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SCHOOL OF BUSINESS, ECONOMICS AND LAW

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Central Counterparties

A Numerical Implementation of the Default Waterfall

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Abstract

This thesis studies so called Central Counterparties (CCP), financial institutions which consist of clearing members, such as large banks. CCPs have the role of centralizing, mutualizing and reducing counterparty risk, by acting as an intermediate in financial transactions. CCPs have existed for a while, however after the 2007-2009 financial crisis regulators have pushed for all OTC-derivatives to be cleared by CCPs. In order to be risk mitigating, the CCPs must have sufficient funds to be able to absorb losses from member defaults. To increase the resilience of the CCP, the loss-absorbing safety buffer exists in several layers, often denoted as the default waterfall. In this thesis we numerically implement the CCP model by Ghamami (2015). We use two different static credit models to quantify the various layers of the default waterfall. Our model is found to adjust to different default probabilities and default correlations by increasing the fund requirements in stressed scenarios in both settings. Finally, we perform a sensitivity analysis in which we change the number of clearing members, the time period considered and the interest rate setting. In each stress test the model reacts to extreme scenarios by increasing the layers accordingly.

Keywords: *Risk Management, Central Counterparty Risk, Stochastic Models, Monte Carlo Simulation, Mixed Binomial Models, Interest Rate Swap*

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Contents

List of Abbreviations

- AIG - American International Group
- BIS - Bank of International Settlements
- CCP - Central Counterparty
- CCPs - Central Counterparties
- CDS - Credit Default Swap
- CIR - Cox-Ingersoll-Ross
- CM - Clearing Member
- CVA - Credit Value Adjustment
- DF - Default Fund
- EE - Expected Exposure
- EPE - Expected Positive Exposure
- EMWA - Exponentially Weighted Moving Average
- ES - Expected Shortfall
- IM - Initial Margin
- IRS - Interest Rate Swap
- LPA - Large Portfolio Approximation
- MC - Monte Carlo (simulation)
- MPOR - Margin Period of Risk
- OECD - Organization for Economic Co-operation and Development
- OTC - Over-the-Counter
- QCCP - Qualified Central Counterparty
- SSBs - Standard Setting Bodies
- VaR - Value-at-Risk
- VM - Variation Margin
- VMGH - Variation Margin Gains Haircutting

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1 Introduction

After the Credit Crisis of 2007-2009, it become clear that the transparency and safety systems of the financial industry had to be revised. One area that has attracted a lot of interest is the concept of Central Counterparties. The role of a CCP is to mitigate counterparty credit risk, acting as a *seller to the buyer and a buyer to the seller* in a bilateral financial contract. This means that in case of a default of one of the parties, the CCP will guarantee the payments that the counterparty has on the defaulter.

There is an ongoing debate whether CCPs will improve the stability in the financial system as a whole. Pirrong (2014) is critical of the concept of Central Counterparties, and claims that Central Counterparties does not reduce the credit risk, but only redistributes it. The criticism is due to the fact that firms are connected not only through derivatives, but through other contracts as well. These connections will remain even after clearing mandates go into effect. Duffie & Zhu (2011) are more positive, and they show that CCPs can in principle achieve significant reductions in counterparty risk. However, there are a number of legal and financial engineering challenges. Duffie & Zhu (2011) also concludes that one clearing house should clear all standard interest-rate swaps. Cont & Kokholm (2014) find that the highest reduction in exposure is obtained if one CCP clears all asset classes. However, the monopoly of a CCP can lead to a concentrated systemic risk as well as a high level of operational risk. At the time of writing, all research states that this is a topic of high interest and that serious examination of the benefits and drawbacks of having multiple CCPs requires a solid model. The 2009 G-20 OTC derivatives reform program did however include Central Counterparties as a way to improve the financial risk management. (BIS 2010)

In order to make sure that a CCP can withstand defaults of its clearing members, the CCP needs to have sufficient resources, a so-called default waterfall. The default waterfall consists of several levels. The defaulters resources are used first. These are the defaulter-pay variation margin, the initial margin and the defaulters prefunded default fund contribution. Losses exceeding this will be taken from the CCPs equity and the surviving members default fund contributions. If these layers do not cover the losses, additional measures can be taken, so called unfunded default funds.

For CCPs, the question of how much that should be in each layer is of highest importance. To specify the first layers of the waterfall, variation margin and initial margin, the CCPs usually has relatively well-defined and model-based methods. The following layers, however, are often specified qualitatively and are not well defined. This is mainly due to the fact that the international standard setting bodies responsible for regulation has broad and non-mathematical models for CCP risk management. (Ghamami 2015)

In order to quantify the risk management of all layers in the default waterfall, Ghamami (2015) has proposed a framework which specifies all parts of the default waterfall in a mathematically coherent way. He gives a risk sensitive definition of the CCP risk capital, as well as a definition of the total prefunded default funds. The paper Ghamami (2015) will be the foundation of our thesis. However, we will make some additional contributions, which are:

- We model the clearing members defaults via Merton and beta framework.
- We obtain numerical results for the different waterfall layers.
- We obtain numerical values for the CCP default probability in different settings.

The rest of this thesis is organized as follows. In Section 2, some central concepts such as Credit Risk and Counterparty Risk are explained. Section 3 gives a detailed explanation of the concept of Central Counterparties, studying the different layers of the default waterfall thoroughly as well as explaining the advantages and disadvantages of CCPs. In Section 4 we give a detailed description of the static credit models used in the numerical studies. Here we present the mixed binomial model, both with a Merton and a beta framework. This examination is essential in order to be able to apply the CCP models numerically. In Section 5, we review Ghamami (2015) in detail. Since our model is based on Ghamami paper, it is the main foundation on which we will base our numerical results. In Section 6 we will describe our implementation of the model, and finally present our numerical results in Section 7.

2 Central concepts

In this section we will present some central concepts regarding the risk management of financial corporations. We explain terms such as credit risk, counterparty risk and model risk. The risk definitions that we use in this section is mainly taken from Gregory (2008).

2.1 Financial Risk Management

It is of importance for a corporation to manage risk carefully. A good understanding and management of risks can be seen as a strong comparative advantage. Financial risk is often broken down into many different components. In order to understand a specific type of risk, other risk types should also be considered and understood. Some different types of risks are:

- *Market risk* arises from movements in market prices. It can come from stock prices, interest rates, commodity prices etc. It has been studied intensely traditionally, and has lead to the introduction such as the Value-at-Risk etc.
- *Liquidity risk* usually comes in two forms. Asset liquidity risk is the risk that an asset cannot be sold at market prices, due to it being an illiquid asset. Funding liquidity risk refers to the risk that you are unable to fund payments, forcing you to sell assets to get cash flow.
- *Operational risk* arises from people, systems and events. Examples can be human errors, model risk and legal risks. It is a risk that is relatively hard to quantify, however quantitative techniques are increasingly being applied.

2.1.1 Credit Risk

Credit risk, sometime referred to as default risk, can be defined as the overall risk of loss arising from the nonpayments from a debtor to a creditor. A default only occurs if the obligor really cannot pay his obligations. In the event of a default, a workout procedure is entered where the obligor loses control of his assets and a third party tries to pay of the creditors. This procedure follows bankruptcy law, and all obligors must be treated fairly; the obligor cannot choose which claims he honors. Even though these guidelines are followed, it is hard to predict how much losses you will incur in the event of an obligors default. Thus, credit risk has some properties that make it difficult to model quantitatively. (Schönbucher 2003)

Firstly, defaults are rare, and often occur unexpectedly. Arrival risk is the term for the uncertainty whether a default will occur or not, the so-called probability of default. It is usually specified within a certain time period.

Timing risk refers to the precise time of default. It is thus more specific than arrival risk, since it considers not only if, but also when, a default occurs. Secondly, defaults usually involve significant losses for the creditors, and these losses are unknown before the default. Recovery risk is the uncertainty about the severity of losses in the event of a default. Market convention is to set the recovery rate to a fraction of the notional value. Also, the market risk has some influence on the credit risk. For example, the default correlation risk is the risk that several obligors default together. This can be due to a market shock, for example. (Schönbucher 2003)

2.1.2 Systemic Risk

In financial terms, the systemic risk is the risk of a failure creating a domino effect, eventually threatening the financial system as a whole. The losses do not even have to actually occur - a higher perception of risk in general might be enough to cause serious disruptions. The systemic risk could arise from several different situations and is usually thought of having an initial spark, and thereafter some sort of chain reaction. Therefore, ways to prevent systemic risk are:

- Minimize the risk of the initial problem.
- Make sure that the chain reaction does not occur.
- Control the chain reaction and make sure that the damage is limited.

In order to decrease the systemic risk, all ways must be taken into account. Our area of interest, Central Counterparties, is mainly a way of making sure that the chain reaction does not occur. This is by trying to make sure that a potential default of one company does not affect the balance sheet of another company in a way that causes a consecutive default. (Gregory 2008)

2.1.3 Model Risk

When working with financial risk modeling, one has always to consider the model risk. Models can often be very useful, doing quick calculations of prices and risks, which is essential in the fast paced financial industry. In good times, they usually work very well, and enable a dynamic approach to risk management. In the bad times, however, the models are really put to their test. The risks of things going really wrong might sometimes be underestimated, leading to catastrophic results.

2.1.4 Counterparty Risk

Counterparty risk is traditionally thought of as a subset of credit risk. It is defined as the risk that the counterparty in a derivatives transaction will default and therefore not make the payments required by the contract. It mainly differs from credit risk in two aspects. (Gregory 2008)

- The value of a derivatives contract in the future is uncertain, and can be both positive and negative.
- Because of the fact that the value of the contract can be both positive and negative, the risk is typically bilateral.

Historically, counterparty risk has been neglected for several reasons, due to the following flawed notions among institutions trading OTC derivations

- The counterparty will never default.
- The counterparty is too big to be allowed to default.
- If the counterparty were to default, the financial system will already have broken down by then.

Since the financial crisis of 2007-2009, the emphasis of the area has increased, since the concept of a "too big to fail" concept has been shattered. This has made the interest to mitigate counterparty risk more interesting. There are several ways to mitigate counterparty risk. These include diversification, offsetting positive and negative contracts against each other (netting), and holding collateral against exposures. In the next section, we will introduce the Central Counterparties (CCP) as a way to mitigate counterparty risk.

3 Central Counterparties

In the following section, we define the concept of large central counterparties as a way to centralize, mutualize and reduce counterparty risk. The aim of a central counterparty (CCP) is to create an entity that stands between buyers and sellers. Doing this, it bears no market risk, however it centralizes the counterparty risk. We begin by explaining the Clearing concept in Subsection 3.1. Subsection 3.2 presents a historical background of financial regulations as well as the impact of the 2007-2009 financial crisis. In Subsection 3.3 we present some general settings for Central Counterparties. Finally, Subsection 3.4 gives a detailed explanation of the different layers of funds that a CCP carries - the so called default waterfall.

3.1 Clearing Concept

In this subsection we explain the concept of clearing. The main references in this subsection is Gregory (2014) and Gregory (2008). The concept of clearing is broadly defined as the period between execution and settlement of a financial transaction. The execution is the part where the parties agree to legal obligations of trading securities or exchanging cash flows, whereas the settlement refers to the completion of those obligations, as shown in Figure 3.1

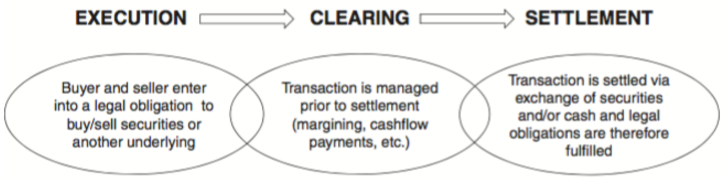

Figure 3.1: Illustration of the clearing concept (Gregory 2014)

The key concept in central clearing is that of so called novation, which is the positioning between buyers and sellers, visualized in Figure 3.2

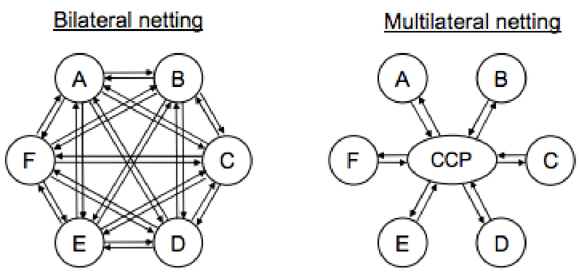

Figure 3.2: Netting concept for a CCP (Gregory 2008)

Novation means that the CCP steps in and acts as an insurer of counterparty risk in both directions, bearing all the counterparty risk. Because of its position between buyers and sellers, the CCP has a "matched book", i.e. no net market risk. In the event of a clearing member default, the CCP has various methods to return to a matched book, for example by auctioning out the defaulting members positions.

Today, a large proportion of the OTC derivatives market is already cleared, and this proportion will increase in the coming years. In order for a transaction to be centrally cleared, some conditions are generally important. Firstly, the product has to be relatively standardized in legal and economic terms, due to the fact that clearing involves contractual responsibility for cash flows. Also, the complexity of the instrument cannot be too high, since the valuation has to be simple in order to be able to calculate margins. Finally, the product should be liquid in order to reduce the liquidity risk. (Gregory 2014)

3.2 Historical Background

In this subsection we overview the regulatory history, focusing on the Basel accords. We subsequently explain the financial crisis of 2007-2009, and continue with describing the regulatory changes made after said crisis.

3.2.1 The Basel Accords

The purpose of bank regulations is to ensure that the banks have enough capital for the risks that it is taking. Prior to 1988, the regulations mainly focused on the ratio of capital to total assets. The regulations varied between different countries, making competition distorted. This, and the fact that the financial derivatives were becoming more complicated, lead to the forming of the "1988 BIS Accord", commonly referred to as Basel I. Basel I was the first attempt to an international risk-based standard for capital requirements. Even though it can be criticized of being too simplistic, it was a huge step at the time. The main focus was for the banks to have a certain percentage of capital, compared to its risk-weighted assets. For example, cash had a risk weight of zero, claims on OECD banks a weight of 20%, whereas uninsured mortgage loans had a risk weight of 50%. The Basel I accord, however made no difference between claims on banks with AAA rating compared to claims on banks with B rating. This, and other weaknesses, led to a new proposition of rules from the Basel Committee, commonly referred to as Basel II. The Basel II accord was first proposed in 1999; however the final rules were not starting to be implemented until 2007. Basel II is based on a three-pillar approach: (Hull 2012)

1. Minimum Capital Requirements: The minimum capital requirements are calculated in a new way which takes credit ratings into account.

2. Supervisory Review: Special supervisors should review and evaluate banks capital adequacy, to make sure they comply with the regulatory standards. If the supervisors are not satisfied, they sure take measures at an early stage.

3. Market Discipline: This requires banks to increase disclosure about how they allocate capital and what risks they take.

As the Basel II was starting to be implemented, the credit crisis of 2007-2009 commenced, which in turn lead to new regulations.

3.2.2 Securitization and the Credit Crisis of 2007-2009

In 2007-2009, the United States experienced the worst financial crisis since the 1930s, and it spreads rapidly to other countries, and from the financial economy to the real economy. The crisis led to a major overhaul of the regulations of financial institutions.

The financial crisis originated in the US housing market. In the early 2000s, mortgage lenders started to relax their lending standards, leading to many loans with low quality, so called subprime loans. The relaxed lending standards increased the demand of real estate, with an increase in prices as shown in Figure 3.3.

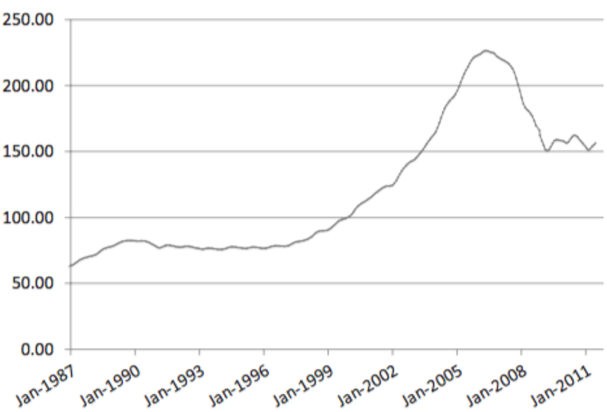

Figure 3.3: Real estate price index from 1987 to 2011 (Hull 2012)

The rise in prices meant that the collateral always covered the lending. Thus, a default did result in very small losses. The rising house prices lead to difficulties for new entrants to the housing market, so in order to continue the lending standards were lowered even further. This behavior of course could have been regulated, however the US government had since the 1990s tried to expand home ownership. For example, in 2002 George W. Bush stated the following in a speech.

> It's a clear goal, that by the end of this decade, we'll increase the number of minority homeowners by at least 5 1/2 million families. [...] One of the programs is designed to help deserving families who have bad credit histories to qualify for homeownership loans. [...] If you put your mind to it, the first-time homebuyer, the low-income homebuyer can have just as nice a house as anybody else. (Bush 2002)

Furthermore, the mortgage originators seldom kept the mortgages themselves. Portfolios of mortgages were sold to companies, which created products of them that investors could buy. Since the mortgages could be sold to a third party, this led to further relaxations of lending standards. The credit quality of the lender did not matter - what mattered was if the mortgage could be sold to someone else. When these mortgages were securitized, they were often given much higher ratings than they deserved, and were thus considered more safe than they were. The relaxed credit quality requirements lead to a housing price bubble, which eventually burst. When mortgage holders were not able to pay their debts, foreclosures occurred and a large number of houses came to the market. (Hull 2012)

As foreclosures increased, so did the losses on mortgages. The losses eventually exploded, and average losses as high as 75% was reported for mortgages on houses foreclosures in some areas. Financial Institutions such as Merrill Lynch, Citigroup and UBS had large positions in some sub-prime loans and incurred huge losses, as did the insurance company AIG. During the financial crisis, AIG was heavily involved in issuing CDS contracts. AIG had not set aside enough capital for their CDS-exposures, and still sold CDS protection without enough collateral requirements. Still, AIG had excellent credit quality, and was able to write out CDS protection on five hundred billion $USD of notional debt. In 2008, AIG suffered a $99.3 billion loss, and failed due to liquidity problems. The US Department of Treasury and the Federal Reserve Bank of New York had to arrange loans as support. Because of AIG's excessive risk taking, they required $100 billion of tax money. AIG was one of many institutions that were rescued by government bailouts. However, many financial institutions did fail, such as Lehman Brothers, Washington Mutual etc. (Hull 2011)

3.2.3 Regulations after the Financial Crisis

Basel III

After the crisis of 2007-2009, it was clear that a revision of the Basel II was necessary. The Basel III proposals were first published in 2009, and a final version was released in 2010. It dramatically increased the amount of equity capital banks were required to keep. Also, it imposed new liquidity requirements since a lot of the problems in the crisis were related to liquidity. (Hull 2012) Apart from this, a main area of interest in the new Basel accords is counterparty credit risk. According to the Bank for International Settlements (BIS), roughly two thirds of the losses attributed to counterparty credit risk were due to CVA losses, and only about one third due to actual losses. (BIS 2011) Credit Value Adjustment (CVA) is a way to include counterparty credit risk into derivative prices. It should cover the potential losses due to a default of the counterparty in a derivative transaction. The CVA-amount is the basis for putting aside collateral to be used in financially distressed periods. (Brigo et al. 2013) (BIS 2011)

In 2014, the Basel Committee presented the final version of a revised framework for capital treatments of banks exposed to central counterparties. The final standard will apply as of 1 January 2017. In these rules, the concept of a Qualified CCP (QQCP) is defined. A QCCP must comply with the current regulations, and is obligated to inform their members with information required to calculate their capital requirements. If a financial institution is exposed to a QCCP, it will receive preferential capital treatment as opposed to otherwise, meaning the bank will be able to hold a smaller capital buffer. For a QCCP, capital requirements for trade-related exposures have a relatively low risk weight of between 0 and 2%. A risk weight of e.g. 2% means that e.g. a bank needs to hold safety capital equal to 2% of the total value. In comparison, a clearing member trading with a non-qualifying CCP is required to capitalize in accordance to a bilateral framework. This will give risk weight of at least 20%. This large discrepancy can of course be dramatic, if a CCP were to lose its qualifying status. (BIS 2014)

Dodd-Frank

In the US, president Obama signed the Dodd-Frank act in July 2010. Its goal is to protect the consumer, and prevent future bailouts of financial institutions. An important goal is to increase the transparency and avoid a new AIG scenario, where large positions in credit derivatives are unknown to regulators. Some regulations are:

- New entities were created in order to monitor systemic risk and research the state of the economy.
- Issuers of securitized products were required to keep at least 5% of each product created.
- Mortgage lenders have to base loans on verified information that the borrower has the ability to repay the loan.
- Standardized OTC derivatives were required to be cleared by Central Counterparties.

The Securities and Exchange Commission (SEC) or the Commodity Futures Trading Commission (CFTC) regulates the CCPs used for clearing in the United States. In addition to this, the trade must be done electronically and reported to a trade repository. (Hull 2012)

3.3 CCP Setup

The CCP idea can be traced back all the way to the 19th century, where exchanges where used for futures trading. In the late 19th century, there existed clearing rings, which organized loss mutualisation in clearings via financial contributions to absorb member default losses. Despite the growth of the financial markets in recent decades, counterparty risk has remained primarily a bilateral matter. As of late, however, the counterparty risk has received more attention, due to its role in the 2007-2009 financial crisis. (Gregory 2008) Today, the global association of CCPs consists of 31 different CCP organizations across the world. According to Gregory (2014), some big players today when it comes to clearing OTC derivatives are:

- *LCH.Clearnet:* A major independent CCP, which through SwapClear is dominant in the interest rate swap market. In May 2016 it had around 100 members.
- *ICE:* Offers clearing for energy products, and some CDS products. In May 2016, it had around 30 members.
- *CME:* Acts as CCP for energy derivatives, and also has an interest rate swap clearing service.
- *Eurex:* Clears mainly in the equity area, but also manages interest rate swaps.

CCP ownership and operation can have either a horizontal or vertical setup. A vertical CCP can be specialized for a particular type of financial product, and these entities are usually more efficient and cheaper. Examples of vertical CCPs are Eurex and CME. A horizontal CCP are typically jointly owned by the clearing members. This encourages market competition, however it can be considered less efficient and more expensive. Two examples of horizontal CCPs are LCH.Clearnet and OCC.(Gregory 2014). In this paper, we only consider horizontal CCPs, which are owned by the clearing members themselves.

3.3.1 CCP Risk Management

The CCP stands between buyers and sellers and guarantees the performance of trades. Thus, all counterparty risk is centralized within the CCP. This means that the original counterparties has no need to monitor one another in terms of credit quality. Consequently, this puts the emphasis on the operation of the CCP itself. In order to mitigate counterparty risk, the CCP performs a number of related functions. For example, by centralization of trades, netting benefits can reduce the exposures for the members. The CCP is also responsible for membership review and background checks. The CCP has strict requirements for admission, to make sure that members have a low probability of insolvency and that it can participate in the clearing. For example, the members should have a sufficient capital base, and a solid credit rating (e.g. BBB or higher). A solid background check of the members ensures that the CCP has a low probability of facing a defaulting member.(Gregory 2014)

In the event of a member default, the CCP will manage this. The CCP has also the right to declare a member defaulted. Default can occur if a member is insolvent and fails to make margin payments. However, the CCP can also declare a default if a member appears to be unlikely to meet its obligations in respect of its contracts. Once default is declared, the CCP will manage the markets risks associated with the defaulted member. In this part, the loss allocation will take place, which we will now explain further. (Gregory 2014)

3.4 The Default Waterfall and Loss Allocation

Central Counterparties usually rely on a waterfall of financial resources to absorb losses from defaults among clearing members. In this section, we present a typical structure of a CCP waterfall, defining the different layers thoroughly, with the definitions mainly found in Gregory (2014). A solid understanding of the different layers in the default waterfall is essential in order to implement the model presented by Ghamami (2015). Figure 3.4 illustrates the waterfall concept.

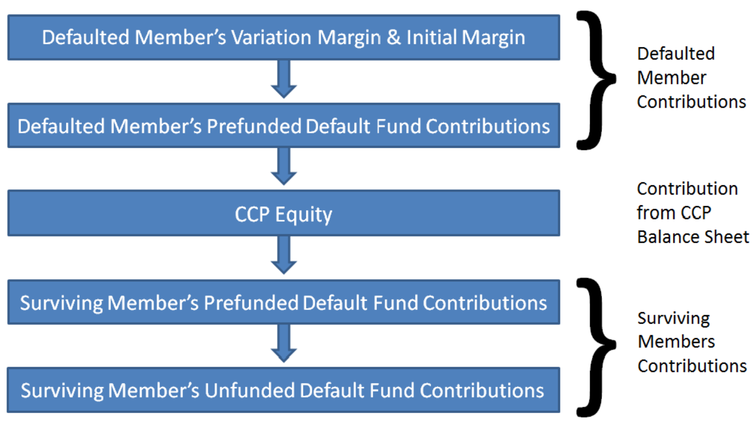

Figure 3.4: Illustration of a typical default waterfall for CCP

3.4.1 Variation Margin and Initial Margin

Variation Margin

Variation margin is an adjustment for the change in mark-to-market value of the relevant positions. Valuation of the variation margin is rather straightforward, since a prerequisite for clearing is that the underlying trades are standardized and liquid. Since the CCP is counterparty for all trades, CCP calculates the value of all positions, and subsequently collects or pays the respective margin amount.

Initial Margin

The Initial Margin is a key concept for clearing. It is an additional margin that should cover the largest projected loss on a given transaction. It is usually determined quantitatively, and can be quite complex. Excessive margins raise costs of trading, however under-margining will impose too much riskiness.

A common approach for initial margin calculations is Value-at-Risk type of approaches. Value-at-Risk (VaR) is a convenient way of summarizing the risk of a loss distribution, where the definition is that we are α % confident that we will not lose more than VaR currency units during a certain time period, where α is often defined to either 95% or 99%. After having defined VaR, one can also calculate the Expected Shortfall, which is the expected loss given that the loss exceeds the Value-at-Risk. However, Initial Margins are typically generated with the intention of covering 'normal market conditions'. In Table 3.1 , we show some examples of how real world CCPs calculate their initial margins. (Gregory 2014)

	LCH.Clearnet	CME	Eurex
History	10 years	5 years	3 year + 1 year stress period
Measure	Expected shortfall (average of 6 worst)	99.7% VaR (4th highest loss)	At least 99% (average of five VaR measures)
Volatility Scaling	Filtered historical simulation	EMWA with volatility scaling and volatility floor	Filtered historical simulation
Liquidity Period	5 days	5 days	5 days

Table 3.1: Various Methods for Initial Margin Calculation

3.4.2 Default Fund

The role of the default fund is to absorb losses beyond the margins posted. This distribution of losses can be very heavy tailed, as shown in Figure 3.5

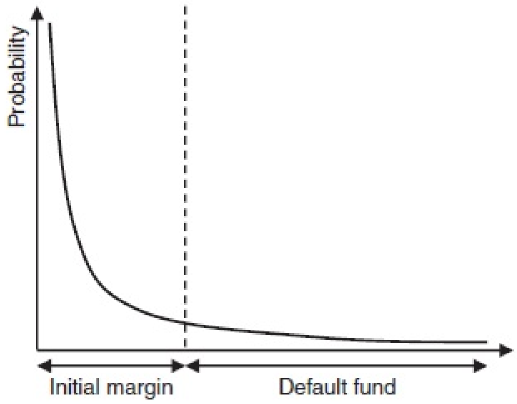

Figure 3.5: Relation between initial margin and default fund (Gregory 2014)

The default fund is a key component in clearing since it is mutualised, meaning that losses are shared among the CCP members. This loss mutualisation means that the total default fund provides a much higher coverage of losses than the initial margin, even though the size of the initial margin is usually higher. However, increased default funds can induce moral hazard, since you pay for less of your own losses. The size of the default fund thus has to be considered carefully. Since its purpose is to cover risks from very extreme scenarios, the calculations can be quite complex. The probability of a CCP exceeding their default contributions is nontrivial to quantify, since it is linked to scenarios that involves several member defaults as well as extreme market movements. CCPs typically calibrate their total default funds through stress tests. For example, SwapClear wants their default fund to cover losses from their two largest clearing members, using extreme stress testing scenarios. (Gregory 2014)

3.4.3 CCP Equity

If the initial margin and default fund has been exhausted, further losses can be taken from the CCP itself. These so called equity contributions can be taken from the CCPs current profits etc. This is to make sure that the CCP also has so called 'skin-in-the-game', and therefore has incentives to make sure that the initial margins and default funds are sufficient. How much equity that the CCP should add is a hot area of discussion. The banks, of course, have an interest in the CCP to add more capital to the default waterfall. Both JP Morgan and Citi have made recent expressions saying that CCPs should increase their contributions. JP Morgan, for example, suggested that this at least should be the maximum amount of 10% of the default fund or the contribution from the largest single clearing member. (JP Morgan 2014). If we however look at the current industry average, the current OTC 'skin-in-the-game' average is 2.6% of the combined default funds. (Risk.net 2016).

3.4.4 Remaining Loss Allocation Methods

When the losses go beyond the default waterfall layers previously mentioned, further loss allocation methods are needed. In theory, the CCP could demand that the members should make unlimited contributions to the default fund. However, the possibility of an unlimited call on more contributions can lead to more defaulting members, and is sometimes not even possible due to regulations. Therefore, there are other methods that are usually preferred. Examples of such a methods are Variation Margin Gains Haircutting (VMGH) or tear-ups, partial or complete. We will not discuss these concepts here since they will not be considered in our implementation, neither are they treated in Ghamami (2015). More information about loss allocation methods, and more on the area, can be found in Gregory (2014).

3.5 Advantages and Disadvantages of CCPs

The goal of the clearing-house is to drastically reduce the counterparty risk. The main pros are in particular: (Herbertsson 2015)

- CCP manages all margin calls and this reduces sudden dry up of liquidity. The CCP can provide temporal liquid collateral for companies in liquidity distress.
- The CCP will guarantee the protection in a derivatives contract in the scenario in which the protection seller defaults.
- CCP will have all the information about all financial transactions made on its platform. Thus, it can identify dangerous asymmetric positions among members and in that case report such patterns to the regulators. Moreover, thanks to the overall information available, CCP will be able to create a more efficient netting of collateral.
- The CCP disclosing public information about the transaction can increase the transparency and facilitate the use of reliable information.

On the other hand, some problems can arise after the introduction of CCP. To be more specific, these include: (Herbertsson 2015)

- CCP may act as intermediaries and reduce counterparty risk. In the same time, this can create moral hazard issues.
- The introduction of CCPs may give rise to greater systemic risk linked to their own failure. A scenario where the CCP defaults could have disastrous consequences.
- Mitigation of counterparty risk may create other financial risk such as operational and liquidity risk.
- In order to assess the risk toward a CCP, clearing members need information about the portfolio composition of each member, and this information is generally confidential. The clearing members have information only on the details of their own portfolio, total margin levels held by CCP and identities of the other clearing members.

3.6 Importance of CCP Framework

CCPs collectively take a well-defined model-based approach of margin requirements. However, the CCP risk capital depends on all layers of the default waterfall, and prefunded and unfunded default funds are often specified more qualitatively or on ad hoc basis. The main reason for the lack of quantitative models is that the international standard setting bodies (SSB) responsible for CCP regulation has set broad, risk insensitive and non-model based principles for CCP risk management, where it is not taken in consideration the portfolios heterogeneity and the correlation effect among clearing members. Since the CCP risk capital is dependent on all layers in a complicated way, an unified framework is essential in order to have a confident risk management. (Ghamami 2015)

4 Static Credit Risk Portfolio Models

In this section we outline the theory for static credit portfolio risk, traditionally used for credit risk management. Later the theory presented in this section will be used to compute the different risk measures for CCP outlined in Section 3. First, in Subsection 4.1, we describe the mixed binomial model, and its usage in credit risk management. Subsequently in Subsection 4.2, two examples of the mixed binomial model are presented, namely the mixed binomial model in the Merton framework and the mixed binomial model with a beta distribution. Finally, in Subsection 4.3, a model comparison of the beta and the Merton model is made.

When applying quantitative credit risk models, there are two main settings. The first area regards the analysis of credit-risky securities and implemented models for the valuation of portfolio credit derivatives. When pricing credit risk securities, dynamic models are required and thus the timing of defaults plays a central role. The second area is represented by the credit risk management, which uses applied measures such as Value-at-Risk and Expected Shortfall. These models are typically static and only consider the arrival risk, which is the risk connected to whether or not an obligor will default in a given time period. In our thesis we will, just as Ghamami (2015), consider static credit risk models, meaning that we are only interested in the arrival risk, and not concern the timing risk of defaults. In this section, we closely follow the notation and setup presented in Chapter 5 in Herbertsson (2015). For more on static credit risk models, see e.g Lando (2004) or McNeil et al. (2005).

4.1 The Mixed Binomial Model

In the binomial loss model for m obligors it is assumed that $X_i = 1$, if obligor i defaults up to a fixed time point T, and $X_i = 0$, otherwise. Further, assume that $X_1, \ldots, X_m$ are i.i.d where $\mathbb{P}[X_i = 1] = p$ and $\mathbb{P}[X_i = 0] = 1 - p$. As we will see later, the binomial loss model will lead to unrealistic results. Therefore, we will in this section discuss the so-called mixed binomial models, which introduce the dependence among the default of the obligors, getting closer to real scenarios. In order to introduce dependence among $X_1, \ldots, X_m$, the binomial model is substituted with a Bernoulli mixture model which can be seen as a conditional binomial model. The dependence is reached by randomizing the individual default probability $p(Z)$ in the standard binomial model, where Z represents some common underlying stochastic factors that affect all obligors in the portfolio. A mixed binomial loss model works as follows.

Let $X_1, X_2, \ldots, X_m$ be identically distributed random variables that takes on the value of 1 if the respective obligor defaults up to time T, and a value of 0 otherwise. Moreover, conditional on Z the X_i's are independent and each X_i has default probability $p(Z) = \mathbb{P}[X_i = 1|Z]$. Letting the individual default probability of X_i be denoted by $\bar{p}$, i.e $\bar{p} = \mathbb{P}[X_i = 1]$, we have

$$\mathbb{P}[X_i = 1] = \mathbb{E}[X_i] = \mathbb{E}[\mathbb{E}[X_i|Z]] = \mathbb{E}[p(Z)] = \bar{p}. \tag{4.1}$$

This means that the expected value of the conditional default probability is equal to the individual default probability. Furthermore, the variance of X_i and the covariance among the obligors default will be:

$$\text{Var}(X_i) = \bar{p}(1 - \bar{p}) \tag{4.2}$$

and

$$\text{Cov}(X_i, X_j) = \mathbb{E}[p(Z)^2] - \bar{p}^2 = \text{Var}(p(Z)). \tag{4.3}$$

Assuming homogeneous credit portfolio model with m obligors and letting the loss rate ℓ be constant for each obligor, the total credit loss in the portfolio at the arrival time T will be

$$L_m = \sum_{i=1}^{m} \ell X_i = \ell \sum_{i=1}^{m} X_i = \ell N_m \tag{4.4}$$

where $N_m = \sum_{i=1}^{m} X_i$ represents the number of defaults in the portfolio up to time T. Since the default indicators X_1,X_2,...,X_m are independent conditional on Z, we have a conditional binomial model where

$$\mathbb{P}[N_m = k|Z] = \binom{m}{k} p(Z)^k (1 - p(Z))^{m-k}. \tag{4.5}$$

From Equation (4.1), we have $\mathbb{P}[N_m = k] = \mathbb{E}[\mathbb{P}[N_m = k|Z]]$, so if Z is continuous with density $f_Z(z)$ we get

$$\mathbb{P}[N_m = k] = \int_{-\infty}^{\infty} \binom{m}{k} p(z)^k (1 - p(z))^{m-k} f_Z(z) dz. \tag{4.6}$$

Furthermore, due to the dependence, the variance of N_m is given by

$$\text{Var}(N_m) = \sum_{i=1}^{m} \text{Var}(X_i) + \sum_{i=1}^{m} \sum_{j=1, j \neq i}^{m} \text{Cov}(X_i, X_j). \tag{4.7}$$

By homogeneity assumption and using Equation (4.3) we can rewrite (4.7) as

$$\text{Var}(N_m) = m\bar{p}(1 - \bar{p}) + m(m - 1)(E[p(Z)^2] - \bar{p}^2). \tag{4.8}$$

As shown in Herbertsson (2015), when $m \to \infty$ $\text{Var}(\frac{N_m}{m}) \to \mathbb{E}[p(Z)^2] - \bar{p}^2$, see also in e.g Lando (2004). In particular, when $p(Z)$ is constant, we are back in the standard binomial model, where $\text{Var}(\frac{N_m}{m}) \to p^2 - p^2 = 0$ and the law of large numbers can be applied. Thus, in the binomial loss model with constant default probability, that is $p(Z) = \bar{p}$ for some constant $\bar{p} \in [0, 1]$, the average number of defaults converge to the constant p. In the mixed binomial model, however, the law of large numbers do not hold, except in the case where $p(Z)$ is constant. Finally, as shown in Herbertsson (2015) and McNeil et al. (2005), the correlation ρ_x in a mixed binomial model is given by

$$\rho_x = \frac{\mathbb{E}[p(Z)^2] - \bar{p}^2}{\bar{p}(1 - \bar{p})}. \tag{4.9}$$

4.1.1 Asymptotic Behavior in Large Portfolios

Since the variance of $\frac{N_m}{m}$ does not converge to zero in the mixed binomial model, the default fraction do not converge to p as $m \to \infty$. As mentioned previously, Bernoulli mixture models are an extension of the standard binomial model. Even though the default indicators are not independent, they will become independent if we condition on the common factor Z. Consequently, the law of large numbers can be applied, and thus it can be shown that given a fixed outcome of Z we have

$$\lim_{m \to \infty} \frac{N_m}{m} = p(Z) \tag{4.10}$$

and this event occurs with probability one conditional on Z. Since convergence almost surely implies convergence in distribution we have

$$\mathbb{P}\left[\frac{N_m}{m} < x\right] \to \mathbb{P}[p(Z) \leq x] \text{ as } m \to \infty. \tag{4.11}$$

The result in Equation (4.11) is often referred to as the large portfolio approximation (LPA) in a mixed binomial model. Hence, the large portfolio approximation guarantees that in large portfolios the fraction of defaults $\frac{N_m}{m}$ converges to the distribution of $p(Z)$. Thus, in large portfolios, if $p(Z)$ has fat tails, $\frac{N_m}{m}$ will have fat tails as well. (McNeil et al. 2005) (Lando 2004) (Herbertsson 2015)

4.2 Examples of the Mixed Binomial Model

In order to get numerical results in our model implementation we will use two examples of the mixed binomial model: the mixed binomial model inspired by the Merton framework, and the mixed binomial model with a beta distribution. In this section we will discuss the theory behind both these models. Later we will present the results we get when we implement them in a static central counterparty risk framework. This subsection closely follows the notation and outline presented in Herbertsson (2015) and Lando (2004).

4.2.1 The Mixed Binomial Model in the Merton Framework

The Merton model assumes that the asset value of each obligors i follows a geometric Brownian motion

$$dV_{t,i} = \mu_i dt + \sigma_i V_{i,t} dB_{t,i} \tag{4.12}$$

where the stochastic process $B_{t,i}$ is defined as follows

$$B_{t,i} = \sqrt{\rho} W_{t,0} + \sqrt{1-\rho} W_{t,i} \tag{4.13}$$

and $W_{t,0}$,$W_{t,1}$,......$W_{t,m}$ are independent standard Brownian motions. Moreover, by applying Ito's lemma we have

$$V_{t,i} = V_{0,i} e^{(\mu_i - \frac{1}{2}\sigma_i^2)t + \sigma_i B_{t,i}}. \tag{4.14}$$

Merton proposes a way of analyzing risk using the company's balance sheet as starting point. Following this idea, the default will occur when the value of the assets are lower than the value of the liabilities. Letting D_i be the debt level of obligor i, than the defaults will occur if

$$V_{o,i} e^{(\mu_i - \frac{1}{2}\sigma_i^2)T + \sigma_i B_{T,i}} < D_i. \tag{4.15}$$

Since the logarithm is a strictly increasing function we can rewrite the disequality as

$$\ln(V_{0,i}) - \ln(D_i) + (\mu_i - \frac{1}{2}\sigma_i^2)T + \sigma_i(\sqrt{\rho} W_{T,0} + \sqrt{1-\rho} W_{T,i}) < 0. \tag{4.16}$$

Recall that $W_{T,i} \sim N(0,T)$. If $Y_i \sim N(0,1)$, then $W_{T,i}$ has the same distribution of $\sqrt{T} Y_i$ for each $i = 0, 1, ..m$, when $Y_0, ...Y_m$ are independent. Moreover, defining Z as $Z = Y_0$ and dividing with $\sigma\sqrt{T}$ we have

$$\frac{\ln V_{0,i} - \ln D_i + (\mu_i - \frac{1}{2}\sigma_i^2)T}{\sigma_i \sqrt{T}} + \sqrt{\rho} Z + \sqrt{1-\rho} Y_i < 0. \tag{4.17}$$

Rearranging Equality (4.17) we get

$$Y_i < \frac{-(C_i + \sqrt{\rho} Z)}{\sqrt{1-\rho}} \tag{4.18}$$

where C_i is a constant equal to

$$\frac{\ln(\frac{V_{0,i}}{D_i}) + (\mu_i - \frac{1}{2}\sigma_i^2)T}{\sigma_i \sqrt{T}}. \tag{4.19}$$

Since $V_{T,i} < D_i$ implies default of obligor i, and it is equivalent to $Y_i < \frac{-(C_i + \sqrt{\rho}Z)}{\sqrt{1-\rho}}$ we have that

$$\begin{aligned}\mathbb{P}[X_i = 1|Z] &= p(Z)\\ &= \mathbb{P}[V_{T,i} < D_i|Z]\\ &= \mathbb{P}\left[Y_i < \frac{-(C_i + \sqrt{\rho}Z)}{\sqrt{1-\rho}}\middle| Z\right]\\ &= N\left(\frac{-(C_i + \sqrt{\rho}Z)}{\sqrt{1-\rho}}\right)\end{aligned}$$

where the last equation is given from the fact that Y_i is normally distributed with mean zero, variance one and independent of Z, and $N(x)$ is the distribution function of a standard normal random variable.

As shown in the Herbertsson (2015), $C = -N^{-1}(\bar{p})$, so we have:

$$\mathbb{P}[X_i = 1|Z] = p(Z) = N\left(\frac{N^{-1}(\bar{p}) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right). \tag{4.20}$$

From Equation (4.6) we can derive the probability of having k defaults:

$$\begin{aligned}\mathbb{P}[N_m = k] = \int_{-\infty}^{\infty} \binom{m}{k} N\left(\frac{N^{-1}(\bar{p}) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right)^k\\ \cdot \left(1 - N\left(\frac{N^{-1}(\bar{p}) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right)\right)^{m-k} \frac{1}{2\pi} e^{-\frac{y^2}{2}} du.\end{aligned} \tag{4.21}$$

Furthermore, $\mathbb{P}[N_m \leq n] = \sum_{i=0}^{m} \mathbb{P}[N_m = k]$ for $n = 1, 2, ..., m$. This formula can be problematic to implement in a numerical software environment due to numerical approximation errors, for example in Equation (4.21) when computing $\binom{m}{k}$. Recalling Equation (4.11), when m is large enough we can approximate $\mathbb{P}[N_m \leq x]$ with its LPA-formula. Thus, for any x we have:

$$\mathbb{P}[N_m \leq x] = \mathbb{P}\left[\frac{N_m}{m} \leq \frac{x}{m}\right] \approx F\left(\frac{x}{m}\right). \tag{4.22}$$

Hence, in the mixed binomial model Merton framework we have

$$F(x) = \mathbb{P}[p(Z) \leq x] = \mathbb{P}\left[N\left(\frac{N^{-1}(\bar{p}) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right) \leq x\right] \tag{4.23}$$

and this means that our final equation, as shown in Herbertsson (2015) is:

$$F(x) = N\left(\frac{1}{\sqrt{\rho}}\left(\sqrt{1-\rho}N^{-1}(x) - N^{-1}(\bar{p})\right)\right). \tag{4.24}$$

4.2.2 The Beta Distributed Mixed Binomial Model

Another example of the mixed binomial model is when $p(Z) = Z$ where Z is beta distributed with parameters a and b, where the density $f_Z(z)$ is given by

$$f_Z(z) = \frac{1}{\beta(a,b)} z^{a-1}(1-z)^{b-1} \tag{4.25}$$

for $a, b > 0$ and $0 < z < 1$ where

$$\beta(a,b) = \int_0^1 z^{a-1(1-z)^{b-1}} dz = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}. \tag{4.26}$$

Here $\Gamma(x)$ is the so called gamma function, see further in e.g. Lando (2004). It can be shown that using beta distribution we have the following properties:

$$\mathbb{E}[Z] = \mathbb{E}[p(Z)] = \bar{p} = \frac{a}{a+b} \tag{4.27}$$

$$\mathbb{E}[Z^2] = \frac{a(a+1)}{(a+b)(a+b+1)} \tag{4.28}$$

$$\text{Var}(Z) = \frac{ab}{(a+b)^2(a+b+1)}. \tag{4.29}$$

Furthermore, by using Equations (4.27)-(4.29) in Equation (4.9) one can show that the default correlation ρ_x in a beta distributed mixed binomial model is:

$$\rho_x = \frac{1}{a+b+1}. \tag{4.30}$$

The probability of having k defaults is given by:

$$\mathbb{P}[N_m = k] = \binom{m}{k} \frac{\beta(a+k, b+m-k)}{\beta(a,b)} \tag{4.31}$$

see e.g in Lando (2004). Finally, since $p(Z) = Z$, using the LPA, $\frac{N_m}{m}$ converges to the beta distribution, meaning that

$$\mathbb{P}\left[\frac{N_m}{m} \le x\right] \to \frac{1}{\beta(a,b)} \int_0^x z^{a-1}(1-z)^{b-1} dz \tag{4.32}$$

where the right hand side in (4.32) is found in any mathematical or statistical software package, see e.g the function "betacdf" in MATLAB or Excel.

For the beta mixed binomial model and the mixed binomial model inspired by Merton framework, the behavior of $\mathbb{P}[\frac{N_m}{m} \le x]$ for some $\bar{p}$ and ρ is similar in both models up to the 99%-quantile, see e.g McNeil et al. (2005). Increasing ρ_x, ceteris paribus, we expect that the probability mass of the distribution will tend to concentrate more on the extreme events, namely fatter tails in the density function. In Figure 4.1 we plot different portfolio credit loss distribution and density functions in the beta mixed binomial model, according to different values of default correlation ρ_x. From Figure 4.1 we can conclude that higher default correlation implies higher probability of having extreme scenarios in terms of losses. By using a lower ρ_x, the loss distribution is centered more around its expected value. In particular, when default correlation is 0.8 choices in terms of confidence level α have higher impact in the overall portfolio credit risk when computing for example Value-at-Risk at α-level. Since more probability mass is concentrated in the tail, a value at risk computed with $\alpha = 0.95$ will not be sufficient and will be way smaller than a value at risk using higher confidence levels.

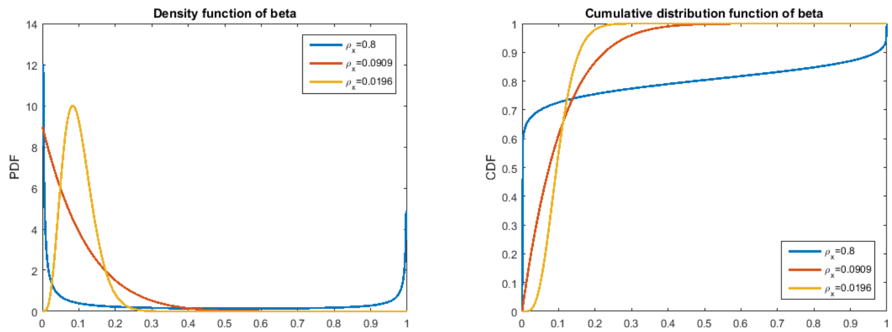

Figure 4.1: Beta density and cumulative distribution function for different values of ρ_x, when $\bar{p} = 0.1$.

4.3 Comparison Between the Merton and Beta Model

In finance, model risk is the risk of loss resulting from using models that fail to take certain scenarios into account. In order to be able to draw valid conclusions, we must assure that the models are comparable. In our setting we will implement Merton and beta models using the same default probability $\bar{p}$ and default correlation ρ_x in both models. Knowing $\bar{p}$ and ρ, we are able to derive the default correlation ρ_x from the Merton model, that is

$$\rho_x^{Merton} = \frac{\mathbb{E}[p(Z)^2] - \bar{p}^2}{\bar{p}(1-\bar{p})} \tag{4.33}$$

where the second moment of $p(Z)$ will be

$$\mathbb{E}[p(Z)^2] = \mathbb{E}\left[\left(N\left(\frac{N^{-1}(\bar{p}) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right)\right)^2\right]. \tag{4.34}$$

Once we have the default correlation, we set $\rho_x^{Merton} = \rho_x^{Beta}$ and then also use Equation (4.27) and (4.30) in order to derive the parameters a, b in the Beta model, which are found to be the following:

$$a = \frac{(1-\rho_x)\bar{p}}{\rho_x} \tag{4.35}$$

$$b = \frac{(1-\rho_x)(1-\bar{p})}{\rho_x} \tag{4.36}$$

With these derived a and b values we are able to analyze tail events in both models and compare the relative differences in the two scenarios, where we know that $\bar{p}$ and ρ_x are the same in both models. Hence the comparison is "fair" in the sense that the key quantities such as default correlation and default probability are identical in both models.

5 Ghamami Model for CCP Risk

The research on the optimal CCP design and the impact derivative CCPs have on the system risk is not conclusive, and there is still an ongoing debate on whether CCPs would make the financial system as a whole more stable. Different views are presented by e.g. Cont & Kokholm (2014), Duffie & Zhu (2011), Pirrong (2014), and the references therein.

The international standard setting bodies responsible for regulation of CCPs has broad and non-model based principles for risk management, particularly for the waterfall resources beyond the initial margin. In the absence of a standardized framework for the default waterfall resources, the different risk management framework among CCPs' can create inconsistencies among CCPs and their clearing members. For this reason, Ghamami (2015) proposes a framework for CCP risk management in a static credit risk setting. He specifies the default waterfall in a mathematically coherent way. In particular, the contribution of Ghamami (2015) is two-fold:

- A risk sensitive definition of the CCP risk capital
- A risk sensitive definition of total prefunded default funds and a technique to specify each clearing members default fund contribution

Ghamami (2015) does not, however, present any numerical results of his model. The main contribution of this thesis is to numerically implement the CCP model outlined by Ghamami (2015), and in order to be able to do so, we will in the coming section present the proposed model thoroughly.

5.1 One-Period Model

In Section 4, we used the notation X_i for the default indicators of obligor i where $i = 1, 2 \ldots, m$. In this section, we will follow the notation and setup of Ghamami (2015) and therefore denote the default indicators with Y_i for each clearing member. Hence, consider n different clearing members indexed by $i = 1, 2, \ldots, n$. Furthermore, let $Y_1, \ldots, Y_n$ be the default indicator functions for these clearing members for a fixed time period of T years, so that $Y_i = 1$ if clearing member defaults up to time T, and 0 otherwise. Also, let $\bar{p}_i$ be the T-year default probability for clearing member i, i.e $\bar{p}_i = \mathbb{P}[Y_i = 1]$, or $\bar{p} = \mathbb{E}[Y_i]$. In this thesis we will assume a homogeneous group of clearing members so that $\bar{p}_1 = \bar{p}_2 = \ldots = \bar{p}_n = \bar{p}$. However, many of the calculations can be done for both homogeneous and inhomogeneous groups of clearing members. Furthermore, we want to emphasize that the CCP model outlined in Ghamami (2015) is independent of how the default indicators are explicitly modeled as long as the exposures are a function of the Y_i's.

We will model $Y_1, \ldots, Y_n$ by using two different mixed binomial models as described in Section 4, namely the mixed Merton binomial model and the mixed beta binomial model, in contrast to Ghamami (2015), who uses the concept of copulas.

5.1.1 First Level Loss

Let C_i denote the CCP's collateralized credit exposure to clearing member i, below denoted by CM_i, at its default in the presence of the CM_i's variation margins and initial margins (VM and IM). A dynamic counterparty risk measure, e.g. Expected Exposure (EE) or Expected Positive Exposure (EPE), can be used to define the C_i. The CCP's collateralized exposure in the presence of variation and initial margin is defined as follows:

$$C_i \equiv (1-\delta_i)\int_0^T \mathbb{E}[e_i(t)]dt \tag{5.1}$$

where δ_i is the CM_i's recovery rate, and $e_i(t)$ denotes the collateralized exposure to CM_i at time t, defined as follows:

$$e_i(t) \equiv \max\Big\{\big(V_i^+(t+\Delta) - VM_i(t) - IM_i(t)\big), 0\Big\} \tag{5.2}$$

where $V_i(t)$ denotes the value of the derivatives portfolio that the CCP holds with CM_i at time t, and $V_i^+(t) = \max\{V_i(t), 0\}$. Thus, $VM_i(t) \equiv V_i^+(t-\hat{\Delta})$, whereas $IM_i(t)$ is the VaR or ES associated with $V_i^+(t+\Delta) - V_i^+(t-\hat{\Delta})$. While Arnsdorf (2012) considers negative value for the losses as well, i.e gains, Ghamami (2015) only uses positive losses for the CCP in the case of clearing member default, and also does not take netting into account. In addition to this, Ghamami (2015) uses max in $e_i(t)$ and takes an average over time with the integral $\int_0^T \mathbb{E}[e_i(t)]dt$, which further suggests that a more conservative approach is used.

The frequency of variation margin calls is denoted by $\hat{\Delta}$, and is usually set to one day. Furthermore, the time interval $\hat{\Delta}+\Delta$ is pre-specified (common is 5 days) and referred to as the margin period of risk (MPOR). It represents the time within the CCP is required to replace a defaulting member's portfolio. Finally, the CCP's total credit loss L in first level scenario is specified as

$$L = \sum_{i=1}^{n} C_i Y_i \tag{5.3}$$

where $Y_1, \ldots, Y_n$ are defined as in Subsection 5.1.

5.1.2 Second Level Loss

When computing the pre-funded defaulted funds DF, one can either use VaR or ES as measure, see e.g page 9 in Ghamami (2015). The pre-funded defaulted fund of clearing member i in terms of VaR or ES is defined in the following way:

$$DF_i = C_i\mathbb{E}[Y_i|L = VaR_\alpha] \tag{5.4}$$

or

$$DF_i = C_i\mathbb{E}[Y_i|L \geq VaR_\alpha] \tag{5.5}$$

Moreover, the DF based on VaR and ES respectively is defined as:

$$DF = VaR_\alpha(L) \tag{5.6}$$

or

$$DF = ES_\alpha(L) = \mathbb{E}[L|L \geq VaR_\alpha(L)]. \tag{5.7}$$

Let $U_i = (C_i - DF_i)^+$ denote the CCP exposure to CM_i in presence of VM_i, IM_i and DF_i. The CCP's counterparty credit loss after the defaulter-pay resources is

$$L^{(1)} = \sum_{i=1}^{n} U_i Y_i. \tag{5.8}$$

The second level loss adds equity contributions E and survivor-pay prefunded funds. Thus, the second level total credit loss becomes

$$L^{(2)} = \left(\sum_{i=1}^{n} U_i Y_i - E - DF_s\right)^+ \tag{5.9}$$

where

$$DF_s \equiv DF - \sum_{i=1}^{n} DF_i Y_i \tag{5.10}$$

and DF_s is the sum of the surviving members default fund contributions. When the total prefunded default funds are obtained using Equations (5.6) and (5.7) we can write $L^{(2)}$ as follows

$$L^{(2)} = \left(\sum_{i=1}^{n} C_i Y_i - E - DF\right)^+. \tag{5.11}$$

The relation holds since $DF_i = C_i\mathbb{E}[Y_i|O] < C_i$, where O indicates the event $\{L > VaR_\alpha(L)\}$ or $\{L = VaR_\alpha(L)\}$.

Note that $L^{(2)}$ represents the loss to the CCP exceeding the defaulter-pay resources, equity contributions and prefunded funds from surviving members. Recall that $L^{(2)}$ starts to be used when all the funded resources has been used, as seen in Figure 3.4.

5.2 The Unfunded Default Funds

If $L^{(2)}$ is fully allocated to the surviving members, the capital calls from the CCP to the clearing members is referred to as uncapped. The unfunded default fund of each member CM_i in the uncapped case will be

$$\tilde{DF_i}^{uc} \equiv \frac{DF_i(1-Y_i)}{DF_s} L^{(2)} \geq 0 \tag{5.12}$$

where $DF_s = \sum_{j=1}^{n} DF_j(1-Y_j)$. Note that $\tilde{DF_1}^{uc} + \ldots + \tilde{DF_n}^{uc} = L^{(2)}$.

In the capped case, the capital calls are capped by a multiple of each members prefunded default fund. In the capped case $L^{(2)}$ is allocated to the surviving members proportional to their prefunded default fund contributions DF_i, so that

$$\tilde{DF_i} \equiv \min\{\tilde{DF_i}^{uc}, \beta DF_i(1-Y_i)\} \tag{5.13}$$

where β is a positive constant.

From the CM_i's and bank regulators perspective, it will be more conservative to specify the CM_i's CCP risk capital assuming that CM_i has not defaulted at time T. In this setting the new unfunded default fund in the uncapped case will be

$$\tilde{DF_i}^{uc,s} = L_i^{uc,s} = \frac{DF_i}{DF_{s,i}} \left(\sum_{j \neq i} C_j Y_j - E - DF \right)^{+} \tag{5.14}$$

where $DF_{s,i} = DF - \sum_{j \neq i} DF_j Y_j$. In the capped case we have

$$\tilde{DF_i} \equiv \min\{\tilde{DF_i}^{uc,s}, \beta DF_i(1-Y_i)\} \tag{5.15}$$

5.3 Margins Procyclicality

During times of financial stress, the market volatility increases. This in turn requests higher initial margins, affecting the funding and market liquidity, and making the financial situation even worse. Using the Ghamami (2015) setting, it is possible to reduce margins procyclicality in a more risk sensitive way. The reduction in the frequency of variation margins calls can be one way of lowering the required initial margin. A further solution is the redefinition of prefunded and unfunded default funds in a way that reduces the initial margin, keeping constant the financial resiliency of the CCP.

Since unfunded default fund are considered unanticipated losses on survival members, they are a new source of procyclicality in periods of financial stress. From this brief analysis it is clear that the only way to reduce initial margins keeping constant the overall situation is increasing the prefunded defaulted funds and/or the CCP's equity contribution.

5.4 Total Losses to Clearing Members and CCP Risk Capital

In this subsection, the possibility that even the central counterparties can default is taken into consideration. Given this, each clearing members should hold so called CCP risk capital. In order to determine the risk capital, the loss a clearing member can suffer in the event of a CCP default has to be defined. Firstly, a replacement cost depending on the value of the derivatives portfolio the member holds with the CCP in case of CCP's default has to be considered. Secondly, in case the initial margin is not held in a bankruptcy remote manner, the initial margin can be lost when the CCP defaults. Moreover, there are the losses in the prefunded default fund for the surviving members, especially in the uncapped case and when a lot of members have defaulted. Finally, the survival members can incur unanticipated losses given by the unfunded default fund capital calls of the CCP.

5.4.1 Total Clearing Member Losses

The possible losses of CM_i can be determined considering three different cases in which CM_i is surviving at time T. In the first case, the CCP's potential loss in the presence of only the defaulter-pay-resources is less than the CCP's equity contribution, that is

$$E > \sum_{j \neq i} (C_j - DF_j) Y_j. \tag{5.16}$$

This means that the prefunded default fund of CM_i is not used. In the second case, it is assumed that the CCP's losses after the defaulter-pay resources are greater than the CCP's equity but less than the survivor-pay-resources plus the CCP's equity

$$E \leq \sum_{j \neq i} (C_j - DF_j) Y_j < E + DF_i + \sum_{j \neq i} DF_j (1 - Y_j). \tag{5.17}$$

In this case, part of DF_i is used by CCP's. The amount $\sum_{j \neq i} (C_j - DF_j) Y_j - E$, will be allocated proportional to the prefunded default fund. Thus, in the second scenario CM_i will lose

$$\frac{DF_i}{DF_{s,i}} \left(\sum_{j \neq i} (C_j - DF_j) Y_j - E \right) \tag{5.18}$$

where $DF_{s,i} = DF - \sum_{j \neq i} DF_j Y_j$.

Finally, in the third case the CCP's potential loss in the presence of the defaulter-pay resources exceeds the sum of prefunded default fund of the survivors and the CCP's equity contribution, namely

$$\sum_{j \neq i} (C_j - DF_j) Y_j \geq E + DF_i + \sum_{j \neq i} DF_j (1 - Y_j). \tag{5.19}$$

In the third case, from survival clearing members point of view, the entire prefunded default fund is used and the unfunded default funds are needed. According to these three cases, we can define the total losses to CM_i. We have to distinguish two different scenarios:

1. Uncapped and CM_i surviving at time T
2. Capped and CM_i surviving at time T

In the first scenario, i.e the uncapped case, the total potential loss is given by the losses to the $CM_i's$ prefunded default fund and the unanticipated losses due to the unfunded default fund, given that CM_i is alive at time T, is

$$L_i^{t,uc,s} = L_i^{df,s} + L_i^{uc,s} \tag{5.20}$$

where

$$L_i^{df,s} = \min\Big\{ \frac{DF_i}{DF_{s,i}} \Big(\sum_{j \neq i} (C_j - DF_j) Y_j - E \Big)^+ , DF_i \Big\} \tag{5.21}$$

and

$$L_i^{uc,s} = \frac{DF_i}{DF_{s,i}} \Big(\sum_{j \neq i} C_i Y_i - E - DF \Big)^+ . \tag{5.22}$$

In the capped case, the total potential loss is influenced by the possibility that other members default and by the default of the CCP. In this case the total credit losses will be

$$L_i^{t,s} = L_i^{df,s} + L_i^s + \tilde{U}_i \tilde{Y}_i \tag{5.23}$$

where $L_i^s = \min\{L_i^{uc,s}, \beta DF_i\}$ and $\tilde{Y}_i$ is the indicator function that takes on a value of 1 in the case where the CCP defaults and $\tilde{U}_i$ is the member's loan-equivalent exposure to the CCP at its default, given by:

$$\tilde{U}_i = (1 - \delta) \int_0^{\tilde{T}} \mathbb{E}[\tilde{e}_i(t)] dt. \tag{5.24}$$

where δ is the CCP's recovery rate

In previous derivation, it is assumed that the CM_i's initial margin held in a bankruptcy-remote manner. If this assumption is relaxed the total credit losses (in the capped case) would be:

$$L_i^{t,s} = L_i^{df,s} + L_i^s + (\tilde{U}_i + IM_i) \tilde{Y}_i. \tag{5.25}$$

5.4.2 The CCP Risk Capital

In the Ghamami (2015) setting, the total loss is modeled using the one-period model. Thus, the VaR-based unexpected portfolio credit risk is $\mathrm{VaR}_\alpha(\tilde{L}) - \mathbb{E}[\tilde{L}]$. In particular, the CCP risk capital of CM_i based on the total expected capped losses is

$$\mathbb{E}[L_i^{t,s}] = \mathbb{E}[L_i^{df,s}] + \mathbb{E}[L_i^s] + \tilde{U}_i P_{ccp,i} \tag{5.26}$$

where $P_{ccp,i}$ is the default probability of the CCP up to time T assuming CM_i is alive at time T

$$P_{ccp,i} = \mathbb{P}\bigg(\sum_{j \neq i} C_j Y_j > E + DF + D\tilde{F}_i^s + \sum_{i \neq j} D\tilde{F}_j \bigg) \tag{5.27}$$

and finally the CCP risk capital based on unexpected losses is

$$VaR_\alpha(L_i^{t,s}) - \mathbb{E}[L_i^{t,s}]. \tag{5.28}$$

6 Model implementation

In this section we describe our implementation of the model designed by Ghamami (2015). In the implementation, we will assume that the clearing members are homogeneous. Furthermore, when simulating the default indicators for the clearing members we will use the mixed binomial model described in Section 4, both in the Merton framework and with a beta distribution. Our implementation is outlined in the steps as follows:

1. In the first step we show how to compute the initial margin, variation margin and the CCP's collaterized exposure at time t in the presence of said margins. In order to determine the exposure, we will simulate a short-term interest rate process and thereafter derive the value of an interest rate swap with unitary notional.

2. In this step it is shown how to perform Monte Carlo simulations in Merton and beta models to derive the default indicator Y_i, which we use to obtain the unfunded default funds and the CCP default probability from the surviving members point of view.

3. Having obtained the CCP's credit exposure we can derive the prefunded default fund based on different values of default correlation ρ, default probability $\bar{p}$ and different α in both the Merton and Beta setting.

4. In the fourth step, we compute the unfunded default fund, as well as the default probability of the CCP. We also discuss the level losses.

5. As a final step, we do some sensitivity analysis of our model. We compute the prefunded default funds as a function of clearing period T as well as for different number of clearing members, and we also control what happens if the interest rate setting is changed.

6.1 Swap Valuation, Margins and Credit Exposure

First, we wish to find the expected exposure, $C_i \equiv (1 - \delta_i) \int_0^T \mathbb{E}[e_i(t)]dt$, which in this homogeneous setting will be the same for all obligors.

In order to find our expected exposure, we will mimic an interest swap process. This will be done in a Cox-Ingersoll-Ross (CIR) framework. The CIR model provides a way to determine the term structure of interest rates. In particular, the model gives a instantaneous interest rate r_t, which follows a stochastic differential equation given by

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dZ_t \tag{6.1}$$

where Z_t is a Brownian motion, κ is the speed of adjustment towards the long term mean θ, and σ is the volatility. (Cox et al. 1985). We will use the findings of Broadie & Kaya (2006) to perform the simulation, where the value of r_t given r_s where $s < t$, is given by

$$r_t = c_t \chi_d^2(a) \tag{6.2}$$

where $\chi_d^2(a)$ is a non-central chi-squared random variable centered at a with d degrees of freedom and c_t being a scaling factor. The parameters are estimated using the following equations:

$$a = \frac{4\kappa e^{-\kappa(t-s)}}{\sigma^2\left(1 - e^{-\kappa(t-s)}\right)} r_s \tag{6.3}$$

$$d = \frac{4\theta\kappa}{\sigma^2} \tag{6.4}$$

$$c_t = \frac{\sigma^2\left(1 - e^{-\kappa(t-s)}\right)}{4\kappa}. \tag{6.5}$$

The process is estimated with the restriction $2\theta\kappa > \sigma^2$, which leads to $d > 1$. When $d > 1$, a non-central chi-squared random variable can be rewritten as the sum of a non-central χ^2 random variable and a normal χ^2 variable with $d - 1$ degrees of freedom.

$$r_t = c_s(\chi_1^2(a) + \chi_{d-1}^2) \tag{6.6}$$

where

$$\chi_1^2(a) = (Z + \sqrt{a})^2 \tag{6.7}$$

and Z is a standard normal random variable. In Figure 6.1, we show some trajectories of simulated CIR-processes.

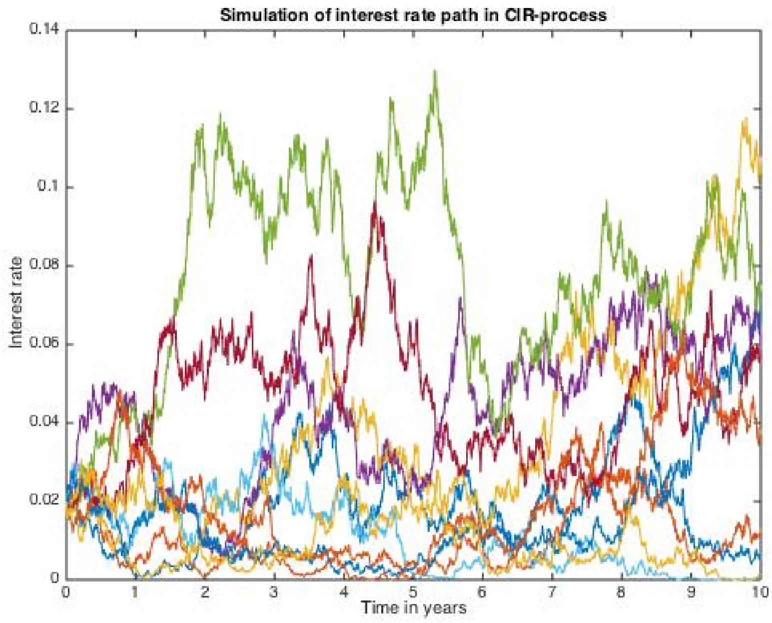

Figure 6.1: CIR-process simulation with $\kappa = 0.1, \theta = 0.03, \sigma = 0.1, r_0 = 0.02$

After simulating the interest rate processes, we can compute the bond prices when r_t follows Equation (6.1) by using closed formulas. The following formulas can be found in Hull (2011):

$$P(t,T) = A(t,T)e^{-B(t,T)r_t} \tag{6.8}$$

$$B(t,T) = \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma} \tag{6.9}$$

$$A(t,T) = \left[\frac{2\gamma e^{(a+\gamma)(T-t)/2}}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma}\right]^{2ab/\sigma^2} \tag{6.10}$$

with $\gamma = \sqrt{a^2 + 2\sigma^2}$. In Figure 6.2, we plot the bond prices for our interest rate paths from Figure 6.1

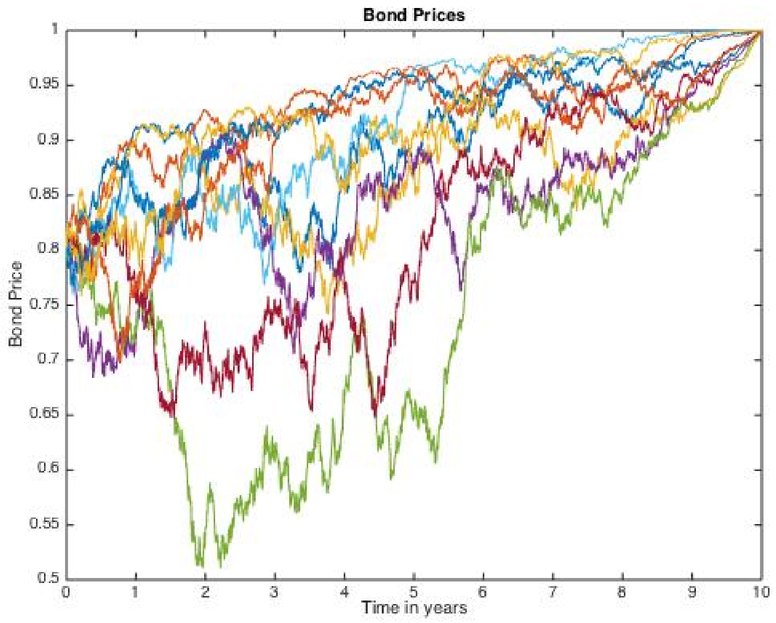

Figure 6.2: Simulated bond prices

Now that we have the bond prices at each point in time, we can find the values of our interest rate swaps. An interest rate swap (IRS) is a contract between two parties, where one pays a fixed interest rate on a notional N and receives a floating interest rate on the same notional, and vice versa. The floating rate is based on an real world interest rate, such as the LIBOR. The cash flows are exchanged at specified settlement dates (T_i), for a period of time (T_n). Assuming that coupons are paid quarterly and the first payment is at time $t = 0.25$, the fixed rate in the contract is the rate that makes the value of the IRS zero at time 0. Using a reinvestment strategy or considering the IRS as a portfolio of Forward Rate Agreements, it can be shown that the IRS rate is the following, see e.g Brigo & Mercurio (2006) or Hull (2011)

$$R = \frac{1 - P(0, T_n)}{\delta \sum_{i=1}^{n} P(0, T_i)} \tag{6.11}$$

where $\delta = T_i - T_{i-1}$. The rate R will be computed once and used in the IRS valuation at each point in time t. The value for the part paying the fixed interest rate is equal to:

$$\Pi_{payer}(t) = N\bigg(P(t, T_\alpha) - P(t, T_n) - R\delta \sum_{i=\alpha+1}^{n} P(t, T_i)\bigg) \tag{6.12}$$

with cash flows taking place at the date of the coupons $T_\alpha+1, T_\alpha+2, \ldots, T_n$. The part $P(t, T_n) - R\delta \sum_{i=\alpha+1}^{n} P(t, T_i)$ is all future discounted cash flows from the fixed payment stream. Thus, it can be seen as a coupon bearing bond, where $NR\delta\sum_{i=\alpha+1}^{n}P(t,T_i)$ are the discounted cash flows from the remaining coupon payments, and $NP(t,T_n)$ is the discounted value of the notional value of the bond. (Brigo & Mercurio 2006)

Approaching maturity the terms $NR\delta\sum_{i=\alpha+1}^{n}P(t,T_i)$ decreases in size. In particular at time T_n no cash flows are exchanged and $NR\delta\sum_{i=\alpha+1}^{n}P(t,T_i) = 0$. Furthermore, at maturity $NP(T_n,T_n) = N$. As result we have that the IRS will be valued zero at time 0 and T, and will present jumps at each cash settlement (in our case each quarter). Our simulated swap values are shown below in Figure 6.3, where we use the same trajectories as in Figure 6.1 and Figure 6.2:

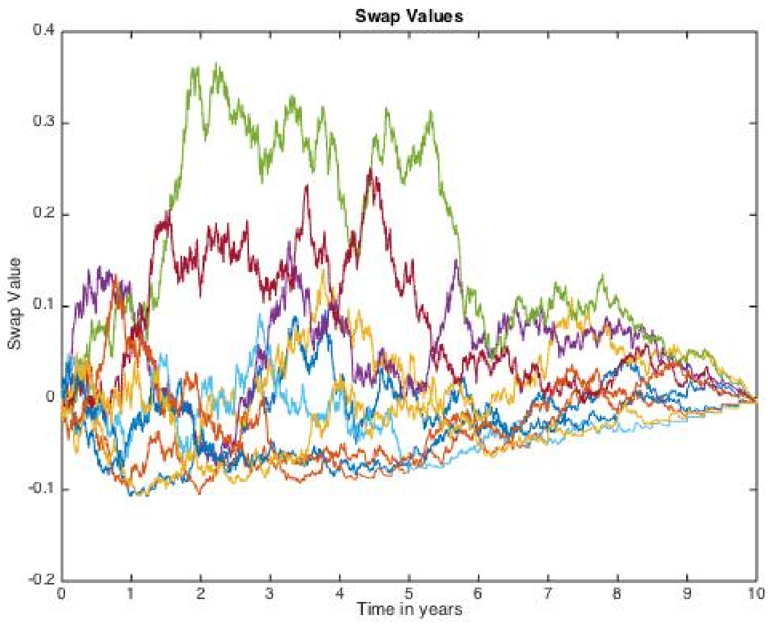

Figure 6.3: Simulated swap values

When we have determined the values of the IRS contract, we can calculate the expected exposure. This is defined as the expected loss given default, with zero recovery rate. In other words, it can be considered the average exposure.

$$EE_t = \frac{1}{M}\sum_{i=1}^{M}\max(0,\Pi_t^i) \tag{6.13}$$

where Π_t^i is the value for swap i at time t, EE_t is the Monte Carlo estimated value of $\mathbb{E}[\max(0,\Pi_t)]$, and M is the number of simulations. A plot of our

Expected Exposure for $T = 10$ is shown in Figure 6.4, based on the same trajectories as in Figure 6.1

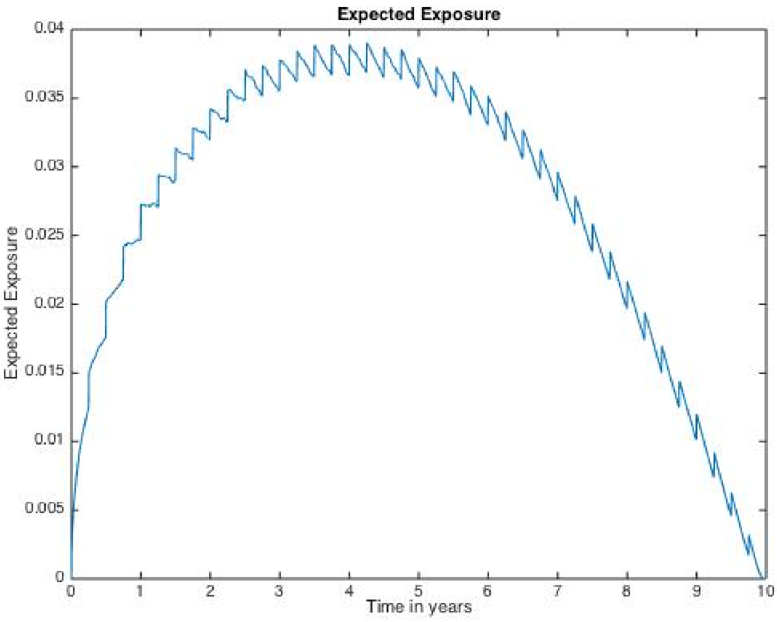

Figure 6.4: 10-year Expected exposure for the IRS

The next step is to find the collateralized exposure for the CCP. If we let $V_i^+(t)$ be the positive part of the value of the derivatives portfolio that the CCP holds with CM_i at time $t > 0$, recall that the CCP's collateralized exposure at time t is

$$e_i(t) \equiv \max\Big\{\big(V_i^+(t+\Delta) - VM_i(t) - IM_i(t)\big), 0\Big\} \tag{6.14}$$

where $VM_i(t) \equiv V_i^+(t-\hat{\Delta})$ and $IM_i(t)$ is the VaR or ES associated with $V_i^+(t+\Delta) - V_i^+(t-\hat{\Delta})$. Using these definitions we have

$$e_i(t) \equiv \max\Big\{\big(V_i^+(t+\Delta) - V_i^+(t-\hat{\Delta}) - IM_i(t)\big), 0\Big\} \tag{6.15}$$

The time interval $\Delta + \hat{\Delta}$, denoted by the marginal period of risk, is set equal to 5 days. Defining $R_i = V_i^+(t+\Delta) - V_i^+(t-\hat{\Delta})$, and given some confidence level α, the initial margin will be

$$IM_i = VaR_\alpha(R_i). \tag{6.16}$$

In order to estimate the CCP's exposure we perform Monte Carlo simulation looking at the difference between the IRS value at $t + 4$ and the IRS value at time $t-1$. Defining IM_i as in Equation 6.16 we can compute $\mathbb{E}[e_i(t)]$ via Monte Carlo simulation, where $\mathbb{E}[e_i(t)]$ is

$$\mathbb{E}[e_i(t)] = \mathbb{E}[(R_i - VaR_\alpha(R_i))^+] = \mathbb{E}[R_i 1\{R > VaR_\alpha(R_i)\}] - \alpha VaR_\alpha(R_i) \tag{6.17}$$

In Figure 6.5 we show the Expected Exposure of the IRS as well as the CCP's collateralized exposure.

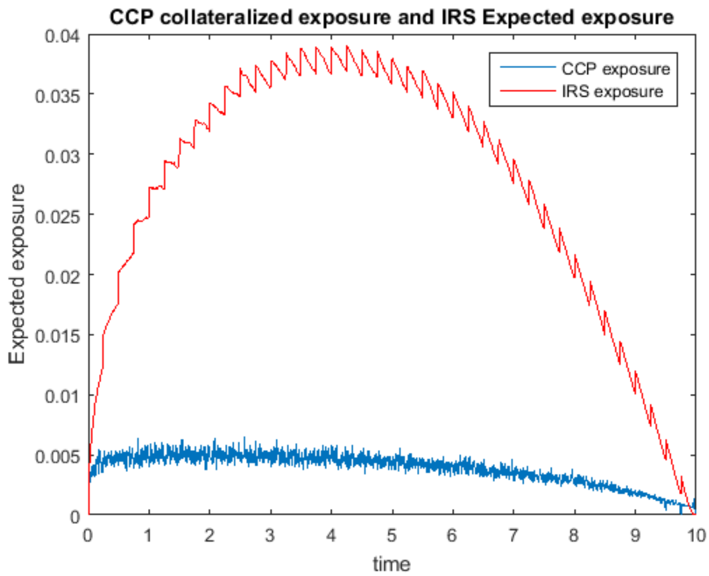

Figure 6.5: Expected exposure for the IRS and expected exposure of the CCP after considering initial margins and variation margins.

From Figure 6.5 we can clearly see the benefits of initial margins and variation margins in how they are reducing the CCP exposure, by adding a capital buffer for losses. In the next step we compute the CCP's EPE-based time-T loan-equivalent collateralized exposure due to the $CM_i's$ default, that is,

$$C_i \equiv (1 - \delta_i) \int_0^T \mathbb{E}[e_i(t)]dt. \tag{6.18}$$

To clarify, C_i is the amount the CCP loses due to the default of clearing member i. By homogeneity, C_i is the same for all clearing members, meaning $C_i = C$. In Figure 6.6 we plot the exposure C for a 10-year IRS with the same parameters as in Figure 6.1-6.4 as a function of different confidence levels when the recovery rate is set equal to 40%.

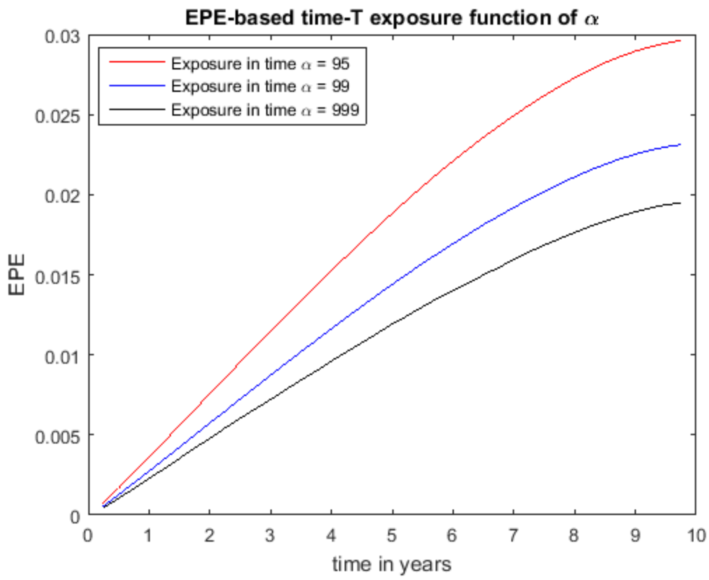

Figure 6.6: EPE-based collateralized exposure for a 10-year IRS as a function of VaR-confidence for three different α.

As we expect, the exposure increases with time and decreases with more conservative choices of confidence levels.

6.2 Simulation of the Default Indicators

In the following subsection we discuss how to obtain the default indicator Y_i, needed in order to compute the first level loss $L = \sum_{i=1}^{n} C_i Y_i$.

6.2.1 Default Indicators in the Merton Framework

As discussed in Section 4, the Merton model has the following properties:

$$V_{T,i} < D_i \text{ is equivalent with } X_i \leq \frac{-(C + \sqrt{\rho} Z)}{\sqrt{1-\rho}} \tag{6.19}$$

where X_i is a random vector drawn from the standard normal distribution and it is independent of Z, while $x_i = \frac{-(C+\sqrt{\rho}Z)}{\sqrt{1-\rho}}$ is our threshold. Thus, $Y_i = 1\{X_i \leq x_i\}$ will be the default indicator and $N_n = \sum_{i=1}^{n} Y_i$ the number of defaults. In Figure 6.7 we plot $\mathbb{E}[N_n]$ via MC-simulations as a function of ρ_x and $\bar{p}$.

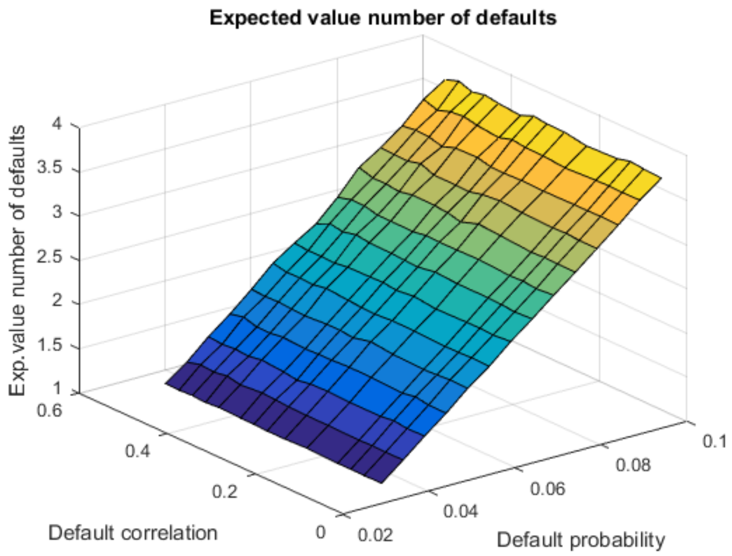

Figure 6.7: Estimate of $\mathbb{E}[N_n]$ for different ρ_x and $\bar{p}$ using 10^5 simulations in the Merton model.

From Figure 6.7 it is clear that the number of defaults are independent of the default correlations, and solely increases as long a function of the default probability. The expected value of number of defaults divided by the number of companies goes towards the default probability, namely $\mathbb{E}\left[\frac{N_n}{n}\right] = \bar{p}$, as the number of simulations goes to ∞.

6.2.2 Default Indicators Under Beta Distribution

To obtain the default indicators in the beta setting, we let M be the number of simulations. For each $j = 1, 2, ..., M$, we repeat the following steps:

1. Simulate the random variable Z that follows a Beta distribution and compute the $p(Z)$ that in the Beta model is equivalent to Z.

2. Simulate the i.i.d sequence $U_1, U_2, ..., U_m$ of uniformly distributed random variables, independent of Z.

3. For each i the default indicator Y_i will be

$$Y_i = \begin{cases} 1 & \text{if } U_i \leq Z \\ 0 & \text{if } U_i > Z \end{cases}$$

As seen in Figure 6.8, the procedure yields similar results to the Merton setting.

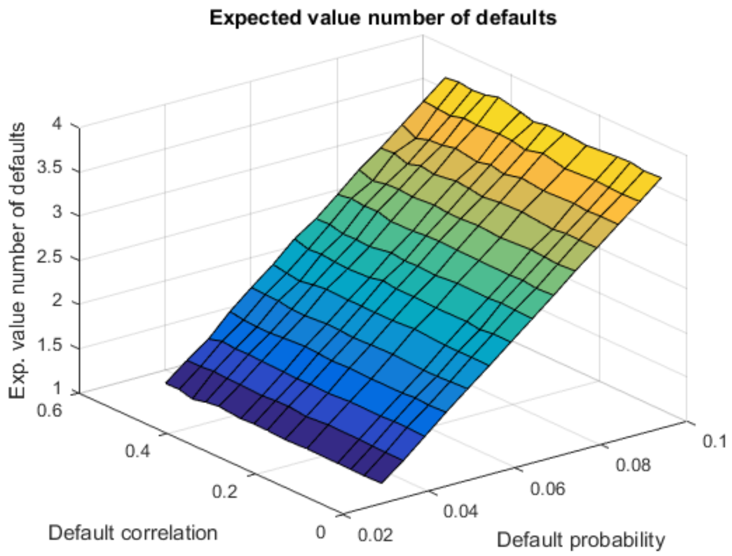

Figure 6.8: Estimate of $\mathbb{E}[N_n]$ for different ρ_x and $\bar{p}$ using 10^5 simulations in the beta model.

6.3 Derivation of Prefunded Default Funds

In this subsection, we derive a closed form formula for DF_i, which can be found in Herbertsson (2016). Recall that the prefunded default funds based on ES is defined as follows

$$DF_i = C_i \mathbb{E}[Y_i | L \geq VaR_\alpha] \tag{6.20}$$

where by definition we have

$$\mathbb{E}[Y_i | L > VaR_\alpha] = \frac{\mathbb{E}[Y_i 1\{L > VaR_\alpha\}]}{\mathbb{P}[L > VaR_\alpha]} \tag{6.21}$$

$$= \frac{\mathbb{E}[Y_i 1\{L > VaR_\alpha\}]}{1-\alpha}. \tag{6.22}$$

where $1\{A\}$ is the indicator function for the event A. Next, note that:

$$\begin{aligned}\mathbb{E}[Y_i 1\{L > VaR_\alpha\}] = \mathbb{E}[Y_i 1\{L > VaR_\alpha\} | Y_i = 1]\mathbb{P}[Y_i = 1] \\ + \mathbb{E}[Y_i 1\{L > VaR_\alpha\} | Y_i = 0]\mathbb{P}[Y_i = 0]\end{aligned} \tag{6.23}$$

where the second term is zero since $Y_i = 0$ on $\{Y_i = 0\}$. Hence:

$$\mathbb{E}[Y_i 1\{L > VaR_\alpha\}] = \mathbb{E}[Y_i 1\{L > VaR_\alpha\} | Y_i = 1]\bar{p} \tag{6.24}$$

where $\bar{p}$ is same for all obligors in the homogeneous setting. We further note that:

$$\mathbb{E}[Y_i 1\{L > VaR_\alpha\}|Y_i = 1] = \mathbb{E}[1\{L > VaR_\alpha\}|Y_i = 1] \tag{6.25}$$
$$= \mathbb{P}[L > VaR_\alpha|Y_i = 1] \tag{6.26}$$

Next, in a homogeneous portfolio we have $C_i = C$ for all obligors and thus:

$$L = CN_n \tag{6.27}$$

where $N_n = \sum_{i=1}^{n} Y_i$. Thus, we have that $L > VaR_\alpha \Leftrightarrow N_n > \frac{VaR_\alpha}{C}$. For notational convenience, we let γ denote $\frac{VaR_\alpha}{C}$, which implies that:

$$\begin{aligned} \mathbb{P}[L > VaR_\alpha|Y_i = 1] &= \mathbb{P}[N_n > \gamma|Y_i = 1] \\ &= \sum_{k=\lfloor\gamma\rfloor+1}^{n} \mathbb{P}[N_n = k|Y_i = 1]. \end{aligned} \tag{6.28}$$

So what is left to compute is $\mathbb{P}[N_n = k|Y_i = 1]$, but note that

$$\begin{aligned} \mathbb{P}[N_n = k|Y_i = 1] &= \frac{\mathbb{P}[N_n = k, Y_i = 1]}{\mathbb{P}[Y_i = 1]} \\ &= \mathbb{P}[Y_i = 1|N_n = k]\frac{\mathbb{P}[N_n = k]}{\bar{p}}. \end{aligned} \tag{6.29}$$

Note that $\bar{p}$ is exogenously given and $\mathbb{P}[N_n = k]$ is determined as:

$$\mathbb{P}[N_n = k] = \mathbb{E}[\binom{n}{k} p(Z)^k (1 - p(Z))^{n-k}]. \tag{6.30}$$

Using the proposition of Herbertsson (2008), we have that

$$\mathbb{P}[Y_i = 1|N_n = k] = \frac{k}{n} \tag{6.31}$$

and thus

$$\mathbb{P}[N_n = k|Y_i = 1] = \frac{\mathbb{P}[N_n = k]}{\bar{p}}\frac{k}{n}. \tag{6.32}$$

From this derivation we have that DF_i is given by the following formula

$$DF_i = C\frac{\sum_{k=\lfloor\gamma\rfloor+1}^{n} \mathbb{P}[N_m = k]k}{(1-\alpha)n}. \tag{6.33}$$

where C is the exposure of the IRS.

6.4 Unfunded Default Funds, Level Losses and Default Probability of the CCP

As discussed in Section 5, we can consider four settings when calculating the unfunded defaulted funds $\tilde{DF}_i$.

1. Uncapped, with CM_i survival
2. Capped, with CM_i survival
3. Uncapped, with CM_i default
4. Capped, with CM_i default

In our implementation we compute $\tilde{DF}_i$ in the first and second setting. Using this version of unfunded default funds, CM_i should be considered as the financial institution that, independent of its credit quality, values the credit quality of $CM_j's$, $j \neq i$, representing them by dependent default indicators. As seen in the previous section, the uncapped unfunded default funds assuming survival at time T are:

$$\tilde{DF}_i^{uc,s} = L_i^{uc,s} = \frac{DF_i}{DF_{s,i}} \left(\sum_{j \neq i} C_j Y_j - E - DF \right)^+ \tag{6.34}$$

where $DF_{s,i} = DF - \sum_{j \neq i} DF_j Y_j$. On the other hand, the capped unfunded default funds assuming survival are:

$$\tilde{DF}_i =\equiv \min\{\tilde{DF}_i^{uc,s}, \beta DF_i(1 - Y_i)\} \tag{6.35}$$

where $\beta > 0$. Once we have defined the waterfall resources, we are ready to check the difference in terms of loss before and after the introduction of each layer. The first level loss is given by

$$L = \sum_{i=1}^{n} C_i Y_i = C \sum_{i=1}^{n} Y_i \tag{6.36}$$

where $C = C_i$, due to our homogeneous setting. In the second level loss, we add the prefunded default funds and the equity contributions of CCP, which in our implementation is equal to 2% of the total prefunded default funds. Thus, we have that

$$L^{(2)} = \left(C \sum_{i=1}^{n} Y_i - E - DF \right)^+. \tag{6.37}$$

Finally we can define the third level loss. Given $\tilde{DF} = \sum_{i=1}^{n} \tilde{DF}_i$, we have that

$$L^{(3)} = \left(C \sum_{i=1}^{n} Y_i - E - DF - \tilde{DF} \right)^+ \tag{6.38}$$

If the model we implement is well designed, we expect the third level loss to be very small and in particular lower than $1 - \alpha$. After defining the level losses is possible to compute the CCP's default probability from $CM_i's$ perspective, i.e, assuming CM_i survival at time T. The probability will be

$$P_{ccp,i} = \mathbb{P}\left(\sum_{j \neq i} C_j Y_j > E + DF + \tilde{DF}_i^s + \sum_{j \neq i} \tilde{DF}_j \right) \tag{6.39}$$

where $P_{ccp,i} \leq \mathbb{P}\left(\sum_{j \neq i} C_j Y_j > E + DF \right)$.

7 Numerical Results

In the following section we present the numerical results for our implementation. In Subsection 7.1 we present the results in both Merton and Beta models as function of default correlation, default probability and confidence levels of 95%, 99% and 99.9%. We let the default probability take on values from 3% to 9.5%, whereas we assume default correlation values as shown in Table 7.1:

0.01	0.05	0.08	0.11	0.18	0.23	0.27
0.3	0.33	0.38	0.41	0.44	0.47	0.501

Table 7.1: Values of ρ_x

We will show how the prefunded default funds, unfunded default funds, levels losses and CCP default probability will change according to different values of default correlation and default probability. Moreover, in Subsection 7.2 we perform different robustness checks of our model. For examples, changing the time period considered, changing the number of clearing members and changing the yield curve. Finally, relative errors for the differences between beta and Merton model are shown in the Appendix.

7.1 Prefunded and Unfunded Default Funds

In the basic setting, we have 40 clearing members (i.e $n = 40$) and we set the time period T equal to 5 years. Figure 7.1 shows an estimation of $\mathbb{E}[L]$, that is

$$L^{MC} = \frac{1}{M}\sum_{j=1}^{M} L^{MC,j} \tag{7.1}$$

where $L^{MC,j}$ is the loss in simulation number j. Note that $\mathbb{E}[L^{MC}] = \mathbb{E}[L] = C \cdot n \cdot \bar{p}$.

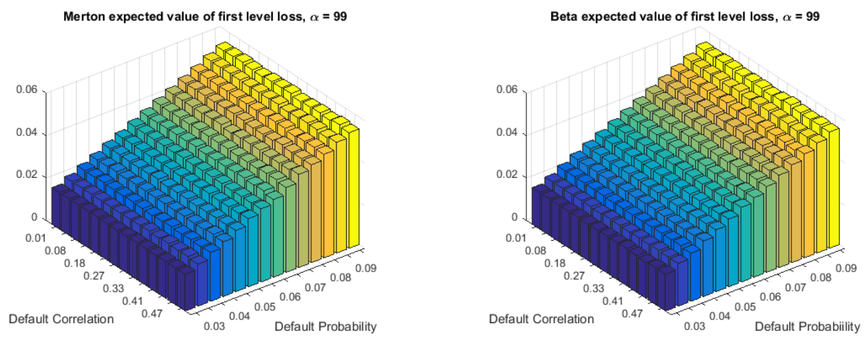

Figure 7.1: Expected value of the first level loss as function of ρ_x and $\bar{p}$ in Beta and Merton models with 10^5 simulations.

The first layer used to reduce the first level loss is the prefunded default funds. In Figure 7.2 we plot the prefunded default funds as a function of default correlation, default probability and confidence level α as an MC-estimation of Equation (6.33)

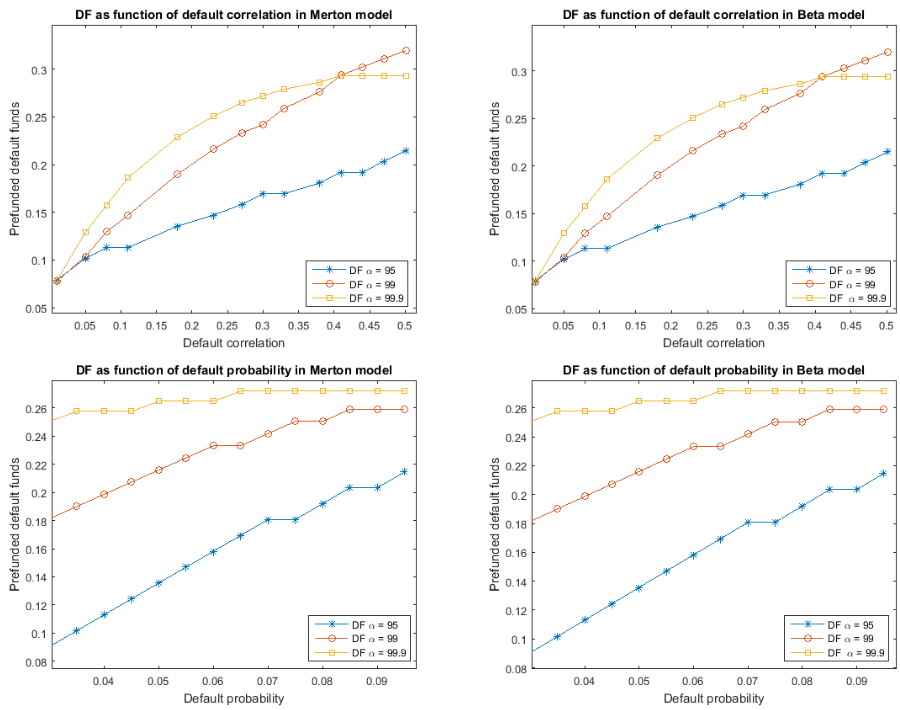

Figure 7.2: Total prefunded default funds as function of ρ_x and α when $\bar{p} = 6\%$ and as function of $\bar{p}$ and α when $\rho_x = 27\%$.

From Figure 7.2 is clear that in both models the prefunded default funds increases with default correlation and default probability. Moreover, the allocation to the funds are higher for $\alpha = 0.99$ and $\alpha = 0.999$ despite the fact that the exposure C is lower for higher confidence levels.

Once we determine the prefunded default funds, we can derive the second level loss, that is the loss a CCP incurs after initial margins, variation margins, prefunded default funds and CCP equity has been posted. According to industry benchmark, we set the CCP equity to 2% of the total prefunded default funds. In Figure 7.3 the second level loss as a function of ρ_x and $\bar{p}$ is shown.

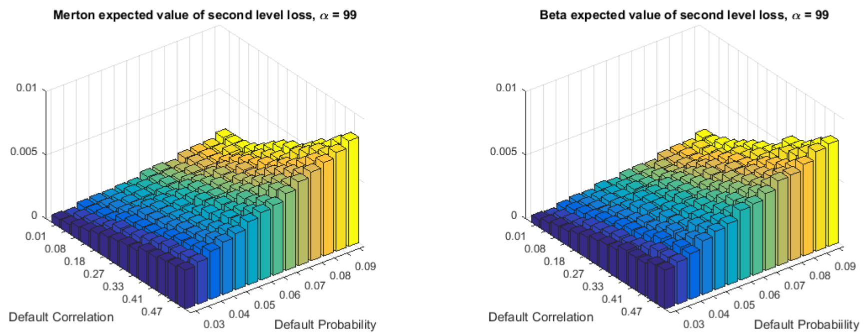

Figure 7.3: Expected value of second level loss as function of ρ_x and $\bar{p}$ in Merton and beta models with 10^5 simulations.

From Figure 7.3 it is clear that the prefunded default funds, together with CCP equity, are calculated in a conservative and risk sensitive way, meaning that they increase with $\bar{p}$ and ρ_x. Moreover, Figure 7.4 compares expected value of the first and second level loss when $\alpha = 0.99$ for different default probabilities. For example, when $\alpha = 99$, $\rho_x = 0.84$ and $\bar{p} = 0.095$, the second level loss are 3.5 times lower than the first level loss. This result confirms the effectiveness of default funds.

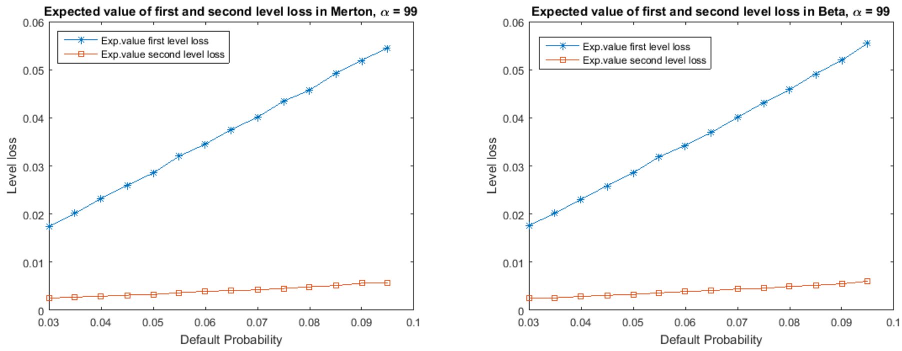

Figure 7.4: Expected value of first and second level loss as function of $\bar{p}$ when $\alpha = 0.99$ and $\rho_x = 38\%$ with 10^5 simulations

As suggested by Ghamami (2015), the third layer we compute is the unfunded default fund. We compute the funds assuming survival at time T in the capped case. Since Ghamami (2015) presents no value for the parameter β, we have to reason ourselves in order to find a suitable parameter value. We end up setting $\beta = 0.04$ in our homogeneous portfolio, meaning that each obligor contributes to the unfunded funds slightly more than its percentage contribution in the total prefunded defaulted funds. In Figure 7.5, we display the Monte Carlo estimation of the unfunded funds in both Merton and Beta as function of ρ_x and $\bar{p}$ when $\alpha = 0.99$.

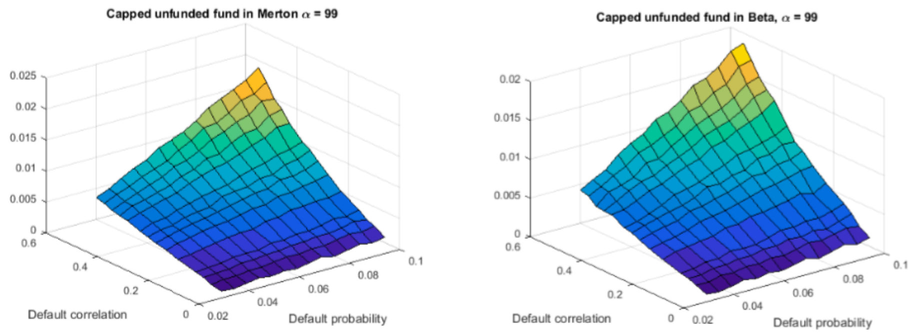

Figure 7.5: Expected value of Unfunded default funds as function of ρ_x, $\bar{p}$ for $\alpha = 0.99$ with 10^5 simulations.

In order to understand the impact of the unfunded funds, we show in Figure 7.6 the Monte Carlo estimation of the third level loss. That is, the loss after initial margins, variation margins, prefunded default funds, CCP equity and unfunded default funds have been posted.

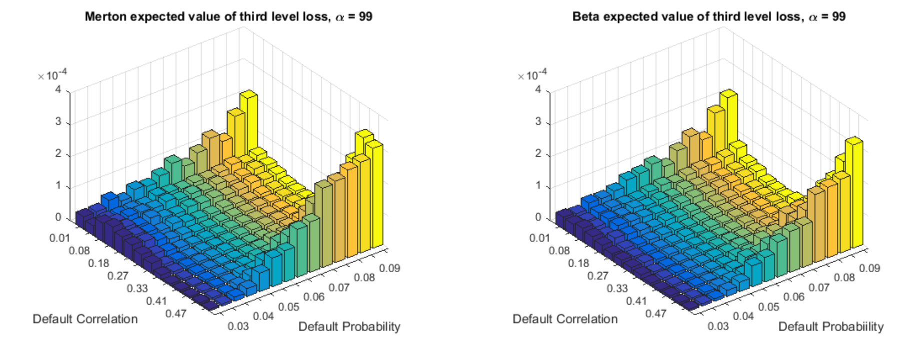

Figure 7.6: Expected value of third level loss as a function of ρ_x, $\bar{p}$ and for $\alpha = 0.99$ with 10^5 simulations.

Moreover, in Figure 7.7 we compare the expected value of the first, second and third level loss to further stress the impact of prefunded and unfunded default funds.

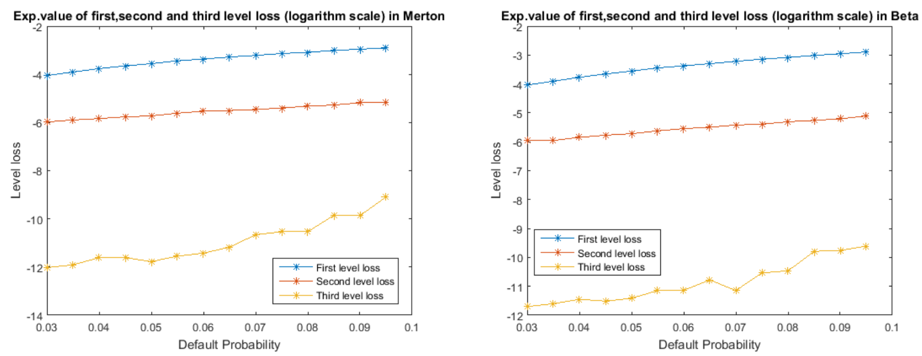

Figure 7.7: Level loss comparison in logarithmic scale for different $\bar{p}$ in both the Merton and beta setting when $\alpha = 0.99$ and $\rho_x = 38\%$.

7.1.1 CCP default probability

As discussed in Section 6, the CCP's default probability assuming CM_i survival at time T is

$$P_{ccp,i} = \mathbb{P}\left(\sum_{j \neq i} C_j Y_j > E + DF + \tilde{DF}_i^s + \sum_{j \neq i} \tilde{DF}_j \right). \tag{7.2}$$

In Figure 7.8, we plot the CCP default probability in both the Beta and the Merton framework.

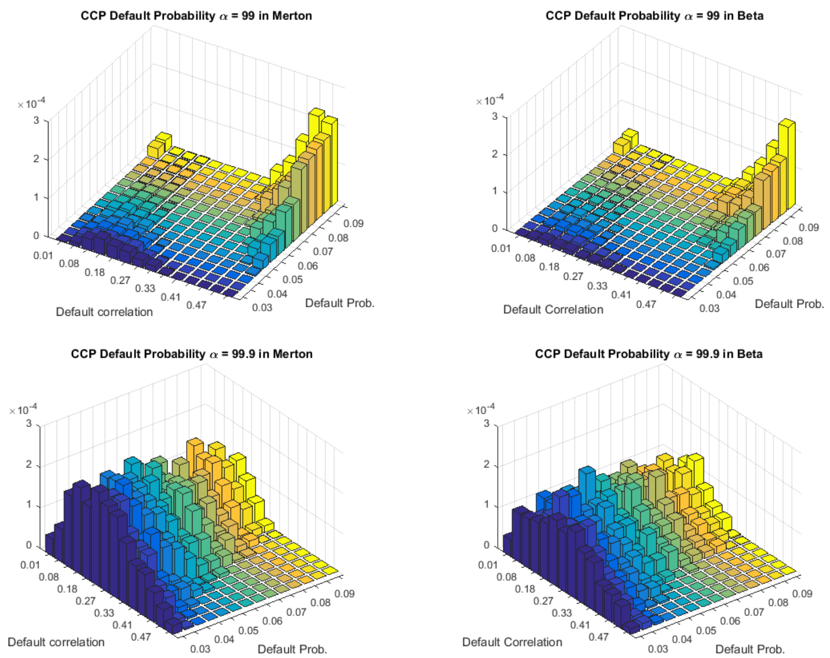

Figure 7.8: CCP default probability as function of ρ_x and $\bar{p}$ for $\alpha = 0.99$ and $\alpha = 0.999$.

In Tables 7.2-7.5 we present numerical values for the CCP default probability when α 0.99 and 0.999.

	$\bar{p} = 3\%$	$\bar{p} = 4\%$	$\bar{p} = 5\%$	$\bar{p} = 6\%$	$\bar{p} = 7\%$	$\bar{p} = 8\%$	$\bar{p} = 9\%$
$\rho_x = 0.01$	0	0	0	0	0	0	0.003%
$\rho_x = 0.08$	0.001%	0.001%	0.001%	0	0	0	0
$\rho_x = 0.18$	0.001%	0.001%	0	0	0	0	0
$\rho_x = 0.27$	0.001%	0	0	0	0	0	0
$\rho_x = 0.30$	0	0	0	0	0	0	0
$\rho_x = 0.41$	0	0	0	0	0.001%	0.003%	0.004%
$\rho_x = 0.47$	0	0	0	0	0.004%	0.007%	0.008%

Table 7.2: Beta model with $\alpha = 99\%$

	$\bar{p}=3\%$	$\bar{p}=4\%$	$\bar{p}=5\%$	$\bar{p}=6\%$	$\bar{p}=7\%$	$\bar{p}=8\%$	$\bar{p}=9\%$
$\rho_x=0.01$	0.004%	0.002%	0.001%	0	0.008%	0	0
$\rho_x=0.08$	0.012%	0.013%	0.014%	0.015%	0.013%	0.011%	0.010%
$\rho_x=0.18$	0.014%	0.018%	0.014%	0.009%	0.009%	0.009%	0.007%
$\rho_x=0.27$	0.015%	0.010%	0.011%	0.007%	0.007%	0.004%	0.003%
$\rho_x=0.33$	0.013%	0.008%	0.007%	0.004%	0.002%	0.001%	0
$\rho_x=0.41$	0.007%	0.005%	0.002%	0	0	0	0
$\rho_x=0.47$	0.005%	0	0	0	0	0	0

Table 7.3: Beta model with $\alpha=99.9\%$

	$\bar{p}=3\%$	$\bar{p}=4\%$	$\bar{p}=5\%$	$\bar{p}=6\%$	$\bar{p}=7\%$	$\bar{p}=8\%$	$\bar{p}=9\%$
$\rho_x=0.01$	0	0	0	0	0	0	0.003%
$\rho_x=0.08$	0.003%	0.002%	0.001%	0.001%	0.001%	0	0
$\rho_x=0.18$	0.003%	0.002%	0.001%	0	0	0	0
$\rho_x=0.27$	0.002%	0	0	0	0	0	0
$\rho_x=0.33$	0	0	0	0	0	0	0
$\rho_x=0.41$	0	0	0	0	0	0.004%	0.004%
$\rho_x=0.47$	0	0	0	0.002	0.006%	0.012%	0.013%

Table 7.4: Merton model with $\alpha=99\%$

	$\bar{p}=3\%$	$\bar{p}=4\%$	$\bar{p}=5\%$	$\bar{p}=6\%$	$\bar{p}=7\%$	$\bar{p}=8\%$	$\bar{p}=9\%$
$\rho_x=0.01$	0.004%	0.002%	0.001%	0	0.008%	0	0
$\rho_x=0.08$	0.018%	0.014%	0.013%	0.014%	0.012%	0.008%	0.016%
$\rho_x=0.18$	0.018%	0.016%	0.018%	0.013%	0.014%	0.009%	0.009%
$\rho_x=0.27$	0.021%	0.019%	0.015%	0.007%	0.008%	0.005%	0.002%
$\rho_x=0.33$	0.015%	0.015%	0.009%	0.003%	0.003%	0.001%	0
$\rho_x=0.41$	0.010%	0.004%	0.002%	0	0	0	0
$\rho_x=0.47$	0.005%	0.001%	0	0	0	0	0

Table 7.5: Merton model with $\alpha=99.9\%$

From Table 7.2-7.5 is clear that the CCP default probability is very low, and for most values of ρ_x and $\bar{p}$ the following test does hold:

$$P_{ccp,i}=\mathbb{P}\left(\sum_{j\neq i}C_jY_j>E+DF+\tilde{DF}_i^s+\sum_{j\neq i}\tilde{DF}_j\right)<1-\alpha. \tag{7.3}$$

As we see, even in the extreme scenarios when ρ_x and $\bar{p}$ are high the CCP default probability is close to zero. The results confirm the robustness of our models and that they are conservative. In a crisis period when default correlation and default probability increases, the layers also increases to protect against drastic financial scenarios.

7.1.2 Margin Procyclicality

As discussed in Section 5, during times of financial stress we need to lower the third level loss. An increase in the margin requirements would dry up liquidity in the market and thus worsen the stress. On the other hand, unfunded default funds are considered unanticipated losses and would have a negative impact in the market. The only way of reducing the third level loss and not increase margin requirements is to increase the prefunded default funds. In Figure 7.9, we show the proportion among all default layers as a function of $\bar{p}$ in both the uncapped and capped case when $\alpha = 0.99$.

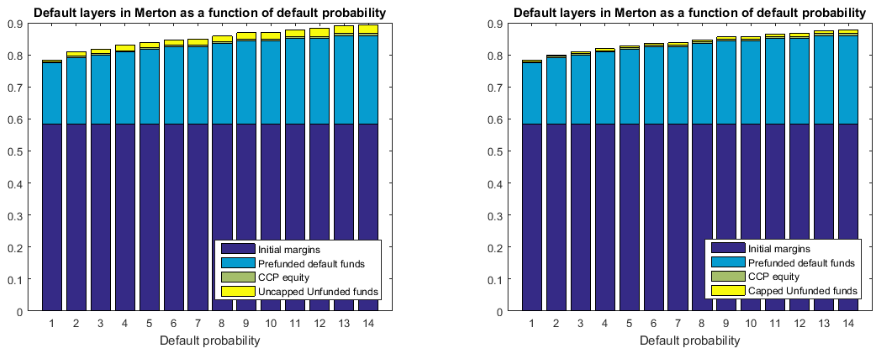

Figure 7.9: Initial margins, prefunded and expected unfunded default funds and CCP equity contribution as function of $\bar{p}$ for $\alpha = 0.99$, $\rho_x = 0.3$ in the uncapped and capped case.

Next in Figure 7.10, we show the proportion of all default layers as a function of ρ_x, in both the uncapped and the capped case, for a fixed $\alpha = 0.99$.

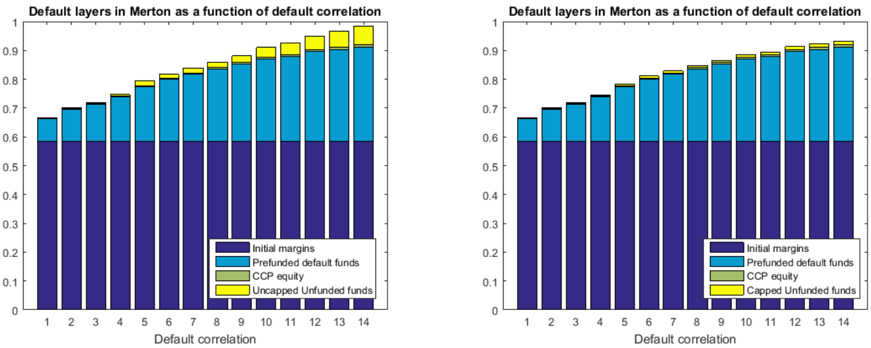

Figure 7.10: Initial margins, prefunded and expected unfunded default funds and CCP equity contribution as function of ρ_x for $\alpha = 0.99$, $\bar{p} = 0.065$ in the uncapped and capped case.

As we can see from Figures 7.9 and 7.10, the mixed binomial models fits well with the margin procyclicality problem, especially when the unfunded funds are capped. In the extreme scenarios, we have constant initial margins and the biggest contribution in terms of funds is given by the prefunded default funds, while the unfunded funds are a small percentage of prefunded funds.

7.2 Sensitivity Analysis and Robustness Checks

In this subsection we present some different stress test scenarios for the prefunded default funds. In particular we check the funds behavior for different number of clearing members, for different clearing period T, and finally we see the impacts of a high-yield scenario where the 10-year yield is around 10%.

7.2.1 Sensitivity to Number of Clearing Members

An interesting sensitivity analysis is how the layers change when we increase or reduce the number of clearing members. In Figure 7.11 we show the individual prefunded funds when the number of clearing members go from 5 to 55 with parameters $\rho_x = 0.38$ and $\bar{p} = 0.04$ and $\alpha = 0.99$

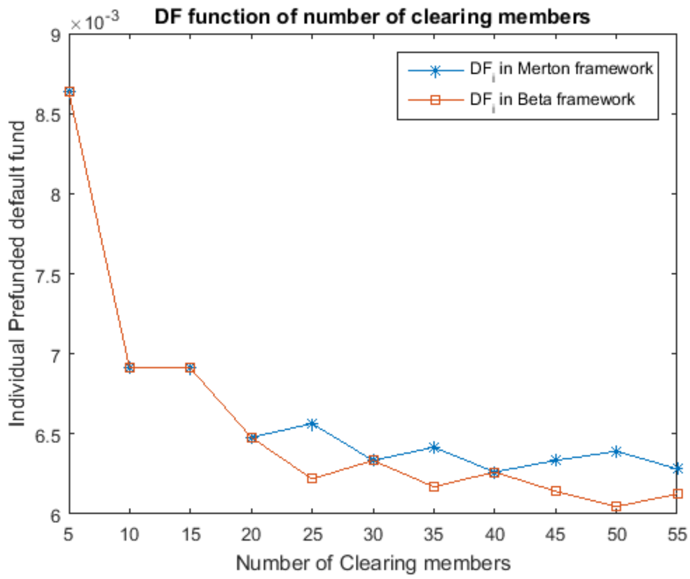

Figure 7.11: Individual prefunded default funds for different number of companies in Beta and Merton models.

Also, in Figure 7.12 we show the individual prefunded default funds as function of ρ_x and number of clearing members.

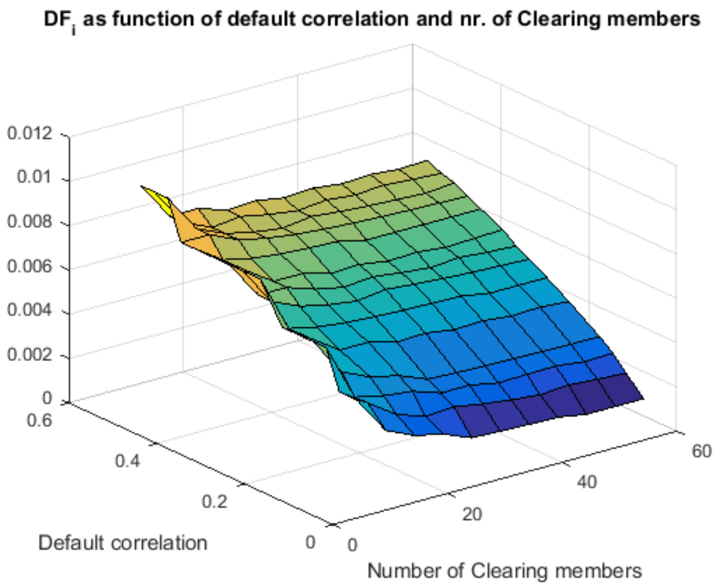

Figure 7.12: Individual prefunded default funds as function of number of companies and default correlation when $\alpha = 0.99$ and $\bar{p} = 0.065$.

As we see, using conditional models we can observe somewhat of a diversification effect, where the default funds decrease drastically from 5 to 20 members, and thereafter flattens out. As a comparison to stock portfolios, Elton & Gruber (1977) found that almost full diversification effect is reached after 30 stocks, which seems in line with our findings.

7.2.2 Sensitivity to the clearing period T

The prefunded funds are a function of the exposure C, that is the exposure for the IRS contract after considering initial margins and variation margins. As shown in Section 6 the exposure is increasing in time. In our basic setting we fix the clearing period equal to 5 years. Intuitively, we expect that if we consider 10 years as clearing period we would have an higher exposure, and consequently higher guarantee funds. In Figure 7.13 we show prefunded default funds as a function of time, default probability and default correlation in the Merton setting.

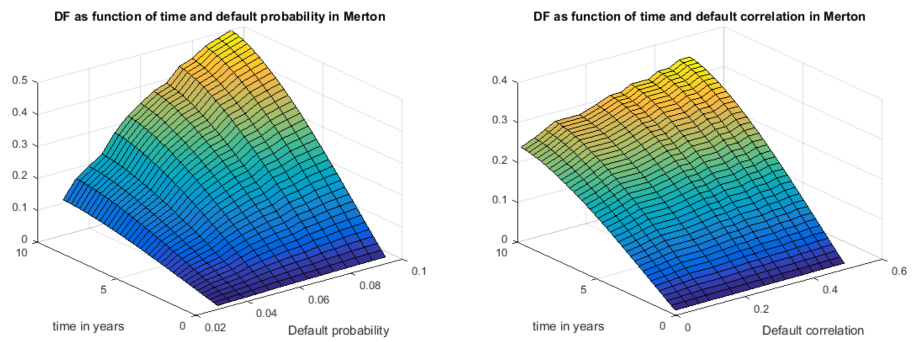

Figure 7.13: Prefunded default funds as function of ρ_x and clearing period when $\bar{p} = 0.05$ and $\alpha = 0.99$ and as function of $\bar{p}$ and clearing period when $\rho_x = 0.18$ and $\alpha = 0.99$.

As we expect, Figure 7.13 confirms that prefunded default funds are increasing with time. Furthermore, we are interested to see the impact of time on the CCP default probability. In Table 7.6 we present the CCP default probability in the Merton setting when $\alpha = 0.99$ and the clearing period is set to 10 years.

	$\bar{p} = 3\%$	$\bar{p} = 4\%$	$\bar{p} = 5\%$	$\bar{p} = 6\%$	$\bar{p} = 7\%$	$\bar{p} = 8\%$	$\bar{p} = 9\%$
$\rho_x = 0.01$	0	0	0	0	0	0	0
$\rho_x = 0.08$	0	0	0	0	0	0	0
$\rho_x = 0.18$	0	0	0	0	0	0	0
$\rho_x = 0.27$	0	0	0	0	0	0	0
$\rho_x = 0.33$	0	0	0	0	0	0	0
$\rho_x = 0.41$	0	0	0	0	0	0.003%	0.004%
$\rho_x = 0.47$	0	0	0	0.003%	0.006%	0.013%	0.015%

Table 7.6: Default probability as function of $\bar{p}$ and ρ_x when $T = 10$

Comparing Table 7.6 and Table 7.4 we can see that the CCP default probability decreases when the clearing period increases. This decrease is simply due to the fact that the funds increase with time, as seen in Figure 7.13, meaning that the model responds well to an increase in considered time period.

7.2.3 Sensitivity to Interest Rates

Since the derivative contract we have considered in our exposure calculation is an interest rate swap, it is of interest to perform a sensitivity analysis of the impact of increased interest rates on the default waterfalls of the CCP. In

Figure 7.14 we show the CCP collateralized exposures in two yield settings. In the first setting the 10-year yield curve is circa 3%, whereas in the second case the 10-year yield is around 10%. Recall from Section 6 that the CCP collateralized exposure is

$$e_i(t) \equiv \max\Big\{\big(V_i^+(t+\Delta) - V_i^+(t-\hat{\Delta}) - IM_i(t)\big), 0\Big\} \tag{7.4}$$

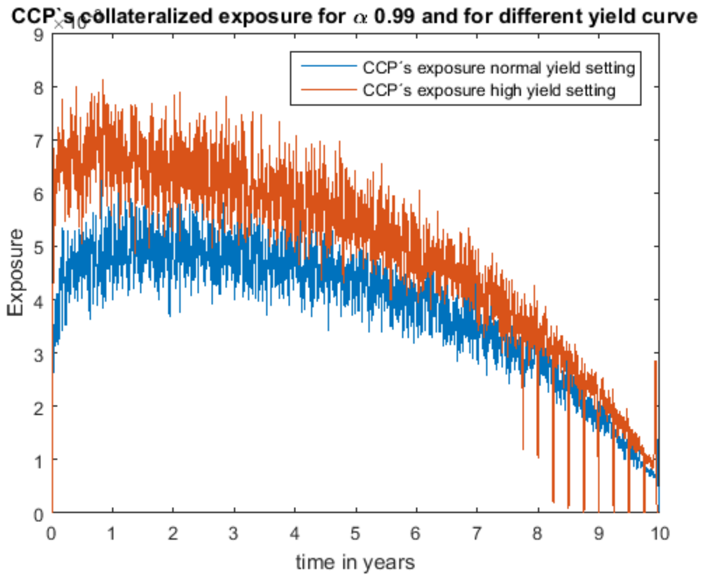

Figure 7.14: CCP collateralized exposure for different yield curves with $\alpha = 0.99$.

In addition to this, recall that the EPE-based time-T loan equivalent collateralized exposure is:

$$C_i \equiv (1 - \delta_i) \int_0^T \mathbb{E}[e_i(t)]dt. \tag{7.5}$$

In Figure 7.15, we show the behavior of the collateralized exposure for our two different interest rate settings.

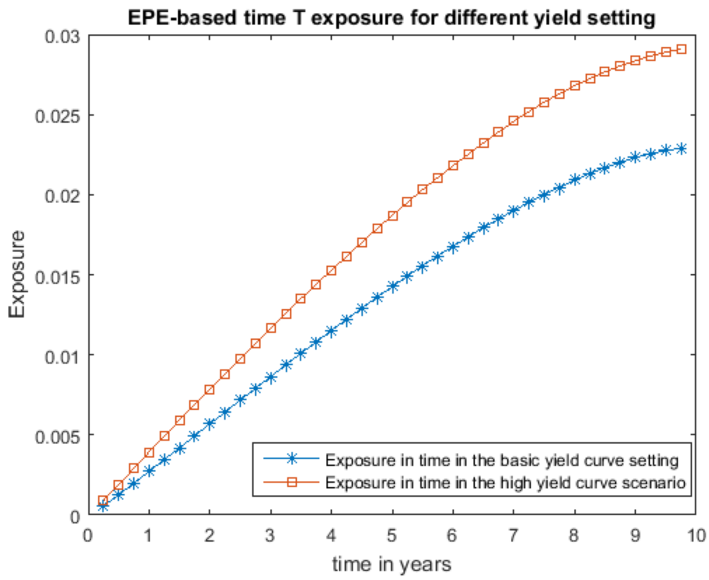

Figure 7.15: EPE-based time-T loan equivalent collateralized exposure for different yield curves with $\alpha = 0.99$

Clearly, the exposure will increase with the interest rate, meaning higher initial margins per definition. As we wish to see the impact of a rate shift on the prefunded default funds, we display in Figure 7.16 the prefunded default funds as a function of default probability, default correlation with $\alpha = 0.99$ for the two different interest rate settings.

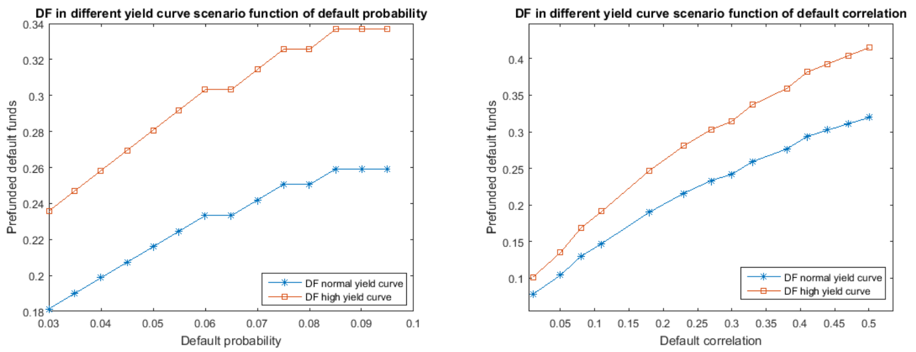

Figure 7.16: Prefunded default funds as function of ρ_x and yield curve when $\bar{p} = 0.06$ and as a function of $\bar{p}$ and yield curve, when $\rho_x = 0.18$ and $\alpha = 0.99$.

From Figure 7.16 we can conclude that the default funds are sensitive to a change in the interest rate, and will increase along with the interest rates.

8 Conclusions

We base our thesis on the CCP static risk model by Ghamami (2015), and implement this model in a numerical software environment. We find that the model increases the layers of the default waterfall as conditions worsen, that is higher default probability and default correlation increase the unfunded and prefunded default funds. Also, an interesting finding is that in stressed scenarios, the CCP default probability is lower than for more normal scenarios, which might seem contradictory to intuition. However, the lower CCP default probability is due to the fact that the model is conservative, and posts very high funds in the stressed scenarios, meaning that a scenario going beyond our funds is extremely unlikely. Regarding the model comparison of our two different mixed binomial models, we find that beta and Merton models yield similar results.

Finally, by performing a number of stress test scenarios we can confirm the robustness of our models. In particular,

- We observe a diversification effect when the number of clearing members increases.
- The models is robust to a change in time horizon, since the prefunded default funds increase with the time.
- The funds are higher in the case of a higher yield curve, since it takes into account the higher risk arising from higher exposure for each clearing member.

All of the above conclusions are in line with economic intuition.

A potential drawback in our model is that we have considered a homogeneous portfolio, which of course is a simplistic assumption. For future research, an interesting application would be to implement the model for a heterogeneous portfolio. As another suggestion, Ghamami (2015) has presented ways to further co-integrate the model with regulations by e.g. the Basel Accords, which could be an interesting addition to our paper. Also, one could certainly tweak our methodology, by for example using other distributions for the default indicators, or other OTC derivatives to calculate the CCP exposure.

Appendix

	$\bar{p} = 3\%$	$\bar{p} = 4\%$	$\bar{p} = 5\%$	$\bar{p} = 6\%$	$\bar{p} = 7\%$	$\bar{p} = 8\%$	$\bar{p} = 9\%$
$\rho_x = 0.01$	0	0	0	0	0	0	0
$\rho_x = 0.08$	0	8.33%	7.69%	7.14%	6.67%	6.25%	0
$\rho_x = 0.18$	0	5.56%	0	4.76%	4.55%	4.35%	4.17%
$\rho_x = 0.27$	0	0	4.17%	3.85%	3.70%	3.57%	3.45%
$\rho_x = 0.33$	4.35%	0	3.70%	3.45%	3.33%	3.23%	3.12%
$\rho_x = 0.41$	0	3.45%	3.23%	3.03%	2.94%	2.86%	2.86%
$\rho_x = 0.47$	3.45%	3.12%	2.94%	2.86%	2.78%	2.70%	2.70%

Table 8.1: Relative error between the Merton and beta model for the pre-funded default fund when $\alpha = 0.99$

	$\bar{p} = 3\%$	$\bar{p} = 4\%$	$\bar{p} = 5\%$	$\bar{p} = 6\%$	$\bar{p} = 7\%$	$\bar{p} = 8\%$	$\bar{p} = 9\%$
$\rho_x = 0.01$	0	0	0	0	0	0	0
$\rho_x = 0.08$	11.76%	11.11%	10.53%	10%	9.52%	4.55%	4.35%
$\rho_x = 0.18$	11.54%	7.14%	6.90%	10.34%	10%	6.45%	6.25%
$\rho_x = 0.27$	9.37%	9.09%	8.82%	5.71%	5.56%	5.56%	2.70%
$\rho_x = 0.33$	8.57%	8.33%	5.41%	2.63%	5.26%	2.56%	2.56%
$\rho_x = 0.41$	5.26%	2.56%	2.50%	2.50%	2.50%	2.50%	2.50%
$\rho_x = 0.47$	2.50%	2.50%	0	0	0	0	0

Table 8.2: Relative error between the Merton and beta model for the pre-funded default fund when $\alpha = 0.999$

	$\bar{p} = 3\%$	$\bar{p} = 4\%$	$\bar{p} = 5\%$	$\bar{p} = 6\%$	$\bar{p} = 7\%$	$\bar{p} = 8\%$	$\bar{p} = 9\%$
$\rho_x = 0.01$	5.08%	10.32%	1.15%	1.06%	4.12%	0.58%	2.30%
$\rho_x = 0.08$	1.81%	10.42%	5.02%	6.18%	6.95%	5.57%	8.72%
$\rho_x = 0.18$	1.96%	2.20%	4.09%	6.19%	2.28%	1.14%	3.64%
$\rho_x = 0.27$	4.14%	1.34%	5.66%	1.70%	1.75%	4.30%	0.41%
$\rho_x = 0.33$	1.76%	0.67%	4.10%	0.56%	1.96%	2.58%	0.55%
$\rho_x = 0.41$	1.11%	4.08%	2.10%	1.92%	5.18%	4.09%	0.91%
$\rho_x = 0.47$	3.27%	3.80%	6.53%	0.66%	1.51%	0.57%	3.02%

Table 8.3: Relative error between the Merton and beta model for the un-funded default fund when $\alpha = 0.99$

	$\bar{p} = 3\%$	$\bar{p} = 4\%$	$\bar{p} = 5\%$	$\bar{p} = 6\%$	$\bar{p} = 7\%$	$\bar{p} = 8\%$	$\bar{p} = 9\%$
$\rho_x = 0.01$	0.26%	9.12%	7.43%	0.22%	2.93%	4.21%	6.83%
$\rho_x = 0.08$	7.20%	2.17%	12.02%	2.01%	5.30%	12.92%	9.83%
$\rho_x = 0.18$	2.07%	20.75%	21.06%	16.51%	3.94%	1.23%	9.82%
$\rho_x = 0.27$	5.63%	6.28%	3.95%	7.19%	8.42%	6.83%	16.73%
$\rho_x = 0.33$	10.63%	4.21%	6.22%	4.74%	8.55%	10.34%	3.74%
$\rho_x = 0.41$	1.91%	11.30%	9.65%	9.53%	0.88%	4.91%	1.77%
$\rho_x = 0.47$	14.12%	2.99%	6.13%	7.60%	9.28%	7.59%	8.05%

Table 8.4: Relative error between the Merton and beta model for the unfunded default fund when $\alpha = 0.999$

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