



**UNIVERSITY OF GOTHENBURG**  
**SCHOOL OF BUSINESS, ECONOMICS AND LAW**

Master Degree Project in Finance

## **CVA for IR-Swaps under Wrong Way Risk**

A numerical evaluation using a semi-analytical model

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Master Degree Project No. 2016:124  
Graduate School

# Abstract

This thesis examines the background and nature of credit value adjustment (CVA), a concept that has heightened in its importance in the financial market after the 2008 financial crisis. Credit value adjustment is defined as a price deducted from the risk-free value of a bilateral derivative to adjust for the counterparty credit risk (CCR). The focus of this thesis is to quantify CVA of an interest rate swap (IRS) under wrong way risk (WWR). Interest rate swap is an agreement between counterparties to exchange future interest rate payments, and WWR is the risk of negative correlation between the credit exposure and the counterparty's credit quality. The numerical studies in this thesis are conducted using the semi-analytical formula derived by Cerny and Witzany (2015). We investigate the behavior of CVA with respect to two factors, the default intensity and the WWR, where the results show that CVA increases with both factors, as has been proven in earlier studies such as Cerny and Witzany (2015) and Brigo & Pallavicini (2007). Additionally, we look at the evolution of CVA before, during and after the 2008 financial crisis where we see that CVA was negligible before June 2007, but then it surged rapidly, which resulted in substantial financial losses for many institutions. Furthermore, we examine the possibility to compute CVA for a heterogeneous portfolio by regarding it as a homogeneous one, in which all obligors in the portfolio are considered to have identical parameters. The results show that the homogenous method works relatively well for portfolios that consist of similar obligors.

*Key words: Credit Value Adjustment, Wrong Way Risk, Interest Rate Swap, Credit Default Swap, Homogeneous CVA Portfolio, Heterogeneous CVA Portfolio, Semi-Analytical Model*

# Acknowledgement

We would like to profusely thank our supervisor, Alexander Herbertsson, for his guidance and encouragement throughout this process. He has not only provided us invaluable inputs but has also ignited our interest in the field of quantitative finance. We would also like to thank the Humanities Library for accommodating us for the majority of the period.

Berglind Halldórsdóttir and Weili Zhang

June 2016

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# List of Abbreviations

BS = Black Scholes

CCR = Counterparty Credit Risk

CDS = Credit Default Swap

CRAs = Credit Rating Agencies

CVA = Credit Value Adjustment

$CVA_{heterog}$  = CVA for a heterogeneous portfolio

$CVA_{homog}$  = CVA for a homogeneous portfolio

$CVA_{IRS}$  = CVA for an IRS contract

DVA = Debit Value Adjustment

EE = Expected Exposure

FRA = Forward Rate Agreement

GBM = Geometric Brownian Motion

IMM = Internal Model Method

IRS = Interest Rate Swap

OIS = Overnight Indexed Swap

OTC= Over-The-Counter

RWR = Right Way Risk

SFT = Securities Financing Transactions

$Std_{\emptyset}$  = Standard Deviation of Recovery Rate in portfolio

$Std_{CDS}$  = Standard Deviation of CDS Spreads in portfolio

VaR = Value at Risk

WWR = Wrong Way Risk

# 1 Introduction

In the wake of the 2008 financial crisis, financial institutions eventually recognized the importance of counterparty credit risk (CCR), which is the risk that counterparties in a bilateral derivative contract default. The market price of CCR is credit value adjustment (CVA) which is the difference between the value of a portfolio when the counterparty is default-free, and the value when the portfolio has been adjusted for the counterparty's default risk.

Counterparty credit risk is only subject to bilateral contracts, which are Over-The-Counter (OTC) and Securities Financing Transactions (SFT). The two most traded OTC derivatives in the market are interest rate swaps (IRS), an agreement between two parties to exchange future interest rate payments, and credit default swaps (CDS), an insurance against default.

The Basel Committee highlights that two-thirds of the CCR losses that accrued during the 2008 financial crisis were due to CVA losses, which is the write-downs on outstanding derivatives caused by the increase in the default probability of counterparties. Even though CCR was conceptually understood, many overlooked this risk on large institutions due to their high credit quality and low CDS spreads. The 2008 financial crisis pulled investors and banks back to reality when the CDS spreads of some major financial institutions surged drastically. Hence, since 2008, CVA has caught the deserved attention of both researchers and practitioners. The Basel committee is, at the moment of writing, implementing capital charges for CVA in Basel III, which is one of the largest changes in the wake of the 2008 financial crisis. Furthermore, many accounting bodies have implemented the concept in their standards for fair value accounting.

Another concept that has gained much attention after the 2008 financial crisis is wrong way risk (WWR), which is the risk of negative correlation between the credit exposure and the counterparty's credit quality. Taking the WWR into account when valuing the derivative leads to an increase in the CVA, as shown in Brigo and Capponi (2008), Brigo and Pallavicini (2007), Hull and

White (2012) and Rosen and Saunders (2012). The same results are exhibited in Cerny and Witzany (2015) where the CVA of an interest rate swap ( $CVA_{IRS}$ ) that is subjected to perfect correlation is almost three times larger than the  $CVA_{IRS}$  when WWR is ignored. Thus wrong way risk in CVA should not be neglected.

Many large financial institutions have derivative transactions with numerous counterparties. Reuters (2008) reported that Lehman Brothers, at the time of its default, had about 1.5 million derivative transactions with over eight thousand counterparties. CVA is calculated separately for each obligor, therefore, to calculate the CVA for thousands of obligors is very time consuming and computationally intensive. One alternative is to assume that all obligors have identical parameters, i.e. the obligors are homogeneous. Herbertsson and Rootzen (2008) apply this homogeneous method on pricing basket default swaps and find this method works relatively well. In this thesis, the same idea is applied on pricing CVA portfolio instead of pricing basket default swaps.

This thesis uses a semi-analytical model that is derived by Cerny and Witzany (2015) for  $CVA_{IRS}$  in the presence of WWR. Semi-analytical methods are good because they are not as computationally heavy and time consuming as Monte Carlo simulation methods. In this semi-analytical model, Cerny and Witzany use the Gaussian copula to model the dependence between the underlying interest rate and the default probability, capturing both WWR and the so-called right way risk (RWR). Furthermore, the dependence in the model represents the correlation between the level of interest rate and the default time, which captures the dynamic relationship between these two factors. The drawback of the model is that it neglects how to transform the default intensity from risk-neutral measure to risk-neutral annuity measure, the underlying measure of the model. Therefore, we add a scalar to the model to adjust the difference between the measures, which we call the change of measure parameter. The purpose of this thesis is to study the effects of WWR and the change of measure parameter on a  $CVA_{IRS}$ . We also look at the movement of  $CVA_{IRS}$  before, during and after the 2008 financial crisis. Finally, this thesis investigates the possibility of using a homogenous method for a  $CVA_{IRS}$  portfolio. These objectives are achieved by answering the following research questions:

- *How will WWR and change of measure parameter affect the value of a single  $CVA_{IRS}$  ?*
- *How did the  $CVA_{IRS}$  evolve before, during and after the 2008 financial crisis?*
- *Under what conditions will the  $CVA_{IRS}$  on a homogeneous portfolio ( $CVA_{homog}$ ) and the  $CVA_{IRS}$  on a heterogeneous portfolio ( $CVA_{heterog}$ ) have a relatively small difference?*

In general, the obligors within a portfolio differ in recovery rates, CDS spreads and correlation parameters. Because obligors' correlation parameters are difficult to estimate as stated by Rosen and Saunders (2012), we keep the correlations constant and same for each obligor. Hence, we answer the last research question by investigating the following two sub-questions:

- *Do changes in the standard deviation of the recovery rates in a portfolio change the difference between  $CVA_{homog}$  and  $CVA_{heterog}$ ?*
- *Do changes in the standard deviation of the CDS spreads in a portfolio change the difference between  $CVA_{homog}$  and  $CVA_{heterog}$ ?*

The rest of the thesis is organized as follows: Section 2 introduces the central concepts that are essential for the understanding of the counterparty risk and the CVA, which are the gist of this thesis. Section 3 provides an overview on OTC derivatives and elaborates on the definition and construction of IRS and CDS. Section 4 proceeds to introduce the main methodologies for both credit risk modeling and dependency modeling in credit risk. In Section 5, the general methodology of a unilateral CVA excluding WWR and the computation of  $CVA_{IRS}$  using swaptions' prices are presented. Section 6 introduces the Cerny and Witzantny (2015) model and the adjustment made on the model in this thesis. Section 7 uses the model that is presented in Section 6, to conduct three numerical studies to answer the research questions. Lastly, in Section 8 we summarize a set of answers to the outlined research questions, along with a discussion on the drawbacks and further research proposals.

## 2 Central Theory

This section introduces the central concepts that are essential for conducting the thesis. In the first two subsections, we provide a rudimentary definition of credit risk and credit value adjustment respectively. Then we outline the three most common CVA measures and lastly we introduce the concept of WWR.

### 2.1 Introduction to Credit Risk

This subsection covers the credit risk in general, which is then followed by a detailed discussion on a special type of credit risk, the counterparty credit risk (CCR).

#### 2.1.1 Credit Risk

Credit risk is defined as the risk of experiencing a financial loss in the event that an obligor defaults. Credit risk can be split into two categories: the default risk and the spread risk. Default risk refers to when the obligor is unable to make the contractual payments on its debt obligations and is the same as the risk of an obligor going bankrupt. On the other hand, spread risk concerns the changes in obligor's credit quality that implies the obligor's ability to meet its contractual obligations (Duffie & Singleton, 2003). The spread is defined as the difference between the interest rate of a default-free security and the interest rate of a defaultable security. Additionally, the spread is the compensation for risk-taking and is negatively correlated to the obligor's credit quality. (Hull, 2012)

Schönbucher (2003) splits credit risk into five components:

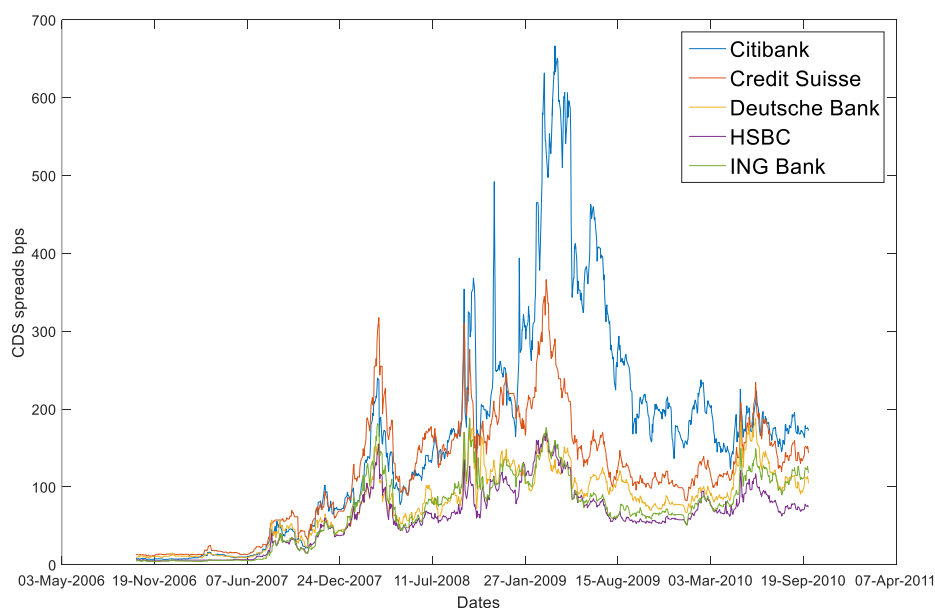
- Arrival risk: The uncertainty whether a default will occur, which is measured by default probability.
- Timing risk: The uncertainty of the exact default time. It is more detailed than arrival risk since it examines arrival risk within all possible time horizons.
- Recovery risk: The uncertainty concerning the size of the loss, given that there is a default.
- Market risk: The risk that changes in the economic environment affect the default risk.

- Default correlation risk: The risk that several obligors default at the same time, or the “domino-effect”, which occurred during the 2008 financial crisis.

### 2.1.2 Counterparty Credit Risk

Counterparty credit risk (CCR) is a risk that the counterparties in a bilateral contract default on their contractual payments. CCR is mainly subject to contracts that are privately negotiated: OTC and SFT derivatives (Pykhtin & Zhu, 2007). A CCR is a combination of the market risk, defined as the exposure, and the credit risk, defined as the counterparty credit quality (Gregory, 2013).

Since the 2008 financial crisis, counterparty credit risk has become one of the key financial risks for major financial institutions and large corporations that deal with OTC contracts. Before the crisis, institutions ignored CCR on large institutions, sovereigns, supranational or collateral posting counterparties, which had good ratings and low CDS spreads indicating negligible CCR. However, as shown in the 2008 crisis, usually these counterparties represented the majority of CCR (Gregory, 2013). Figure 1 illustrates the banks’ CDS spreads were almost zero before June 2007, and how they dramatically changed during the tumultuous period.



**Figure 1: The 5-year CDS spreads on senior debt of five major banks from Oct. 2006 to Sep. 2010.**

Counterparty credit risk is not the same as lending risk, the traditional credit risk. The value at risk (VaR) for CCR is usually significantly uncertain because the underlying market value of the outstanding bilateral contract is unknown. Meanwhile, the VaR for lending risk is known with some degree of certainty. Additionally, CCR occurs in a bilateral contract where both parties are at risk of default, i.e. both parties bear the risk (bilateral risk), contrarily, lending risk occurs in loan contracts where only the lending party bears the risk (unilateral risk). (Gregory, 2013)

To account for CCR in OTC derivatives, institutions estimate the market price of CCR, and deduct it from the risk-free value of the derivative (Pykhtin & Zhu, 2007). The market price of CCR is called credit value adjustment (CVA), which is the main focus of this thesis and is described in the next subsection. In the 2008 financial crisis approximately 70% of CCR losses were due to CVA losses, i.e. write-downs, and only 30% were due to the actual default (Basel Committee on Banking Supervision, 2011). As a result of this substantial financial loss, the Basel committee and some accounting standards have implemented a CVA measure in their frameworks, along with banks computing CVA for pricing and risk management purposes. Therefore, it is common for a bank to have three different CVA values; the regulatory CVA, the accounting CVA and the trading book CVA (Gregory, 2013), which all are discussed in Subsection 2.3.

## **2.2 Introduction to Credit Value Adjustment**

This subsection explains the credit value adjustment (CVA) concept and how CVA is quantified. Additionally, we outline the difference between a unilateral and a bilateral CVA.

The concept of CVA is defined as the difference between the value of a portfolio when the counterparty is default-free, and the value of the portfolio that has been adjusted for the default risk of the counterparty (Gregory, 2013).

To quantify the CVA, consider a bilateral OTC derivative (e.g. an IRS or CDS) between two counterparties  $A$  and  $B$  with maturity  $T$ . Seen from  $A$ 's

perspective, let  $V(t, T)$  be the risk-free value of this contract at time  $t$ ,  $0 \leq t \leq T$ , meaning that neither  $A$  nor  $B$  can default. Furthermore, let  $V^*(t, T)$  be the risky value of the same contract, in which  $B$  can default before  $T$  and  $A$  is default-free. Then the CVA at time  $t$ , for the above contract is denoted by

$$CVA(t, T) = V(t, T) - V^*(t, T).$$

The above equation defines the so-called unilateral CVA, which means that the valuing party  $A$  assumes itself to be default-free (Brigo, et al., 2013). This model is elaborated further in Section 5.

The adjustment with respect to CCR on a bilateral contract is either a bilateral CVA or a unilateral CVA. Due to the bilateral nature of OTC derivatives, the contract parties are exposed to each other's default probability. For a bilateral CVA, party  $A$ , in the above example, takes into account both the counterparty  $B$ 's and its own default risk. The latter is called the Debt Value Adjustment (DVA), which is CVA seen from  $B$ 's perspective.

Hull (2015) points out that DVA is controversial because it can only be monetized when  $A$ , in the above example, actually defaults. Theoretically, the increasing default probability of  $A$  is its own benefit. That is, the DVA increases as  $A$ 's credit spread increases, leading to an increase in the derivatives' reported value in  $A$ 's books, and thus, an increase in its profits. In 2008, some banks reported billion dollars of profits from this practice. This controversial concept has been deducted from some accounting standards (Onaran, 2016) and Basel III (Basel Committee on Banking Supervision, 2012).

## **2.3 Three Different CVA Measures**

In the following subsection, we describe the three most prevalent measures for CVA: the accounting, the trading book and the regulatory.

### **2.3.1 The accounting CVA**

Due to the controversy of DVA, many accounting bodies, such as Financial Accounting Standards Board, have decided to remove DVA from the companies' earnings statements (Onaran, 2016). However, IFRS, the accounting standard



that is used in most parts of the world except in China and the US<sup>1</sup>, has the requirement to reflect both the CVA and the DVA. In addition, IAS 39, a standard adopted in IFRS, requires changes in the fair values of those instruments to be recognized as a profit or a loss. Although IFRS requires no specific method to calculate the CVA and the DVA, the methods chosen should maximize the use of observable inputs and minimize the use of unobservable inputs. (European Banking Authority, 2015)

### **2.3.2 The trading book CVA**

The trading book CVA depends on the extent of adjustment on CCR in the derivative prices. The price should be competitive and at the same time ensure that the banks can absorb or hedge potential CCR losses during the life of the transaction. (European Banking Authority, 2015)

The EBA report (2015) on CVA shows that in most cases institutions applying the IFRS use the same CCR adjustment (unilateral or bilateral) for trading book and accounting purposes. Most of those who do not use the same method, do not compute the DVA for the trading book, while doing so for accounting CVA as enforced by IFRS standards.

### **2.3.3 The regulatory CVA**

The introduction of capital reserves for CCR is in Basel II (Basel Committee on Banking Supervision, 2008) where it is based on the credit risk framework and designed to capitalize for default and mitigation risk rather than the fair value adjustment of the banks' derivatives, as in Basel III. Additionally, under the Basel II market risk framework, banks are required to capitalize against variability in market value of their derivatives, but not against variability in their CVA.

As mentioned earlier, the majority of CCR losses on banks' OTC derivative portfolios suffered in the financial crisis was due to CVA losses (Basel Committee on Banking Supervision, 2011). In the wake of these events, the capital charge

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<sup>1</sup> Required or permitted for listed companies. Of the 137 countries that PwC investigated, 9 of them did not have central exchange and 14 did not require or permit IFRS. Of those 14 were the US, China, Indonesia, Thailand and the rest was mainly located in Africa. (PricewaterhouseCoopers, 2015)

for the CVA variability is introduced in Basel III (Basel Committee on Banking Supervision, 2010). The Basel committee has introduced two methods to calculate capital charge for CVA, the current Standardized Approach and the current Advanced Approach. The latter is only available to banks that have the Internal Model Method (IMM) approval. Both approaches have the objective to capture the variability in the regulatory CVA that are solely due to the changes in the credit spreads, but do not take into account the changes in the exposure due to the daily changes in the market risk factors. This shortage is taken into account in a proposed CVA framework, which was published in July 2015 by the Basel Committee. This framework was derived from a revised market risk framework which is based on fair value sensitivities to market risk factors and is called *Fundamental Review of the Trading Book* (Basel Committee on Banking Supervision, 2015).

## **2.4 Wrong Way Risk**

In this subsection we introduce wrong way risk (WWR) and its effect on CVA. Additionally, we outline some of the complications associated with quantifying the WWR.

Generally, WWR refers to a negative correlation between the credit exposure and the counterparty's credit quality, leading to a further increase in the derivatives' CVA. Consider two counterparties,  $A$  and  $B$ , who have entered into a bilateral contract running up to time  $T$ . Seen from  $A$ 's perspective, during this trade period, the credit exposure of the contract increases as the creditworthiness of  $B$  deteriorates. Thus, the dependence between these two factors moves in the "wrong way" for  $A$ . When the exposure decreases as counterparty credit quality worsens we have the so-called right way risk (RWR) (Rosen & Saunders, 2012). The concept of RWR is disregarded in this thesis.

Wrong way risk is classified into general WWR and specific WWR. General WWR is when the counterparty's credit quality is correlated with a general market risk factor, e.g. interest rate or inflation, which further increases the exposure of the contract. For example,  $A$  enters into a bilateral contract with  $B$  where the underlying is an interest rate or an index, which is correlated in the

“wrong way” with *B*’s credit quality. Specific WWR is when the credit exposure is correlated to the counterparty’s default probability that is caused by some idiosyncratic factors. For example *B* writes an American put option on its own stock to *A*. When *B*’s credit rating is degraded, its stock price will decrease, thus the option is likely to be further in-the-money for *A*. This implies higher exposure of the contract for *A*, and at the same time, higher probability that *B* will default, i.e. not able to pay its contractual obligations (Rosen & Saunders, 2012). Figure 2 summarizes the general definition for both general and specific WWR.



**Figure 2: WWR is when the movement of a general or a specific factor causes both the default probability and the exposure to increase, which means there is more risk of default and more value that can be loss than else.**

Rosen and Saunders (2012) point out some complications associated with WWR. First of all, there is no standard approach to treat WWR in the industry, which leads different banks to come up with different capital charges for CVA when incorporating WWR. Another problem is how to estimate the correlation. Risk-neutral CVA requires implied market-credit correlations, which are very difficult to find, because there are no market prices to imply these correlations from.

Since the 2008 financial crisis, CVA and WWR have gained much attention both in the financial industry and in academia. Although no specific method has been adopted in the industry, many methods have been proposed by, e.g. Hull and White (2012), Brigo and Chourdakis (2009) and Cerny and Witzany (2015). This thesis focuses on the semi-analytical model derived by Cerny and Witzany for CVA analysis.

### 3 Over-The-Counter Derivatives

This section describes two commonly traded OTC derivatives, namely, credit default swap (CDS) and interest rate swap (IRS). First, we give a short introduction to the OTC market. In Subsection 3.1 we provide a definition of the

IRS, along with the construction and valuation method for IRS. Furthermore, in Subsection 3.2, a thorough description of CDS is presented. Lastly, Subsection 3.3 presents the relevant interest rate theory that is critical when valuing derivatives.

OTC derivatives are contracts that trade directly (privately) between two counterparties, and do not need to go through a centralized exchange. However, following the 2008 financial crisis, several countries have passed legislation that some OTC derivatives must go through the clearing houses (Hull, 2012). Figure 3 illustrates that IRS is the most traded derivative in the OTC market. In 2015, the outstanding market size for IRS was 320 trillion US\$ ( $= 3.2 * 10^{14}$  US\$), and the average daily trading value in IRS was 1.27 trillion US\$, which approximately 58% of the total trading value, while CDS was approximately 3% of the total value. (Bank for International Settlements)

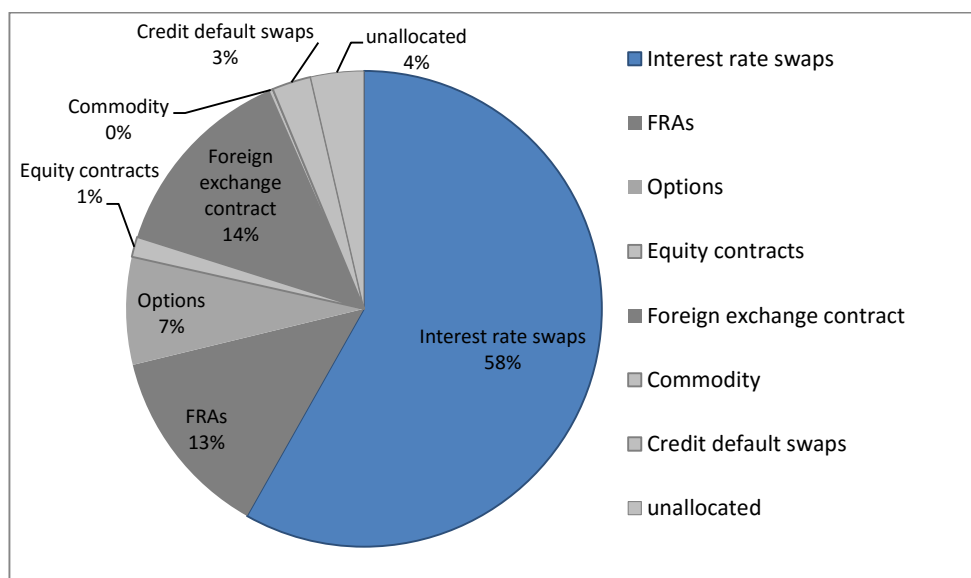


Figure 3: The notional outstanding and gross market value of instruments in the global OTC derivatives market in 2015 (Bank for International Settlements).

### 3.1 Interest Rate Swaps

This subsection starts with a general definition of a swap and proceeds to a discussion of a forward rate agreement. Subsection 3.1.2 covers the construction and valuation of interest rate swaps and how the swap rate is determined.

A swap is an agreement between two parties to exchange future cash flows at a certain exchange rate, typically at several future dates. An interest rate

swap is a financial derivative, used by institutions as an instrument to hedge and to manage interest rate risk. (Hull, 2012)

### 3.1.1 Forward rate agreements

A forward rate agreement (FRA) is an OTC agreement between two parties to ensure a predetermined interest rate will apply to a certain notional for a specific future time. In practice only the interest rate cash flows are exchanged, because the notional cancels out. (Hull, 2012)

Consider a FRA, which starts at  $T_1$  and matures at  $T_2$ , in which a predetermined fixed interest rate,  $R_K$ , is exchanged for a floating rate  $R_M$ . At  $T_2$ ,  $R_M(T_1, T_2)$  is equal to the market rate (Libor) observed at time  $T_1$  for the period  $\delta(T_1, T_2) = T_2 - T_1$ . The Libor rate will be discussed in Subsection 3.3.1. The value of a FRA for a payer  $V_{FRA_P}$  and a receiver  $V_{FRA_R}$  at time  $t < T_1$ , with notional  $N$ , and the predetermined rate  $R_K$  are given by

$$V_{FRA_P}(t) = N(R_F - R_K)(T_2 - T_1)e^{-R_2(T_2-t)}$$

$$V_{FRA_R}(t) = N(R_K - R_F)(T_2 - T_1)e^{-R_2(T_2-t)}$$

where  $R_2$  is the return for an investment for time period  $(t, T_2)$  and  $R_F$  is the forward rate observed in the market at time  $t$ , which is the interest rate that can be locked in at time  $t$  for a future investment in the period  $\delta(T_1, T_2)$ . At inception, the fair value of the contract equals to zero, thus,  $R_K$  equals  $R_F$  at  $t = 0$ . (Hull, 2012)

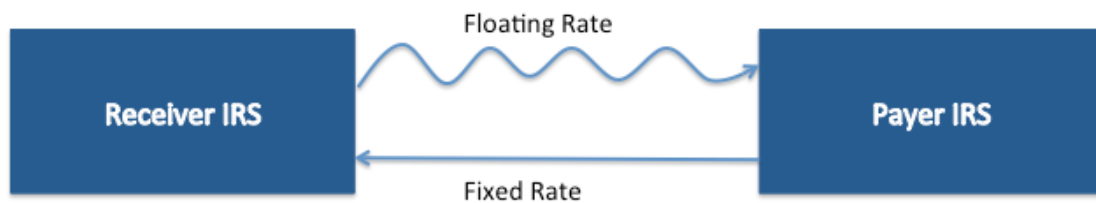
### 3.1.2 Construction and valuation of interest rate swaps

An interest rate swap can be seen as a portfolio of forward rate agreements with a sequence of payment dates  $T_1 < T_2 < \dots < T_n$ . The most common IRS is the “plain vanilla” swap, which is illustrated in Figure 4, where a fixed rate is swapped for a floating rate. The party who receives the floating leg and pays the fixed leg is said to hold a payer IRS, while the receiver IRS pays the floating leg and receives the fixed leg. The floating rate is usually determined by the Libor rate. The fixed rate is predetermined to ensure that the value of the contract is zero at the inception date, and this rate is referred to as the swap rate  $s_K$ . (Hull, 2012)

According to Brigo and Mercurio (2006), a forward swap rate at time  $t$  is the fixed rate that makes IRS a fair contract at time  $t$ . The forward swap rate for a swap with the first reset date at  $T_0$ , where  $t < T_0$ , and the last payment date at  $T_n$  is given by

$$s_{0,n}(t) = \frac{P(t, T_0) - P(t, T_n)}{\sum_{i=1}^n \delta_{fi} P(t, T_{fi})} \quad (1)$$

where  $P(t, T_i)$  is a zero-coupon bond's value at time  $t$  with maturity  $T_i$ , and  $\delta_{fi}$  is the payment frequency for the floating rate.



**Figure 4: Cash flows for a plain vanilla interest rate swap.**

Let the fixed leg payments,  $F_1 = F_2 = \dots = F_n$ , occur at the dates  $T_1 < T_2 < \dots < T_n$ , and the floating leg payments,  $L_1, L_2, \dots, L_n$ , occur at the dates  $T_1^f < T_2^f < \dots < T_n^f$ , where the floating legs are re-adjusted at reset dates  $T_0^f < T_1^f < \dots < T_{n-1}^f$ . This means that each floating leg payment is known at the reset date, which is one period prior to the payment date. Note that the fixed leg and floating leg are not required to have the same payment frequency. (Hull, 2012)

The value of an IRS is the difference between the present value of the fixed cash flows,  $PV_{fixed}$  and the expected present value of the implied floating rate payments,  $PV_{floating}$ . The value of the payer and that of the receiver are respectively denoted by

$$V_P = PV_{floating} - PV_{fixed}$$

$$V_R = PV_{fixed} - PV_{floating}$$

Just as for a FRA, the value of an IRS is zero at inception  $t = 0$ , thus  $s_K = s_{0,n}(0)$ . (Hull, 2012)

## 3.2 Credit Default Swaps

This subsection outlines the definition and the nature of the credit default swap (CDS). We start by defining a CDS and discuss its origin and historical role in the financial market. In Subsection 3.2.1 we describe the construction of a CDS and how to quantify a CDS spread.

A credit default swap is a credit derivative that provides an insurance against a credit loss in the event of a default by a specific firm, the so-called reference entity. With a CDS, the protection seller is obligated to cover the protection buyer's credit loss if the reference entity defaults before the maturity of the CDS. Credit default swap was solely an OTC contract, but after the 2008 financial crisis, regulators started to force financial institutions to trade credit derivatives via clearing houses or so-called central counterparties. The CDS is the most liquid single-name credit derivative and is therefore widely used to find the implied default probability of the firms. (Herbertsson, 2015)

JP Morgan bankers pioneered the CDS market by providing a hedging alternative for risk managers to efficiently manage credit risk without moving the asset position (Alloway, 2015). However, traders also saw large opportunities to use CDS to speculate on the future performance of the credit market. By mid-2000, traders and banks used CDS to hedge and speculate on the future value of mortgage securities. When the housing market crashed in 2007, many large financial institutions, or CDS sellers, were indebted up to hundreds of billions of US dollars (Yoon, 2011).

### 3.2.1 Construction and valuation of CDS

Herbertsson (2015), McNeil (2005) and Lando (2004) describe a CDS's structure as follows: Company  $C$ , the so-called reference entity, issues bonds that have the default time  $\tau_C$ . The protection buyer  $A$  buys the protection from  $B$ , the protection seller, on the notional  $N$  against the potential credit loss of bonds issued by  $C$  within the coming  $T$  years.  $B$  promises to cover the credit loss if  $C$  defaults. As a compensation,  $A$  pays  $B$  a fee, usually a quarterly premium, that equals to  $\frac{R(T)N}{4}$  at time  $0 < t_1 < t_2 < \dots < t_{nT} = T$ , where  $R(T)$  is the so-called CDS spread on obligor  $C$  up to time  $T$ . The quarterly premium is paid up to

maturity  $T$ , or until  $C$  defaults at time  $\tau_C$ , whichever comes first. If  $C$  defaults before maturity,  $\tau_C < T$ , then  $B$  pays  $A$  the guarantee  $N(1 - \phi)$ , where  $\phi$  is the recovery rate of the notional value of bonds issued by  $C$ . If the default happens between the quarterly premium payment dates,  $A$  pays  $B$  the accrued premium payment up to  $\tau_C$ , which is the amount  $A$  owes  $B$  for covering the period from the last quarterly payment to the default time  $\tau_C$ . Figure 5 illustrates the payment streams in the case of no default and in the case of default for a CDS with maturity  $T$ .



**Figure 5: (t.l.): Cash flows from A to B if C does not default. (t.r.): Cash flows between A to B if C defaults. .**

The CDS spread  $R(T)$  is settled so that the expected present value of  $A$ 's and  $B$ 's cash flows are equal at inception and is expressed as:

$$R(T) = \frac{\mathbb{E}[1_{\{\tau_C \leq T\}} D(\tau_C)(1 - \phi)]}{\sum_{n=1}^{nT} \mathbb{E}[D(t_n) \delta_n 1_{\{\tau_C > t_n\}} + D(\tau_C)(\tau_C - t_{n-1}) 1_{\{t_{n-1} < \tau_C \leq t_n\}}]}$$

where  $\delta_n = t_n - t_{n-1}$ ,  $1_{\{\tau_C \leq T\}}$  is an indicator function taking the value of 1 if the default time  $\tau_C$  happens before or at the maturity  $T$ , and zero otherwise. The discount factor  $D(t)$  and the short term risk-free rate  $r_t$  at time  $t$  are explained in Subsection 3.3. Furthermore, the expectations are under risk-neutral measure, which exist by the assumption of arbitrage free theory. The nominator is the so-called default leg, representing the expected cash flow from  $B$  to  $A$ , and the denominator is the premium leg, the expected cash flow from  $A$  to  $B$ . The term  $D(\tau_C)(\tau_C - t_{n-1}) 1_{\{t_{n-1} < \tau_C \leq t_n\}}$  in the denominator is the accrued premium that  $A$  pays  $B$  in the case of default, as described earlier. (Herbertsson, 2015)

If we assume a constant recovery rate and  $r_t$  is a deterministic function of time,  $r_t = r(t)$ , then  $R(T)$  is given by



$$R(T) = \frac{(1 - \phi) \int_0^T D(t) f_{\tau_C}(t) dt}{\sum_{n=1}^{4T} \left( D(t_n) \frac{1}{4} (1 - F(t_n)) + \int_{t_{n-1}}^{t_n} D(s) (s - t_{n-1}) f_{\tau_C}(s) ds \right)} \quad (2)$$

where  $F(t_n) = \mathbb{P}[\tau_C \leq t_n]$  i.e. the default probability at time  $t_n$ ,  $f_{\tau_C}(t)$  is the density of default time  $\tau_C$ , and  $D(t) = \exp(\int_0^t r(s) ds)$ . The assumption that  $r_t = r(t)$  is deterministic implies that the interest rate is independent of the default time  $\tau_C$ . (Herbertsson, 2015)

Herbertsson (2015) derives a semi-closed formula for  $R(T)$  from Equation (2), by making the following two additional assumptions:

- a. Ignore the accrued premium term.
- b. If the default is in the period  $\left[\frac{n-1}{4}, \frac{n}{4}\right]$ , the loss is paid at time  $t_n = \frac{n}{4}$ , i.e. at the end of the quarter, instead of immediately at  $\tau_C$ .

As a result, Equation (2) is simplified to the following:

$$R(T) = \frac{(1 - \phi) \sum_{n=1}^{4T} D(t_n) (F(t_n) - F(t_{n-1}))}{\sum_{n=1}^{4T} D(t_n) (1 - F(t_n)) \frac{1}{4}}. \quad (3)$$

### 3.3 Interest Rate Theory

The first part of this subsection discusses the concept of risk-free rate, its role in derivative valuation and the proxies used by practitioners before and after the 2008 financial crisis. The two last parts discuss and describe the discount factor and the yield curve respectively.

#### 3.3.1 The risk-free rates

The so-called risk-free rate refers to a return rate that an investor expects from a secure investment over a specific period of time. A risk-free rate is required as an input for all derivative valuation models, such as the Black-Scholes model. Consider a risk-free deposit account that has a value of  $B(0) = 1$ , at time 0, then its value at  $t > 0$  is given by

$$B(t) = \exp\left(\int_0^t r_s ds\right).$$

where  $r_f$  is the deterministic risk-free rate. (Brigo & Mercurio, 2006)

Financial institutions traditionally used the Libor rate as the proxy for risk-free rate, but after the 2008 financial crisis, many have switched to overnight indexed swap (OIS) rate as the proxy. Both the Libor rate and the OIS rate are further discussed in the next two subsections. Figure 6 shows the historical values of Euribor-OIS spreads, which are used to measure stress in financial markets. In normal circumstances the spread is about 10 bps, but during the 2008 financial crisis, it rose sharply, due to the increase in the Libor rates. The rise in the spread during the crisis implies that the Libor rates are in fact not risk-free. (Hull, 2012)

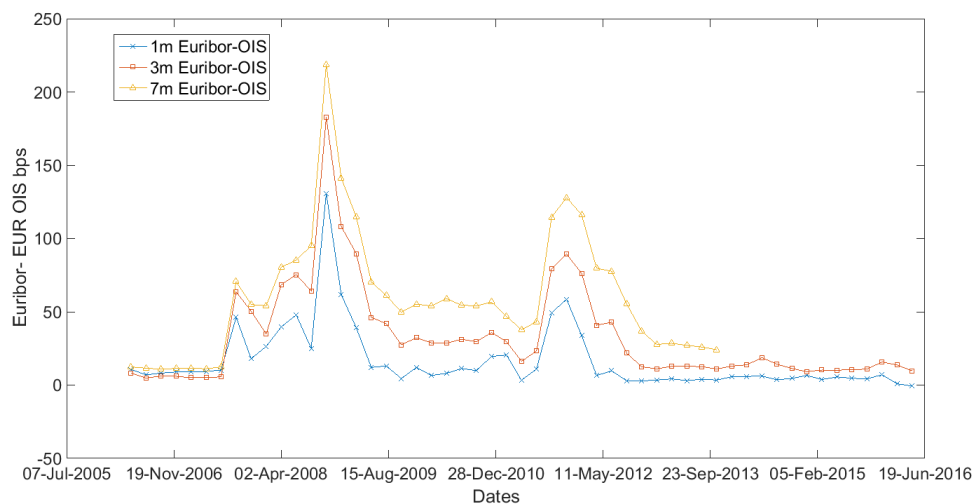


Figure 6: The Euribor-OIS spreads in 2006 -2010. Source: Datastream

### **Libor rate**

The London Interbank offered rate (Libor) is a reference interest rate that is designed to reflect the rate at which banks are prepared to make large wholesale deposits without collateral with at least AA rated banks. These loans have specific maturities, between overnight to one year. The Libor rate is calculated daily by the British Bankers' Association, as the average of the estimated rates for those wholesales deposits, with the exclusion of the highest and lowest quartile rates. The provided rates are for major currencies in 15 different maturities. (Hull, 2012)

The market Libor rate  $L(t, T)$  is discretely compounded and can be used to price a discretely compounded zero-coupon bond with maturity  $T$  at time  $t$  by

$$P(t, T) = \frac{1}{1 + L(t, T)\delta(t, T)}$$

where  $\delta(t, T)$  is the year fraction between  $t$  and  $T$ , typically calculated by the actual/360 day convention. (Brigo & Mercurio, 2006)

### ***Overnight indexed swap rate***

The overnight indexed swap (OIS) rate is the fixed rate of an OIS. An OIS is an interest rate swap where a fixed rate for a period, e.g. 1 month, is exchanged for the geometric average<sup>2</sup> of overnight rates during that period. These overnight rates, which are often the central bank's target rates, are the rates that banks lend to and borrow from each other overnight to fulfill their liquidity needs for the day. (Hull, 2012)

### **3.3.2 Discount factor**

As defined by Brigo and Mercurio (2006) the so-called discount factor is required to adjust the time value of money, because one unit of currency today is greater than one unit of currency tomorrow. To equate the value of currency between time  $t$  and future time  $T$  the discount factor is defined as

$$D(t, T) = \frac{B(t)}{B(T)} = \exp\left(-\int_t^T r_s ds\right)$$

which is also equal to a zero-coupon bond  $P(t, T)$  with a face value of 1. Brigo and Mercurio (2006) also note that if the risk-free rate is stochastic then  $D(t, T)$  is also stochastic. Therefore,  $P(t, T)$  can be viewed as the expectation of a random variable  $D(t, T)$  under a particular probability measure, conditional on full information available at time  $t$ ,  $\mathcal{F}_t$ . Under the risk neutral probability measure  $\mathbb{Q}$ , the zero-coupon bond  $P(t, T)$  is therefore defined as

$$P(t, T) = E^{\mathbb{Q}}\left[\exp\left(-\int_t^T r_s ds\right) \middle| \mathcal{F}_t\right].$$

---

<sup>2</sup> Geometric mean is not a simple average, it refers to the  $n^{th}$  root of the product of  $n$  rates which is denoted as  $(\prod_{n=1}^k x_n)^{1/k}$

### 3.3.3 Yield curve

The yield curve plots the rates of financial instruments, such as bonds or swaps that have similar risk-level, against their maturities. The yield curve is also known as the current term structure of interest rates. A yield curve is composed of risk-free rates that are needed to value derivatives. Additionally, a yield curve provides an indication of the future rates and can therefore imply the future prospects of the financial market (Hull, 2012). In this thesis, the yield curves are built using the OIS spot rates.

## 4 Credit Risk Models

In this section, we discuss the two most prevalent credit risk models which are structural models (firm value approach) and intensity-based models (reduced-form approach). Subsection 4.1 describes the structural model and Subsection 4.2 defines and outlines the nature of the intensity-based model, which is used in this thesis. Lastly, Subsection 4.3 introduces dependency modeling in credit risk.

### 4.1 Structural Model

This subsection introduces one of the most popular structural models, the Merton model. The structural model infers the default probability by modeling the future evolution of the firm's value and capital structure, in which credit events are triggered when the firm's value drops below a certain threshold. (T.R & Rutowski, 2004)

The set up in the Merton model is the same as in the standard Black-Scholes model, i.e. the market is analyzed with continuous trading which is frictionless and competitive<sup>3</sup>. The value of the company's assets  $V_t$  follows a geometric Brownian motion (GBM). Under the risk-neutral measure  $\mathbb{Q}$ , the dynamics of  $V_t$ , or the market value of the firm, is given by

$$V_t = V_0 e^{\left(r - \frac{1}{2}\sigma^2\right)t + \sigma W_t^{\mathbb{Q}}}$$

---

<sup>3</sup> 1) Agents are price takers 2) no transaction costs and taxes 3) no restriction on short selling and borrowing 4) borrowing and lending are done on a risk-free interest rate

where  $r$  denotes the risk-free rate,  $\sigma$  is the annual asset volatility and  $W_t$  is a random variable that follows a Wiener process. (Lando, 2004)

The Merton model assumes the firm as a limited liability company that has issued two types of claims: equity and debt. The debt is a zero-coupon bond, with maturity  $T$  and face value  $D$ . Let  $S_t$  and  $B_t$  denote the value of the equity and the bond at time  $t$  for  $0 \leq t \leq T$ , then the company's corporate structure is given by:

$$\text{Assets} = \text{equity} + \text{debt}, \quad \text{or} \quad V_t = S_t + B_t.$$

Due to the firm's limited liability structure the equity owners have the incentive and option to abandon the firm if  $V_T < D$  at maturity  $T$ . However, if  $V_T > D$ , the owners pay the full debt to the bondholders and keep the residual. The payoff structure of the equity owners and the debt holders can therefore be described in the following way:

$$B_T = \min(D, V_T) = D - \max(D - V_T, 0)$$

and

$$S_T = \max(V_T - D, 0).$$

Then the two claims can be priced in terms of Black-Scholes option pricing model as

$$S_t = C^{BS}(V_t, D, T - t, \sigma, r)$$

and

$$B_t = De^{-r(T-t)} - P^{BS}(V_t, D, T - t, \sigma, r)$$

where  $C^{BS}(V_t, D, T - t, \sigma, r)$  is the price of a call option and  $P^{BS}(V_t, D, T - t, \sigma, r)$  is the price of a put option. One of the drawbacks of this model is that the credit event can only occur at the maturity of the debt, not prior to the maturity date. (Lando, 2004)

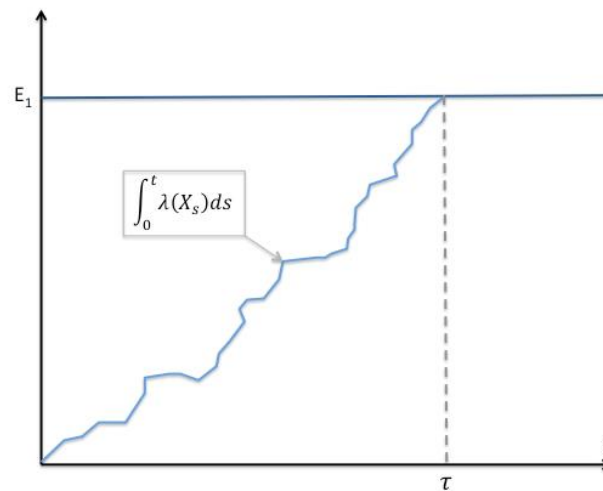
## 4.2 Intensity-Based Models

This subsection describes the structure of an intensity-based model and how to use this model to calibrate the default intensity from the market CDS spread.

In an intensity-based credit risk model, the default probability is seen as an unexpected event and is modeled by a stochastic process. The intensity-based models are based on the default intensity  $\lambda_t$  of the default time  $\tau$ , in which  $\lambda_t$  is modeled as the first jump of a point-process. Let  $(X_t)_{t \geq 0}$  be a  $d$ -dimensional stochastic process which models the underlying economic factors that drive the default time  $\tau$ , where  $d$  is an integer and  $\mathcal{G}_u^X$  is the filtration generated by  $X_t$  up to time  $t$ . If we let  $\lambda: \mathbb{R} \rightarrow [0, \infty)$  be a function from  $\mathbb{R}^d$  to  $\mathbb{R}^+$ , then we define the default intensity  $\lambda_t$  of the default time  $\tau$  as  $\lambda_t = \lambda(X_t)$ . Given that the random level  $E_1 \sim \exp(1)$  is the default threshold, then the random variable  $\tau$  is given by

$$\tau = \inf \left\{ t \geq 0 : \int_0^t \lambda(X_s) ds \geq E_1 \right\}. \quad (4)$$

The random variable  $\tau$  is the first time the increasing process  $\int_0^t \lambda(X_s) ds$  reaches the default threshold  $E_1$  (Lando, 2004). The construction in Equation (4) is visualized in Figure 7.



**Figure 7: The random variable  $\tau$  is the first time the increasing process reaches the random level  $E_1$ , the default threshold (Herbertsson, 2015, p. 39)**

From the above definition of the default time  $\tau$  in Equation (4), McNeil (2005), Herbertsson (2015) and Lando (2004) prove that the survival probability at time  $t$  is given by

$$\begin{aligned}
\mathbb{P}[\tau > t] &= \mathbb{E}[1_{\{\tau > t\}}] \\
&= \mathbb{E}[\mathbb{P}[\tau > t \mid \mathcal{G}_u^X]] = \mathbb{E}\left[\mathbb{P}\left[\int_0^t \lambda(X_s) ds < E_1 \mid \mathcal{G}_u^X\right]\right] \\
&= \mathbb{E}\left[1 - \mathbb{P}\left[E_1 \leq \int_0^t \lambda(X_s) ds \mid \mathcal{G}_u^X\right]\right].
\end{aligned}$$

Since  $E_1$  is independent of  $\mathcal{G}_u^X$  we therefore get

$$\mathbb{P}[\tau > t] = \mathbb{E}\left[e^{-\int_0^t \lambda(X_s) ds}\right]. \quad (5)$$

According to Lando (2004), the default intensity can be constructed as any kind of non-negative stochastic process. If the default intensity from Equation (5) is a constant, the survival probability is given by

$$H(t) = \mathbb{P}[\tau > t] = e^{-\lambda t}$$

since the integral of a constant is defined as:  $\int_0^t \lambda(X_s) ds = \lambda \cdot t - \lambda \cdot 0 = \lambda t$ . The corresponding default probability is

$$F(t) = \mathbb{P}[\tau \leq t] = 1 - \mathbb{P}[\tau > t] = 1 - e^{-\lambda t}.$$

In this thesis we only use the constant default intensity to compute the default probability.

#### 4.2.1 The default intensity and CDS spreads

As mentioned in Subsection 3.2, CDSs are highly liquid instruments and therefore they are very useful when calibrating the risk-neutral default probability. The easiest way to calibrate the default intensity is to assume it as a constant  $\lambda$ . With that assumption, Herbertsson (2015) derived and simplified Equation (3) Subsection 3.2.1 to

$$R(T) = 4(1 - \phi) \left(e^{\frac{\lambda}{4}} - 1\right)$$

and if  $\lambda$  is “small”,  $R(T)$  can be simplified further to:

$$R(T) \approx (1 - \phi)\lambda.$$

Using the market CDS spread  $R_M(T)$  and an estimated recovery rate  $\phi$ , the default intensity is calibrated by

$$\lambda \approx \frac{R_M(T)}{(1 - \phi)} \quad (6)$$

where Equation (6) is the so-called credit triangle.

### 4.3 Dependency modeling in credit risk

In the following subsection, we give a short description of the copula concept, followed by a detailed discussion of the Gaussian copula, which is used in this thesis, to model WWR when computing  $CV A_{IRS}$ .

#### 4.3.1 Copula

One way to model dependence is to use copulas. A copula is a function that links several marginal distributions which are uniform on an interval  $[0,1]$  to form a cumulative multivariate distribution function. Sklar's theorem proves that any multivariate distribution function can be written as a copula, and that a copula representation is unique when the marginal distributions are continuous. With the copula function we can model the dependence by specifying the marginal function and a copula, rather than specifying the multivariate distribution. (O'Kane, 2008)

O'Kane (2008) states that in the form of credit modeling  $F_j(t_j)$  is the probability of obligor  $j$  defaulting before time  $t_j$ . Then the  $M$ -dimensional copula function  $C$  with  $M$  uniform marginals ( $\tau_j \leq t_j$ ) can be described as

$$C(F_1(t_1), F_2(t_2), \dots, F_M(t_M)) = \Pr(\tau_1 \leq t_1, \tau_2 \leq t_2, \dots, \tau_M \leq t_M).$$

When modeling portfolio credit risk a copula  $C$  is a convenient tool, because the obligor's default probability curve  $F_j(t_j)$  can be calibrated directly from the market CDS curve and the multivariate dependence of the default times is specified by only few parameters. More on this see McNeil (2005), Schönbucher (2003) and O'Kane (2008).

#### 4.3.2 Gaussian copula

There are numerous types of copulas that can be broadly categorized into either a one-parameter or a two-parameter copula. The one-parameter copulas are classified into the Archimedean copula family and the Gaussian copula, where the Gaussian copula is the most applied copula in finance (Meissner, 2014). The



Gaussian copula model assumes normal distribution (Gaussian distribution), with zero mean and unit variance and can be described as:

$$C^{GC}(F_1(t_1), F_2(t_2), \dots, F_M(t_M)) = \Phi(\Phi^{-1}(F_1(t)), \Phi^{-1}(F_2(t)), \dots, \Phi^{-1}(F_M(t)))$$

where  $\Phi(\cdot)$  is the cumulative normal distribution and its inverse is  $\Phi^{-1}(\cdot)$  (O'Kane, 2008).

O'Kane (2008) derives the above formula by first defining  $X_j$ , the default indicator for obligor  $j$ , as a function of the common factor  $Z$ , which can represent the economic environment, and the idiosyncratic factor  $Y_j$ . Thus,  $X_j$  can be described as

$$X_j = \sqrt{\rho}Z + \sqrt{1 - \rho}Y_j$$

where  $\{Y_1, Y_2, \dots, Y_M\}$  is an independent and identically distributed standard normal sequence,  $Z$  is a standard normal random variable which is independent of  $Y_j$ , and  $\rho$  is the default correlation factor. One can easily show that  $Cov(X_j, X_l) = \rho$  when  $j \neq l$  (see Herbertsson (2015)). If  $\Phi^{-1}(F_j(t))$  is the default threshold, then the default time for each obligor  $j$  is defined as:

$$\tau_j = \inf\{t > 0: X_j \leq \Phi^{-1}(F_j(t))\}.$$

As a result, the default probability equals to

$$\mathbb{P}[\tau_j \leq t] = \mathbb{P}[X_j \leq \Phi^{-1}(F_j(t))] = \mathbb{P}[\sqrt{\rho}Z + \sqrt{1 - \rho}Y_j \leq \Phi^{-1}(F_j(t))].$$

Given that the default times  $\tau_j$ , are conditionally independent of the given  $Z$  we get:

$$\mathbb{P}[\tau_j \leq t | Z] = \mathbb{P}\left[Y_j \leq \frac{\Phi^{-1}(F_j(t)) - \sqrt{\rho}Z}{\sqrt{1 - \rho}} \mid Z\right]$$

and since  $Y_j \sim N(0,1)$  and independent of  $Z$ , the conditional default probability is given by

$$\mathbb{P}[\tau_j \leq t | Z] = \Phi \left( \frac{\Phi^{-1}(F_j(t)) - \sqrt{\rho}Z}{\sqrt{1-\rho}} \right).$$

See O’Kane (2008) and Herbertsson (2015).

## 5 Modeling Credit Value Adjustment

In this section we extend the discussion on unilateral CVA, and present the relevant models that are essential to price CVA. Subsection 5.1 elaborates on the unilateral CVA formula, while Subsection 5.2 explains how to calculate credit value adjustment for an IR-swap.

### 5.1 Unilateral CVA Formula

In this subsection unilateral CVA is further quantified. As mentioned earlier, a unilateral credit value adjustment is the difference between risk-free and risky value of a bilateral contract where one of the counterparties is assumed to be default-free. If we assume counterparty *A* as default-free, then the unilateral CVA formula seen from *A*’s perspective is given by

$$CVA(t, T) = \mathbb{E}^{\mathbb{Q}}[(1 - \phi)1_{\{\tau \leq T\}} \max(V(\tau, T), 0) D(t, \tau) | \mathcal{F}_t] \quad (7)$$

where,

- $V(\tau, T)$  is the value of the risk-free contract at default time  $\tau$
- $D(t, \tau)$  is the discount rate between  $t$  and default time  $\tau$
- $\mathbb{E}^{\mathbb{Q}}$  is the expected value under the risk-neutral measure
- $1_{\{\tau \leq T\}}$  denotes the indicator function, taking the value of 1 if the default time is before the maturity  $T$ , and zero otherwise
- $\mathbb{E}^{\mathbb{Q}}[1_{\{\tau \leq T\}}] = \mathbb{P}[\tau \leq T]$  is the probability that the counterparty defaults before a maturity date  $T$
- $\phi$  denotes the recovery rate of the defaultable counterparty in the bilateral contract in the case of default
- $\mathcal{F}_t$  is the full information available at time  $t$ .

Furthermore, Equation (7) assumes the contract has a risk-free closeout<sup>4</sup>, no collateral, no re-hypothecation<sup>5</sup> and no funding cost. (Brigo, et al., 2013)

## 5.2 CVA for Interest Rate Swaps

In this subsection we describe how to use swaptions' prices to compute the CVA for an IRS, hereafter referred to as  $CVA_{IRS}$ .

A swaption is an option on a swap, which gives the holder the right to enter into a swap contract at a future time  $T_0$ , with a predetermined rate  $s_K$ . This swap has its first payment date at time  $T_1$  and lasts up to  $T_n$ . Like a swap, a swaption has two types of holders, payer and receiver. A payer swaption, with maturity  $T_0$ , notional  $N$ , forward swap rate  $s_{0,n}(t)$  (see Equation (1)) and where  $t < T_0 < T_n$ , can be priced at time  $t$  using Black's model, which is discussed in Appendix A.1, by

$$V_P(t, T_0, T_n) = N \cdot X(t, T_1, T_n) \cdot (s_{0,n}(t) \Phi(d_1) - s_K \Phi(d_2)) \quad (8)$$

and for the receiver of the same contract

$$V_R(t, T_0, T_n) = N \cdot X(t, T_1, T_n) \cdot (s_K \Phi(-d_1) - s_{0,n}(t) \Phi(-d_2)) \quad (9)$$

where

$$d_1 = \frac{\ln\left(\frac{s_{0,n}(t)}{s_K}\right) + \frac{1}{2}\sigma^2(T_0 - t)}{\sigma\sqrt{T_0 - t}}$$

$$d_2 = d_1 - \sigma\sqrt{T_0 - t}.$$

Additionally,  $\sigma$  is the volatility of the underlying swap and  $\Phi(\cdot)$  is the cumulative normal distribution. The so-called annuity,  $X(t, T_1, T_n)$ , replaces the zero-coupon discounting and is found by

---

<sup>4</sup> The value, when contract is terminated, is discounted at a risk-free rate.

<sup>5</sup> Clients who permit re-hypothecation of their collateral may be compensated either through a lower cost of borrowing or a rebate on fees (Investopedia).

$$X(t, T_1, T_n) = \sum_{i=0}^{n-1} \delta_{i+1} P(t, T_{i+1}) \quad (10)$$

where  $\delta_{i+1}$  is the time fraction between  $T_i$  and  $T_{i+1}$  and  $P(t, T_{i+1})$  is a zero-coupon bond at time  $t$  with maturity  $T_{i+1}$ . This type of discounting leads the swaption to be valued under a risk-neutral annuity measure (Cerny & Witzany, 2015). A risk-neutral annuity measure is a forward measure, which is useful when pricing random interest rates under the risk-neutral measure. It is also used to determine the forward swap rate when pricing swaptions (Privault, 2013).

When computing the  $CVA_{IRS}$  we make two assumptions:

- When the default time  $\tau$  occurs in the interval  $[T_i, T_{i+1}]$  the loss will be paid at time  $T_{i+1}$ .
- The recovery rate  $\phi$  is deterministic.

As stated in Brigo, Morini and Pallavicini (2013) the  $CVA_{IRS}$  is approximately the sum of swaptions prices with maturities at each payment date  $(T_1, T_2, \dots, T_n)$ , where each swaption is weighted by the probabilities that the entity will default in the interval  $[T_i, T_{i+1}]$ . For each swaption, there is a new forward swap rate,  $s_{i+1,n}(t)$  that is calculated by Equation (1). The  $CVA_{IRS}$  for both receiver and payer can be described by

$$CVA_{IRS}(t, T_n) \approx (1 - \phi) \sum_{i=0}^{n-1} \mathbb{E}^{\mathbb{Q}}[1_{\{T_i < \tau \leq T_{i+1}\}} V(t, T_{i+1}, T_n) | \mathcal{F}_t] \quad (11)$$

where  $V(t, T_{i+1}, T_n)$  is given by either Equation (8) or Equation (9) depending on whether the swap is a payer or a receiver.

## 6 A Semi-Analytical Model for CVA of IRS under WWR

In this section we outline the model introduced in Cerny and Witzany (2015) for computing  $CVA_{IRS}$  under wrong way risk. This is the model implemented in our numerical studies given in Section 7.

Cerny and Witzany (2015) derive a semi-analytical formula to value the  $CVA_{IRS}$ , taking into account WWR. The authors examine the WWR between the underlying interest rates of the swap and the default time using the Gaussian copula model, which is explained in Subsection 4.3.2.

A major advantage of the model of Cerny and Witzany (2015) is that it is semi-analytical, and thus avoids Monte Carlo simulations, which can be time-consuming and computationally burdensome. Furthermore, the correlation coefficient in the model of Cerny and Witzany (2015) is between the levels of interest rates and default time, which captures the dynamic relationship between these two factors. Cerny and Witzany believe that higher interest rates may have a negative impact on the market economically; hence more corporate defaults in the short term. On the contrary, in the long run higher interest rates could also produce lower default rates as it induces lower target leverage across all firms (Gonzalez-Aguado and Suarez, 2013). The instantaneous correlation that is used in other studies such as Brigo and Pallavicini (2007), captures only how the interest rates and default time move together, not how they affect each other's levels.

The model of Cerny and Witzany (2015) is based on the fact that the CVA of an interest rate swap can be expressed using swaption prices, as is shown in Equation (11). As described in Section 5 when valuing a swaption, the zero-coupon bond discounting is replaced by annuity, leading the swaption on an IRS to be valued under the risk-neutral annuity measure. To be consistent, Cerny and Witzany (2015) define the survival probability function, which has an exponential distribution, with respect to risk-neutral annuity measure as

$$H(t) = e^{-ht}$$

where  $h$  is the constant default intensity. However, Cerny and Witzany do not state how the default intensity under the risk-neutral annuity measure is derived. In practice the default intensity of CDS spreads are calibrated from the market under the risk-neutral measure and not under the risk-neutral annuity measure.

The forward swap rate  $s_{0,n}(T_0)$ , with  $T_0 > 0$ , is defined as:

$$s_{0,n}(T_0) = s_{0,n}(0) \exp \left\{ -\frac{\sigma^2 T_0}{2} + \sigma \sqrt{T_0} Y \right\}$$

where  $Y$  is a standard normal random variable,  $s_{0,n}(0)$  is the forward swap rate at time 0,  $\sigma$  is the volatility of the underlying swap and  $T_0$  is the exercise date of the swaption. The default time distribution is defined as:

$$\tau = H^{-1}(\Phi(-Z)).$$

where  $Z$  is a standard normal random variable,  $H^{-1}(\cdot)$  is the inverse of the survival probability function and  $\Phi(\cdot)$  is the cumulative normal distribution.

Both  $Y$  and  $Z$  are decomposed into a common systematic factor  $U$ , and different idiosyncratic factors  $\varepsilon_1, \varepsilon_2$  respectively.

$$Y = aU + \sqrt{1 - a^2} \varepsilon_1, \quad a \in [-1, 1].$$

$$Z = bU + \sqrt{1 - b^2} \varepsilon_2, \quad b \in [-1, 1].$$

The parameters  $U, \varepsilon_1, \varepsilon_2$  are assumed to be independent standard normal (Gaussian) random variables. Furthermore, the underlying interest rate of the swap and default time are correlated through the common factor  $U$ , and the correlation coefficient  $\rho = ab$ , where  $|a| = |b|$ .

Under the assumption that the correlation construction between the default time and the underlying interest rate holds, Cerny and Witzany (2015) derive a formula for a risky swaption price  $V_{RS}(t, \tilde{T}, T_0, T_1, T_n)$ , which includes both default time and WWR. The formula is given in Appendix A.2. The notations are the same as in Equation (8) and (9) with the addition of  $\tilde{T}$ , representing a time point to which the counterparty has survived. From the risky swaption prices, Cerny and Witzany derived a semi-analytical formula for  $CVA_{IRS}$  by making the assumptions that the recovery rate is deterministic and the loss will be paid at  $T_{i+1}$  when default time  $\tau$  occurs in  $(T_i, T_{i+1}]$ . The derivation of  $CVA_{IRS}$  in term of risky swaptions prices  $V_{RS}(t, \tilde{T}, T_0, T_1, T_n)$  is as follows

$$\begin{aligned}
CVA_{IRS}(t, T_n) &= (1 - \phi) \left( \sum_{i=0}^{n-1} \mathbb{E}^{\mathbb{Q}}[1_{\{T_i < \tau \leq T_{i+1}\}} V(\tau, T_{i+1}, T_n)^+ D(t, \tau) | \mathcal{F}_t] \right) \\
&\approx (1 - \phi) \left( \sum_{i=0}^{n-1} \mathbb{E}^{\mathbb{Q}}[1_{\{T_i < \tau \leq T_{i+1}\}} V(T_{i+1}, T_{i+1}, T_n)^+ D(t, T_{i+1}) | \mathcal{F}_t] \right) \\
&= (1 - \phi) \left( \sum_{i=0}^{n-1} \mathbb{E}^{\mathbb{Q}}[1_{\{\tau > T_i\}} V(T_{i+1}, T_{i+1}, T_n)^+ D(t, T_{i+1}) | \mathcal{F}_t] \right. \\
&\quad \left. - \mathbb{E}^{\mathbb{Q}}[1_{\{\tau > T_{i+1}\}} V(T_{i+1}, T_{i+1}, T_n)^+ D(t, T_{i+1}) | \mathcal{F}_t] \right) \\
&= (1 - \phi) \left( \sum_{i=0}^{n-1} V_{RS}(t, T_i, T_{i+1}, T_{i+1}, T_n) - V_{RS}(t, T_{i+1}, T_{i+1}, T_{i+1}, T_n) \right) \quad (12)
\end{aligned}$$

where  $V(\tau, T_{i+1}, T_n)$  is the risk-free swaption price from Equation (8) and (9) at default time  $\tau$ ,  $D(t, \tau)$  is the discount rate between time  $t$  and default time  $\tau$ , and  $1_{\{T_i < \tau \leq T_{i+1}\}}$  is the indicator function, taking the value 1 if the default time is in the interval  $[T_i, T_{i+1}]$  and 0 otherwise. In other words,  $CVA_{IRS}$  is approximated as the sum of the differences between two risky swaption prices, which are when the counterparty survives up to time  $T_i$  and when the counterparty survives up to  $T_{i+1}$ . Thus, combining the derivation of the  $CVA_{IRS}$  in Equation (12) with the risky swaption prices formulas in Appendix A.2, the semi-analytical  $CVA_{IRS}$  equation in the presence of WWR for a payer IRS and a receiver IRS are respectively given as,

$$CVA_{P,IRS}(t, T) \approx N \cdot (1 - \phi) \sum_{i=0}^{n-1} X(t, T_{i+1}, T) [s_{i,n}(t) (A_{1,i} - B_{1,i}) - s_K (A_{2,i} - B_{2,i})]$$

$$CVA_{R,IRS}(t, T) \approx N \cdot (1 - \phi)$$

$$\sum_{i=0}^{n-1} X(t, T_{i+1}, T) [s_K (A_{-2,i} - B_{-2,i}) - s_{i,n}(t) (A_{-1,i} - B_{-1,i})] \quad (13)$$

where

$$A_{\pm 1,i} = \int_{-\infty}^{\infty} \exp \left\{ au\sigma\sqrt{T_{i+1}-t} - \frac{a^2\sigma^2(T_{i+1}-t)}{2} \right\} \\ \cdot \Phi \left( \pm \frac{(d_{1,i} + au - a^2\sigma\sqrt{T_{i+1}-t})}{\sqrt{1-a^2}} \right) \\ \cdot \Phi \left( \frac{bu - \Phi^{-1}(1 - H_i(T_i - t))}{1-b^2} \right) \varphi(u) du$$

$$B_{\pm 1,i} = \int_{-\infty}^{\infty} \exp \left\{ au\sigma\sqrt{T_{i+1}-t} - \frac{a^2\sigma^2(T_{i+1}-t)}{2} \right\} \\ \cdot \Phi \left( \pm \frac{(d_{1,i} + au - a^2\sigma\sqrt{T_{i+1}-t})}{\sqrt{1-a^2}} \right) \\ \cdot \Phi \left( \frac{bu - \Phi^{-1}(1 - H_i(T_{i+1} - t))}{1-b^2} \right) \varphi(u) du$$

$$A_{\pm 2,i} = \int_{-\infty}^{\infty} \Phi \left( \pm \frac{(d_{2,i} + au)}{\sqrt{1-a^2}} \right) \Phi \left( \frac{bu - \Phi^{-1}(1 - H_i(T_i - t))}{\sqrt{1-b^2}} \right) \varphi(u) du$$

$$B_{\pm 2,i} = \int_{-\infty}^{\infty} \Phi \left( \pm \frac{(d_{2,i} + au)}{\sqrt{1-a^2}} \right) \Phi \left( \frac{bu - \Phi^{-1}(1 - H_i(T_{i+1} - t))}{\sqrt{1-b^2}} \right) \varphi(u) du$$

$$d_{1,i} = \frac{\ln \left( \frac{s_{i,n}(t)}{s_K} \right) + \frac{\sigma^2(T_{i+1}-t)}{2}}{\sigma\sqrt{T_{i+1}-t}}$$

$$d_{2,i} = d_{1,i} - \sigma\sqrt{T_{i+1}-t}$$

furthermore,  $\varphi(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}$  is the normal pdf density function,  $\Phi$  is the cumulative normal distribution,  $\Phi^{-1}(\cdot)$  is the inverse cumulative normal distribution,  $s_K$  is the predetermined swap rate and  $s_{i,n}(t)$  is the forward swap rate computed using Equation (1). Note that when  $t > 0$  the expression of  $CVA_{IRS}$  in Equation (13) is random, because the forward swap rate  $s_{i,n}(t)$  is random. In this thesis we assume  $t = 0$  at all times, making the formula deterministic.

WWR is outlined in the Cerny and Witzany (2015) as follows: a decrease in the Z parameter corresponds to an increase in the default probability. Then, for an receiver IRS, the WWR is captured when  $\rho > 0$ , since receiver's exposure increases as interest rates decrease. Contrarily WWR is captured for an payer



IRS when  $\rho < 0$ , since its exposure increases as interest rates increase. Table 1 shows in which cases the receiver IRS and payer IRS have WWR and RWR.

**Table 1: WWR and RWR depending on the correlation factor and contract side (Cerny & Witzany, 2015)**

Correlation	IRS Driver	$\tau$ Driver	RWR	WWR
$\rho > 0$	$Y \uparrow$	$Z \uparrow$	Payer IRS	
	$Y \downarrow$	$Z \downarrow$		Receiver IRS
$\rho < 0$	$Y \downarrow$	$Z \uparrow$	Receiver IRS	
	$Y \uparrow$	$Z \downarrow$		Payer IRS

## 6.1 Our adjustment to the model

A major drawback in the Cerny and Witzany (2015) approach is that modeling the default intensity under the risk-neutral annuity measure is unknown. As described in Subsection 4.2.1, the most common way in practice is to calibrate the default intensity for the counterparties using their respective CDS spreads which are obtained from the market. Since the implied intensity from the CDS market is under the risk-neutral measure, we need to make some adjustments to the default intensity.

According to McNeil, et al (2005) in practice, when transforming the default intensity from a risk-neutral measure  $\lambda$  to a physical measure  $\lambda^{\mathbb{P}}$ , one usually assumes that default intensity under both measures belongs to a given parametric family of functions from  $\mathbb{R}^d$  to  $\mathbb{R}^+$  and that  $\lambda = v\lambda^{\mathbb{P}}$ , for some scaling factor  $v > 0$ . In this thesis we perform a similar scaling method to transform the calibrated risk-neutral default intensity  $\lambda$  found by Equation (6) to a risk-neutral annuity measure  $h$ . We multiply  $\lambda$  by a scaling factor, which is called the change of measure parameter  $c$ . Hence, the default intensity under the risk-neutral annuity measure equals to:  $h = \lambda c$ . We assume the change of measure parameters take any positive values, since the real value is unknown and the probability is never negative.

## 7 Numerical Study

This section conducts three different numerical studies using the model of Cerny and Witzany (2015) on CVA of receiver IRS with respect to WWR. Recall that WWR is when the credit exposure and default probability increase simultaneously. Exposure rises for a receiver IRS when the underlying interest rates decline, and the default probability increases when the default time driver decreases, as described in Section 6. Therefore we will only look at positive values for the correlation parameter, measured by  $\rho$ . Furthermore the change of measure parameter is denoted by  $c$  and can take any positive values as mentioned in Subsection 6.1.

Study 1 investigates the behavior of a single  $CVA_{IRS}$  with respect to WWR, and the change of measure parameter. These two quantities are difficult to estimate properly, therefore studying  $CVA_{IRS}$  as a function of WWR and change of measure parameter will give a better understanding of their impact.

- *How will WWR and change of measure parameter affect the value of a single  $CVA_{IRS}$  ?*

Study 2 looks at the running  $CVA_{IRS}$  before, during and after the 2008 financial crisis for five major investment banks. It is interesting to look at the evolvement of  $CVA_{IRS}$  for this period because the realized CVA losses were substantial after this period. Furthermore, this study also looks at the effects by the two uncertain parameters,  $\rho$  and  $c$ , on the running  $CVA_{IRS}$ .

- *How did the  $CVA_{IRS}$  evolve before, during and after the 2008 financial crisis?*

Study 3 investigates the possibility to calculate the  $CVA_{IRS}$  for a heterogeneous portfolio ( $CVA_{heterog}$ ) by regarding it as a homogeneous portfolio ( $CVA_{homog}$ ) since the computation of the latter is faster, easier and much more convenient from a practical point of view. If the difference between  $CVA_{heterog}$  and  $CVA_{homog}$  is small, then financial institutions can calculate portfolio  $CVA_{IRS}$  more efficiently using the homogenous method.

- *Under what conditions will the  $CVA_{IRS}$  on a homogeneous portfolio and the  $CVA_{IRS}$  on a heterogeneous portfolio have a relatively small difference?*

We assume that all obligors hold the same IRS contract, but differ in recovery rates and CDS spreads, which are easily estimated from market data. The correlation parameter is also a factor which differs amongst obligors, but we keep it constant and same for all obligors because it is hard to estimate, as stated by Rosen and Saunders (2012). Hence, we answer the last research question by investigate the following two sub-questions:

- *Do changes in the standard deviation of the recovery rate in a portfolio change the difference between  $CVA_{heterog}$  and  $CVA_{homog}$  ?*
- *Do changes in the standard deviation of the CDS in a portfolio change the difference between  $CVA_{heterog}$  and  $CVA_{homog}$ ?*

Input parameters for all studies are summarized in Appendix B.1.

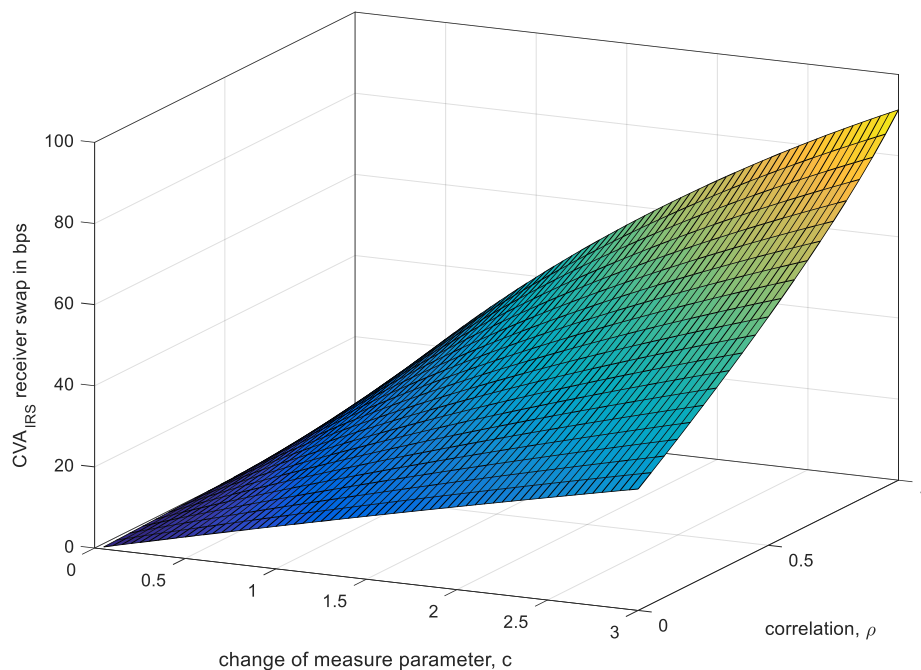
### **Study 1: CVA under WWR and Change of Measure Parameter**

As mentioned in Subsection 2.4, it is difficult to extract the implied correlation from market instruments. Therefore, it is interesting to investigate the behavior of  $CVA_{IRS}$  with respect to different correlation coefficients  $\rho$ . In addition, as mentioned in Subsection 6.1, the change of measure parameter  $c$  can take any positive value, making it an essential parameter to investigate further relative to the behavior of  $CVA_{IRS}$ .

Study 1 and Study 3 use 10-year IRS with settlement date on March 13<sup>th</sup> 2014. This contract has the annual fixed payments based on the contract swap rate,  $s_K = 1.87\%$ , and has the semi-annual payments based on the 6-month Euribor. The annualized volatility,  $\sigma = 23.2\%$ , is estimated using the daily swap rate for a 10-year IRS using six month historical data prior from the settlement date. The zero curve that is used to compute the zero-coupon bond's value in Equation (10) and the forward swap rate in Equation (1) in Subsection 3.1.2, is observed on March 13<sup>th</sup> 2014 and is exhibited in Appendix B.2.

The counterparty of the contract in Study 1 is Citibank and its 5-year CDS spread  $R_M(5)$  for a senior debt is 79 bps on the settlement date; this spread is used to calibrate the risk-neutral default intensity  $\lambda$  using Equation (6) in Subsection 4.2.1. The recovery rate  $\phi$  is kept at zero, which is reasonable since banks have few tangible assets, i.e. nothing is recovered, everything is lost at the default. Furthermore,  $\rho$  is ranged from 0 to 1 with a 0.05 interval. Since we do not know whether the default intensity under the risk-neutral annuity measure is higher or lower than the risk-neutral default intensity, we look at the  $c$  parameter both lower and higher than 1. Therefore, the  $c$  is ranged from 0.05 to 3 with a 0.05 interval.

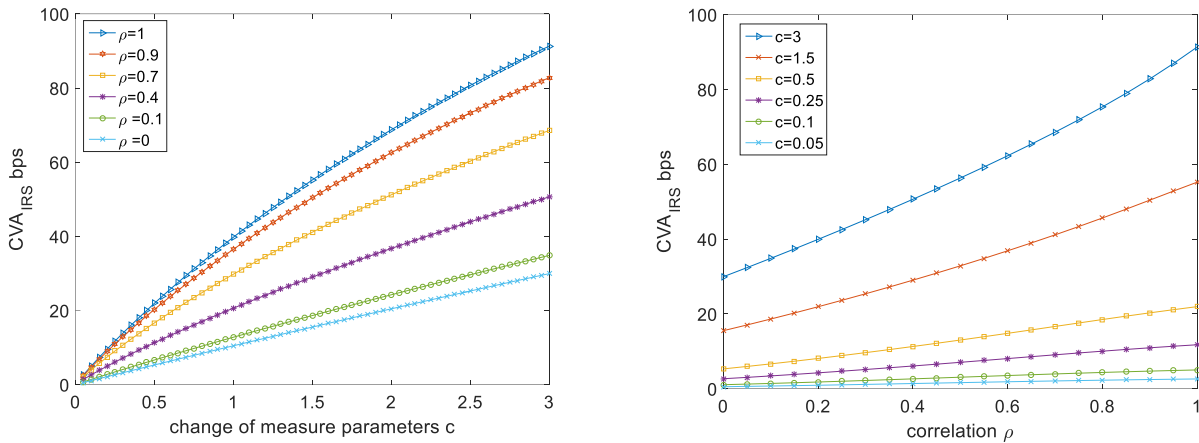
From Figure 8 it is clear that the absolute  $CVA_{IRS}$  increases with both  $\rho$  and  $c$ . In the case of no WWR, i.e.  $\rho = 0$ ,  $CVA_{IRS}$  increases linearly with  $c$ . The  $CVA_{IRS}$  increases more rapidly with  $c$  for higher levels of  $\rho$  but at a decreasing rate, due to the exponential distributed survival probability function.



**Figure 8:  $CVA_{IRS}$  with respect to the change of measure parameter  $c$  and correlation  $\rho$**

In the left subplot in Figure 9 we confirm that for higher levels of  $\rho$  the rate of increase in  $CVA_{IRS}$  is decreasing. The right side of Figure 9 exhibits that when  $c < 1$ , the increase in  $CVA_{IRS}$  with  $\rho$  is small. The figure also shows that when  $c > 1$ ,  $CVA_{IRS}$  increases at an increasing rate with  $\rho$ , which indicates that

correlation has more impact on derivative's CVA when the default probability is higher. This is due to the fact that the correlation is through the levels of interest rate and default time.



**Figure 9: (t.l.) Shows  $CVA_{IRS}$  as a function of  $c$  for different levels of the correlation coefficient  $\rho$ . (t.r.) Shows  $CVA_{IRS}$  as a function of the correlation coefficient  $\rho$  for different levels of  $c$ ,**

Inarguably, there is dependence between banks and interest rates; therefore, the WWR should be taken into account. However, the real level of dependence between banks' default probability and interest rates is difficult to estimate. Table 2 exhibits that ignoring WWR, i.e.  $\rho = 0$ , always results in underestimating the  $CVA_{IRS}$ , regardless of the true level of  $\rho$ . For Citibank, when  $c = 3$  the  $CVA_{IRS}$  is underestimated by 61.37 bps if true level of correlation parameter was  $\rho_{real} = 0.9$ , by 52.89 bps if  $\rho_{real} = 0.7$ , and by 20.70 bps if  $\rho_{real} = 0.4$ .

**Table 2:  $CVA_{IRS}$  in bps of the 10-year interest rate swap as a function of  $\rho$  and  $c$ .**

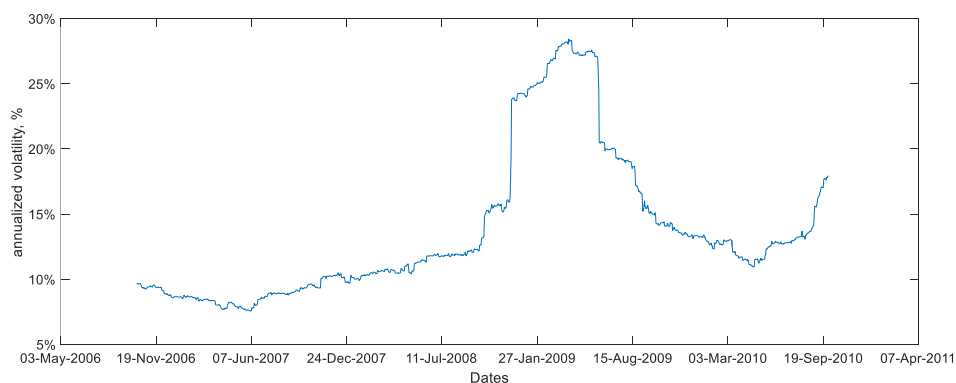
$\rho$	$c = 0.05$	$c = 0.5$	$c = 1.5$	$c = 3$
0	0.5356	5.2969	15.5099	29.9325
0.1	0.7208	6.6367	18.6207	34.8398
0.4	1.3855	11.2824	29.0060	50.6288
0.7	2.0773	16.6403	41.1145	68.5523
0.9	2.4534	20.2755	50.3928	82.7875
1	2.6091	21.9368	55.2018	91.3030

## Study 2: Running CVA

Study 2 investigates the running CVA for the receiver of an interest rate swap, whose counterparties are five major investment banks. The study shows how  $CVA_{IRS}$  varies before, during and after the 2008 financial crisis. In addition, the study also looks at the impact of  $\rho$  and  $c$  on the running  $CVA_{IRS}$ , in which Citibank is the counterparty.

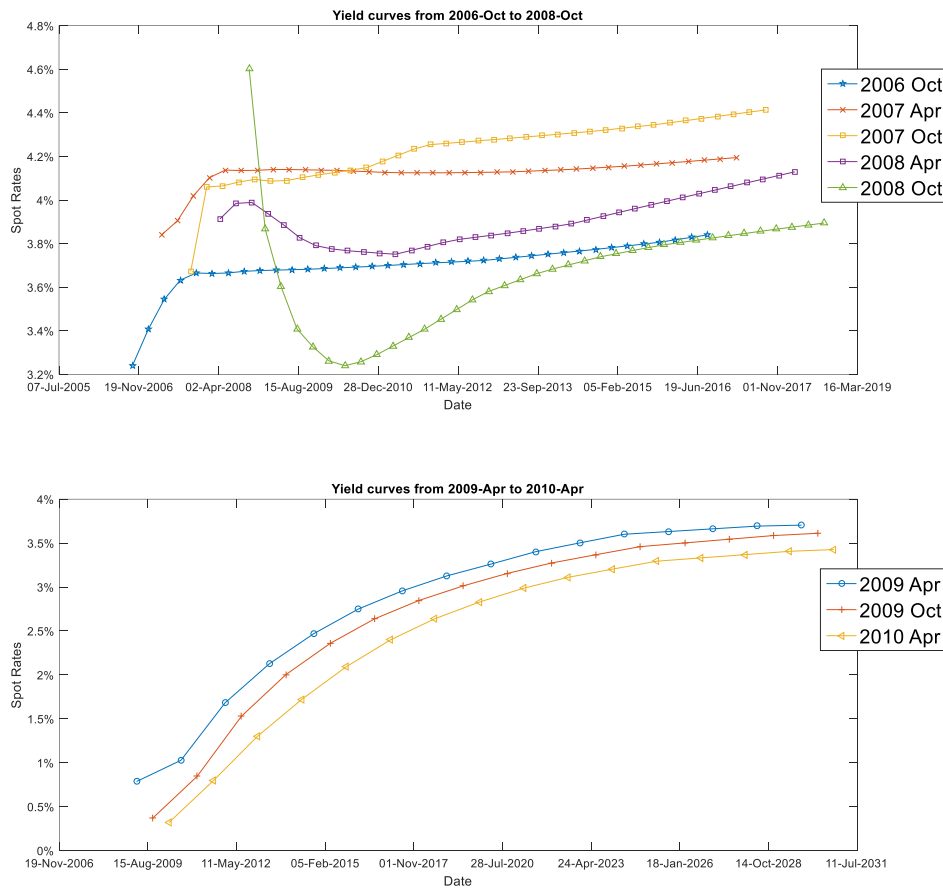
This study evaluates the CVA of 10-year IR-swaps with settlement dates running from October 10<sup>th</sup> 2006 to September 30<sup>th</sup> 2010. The running  $CVA_{IRS}$  is calculated by changing all input factors in Equation (13) daily, except the correlation coefficient  $\rho = ab$  and recovery rate  $\phi$ , which are kept constant at 0.6 and 0 respectively. The daily swap rate  $s_K$  is found using historical daily swap rates of a 10-year IRS for the observed period. The contracts pay annual fixed payments at a swap rate and semi-annual floating payments at rates based on the 6-month Euribor. Furthermore, the survival probability under the risk-neutral annuity measure,  $H(t) = e^{-(\lambda \cdot c)t}$ , is found by daily observed 5-year CDS spreads  $R_M(5)$  for senior debt for all five banks by using Equation (6) together with the chosen value of parameter  $c$ .

The annualized volatility  $\sigma$  is estimated from the running volatility of historical 10-year swap rates. Thus, each observed volatility corresponds to the previous six month interval, which can be seen in Figure 10. For example, the volatility on December 7<sup>th</sup> 2006 is calculated using the daily swap rate observed from June 7<sup>th</sup> 2006 to December 6<sup>th</sup> 2006.



**Figure 10: Running volatility of 10-year swap rates. Each observed volatility corresponds to the previous six month interval**

The zero curve, used to compute the annuity  $X(t, T_{i+1}, T)$  and the forward swap rate  $s_{i,n}(t)$ , also changes daily. For every six months interval, the EUR OIS yield curves in Figure 11 are used as proxies. This assumes that the yield curve observed at the first date of each interval remains the same for the rest of the interval.



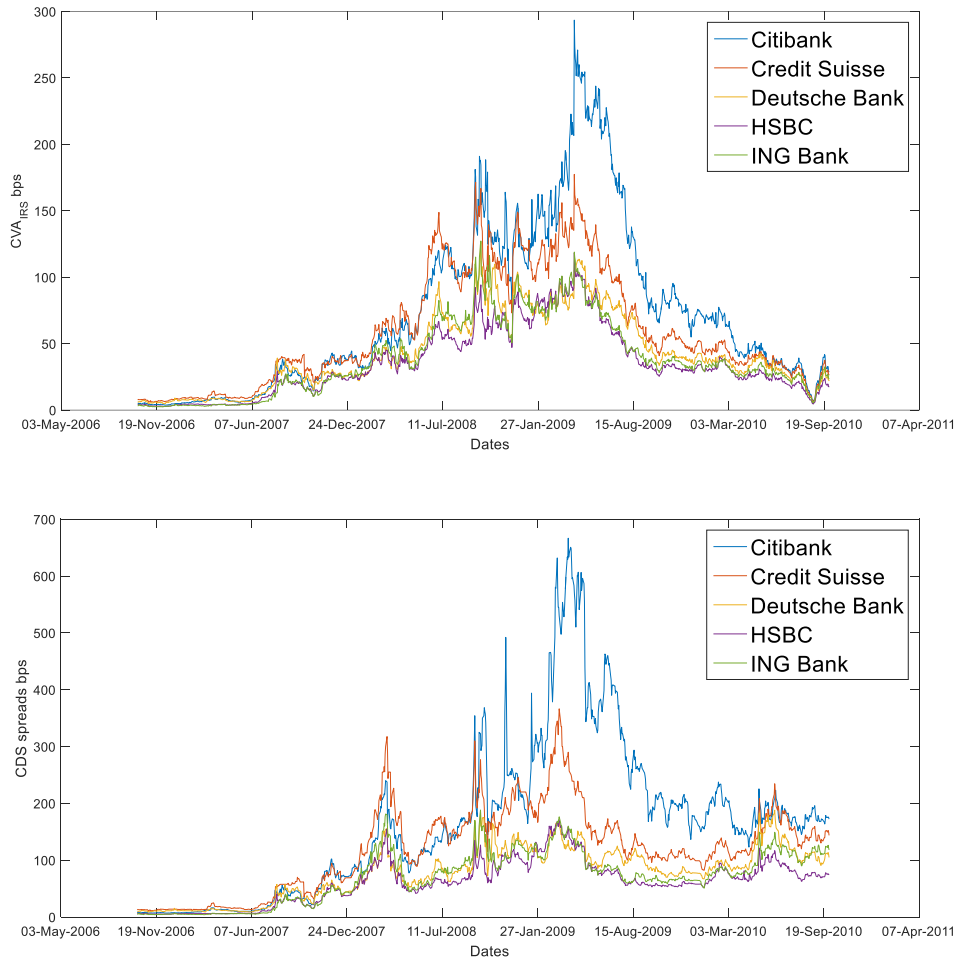
**Figure 11: Yield curves on the upper graph are from October 2006 to October 2008. The lower graph exhibits the yield curves from April 2009 and April 2010**

### Test 2.1: Running CVA

In this computation the change of measure parameter is  $c = 1.5$  and the correlation parameter is  $\rho = 0.6$ . Figure 12 shows that the banks' CDS spreads are relatively low and stable before June 2007, and increase gradually from 2007 and 2008, reaching their highest level for the period around mid-April 2009. Figure 12 also shows that the running  $CVA_{IRS}$  rises simultaneously with the corresponding CDS spread, and falls as CDS spread declines.

Having the position as a receiver in a 10-year IRS with these banks during the 2008 crisis clearly resulted in substantial CVA losses. For example, if

company  $A$  enters into a 10-year IRS swap with Citibank on April 13<sup>th</sup> 2009 and the nominal is 10 million monetary units, ignoring CVA would result in CVA loss of 293 thousand monetary units, since the  $CVA_{IRS}$  at that date is 293.49 bps.



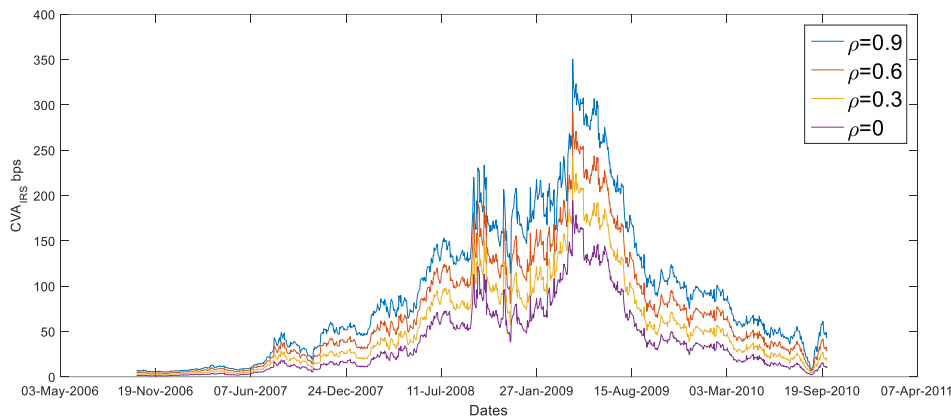
**Figure 12:** The upper graph shows the running  $CVA_{IRS}$ , while the lower one shows the corresponding CDS spreads.

### Test 2.2: Effects of WWR on running CVA

The second part of this study investigates the effect of  $\rho$  on  $CVA_{IRS}$ . Figure 13 shows that running  $CVA_{IRS}$  of Citibank increases with  $\rho$ , consistent with the results in Study 1. However, the highest relative difference in  $CVA_{IRS}$  between  $\rho = 0.9$  and  $\rho = 0$  is 449.45% and is at December 4<sup>th</sup> 2006 when Citibank's CDS spread is at the lowest level for the period. On the contrary, the least relative difference is observed on April 1<sup>st</sup> 2009, where Citibank's CDS spread is the highest level for the period. This contradicts to our results in Study 1, where we state that for higher levels of  $c$ , i.e. higher default probability, the  $CVA_{IRS}$  increases with  $\rho$  at an increasing rate. The reason for the inconsistent results is



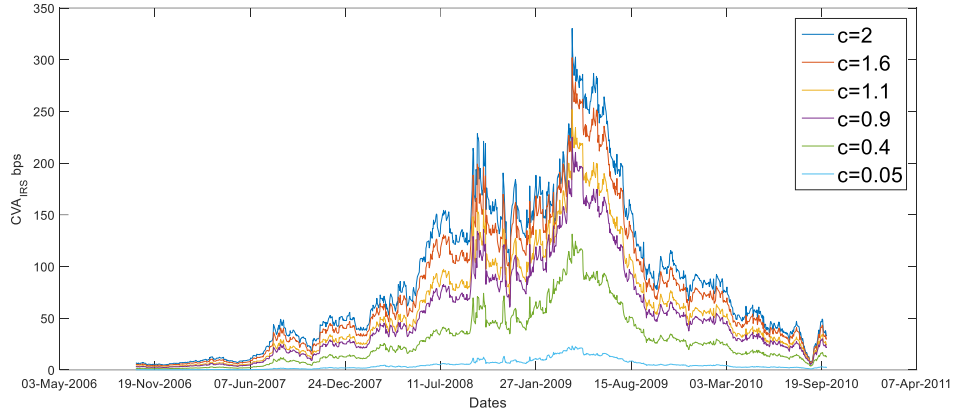
that in this study the volatility, the swap rate and the yield curves are also different between the two dates. Even though the relative difference in  $CVA_{IRS}$  is largest on December 4<sup>th</sup> 2006, the absolute difference is almost negligible. In practice, the absolute difference is more important, since it represents the loss from underestimating the  $CVA_{IRS}$  due to overlooking WWR. The maximum absolute difference between  $CVA_{IRS}$  when  $\rho = 0.9$  and when  $\rho = 0$  is observed on May 28<sup>th</sup> 2009, with 162.23 bps. This means that if the  $\rho_{real}$  is 0.9 but WWR is ignored, then entering into a 10-year receiver IRS with nominal of 10 million monetary units on May 28<sup>th</sup> 2009 would result in an underestimation of  $CVA_{IRS}$  of 162 thousand monetary units.



**Figure 13: Running CVA of a 10-year receiver IRS under different levels of WWR with Citibank as counterparty from 2006 to 2010**

### **Test 2.3: Effects of market price of risk on running CVA**

The last test in this study examines the impact of the parameter  $c$  on the running  $CVA_{IRS}$ , by looking at six different parameters; three are set lower than 1 and three are greater than 1. Figure 14 illustrates that increasing the change of measure parameter  $c$  increases the  $CVA_{IRS}$ , which is expected. The figure also shows that at a particular time the difference between  $CVA_{IRS}$  at each level of  $c$  decreases as  $c$  becomes larger. This phenomenon is due to the exponential distribution of the survival probability function and is shown in Study 1.



**Figure 14: Running  $CVA_{IRS}$  of the 10-year receiver IRS under different change of measure parameters  $c$  with Citibank as the counterparty from 2006 to 2010..**

### Study 3: CVA portfolio calculation

In the final study we examine the impact of assuming a homogeneous portfolio for a non-homogeneous portfolio using the same 10-year IRS contract as in Study 1. We start this section by defining the  $CVA_{heterog}$  and the  $CVA_{homog}$ , followed by investigating to what extent the  $CVA_{heterog}$  differs from the  $CVA_{homog}$  by conducting two tests; in the first test we vary the standard deviations of the recovery rates ( $Std_{\phi}$ ) and in the second test we diverge the standard deviations of the CDS spreads ( $Std_{CDS}$ ). In this study, the change of measure parameter  $c$  is assumed to be 1.5.

#### Introduction to methods to compute CVA portfolio

The computation for CVA portfolio is easier and faster when assuming the obligors are homogeneous, i.e. all obligors have identical parameters. This idea is used in A. Herbertsson and H. Rootzen (2008), where the authors calculate the spreads for basket default swaps. We use this idea on pricing CVA for an IRS.

Consider company  $A$  that has a heterogeneous portfolio, which consists of  $m$  obligors where each obligor has different default intensity and recovery rate due to different underlying idiosyncratic factors. Assuming all the obligors hold the same IR swaps, then  $A$ 's accumulated CVA with all its counterparties for this portfolio is given by

$$CVA_{heterog} = \sum_{j=1}^m CVA_j.$$

Calculating the  $CVA_{heterog}$  is computational intensive and highly time consuming, especially when involving large numbers of obligors. A simple alternative is to assume all obligors in the portfolio are identical in which each has the same recovery rate and default intensity. This portfolio is called a homogenous portfolio and has the recovery rate and default intensity equal to the average of the heterogeneous rates. The CVA of a homogeneous portfolio is given by

$$CVA_{homog} = \sum_{j=1}^m CVA_j = m CVA_j.$$

### **Test 3.1: Standard deviation of recovery rates**

This test is conducted on 3 different portfolios. Each consists of five obligors that have different CDS spreads observed in DataStream, see Appendix B.3. Portfolio 1 consists of only banks that have very similar CDS spreads on average 77.14 bps, Portfolio 2 is composed of firms that have similar CDS spreads with average of 24.32 bps, and Portfolio 3 contains firms from different industries where each obligor has very different CDS spreads ranging from 20 bps to 300 bps and average of 88.52. Each portfolio's respective homogeneous data, the average of the portfolio, is also exhibited in Appendix B.3.

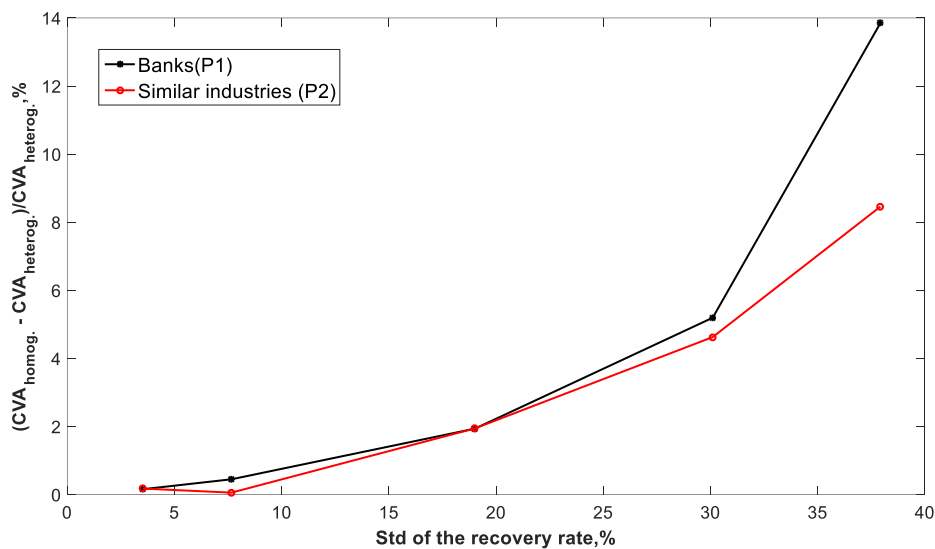
Each numerical experiment looks at five different scenarios where standard deviation of recovery rates ( $Std_{\phi}$ ) changes, while the mean of the recovery rate remains constantly at 45%. The recovery rates  $\phi_j$  are randomly chosen for each obligor in the portfolio and are displayed in Appendix B.4. All obligors are assumed to have the same correlation parameter  $\rho = 0.6$ .

The results that are displayed in Table 3 show that for Portfolio 1 and 2, the difference between  $CVA_{homog}$  and  $CVA_{heterog}$  relative to  $CVA_{heterog}$  increases with  $Std_{\phi}$ . The maximum of the two relative differences are below 2% at  $Std_{\phi} = 19.01\%$ , however, the relative differences are four times larger when the  $Std_{\phi} = 37.93\%$ .

**Table 3: The relative difference in percentage between  $CVA_{homog}$  and  $CVA_{heterog}$  with  $CVA_{heterog}$  as denominator for the five different scenarios of recovery rates for the three portfolios**

	$Std_{\phi} = 3.54\%$	$Std_{\phi} = 7.66\%$	$Std_{\phi} = 19.01\%$	$Std_{\phi} = 30.11\%$	$Std_{\phi} = 37.93\%$
Portfolio 1	0.16%	0.45 %	1.94%	5.19%	13.85%
Portfolio 2	0.17 %	0.06%	1.95%	4.62%	8.45%
Portfolio 3	15.11%	13.86%	37.51%	8.89%	9.63%

The positive relationship between  $Std_{\phi}$  and the relative difference between  $CVA_{homog}$  and  $CVA_{heterog}$  for Portfolio 1 and Portfolio 2 is exhibited clearly in Figure 15. Furthermore, the rate of increase is larger in Portfolio 1 which has higher average CDS spread than Portfolio 2.



**Figure 15: The relative difference between homogeneous CVA portfolio and heterogeneous CVA portfolio as a function of standard deviation of recovery rates for P1 & P2.**

However, this obvious positive relationship between the relative difference and  $Std_{\phi}$  does not occur for Portfolio 3, in which the relationship between these two factors is unclear. Figure 16 shows that Sprint Corp, the obligor that has the highest CDS spread (264.68 bps), has the most influence on the  $CVA_{IRS}$  on Portfolio 3 compared to other obligors in the portfolio.

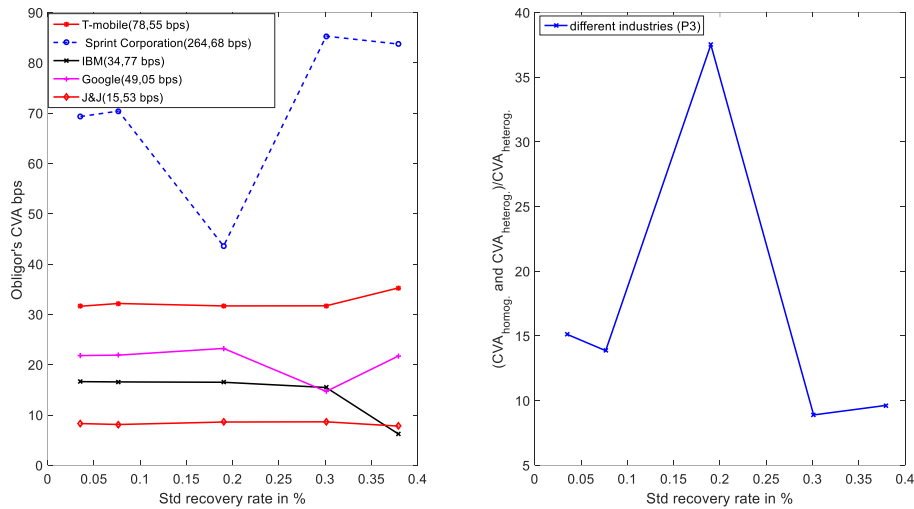


Figure 16: (t.l.): The relative difference between  $CVA_{homog}$  portfolio and  $CVA_{heterog}$  portfolio. (t.h.)  $CVA_{IRS}$  as a function of standard deviation of the recovery rates for each obligor in Portfolio 3.

### Test 3.2: Standard deviation of CDS spreads

This test examines five scenarios where standard deviation of the CDS spreads ( $Std_{CDS}$ ) changes while the average CDS spread for each scenario remains the same, see Appendix B.5. The test is performed on Portfolio 1 and 2, where the CDS spreads for all scenarios are randomly chosen and have an average CDS spread of 77.2 bps and 24.32 bps, respectively. To isolate the effect of the CDS spread, all obligors are assumed to have the same correlation parameters,  $\rho=0.6$ , and have recovery rate  $\phi = 0$ , as displayed in Appendix B.5.

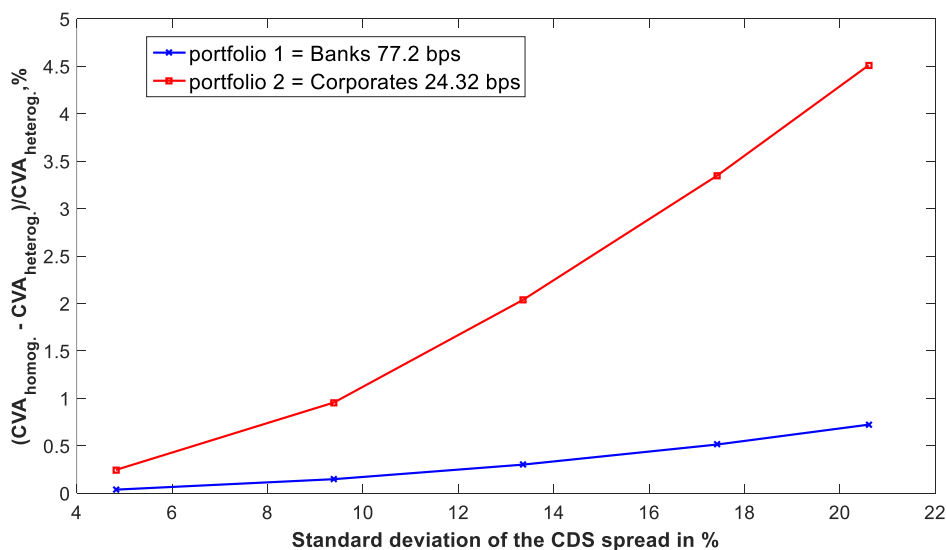


Figure 17: The relative difference between  $CVA_{homog}$  portfolio and  $CVA_{heterog}$  portfolio as a function of standard deviation of the CDS spread for P1 & P2.

Figure 17 shows that increasing the standard deviation of the CDS spreads increases the relative difference between  $CVA_{homog}$  and  $CVA_{heterog}$ . The rate of increase is larger in Portfolio 2, which has a lower average CDS spread. Table 4 shows that in Portfolio 1 the relative difference is at most 0.72%, which is when  $Std_{CDS}$  is 20.61%. However, for Portfolio 2 when  $Std_{CDS}$  is 20.61%, the relative difference is approximately six times larger than in Portfolio 1.

**Table 4: The relative difference between  $CVA_{homog}$  and  $CVA_{heterog}$ , where  $CVA_{heterog}$  is the denominator. The relative difference is as a function of standard deviation of the CDS spread for P1 & P2.**

Portfolios	$Std_{CDS} = 4.82\%$	$Std_{CDS} = 9.39\%$	$Std_{CDS} = 13.35\%$	$Std_{CDS} = 17.43\%$	$Std_{CDS} = 20.61\%$
P1: avg. CDS (77.2 bps)	0.038%	0.15%	0.30%	0.51%	0.72%
P2: avg. CDS (24.32 bps)	0.25%	0.96%	2.04%	3.35%	4.51%

## 8 Discussion and Conclusion

In this thesis we investigated the CVA for interest rate swaps under wrong way risk using the semi-analytical model of Cerny and Witzany (2015). Three different studies have been conducted; Study 1 examined the impacts of WWR and the change of measure parameter on  $CVA_{IRS}$ , Study 2 investigated how  $CVA_{IRS}$  evolves between 2006-2010, and Study 3 compared  $CVA_{IRS}$  under the homogeneous method and heterogeneous method. The following conclusions are inferred from the results obtained from each of the studies.

The results in Study 1 show that  $CVA_{IRS}$  increases substantially as the WWR becomes higher. Therefore we conclude that WWR should not be neglected, otherwise  $CVA_{IRS}$  might be underestimated. This positive relationship is also shown between the  $CVA_{IRS}$  and the change of measure parameter, which implies that  $CVA_{IRS}$  increases with the default probability of the obligor. Both these results are expected, as shown in Cerny and Witzany (2015) and Brigo & Pallavicini (2007).

Study 2 confirms the importance of considering CVA in fair value calculations of OTC derivatives. Since CVA was overlooked before the financial crisis, our figures in Study 2 shows graphically the enormous losses that occurred at that time, due to write downs on outstanding derivatives.

The results in Study 3 indicate that using the homogenous method works well for a portfolio that consists of obligors with similar recovery rates and CDS spreads, which is often the case for intra-industry portfolios. However, to be able to state this soundly, it is relevant to conduct this test on larger portfolios where obligors can hold more than one OTC derivatives, which are more realistic. Additionally, it is not conclusive that homogenous method is better than heterogeneous method without financially weighing the time saved against the preciseness of CVA. This is a subject for further research.

One of the drawbacks of the model of Cerny and Witzany (2015) is that it does not allow for analyzing the WWR and obligors' default dependence separately, since this model uses the one-factor Gaussian copula. The S&P

correlation matrix shows that the default dependence within the same industry is positive for almost all the observed industries. This indicates that to compute a CVA portfolio properly, it is important to account for the dependence between the defaults probabilities amongst the obligors. The obligors' dependence is, though, captured to some extent through the WWR, since all obligors are correlated with the same economic factor.

Another drawback of the Cerny and Witzany (2015) model is that changing the measure from risk-neutral annuity to risk-neutral is ignored. We have adjusted this matter by assuming the change of measure parameter as a scalar. Finding the exact change of measure parameter is a subject for further research on extension of the model.

Lastly, the correlation parameter  $\rho$  and the recovery rate  $\phi$  in the semi-analytical model are assumed to be deterministic and constant, but in reality these two parameters are stochastic. The correlation is hard to estimate and tends to be lower when the economic environment is stable than when it is unstable. Furthermore, the economic intuition tells us that the recovery rate is lower in bad times and higher in good times. Therefore, another interesting study would be to derive a semi-analytical model that allows  $\rho$  and  $\phi$  to be stochastic as shown in Andersen and Sidenius (2004).



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## Appendix A: Formula Derivations and Elaborations

### Appendix A.1: Black's Model to price a swaption

Black's model is used to price future options by assuming the future price follows a lognormal process,  $dF_t = \sigma F dz_t$  when  $dz_t$  is a Brownian motion. Black's model is extended to value options underlying different assets, such as Swaption. Black's Model prices European options in terms of forward or future price of an underlying asset, both when interest rate is constant and when it is stochastic.

A payer's swap can be valued as a European call option and a receiver's swap can be valued as a European put option. Consider a European call option on an asset with strike price  $K$  and matures at  $T$ . The future price of the asset at  $T$  is  $F$ , hence the option's price at time  $t$  is given

$$\text{A.1.1} \quad c(t) = X(t, T) \mathbb{E}_T[\text{Max}((F - K), 0)]$$

The corresponding put is given

$$p(t) = X(t, T) \mathbb{E}_T[\text{Max}((K - F), 0)]$$

where  $X(t, T)$  is the discount factor,  $\mathbb{E}_T$  denoted the so-called forward risk neutral expectation with respect to  $X(t, T)$ , See Hull (2012)  $\mathbb{E}_T(F) = S_0$ , so the expected value of the future price is equal to the price today

$$\text{A.1.2} \quad c = X(t, T) \text{Max}((S_0 - K), 0)$$

$$p = X(t, T) \text{Max}((K - S_0), 0)$$

### Appendix A.2: Risky Swaption Price

We have a swaption that has maturity  $T_0$  and strike rate  $s_K$  and a forward swap rate for the maturity  $s_{T_0}$ . The underlying swap payments at dates are  $T_1, T_2, \dots, T_n$

$$N \cdot X(\tilde{T}, T_1, T_0) \cdot 1_{[\tau > \tilde{T}]} (s_{0,n}(T_0) - s_K)^+ \text{ for payer}$$

$$N \cdot X(\tilde{T}, T_1, T_0) \cdot 1_{[\tau > \tilde{T}]} (s_K - s_{0,n}(T_0))^+ \text{ for receiver}$$

Then the risky swaption price at time  $t$  for a payer and receiver, respectively is given by

$$V_{RS}(0, \tilde{T}, T_0, T_1, T_n) = N \cdot X(0, T_1, T_n) \cdot [s_{0,n}(0) \cdot A_1 - s_K \cdot A_2]$$

$$V_{RS}(0, \tilde{T}, T_0, T_1, T_n) = N \cdot X(0, T_1, T_n) \cdot [s_K \cdot A_{-1} - s_{0,n}(0) \cdot A_{-2}]$$

where

$$A_{\pm 1} = \int_{-\infty}^{\infty} \exp \left\{ au\sigma\sqrt{T_0} - \frac{a^2\sigma^2 T_0}{2} \right\} \cdot \Phi \left( \pm \frac{d_1 + au - a^2\sigma\sqrt{T_0}}{\sqrt{1-a^2}} \right) \cdot \Phi \left( \frac{bu - \Phi^{-1}(1-H(\tilde{T}))}{\sqrt{1-b^2}} \right) \varphi(u) du$$

$$A_{\pm 2} = \int_{-\infty}^{\infty} \Phi \left( \pm \frac{d_2 + au}{\sqrt{1-a^2}} \right) \cdot \Phi \left( \frac{bu - \Phi^{-1}(1-H(\tilde{T}))}{\sqrt{1-b^2}} \right) \varphi(u) du$$

$$d_1 = \frac{\ln \left( \frac{s_{0,n}(0)}{s_K} \right) + \frac{\sigma^2 T_0}{2}}{\sigma\sqrt{T_0}}$$

$$d_2 = d_1 - \sigma\sqrt{T_0}$$

$$\varphi(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}, \quad u \in \mathbb{R}$$

## Appendix B: Numerical study data

### Appendix B.1: The input parameters for all numerical studies

#### Study 1:

Study 1	
Swap rate $s_K$	1.87 %
Volatility, $\sigma$	23.2 % (Sep 13 <sup>th</sup> 2013 – Mar 13 <sup>th</sup> 2014)
5-year CDS spread $R_M(5)$	0.079 % (Mar 13 <sup>th</sup> 2014)
Recovery rate $\phi$	0
Correlation coefficient	From 0 to 1, with 0.05 interval
Change of measure parameter $c$	From 0.05 to 3, with 0.05 interval
Yield curve	Appendix B.1

$s_K$ ,  $\sigma$  and  $R_M(5)$  and yield curve Source: Bloomberg

#### Study 2:

	Test 2.1	Test 2.2	Test 2.3
Swap rate $s_K$	Min value: 0.44% ; Max value: 5.12%	Min value: 0.44%; Max value: 5.12%	Min value: 0.44%; Max value: 5.12%
Volatility, $\sigma$	Min value: 7.57% ; Max value: 28.44%	Min value: 7.57% ; Max value: 28.44%	Min value: 7.57% ; Max value: 28.44%
Recovery rate $\phi$	0	0	0
Correlation coefficient	0.6	[0, 0.3, 0.6, 0.9]	0.6
Change of measure parameter $c$	1.5	1.5	[0.05, 0.4, 0.9, 1.1, 1.6, 2]
Yield curve	Section 7 Figure 11	Section 7 Figure 11	Section 7 Figure 11
5-year CDS spread $R_M(5)$	Counterparties: <b>Citibank</b> : Min value: 6.8 bps; Max value: 666.57 bps <b>Credit Suisse</b> : Min value : 9 bps; Max value: 366.32 bps <b>Deutchbank</b> : Min value : 8.7 bps; Max value: 187.95 bps <b>ING</b> : Min value : 4 bps; Max value: 188.3 bps <b>HSBC</b> : Min value : 4.9 bps; Max value: 170.59 bps	Counterparty: <b>Citibank</b> Min value: 6.8 bps; Max value: 666.57 bps	Counterparty: <b>Citibank</b> Min value: 6.8 bps; Max value: 666.57 bps

$s_K$ ,  $\sigma$ ,  $\phi$  and  $R_M(5)$  historical daily data (Oct 10<sup>th</sup> 2006 to Sep 30<sup>th</sup> 2010). Source: Datastream

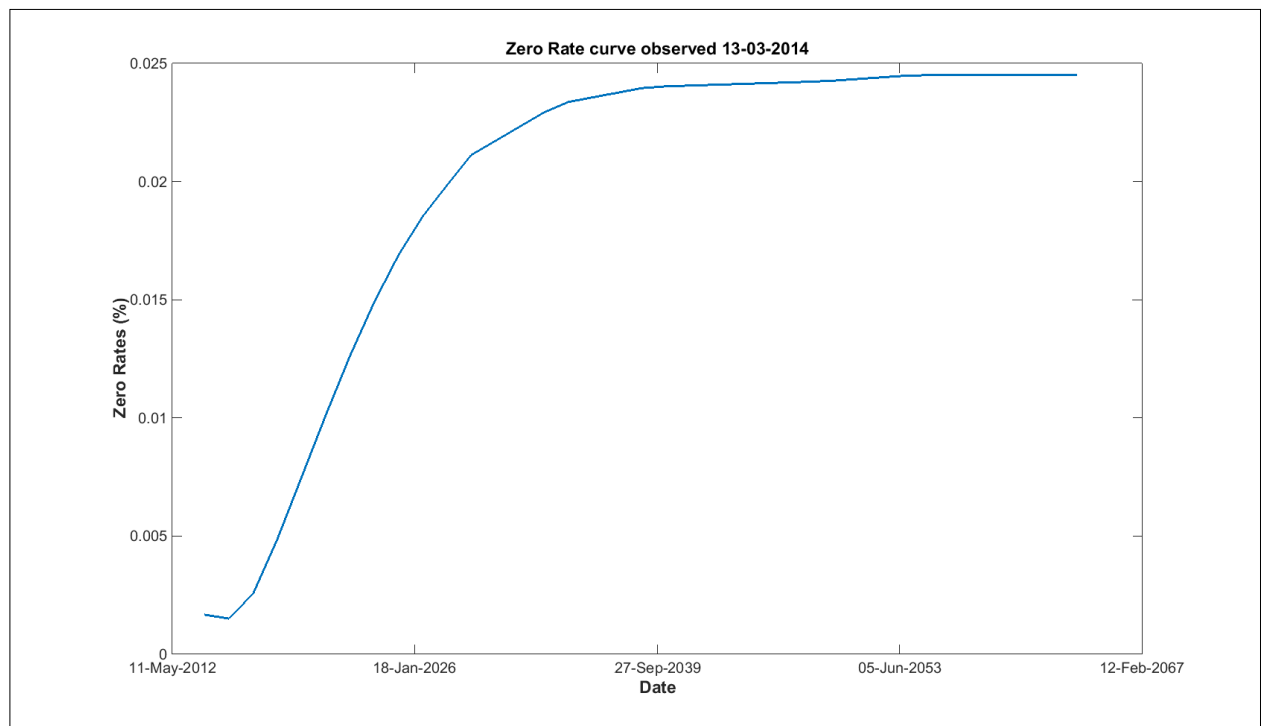
### Study 3:

	Test 3.1	Test 3.2
Swap rate $s_K$	1.87 %	1.87 %
Volatility, $\sigma$	23.2 % (Sep 13 <sup>th</sup> 2013 – Mar 13 <sup>th</sup> 2014)	23.2 % (Sep 13 <sup>th</sup> 2013 – Mar 13 <sup>th</sup> 2014)
5-year CDS spread $R_M(5)$	Appendix B.2	Appendix B.4
Recovery rate $\phi$	Appendix B.3	0
Correlation coefficient	0.6	0.6
Change of measure parameter $c$	1.5	1.5
Yield curve	Appendix B.1	Appendix B.1

Source for  $s_K$ ,  $\sigma$  and  $R_M(5)$  and Yield curve : Bloomberg

### **Appendix B.2: OIS zero yield curve for Study 1 and Study 3**

Observed on March 13<sup>th</sup> 2014 and date convention actual/360.



Source: Bloomberg



### Appendix B.3: The three portfolios used in Study 1 and Study 3.

#### Portfolio 1:

Obligor	Banks	CDS (bps)	Rho
1	Citibank	79.26	0.6
2	ING	75.96	0.6
3	Credit Suisse	73.21	0.6
4	HSBC	71.46	0.6
5	Deutchebank	85.81	0.6
	<b>Mean</b>	<b>77.14</b>	<b>0.6</b>
	<b>Std dev.</b>	<b>5.67 %</b>	

Source: Datastream, 5-year CDS spreads observed on March 13<sup>th</sup> 2014

#### Portfolio 2:

Obligor	Company name	CDS(bps)	Rho
1	IBM	29.77	0.6
2	Unilever NV	25.89	0.6
3	Procter & Gamble	21.70	0.6
4	Novartis	26.82	0.6
5	Johnson & Johnson	17.43	0.6
	<b>Mean</b>	<b>24.32</b>	<b>0.6</b>
	<b>Std dev.</b>	<b>7.15 %</b>	

Source: Datastream, 5-year CDS spreads observed on March 13<sup>th</sup> 2014

#### Portfolio 3:

Obligor	Company name	CDS (bps)	Rho
1	T-Mobile US	78.55	0.6
2	Sprint Corporation	264.68	0.6
3	IBM	34.77	0.6
4	Google INC	49.05	0.6
5	Johnson & Johnson	15.53	0.6
	<b>Mean</b>	<b>88.52</b>	<b>0.6</b>
	<b>Std dev.</b>	<b>10%</b>	

Source: Datastream, 5-year CDS spreads observed on March 13<sup>th</sup> 2014

## Appendix B.4: Different recovery rate scenarios used in Test 3.1

Recovery rates for each obligor  $j$  in each portfolio in Appendix B.2

	Scenario	1	2	3	4	5
Obligor 1	$\phi_1$	42.0%	38.0%	41.6%	41.4%	7.98%
Obligor 2	$\phi_2$	43.0%	41.0%	76.0%	5.00%	9.70%
Obligor 3	$\phi_3$	44.0%	45.0%	46.2%	61.0%	96.4%
Obligor 4	$\phi_4$	45.0%	44.0%	25.0%	85.5%	45.8%
Obligor 5	$\phi_5$	51.0%	58.0%	36.5%	34.2%	67.3%
	<b>Mean</b>	45.0%	45.2%	45.1%	45.4%	45.4%
	<b>Std. dev.</b>	3.50%	7.70%	19.0%	30.1%	37.9%

*The data is randomly assigned*

## Appendix B.5: Different CDS spreads scenarios used in Test 3.2.

**Portfolio 1:** Five scenarios of CDS spreads, all with same mean but different standard deviations.

Obligor	Banks	CDS1	CDS2	CDS3	CDS4	CDS5	Rho
1	Citibank	81.26	73	67	60	55	0.6
2	Ing	73.6	82	86	86	88	0.6
3	Credit Suisse	74.21	87	85	88	90	0.6
4	Hsbc	73.4	63	59	57	55	0.6
5	Deutchebank	83.53	81	89	95	98	0.6
	<b>Mean</b>	77.2	77.2	77.2	77.2	77.2	0.6
	<b>Std. dev.</b>	4.82%	9.39%	13.35%	17.43%	20.61%	

*The data is randomly assigned for all scenarios. The CDS spreads are expressed in basis points*

**Portfolio 2:** Five scenarios of CDS spreads, all with same mean but different standard deviations.

Obligor	Company name	CDS1	CDS2	CDS3	CDS4	CDS5	Rho
1	IBM	29.77	23.27	32.27	41.27	21.27	0.6
2	Unilever NV	25.89	32.89	28.39	12.39	32.39	0.6
3	Procter & Gamble	21.70	19.20	13.70	20.70	9.70	0.6
4	Novartis	26.82	34.32	39.82	42.82	54.82	0.6
5	J & J	17.43	11.93	7.43	4.43	3.43	0.6
	<b>Mean</b>	24.32	24.32	24.32	24.32	24.32	0.6
	<b>Std. dev.</b>	4.8%	9.4%	13.4%	17.2%	20.3%	

*The data is randomly assigned for all scenarios. The CDS spreads are expressed in basis points*