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Risk, Occupational Choice, and Inequality

by

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Abstract

This essay presents a new theory explaining increased wage inequality. A standard endogenous growth model is augmented with occupational choice of high-skill workers. Depending on the occupational choice, high-skill workers earn either a certain or uncertain income. Wage inequality, measured by the average wage of high-skill workers divided by the average wage of low-skill workers, can increase or decrease due to an *increased* supply of high-skill workers.

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Keywords: Distribution, Wages, Cooperatives,
               Technological Change and Economic Growth

1 Introduction

There is a growing consensus that over the last 25 years, the dispersion of income and wages have increased in developed countries. The U.S. and the U.K experienced a larger dispersion earlier then most other countries but later several other countries fell in line. Those changes are well documented by among others Förster and Pearson (2002). The dispersion of wages can be decomposed into dispersion among individuals with similar characteristics (residual wage inequality) and dispersion between individuals with different characteristics, such as for example skill, experience or gender.

In order to briefly exemplify the changes, consider the changes in the U.S. during the period 1973 to 1989. The wage rate for the 10 percent at the top of the wage distribution increased by approximately 20%, greatly surpassing the growth rate of wages at the median, which was approximately 5%. Even more strikingly, the wage rate for the poorest 10% *decreased* by about 25% during the same period (Juhn et al. 1993). Those divergent trends for the wage rates of the lowest paid and the highest paid workers are likely to have a profound impact on income. This is confirmed by Gottschalk, reporting that the real income ratio between the 80th and 20th percentiles in the distribution shows a clear upward trend in the period 1968 to 1992 (Gottschalk 1997, p. 23).

This paper focuses on two components of inequality. On the one hand, the distribution of wages *between* individuals with different skill levels. On the other hand, the distribution of wages *among* high-skill workers.

The dispersion of wages between high-skill workers and low-skill workers in the U.S. has increased. Gottschalk's annual (log) wage regressions, including a dummy variable for college graduates shows a decreasing trend during the first part of the period, 1970 – 1980, and there after an increasing trend during the 1980s (Gottschalk 1997).

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However, as the difference in wages *between* college graduates and high school graduates increased, wage dispersion *among* both groups' members also widened. There is a very small fraction at the bottom of the wage distribution of college graduates that experienced decreased real wages whereas college graduates at the top of the distribution gained more then 20 percent in real terms (Juhn et al. 1993, p. 422, fig. 6).

1.1 Contribution

The central idea in this paper is that workers can make an active choice concerning risk exposure. Exposing oneself to more risk in the model is considered a substitute for decreased wage earnings. Endogenous choices of occupation, i.e. whether or not to expose oneself to risk, changes the economy's distribution of wages. Workers characterized by relative risk aversion would never substitute lower earnings for increased risk unless paid a risk premium. Hence there must exist a sector in the economy to which workers can switch and which is characterized by higher but more uncertain returns. The research sector is assumed to be such a sector in this paper.

By this approach, this paper draws heavily on the literature on entrepreneurship, which can be traced back to the writings of Knight (1921). Knight's ideas have been formalized, at least partially, by Kanbur (1979) and Kihlstrom and Laffont (1979). A fundamental property of those models is that entrepreneurs bear risk. Workers chose to become entrepreneurs only if the *expected* utility of being an entrepreneur exceeds the *certain* utility of ordinary work with a certain wage rate. The difference in this paper is that entrepreneurs are high-skill workers that form co-operatives, and hence do not employ workers.

A second strand of literature which this paper relies on is the endogenous growth

models where growth is driven by development of new intermediate goods. This idea is formalized by Romer (1990). The model presented in this paper augments Romer's model by introducing stochastic development of new intermediate goods.

The model postulates to two main characteristics of research activity. First, it is assumed that only high-skill workers can work in the research sector. This is a crude enforcement of the assumption that research is human capital intensive (Barro and Xavier 1995, p. 179) and hence high-skill workers have an advantage over low-skill workers. Second, research is stochastic. Researchers directly face uncertainty concerning the products ex post productivity or ex post capacity to generate utility and therefore its value. It is common to model firms as risk neutral. Risk neutral firms maximize expected profits and, hence, Pareto optimality implies that risk averse agents negotiate wage contracts with no uncertainty. This paper models research firms as co-operatives. The members share the revenues and the firm's decisions are determined by the representative member, trying to maximize his or her utility.

The choice to model research firms as co-operatives is not to be taken literally. Co-operatives are used in order to keep the analysis simple and emphasis research firms' need to share risk with their employees. Risk sharing between firms and employees can take on several shapes ranging from for example flexible working hour arrangements to profit bonuses and options programs where employees are offered stock shares.

The model abstracts from all kinds of risk sharing by financial markets. Not allowing any insurance possibilities via financial markets unrealistic but this assumption is made to simplify the model. The key assumption is that firms and workers can benefit from risk sharing. It is however to important to recognize that precluding risk-sharing can have strong implications. As is shown by Newman (1999), combining the standard theory of entrepreneurship (Kanbur 1979; Kihlstrom and Laffont 1979) with moral haz-

ard considerations and some risk sharing can reverse some of the standard results. Since the analysis is very similar to the standard entrepreneur theory some caution should be applied.

Within the framework in this paper, increasing the supply of high-skill workers shifts high-skill workers into research co-operatives, paying a stochastic wage rate. On the one hand, more high-skill workers earn a stochastic wage rate, increasing the residual wage inequality among high-skill workers. On the other hand more high-skill workers earn a risk premium, tending to increase the average wage rate for high-skill workers. To summarize, the two main hypotheses investigated in this paper are:

1. An increased supply of high-skill workers *increases* the wage dispersion *among* high-skill workers.

2. An increased supply of high-skill workers *increases* the wage dispersion *between* high-skill and low-skill workers.

The first hypothesis refers to the residual wage inequality for high-skill workers, while the second hypothesis concerns the skill premium.

1.2   Related Literature

The analysis in this paper does not fit into any of the three broad categories generally used to explain changes in the distribution, namely skill-biased technological change (Acemoglu 1998; Krusell et al. 2000), increased trade (Aghion et al. 1999; Borjas and Ramey 1995; Wood 1995, 1998), or institutional change (DiNardo and Lemieux 1997; Fortin and Lemieux 1997).

The idea that an increased supply of high-skill workers can *increase* the skill premium is not new. In Acemoglu (1998), and also Kiley (1999), an increased supply of

high-skill workers can increase the skill premium due to an increased market size for inventions, *directed towards* high-skill workers, possibly increasing the skill premium in the long run. Acemoglu's paper is remotely connected to this paper in the sense that the skill premium increases due to changes in the research process.

In Machin and Manning (1997) and Acemoglu (1999), increasing the supply of high-skill workers motivates firms to open vacancies taylored to high-skill workers, thereby increasing the productivity of high-skill workers relative to low-skill workers. This in turn increases the relative wage for high-skill workers.

In the paper by Rosén and Wasmer (2005), the skill premium is positively correlated with the relative supply of high-skill workers due to the increased outside option of firms in their wage negotiations. In Rosén and Wasmer's paper wages are determined in Nash-bargains and increases in firms' outside option hurts low paid (i.e. low-skill) workers more then high-skill workers, increasing the skill premium.

It is reasonable to assume that there is an asymmetry between high-skill and low-skill workers. While high-skill workers can occupy low-skill jobs, low-skill workers cannot occupy high-skill jobs. Auerbach and Skott (2005) investigate the impact of a skill neutral productivity slow down given this asymmetry. As productivity decreases, more high-skill workers occupy low-skill jobs and residual wage inequality of high-skill workers increases.

A somewhat similar argument is presented in Mendez (2002). For incentive reasons, workers producing goods in the early stage of the product cycle are paid efficiency wages, while workers in the later stage are paid competitive wages. This creates a wage gap between workers in the early and late stage of the product cycle, and workers that fail to find work producing products in the early stage earn a lower wage rate.

The paper is organized as follows. Section 2 describes the formal model. Section

3 summarizes the results obtained. Section 4 concludes and summarizes the findings. Appendix A contains a full record of notation. The subsequent appendix contain proofs and derivations.

2 Model

The model used to study the occupational choices of high-skill workers originates from Romer (1990)'s endogenous growth model. Romer's model is characterized by horizontal innovations, that is new innovations do not replace innovations made earlier but complement them. A vertical innovation process is modeled in, for example, Aghion and Howitt (1992). In their model inventions lead to monopoly power, but by creative destruction, any new invention erodes the previous monopolist's profit, whereas in Romer's model, monopoly profits last forever. This paper simplifies Romer's model by assuming that monopoly profits are eroded exogenously after one period and leaving out the distinction between designs and intermediate goods.

Leaving out the distinction between intermediate goods and designs simplifies the presentation of the model by reducing the number of concepts but can also be confusing. It is important to realize that already produced intermediate goods can not be stored and used in the subsequent period. Therefore, at the beginning of each period the number of intermediate goods ready to be used in production is zero. However there exists a variety of old intermediate goods that can be produced, i.e. in Romer's words, there exists a variety of designs for intermediate goods.

Workers live for one period and consume their entire wage. They derive utility from consuming w units of the single consumption good. Preferences over consumption are

described by expected utility under the CRRA utility function:

$$u(w) = \frac{w^{1-\theta}}{1-\theta} \quad \theta > 0.$$

At the expense of realism but for the benefit of simplicity all forms of non fully depreciating capital are excluded from the model. However, the model includes capital that is fully used up in the production process and therefore called *intermediate goods*.

The economy's total endowment of labor is normalized to unity. Workers can be divided into two categories depending on the worker's level of human capital, high-skill workers and low-skill workers. The fraction of high-skill workers is denoted ϕ, and hence $1 - \phi$ denotes the fraction of low-skill workers. The fraction of the labor force being considered high-skill is taken to be exogenous. While in reality, ϕ is endogenous in this analysis ϕ is exogenous. This is a reasonable assumption as long as the changes in the skill composition of the work force is slow relative to other responses. This seems reasonable in the present context, since switching occupation can be done several times during a lifetime while human investment in general are made once at young age.

It will be assumed that high-skill workers can replace low-skill workers, but not the opposite. This implies that high-skill workers never earn less than low-skill workers in equilibrium. If wages for low-skill workers were higher than wages for high-skill workers, some high-skill workers would switch to the low-skill occupation until both groups' wages were equalized. It will be assumed that there are enough low-skill workers, to ensure that wages of high-skill workers are higher than wages of low-skill workers.
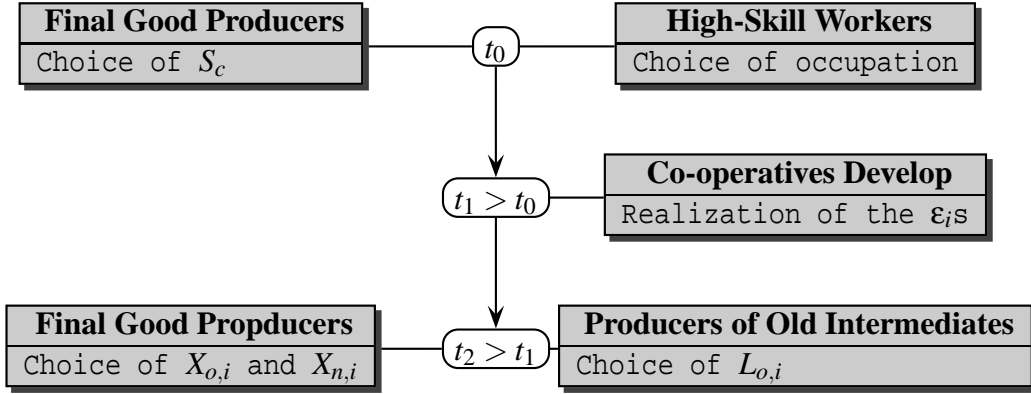
The sectors in the economy can be divided into two categories depending on what they produce. One of the sectors produce the single consumption good by employing high-skill workers and purchasing a variety of fully depreciating intermediate goods.

The other sector produces the set of intermediate goods. While low-skill workers only have the possibility to work in the intermediate goods sector, high-skill workers have the possibility to work in either sector, either as a worker in the consumption good sector or as a member of a co-operative in the intermediate goods sector. Co-operatives invent and produce new intermediate goods. A larger variety of intermediate goods increases the economy's total output, i.e. generates growth. Research co-operatives invent new intermediate goods and sell these to consumption good producers. Therefore the number of intermediate goods is endogenous. The gains from inventing a new intermediate good arise due to the one period monopoly profit derived from selling it to the producers of final goods. The share of high-skill workers doing research is denoted μ.

Intermediate goods are categorized as *old* or *new*. A new intermediate good is considered as an old intermediate good the subsequent period. Hence a research co-operative have monopoly for one period. The number of old intermediate goods, denoted by k_o, is predetermined while the number of new intermediate goods, denoted by k_n, is endogenous. Old intermediate goods are available at the beginning of the period and produced by low-skill labor while new intermediate goods are invented and produced by high-skill workers in the research sector. $X_{o,i}$ denotes the quantity of the ith old intermediate good and $X_{n,i}$ denotes the quantity of the ith new intermediate good.

Since high-skill workers can choose to work in the stochastic research sector and invent and produce new intermediate goods, at the time of production the set of intermediate goods is extended by those newly invented intermediate goods, i.e. k_n newly invented intermediated goods can be used by consumption good producers. Hence, at the time of production, consumption good producers can combine $k_o + k_n$ different intermediate goods to produce the consumption good.

Figure 1: Choice Sequence

Final Good Producers			**High-Skill Workers**
Choice of S_c	t_0		Choice of occupation

		Co-operatives Develop
	$t_1 > t_0$	Realization of the ε_is

Final Good Propducers		**Producers of Old Intermediates**
Choice of $X_{o,i}$ and $X_{n,i}$	$t_2 > t_1$	Choice of $L_{o,i}$

The figure shows the sequence of choices, within a single period.

2.1 The Consumption Good Sector

The model's single consumption good is produced by combining high-skill labor, and intermediate goods. Hence each firm producing the consumption good must decide upon hiring a certain *quantity* of high-skill labor, denoted S_c, and purchase a certain *quantity* of each available intermediate good, i.e. chose values for all members of the set $X = \{X_{o,i} \mid i = 1, 2, ..., k_o\} \cup \{X_{n,i} \mid i = 1, 2, ..., k_n\}$. The production function obeys to the standard properties, such as diminishing marginal productivity in each input and constant returns to scale. These properties eliminate profits in the consumption good sector. The old intermediate good *i*'s productivity is measured by γ_i and the new intermediate good *i*'s productivity is measured by ε_i. Formally the technology is described by:

$$Y = S_c^\alpha \left[\sum_{i=1}^{k_o} \gamma_i X_{o,i}^{1-\alpha} + \sum_{i=1}^{k_n} \varepsilon_i X_{n,i}^{1-\alpha} \right] \quad \alpha \in (0,1). \tag{1}$$

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The constant returns to scale property and perfect competition implies that firms in the consumption good sector can be modeled as a single price taking firm. In what follows, let $P_{o,i}$ denote the price of the ith old intermediate good, let $P_{n,i}$ denote the price of the ith new intermediate good, and let w_c denote the wage rate for high-skill workers employed in the consumption good sector. Competitive behavior implies that prices of intermediate goods and the wage rate are taken as given by producers of the final good.

In every period, a sequence of choices are made by different agents. The order in which choices are made is illustrated in Figure 1. At the start of every period, high-skill workers choose either to work for firms producing the final good or to start a co-operative and final good producers choose the amount of high-skill workers, S_c, to employ.

Next, every co-operative develops a new intermediate good and its productivity is revealed, i.e. the value of ε_i is revealed. Finally, final good producers choose the quantity of every intermediate input to use, and producers of old intermediate goods hire low-skill labor.

The objective function for the competitive final good producer is

$$
\begin{aligned}
\pi \;=\; & S_c^{\alpha}\left[\sum_{i=1}^{k_o}\gamma_i X_{o,i}^{1-\alpha}+\sum_{i=1}^{k_n}\varepsilon_i X_{n,i}^{1-\alpha}\right] \\
& - w_c S_c - \sum_{i=1}^{k_o}P_{o,i}X_{o,i}-\sum_{i=1}^{k_n}P_{n,i}X_{n,i},
\end{aligned}
\tag{2}
$$

where the price of the consumption good is normalized to unity. Profit maximizing behavior yields the following inverse factor demand functions:

$$
P_{o,i} \;=\; (1-\alpha)\gamma_i\frac{S_c^{\alpha}}{X_{o,i}^{\alpha}}
\tag{3a}
$$

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$$P_{n,i} = (1-\alpha)\varepsilon_i \frac{S_c^\alpha}{X_{n,i}^\alpha} \tag{3b}$$

$$w_c = \frac{\alpha Y}{S_c} = \alpha S_c^{\alpha-1} \left[\sum_{i=1}^{k_o} \gamma_i X_{o,i}^{1-\alpha} + \sum_{i=1}^{k_n} E\varepsilon_i X_{n,i}^{1-\alpha} \right]. \tag{3c}$$

To obtain those first order conditions, first maximize the objective function in (2), given the realizations of every γ_i and ε_i, with respect to every $X_{o,i}$ and every $X_{n,i}$, taking S_c as given. Those first order conditions are the profit maximizing choices corresponding to node t_2 in Figure 1, for *any* choice of S_c and *any* realization of the productivity variables, i.e. the different ε's.

At the t_0 node in Figure 1, the price taking producer of the final good maximizes the expected value of the objective function in (2), where every occurrence of $X_{o,i}$ and $X_{n,i}$ have been replaced, using the first order conditions in (3a) and (3b). This first order condition implies a zero profit condition, but by, again, using the first order conditions in (3a) and (3b) to eliminate every $P_{o,i}$ and $P_{n,i}$, the first order condition in (3c) is obtained. Those first order conditions define the cost minimizing mix of high-skill labor and intermediate goods. Since the tecnology is characterized by constant returns to scale and the firm act as a price taker, the scale of production is not determined by the first order conditions but from a set of equilibrium market clearing conditions.

The choice of technology ensures that high-skill workers in the consumption good sector share a constant fraction, α, of total output. The inverse demand functions are intuitive, a more productive intermediate good, γ or ε large, increases the expenditure on the intermediate good.

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2.2 The Intermediate Goods Sector

The knowledge necessary to produce old intermediate goods is freely available to all workers and hence it is most appropriate to model the market for old intermediate goods as a perfectly competitive market with zero profits. The knowledge necessary to produce new intermediate goods is only available for the workers that developed the new intermediate. Hence, a co-operative that develops a new intermediate good becomes the sole producer of that good. Therefore the market for a new intermediate good is characterized by monopoly.

2.2.1 Old Intermediate Goods

Old intermediate goods are produced by low-skill labor. To keep the analysis simple it is assumed that one unit of low-skill labor produces one unit of the intermediate good. Formally:

$$X_{o,i} = L_{o,i}. \tag{4}$$

The linear technology and the competitive market implies that the size of the firm is indeterminate but the industry can be modeled as if there is a single competitive firm. Hence the production of a specific old intermediate good $X_{o,i}$ is modeled as such. The profit maximization problem for a competitive firm producing an old intermediate i is:

$$\max_{L_{o,i}} \quad P_{o,i}L_{o,i} - w_o L_{o,i}. \tag{5}$$

The price taking firm takes $P_{o,i}$ and w_o as given but chooses the quantity of low-skill labor, $L_{o,i}$, to employ. The first order condition for this problem ensures zero profit,

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but does not, as noted above, pin down the number of employees in the firm. The first order/zero profit condition is:

$$w_o = P_{o,i}. \tag{6}$$

In order to find the number of employees engaged in producing an old intermediate good i, combine the zero profit equation (6) with the inverse demand function for old intermediates, given by (3a). Total output and total employment in a industry producing the old intermediate good $X_{o,i}$ become:

$$X_{o,i} = L_{o,i} = S_c \left[\frac{(1-\alpha)\gamma_i}{w_o} \right]^{\frac{1}{\alpha}}. \tag{7}$$

Wages for low-skill workers must be equal across all firms producing intermediate goods since there is no stickiness in the economy. Therefore, it is not necessary to index the wage rate for low-skill workers by the specific old intermediate good they produce. As seen by (6), all old intermediate goods are sold at the same price, but more productive intermediate goods are sold in larger quantities, see (7). Hence it is not necessary to index the price of old intermediates. Henceforth P_o will denote the price of any old intermediate good.

2.2.2 New Intermediate Goods

A co-operative producing a new intermediate good must spend a fixed amount, l, of labor units in order to develop the new intermediate good. Ex ante, research co-operatives cannot perfectly foresee the ex post productivity of the intermediate good they plan to develop. Hence there is some uncertainty concerning future revenues. Formally the uncertainty is modeled by the log-normal random variable ε_i. For shorter notation, σ^2 will

be used to denote the variance of ε_i, i.e. $var(\varepsilon_i)$. Hence:

$$\ln \varepsilon_i \sim N \left(\ln \left[\{E\varepsilon\}^2 \right] - \frac{1}{2} \ln \left[\{E\varepsilon\}^2 + \sigma^2 \right], \ln \left[1 + \frac{\sigma^2}{\{E\varepsilon\}^2} \right] \right). \tag{8}$$

Besides being non-negative, the log-normal distribution is chosen because its mathematical properties makes it easy to work with. The intuitive assumption that development requires some high-skill labor, $l > 0$ introduces the fixed cost necessary to assure that co-operatives do not produce an infinitely small output.

Once the new intermediate good is developed it takes one unit of high-skill labor to produce one unit of the new intermediate good. Formally the technology for research firms are:

$$X_{n,i} = S_{n,i} - l. \tag{9}$$

It is assumed that research firms are managed as co-operatives where total revenue is distributed uniformly among its members. Since research firms hire only high-skill workers, and all high-skill workers are identical, each member has the same objective. The objective of the co-operative is described by the maximization of the expected utility of the representative member. The co-operative's revenue is the quantity produced times the price. The price is given by the inverse demand function in (3b)

The maximization problem for the representative co-operative member is:

$$\max_{S_{n,i}, P_{n,i}, X_{n,i}} Eu \left[\frac{P_{n,i} \times X_{n,i}}{S_{n,i}} \right]$$

$$s.t. \quad \text{(3b)}, \text{(9)}. \tag{10}$$

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The first constraint, (3b), ensures that the research co-operative's price-quantity combination lies somewhere on the demand schedule of final good producers. The second constraint, (9), ensures that the research firm uses the only feasible production technology. Solving problem (10) defines the optimal co-operative size for research co-operatives, $S_{n,i}$, the optimal quantity to produce, $X_{n,i}$, the monopoly price, $P_{n,i}$ and the wage for the co-operative member, $w_{n,i}$:

$$S_{n,i} = \frac{l}{\alpha} \tag{11a}$$

$$X_{n,i} = \frac{1-\alpha}{\alpha} \times l \tag{11b}$$

$$P_{n,i} = (1-\alpha)^{1-\alpha} \left[\frac{\alpha S_c}{l}\right]^{\alpha} \varepsilon_i \tag{11c}$$

$$w_{n,i}(\varepsilon_i) = \alpha^{\alpha}(1-\alpha)^{2-\alpha} \left[\frac{S_c}{l}\right]^{\alpha} \varepsilon_i. \tag{11d}$$

The derivations are shown in Appendix B.1. Note that the co-operative's size, $S_{n,i}$, and quantity produced, $X_{n,i}$ is independent of i. This is intuitive, all research firms are identical ex-ante. Hence, the size of the co-operative, which is determined before the productivity of the new intermediate is realized, is equal across all research co-operatives. Further, since the co-operative members share the revenues all members are engaged in the production process, unconditioned on the ex-post productivity, ε_i. Hence, the quantity produced is equal across all research co-operatives. However, depending on the productivity, each co-operative sell their intermediate good at a different price and hence earn idiosyncratic revenues.

2.3 Occupational Choice

Based on the assumption that research is a human capital intensive activity, the model precludes low-skill workers to take any part in the economy's research process. High-skill workers, on the other hand, have the opportunity to work in the consumption sector earning a certain wage or to work in the research sector as a member of a research co-operative. A rational worker chooses the occupation that yields the highest expected utility. Since all high-skill workers have identical endowments and identical preferences all high-skill workers make the same occupational choice unless they are indifferent between the two choices.

It is possible to have an equilibrium where all high-skill workers are employed in the consumption good sector. With no research sector this model becomes a standard two sector model and nothing new is added. Hence, it is assumed that if all high-skill workers are employed in the consumption good sector the expected utility of forming a research co-operative will exceed the expected utility of working in the consumption good sector, formally:

$$u(w_c)\Bigg|_{S_c = \phi} \quad < \quad Eu\left[w_{n,i}(\varepsilon_i)\right]\Bigg|_{S_c = \phi} \quad . \tag{12}$$

With this additional assumption it is possible to show that a feasible equilibrium requires that the expected utility of working in either the research sector or in the consumption good sector is equal. *It is very important to note that this implies that every result derived later, must by checked against this condition, or equivalently, that the share of high-skill workers doing research, μ, is greater than zero.*

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If the share of high-skill workers doing research work, which is determined endogenously, is solved for and turns out negative, i.e. $\mu < 0$, then the utility of working in the consumption good sector exceeds the expected utility working in the research sector.

If the expected utility of working in the consumption good sector exceeds the expected utility of working in the research sector more high-skill workers choose to work in the consumption good sector. This increases S_c, and given (3c) and (11d) decreases the wage rate in the consumption good sector but increases the expected wage in the research sector. Hence S_c must become larger as long as the expected utility of working in the consumption good sector exceeds the expected utility of working in the research sector. In equilibrium the expected utility of working in the consumption good sector can not exceed the expected utility of working in the research sector unless all high-skill workers are employed in the consumption good sector. That scenario is discarded due to the assumption stated in (12).

If the expected utility of working in the research sector exceeds the expected utility of working in the consumption good sector all high-skill worker will choose to work in the research sector. To see why that is impossible in equilibrium see equations (3c) and (11d). If all high-skill workers are employed in the research sector S_c equals zero. From (3c) it is clear that the wage rate in the consumption good sector equals $+\infty$ and from (11d) it is clear that the expected income from research work equals zero. Hence the expected utility of research work can not exceed the expected utility of working in the consumption good sector in equilibrium.

Hence, a feasible equilibrium requires that the utility of working in the consumption sector equals the expected utility of working in the research sector, $u(w_c) = Eu(w_{n,i})$. Elaborating on this condition, see Appendix B.2, implies the following equilibrium con-

dition, from now on called *the high-skill arbitrage condition*:

$$w_c = \alpha^\alpha (1-\alpha)^{2-\alpha} \left[\frac{S_c}{l}\right]^\alpha \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{13}$$

2.4   Equilibrium

Table 1 reviews the important equilibrium variables. The endogenous variables of inter-est are the income of low-skill workers, high-skill workers in the consumption sector, high-skill workers in the research sector and the fraction of the high-skill workers that choose to become researchers, denoted w_o, w_c, $w_{n,i}(\varepsilon_i)$ and μ respectively. In Appendix B.3 it is shown how to derive expressions for those endogenous variables by combining full employment assumptions and the high-skill arbitrage condition.

$$\mu = \frac{(1-\alpha)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} - \left[\frac{\alpha}{1-\alpha}\right]^{1-\alpha}[k_o l]^\alpha \frac{(1-\phi)^{1-\alpha}}{\phi}\left[\gamma_i^{\frac{1}{\alpha}}\right]^\alpha}{(1-\alpha)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} + \alpha E\varepsilon_i} \tag{14a}$$

$$w_o = (1-\alpha)\left[\frac{\phi(1-\mu)}{1-\phi}\right]^\alpha \left[\gamma_i^{\frac{1}{\alpha}}\right]^\alpha k_o^\alpha \tag{14b}$$

$$w_c = \alpha^\alpha (1-\alpha)^{2-\alpha}\left[\frac{\phi(1-\mu)}{l}\right]^\alpha \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{14c}$$

$$w_{n,i}(\varepsilon_i) = \alpha^\alpha (1-\alpha)^{2-\alpha}\left[\frac{\phi(1-\mu)}{l}\right]^\alpha \varepsilon_i. \tag{14d}$$

2.4.1   Properties of μ

Equation (14a) gives an expression for the fraction of the high-skill workers in the re-search sector. Hence this expression is constrained to be greater than zero but less than one. Since the denominator is greater then the numerator μ can never exceed one. It is,

Table 1: Equilibrium Variables

Producing	Wage	Skill Level	Employees/Members	Fraction
$X_{o,i}$	w_o	Low-Skill	$L_{o,i}$	$(1-\phi)$
Y	w_c	High-Skill	S_c	$\phi(1-\mu)$
$X_{n,i}$	$w_{n,i}(\varepsilon_i)$	High-Skill	$S_{n,i}$	$\phi\mu$

however, quite possible that μ falls below zero. This is likely to happen if the variety of old intermediate goods possible to produce in the period is large, i.e. k_o large. A large variety of old intermediate goods depresses the profitability of research co-operatives producing new intermediate goods.
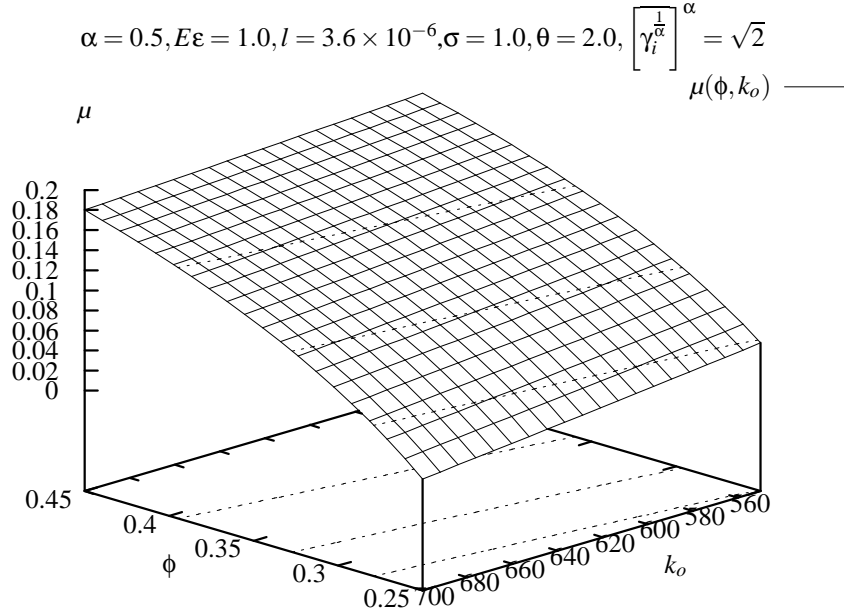
At a corner, solution, i.e. μ constrained to zero, both hypotheses investigated in this paper can be rejected. It is immediately clear that an increase in the fraction of high-skill workers in the economy, ϕ greater, increases the fraction high-skill workers that choose to work in the research sector. Figure 2 plots μ, the fraction high-skill workers that choose to work in the research sector.

Figure 2 verifies that as ϕ increases, the fraction of high-skill workers that choose to work in the research sector increases. More old intermediate goods tend to decrease the fraction of high-skill workers choosing to work in the research sector, by depressing the profitability of new intermediate goods. Also, Figure 2 is important because for the given parameter values it shows that μ is positive, thereby verifying the assumption in (12).

2.4.2 Properties of w_o

Equation (14b), describing the wage rate of low-skill workers, have some interesting properties. Increasing the fraction of high-skill workers, ϕ, has several effects. The term, $\left(\frac{\phi(1-\mu)}{1-\phi}\right)^{\alpha}$, captures the supply and demand effects. Note that since more high-skill

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Figure 2: Share of High-Skill Workers in Research Sector

$\alpha = 0.5, E\varepsilon = 1.0, l = 3.6 \times 10^{-6}, \sigma = 1.0, \theta = 2.0, \left[\overline{\gamma_i^{\frac{1}{\alpha}}}\right]^{\alpha} = \sqrt{2}$

$\mu(\phi, k_o)$ ——

The figure shows the fraction of high-skill workers employed by research co-operatives, for different combinations of the relative supply of high-skill workers, ϕ, and the number of old intermediate goods, k_o. For every combination of ϕ and k_o, $\mu > 0$.

workers choose to work in the research sector the effect is hampered, as ϕ increases, $1 - \mu$ decreases. The term k_o measures the higher productivity gains from more intermediate goods, obtained by low-skill workers.

In a static setting, low-skill workers do not benefit from a larger variety of intermediate goods due to more research, i.e. k_n higher. However, in the long run, old intermediate goods must be the result of new intermediate goods developed in earlier periods. Hence, in the long run both high-skill and low-skill workers benefit from a larger variety of intermediate goods.

2.4.3 Properties of w_c and $w_{n,i}$

Together equations (14c) and (14d) show that on average $Ew_{n,i}$ is proportional to w_c, that is:

$$\frac{w_c}{Ew_{n,i}(\varepsilon_i)} = \frac{\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}{E\varepsilon_i}. \tag{15}$$

Hence the wage levels for high-skill workers in the consumption sector and high-skill workers in the research sector move together and the log differential is determined solely by the relative risk aversion, θ, and the mean and variance of the log-normal distribution, $E\varepsilon$ and σ^2. As in the case of w_o, w_c and $w_{n,i}$ are affected by supply and demand effects, via the term $(\phi(1-\mu))^\alpha$. However, wages in the consumption sector are only indirectly dependent on the supply of low-skill workers, via μ.

3 Results

The following section makes use of the previously defined and solved model to draw conclusions about wage differences in the growth economy. Recall that the paper's two hypotheses are:

1. Increasing the supply of high-skill workers *increases* the wage dispersion *among* high-skill workers

2. Increasing the supply of high-skill workers *increases* the wage dispersion *between* high-skill workers and low-skill workers

3.1 High-Skill Workers Wage Distribution

Defining the income dispersion among high-skill workers, i.e. residual wage inequality for high-skill workers, as the ratio of expected wage rate for workers in the research sector to the certain wage rate of workers in the consumption sector implies a measure of inequality which, for this model, is independent of the relative supply of high-skill workers. This claim is easily verified by the equilibrium relation between w_c and $Ew_{n,i}$, given by (14c) and (14d). The expected income levels of researchers and high-skill workers are linked and their relative magnitude depends only on the properties of $E\varepsilon_i / \left[E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}$.

Inspecting the expression $\frac{Ew_{n,i}}{w_c} = \frac{E\varepsilon_i}{\left[E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}}$ more closely, see Appendix B.2, confirms some standard economic results concerning the risk premium:

$$\frac{\partial}{\partial \theta} \frac{Ew_{n,i}}{w_c} = \left[\ln \frac{\sqrt{\{E\varepsilon\}^2 + \sigma^2}}{E\varepsilon} \right] \frac{E\varepsilon}{\left[E\varepsilon_i^{1-\theta} \right]^{\frac{1}{1-\theta}}} > 0 \qquad (16a)$$

$$\frac{\partial}{\partial \sigma} \frac{Ew_{n,i}}{w_c} = \theta\sigma \frac{\left[\{E\varepsilon\}^2 + \sigma^2 \right]^{\frac{\theta}{2}-1}}{E\varepsilon^\theta} > 0 \qquad (16b)$$

$$\frac{\partial}{\partial E\varepsilon} \frac{Ew_{n,i}}{w_c} = -\theta\sigma^2 \{E\varepsilon\}^{-(1+\theta)} \left(\{E\varepsilon\}^2 + \sigma^2 \right)^{\frac{\theta-2}{2}} < 0. \qquad (16c)$$

Income inequality among high-skill workers increases with stronger relative risk aversion, (16a), and more uncertain returns to research, (16b). All else equal, increasing the expected productivity of a new intermediate good invented by a co-operative makes research more profitable, which tends to increase inequality. However, at the same time, in equilibrium, the higher profitability in research firms implies that more high-

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Table 2: Distribution of W^S

Realization, w^S, of W^S	Probability/Share
$w^S < w_c$	$\mu P\left(w_{n,i}(\varepsilon_i) < w^S\right)$
$w^S \geq w_c$	$(1-\mu) + \mu\left[1 - P\left(w_{n,i}(\varepsilon_i) < w^S\right)\right]$

skill workers choose to work in the research sector. An increasing number of workers in the research sector decreases the number of high-skill workers in the consumption good sector, increasing wages in the consumption good sector. Hence in equilibrium an increase in the expected productivity of new intermediate goods have two counteracting effects. As (16c) shows, the net effect is decreased inequality.

The independence between the income dispersion *among* high-skill workers and the relative supply of high-skill workers is fragile. Measuring inequality among high-skill workers by the variance wages for all high-skill workers shows that inequality can increase or decrease with an increased relative supply of high-skill workers. Let W^S denote the wage rate of a high-skill worker *unconditional* on being employed in the consumption good sector or in the research sector. Formally the distribution of W^S is summarized in Table 2. The variance of W^S can be decomposed as:

$$\mathrm{var}\left(W^S\right) = (1-\mu)\sigma_c^2 + \mu\sigma_n^2 + \mu(1-\mu)\left(Ew_{n,i}(\varepsilon_i) - Ew_c\right)^2. \tag{17}$$

Equation (17) decomposes the variance of *all* high-skill workers into three parts, variance *among* workers in the consumption good sector, σ_c^2, variance *among* workers in the research sector, σ_n^2, and lastly the part of total variance due to the wage differential *between* consumption good workers and research workers, $(Ew_{n,i}(\varepsilon_i) - Ew_c)^2$. High-skill workers producing the consumption good are paid non-stochastic wages, therefore $Ew_c = w_c$ and $\sigma_c^2 = 0$. Increasing the proportion of high-skill workers has several im-

plications for the overall variance. First, from the second term of (17) more high-skill workers earn a stochastic income, $d\mu/d\phi > 0$, increasing the overall variance. Second, the weight of the third term, $\mu(1-\mu)$ changes. The weight of the this term is maximized for $\mu = 1/2$ so it can increase or decrease depending on the number of high-skill workers that already choose to do research work.

Apart from changing the weights in (17), increasing the supply of high-skill workers also changes the variance among research workers, σ_n^2, and the wage difference between research and consumption workers, $Ew_{n,i}(\varepsilon_i) - w_c$. Since the expression for $w_{n,i}(\varepsilon_i)$ is multiplicative separable and the sign of $dw_{n,i}(\varepsilon_i)/d\phi$ is ambiguous, σ_n^2 can either increase or decrease due to increases in ϕ. The same is true for the wage difference between high-skill workers working with research and high-skill workers employed by final good producers. Hence, increasing the share of high-skill workers, ϕ, can either increase or decrease the variance of all high-skill workers' wages.

To avoid this ambiguity and simplify the decomposition of total variance for high-skill workers it is useful to investigate the distribution of $\ln W^S$. This also brings the analysis closer to the empirical literature concerning wage inequality which concentrates on the distribution of the natural logarithm of wages. Let $\hat{\sigma}_n^2$ denote the variance of the natural logarithm of wages in the research sector, i.e. $\text{var}\{\ln[w_{n,i}(\varepsilon_i)]\}$. The expression decomposing total variance becomes:

$$\text{var}\left(\ln W^S\right) = \mu\hat{\sigma}_n^2 + \mu(1-\mu)\left[E\ln\left(w_{n,i}(\varepsilon_i)\right) - \ln\left(w_c\right)\right]^2. \tag{18}$$

The multiplicative separability of the wage expressions turns into additive separa-

bility for the natural logarithm of the wage expressions. This in turn implies that:

$$\frac{d\hat{\sigma}_n^2}{d\phi} = 0 \quad , \quad \frac{d\left[E\ln\left(w_{n,i}(\varepsilon_i)\right) - E\ln\left(w_c\right)\right]}{d\phi} = 0.$$

Hence the effect of changes in the supply of high-skill workers, ϕ, on the variance of the distribution of the natural logarithm of wages for all high-skill workers operates only via changes in the fraction, μ, of high-skill workers choosing to work in the research sector. Proposition 3.1.1 summarizes how the variance of high-skill wages changes with the relative supply of high-skill workers. The proof can be found in Appendix B.4.

Proposition 3.1.1 (Residual Wage Inequality) *The variance of the natural logarithm of wages for high-skill workers, derived from the theoretical distribution of* $\ln W^S$ *is*

$$var\left(\ln W^S\right) = \mu\ln\left(1 + \frac{\sigma^2}{\{E\varepsilon\}^2}\right) + \mu(1-\mu)(1-\theta)^2\left(\ln\frac{E\varepsilon}{\sqrt{\{E\varepsilon\}^2 + \sigma^2}}\right)^2 \quad (19a)$$

and its derivative with respect to ϕ *is:*

$$\begin{aligned}\frac{dvar\left(\ln W^S\right)}{d\phi} &= \left[\ln\left(1 + \frac{\sigma^2}{\{E\varepsilon\}^2}\right) + (1-2\mu)(1-\theta)^2\left(\ln\frac{E\varepsilon}{\sqrt{\{E\varepsilon\}^2 + \sigma^2}}\right)^2\right] \\ &\quad \times \quad \frac{d\mu}{d\phi}.\end{aligned} \quad (19b)$$

Hence for sufficient low shares of researchers among high-skill workers, $\mu < 1/2$, an increased supply of high-skill workers increases wage dispersion among high-skill workers. For larger shares of researchers among high-skill workers, $\mu > 1/2$, the wage dispersion might increase or decrease with an increased supply of high-skill workers.

On the one hand, increasing the number of high-skill workers doing research always

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increases residual wage inequality, due to the fact that more high-skill workers earn a stochastic wage rate. On the other hand, the discrepancy between the expected research wage and the certain wage rate paid to workers employed by final good producers, i.e. the risk premium, also increases residual wage inequality. However, the latter contribution is maximized as number of workers in the research sector and the number of workers employed by final good producers are equalized. Therefore this latter effect can increase or decrease the residual wage inequality as the number of research workers increases.

By inspecting the expression for μ, given by (14a), it is immediately clear that μ is more likely to be less then $1/2$ if the number of old intermediate goods, k_o, is large, the time necessary to develop a new intermediate good, l, is large, or the fraction of high-skill workers, ϕ, is small.
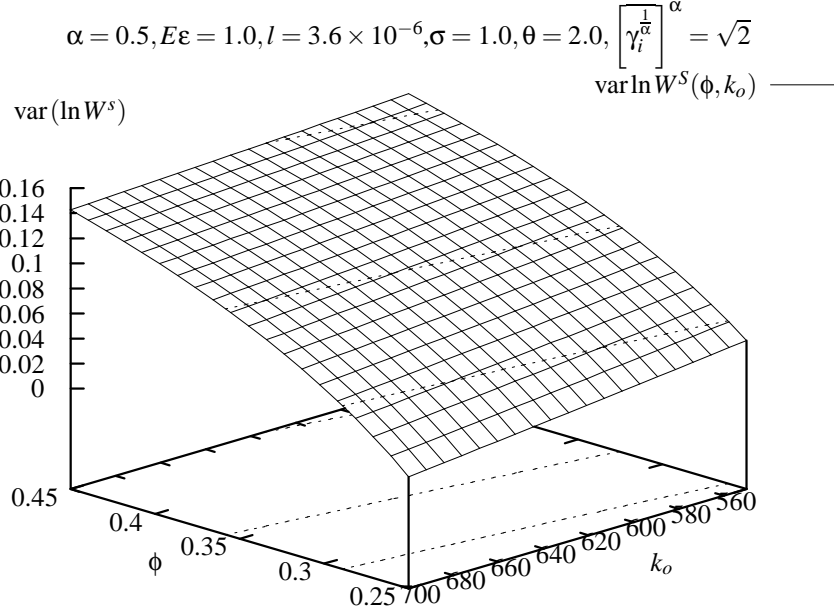
Note that a low fraction of high-skill workers, ϕ, is associated with a low fraction of high-skill workers choosing to work in the research sector, μ. Hence, economies starting with a low fraction of high-skill workers, but increasing the fraction, are likely to experience increased wage inequality for high-skill workers.

Figure 3 plots the residual wage inequality, as defined in Proposition 3.1.1 for the same parameters values as in Figure 2. As is clear from the latter figure, μ is less then $1/2$ and consequently, the residual wage inequality for high-skill workers increases with the relative supply of high-skil workers.

3.2   The Skill Premium

Let τ denote the ratio of high-skill workers' expected wage rate and low-skill workers' wage rate, that is $\tau = \frac{EW^S}{w_o}$. After some algebraic manipulations, see Appendix B.5, the

Figure 3: Residual Wage Inequality for High-Skill Workers

$$\alpha = 0.5, E\varepsilon = 1.0, l = 3.6 \times 10^{-6}, \sigma = 1.0, \theta = 2.0, \left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha} = \sqrt{2}$$

$$\text{var}\ln W^S(\phi, k_o) \quad \text{———}$$

The figure graphs the variance of wages, for high-skill workers, given different combinations of the relative supply of high-skill labor, ϕ, and the number of old intermediate goods, k_o.

expression for τ turns out to be:

$$\tau = \frac{\alpha}{1-\alpha} \times \frac{1-\phi}{\phi} \times \frac{(1-\mu)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} + \mu E\varepsilon_i}{(1-\mu)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} - \frac{\alpha}{1-\alpha}\mu E\varepsilon_i}. \tag{20}$$

The following lemma is useful for comparing the impact of augmenting a standard equilibrium model with a stochastic research sector, the proof is given in Appendix B.6.
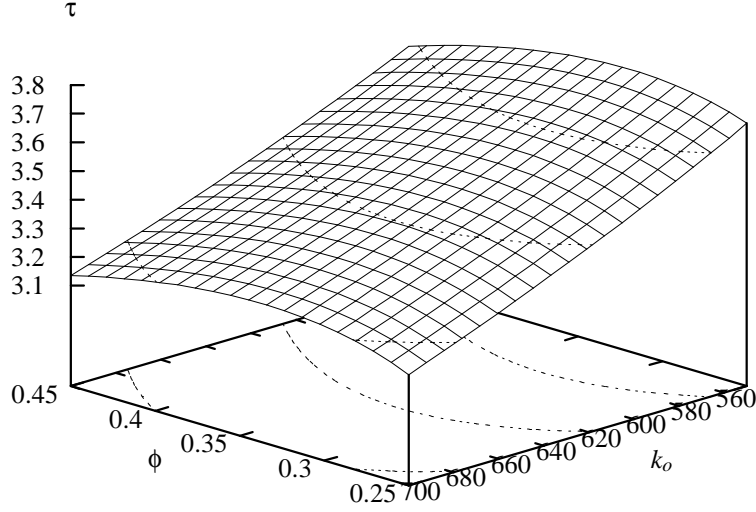
Lemma 3.2.1 *The expression*

$$\frac{\alpha}{1-\alpha} \times \frac{1-\phi}{\phi}.$$

describes the wage dispersion between high-skill workers and low-skill workers if there

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Figure 4: Skill Premium

$$\alpha = 0.5, E\varepsilon = 1.0, l = 3.6 \times 10^{-6}, \sigma = 1.0, \theta = 2.0, \left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha} = \sqrt{2}$$

$\tau(\phi, k_o)$ ———

The figure illustrates the skill premium for different combinations of relative supply of high-skill workers, ϕ, and the number of old intermediate goods, k_o.

is no research sector.

To investigate the second hypothesis, that an increased supply of high-skill workers increases wage dispersion between high-skill workers and low-skill workers, it is necessary to investigate how τ changes with ϕ. That is, it is necessary to find the derivative of τ with respect to ϕ. Proposition 3.2.2 summarizes the results on wage dispersion between high-skill and low-skill workers. The proof is found in Appendix B.7.

Proposition 3.2.2 (The Skill Premium) *Define the skill premium as the ratio of high-skill workers' expected wage rate to low-skill workers' wage rate, then in equilibrium:*

1. *The skill premium increases with the number of high-skill workers choosing to work in the research sector.*

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2. *Increased supply of high-skill workers, relative to the supply of low-skill workers, increases the skill premium if and only if:*

$$\left[\frac{\alpha}{lk_o}\right]^{\alpha} \times \left[\frac{1-\alpha}{1-\phi}\right]^{1-\alpha} \times \frac{E\varepsilon_i\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha}\left(E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}\right)} \times \phi^2 < 1 \qquad (21a)$$

and

$$(1-\alpha)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} - \left[\frac{\alpha}{1-\alpha}\right]^{1-\alpha}[k_o l]^{\alpha}\frac{(1-\phi)^{1-\alpha}}{\phi}\left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha} \geq 0. \qquad (21b)$$

Corollary 3.2.3 *If there is no uncertainty, $\sigma = 0$, or high-skill workers are not risk averse, $\theta = 0$, increased relative supply of high-skill workers decreases the skill premium.*

The first condition in the second part of proposition ensures that the skill premium increases as the relative supply of high-skill workers increase. The second condition in the second part of the proposition assures that in equilibrium, the number of research workers is non negative, see (14a). Both those conditions must be satisfied, but it is not easy to prove that such an equilibrium exist.

To prove the existence of such an equilibrium, Figure 4 plots the skill premium for the same parameter values as in Figure 2. From Figure 2 it is clear that for all those parameter values there is a non negative number of research workers and the second condition in Proposition 3.2.2 is fullfilled. Further as is seen in Figure 4, for some range of values the skill premium increases as the fraction of high-skill workers increases.

In general, large values of l, $\left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha}$, and large a number of old intermediate goods, k_o, clearly ensures that expression (21a) is less then unity, and hence the wage dispersion

increases as the number of high-skill workers increases.

Corollary 3.2.3 highlights the importance of risk and risk aversion for the results to hold. If agents are not risk averse or there is no risk, an increased relative supply of high-skill workers decreases the wage inequality between high-skill and low-skill workers, even though high-skill workers have the opportunity to make an occupational choice.

4   Conclusions

This paper presents a stylized general equilibrium model, augmented by a profit driven research sector. The labor force is divided into two categories, low-skill workers and high-skill workers. Capital is excluded and the time frame is collapsed into a single period during the entire analysis. The research sector is characterized by uncertain pay-offs. Due to the model's limited insurance possibilities for research workers, they have to bear all risk associated with inventing new intermediate goods, their opportunity cost being a foregone certain wage, paid by producers of consumption goods.

In a standard general equilibrium model, without a risky research sector, increased supply of a specific production factor, ceteris paribus, tends to lower the returns to that factor. If the research sector is excluded, research is non-stochastic or workers are risk neutral, the model presented in this paper also predicts that increasing the supply of high-skill workers lowers the average wage rate for high-skill workers.

However, since high-skill workers can choose to work in the research sector, as their wages tend to fall due to an increased supply of high-skill workers, there is a reallocation such that the fraction of high-skill workers choosing to work in the research sector increases. This shift implies that more high-skill workers are paid a risk premium for

bearing risk. The reduction in the wage rate for high-skill workers, due to the increased supply of high-skill workers, is partly counteracted by a flow of high-skill workers from the consumption goods sector to the research sector. Due to the flow of workers to the research sector, more high-skill workers earn a risk premium. Therefore the average wage rate for high-skill workers can increase due to an increased supply of high-skill workers.

The formal analysis shows that the intuition outlined above is correct. There is no simple relationship between an increased supply of high-skill workers and reduction in wages for the same group. The comparative advantage of high-skill workers in producing knowledge, a good that is human capital intensive, combined with the uncertainty associated with research activity blurs the standard increased supply, decreased wage argument.

Appendices

A   Record of Notation

In Table 3 the symbols used in the paper are listed and briefly explained.

B   Proofs and Derivations

The following section contains the derivations and proofs of various results in the paper.

Symbol	Meaning
α	Cobb-Douglas exponent.
ε	Productivity of some new intermediate good.
γ	Productivity of some old intermediate good.
μ	Fraction of high-skill workers in developing co-operatives.
k	Number of intermediate goods.
l	Hours necessary to develop a new intermediate good.
L	Quantity of low-skill labor.
P	Price of some good.
ϕ	Share of high-skill workers.
π	Profit rate.
σ^2	Variation in productivity for a new intermediated good.
S	Quantity of high-skill labor.
θ	Relative risk aversion.
τ	Skill premium.
w	Wage rate.
X	Quantity of an intermediate good.
Y	Quantity of the consumption good.

B.1 The Co-operative Problem

A co-operative aiming at producing a new intermediate good $X_{n,i}$, must spend a fixed amount of labor units, l, in order to invent the new good. Once the new intermediate good is invented it takes one unit of high-skill labor to produce one unit of the intermediate good. Let $S_{n,i}$ denote of size of the ith co-operative, i.e. the total number of high-skill labor units supplied by all its members. The co-operatives technology is described formally by (9).

The co-operative's revenue is given by the price times the quantity produced. Since the co-operative's intermediate good is unique, co-operatives face a monopoly situation. Hence, co-operatives exploit the price-quantity relation given by the consumption good producer's demand function, given by (3b).

The earnings are shared uniformly across the co-operative's members. Hence the earning of each member equals total revenues divided by the number of members. Since the co-operative members are identical they all share the same objective; maximizing the expected utility of total reveue per member. By using (9) and (3b) to substitute out $X_{n,i}$ and $P_{n,i}$. Given that the utility function is monotonically increasing and multiplicative separable, the following simplifying steps are feasible:

$$\max_{P_{n,i},X_{n,i},S_{n,i}} \quad Eu\left[\frac{P_{n,i} \times X_{n,i}}{S_{n,i}}\right]$$

$$s.t.\ (3b)\ and\ (9)$$

$$\max_{S_{n,i}} \quad Eu\left[(1-\alpha)S_c^{\alpha}\frac{(S_{n,i}-l)^{1-\alpha}}{S_{n,i}}\varepsilon_i\right]$$

$$\max_{S_{n,i}} \quad \frac{(S_{n,i}-l)^{1-\alpha}}{S_{n,i}}. \tag{22}$$

The last optimization problem is simple to solve. Substituting back gives the results for the size of the co-operative, the quantity produced, the price, and the wage, as listed in (11a), (11b), (11c), and (11d) respectively.

B.2 The High-Skill Arbitrage Condition

The derivation and simplification of the arbitrage condition, ensuring equalization of high-skill workers' expected utility from co-operative research work and working for a certain wage in the consumption good sector, follows (note that $w_{n,i}$ is given by (11d)):

$$Eu[w_c] = E_{\varepsilon_i}u\left[w_{n,i}(\varepsilon_i)\right]$$

$$w_c^{1-\theta} = E_{\varepsilon_i}\left[w_{n,i}(\varepsilon_i)^{1-\theta}\right]$$

$$w_c = \alpha^{\alpha}(1-\alpha)^{2-\alpha}\left[\frac{\phi(1-\mu)}{l}\right]^{\alpha}\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{23}$$

If ε_i is lognormally distributed with mean $E\varepsilon$ and variance σ^2. Then $\ln\varepsilon_i$ is normally distributed with mean m and variance s^2, where

$$
\begin{aligned}
m &= \ln\left[\{E\varepsilon\}^2\right] - \frac{1}{2}\ln[\{E\varepsilon\}^2 + \sigma^2] \\
s^2 &= \ln\left[1 + \frac{\sigma^2}{\{E\varepsilon\}^2}\right]
\end{aligned}
\tag{24}
$$

and $\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}$ can written as:

$$
\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} = e^{m + \frac{(1-\theta)s^2}{2}}.
\tag{25}
$$

Substituting out m and s^2 by use of (24) the final expression for $\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}$ become:

$$
\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} = \frac{\{E\varepsilon\}^{1+\theta}}{\sqrt{\{E\varepsilon\}^2 + \sigma^2}^{\theta}}.
\tag{26}
$$

To verify that $E\varepsilon > \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}$ note that if $\sigma^2 = 0$ then $\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} = E\varepsilon$. Since $\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}$ is monotonically decreasing in σ^2 and $\sigma^2 > 0$ the statement is verified.

B.3 Equilibrium

In equilibrium there is full employment and hence the market for low-skill workers must clear. Since the economy's total labor endowment is normalized to unity, $(1 - \phi)$ is the share and the total number of low-skill workers. The number of high-skill workers employed in the consumption good sector, S_c, equals $\phi(1 - \mu)$. That is, ϕ is the share of high-skill workers, μ is the share of high-skill workers that are members of research co-

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operatives and the economy's total labor endowment is normalized to unity. Low-skill labor market equilibrium imply:

$$(1 - \phi) = \sum_{i=1}^{k_o} L_{o,i}.$$

Substituting the low-skill labor demand, (7), and simplifying gives:

$$(1 - \phi) = \sum_{i=1}^{k_o} S_c \left[\frac{(1-\alpha)\gamma_i}{w_o} \right]^{\frac{1}{\alpha}}$$

$$w_o = (1-\alpha) \left[\frac{\phi(1-\mu)}{1-\phi} k_o \right]^{\alpha} \left[\sum_{i=1}^{k_o} \frac{\gamma_i^{\frac{1}{\alpha}}}{k_o} \right]^{\alpha}$$

$$w_o = (1-\alpha) \left[\frac{\phi(1-\mu)}{1-\phi} k_o \right]^{\alpha} \left[\overline{\gamma_i^{\frac{1}{\alpha}}} \right]^{\alpha}. \tag{27}$$

Using the inverse demand function for high-skill workers in the consumption good sector, (3c), is equivalent to clearing the market for high-skill workers in the consumption good sector. By substituting the quantity of each intermediate good used, (7) and (11b), this condition becomes:

$$w_c = \alpha \left[\frac{1-\alpha}{w_o} \right]^{\frac{1-\alpha}{\alpha}} k_o \left[\overline{\gamma_i^{\frac{1}{\alpha}}} \right] + \alpha^{\alpha} \left[\frac{(1-\alpha)l}{\phi(1-\mu)} \right]^{1-\alpha} k_n E \varepsilon_i. \tag{28}$$

The number of new intermediate goods, k_n is endogenous. Each research co-operative employs l/α high-skill labor units, see (11a), and there are $\phi\mu$ high-skill labor units in the research sector. Hence there are

$$\alpha \frac{\phi\mu}{l} = k_n. \tag{29}$$

research co-operatives and new intermediate goods.

Using the arbitrage condition for high-skill workers, (13), the expression for low-skill workers' wages, (27), and the expression for the number of new intermediate goods, (29), to substitute out w_c, w_o and k_n, respectively, after simplification μ is the only unknown:

$$\alpha^\alpha (1-\alpha)^{2-\alpha} \left[\tfrac{1}{l}\right]^\alpha \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} \phi - \alpha(1-\phi)^{1-\alpha} \left[\gamma_i^{\frac{1}{\alpha}}\right]^\alpha k_o^\alpha$$

$$=$$

$$\alpha^\alpha (1-\alpha)^{2-\alpha} \left[\tfrac{1}{l}\right]^\alpha \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} \phi\mu + \alpha^{1+\alpha}(1-\alpha)^{1+\alpha}(1-\alpha)^{1-\alpha} l^\alpha \phi\mu E\varepsilon_i$$

$$\mu = \frac{(1-\alpha)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} - \left[\frac{\alpha}{1-\alpha}\right]^{1-\alpha}[k_o l]^\alpha \frac{(1-\phi)^{1-\alpha}}{\phi}\left[\gamma_i^{\frac{1}{\alpha}}\right]^\alpha}{(1-\alpha)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} + \alpha E\varepsilon_i}. \tag{30}$$

By substituting this expression into (27) and (28) the wage for low-skill workers and high-skill workers in the consumption sector can be obtained as a function of exogenous variable and parameters, only. However, the results are quite complex therefore it is better to keep μ and remember that μ is endogenous.

Finally to find the earnings for a co-operative member of the co-operative indexed by i, replace S_c by $\phi(1-\mu)$ in (11d). Again substituting out μ by use of (14a) makes the expression unnecessary complicated.

B.4   Proof of Proposition 3.1.1

Let $\ln W^S$ denote the random variable describing the natural logarithm of the wage rate for any high-skill worker. The variance of $\ln W^S$ can be decomposed into three parts: the variance among high-skill consumption workers, the variance among high-skill research workers and the difference between the average research and consumption wage.

The consumption worker wage is non-stochastic so the first part vanishes from the decomposition, hence:

$$\text{var}\left(\ln W^S\right) = \mu\hat{\sigma}_n^2 + \mu(1-\mu)\left[E\ln w_{n,i}(\varepsilon_i) - \ln w_c\right]^2. \tag{31}$$

$\hat{\sigma}_n^2$ denotes the variance of the natural logarithm of the wage of research workers, which by (14d) equals $\text{var}(\ln\varepsilon_i)$. By (14d) and (14c):

$$E\ln w_{n,i}(\varepsilon_i) - \ln w_c = E\ln\varepsilon_i - \ln\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

By using (8), describing the parameterization of the log-normal distribution of ε_i and (26) the variance decomposition can be shown, by simple substitution and straight forward simplification, to equal the expression in (19a). The differentiation is straightforward.

B.5 Wage Dispersion

First note that the average (i.e. expected) level of income for high-skill workers, EW^S, can be written in terms of w_c:

$$EW^S = \frac{w_c\left[(1-\mu)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} + \mu E\varepsilon_i\right]}{\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}. \tag{32}$$

Defining wage dispersion as the ratio between the average high-skill worker's wage,

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EW^S, and the wage of low-skill workers, w_o, implies:

$$\frac{EW^S}{w_o} = \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}(1-\phi)^\alpha}{l^\alpha k_o^\alpha \left[\gamma_i^{\frac{1}{\alpha}}\right]^\alpha} \times \frac{(1-\mu)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} + \mu E\varepsilon_i}{\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}. \tag{33}$$

Using the equilibrium value of μ given by (14a) to find an expression for $k_o^\alpha l^\alpha \left[\gamma_i^{\frac{1}{\alpha}}\right]^\alpha$ as:

$$k_o^\alpha l^\alpha \left[\gamma_i^{\frac{1}{\alpha}}\right]^\alpha = \left[\frac{1-\alpha}{\alpha}\right]^{1-\alpha} \frac{\phi}{(1-\phi)^{1-\alpha}} \left[(1-\alpha)(1-\mu)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} - \alpha\mu E\varepsilon_i\right]. \tag{34}$$

Substituting this expression into (33) and performing some straightforward algebraic manipulations, results in the expression for τ given by (20).

B.6 Proof of Lemma 3.2.1

By the inverse demand function for high-skill labor, (3c), the total wages payment for all high-skill workers in the consumption sector, $w_c S_c$, are: αY. The total earnings of *all* low-skill workers, L_o must equal the remaining part:

$$w_o L_o = Y - w_c S_c = (1-\alpha)Y.$$

Now, given that $S_c = \phi$ and $L_o = 1 - \phi$:

$$\frac{w_c}{w_o} = \frac{\alpha}{1-\alpha} \times \frac{1-\phi}{\phi}. \tag{35}$$

B.7 Proof of Proposition 3.2.2

The expression for the income dispersion, τ, given by (20), depends on μ and μ depends on ϕ. Hence to find the derivative $\frac{d\tau}{d\phi}$ it is necessary to replace μ by the expression (14a) or use the chain rule. Replacing μ by (14a) gives:

$$\tau = \frac{\alpha}{\left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha}} \times \frac{E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\alpha E\varepsilon_i + (1-\alpha)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}} \times \tag{36}$$

$$\times \left[\frac{E\varepsilon_i\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}{E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}\left[\frac{1-\alpha}{\alpha}\right]^{1-\alpha}\left[\frac{(1-\phi)}{k_o l}\right]^{\alpha} - \frac{1-\phi}{\phi}\left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha}\right].$$

Simple derivation and simplification of (36) gives

$$\frac{d\tau}{d\phi} = \frac{\alpha}{\phi^2} \times \frac{E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\alpha E\varepsilon_i + (1-\alpha)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}} \times \tag{37}$$

$$\times \left[1 - \left[\frac{\alpha}{lk_o}\right]^{\alpha}\left[\frac{1-\alpha}{1-\phi}\right]^{1-\alpha}\frac{E\varepsilon_i\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}\phi^2}{\left(E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}\right)\left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha}}\right].$$

which is positive only if (21a) is fulfilled. Condition (21b) is easily obtained by simplifying $\mu \geq 0$ using (14a).

To prove Corollary 3.2.3 note that if $\sigma = 0$ or $\theta = 0$:

$$E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} = 0.$$

De-factorizing $E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}$ in (37) and *imposing* $E\varepsilon_i - \left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}} = 0$ gives:

$$
\begin{aligned}
\frac{d\tau}{d\phi} &= -\frac{\alpha}{\left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha}} \times \frac{E\varepsilon_i\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\alpha E\varepsilon_i + (1-\alpha)\left[E\varepsilon_i^{1-\theta}\right]^{\frac{1}{1-\theta}}} \times \left[\frac{\alpha}{lk_o}\right]^{\alpha}\left[\frac{1-\alpha}{1-\phi}\right]^{1-\alpha} \\
&= -\frac{\alpha}{\left[\gamma_i^{\frac{1}{\alpha}}\right]^{\alpha}} \times E\varepsilon_i \times \left[\frac{\alpha}{lk_o}\right]^{\alpha}\left[\frac{1-\alpha}{1-\phi}\right]^{1-\alpha} < 0.
\end{aligned}
$$

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