to exploit expectations of high volatility (Ben Dor et al., 2006). Earlier research has established that convexity is priced according to volatility expectations (Smit & Swart, 2006). What happens if volatility exceeds these expectations, is however so far unobserved. Even if markets are efficient, convexity should logically yield excess returns if volatility exceeds market expectations. However, no earlier convexity research has exhibited findings of such excess return, leading to the compelling suggestion that convexity might even be overpriced. Overpricing of convexity would actually indicate inefficient markets where shorting convexity results in arbitrage profits.

Convexity is of special concern to insurance portfolio managers and pension fund managers with specified liabilities in the future. These kind of investors can immunize their portfolios to interest rate risk by ensuring equal duration in assets and liabilities (Fabozzi, 2006). As a consequence of this strategy, the only way to generate returns through interest rate movements is to exploit convexity. Insurance portfolio managers and pension fund managers are thus awaiting empirical evidence on how convexity affects returns in volatile periods.

1.3 Purpose

The main purpose of our thesis is to further extend convexity theory by testing how the convexity phenomenon affect bond returns in a volatile period like 2007 to 2012. Traditional theory states that convexity is return enhancing in periods of large yield shifts. If convexity in practice was return enhancing during the crisis is so far unobserved, and investors await extended convexity research through empirical studies in a modern and volatile environment. By achieving our purpose of extending convexity theory, we hope to attain greater understanding of bond pricing in the context of modern interest rate environment and furthermore provide investors with accurate information about convexity and its effect on returns. In summary we want to contribute to the research on bond portfolio management in general and convexity in particular, which implies a theory developing character of our purpose.

Research Question

”How did convexity affect bond returns in the volatile period 2007-2012?”

Progress in the specific research question is valuable since the area of individual bond returns and risks in unconventional market conditions, like abnormal volatility due to a financial crisis and quantitative easing, has not been sufficiently researched. This is true despite the fact that treasury bonds represent a substantial part of total investments. In mid-2015 the size of the U.S. bond market was $39.9 trillion compared to the size of the U.S. stock market, $26.3 trillion (SIFMA, 2016).

2. Delimitations

We limit our research to U.S. treasury bullet bonds issued in U.S. dollars and active the entire period between 2007 and 2012. The reason for using U.S. treasury fixed-income securities is that it is the most heavily traded fixed-income market in the world. A highly liquid market results in highly efficient pricing which is relevant for our research. With treasury securities we can ignore aspects of credit risk in pricing of the securities, since they are considered “risk-free”. The same analogy could be made with the reason for only using bonds issued in U.S. dollars, any concerns of currency effects on pricing are now avoided. Bullet bonds without any embedded options simplify the pricing mechanism and make the calculation of yield-to-maturity more reliable. The reason for limiting our research to a six-year period in the 21st century is that most of the research we found on convexity was conducted in the twentieth century, with completely different market conditions.

3. Theoretical and Conceptual Framework

Our results for how convexity affect bond returns are generated using quantitative approaches where duration, convexity, and yield-to-maturity are main concepts which have to be explained and motivated by accepted theories. Before these core concepts are explained, the first sections of the chapter describe our theoretical references and empirical studies in the field.

3.1 Theoretical References

For explanation of the concepts of duration and convexity we mainly use literature by Frank J. Fabozzi, a professor of finance at Yale University who is acknowledged in the area of fixed-income securities market. His books *Duration, Convexity, and Other Bond Risk Measures* (Wiley, 1999) and *Advanced Bond Portfolio Management* (Wiley, 2006) are of particular interest for us when it comes to bond pricing. The paper *A Closed-Form Equation for Bond Convexity,* Financial Analysts Journal 1989, by Robert Brooks and Miles Livingston provides us with equations for calculating the convexity of coupon-bearing bonds. The paper also states that high convexity is preferable – a statement that we want to test if it holds true in reality.

The question if high-convexity bonds in reality outperform low-convexity bonds with similar duration has been empirically studied earlier. Theory suggests that more convexity should generate higher returns with above stated arguments, but empirical studies show that it might not imply a successful investment strategy. Among others, Kahn and Lochoff performed a study in 1990 and was followed by Lieu and Shyy in 1994, both analyzing convexity for U.S. treasuries in their papers. The motivation behind the choice of earlier papers is that they perform similar tests and provide good guidelines for our own research. The studies are presented in 3.2 below.

3.2 Empirical Studies

Lieu and Shyy study the convexity effect on returns in the U.S. treasury bond market between 1983-1987 by performing a regression analysis. Returns are collected based on monthly price data. The bonds are further divided into 13 classes with similar duration, and within each duration class, classifications into high-or-low convexity groups are done. The traditional idea of the positive relationship between convexity and excess returns is thereby tested by comparing the high- and low-convexity groups. No such behavior of convexity generating excess returns is significantly supported in this empirical test. This result suggests an efficient market where any gains from convexity is priced by the market.

Kahn and Lochoff perform a study in 1990 based on the same assumptions about convexity. This is done in a return attribution analysis where many factors, convexity among others, was assigned impact on excess returns. But the convexity factor is not found to generate significant excess return. In their paper, they also criticize the assumption of parallel yield curve shifts, which rarely holds.

Ben Dor et al. (2006) come to a similar conclusion in their research at Lehman Brothers from 2006, where they study whether an immunized strategy only exposed to convexity can yield excess returns in the U.S. treasury market in 1989 to 2006. Lacey & Nawalkha (1993) continue the argument for arbitrage-free pricing of convexity, when they establish that convexity was priced between 1976 and 1987. Our study will examine if the results of these empirical studies held during the financial crisis and in the volatile period that followed.

3.3 Efficient Market

Fama (1970) introduced the assumption that capital markets are efficiently priced and his Efficient Market Hypothesis is still a used assumption in monitored and liquid markets. The theory states that all available information is reflected in the price of the security. Hence, a market where all public information is exploited is considered efficient, and investors cannot outperform the market based on available information. Fama concludes in 1970 that the efficient market hypothesis “stands up well” but a few exceptions. These conclusions should be transferable to present time and the most liquid market of them all - the U.S. Treasury market.

For bond investors the convex price response to yield shifts is available information, but information about future yield shifts are unknown and often unforeseen in volatile periods. So in terms of convexity, an efficient market implies arbitrage-free pricing where convexity is priced according to current yield volatility expectations (Fabozzi, 1999). Unless shifts in yield exceed market expectations, any gains attributable to convexity would diminish through an efficient pricing.

3.4 The Mechanics of Bond Pricing

Before studying the mechanics of bond pricing its crucial to define what *type* of bond to be valued. The bond of interest is the most common type of bond; the option-free U.S. treasury bullet bond with straightly scheduled coupon payments, simply referred to as *bond* in the future. Options-free bullet bonds guarantee certain cash flows which simplifies the valuation since the price of a bond equals the present value of *expected* future cash flows. Bullet bonds pay the face value at the maturity date, compared to amortizing bonds with continuous repayments. The choice of bonds with understandable price mechanisms and matching it with corresponding convexity- and duration-formulas suitable for our delimited type of bonds will generate as appropriate results as possible.

The price of a bond is dependent on a few factors where the yield-to-maturity, or interest rate in other words, is the single most important factor. This is evident since the value of a bond is the present value of the expected future cash flows. The cash flows of a coupon paying bond are known in advance, but to obtain the present value the investor has to consider the discounting rate that the cash flows are discounted at. The relation between bond price and interest rate is reverse – if rates goes down, bond prices goes up. This is true because the future cash flows will be discounted at a lower rate and the value of them therefore increases. Coupons are fixed, so when the bond moves towards maturity or market rates changes, the price is the factor alone that can change in order to keep the bond competitive to comparable bonds. The rate affects how the cash flows are discounted, but also at what rate cash flows can be reinvested. Thus interest rate directly affects the value of bonds and the exposure must be measured to control the risk of interest rate changes.

$C = Coupon\ paid\ at\ time\ t$

$M = Principal\ value\ paid\ at\ maturity\ date$

$y = Yield\ to\ maturity$

$$Bond\ Price = \sum_{t=1}^{n} \frac{C_1}{y^1} + \frac{C_2}{y^2} + \frac{C_t}{y^t} + \cdots + \frac{C_n+M}{y^n}$$

[Equation 1] (Fabozzi, 1999)

It is again important to point out that in the field of fixed-income, yield-to-maturity and interest rate are completely interchangeable. Interest rate exposure can be measured in two ways: Either in a scenario analysis or by obtaining estimates from risk parameters such as duration and convexity (Fabozzi, 1999). Since we are interested in analyzing convexity, the second approach is more suitable for us, especially since we use historic data. The advantage of these measures is that it is not necessary to revalue the entire bond since the parameters give the approximate price change. The disadvantage is that they can only provide *approximate* measures. Also, the duration and convexity measures rely on assumptions of infinitesimal parallel yield curve shifts, that have to be fulfilled for the model to hold (Reitano, 1992). The estimates provided under these assumptions are therefore exposed to *modelling risk* (Fabozzi, 1999).

3.5 Yield-to-Maturity

As described above, interest rate has a direct effect on bond values since it affects both the discounting rate and the reinvestment rate. The interest rate is also referred to as yield-to-maturity or required yield, since it is the rate that the market requires to invest in the bond. The appropriate discount rate for the cash flows is the risk-free rate from U.S. Treasuries plus a spread that reflects the risk of the security (Fabozzi, 1999). Since we are analyzing U.S. treasury bonds, no risk premium has to be accounted since U.S. treasuries are considered risk-free. The only reason for changes in the required yield is thereby fully dependent on treasury yield changes. The chart 3.5.1 shows the 10-year treasury rate during our research period 2007-2012. The volatility is obvious and both the 'flight-to-safety' following Lehman Brothers collapse on the 15th of September 2008, the risk aversion in 2009, and the rate-decreasing effect of "QE" from 2011 and onwards are apparent.

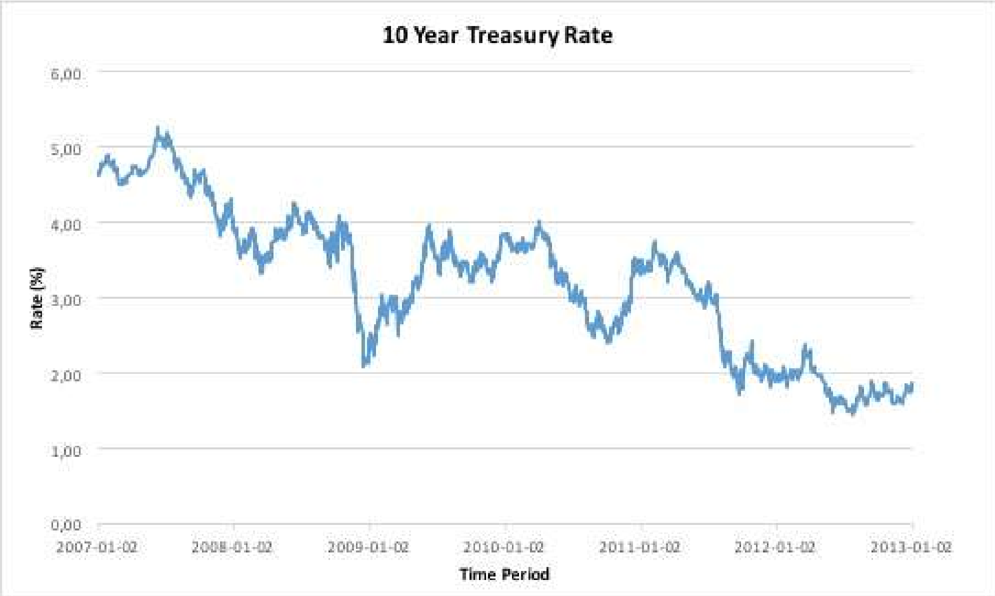

Chart 3.5.1: The 10-year treasury rate for the period 2007-2012. Source: FRED

Yield-to-maturity is the interest rate that satisfies that the present value of future cash flows is equal to the price, if the bond is held to maturity. It is the current rate of return if coupons are reinvested at the current yield until the bond matures. It is thus the implied interest rate at the current bond price. Therefore, yield-to-maturity and interest rate refers to the same thing – the denominator in the bond pricing function.

3.6 Duration

As mentioned above, bond pricing theory tells us that an interest rate decrease results in increasing bond values since the bonds cash flows are discounted at a lower rate. On the contrary, a decreased interest rate results in lower reinvestment returns, since investments in the future are made at a lower rate of return. The investment horizon for when the two effects of a decreased interest rate are neutralized, the price appreciation and the decreased reinvestment returns, is known as duration.

Duration is primarily a measure of interest rate risk on bond prices and is widely used by investors to quantify interest rate risk of bonds. Duration measures the sensitivity of a bond's price of a 100 basis points change in interest rates (Fabozzi, 1999). If the duration of a bond is 4, one percentage point change in rates affects the bond price by approximately 4 percent. Duration is expressed in years – indicating that bonds with higher duration are more exposed to interest rate risk since more future cash flows will be discounted at the new rate.

Duration is a linear approximation as can be seen in Figure 3.6.1 below. It captures the derivative effect of an interest rate change as if the price-yield relationship was linear. Hence, the price estimated by duration is only accurate for very small interest changes, if the actual price-yield relationship in fact has a more convex shape.

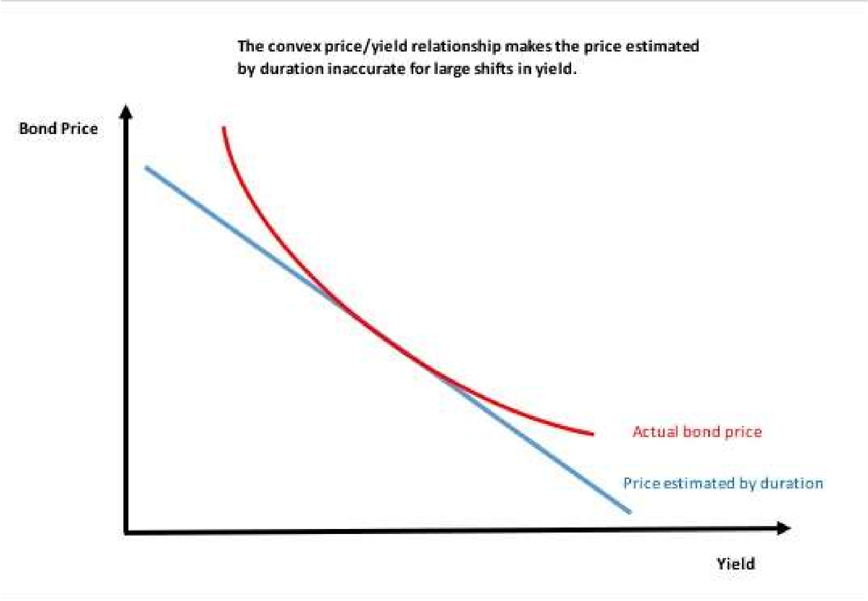

Figure 3.6.1: Price estimated by duration is accurate only for small yield shifts.

Duration can be measured in different ways. Frederick Macaulay first introduced the concept, and *Macaulay duration* can be expressed as below. It can be described as the weighted average term to maturity of the cash flows of a bond.

$$\text{Macaulay Duration} = \frac{\sum_{t=1}^{n} \frac{tC}{(1+y)^t} + \frac{nM}{(1+y)^n}}{P}$$

[Equation 2] (Fabozzi, 2000)

Our data is based on *Modified duration* which is the most appropriate duration measure for option-free bullet bonds. It is calculated by dividing the Macaulay duration by one plus the yield.

$$\text{Modified duration} = \frac{Macaulay\ Duration}{1+y}$$

[Equation 3] (Fabozzi, 2000)

$$\text{Duration as first order condition} = -\frac{1}{P} \times \frac{dP}{dy}$$

[Equation 4] (Fabozzi, 1999)

Bond price change ≈ $-Duration \times (\Delta y) \times 100$

[Equation 5] (Fabozzi, 1999)

The application of duration theory is straightforward for investors with clearly defined expectations of future interest rate changes. An investor who anticipates the rates to decline will increase the duration of the portfolio, to maximize the price appreciation if the rates decline as expected. Investors can also hedge using duration by matching the duration of the portfolio with the liabilities. Duration theory therefore provides investors with a useful tool to control interest rate risk.

3.7 Convexity

Convexity tells us, just like duration, about a bond's interest rate sensitivity on its price. It captures the second order condition of interest rate sensitivity – the non-linear shape of the price-yield relationship. Convexity is therefore an extended measure compared to duration. The price change measured by duration is thought to have the same effect on prices whether the rates increase or decline. An important property of bond values is that the price gain in fact is larger than the price decline for given yield changes (Fabozzi, 1999, 39). The effect is demonstrated in figure 3.7.1. Convexity captures this effect, which duration cannot do for large yield changes.

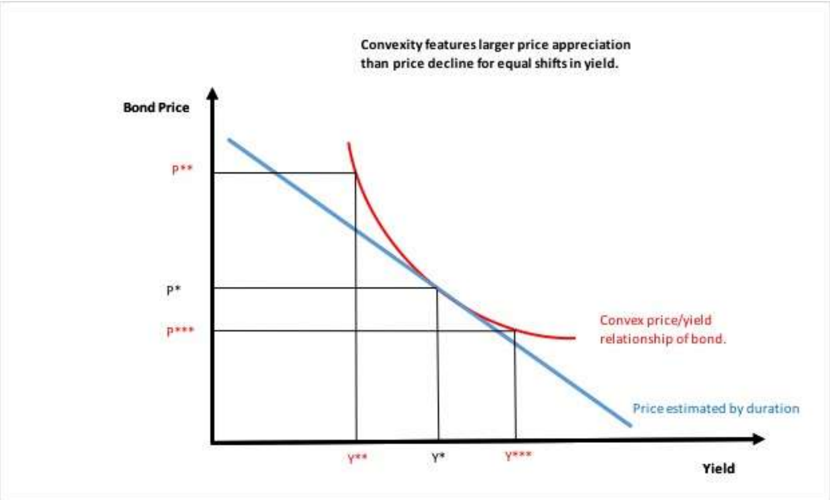

Figure 3.7.1: Bond prices increases more than they decline for equal absolute yield changes, due to convexity

Convexity = $\frac{\sum_{t=1}^{n} PV(C_t) \times t \times (t+1)}{\sum_{t=1}^{n} PV(C_t)}$

[Equation 6] (Fabozzi, 1999)

Bond percentage change $\approx -Duration \times (\Delta y) + Convexity \times \frac{1}{2}(\Delta y)^2$

[Equation 7] (Fabozzi, 1999)

Convexity as second order condition = $\frac{1}{2P} \times \frac{d^2 P}{dy^2}$

[Equation 8] (Brooks & Livingston, 1989)

The *degree of convexity* of the price-yield relationship, is dependent on two main factors. The coupon rate, and term to maturity (Fabozzi, 1999). Other factors held constant; lower coupon rates generate higher convexity, as well as longer terms to maturity increase the convexity.

The new price predicted by duration is always lower than the price calculated with convexity adjustments. With convexity, the bond price reacts less negatively to an interest rate increase than estimated with only duration, and the bond price reacts more positively to an interest rate decrease than estimated with only duration. This suggests that out of two bonds with the same duration; the bond with more convex price-yield relationship should be favorable, ceteris paribus. Convexity suggests, at least in theory, excess returns. The excess return is the difference between the actual return of a security given specific exposures and the expected return (Fabozzi, 1999). In our case, the excess return is defined as the actual return of a bond compared to the actual return of a similar benchmark bond with lower convexity.

The more or less convex curvature of the price-yield relationship has implications for return analysis. Two bonds with the same duration are thought to have the same interest rate exposure – but in fact the exposures can differ if the convexity differs. In other words, two bonds might have the same duration but different convexity and thereby respond differently to interest rate changes. A risk that the duration approximation itself cannot reveal. Observed in Figure 3.7.2 below, Bond A and Bond B have equal duration but respond differently to interest rate changes due to convexity. As can be seen, convexity becomes more valuable for large interest shifts where duration lacks preciseness. This motivates our analysis of convexity in a volatile period with large interest rate changes; 2007-2012. The magnitude of the interest rate shifts is critical for the significance of convexity, so the choice of research period is important to draw any conclusions about convexity and its effects on returns.

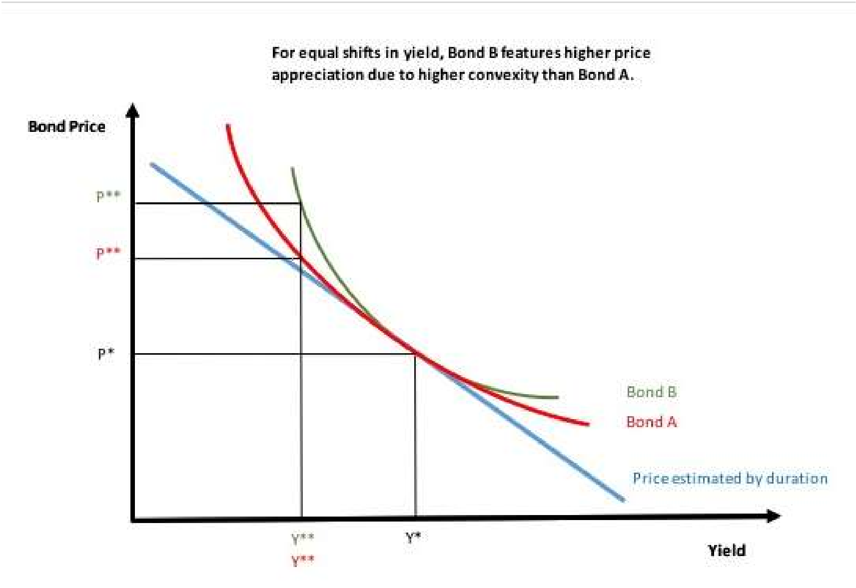

Figure 3.7.2: Comparison of two bonds with equal duration but different convexity.

3.8 Volatility in 2007-2012

The reason we study 2007 to 2012 in specific, is the abnormal volatility both in terms of yield-to-maturity and bond return shown in the period. We define 'abnormal volatility' as volatility significantly above the long term average. To establish that the volatility in fact was abnormal during the financial crisis, we study data from Federal Reserve Economic Data, Stern and Chicago Board Options Exchange. The compiled data is summarized in Table 3.8.1. Engle et al. (2012) confirm that volatility was well above average in the crisis period, in their report for the Federal Reserve Bank of New York.

Volatility 10-Year U.S Treasury	1Y Return Volatility	YTM Change (%) Volatility	Average TYVIX Index
Long-term period	7,69%	5,74%	6,66
2007 - 2012	10,95%	9,58%	7,43
2008 - 2009	15,61%	11,67%	9,64

Table 3.8.1: Measures of Volatility on 10-year U.S. Treasury. Source: FRED, Stern & CBOE

Looking at Chart 3.8.2 of the 10-year U.S. treasury yearly returns, it is not obvious that our research period contains abnormal volatility. But as presented in Table 3.8.1, the standard deviation from the average yearly return is above 3 percentage points higher in our research period than the long-term standard deviation.

Chart 3.8.2: Yearly returns on 10-year treasuries 1928-2012. Source: Stern and FRED

Chart 3.8.3 on the percentage change in yield-to-maturity on 10-year U.S. treasuries also illustrates the volatility in our research period. As a matter of fact, the 2007 to 2012 standard deviation in percentage change in yield to maturity is almost 4 percentage points higher than the long-term standard deviation.

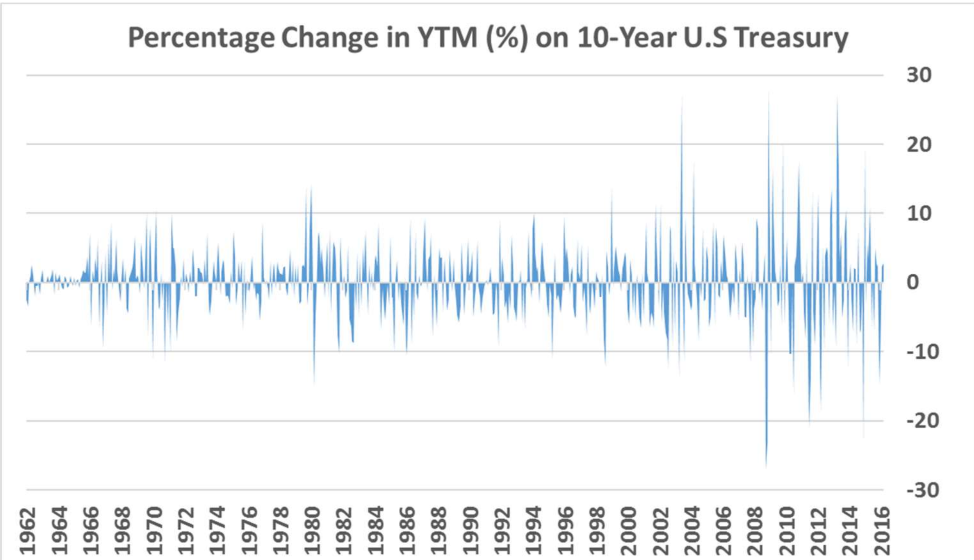

Chart 3.8.3: Percentage change in yield-to-maturity on 10-year U.S. treasuries 1962-2016. Source: FRED

This volatility during the financial crisis also lead to higher expected 30-day volatility, measured by the TYVIX Index on the Chicago Board Options Exchange. TYVIX extracts the expected next 30-day volatility in 10-year U.S. treasuries from prices on options. Chart 3.8.4 illustrates the spike in volatility expectations during the financial crisis.

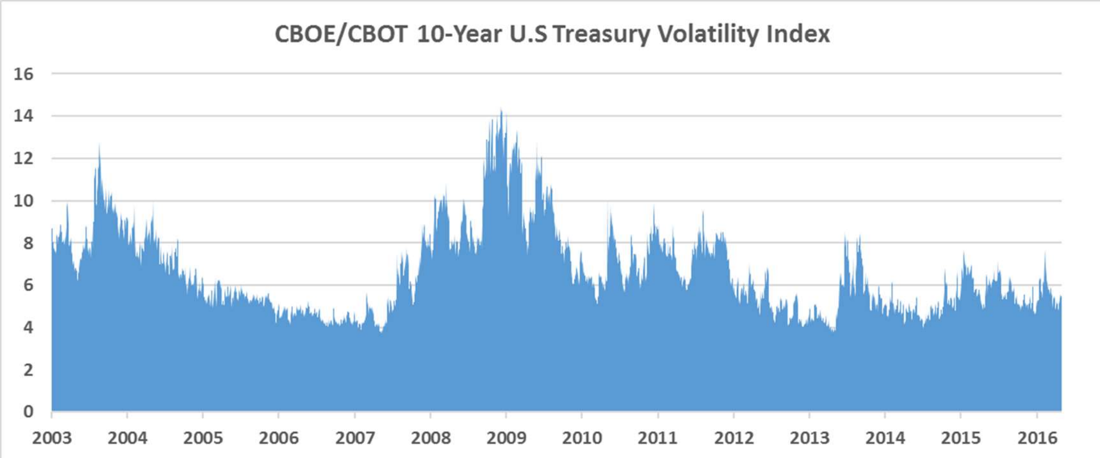

Chart 3.8.4: TYVIX 2003-2016. Source: CBOE

4. Hypotheses

Empirical evidence from earlier studies show that high-convexity bond portfolios do not outperform low-convexity portfolios. Our null hypothesis is that the U.S. treasury market is efficient in a sense that convexity in reality does not generate excess returns, in accordance with earlier research. Since our study is performed in recent years with new circumstances described in chapter 3.8, we doubt that earlier theories will hold. Consequently, the alternative hypothesis proposes excess returns due to convexity. We believe this will be true in this specific time period characterized of high volatility. Important to note is that convexity in theory is more favorable during large interest rate changes, which makes this volatile period relevant to study and new results might be found.

H_0 = Convexity is efficiently priced in 2007-2012 and no excess return can be found.
H_1 = Convexity yields excess returns in 2007-2012, due to the abnormal volatility.

Our results will either contribute to one side or another of the convexity debate – the theory that suggests reward for convexity exposure, or empirical studies that indicates the opposite. If we can't disprove our null hypothesis, the market has priced convexity more or less efficiently, which of course can be further analyzed.

5. Method

We designed our research into a quantitative study. Quantitative methods are since long established as the most accepted and appropriate way to study finance, due to the general availability of data and theories based on math in the field. In contrast, a qualitative research approach can increase the depth of the study but at the same time endanger the objectivity of the report. With the purpose of our research in mind, the prospects of reaching general and conclusive results through quantitative methods were appealing to us.

5.1 Data and Variables

As will be discussed further in the data chapter, we have two different data sets consisting of fifty-five U.S. treasury bonds each. One data set of yearly data running from 2007 to 2012. Another set of data describing the years 2008 and 2009 on a monthly basis. The dependent variable throughout our entire research is 'returns', either yearly or monthly returns depending on the data set. Our independent variable of interest is 'beginning convexity', which we define as the bond convexity at the start of each time period. Independent variables of control are 'change in yield-to-maturity', 'beginning duration', 'time period', 'U.S. Treasury Yield Index change', 'high convexity group dummy' and 'time-to-maturity', with an emphasis on the three first mentioned.

We have 330 and 1318 observations in the respective data sets, ensuring us of large enough samples to draw statistically secured conclusions (Bryman & Bell, 2013). The reason for using two different data sets is to explore the effect of convexity on returns with two different time perspectives, one long-term and one medium-term perspective. We should not hide the fact that we have studied a very specific time frame with great volatility in bond returns due to the financial crisis. But considering the results of earlier empirical studies, we found this time period most scientific relevant in our quest for new contributions to finance theory. Triangulated evidence on abnormal volatility in 2007 to 2012 are presented in chapter 3.8. As observed in Table 3.8.1, the volatility measures are even greater in 2008 to 2009, which validates our monthly data set concentrated to these two years.

The process of data gathering was greatly simplified by the Bloomberg Terminals in the Centre for Finance at Gothenburg University. We used the Bloomberg function 'Fixed Income Search' and screened for bullet bonds issued by the U.S. treasury in U.S. dollars that was active the entire period between 1st January 2007 and 1st January 2013. This search resulted in 55 remaining bonds. As described in the delimitations chapter, we have good reason for all these screening criteria. We then manually copied the Bloomberg information, from the 'Yield and Spread Analysis' function, on clean bond price, bond yield, duration and convexity for each bond on each point in time. From the bond prices on two consecutive time periods, we could calculate the return on the bond in the specified period. In a similar way, we could calculate the change in yield-to-maturity from the bond yields on two consecutive time periods. By subtracting the observation time point from the bond maturity date, we reached time-to-maturity in days. We also downloaded information on a U.S. Treasury Bond Yield Index, consisting of bullet bonds across all maturities, from Bloomberg. The change in index yield was calculated as the current time period index yield subtracted with the former time period index yield, which was then copied into the respective time period on every bond. The results were two Excel files with panel data on 55 bonds.

5.2 Analysis Method

Our model for analyzing the data is the commonly used multiple linear regression with the OLS function in the statistics software Stata. The alpha level of our statistical tests is 0.05, which reflects a 95 % certainty that an existing effect will be found in the sample, or a 5 % probability that a rejected hypothesis is true. Due to the heteroscedasticity discussed in Appendix, we performed robust regressions controlling for heteroscedasticity. The multiple linear regression has been used before in convexity papers. Shyy & Lieu (1994), Lacey & Nawalkha (1993) and Kahn & Lochoff (1990) used it among others. Another popular form of analyzing convexity effects is return attribution analysis, used by Ben Dor, et al. (2006), Smit & Swart (2006) and Kahn & Lochoff (1990). The reasons for us not using return attribution analysis are:

i) We are not interested in constucting non-existing bond portfolios; we rather look at convexity's effect on individual bonds.

ii) We are not nearly as comfortable or familiar with return attribution analysis as we are with multiple linear regressions, and we believe the use of one of them is sufficient in our analysis.

Since convexity is a mathematically complex phenomenon, a last alternative way of analyzing convexity is by creating fictional scenarios and then calculating the impacts of these scenarios on bond returns and bond convexity etcetera. This approach was taken by Chance & Jordan (1996) and Barber & Copper (1997). As we are interested in the empirical effects of convexity in the past and not some hypothetical effects convexity might have in the future, this approach never was an alternative for us.

Like Shyy & Lieu (1994), we divided our sample into 13 'duration classes' and created a dummy variable for the observations with above median convexity in each duration class. The reason for this dummy is theoretically sound, since higher convexity is supposed to outperform given equal duration. However, we find that convexity is highly correlated with duration even within the duration classes. This invalidates the use of the dummy altogether and we rather control for duration by including duration in the regression, to not overcomplicate our interpretation of the results.

By using quantitative methods and statistical models which we are comfortable with and which furthermore have been used in earlier convexity research, we are confident to reach validity of our research.

6. Results

This section presents the results from our statistical tests, with corresponding interpretations and conclusions left for the next two chapters. The tests are performed in the statistical software Stata, and the results will be given in both statistical outputs and complementing descriptions. The tests are performed with two datasets, yearly data from 2007-2012, and monthly data from 2008-2009. Naturally, that is also the sectioning of this chapter. Descriptive statistics of our data sets are presented in Table 6.0.1 and Table 6.0.2. More descriptions of our data sets, along with statistical discussions and non-instrumental results are located in Appendix.

Monthly Data Table	Observations	Average	Max	Min	Standard Deviation
Monthly Return	1318	0,08%	15,17%	-15,24%	3,46%
Change in YTM	1318	-0,02%	0,98%	-1,29%	0,39%
Beginning Duration	1318	8,11	17,07	3,01	2,97
Beginning Convexity	1318	0,98	3,9	0,11	0,74

Table 6.0.1: Summary statistics of the monthly data set

Yearly Data Table	Observations	Average	Max	Min	Standard Deviation
Yearly Return	330	2,62%	32,82%	-26,24%	8,91%
Change in YTM	330	-0,61%	1,85%	-2,39%	0,95%
Beginning Duration	330	7,78	17,07	1,09	3,2
Beginning Convexity	330	0,93	3,9	0,02	0,74

Table 6.0.2: Summary statistics of the yearly data set

Before presenting our results, we would like to repeat our research question:

”How did convexity affect bond returns in the volatile years of 2007 to 2012?”
The null hypothesis for all our statistical test was that convexity has no effect on bond returns in this period. The convexity coefficient would have to show a p-value of below 0.05 in our regressions to be considered significant and thus rejecting our null hypothesis.

6.1 2007-2012 Yearly Results

This test was set up to show the effect of convexity on returns for bonds active during the years between 2007 and 2012. That means 55 bonds and 6 time observations, adding up to 330 observations. The variable of interest in the regression was convexity in the beginning of the period and the control variables were duration, yield-to-maturity change and time period. This was regressed against the yearly returns of the bonds.

The results showed no effect of excess returns due to convexity. The coefficient for convexity is close to zero, with a confidence interval ranging between -0.0334441 and 0.0464594. And above all, it is insignificant due to a p-value of 0.749, which is far from our pre-determined accepted significance level of 0.05. The model has a high degree of explanation, 85.93%, but it is not due to the convexity variable.

Robust regression on yearly returns	Observations	R-square
	330	0,8593
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0843525	0,000
Duration	0,0073846	0,073
Convexity	0,0065076	0,749
Time Period	-0,0024436	0,006

Output 6.1.1: The regression result using yearly data proves no significant effect of convexity.

When the duration variable was left out of the regression, convexity became significant. This proves the close relation between duration and convexity, and duration should be included as a control variable for an accurate and unbiased result.

6.2 2008-2009 Monthly Results

The result from a test using monthly data might be different from the yearly tests. The monthly data provides additional observations but also shorter time periods for any excess returns generated by convexity to be priced by the market. That was the motivation behind performing a test with the same bonds using a dataset with monthly observations in the period 2008-2009, giving us 1318 observations.

Running the same type of test as with the yearly data, we once again found no significant effect of convexity on bond returns. The coefficient was centered around zero and again highly insignificant with a p-value of 0.921. Our null hypothesis that convexity has no effect on bond returns cannot be rejected in this test either.

Robust regression on monthly returns	Observations	R-square
	1 318	0,8809
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0828824	0,000
Duration	0,0005255	0,593
Convexity	-0,000425	0,921

Output 6.2.1: The regression result using monthly data proves no significant effect of convexity.

In accordance with earlier research methods by Lieu and Shyy (1994), we also performed a test with our data divided into 13 groups sorted in order of duration. This test aimed to isolate the effect of convexity exposure by equate the duration within the groups. The bonds in each group were divided into high-convexity bonds and low-convexity bonds based on if they had a convexity exposure above or below the group median. Theory suggests that high-convexity bonds should outperform low-convexity bonds with the same duration, so the method of sorting out high-convexity bonds and include the variable in the regression is motivated. We would, according to theory, expect a positive significant effect of the dummy variable *high-convexity*, but the variable couldn't show any significant effect for the entire period. Since each group now had a tight duration spread, the duration variable was omitted in this regression.

Robust regression on monthly returns	Observations	R-square
	1318	0,8808
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0828664	0,000
Convexity	0,0015934	0,036
High Convexity Dummy	-0,0002163	0,717

Output 6.2.2: Regression using 'high convexity dummy', showing insignificance

6.3 Time Period-by-Time Period Results

Our tests cannot show any significant effect of convexity on returns for the entire period, neither for the yearly or monthly datasets. But there might still be significant effects hidden in single periods. To test this, we set up regressions for every single time period, using the 55 observations we had in each time period. The results were two out of six insignificant time periods in the yearly data. The other four years exhibited negative effect on returns from convexity, with significant p-values. In the monthly data, only seven out of twenty-four time periods were insignificant. Nine months showed that convexity gave significant negative effects on returns. Eight months in the period of 2008 to 2009 exhibited significant positive effects of convexity. The results of all the thirty regressions are presented in Appendix.

7. Analysis and Discussion

Considering the amount of data points we gathered and the established methods we used, we are confident our results are trustworthy and conclusions can be drawn from them. Our results presented in the results section exhibit insignificant effect on returns from convexity in both our yearly and monthly data. Therefore, our null hypothesis cannot be rejected. This is consistent with all earlier empirical studies on convexity and suggests that convexity was efficiently priced on the U.S. treasury market in general between 2007 and 2012.

As stated in our alternative hypothesis, we expected the abnormal volatility in our research period to positively affect the returns associated with convexity. Our results cannot support the alternative hypothesis and our conclusion is that convexity was efficiently priced over the entire research period. Consequently, investment strategies to capitalize on large shifts in yield by increasing the convexity exposure cannot be supported in our empirical study, even in the most volatile periods. These results are important for bond investors and extends the traditional view of convexity.

It is interesting that convexity seems to be precisely priced even during an ongoing financial crisis. Barber & Copper (1997) and Kahn & Lochoff (1990) among others establish that convexity is consistent with arbitrage-free pricing, implying that convexity is priced according to current yield volatility expectations. It is however clear that volatility in yields was abnormal in our research period, as presented in chapter 3.8. In the beginning of 2007, both a financial crisis and quantitative easing were unexpected, if not unheard of. Despite this development of the U.S. treasury market in the time period we studied, convexity failed to generate excess returns.

Like Ben Dor, et al. (2006), we arrive at the conclusion that over a long time horizon, convexity is not the appropriate tool to exploit volatility expectations exceeding the market average. Unlike Ben Dor, et al., we are not ending up in the recommendation to use options strategies through Lehman Brothers services.

Our time period-by-time period analysis is more ambiguous. In four out of six years, convexity had a significant negative effects on returns, implying that convexity from time to time is overpriced. Similar results have been found by Lacey & Nawalkha (1993). According to our results, it would have been beneficial to short high convexity and to go long low convexity given equal duration in 2007, 2009, 2010 and 2011, which is inconsistent with efficient market theories.

The same pattern is shown on the monthly data, where seventeen out of twenty-four time periods exhibit significant effects of convexity on returns. Convexity generated significant positive returns in eight of these months. To our knowledge, this is the first study observing positive effects of convexity empirically. Despite this, we are surprised to see that convexity did not generate more months of positive returns in 2008 and 2009, considering the volatility following the financial crisis. Our conclusion from the time period-by-time period analysis is that convexity might have been mispriced during fractions of our research period 2007 to 2012. The findings imply that convexity can be an appropriate tool to exploit volatility expectations other than the market average, in short time periods.

In short, our research indicates that convexity is efficiently priced over time, but convexity can yield excess returns in shorter time frames. Our real contribution to finance is the conclusion that efficient pricing of convexity generally held even in the mid of a financial crisis.

When replicating Lieu & Shyy's study from 1994, we discovered that convexity and duration were highly correlated even within the duration classes. Duration classes are thus an insufficient way to control for duration. This leads to an invalid dummy of 'high convexity within the duration classes' which is biased and to a large extent only measures the effect of duration on returns. Our findings emphasize the including of duration in convexity regressions and could be seen as critique of Lieu & Shyy.

When it comes to the time period-by-time period analysis, we would like to highlight that the regressions are based on only 55 data points, making the statistical interpretation weaker than our main regressions. On the other hand, these 55 observations make up the entire universe of bullet U.S. treasury bonds traded in U.S. dollars and active in the entire period of 1st January 2007 to 1st January 2013. The question is whether our conclusions hold for a broader spectra of bonds.

As described in the theoretical framework, the traditional convexity theories only hold for infinitesimal parallel shifts of the yield curve. Our use of yearly data is thus questionable as the beginning convexity might not explain very much of the ending return. The fact that our results on the monthly data confirms our yearly results, strengthens the confidence of valid results.

Our 'change in YTM' variable could be a potential source of endogeneity, since yield-to-maturity is a function of the bond price. However, we believe that our argument for exogeneity is greater when we control for 'change in YTM', since it is such an instrumental variable. Omitting 'change in YTM' could be a source of endogeneity in itself due to its obvious impact on returns. Without 'change in YTM', chances are coefficients of 'convexity' are more or less random. The results of our regressions without the 'change in YTM' variable show that the interpretation of our study would not differ if the variable was excluded. The fact that Kahn & Lochoff includes yield-to-maturity in their much cited research from 1990, validates our regression model.

Regarding our data sample, we are relying heavily on Bloomberg for all of our information. The precise return approximation provided in Appendix along with the consistency in results compared to earlier studies, validates the inputs from Bloomberg, to our belief.

With use of accepted norms, we have described the effects of convexity on the world's most liquid bond market in 2007 to 2012. But our results are certainly dependent on the time period and the market of choice. In a less liquid market with less frequently traded bonds-, or in a completely different time period, the effects of convexity may differ.

By studying convexity in a period of extreme volatility, we believe we have closed the circle of convexity research on U.S. treasury securities. How convexity interacts with returns on riskier bonds, like corporate bonds with aspects of credit risk, is relatively unclear. The pricing of convexity is likely to differ across market and further research should aim to identify markets where convexity might be exploited to generate excess returns. Furthermore, it should be possible to extract information about the market average volatility expectation from the average price of convexity. This relationship is yet an unbeaten track and we encourage further research in the field. Our data set was sufficient to establish an average "price" of convexity in terms of given up yield-to-maturity in our time period. But we did not find that question relevant for our purpose of the study. To explore the development of the "price" of convexity over time is a suggestion for further research. Old truths are being questioned in the field of fixed-income investments right now, with yield-to-maturities below zero on plenty of treasury securities. In general, we encourage studies on old fixed-income theories in this new market environment.

8. Conclusion

This thesis studies the effect of convexity on bond returns in the U.S. treasury market between 2007 and 2012. We collected data on 55 U.S. treasury bullet bonds, issued in U.S. dollars, active the entire period between 1st of January 2007 and 1st of January 2013. The data aggregates to two sets, one data set of 330 yearly observations in 2007 to 2012 and one data set of 1318 monthly observations in 2008 to 2009. The abnormal interest rate volatility in our data sets motivates our choice of research period. The set of monthly data complement the yearly data with shorter time horizon, greater amount of data points, and even greater abnormal volatility which is of special interest due to the theoretical properties of convexity.

Our multiple linear regressions exhibit insignificant returns associated with convexity in both our yearly and monthly data. Time period-by-time period, significant returns in both the positive- and negative range is evident in certain time periods. Our conclusion, which is consistent with earlier convexity studies, is that convexity is arbitrage-free priced over longer time frames. In shorter time frames however, convexity can yield excess returns in short periods of abnormal volatility. Convexity as a tool to exploit above market average volatility expectations may thus be useful in the short-run, but in the long-run the efficient pricing makes convexity an inappropriate tool. Our key contribution is that arbitrage-free pricing of convexity generally held even in the abnormal volatility which followed the financial crisis.

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10. Appendix

10.1 Data

Our data consist of two sets with 330 and 1318 observations respectively. Both our data sets are highly heteroscedastic, as can be observed below.

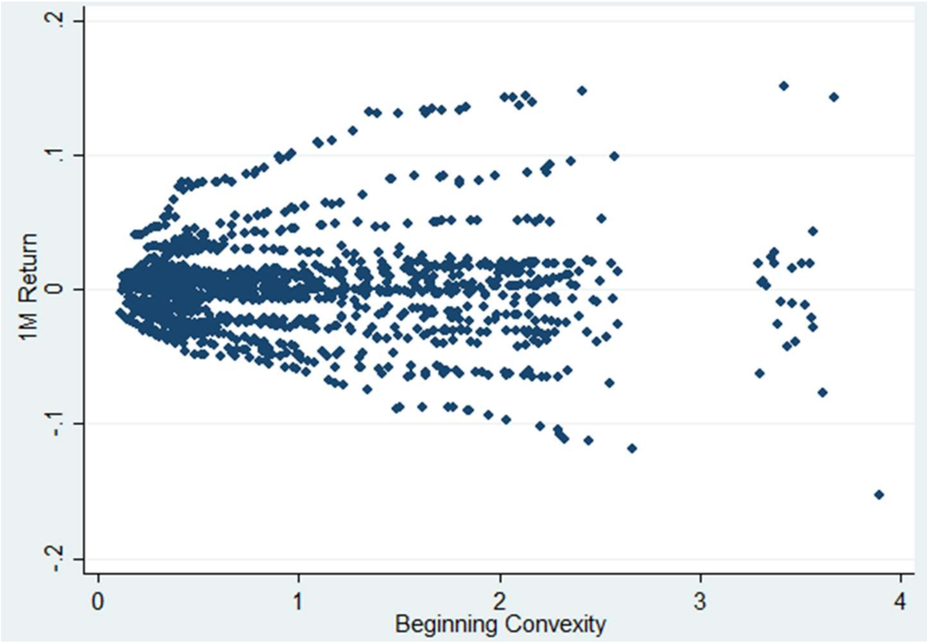

Figure 10.1.1: Scatterplot of monthly data, showing heteroscedasticity

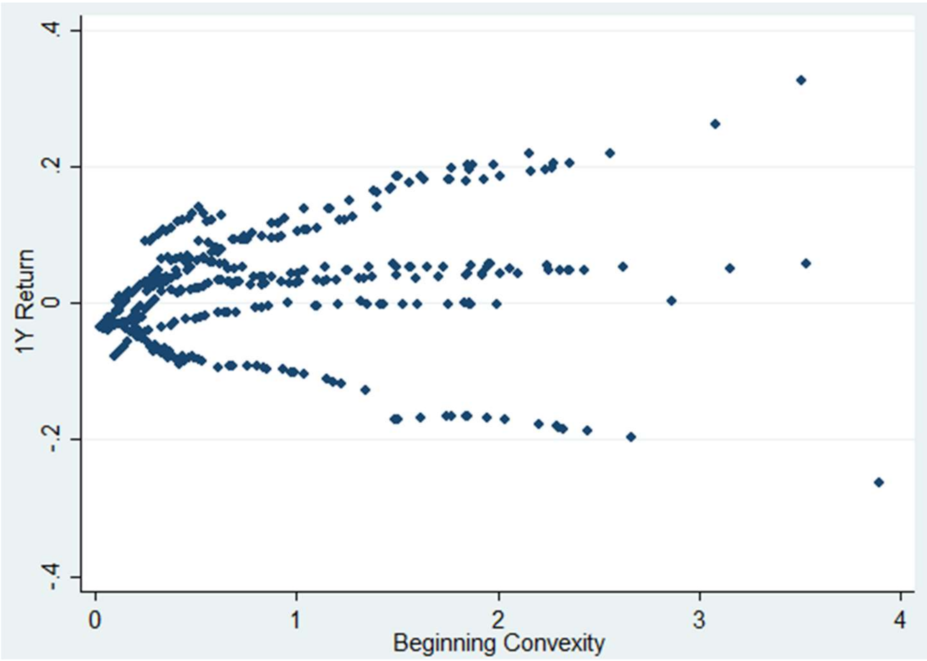

Figure 10.1.2: Scatterplot of yearly data, showing heteroscedasticity

The heteroscedasticity is natural to convexity, considering its properties. As described in the theoretical framework, convexity is the second order condition of duration and is thus highly correlated with duration. Hence, higher convexity leads to higher bond volatility. Higher bond volatility means higher variability in the error term in our OLS regression. Convexity is by definition violating the homoscedasticity criteria since it explains variance in the error term. We come around this problem by using the robustness function in Stata for all our regressions. The robustness function controls for heteroscedasticity by increasing the standard errors of the beta coefficients in the regressions, resulting in higher requirements for significant results.

Another potential challenge with our data is endogeneity, a regression variable explaining the level of the error term. A variable of especial concern is 'yield-to-maturity change'. As explained in the theoretical framework, yield-to-maturity is simply a function of the current bond price. Our dependent variable 'return' is as well closely related to the bond price. Change in yield-to-maturity will explain the bond return to a large extent and could thus explain the error term in the regressions to a large extent. Considering the extremely low R-square values of our regressions when excluding 'yield-to-maturity change', as demonstrated below, we chose to include the variable in our results anyway.

Robust regression on yearly returns	Observations	R-square
	330	0,0512
Regressors	**Coeffecient**	**P-value**
Duration	0,0146438	0,067
Convexity	-0,0385889	0,344

Table 10.1.3: Regression of only duration and convexity on yearly returns, showing low R-square

Robust regression on monthly returns	Observations	R-square
	1 318	0,0003
Regressors	**Coeffecient**	**P-value**
Duration	-0,0008031	0,698
Convexity	0,002614	0,774

Table 10.1.4: Regression of only duration and convexity on monthly returns, showing low R-square

A way for us to get around the potential endogeneity problem was to use a proxy for 'yield-to-maturity change' in a two-stage least squares model, like a U.S. Treasury Yield Index for an example. As you can see below, 'yield-to-maturity change' and 'index yield change' is highly correlated.

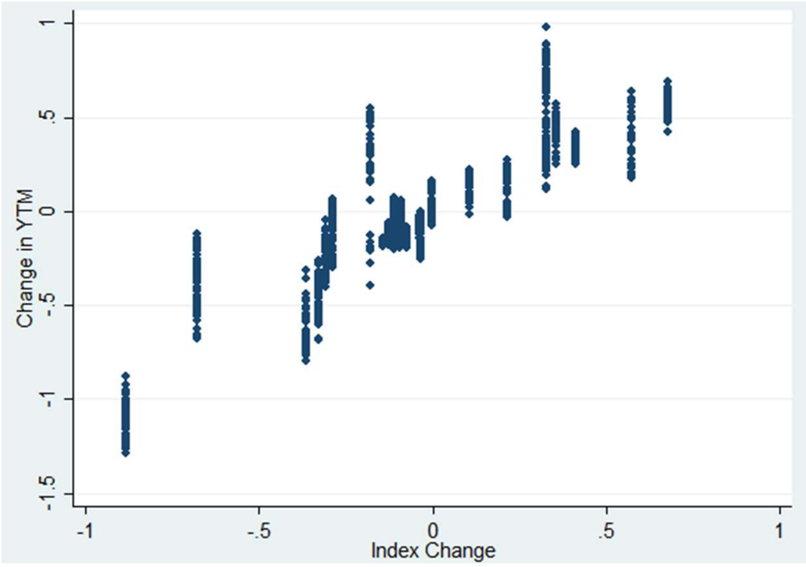

Figure 10.1.5: Scatterplot of 'change in YTM' and 'index yield change' in monthly data, showing correlation

A problem using a 2SLS model with 'index yield change' as an instrument for 'change in yield-to-maturity' could be that they might be too correlated. The 55 bonds in our regressions are as a matter of fact included in the Treasury Yield Index. 'Index yield change' might still provide information about the error term in the regressions and the endogeneity problem might not be avoided. Furthermore, omitting the 'yield-to-maturity change' could in itself be a source of endogeneity, since it is such an instrumental variable and in general explaining around 90 % of the variance in 'returns' in our regressions. In conclusion, we believe that including 'yield-to-maturity change' in our analysis gives our research validity. Controls for yield have been included in Kahn & Lochoff's study from 1990. We will provide results of the regressions including 'index yield change' in Table 10.1.6 and Table 10.1.7, just to prove that they are consistent with our main results. As you can see in Table 10.1.3 and Table 10.1.4, the results would not have changed even if we only regressed 'duration' and 'convexity' on 'returns'.

Robust regression on monthly returns	Observations	R-square
	1318	0,6303
Regressors	**Coeffecient**	**P-value**
Change in U.S Treasury Yield Index	-0,0763966	0,000
Duration	-0,0010203	0,488
Convexity	0,0031826	0,625

Table 10.1.6: 'Change in YTM' replaced by index, convexity still insignificant

Robust regression on yearly returns	Observations	R-square
	330	0,7776
Regressors	**Coeffecient**	**P-value**
Change in U.S Treasury Yield Index	-0,0795803	0,000
Duration	0,0150965	0,003
Convexity	-0,0404605	0,106
Time Period	0,0085029	0,000

Table 10.1.7: 'Change in YTM' replaced by index, convexity still insignificant

A final statistical trap we could fall into is time trend in our data. Since we are analyzing panel data, not considering time trends could result in spurious relationships just due to the passage of time (Greene, 2002). When dealing with time series, we need to ensure weakly dependent variables. Our choice of 'returns' instead of 'price' as dependent variable makes strongly depending variables highly unlikely. Looking at the data in Figure 10.1.8 and Figure 10.1.9, it is doubtful whether there is any observable time trend.

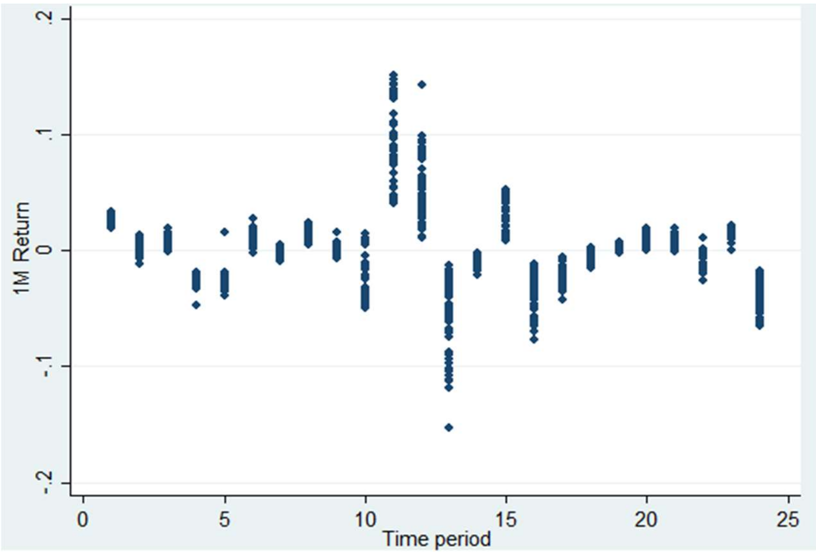

Figure 10.1.8: Plot of monthly returns over 24 time periods, showing no time trend

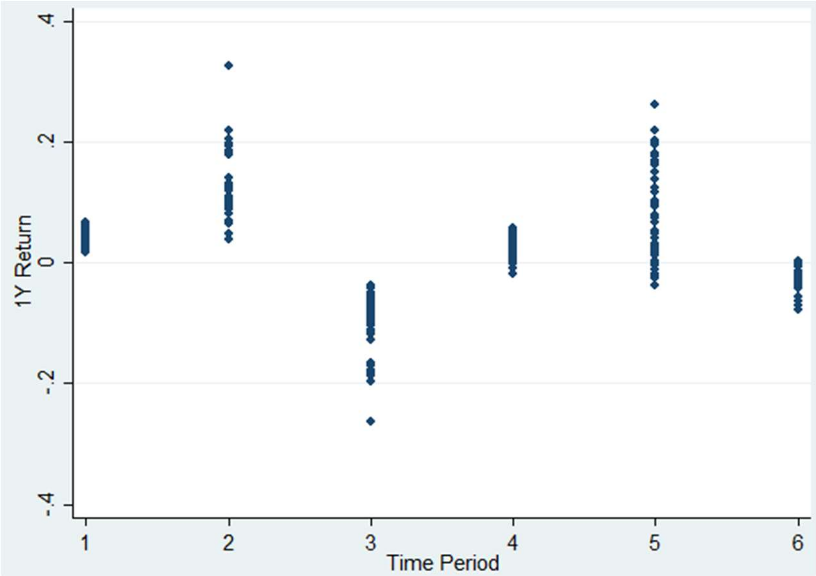

Figure 10.1.9: Plot of yearly returns over 6 time periods, might showing a slightly negative time trend

In order to deal with this potential problem, we created 'time period' variables and included them in the regressions. As it turned out, the 'time period' variable was insignificantly different from zero in the monthly data, while it was significantly different from zero in the yearly data. Proof is shown in Table 10.1.10 and Table 10.1.11 below. As a consequence, the 'time period' variable will be included in the results from the yearly data but excluded in the results from the monthly data.

Robust regression on monthly returns	Observations	R-square
	1 318	0,8809
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0829411	0,000
Duration	0,0005571	0,593
Convexity	-0,0005308	0,921
Time Period	0,0000243	0,491

Table 10.1.10: Regression of monthly data, note the insignificant p-value of 'time period'

Robust regression on yearly returns	Observations	R-square
	330	0,8593
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0843525	0,000
Duration	0,0073846	0,073
Convexity	0,0065076	0,749
Time Period	-0,0024436	0,006

Table 10.1.11: Regression of yearly data, note the significant p-value of 'time period'

As validation of our data set we will provide proof of better return approximation with both duration and convexity as appreciators, than solely duration as appreciator. The formulas and theories behind these approximations are described in the theoretical framework. These regressions are merely evidence that our data set holds for the theoretical properties of convexity, and are not part of our result. Note that the beta coefficient- and the R-square of the convexity approximation are slightly closer to one. A beta coefficient of one implies that a one-unit change in the independent variable, changes the dependent variable with one unit. Similarly, R-square reflects the degree of explanation in the dependent variable's variance, where one is full explanation.

Robust regression on monthly returns	Observations	R-square
	1 318	0,9942
Regressors	**Coeffecient**	**P-value**
Predicted return with convexity	1,022858	0,000

Table 10.1.12: Regression of predicted return with both duration and convexity on monthly returns

Robust regression on monthly returns	Observations	R-square
	1 318	0,9941
Regressors	**Coeffecient**	**P-value**
Predicted return with duration	1,022948	0,000

Table 10.1.13: Regression of predicted return with only duration on monthly returns

Looking at the distributions of our two variables of most interest, returns and convexity, we can conclude that returns are normal distributed while the convexity histogram resembles a log-normal distribution. These facts have no impact on our analysis, but they are important to describe our data.

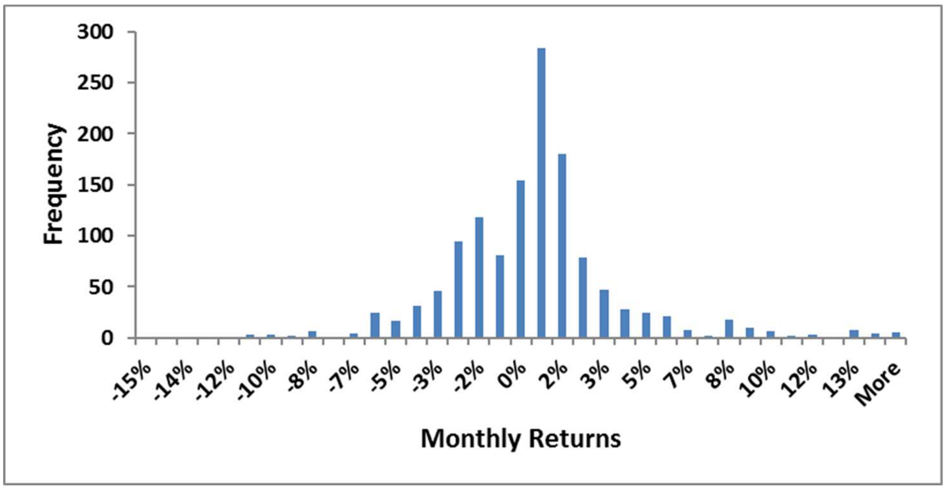

Figure 10.1.14: Histogram of monthly returns, resembling a normal distribution

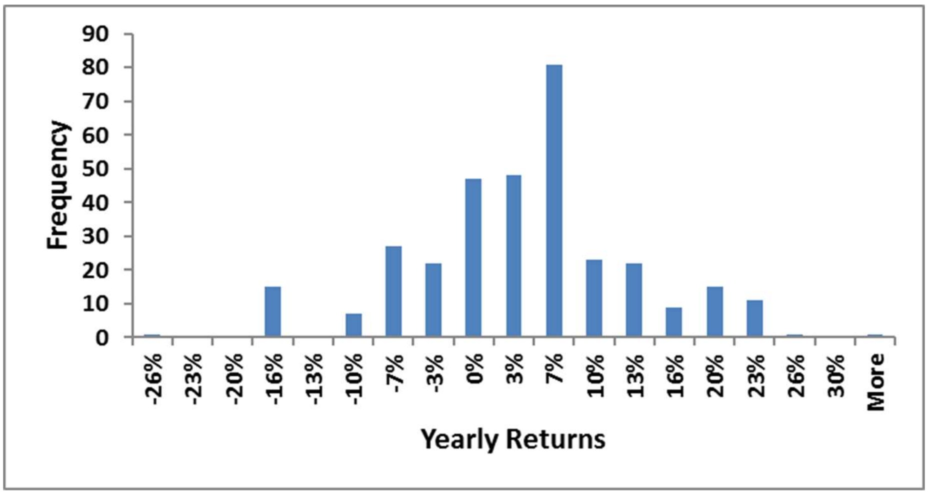

Figure 10.1.15: Histogram of yearly returns, resembling a normal distribution

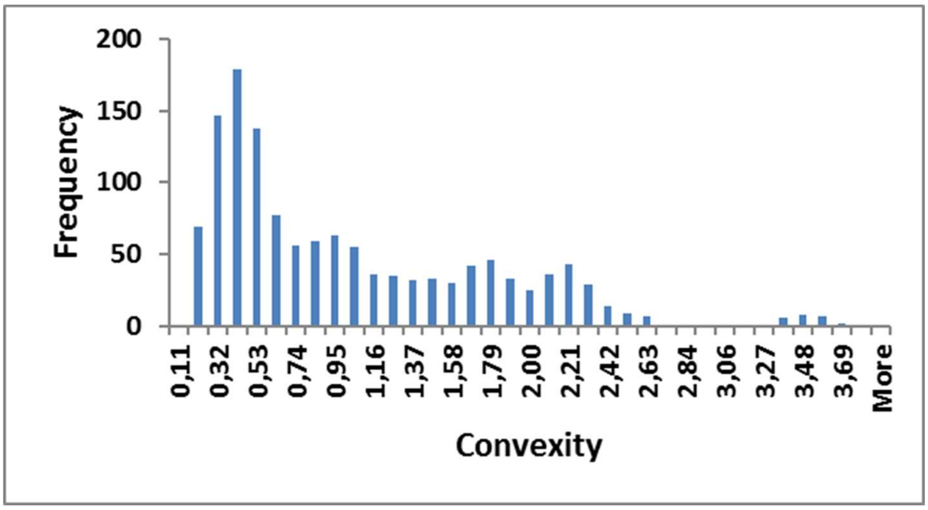

Figure 10.1.16: Histogram of convexity in monthly data set, resembling a log-normal distribution

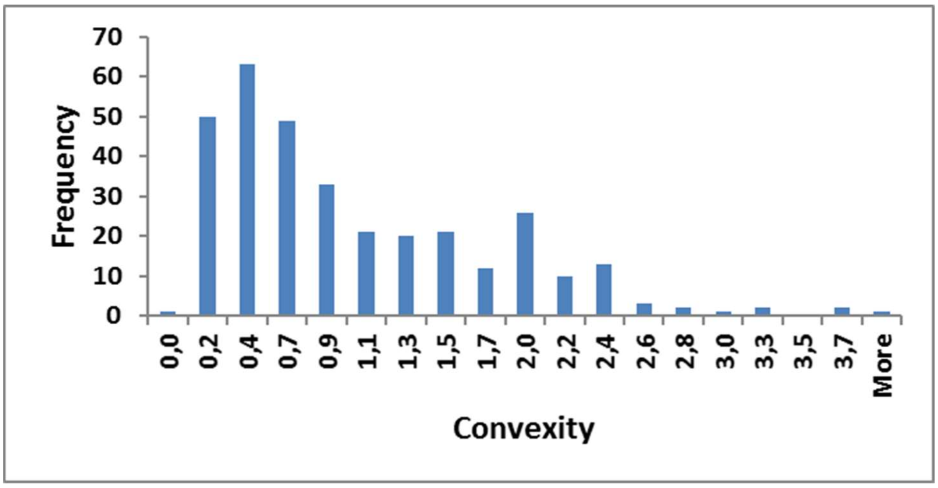

Figure 10.1.17: Histogram of convexity in yearly data set, resembling a log-normal distribution

10.2 Time Period-by-Time Period Results

Robust regression on returns year 1	Observations	R-square
	55	0,8964
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,1456453	0,000
Duration	0,0345493	0,000
Convexity	-0,077998	0,000

Output 10.2.1: Regression of 2007, convexity negatively significant

Robust regression on returns year 2	Observations	R-square
	55	0,9854
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,1002961	0,000
Duration	0,0267643	0,000
Convexity	-0,0092048	0,661

Output 10.2.2: Regression of 2008, convexity insignificant

Robust regression on returns year 3	Observations	R-square
	55	0,9782
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0324983	0,001
Duration	-0,004799	0,000
Convexity	-0,0297901	0,000

Output 10.2.3: Regression of 2009 convexity negatively significant

Robust regression on returns year 4	Observations	R-square
	55	0,8297
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,1471759	0,010
Duration	0,0266528	0,000
Convexity	-0,0450859	0,000

Output 10.2.4: Regression of 2010, convexity negatively significant

Robust regression on returns year 5	Observations	R-square
	55	0,9900
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0322494	0,000
Duration	0,0340723	0,000
Convexity	-0,0374969	0,002

Output 10.2.5: Regression of 2011, convexity negatively significant

Robust regression on returns year 6	Observations	R-square
	55	0,7152
Regressors	**Coeffecient**	**P-value**
Change in YTM	0,0246585	0,220
Duration	0,0084359	0,000
Convexity	-0,0211755	0,062

Output 10.2.6: Regression of 2012, convexity insignificant

Robust regression on returns month 1	Observations	R-square
	55	0,9210
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0770372	0,000
Duration	0,0098517	0,000
Convexity	-0,0262468	0,000

Output 10.2.7: Regression of January 2008, convexity negatively significant

Robust regression on returns month 2	Observations	R-square
	55	0,9884
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0692667	0,000
Duration	0,0038441	0,000
Convexity	-0,0137566	0,000

Output 10.2.8: Regression of February 2008, convexity negatively significant

Robust regression on returns month 3	Observations	R-square
	55	0,9593
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0730663	0,000
Duration	-0,0000282	0,949
Convexity	0,0042494	0,010

Output 10.2.9: Regression of March 2008, convexity positively significant

Robust regression on returns month 4	Observations	R-square
	55	0,4200
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0362778	0,000
Duration	-0,001725	0,097
Convexity	0,0027746	0,439

Output 10.2.10: Regression of April 2008, convexity insignificant

Robust regression on returns month 5	Observations	R-square
	55	0,2670
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0374807	0,053
Duration	-0,0015352	0,087
Convexity	-0,0014455	0,639

Output 10.2.11: Regression of May 2008, convexity insignificant

Robust regression on returns month 6	Observations	R-square
	55	0,8443
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0721689	0,000
Duration	-0,0000755	0,923
Convexity	0,0065554	0,026

Output 10.2.12: Regression of June 2008, convexity positively significant

Robust regression on returns month 7	Observations	R-square
	55	0,9665
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0742466	0,000
Duration	0,0019567	0,000
Convexity	-0,0073489	0,000

Output 10.2.13: Regression of July 2008, convexity negatively significant

Robust regression on returns month 8	Observations	R-square
	55	0,9787
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0385347	0,010
Duration	0,0015606	0,000
Convexity	0,0008136	0,273

Output 10.2.14: Regression of August 2008, convexity insignificant

Robust regression on returns month 9	Observations	R-square
	55	0,9622
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0558092	0,000
Duration	-0,002196	0,000
Convexity	0,010905	0,000

Output 10.2.15: Regression of September 2008, convexity positively significant

Robust regression on returns month 10	Observations	R-square
	55	0,9679
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0513387	0,000
Duration	-0,0088739	0,000
Convexity	0,0256338	0,006

Output 10.2.16: Regression of October 2008, convexity positively significant

Robust regression on returns month 11	Observations	R-square
	55	0,9932
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0640139	0,000
Duration	0,0156621	0,000
Convexity	-0,0148122	0,064

Output 10.2.17: Regression of November 2008, convexity insignificant

Robust regression on returns month 12	Observations	R-square
	54	0,9931
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0630084	0,000
Duration	0,0034921	0,010
Convexity	0,0165538	0,003

Output 10.2.18: Regression of December 2008, convexity positively significant

Robust regression on returns month 13	Observations	R-square
	55	0,9985
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0552884	0,000
Duration	-0,0024579	0,000
Convexity	-0,0195354	0,000

Output 10.2.19: Regression of January 2009 convexity negatively significant

Robust regression on returns month 14	Observations	R-square
	55	0,8641
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0612084	0,000
Duration	-0,0009714	0,097
Convexity	0,0004478	0,854

Output 10.2.20: Regression of February 2009, convexity insignificant

Robust regression on returns month 15	Observations	R-square
	55	0,9898
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,041889	0,000
Duration	0,0098759	0,000
Convexity	-0,0236983	0,000

Output 10.2.21: Regression of March 2009, convexity negatively significant

Robust regression on returns month 16	Observations	R-square
	55	0,9960
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,043372	0,000
Duration	-0,0048559	0,000
Convexity	0,0003933	0,850

Output 10.2.22: Regression of April 2009, convexity insignificant

Robust regression on returns month 17	Observations	R-square
	55	0,9749
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0492114	0,000
Duration	-0,0033054	0,000
Convexity	0,0040068	0,098

Output 10.2.23: Regression of May 2009, convexity insignificant

Robust regression on returns month 18	Observations	R-square
	55	0,9499
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0509648	0,000
Duration	-0,0013181	0,091
Convexity	0,0067182	0,029

Output 10.2.24: Regression of June 2009, convexity positively significant

Robust regression on returns month 19	Observations	R-square
	55	0,8558
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0812162	0,000
Duration	0,0016968	0,000
Convexity	-0,0036053	0,000

Output 10.2.25: Regression of July 2009, convexity negatively significant

Robust regression on returns month 20	Observations	R-square
	55	0,9882
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0855362	0,000
Duration	0,0019823	0,000
Convexity	-0,0014249	0,012

Output 10.2.26: Regression of August 2009, convexity negatively significant

Robust regression on returns month 21	Observations	R-square
	55	0,9734
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0794714	0,000
Duration	0,0007805	0,000
Convexity	0,0020765	0,001

Output 10.2.27: Regression of September 2009, convexity positively significant

Robust regression on returns month 22	Observations	R-square
	55	0,9438
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0470836	0,007
Duration	0,0005312	0,614
Convexity	-0,00758	0,010

Output 10.2.28: Regression of October 2009, convexity negatively significant

Robust regression on returns month 23	Observations	R-square
	55	0,8184
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0776226	0,000
Duration	0,0076288	0,000
Convexity	-0,0219735	0,000

Output 10.2.29: Regression of November 2009, convexity negatively significant

Robust regression on returns month 24	Observations	R-square
	54	0,9917
Regressors	**Coeffecient**	**P-value**
Change in YTM	-0,0538263	0,000
Duration	-0,0078114	0,000
Convexity	0,0130045	0,000

Output 10.2.30: Regression of December 2009, convexity positively significant.