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Firm Fragmentation and the Skill Premium

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Shutdown Threats, Firm Fragmentation, and the Skill Premium

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Abstract

This essay investigates the interaction between demand uncertainty and non-competitive labor markets where firm owners have the option to shut down and relocate. Workers cannot find new jobs instantly and therefore accept wage reductions to avoid unemployment, if firm owners credibly threaten to shut down.

The analysis shows that the expected wage rate is a mix of a competitive wage rate and a bargained wage rate and that this lowers the skill premium. Further, the option of firms to shut down and relocate increases the average size of firms. The analysis also shows that outsourcing or contracting out is more likely if demand is more uncertain, if market power is smaller, and if the markets for intermediate goods are more competitive.

Fragmentation increases the skill premium because it leads to more homogenous firms, with respect to workers' skills. With more homogenous firms, low-skill workers cannot compensate their inferior productivity in wage bargains with high-skill workers.

JEL:	J24, J31, J41, J52, L23, L24,
Keywords:	Distribution, Wages, Outsourcing,
	Fragmentation, Bargaining

1 Introduction

Following the recognition of the massive increase in wage inequality in the U.S. in the 1980–1990 period, economists' slumbering interest in distributional questions was awakened. Several theories have been proposed to understand the changes. The most common revolve around skill-biased technological change (Berman et al. 1998), increased competition from low wage countries (Wood 1995), and institutional changes (Fortin and Lemieux 1997). One purpose of this paper is to augment those explanations by investigating the effect of domestic outsourcing and domestic sub-contracting on the skill premium.

The massive changes in the U.S.wage distribution during the 1970–1990 period are well documented. Wage inequality in U.S. increased rapidly during the 1980–1990 period due to increases in most of the different components of overall wage inequality. The skill premium, or returns to education, increased, returns to experience increased and residual wage inequality, or inequality among individuals with similar characteristics, also increased (Gottschalk 1997; Juhn et al. 1993).

Gottschalk points out that "... the increases in the college premium are being driven more by the decline in real earnings of high school graduates than by the increase in earnings of college workers" (Gottschalk 1997, p. 30). Any full explanation of the changes in the skill premium in the U.S. is therefore obligated to present a plausible case for an absolute *decrease* in earnings of workers with relatively low education.

The rapid increase in U.S. wage inequality during the 1980–1990 period is unmatched by any European country. Gottschalk and Smeeding (1997) summarize the changes in Europe. While the U.K. stands out in the European family by experiencing large increases in earnings inequality during the 1980–1990 period, the European experience is in general mixed. Most, but not all, countries experienced some increases in earnings inequality. For Sweden the results differ depending on choices of periods and measurement, but several studies describe increased inequality (Gottschalk and Smeeding 1997; Gustafsson and Palmer 1997; Gottschalk and Smeeding 2000; Gustafsson and Palmer 2001).

1.1 Contribution

The contribution of this paper is twofold. On the one hand it presents a novel framework for combining the standard marginal analysis, i.e. competitive wages, with rent sharing theories where workers bargain over wages. On the other hand

it hypothesizes that changes in the skill premium can be explained by *domestic* disintegration of production which prohibits workers with different skill levels to negotiate with each other over wage rates. In addition, the model investigates what factors cause outsourcing and contracting out. An important property of the framework is that firms operate under uncertainty. This uncertainty causes firm owners to occasionally threaten to shut down or relocate production. Employees are therefore occasionally subject to the risk of unemployment.

Workers can influence firm owners not to shut down the firm by renegotiating wages, i.e. agreeing on lower wages to avoid unemployment. This assumption introduces wage bargaining in the model. As opposed to many other labor market models, workers do not bargain over profits but rather to avoid unemployment, i.e. workers bargain over losses.

Firm owners always have incentives to threaten to shutdown the firm in order to lower wages and thereby increase profits. However, rational workers only consider *credible threats*. If a firm owner credibly threatens to shut down the firm, workers agree on lowering wages precisely such that firm owners are indifferent between shutting down the firm or continuing production. Credible shut down threats put workers in a bargaining situation. Workers do not primarily bargain with firm representatives since the total reduction of the wage bill necessary for firm owners not to shut down the firm is known to all parties. Instead, workers with different characteristics must agree on the distribution of wage reductions.

The model developed in this paper focuses on two types of workers: high-skill and low-skill. Whether two types of workers, in general, should form a single union that bargains with the firm representative or bargain separately is discussed in Horn and Wolinsky (1988). Their results indicate that high-skill and low-skill workers should form a single union if they are substitutes. The model in this paper is set such that high-skill and low-skill workers bargain over a fixed surplus. That is, the maximum total surplus that can be extracted by all workers together does not depend on whether high-skill and low-skill workers form a single union or not. Therefore it is reasonable to assume that high-skill and low-skill workers form two separate unions. To see why, consider first the case where high-skill and low-skill workers form an alliance. In this case the distribution of the surplus between high-skill and low-skill workers is determined by the political mechanisms within the single union. A median voter outcome would dictate the minority group its outside option. The minority group would then always leave and form a separate union.

Given this basic setting, the model investigates how labor demand and wages are affected by firms' option to default on labor contracts, but also how increased

utilization of external provision of labor by firms affects wage rates and the skill premium. The reliance of external provision can be categorized into two broad categories: *outsourcing* and *contracting out*. In both cases the final goods producer hands over the employment and more or less of the employer responsibilities to a third party. In the outsourcing case the final goods producer can be fully detached from the third party employee, while in the contracting out case, the final goods producer provides capital, like office space, machines or software tools, to the third party employee. Henceforth the term *fragmentation* will be used instead of outsourcing and contracting out.

In a less fragmented economy more firms employ a mix of high-skill and low-skill workers. Low-skill workers benefit from bargains relative to high-skill workers if firm owners threaten to shut down the firm. Therefore, shut down threats tend to *decrease* the skill premium in a *less* fragmented economy.

1.2 Some Supporting Data

The graph in Figure 1 plots the inverse of plant size against the skill premium during the 20th century in the U.S. The correlation is striking:

- 1900–1940: Plant size increased and the skill premium decreased.

- 1940–1980: Plant size and the skill premium were relative stable.

- 1980–2000: Plant size decreased and the skill the premium increased.

Needless to say, Figure 1 does not prove that fragmentation increases the skill premium. First, plant size and firm size are related but not identical. Second, firm size and firm homogeneity,with respect to employees, are different concepts. However, it seems plausible that in an economy with smaller firms, there is a larger number of homogenous firms. This is also confirmed by Kremer and Maskin (1996) who present evidence of a trend where high-skill and low-skill workers are sorted into separate firms.

Recognizing these caveats, the figure hints that fragmentation can be important for explaining changes in the skill premium.

1.3 Related Literature

In the discussion of the impact of unions on wage inequality, Freeman and Medoff (1984) argue that unions favor wage equality because unions prefer single rate

Figure 1: U.S. Skill Premium and Manufacturing Plant Size
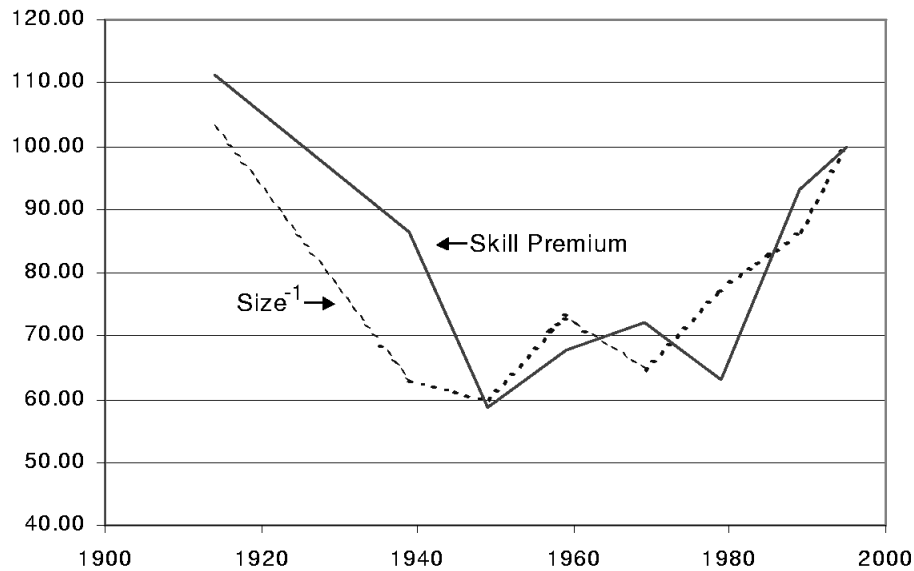
The evolution of U.S. plant size during the 20th century is highly correlated with the evolution of the skill premium. Both series are indexed relative to 1995. Source: Mitchell (2005)

wage policies to individual wage policies. Freeman and Medoff put forth a few arguments: First, because of political mechanisms within the union, unions favor the majority of workers, thereby favoring redistributive contracts. This result follows, for example, by applying the median voter theorem. Second, Freeman and Medoff argue that unions tend to equalize wages due to ideological reasons favoring worker solidarity and organizational unity. This argument parallels the brief discussion in Abraham and Taylor (1996) concerning the possibility that within larger and more heterogenous firms, equity motives play an important role in the wage determination process.

Besides favoring single rates across its members, unions tend to decrease wage inequality by favoring single rates across firms and industries. None of those arguments are applicable for this paper since high-skill and low-skill workers are members in *separate* unions, whereby the political mechanisms within unions are sidestepped, since all members are identical. Further, every worker behaves in a neo-classical way; that is, every worker acts as if maximizing his or her utility without any egalitarian considerations. Finally, unions are firm specific and do not synchronize policies across firms or industries.

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Borjas and Ramey (1995) relate to this paper by discussing the importance of the distribution of rents for the wage distribution. They claim that the industries that are hurt the most by import competition from less developed countries are manufacturing firms earning rents. These firms, according to Borjas and Ramey, employ relatively many low-skill workers. Tougher competition decreases both rents and low-skill employment in manufacturing firms. Hence, the low-skill workers are hurt "twice" from increased import competition.

The analysis in Kremer and Maskin (1996) shows that if the variation in skill levels is sufficiently low, it is efficient to match low- and high-skill workers in production. But with sufficiently large variation in the distribution of skills, efficiency requires that low-skill workers match with low-skill workers, and high-skill workers match with high-skill workers, causing a segregation of firms with respect to skill. With segregation by skill, the skill premium increases since the two production tasks are complementary.

Mitchell (2005) proposes that high-skill workers are superior to low-skill workers in being able to perform a wider variety of tasks. In the first part of the 20th century, mass production led to larger plants and a higher degree of specialization. The demand for high-skill workers diminished as every worker was required to perform a smaller number of tasks. As a result, the skill premium decreased during the first half of the century. During the last part of the 20th century new production technology decreased the cost-efficient plant size and increased the demand for workers who are able to perform a wider variety of tasks, thereby increasing the demand for high-skill workers. The increased demand for high-skill workers during the second half of the century increased the skill premium.

Caroli and Van Reenen (2001) use British and French micro data to investigate the impact of organizational change on the demand for high-skill and low skill-labor. Their definition of organizational change states not only that employees must perform more tasks but also includes flatter organizational hierarchies, implying that employees face more responsibility and have to work more independently. This supposedly benefits high-skill workers. Caroli and Van Reenen's analysis indicates that there is a complementarity between organizational change and skill.

Acemoglu et al. (2001) focus on the distribution of rents as they build a model where high-skill and low-skill workers bargain over rents. However, they do not model vertical disintegration as a choice of firm owners but instead focus on skill biased technological change, increasing high-skill workers' gains from switching to specialized firms, thereby undermining the possibility for low-skill workers to specify redistributive wage contracts.

Harrison and Bluestone (1988) connect U.S. firms' increased use of contingent workers, i.e. temporary employed and third party workers, to the deterioration of low-skill workers' wages. Contingent workers are in general paid lower wages and receive less insurance benefits (Kalleberg et al. 1984).

The analysis in this paper can be seen as extending the analysis of Sap (1993), who integrates unionized workers into two groups – men and women. Standard bargaining theory is applied, highlighting that bargaining strength and outside options determine the wage differentials between men and women. This analysis puts Sap's analysis into a broader context and replaces the gender distinction with a skill distinction.

Thesmar and Thoenig (2004) hypothesize that increased fragmentation can be linked to financial liberalization. Financial liberalization diversifies shareholder portfolios, thereby reducing the cost of risk, ceteris paribus. Shareholders demand more risky assets, relative to the expected returns, and firms respond by relying more heavily on external provision of intermediate goods.

In Burda and Dluhosch (2002) firm's choice of fragmentation is endogenous. By disintegrating the production chain, demand for communication and coordination services, produced solely by high-skill workers, increases but the variable marginal production cost decreases. Burda and Dluhosch show that in the long run, if the growth rate of high-skill workers exceeds the growth rate of low skill workers, fragmentation increases, and the skill premium increases.

1.4   Outline

In Section 2 the basic properties of the model are presented. The section describes the endowments, and parts of the institutional setting. Section 3 presents the fundamental setup and some general results. Section 4 analyzes firms in more detail and derives the necessary expressions to analyze the impact of fragmentation on the skill premium. Section 5 discusses the possible steady state equilibria and verifies the hypotheses of the paper. Section 6 summarizes the findings. Appendix A contains a list of symbols used, Appendix B and Appendix C complement Section 4 and Section 5 with some mathematical derivations.

2   Model

This section describes the fundamental parts of the model. Table 1 depicts the general logic for subscripts used to categorize different variables. Indices over

Table 1: Subscripts

Subscript	Indicates
i	In-house Firm
f	Fragmented Firm
s	Specialized Firm
h	High-Skill
l	Low-Skill

a continuum are written in parentheses. All symbols are listed in Table 3 in Appendix A. Random variables are marked by $\tilde{\cdot}$, $\hat{\cdot}$, or $\check{\cdot}$, depending on the information available. An upper case symbol is used for stochastic variables while lower case symbols are used to denote a particular realization of the corresponding random variables. Upper case letters are also used to denote aggregate quantities while lower case letters are also used to denote micro quantities. Symbols marked by $\cdot^*$ are derived from an optimization problem.

2.1 General Setting

Consider an economy with a single consumption good, the Y good. There is a continuum of firms selling a distinct variation of the Y good. The Y good is assembled using two other goods: the X good and the Z good. The X good is produced using high-skill labor, and the Z good is produced using low-skill labor. Firms that employ workers and produce both the X and Z goods (which are necessary to assemble the Y good) are labeled *in-house* firms. Firms that do not hire any labor but purchase the X and Z goods, which are necessary to assemble the Y good, from *specialized* firms, are called *fragmented* firms. Naturally, asserting that fragmented firms hire no labor, is a crude characterization of firms relying more heavily on outside contractors.

2.1.1 In-house Firms

There is a continuum of *in-house* firms with range K_i. Every in-house firm *produces* and sells a distinct variation of the consumption good. In-house firms produce both intermediate goods, i.e. both the X good and the Z good, necessary to assemble the Y good. To denote the quantity of the Y good sold by the kth in-house firm, the notation $y_i(k)$ is used. The corresponding price is denoted $\tilde{P}_i(k)$.

2.1.2 Fragmented and Specialized Firms

There is a continuum of firms with range K_f that only *assemble* and sell the Y good. Those firms are labeled *fragmented* firms. Fragmented firms purchase intermediate goods, the X and Z goods, necessary to assemble the Y good. Firms producing either an X or a Z good, but not both, are called *specialized* firms. To denote the kth fragmented firm's output of the Y good, $y_f(k)$ is used, and its price is denoted $\tilde{P}_f(k)$.

2.2 Consumers' Preferences

The representative consumer does not care whether or not the consumption good is sold by an in-house or a fragmented firm, hence from the consumer point of view there is a continuum of variations of the Y good with range $K_i + K_f$. Due to a preference for variety, consumers are biased towards spreading their consumption across all the different variations of the Y good, thereby providing producers with some market power. Let C denote the amount the representative consumer spends on the Y good. The representative consumer behaves as if maximizing

$$\left[\int_0^{K_i} \tilde{d}_i(k) y_i(k)^{1-\beta} dk + \int_0^{K_f} \tilde{d}_f(k) y_f(k)^{1-\beta} dk \right]^{\frac{1}{1-\beta}}$$

subject to the budget constraint

$$C = \int_0^{K_i} \tilde{p}_i(k) y_i(k) dk + \int_0^{K_f} \tilde{p}_f(k) y_f(k) dk.$$

The first integral in the objective function sums the utility derived by consuming different variations of the Y good, sold by *in-house* firms. The second integral sums the utility derived by consuming different variations of the Y good, sold by *fragmented* firms. Demand uncertainty is modeled using the stochastic demand variables $\tilde{D}_i$ and $\tilde{D}_f$, and the consumer preference for a variation of the good depends on the realizations of those demand variables, $\tilde{d}_i(k)$ and $\tilde{d}_f(k)$. Consumer preference for variety is parameterized by $\beta \in [0, 1)$. If β equals zero, consumers only purchase the cheapest variation of the Y good, given that the realizations of the demand variables are equal.

The first integral in the budget constraint sums the representative consumer's expenditures on all the K_i different variations of the Y good sold by *in-house* firms. The second integral sums the representative consumer's expenditure on the K_f different variations of the Y good sold by *fragmented* firms.

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Demand uncertainty is modeled using the stochastic variables $\tilde{D}_i$ and $\tilde{D}_f$, with appropriate indices. That is, the demand for every variation of the Y good is stochastic. Demand shocks are deviations from expected demand. The stochastic demand variables are uniformly distributed with an expected value of 1 and range 2Δ. The cumulative density function is therefore:

$$F_i(d) = F_f(d) = \frac{d - (1 - \Delta)}{2\Delta} \qquad 0 < \Delta \leq 1. \tag{1}$$

Solving for the inverse demand functions yields:

$$\forall k \in [0, K_i] \quad : \quad \tilde{p}_i(k) = \left[\frac{C}{\overline{p}}\right]^{\beta} \frac{\tilde{d}_i(k)}{y_i(k)^{\beta}} \tag{2a}$$

$$\forall k \in [0, K_f] \quad : \quad \tilde{p}_f(k) = \left[\frac{C}{\overline{p}}\right]^{\beta} \frac{\tilde{d}_f(k)}{y_f(k)^{\beta}} \tag{2b}$$

$$\left[\frac{C}{\overline{p}}\right]^{\beta} = \frac{C}{K_i \times \overline{\tilde{d}_i y_i^{1-\beta}} + K_f \times \overline{\tilde{d}_f y_f^{1-\beta}}}. \tag{2c}$$

Relations (2a) and (2b) together with (2c) provide the inverse demand function for every firm in the model. The $\overline{p}$ variable is a price index. Since there is a continuum of firms, the price index is unaffected by each firm's price and quantity choice and is therefore taken as given by each firm. The averages in the expression for $\overline{p}$ are taken over the continuum of in-house firms and the continuum of fragmented firms.

Notice that a demand shock by some percentage increases revenues, $\tilde{p}_i y_i$ or $\tilde{p}_f y_f$, by the same percentage, independent of the production levels, y_i or y_f. This in turn implies that even though the revenue function is concave with respect to the production level, the revenue function is linear with respect to the demand shock. Hence a mean preserving spread in demand changes neither the expected profit rate nor the size of the firm, if the firm owner is risk neutral and must commit to an employment choice prior to the realization of the demand shock.

2.3 Firms' Technology

The production of the Y good requires two intermediate goods: the X good and the Z good. The production of the X (Z) good requires high-skill (low-skill) labor. The production functions for the X and Z goods are:

$$x = h \qquad z = l. \tag{3a}$$

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That is, one unit of high-skill labor, h, produces one unit of the X good and one unit of low-skill labor produces one unit of the Z good.

The production of the Y good is described by the Cobb-Douglas production function in the X and Z goods as:

$$y \;=\; x^{\alpha} z^{1-\alpha}. \tag{3b}$$

2.4 Institutional Setting

The following paragraphs define the different institutional settings for in-house and fragmented firms. Both firms are of course subject to the same institutional constraints, but the different ways of organizing production implies some differences.

In-house Firms   In-house firms and their employees are limited by institutional constraints. In-house firms post skill specific job vacancies, either high-skill or low-skill. It is assumed that firms find it easy to fill vacancies with workers with appropriate skills, while workers find it costly or time consuming to find employment. However, once contracted, in-house firms cannot, for whatever reason, replace or dismiss workers, during the contract period, unless workers threaten to strike in order to increase their wages. Employers and employees agree on one period contracts. Hence, firms cannot decrease production levels by changing employment during the period.

During the contract period, firms are subject to a demand shock. Firm owners cannot change employment or lower wages during the contract period without incurring a prohibitive cost, but firm owners always have the option to shut down the firm instantly and thereby avoid paying wages for the remainder of the contract period. Naturally, workers suffer if the firm is shut down, since unemployed workers cannot find work instantly.

To simplify the analysis, the following assumptions are made. First, since it is easy for firms to recruit employees, workers and firm owners agree on competitive wage rates, i.e. standard wage rates determined by marginal productivity.

Second, a demand shock is not realized at any arbitrary point in time during the contract period, but immediately after signing wage contracts. The assumption magnifies the effect of demand uncertainty, but does not alter qualitative results.

Third, firm owners are not allowed to increase production and thereby employment during the period, even if the realization of the demand shock is favorable. This assumption is made only to simplify the analysis but can be rationalized by

Figure 2: Sequence of Events for In-house Firms
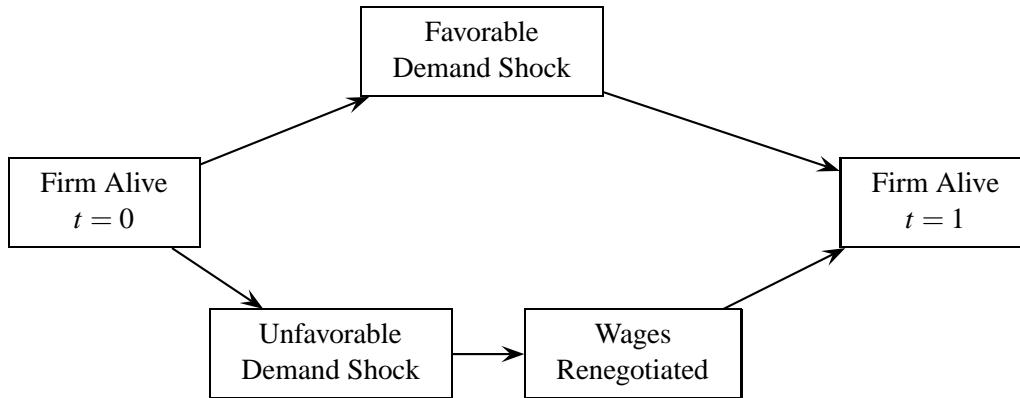
If the demand shock is favorable, the firm produces as planned. If the realization of the demand variable is unfavorable, the in-house firm's employees re-negotiate lower wages and the firm produces as planned. The sequence of events is repeated in the next period.

assuming that new workers need some training before becoming productive. Note that firm owners still benefit from favorable demand shocks as the price of their good increases.

Fourth, high- and low-skill workers do not bargain over employment in order to save the firm. This assumption is not unreasonable since workers find unemployment costly.

Figure 2 illustrates the sequence of events for an in-house firm during a single period. First, firms employ workers and agree on the competitive wage rates. Second, the demand shock is realized. If the demand shock is favorable, the firm produces as planned but if the demand shock is unfavorable, high-skill and low-skill workers renegotiate wages, and the firm again produces as planned. This sequence of events is repeated every period.

Notice that a demand shock is considered to be favorable if the firm owner does not threaten to shut down the firm. The probability that a firm owner does not threaten to shut down the firm is denoted Q_i. Q_i is endogenous and derived from the behavior of rational firm owners, maximizing the discounted profit stream.

Fragmented and Specialized Firms Each specialized firm sells its good, either the X good or the Z good, to a continuum of fragmented firms. To make the

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analysis as simple as possible, it is assumed that fragmented firms can purchase the *X* and *Z* good *after* the realization of the demand variable. This is reasonable only if the *X* and *Z* goods are homogeneous, i.e. identical across fragmented firms, and transport costs are negligible.

While this assumption is questionable it simplifies the analysis because it is possible to apply the mean value theorem for the demand shocks faced by fragmented firms. That is, the demand shocks of the different fragmented firms even out and each specialized firm faces a certain demand. Since the demands for the *X* and *Z* goods are certain, employees of specialized firms never face shutdown threats and never renegotiate wage rates.

It is however worth pointing out that it is not the lack of shutdown threats for specialized firms that drives the results derived ahead. Because specialized firms employ either high-skill or low-skill workers but not both, renegotiating wages in specialized firms would not have any redistributive effect.

3 Intermediate Results

The following sections present some general results governing the decisions of firm owners and workers. Due to the generality of the discussion some terms, such as profit rates or investment costs, are not formally defined or properly subscripted. Formal definitions and proper subscripts follow in later sections where the results, derived in this section, are applied.

3.1 In-house Firms

Starting a firm requires a capital investment of I_i. The depreciation rate of capital, whether used or not, is δ. The demand for the firm's product is uncertain, due to the market demand shock. It is assumed that firm owners observe the realization of demand shocks after hiring employees. The owner of a firm can shut down the firm in order to avoid paying wages, knowing that variable costs will exceed revenues. The possibility for firm owners to terminate operations gives the model a foundation for wage bargains which is a central feature of the setup and necessary to derive the results.

Owners of in-house firms and workers employed by in-house firms face two possible scenarios in every period. Either the firm owner threatens to shut down the firm and workers renegotiate new wages, or the firm owner does not threaten to shut down the firm and workers are paid the wage agreed upon at the beginning

of the period. Uncertainty arises because the demand for the in-house firm's good is uncertain. This uncertainty carries over to profit and wage rates, to price, as well as to revenues.

There exists an endogenous firm-specific threshold, $\underline{d}_i$, such that if the realization of the stochastic demand variable, $\tilde{D}_i$, is greater than or equal to this threshold, $\tilde{d}_i \geq \underline{d}_i$, then the firm *does not* threaten to shut down the firm. Note that the ~ notation is used for $\tilde{d}_i$ since it is a realization of a stochastic variable. The threshold $\underline{d}_i$ on the other hand is non-stochastic and therefore not marked by ~.

If the realization of the demand variable is less than this threshold, $\tilde{d}_i < \underline{d}_i$, then the firm owner *does* threaten to shut down the firm. In the latter case, workers renegotiate wages to motivate the firm owner to continue operations and not shut down the firm.

Demand uncertainty is described by the firm-specific stochastic variable $\tilde{D}_i$, which is uniformly distributed with mean 1 and range 2Δ. The cumulative density function for $\tilde{D}_i$ is denoted by $F(\cdot)$ and is given by (1). Given the threshold $\underline{d}_i$, the probability that the firm owner does not threaten to shut down is $1 - F(\underline{d}_i)$. From here on this probability is denoted by:

$$Q_i \equiv \text{Prob}\left(\tilde{d}_i \geq \underline{d}_i\right) = 1 - F(\underline{d}_i). \tag{4}$$

It follows immediately that the probability that the firm owner does threaten to shut down the firm is $1 - Q_i = F(\underline{d}_i)$.

Due to the stochastic demand, the profit rate, $\tilde{\Pi}_i$, the wage rates, $\tilde{W}_i$, the price of the good, $\tilde{P}_i$, and firm revenue, $\tilde{R}_i$, are also stochastic. In the derivation that follows it is often convenient to rewrite expectations conditionally. For example consider the expected value of the wage rate, $E\{W_i\}$:

$$E\{W_i\} = Q_i E\left\{W_i \mid \tilde{d}_i \geq \underline{d}_i\right\} + (1 - Q_i)E\left\{W_i \mid \tilde{d}_i < \underline{d}_i\right\}.$$

Here, $E\left\{W_i \mid \tilde{d}_i \geq \underline{d}_i\right\}$ is the expected wage rate given that the firm owner does not threaten to shut down (that is, it is known that the realization of the demand variable, $\tilde{d}_i$, is greater than $\underline{d}_i$) and Q_i is the probability that the firm owner does not threaten to shut down the firm; see (4). $E\left\{W_i \mid \tilde{d}_i < \underline{d}_i\right\}$ is the expected wage rate given that the firm owner threatens to shut down the firm and workers renegotiate wages. This happens only if $\tilde{d}_i < \underline{d}_i$, which occurs with probability $1 - Q_i$; again see (4).

Because it becomes cumbersome to write the conditional expectation operator everywhere, the following notations are used. $\tilde{W}_i$ denotes the wage rate, which is stochastic, $\hat{W}_i$ denotes the wage rate given that the firm owner does not threaten to

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shut down the firm, and $\check{W}_i$ denotes the wage rate given that the firm owner does threaten to shut down the firm. Given these definitions, the expected wage rate can be written as:

$$E\{W_i\} = Q_i E\{\hat{W}_i\} + (1 - Q_i)E\{\check{W}_i\}. \tag{5}$$

The wage rate was used as an example above, but the same notational convention is used for the profit rate, $\tilde{\Pi}_i$, firm revenue, $\tilde{R}_i$, the price of the good, $\tilde{P}_i$, and the demand variable, $\tilde{D}_i$. To summarize, $\hat{}$ and $\check{}$ are used to distinguish the scenarios where the firm owner does not and does threaten to shut down the firm, respectively. In terms of information sets, $\hat{}$ is used if the only information given is that the firm owner does not threaten to shut down the firm and $\check{}$ is used if the only information given is that the firm owner does threaten to shut down the firm. Of course this notation is only meaningful before the realization of the stochastic demand variable is known. Once it is known, there is no uncertainty about profit rates, wage rates, or firm revenues and given the notation convention stated earlier, lower case letters are used.

It is possible to simplify the analysis further by noting the following. First, if the firm owner shuts down the firm, the profit rate is simply the replacement cost of capital. If the firm owner threatens to shut down the firm, workers will renegotiate wages such that the firm owner is indifferent about shutting down the firm and keeping it alive. Therefore, the profit rate given that the firm owner threatens to shut down the firm is simply $-\delta I_i$, i.e. the replacement cost for capital.

Second, given that the firm owner does not threaten to shut down the firm, wage rates are not renegotiated and thereby not affected by the realization of the demand shock. Therefore the wage rate given that the firm owner does not threaten to shut down the firm is simply w. Note that w is determined at the start of the period and hence it is not stochastic.

The profit rate with respect to time is a stochastic variable, denoted by $\tilde{\Pi}_i$. Given the realization of the the demand shock, $\tilde{d}_i$, the owner of the firm can either shut down the firm, with a *non-stochastic* profit rate $-\delta I_i$, or keep the firm alive, with the given profit rate $\hat{\pi}_i$.

Let $\tilde{v}_i$ denote the value of a firm, *after* the outcome of the demand shock is realized, let ρ denote the discount rate, and let $\tilde{V}'_i$ denote the value of the firm in the next period. $\tilde{v}_i$ satisfies:

$$\tilde{v}_i = \max\left\{\hat{\pi}_i + \frac{E\left\{\tilde{V}'_i \big| \tilde{d}_i > \underline{d}_i\right\}}{1 + \rho}, -\delta I_i + \frac{I_i}{1 + \rho}\right\}. \tag{6}$$

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The first member of the set is the value of the firm given that the owner, knowing the realization of the demand shock, decides to keep the firm alive. The second member of the set is the value of shutting down the firm.

If the owner decides to keep the firm alive, the instantaneous profit received is $\hat{\pi}_i$ plus the discounted continuation value. The continuation value is $E\left\{\tilde{V}_i' \middle| \tilde{d}_i > \underline{d}_i\right\}$, which is the expectation operator, conditioned on the information that the firm was not shut down. However, it is assumed that demand shocks are serially uncorrelated. Therefore, it is possible to replace $E\left\{\tilde{V}_i' \middle| \tilde{d}_i > \underline{d}_i\right\}$ by $E\left\{V_i\right\}$, i.e. the unconditional expectation operator.

If the owner decides to shut down the firm, he or she earns profits $-\delta I_i$ before selling the capital, worth I_i, at the end of the period.

The continuation value, keeping the firm alive, is the expectation of the next period value, V_i'. The expectation operator is necessary since the demand in the next period is unknown in the current period. However, in equilibrium, the expected value of owning a firm must equal its investment costs. Therefore:

$$E\left\{V_i\right\} = I_i. \tag{7}$$

Hence, in the steady state equilibrium, $E\left\{\hat{V}_i\right\} = E\left\{V_i\right\}$ in (6) can be replaced by I_i. Simplifying this implies:

$$\tilde{v} = \max\left\{\hat{\pi}_i, -\delta I_i\right\} + \frac{I_i}{1+\rho}. \tag{8}$$

Naturally, the owner threatens to shut down the firm only if:

$$\hat{\pi}_i < -\delta I_i. \tag{9}$$

The profit rate if the firm owner does not threaten to shut down, $\hat{\Pi}_i$, is written in lower case letters since the firm owner makes the decision of whether or not to threaten to shut down the firm when the realization of the random variable is known, so that the profit rate is $\hat{\pi}_i$.

Q_i denotes the probability that the firm owner *does not* threaten to shut down the firm. The *expected* value of owning a firm in terms of conditional expectations can be found by rewriting (6):

$$E\left\{V_i\right\} = Q_i\left[E\left\{\hat{\Pi}_i\right\} + \frac{E\left\{\tilde{V}_i' \middle| \tilde{d}_i > \underline{d}_i\right\}}{1+\rho}\right] + (1-Q_i)\left[-\delta I_i + \frac{I_i}{1+\rho}\right]. \tag{10}$$

That is, Q_i is the probability that the firm owner does not threaten to shut down and is defined endogenously from the condition in (9). $E\left\{\hat{\Pi}\right\}$ is the expectation given

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that it is known that the firm owner did not threaten to shut down the firm. That is, the realized profit rate is greater than the profit rate if the firm owner threatens to shut down, i.e. $\hat{\pi}_i \geq -\delta I_i$.

Simplifying $E\{V_i\}$ using the steady state equilibrium condition, $E\{\tilde{V}_i \mid \tilde{d}_i > \underline{d}_i\} = E\{V_i\} = I_i$ implies:

$$E\{V_i\} = \frac{1+\rho}{\rho} \left[Q_i E\{\hat{\Pi}_i\} - (1-Q_i)\delta I_i \right] = I_i. \tag{11}$$

Notice that in a world without uncertainty and continuous time, this condition reduces to $\pi/\rho = I_i$. Remember that this condition stems from the steady state equilibrium condition $E\{V_i\} = I_i$, and holds due to free entry. New firms enter or leave at a rate such that the value of starting a new firm is always zero. Solving for $E\{\hat{\Pi}_i\}$ gives:

$$E\{\hat{\Pi}_i\} = \frac{\rho + (1-Q_i)(1+\rho)\delta}{(1+\rho)Q_i} I_i. \tag{12}$$

3.2 Workers

The economy is populated by $H_i + H_s$ high-skill workers and $L_i + L_s$ low-skill workers. The H_i high-skill workers are employed by in-house firms, and the H_s high-skill workers are employed by firms specialized in producing intermediate goods necessary to assemble the Y good. L_i and L_s are interpreted analogously.

3.2.1 In-house Workers

While the losses of firm owners are limited by the depreciation of capital and foregone interest payments, workers are left without any wage payments if the firm is shut down. By assuming that unemployment benefits are paid only if workers are unemployed at the beginning of the period, workers and firm owners always reach an agreement in order to save the firm. Therefore workers always accept lower wage rates in order to assure that the owner does not shut down the firm.

The firm owner accepts *losses* less than the capital replacement cost δI_i; see (9). Profits are defined including capital replacement costs; that is, revenues minus the wage bill minus capital replacement costs: $\tilde{\Pi}_i = \tilde{R}_i - wb_i - \delta I_i$. The owner therefore shuts down the firm if $\tilde{r} < wb$, i.e. if the revenue realization is insufficient to cover variable costs. Again, lower case letters are used in the condition, since firm owners base their decision on the realization of revenues and the non-stochastic wage bill.

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To save the firm, workers must agree on wage rates such that wage costs are covered by revenues. Utility maximizing employees naturally agree on wage rates such that the owner is indifferent about shutting down the firm and keeping it alive. That is, workers renegotiate wages such that wage costs equal revenues, $wb = \tilde{r}$. Therefore, the firm is never shut down.

The wage rate paid to workers depends on whether the firm owner is inclined to shut down the firm or not. Expected wages of workers satisfy:

$$E\{W_i\} = Q_i w + (1 - Q_i)E\{\check{W}_i\}. \tag{13}$$

The expected wage rate for workers is simply the sum of the expected value if the firm owner does not threaten to shut down and the expected value if the firm owner threatens to shut down, weighted by the appropriate probabilities. From (9) it is clear that the firm owner threatens to shut down if and only if $\hat{\pi}_i < -\delta I_i$, which occurs with probability $1 - Q_i$.

The wage rate paid if the firm owner does not threaten to shut down is non-stochastic and the expected value, conditioned on the information that the firm owner does not threaten to shut down is simply w.

Given only the information that the firm owner threatens to shut down the firm, i.e. that $\hat{\pi}_i < -\delta I_i$, there is a range of possible realizations for the demand variable satisfying this condition. Each such realization implies a different wage rate if the workers of the firm agree on lowering their wage rates. Therefore the wage rate, if the firm owner threatens to shut down, is stochastic and the expectation must be conditioned on the information that $\hat{\pi}_i < -\delta I_i$, hence the use of $E\{\check{W}_i\}$.

3.2.2 Bargaining Positions

In the Nash solution to the bargaining problem, the difference between the parties' outside options is the major determinant of the outcome. In order to determine the outside option of high-skill workers and low-skill workers, every worker's lifetime utility, employed and unemployed, must be derived.

There is frictional unemployment, implying that unemployed workers cannot find employment instantaneously. An unemployed worker receives unemployment benefits. The unemployment benefit is a fraction, u_b, of the worker's average, i.e. expected, wage. Therefore the unemployment benefit is $u_b E\{W_i\}$. Note that an unemployed low-skill worker receives a fraction of the average wage of low-skill workers, while a high-skill worker receives a fraction of the average wage of high-skill workers. In both cases this fraction is u_b. Let $E\{J_i\}$ denote the expected discounted lifetime utility of a currently employed worker , and let

$E\{U_i\}$ denote the expected discounted lifetime utility of a currently unemployed worker. In steady state, $E\{J_i\}$ and $E\{U_i\}$ satisfy:

$$E\{J_i\} = E\{W_i\} + \frac{E\{J_i\}}{1+\rho} \tag{14a}$$

$$E\{U_i\} = u_b E\{W_i\} + \frac{\theta E\{J_i\} + (1-\theta)E\{U_i\}}{1+\rho}. \tag{14b}$$

Employed workers are paid the stochastic wage rate $\tilde{W}_i$ and the expected continuation value is $E\{J_i\}$. Note that firms are never shut down due to adverse demand shocks, since high-skill and low-skill workers always reach an agreement on lower wages. The unemployed worker receives unemployment benefits equal to $u_b E\{W_i\}$, becomes employed in the next period with probability θ, and stays unemployed with probability $1-\theta$. The θ coefficient parameterizes the matching quality in the labor market. Solving for $E\{J_i\}$ and $E\{U_i\}$ implies:

$$E\{J_i\} = \frac{1+\rho}{\rho} E\{W_i\} \tag{15a}$$

$$E\{U_i\} = \frac{1+\rho}{\rho} \frac{\rho u_b + \theta}{\rho + \theta} E\{W_i\}. \tag{15b}$$

This specification implies a logic inconsistency. If workers do not face any risk of becoming unemployed, in the long run the economy must converge to full employment. The common solution to this problem is to add an exogenous shock such that the firm is shut down with some exogenous probability. The analysis in this paper can easily be extended in that direction without changing any of the results. However, to minimize the notation this is not done, and this inconsistency is overlooked.

3.2.3   In-house Bargaining

If an in-house firm is about to be shut down, high-skill and low-skill workers negotiate new wage rates via union representatives in order to motivate the firm owner not to shut down. Let $\check{r}_i$ denote the revenues to be distributed among high-skill and low-skill workers. The lower case notation is used since negotiations are done ex post the realization of the demand variable and the revenue of the firm is known to all parties. The outcome is described by the Nash solution for the bargaining problem where γ denotes the bargaining power of high-skill workers, and $1-\gamma$ the bargaining power of low-skill workers. The share of revenues captured

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by high-skill workers, ψ^*, is the share of revenues which maximizes the Nash product:

$$\psi^* = \underset{\psi}{\text{argmax}} \quad \gamma \log \left[\frac{\psi \check{r}_i}{h_i} + \frac{E\left\{J'_{ih}\right\} - E\left\{U'_{ih}\right\}}{1+\rho} \right] +$$

$$(1-\gamma) \log \left[\frac{(1-\psi)\check{r}_i}{l_i} + \frac{E\left\{J'_{il}\right\} - E\left\{U'_{il}\right\}}{1+\rho} \right]. \quad (16)$$

The expected lifetime utility can be decomposed into an instantaneous pay-off and a continuation value. The instantaneous pay-off for high-skill workers, if the parties reach an agreement, is ψ times the revenues of the firm, $\check{r}_i$, divided by the number of high-skill time units employed, h_i. The continuation value is the discounted lifetime utility being employed.

If the parties cannot reach an agreement, the firm is shut down and the high-skill worker becomes unemployed. His or her continuation value and expected discounted lifetime utility is in this case $E\left\{U'_{ih}\right\}/(1+\rho)$, which is the threat point of high-skill workers. This specification is a consequence of the assumption that unemployed workers do not receive any unemployment benefits the period they become unemployed.

Given that ψ denotes the share of revenues captured by high-skill workers, the share of revenues captured by low-skill workers is $1-\psi$. The interpretation of the second term, i.e. the bargaining position of low-skill workers, is analogous. Solving this problem for ψ^*:

$$\psi^* = \gamma + \gamma l_i \frac{E\left\{J'_{il}\right\} - E\left\{U'_{il}\right\}}{(1+\rho)\check{r}_i} - (1-\gamma)h_i \frac{E\left\{J'_{ih}\right\} - E\left\{U'_{ih}\right\}}{(1+\rho)\check{r}_i}. \quad (17)$$

In steady state, $E\left\{J'_i\right\} = E\left\{J_i\right\}$ and $E\left\{U'_i\right\} = E\left\{U_i\right\}$. Replacing $E\left\{J'_{ih}\right\} - E\left\{U'_{ih}\right\}$ and $E\left\{J'_{il}\right\} - E\left\{U'_{il}\right\}$ using (15a) – (15b) simplifies the steady state bargaining outcome such that:

$$\psi^* = \gamma + \gamma(1-u_b)l_i \frac{E\left\{W_{il}\right\}}{(\rho+\theta_{il})\check{r}_i} - (1-\gamma)(1-u_b)h_i \frac{E\left\{W_{ih}\right\}}{(\rho+\theta_{ih})\check{r}_i}. \quad (18)$$

4   Firms Revisited

The following sections derive the optimal management of firms, or how to maximize the rate of profit given the firm owners decision of whether or not to produce.

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Hence, derivations in the following section pin down the flows generated by firms, such as profit, wage, and employment rates. This is in contrast to the problem of the firm owners, such as whether or not to keep the firm alive or when to invest, which was analyzed in previous sections. It is assumed that there is no conflict in the objectives of owners and managers, so those words can be used interchangeably.

The first sub-section analyzes in-house firms, while the second sub-section analyzes fragmented and specialized firms. In-house firms must commit to an employment choice ex ante the realization of the demand shock, while fragmented firms purchase intermediate goods ex post the realization of the demand shock.

Quantities referring to in-house firms are subscripted by an i and quantities referring to fragmented firms are subscripted by a f. Quantities derived from an optimization problem are superscripted by $*$. As before, $\hat{}$ and $\check{}$ are used to distinguish scenarios where firm owners do not threaten to shut down the firm and where firm owners do threaten to shut down the firm, respectively.

4.1 In-house Firms

The choices of in-house firm owners involve shutting down the firm or keeping it alive. The firm owner must commit to an employment choice prior to deciding whether or not to threaten to shut down the firm. This is a reasonable assumption if demand changes frequently, relative to the turnover rate of workers.

Before deriving the optimal choices of firm owners, remember that the profit rate if the firm owner shuts down the firm is the non-stochastic capital replacement cost, equal to $-\delta I_i$. Given that the firm owner does not threaten to shut down the firm, the wage rate for high-skill workers is non-stochastic and equals w_{ih}, while the wage rate for low skill workers, which is also non-stochastic, is w_{il}.

In-house firms produce the X and Z goods by hiring high-skill and low-skill workers. Augmenting the production functions in (3a) by an i subscript for in-house firms gives

$$x_i = h_i \quad z_i = l_i \quad y_i = x_i^{\alpha} z_i^{1-\alpha} \tag{19}$$

where h_i is the firm's total use of high-skill labor and l_i is the firm's total use of low-skill labor. The firm owner maximizes the expected profit rate:

$$E\left\{\Pi_i\right\} \quad = \quad Q_i E\left\{\hat{\Pi}_i\right\} y_i - (1-Q_i)\delta I_i.$$

The first term captures the expected profit rate, if the firm owner does not threaten to shut down the firm. The second term captures the non-stochastic profit rate, if

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the firm owner threatens to shut down the firm. The profit rate if the firm owner does not threaten to shut down the firm is

$$\hat{\Pi}_i = \hat{P}_i y_i - w_{ih} h_i - w_{il} l_i - \delta I_i$$

where the inverse demand function, given by (2a), restricts the owners feasible choices of y_i. The price of the consumption good is written in upper case since it, via (2a), is stochastic. Note that y_i is certain since the firm owner can control the number of workers to employ and thereby the output of the firm, hence also h_i and l_i are non-stochastic. Even though the wage rates are stochastic, the wage rates conditional on the firm owner not threatening to shut down, are not. Hence w_{ih} and w_{il} are used.

The problem for the firm owner is complicated by the fact that the probability that the firm owner will not find it optimal to threaten to shut down the firm, Q_i, depends on the choice of employment, h_i and l_i. That is, a rational firm owner must take into account the impact of his or her employment choice today, on the probability that he or she will threaten to shut down the firm during the period.

There exists a minimal realization of $\tilde{D}_i$, the stochastic demand variable, such that the firm owner is willing to keep producing. This threshold value, denoted $\underline{d}_i$, is defined by relation (9) as:

$$\hat{\pi}_i \bigg|_{\tilde{d}_i = \underline{d}_i} = -\delta I_i \ . \tag{20}$$

The probability that the firm owner is not inclined to threaten to shut down the firm, given $\underline{d}_i$, is simply $Q_i = 1 - F_i(\underline{d}_i)$. The cumulative density function, $F_i(\cdot)$, is in turn given by (1).

4.1.1 Employment and Firm Size

The problem solved by the firm owner in order to determine employment of high-skill and low-skill workers becomes:

$$\max_{\underline{d}_i, h_i, l_i} \quad [1 - F(\underline{d}_i)] \left[E\left\{\hat{P}_i\right\} y_i - w_{ih} h_i - w_{il} l_i \right] - \delta I_i$$

$$s.t. \quad (2a), (19), (20).$$

The wage rates for high-skill and low-skill labor, taken as given by the firm, are denoted w_{ih} and w_{il}, respectively. These wage rates are called the competitive

wage rates and are paid to workers, only if the firm owner decides to keep the firm alive. Due to capital depreciation, the firm owner must pay δI_i to replace depreciated capital.

The solution to this problem is derived in Appendix B and the unique maximizing choice of $(\underline{d}_i, h_i, l_i)$ is:

$$
\underline{d}_i^* = \begin{cases} \frac{(1-\beta)(1+\Delta)}{1+\beta} & \beta \leq \Delta \\ 1-\Delta & \beta > \Delta \end{cases} \tag{21a}
$$

$$
h_i^{*\beta} = (1-\beta)E\left\{\hat{D}_i^*\right\}\left[\frac{C}{\overline{p}}\right]^\beta \left[\frac{\alpha}{w_{ih}}\right]^{1-(1-\alpha)(1-\beta)}\left[\frac{1-\alpha}{w_{il}}\right]^{(1-\alpha)(1-\beta)} \tag{21b}
$$

$$
l_i^{*\beta} = (1-\beta)E\left\{\hat{D}_i^*\right\}\left[\frac{C}{\overline{p}}\right]^\beta \left[\frac{\alpha}{w_{ih}}\right]^{\alpha(1-\beta)}\left[\frac{1-\alpha}{w_{il}}\right]^{1-\alpha(1-\beta)}. \tag{21c}
$$

As is shown in Appendix B there are two solutions. If the variation in demand is small compared to the degree of market power, $\Delta < \beta$, firm owners never find it optimal to exercise the option to shut down the firm. In this case $\underline{d}_i^* = 1 - \Delta$, $Q_i^* = 1$ and $E\left\{\hat{D}_i^*\right\} = E\left\{D_i\right\} = 1$.

The more interesting case, where firm owners occasionally exercise their right to shut down the firm, applies in the opposite case, when the variation in demand is large compared to the degree of market power, , i.e. $\beta \leq \Delta$. In this case the firm owner threatens to shut down the firm if the realization of $\tilde{D}_i$ is less than $\underline{d}_i^* = (1-\beta)(1+\Delta)/(1+\beta)$ and $E\left\{\hat{D}_i^*\right\} > E\left\{\tilde{D}_i\right\}$.

This of course implies that market power in the product market shelters workers from variation in wages, i.e. risk, and can be welfare improving if insurance markets are absent and workers are risk averse. Given the shutdown threshold, $\underline{d}_i^*$, it is possible to compute the different conditional expectations of the demand variable:

$$
E\left\{\hat{D}_i^*\right\} = E\left\{\hat{D}_i\right\}\Big|_{\underline{d}_i^*} = \begin{cases} \frac{1+\Delta}{1+\beta} & \beta \leq \Delta \\ 1 & \beta > \Delta \end{cases} \tag{22a}
$$

$$
E\left\{\check{D}_i^*\right\} = E\left\{\check{D}_i\right\}\Big|_{\underline{d}_i^*} = \begin{cases} \frac{1-\beta\Delta}{1+\beta} & \beta \leq \Delta \\ 1-\Delta & \beta > \Delta \end{cases}. \tag{22b}
$$

A firm owner threatens to shut down the firm with probability $1 - Q_i$, i.e. only if $\tilde{d}_i < \underline{d}_i^*$. The probability that the firm owner does not threaten to shut down

the firm is therefore $1 - F(\underline{d}_i^*)$. From the definition of the cumulative density function in (1), and the solution to the profit maximization problem in (21a), it follows that:

$$Q_i^* = \left. Q_i \right|_{\underline{d}_i^*} = \begin{cases} \frac{1+\Delta}{\Delta} \frac{\beta}{1+\beta} & \beta \leq \Delta \\ 1 & \beta > \Delta \end{cases}. \tag{23}$$

This implies that as demand uncertainty increases (Δ closer to unity), the probability that firm owners pay the competitive wage rate decreases. Hence, greater demand uncertainty tends to increase the competitive wage rate but also to decrease the probability that the worker receives the competitive wage rate. The neatest property of Q_i^* is that it is independent of endogenous variables. Q_i^* only depends on two parameters: the variation in demand, Δ, and the preference for variety, β.

It is difficult to predict the effect of greater market power, i.e. more preference for variety, on the size of the firm since β is present in the exponents in the expressions for h_i and l_i. However, an interesting result concerning the effect of shutdown threats and firm size is easily obtained:

Proposition 4.1.1 (Firm Size and Demand Uncertainty) *If firm owners have the option to shut down the firm in order to avoid variable costs, i.e. paying the wage bill, greater demand uncertainty (Δ greater) implies larger firms.*

Proof The result that firm size increases with demand uncertainty is easily verified by noting that the derivative of h_i and l_i with respect to Δ is greater than zero. ∎

It might appear surprising that firm size increases with uncertainty. However, as noted in Section 2.2, the revenue function is linear with respect to the demand shock. This in turn implies that the risk neutral firm owner, without the option to shut down the firm, is not affected by a mean preserving spread in demand. Hence, if the variation in demand increases, this firm owner does not change the employment level, and the expected profit rate stays unchanged.

However, given the option to shut down the firm, the expected profit rate must increase, or at least not decrease. This follows because the firm owner can choose to ignore the option to shut down the firm. However, as is shown above, the firm owner does indeed occasionally utilize the option to shut down the firm, if $\beta \leq \Delta$. By hiring more workers and threatening to shut down the firm in case of

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sufficiently low demand, the firm owner can increase the expected profit rate. If $\beta > \Delta$ the firm owner never threatens to shut down the firm.

H_i and L_i denote aggregate employment of high-skill and low-skill workers by in-house firms. Because h_i^* and l_i^* are identical across in-house firms, no variable on the right hand side is firm specific. The competitive wage rates are easily obtained by integrating labor demand over the range of in-house firms and solving for w_{ih} and w_{il}:

$$w_{ih} = \alpha(1-\beta)E\left\{\hat{D}_i^*\right\}\frac{K_i}{H_i}\left[\frac{C}{\overline{p}}\right]^{\beta}\left[\frac{H_i^{\alpha}L_i^{1-\alpha}}{K_i}\right]^{1-\beta} \tag{24a}$$

$$w_{il} = (1-\alpha)(1-\beta)E\left\{\hat{D}_i^*\right\}\frac{K_i}{L_i}\left[\frac{C}{\overline{p}}\right]^{\beta}\left[\frac{H_i^{\alpha}L_i^{1-\alpha}}{K_i}\right]^{1-\beta}. \tag{24b}$$

These wage rates are called competitive since they are derived from the demand of profit maximizing firms, taking the wage rate as given. However, they are not identical to wage rates on perfectly competitive markets since firms do not take the price of their output as given. The relative competitive wage rates reduce to the standard Cobb-Douglas case where the relative wage is determined by relative employment and the elasticity of substitution between high-skill and low-skill labor.

The wage rates are easily interpreted. Given the Dixit and Stiglitz preferences, workers are paid a share of revenues equal to $1-\beta$, while firm owners receive the remaining share, β. Due to the Cobb-Douglas production function, high skill workers, as a group, receive a fraction equal to α while low-skill workers, as a group, receive the remaining part, as will be clear below. Hence, competitive wage rates increase one-to-one with expected productivity. This in turn implies that the competitive wage rates increase with greater demand uncertainty, i.e. Δ closer to unity. The interpretation is straightforward with greater variation in demand, the threshold for not threatening to shut down the firm is higher; see (21a). Therefore the expected competitive wage rate is higher. It is of course important to remember that changing demand uncertainty, Δ, also changes the probability that the firm owner does not threaten to shut down the firm and pays the workers the competitive wage rates.

4.1.2 Entry and Exit

The expected profit rate $E\left\{\hat{\Pi}_i\right\}$ can be reduced using the competitive wage rate expressions, (24a) and (24b), together with the symmetric employment condi-

tions, $h_i = H_i/K_i$ and $l_i = L_i/K_i$:

$$E\{\hat{\Pi}_i\} = \beta E\{\hat{D}_i^*\} \left[\frac{C}{\overline{p}}\right]^{\beta} \left[\frac{H_i^{\alpha}L_i^{1-\alpha}}{K_i}\right]^{1-\beta} - \delta I_i.$$

New firms enter or existing firms leave the market unless the value of owning an in-house firm equals the initial investment cost. This occurs unless (12) is satisfied. The steady state equilibrium number of in-house firms is be:

$$K_i = \left[\beta Q_i^* E\{\hat{D}_i^*\} \frac{1+\rho}{[\rho+\delta(1+\rho)]I_i}\right]^{\frac{1}{1-\beta}} \left[\frac{C}{\overline{p}}\right]^{\frac{\beta}{1-\beta}} H_i^{\alpha} L_i^{1-\alpha}. \tag{26}$$

This relation provides a necessary condition for determining the number, i.e. range, of in-house firms in the steady state equilibrium. Because capital can be resold if the firm is shut down, the only real cost of starting a firm is the capital depreciation, $\delta > 0$, and the inter-temporal cost of giving up I_i while the firm is operating. The latter cost hinges on $\rho > 0$. Without depreciation and without impatience, $\rho = \delta = 0$, the cost of starting a firm is zero, and the steady state equilibrium number of firms must equal infinity, i.e. $K_i \to \infty$.

The following results are easily verified and most of them are intuitive:

- Higher investment costs decrease the number of firms.

- A higher rate of depreciation decreases the number of firms.

- More impatient investors decreases the number of firms.

- More demand uncertainty decreases the number of firms.

The first three results are intuitive while. The last result is an equilibrium result. There is a fixed number of workers and more demand uncertainty increases the firm size, hence in equilibrium the number of firms must decrease. The effect of greater market power, β greater, is again ambiguous since β appears in the exponents in (26).

4.1.3 Wages

If the firm owner does not threaten to shut down the firm, the competitive wage rates, denoted w_{ih} and w_{il}, are paid to high-skill and low-skill workers. These wage rates are non-stochastic but depend on the variation in demand, Δ, and the

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preference for variety, i.e. the degree of market power, β. If the firm owner credibly threatens to shut down the firm, high-skill and low-skill workers negotiate new wage rates. The negotiated wage rate for high-skill workers is stochastic and denoted by $\check{W}_{ih}$ and the negotiated wage rate for low-skill workers, also stochastic, is denoted by $\check{W}_{il}$.

The expected wage rate of high-skill workers, $E\left\{\tilde{W}_{ih}\right\}$, is the probability that the firm owner does not threaten to shut down the firm, Q_i^*, times the non-stochastic competitive wage rate w_{ih}, plus the probability that the firm owner threatens to shut down the firm, $1 - Q_i^*$, times the conditional expectation of the stochastic negotiated wage rate, $E\left\{\check{W}_{ih}\right\}$. The expected wage rate for low-skill workers, $E\left\{\tilde{W}_{il}\right\}$, is analogous. Therefore:

$$E\left\{\tilde{W}_{ih}\right\} = Q_i^* w_{ih} + (1 - Q_i^*) E\left\{\check{W}_{ih}\right\} \tag{27a}$$

$$E\left\{\tilde{W}_{il}\right\} = Q_i^* w_{il} + (1 - Q_i^*) E\left\{\check{W}_{il}\right\}. \tag{27b}$$

The firm's revenue is stochastic and denoted $\tilde{R}_i$. Because all in-house firms are ex-ante identical, they employ the same amount of high-skill and low-skill labor units, namely H_i/K_i and L_i/K_i respectively, implying that revenues for an in-house firm reduce to:

$$\tilde{R}_i = \tilde{D}_i \left[\frac{C}{\overline{p}}\right]^\beta \left[\frac{H_i^\alpha L_i^{1-\alpha}}{K_i}\right]^{1-\beta}. \tag{28}$$

The negotiated wage rates for high-skill and low-skill workers equal $\check{W}_{ih} = \psi^* \check{R}_i K_i / H_i$ and $\check{W}_{il} = (1 - \psi^*) \check{R}_i K_i / L_i$, respectively. The share of revenues captured by high-skill workers, ψ^*, is given by the Nash solution to the bargaining problem in (16). By comparing firm revenue, see (28), and the competitive wage rates, see (24a) and (24b), the competitive wage rates can be rewritten in terms of firm revenue. Hence:

$$\check{W}_{ih} = \psi^* \check{R}_i \frac{K_i}{H_i} \qquad w_{ih} = \alpha(1-\beta) \frac{K_i}{H_i} E\left\{\hat{R}_i\right\} \tag{29a}$$

$$\check{W}_{il} = (1 - \psi^*) \check{R}_i \frac{K_i}{L_i} \qquad w_{il} = (1-\alpha)(1-\beta) \frac{K_i}{L_i} E\left\{\hat{R}_i\right\}. \tag{29b}$$

The expected wage rates for in-house high-skill and low-skill workers can in turn be written:

$$E\left\{W_{ih}\right\} = \left[\alpha(1-\beta) Q_i^* E\left\{\hat{R}_i\right\} + (1-Q_i^*) E\left\{\psi^* \check{R}_i\right\}\right] \frac{K_i}{H_i}$$

$$E\left\{W_{il}\right\} = \left[(1-\alpha)(1-\beta) Q_i^* E\left\{\hat{R}_i\right\} + (1-Q_i^*) E\left\{(1-\psi^*)\check{R}_i\right\}\right] \frac{K_i}{L_i}.$$

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It is now possible to solve for $E\left\{\psi^*\check{R}_i\right\}$ and $E\left\{(1-\psi^*)\check{R}_i\right\}$ by using the Nash solution for the bargaining problem; see (18). To do so, replace $E\left\{W_{ih}\right\}$ and $E\left\{W_{il}\right\}$ in (18) using the expected wage expressions above. After some cumbersome algebra

$$
\begin{aligned}
E\left\{\psi^*\check{R}_i\right\} &= \frac{\gamma\overline{\rho}_{ih}\left[\overline{\rho}_{il}+(1-Q_i^*)(1-u_b)\right]E\left\{\check{R}_i\right\}}{\overline{\rho}_{ih}\overline{\rho}_{il}+(1-Q_i^*)(1-u_b)\left[\gamma\overline{\rho}_{ih}+(1-\gamma)\overline{\rho}_{il}\right]} \\
&\quad - \frac{Q_i^*(1-u_b)(1-\beta)\left[\alpha(1-\gamma)\overline{\rho}_{il}-\gamma(1-\alpha)\overline{\rho}_{ih}\right]E\left\{\hat{R}_i\right\}}{\overline{\rho}_{ih}\overline{\rho}_{il}+(1-Q_i^*)(1-u_b)\left[\gamma\overline{\rho}_{ih}+(1-\gamma)\overline{\rho}_{il}\right]}
\end{aligned}
$$

(31a)

$$
\begin{aligned}
E\left\{(1-\psi^*)\check{R}_i\right\} &= \frac{(1-\gamma)\overline{\rho}_{il}\left[\overline{\rho}_{ih}+(1-Q_i^*)(1-u_b)\right]E\left\{\check{R}_i\right\}}{\overline{\rho}_{ih}\overline{\rho}_{il}+(1-Q_i^*)(1-u_b)\left[\gamma\overline{\rho}_{ih}+(1-\gamma)\overline{\rho}_{il}\right]} \\
&\quad + \frac{Q_i^*(1-u_b)(1-\beta)\left[\alpha(1-\gamma)\overline{\rho}_{il}-\gamma(1-\alpha)\overline{\rho}_{ih}\right]E\left\{\hat{R}_i\right\}}{\overline{\rho}_{ih}\overline{\rho}_{il}+(1-Q_i^*)(1-u_b)\left[\gamma\overline{\rho}_{ih}+(1-\gamma)\overline{\rho}_{il}\right]}
\end{aligned}
$$

(31b)

where

$$
\overline{\rho}_{ih}\equiv\rho+\theta_{ih} \qquad \overline{\rho}_{il}\equiv\rho+\theta_{il}.
$$

The expressions above are quite messy. Most interestingly, equal bargaining power $\gamma=1/2$, similar discount rates and labor market conditions for high-skill and low-skill workers, i.e. $\overline{\rho}_{ih}=\overline{\rho}_{il}$ and $\alpha>1/2$, imply that $w_{ih}>w_{il}$, and $E\left\{(1-\psi^*)\check{R}_i\right\}>E\left\{\psi^*\check{R}_i\right\}$.

With probability $1-Q_i^*$, high- and low-skill workers are paid the negotiated wage rates, $\check{W}_{ih}$ and $\check{W}_{il}$. These are easily rewritten in terms of the competitive wage rates, w_{ih} and w_{il}, by use of (29a) and (29b). The expected negotiated high-skill wage rates are:

$$
\begin{aligned}
E\left\{\check{W}_{ih}\right\} &= \frac{1}{(1-\beta)}\frac{E\left\{\psi^*\check{R}_i\right\}}{\alpha E\left\{\hat{R}_i\right\}}w_{ih} \\
E\left\{\check{W}_{il}\right\} &= \frac{1}{(1-\beta)}\frac{E\left\{(1-\psi^*)\check{R}_i\right\}}{(1-\alpha)E\left\{\hat{R}_i\right\}}w_{il}.
\end{aligned}
$$

Given the expected wage rates in both scenarios, the expected wage rates satisfy:

$$
E\left\{W_{ih}\right\} = \left[Q_i^*+\frac{1-Q_i^*}{(1-\beta)}\frac{E\left\{\psi^*\check{R}_i\right\}}{\alpha E\left\{\hat{R}_i\right\}}\right]w_{ih}
$$

(32a)

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$$E\{W_{il}\} = \left[Q_i^* + \frac{1-Q_i^*}{(1-\beta)}\frac{E\{(1-\psi^*)\check{R}_i\}}{(1-\alpha)E\{\hat{R}_i\}}\right]w_{il}. \tag{32b}$$

The relative competitive wage rate reduces to the standard Cobb-Douglas relative wage. That is, from (24a) and (24b) it follows that:

$$\frac{w_{ih}}{w_{il}} = \frac{\alpha}{1-\alpha}\frac{L_i}{H_i}$$

Using (32a) and (32b) it is straightforward to show that:

$$\frac{E\{W_{ih}\}}{E\{W_{il}\}} \leq \frac{w_{ih}}{w_{il}} \iff \frac{E\{\psi^*\check{R}_i\}}{E\{(1-\psi^*)\hat{R}_i\}} \leq \frac{\alpha}{1-\alpha}. \tag{33}$$

If this condition is fulfilled, then the possibility for owners to shut down the firm increases the relative wage of low-skill workers compared to high-skill workers. By imposing symmetry conditions, i.e. $\bar{\rho}_{ih} = \bar{\rho}_{il}$ and $\gamma = 1/2$, in (31a) and (31b) this is clearly true for $\alpha > 1/2$, while not true for $\alpha < 1/2$. This implies that shutdown threats moderate wage differences across skills since relative wages are determined by bargains if the firm owner threatens to shut down the firm.

In the simplest case with a 100 percent replacement rate in case of unemployment, i.e. $u_b = 1$, $\psi^*/(1-\psi^*)$ reduces to $\gamma/(1-\gamma)$. A lower replacement ratio (u_b closer to zero) decreases the outside option of high-skill and low-skill workers. This benefits low-skill workers relative to high-skill workers, because the surplus to bargain over, which is divided equally if γ equals $1/2$, increases. While it is difficult to verify this claim algebraically, numerical examples (see below) support this intuitive conjecture.

This completes the description of in-house firms. Taking employment, i.e. H_i and L_i, as given, all endogenous in-house firm variables are pinned down, either explicitly or implicitly.

4.2 Fragmented and Specialized Firms

The following section analyzes fragmented and specialized firms. Fragmented firms subcontract production of intermediate goods to specialized firms. There are two types of specialized firms: producers of the X good, i.e. the high-skill intermediate good, and producers of the Z good, i.e. the low-skill intermediate good.

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The markets for intermediate goods are not analyzed in detail. Due to either transaction costs or non-competitive markets, the markup over marginal cost is m_x for the X good and m_z for the Z good. Since the X good is produced by a constant returns to scale technology in high-skill labor only, the price of the X good is simply the markup times the wage rate for high-skill labor, i.e. $m_x w_{sh}$. Analogously, the price of the Z good is $m_z w_{sl}$, where w_{sl} is the wage rate for low-skill workers and m_z is the markup factor over marginal cost.

Specialized firms, i.e producers of the X or Z good, supply the intermediate good to a continuum of fragmented firms. Therefore, by the mean value theorem, the aggregate demand faced by a producer of the X or Z good is certain. It is assumed that any X or any Z good can be sold to any fragmented firm which implies that fragmented firms can choose the quantity of intermediate goods to use ex post the realization of the demand shock. This assumption can be rationalized if transport time and transport costs are negligible, so that specialized firms are indifferent about which fragmented firm purchases their products.

4.2.1 Fragmented Firms

The owner of a fragmented firm maximizes profit by solving:

$$\tilde{\pi}_f = \left. \tilde{p}_f y_f \right|_{\tilde{D}_f = \tilde{d}_f} - m_x w_{sh} h_s - m_z w_{sl} l_s - \delta I_f.$$

The first term is total revenue, given that the realization of the stochastic demand variable is d_f. The next two terms are the costs of purchasing the high-skill intermediate good (X) and the low-skill intermediate good (Z). The last term is the cost of replacing depreciated capital. The quantity purchased of the X good is denoted h_s. Since the production technology for the X good maps one unit of high-skill labor into one unit of the X good, h_s also denotes high-skill labor requirements. The l_s symbol is interpreted analogously.

This specification implies that specialized firms supply intermediate goods on demand, that isale ex post the realization of the demand variables. This is reasonable since every specialized firm supplies intermediate goods to a large number, i.e. a continuum, of fragmented firms. Since there is no aggregate demand uncertainty, the idiosyncratic demand shocks observed by fragmented firms sum to zero and specialized firms face a certain demand.

To solve the problem of the owner of a fragmented firm, replace $\tilde{p}_f y_f$ using the inverse demand function in (2b), then replace y_f using the production function

in (3b), and finally replace x_f and z_f using the production functions in (3a). The problem for the firm owner is to maximize:

$$\tilde{\pi}_f = \tilde{d}_f \left[\frac{C}{\overline{p}}\right]^\beta h_s^{\alpha(1-\beta)} l_s^{(1-\alpha)(1-\beta)} - m_x w_{sh} h_s - m_z w_{sl} l_s - \delta I_f.$$

Be careful to notice that h_s and l_s do not denote employment of high-skill and low-skill workers, instead they denote the quantity purchased of the intermediate goods (the X and Z goods) necessary to assemble the Y good. Solving this problem is straightforward. The first order conditions are:

$$\alpha(1-\beta)\tilde{d}_f \left[\frac{C}{\overline{p}}\right]^\beta h_s^{\alpha(1-\beta)-1} l_s^{(1-\alpha)(1-\beta)} = m_x w_{sh} \tag{34a}$$

$$(1-\alpha)(1-\beta)\tilde{d}_f \left[\frac{C}{\overline{p}}\right]^\beta h_s^{\alpha(1-\beta)} l_s^{(1-\alpha)(1-\beta)-1} = m_z w_{sl}. \tag{34b}$$

Solving this system for h_s and l_s, or the demand for the X and Z good, is straightforward:

$$h_s = \left\{ (1-\beta)\tilde{d}_f \left[\frac{C}{\overline{p}}\right]^\beta \left[\frac{\alpha}{m_x w_{sh}}\right]^{1-(1-\alpha)(1-\beta)} \left[\frac{1-\alpha}{m_z w_{sl}}\right]^{(1-\alpha)(1-\beta)} \right\}^{1/\beta} \tag{35}$$

$$l_s = \left\{ (1-\beta)\tilde{d}_f \left[\frac{C}{\overline{p}}\right]^\beta \left[\frac{\alpha}{m_x w_{sh}}\right]^{\alpha(1-\beta)} \left[\frac{1-\alpha}{m_z w_{sl}}\right]^{1-\alpha(1-\beta)} \right\}^{1/\beta}. \tag{36}$$

The total supply of high-skill labor employed by firms producing the X (Z) good is H_s (L_s), and since the X (Z) technology maps one unit of high-skill (low-skill) labor into one unit of the X (Z) good, H_s (L_s) is also the aggregate supply of the X (Z) good. The reduced first order conditions above give the demand for the X and Z goods by a single fragmented firm. Aggregate demand is easily obtained by integrating over the continuum of fragmented firms. Clearing the market for high-skill and low-skill labor implies that the equilibrium wage rates must satisfy:

$$w_{sh} = \frac{\alpha(1-\beta)}{m_x} \left[\frac{C}{\overline{p}}\right]^\beta \left[E\left\{\tilde{D}_f^{1/\beta}\right\}\right]^\beta \frac{K_f}{H_s} \left[\frac{H_s^\alpha L_s^{1-\alpha}}{K_f}\right]^{1-\beta} \tag{37}$$

$$w_{sl} = \frac{(1-\alpha)(1-\beta)}{m_z} \left[\frac{C}{\overline{p}}\right]^\beta \left[E\left\{\tilde{D}_f^{1/\beta}\right\}\right]^\beta \frac{K_f}{L_s} \left[\frac{H_s^\alpha L_s^{1-\alpha}}{K_f}\right]^{1-\beta}. \tag{38}$$

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4.2.2   Entry and Exit

To find the expected profit rate it is necessary to find each fragmented firm's relative use of the X and Z good, meaning it is necessary to find h_s/H_s and l_s/L_s. The h_s and l_s differ among fragmented firms since different firms experience different demand shocks. Aggregating using (35) and (36), it is easy to see that:

$$\frac{h_s}{H_s} = \frac{\tilde{d}_f^{1/\beta}}{E\left\{\tilde{D}_f^{1/\beta}\right\}}\frac{1}{K_f} \qquad \frac{l_s}{L_s} = \frac{\tilde{d}_f^{1/\beta}}{E\left\{\tilde{D}_f^{1/\beta}\right\}}\frac{1}{K_f}. \tag{39}$$

First inserting the first order conditions in (34a) and (34b) into the objective function, (34), then replacing h_s and l_s using the relative use of the X and Z good above, the profit rate of a fragmented firm becomes:

$$\tilde{\pi}_f = \beta\frac{\tilde{d}_f^{1/\beta}}{\left[E\left\{\tilde{D}_f^{1/\beta}\right\}\right]^{1-\beta}}\left[\frac{C}{\overline{p}}\right]^{\beta}\left[\frac{H_s^{\alpha}L_s^{1-\alpha}}{K_f}\right]^{1-\beta} - \delta I_f. \tag{40}$$

The initial cost for starting a fragmented firm is I_f. Unless the value of owning a fragmented firm equals the initial investment cost, new firms are started or existing firms are shut down. In the steady state equilibrium, (12) must be satisfied. Using (12), letting $Q_i = 1$, $I = I_f$, and $E\left\{\hat{\Pi}_i\right\} = E\left\{\tilde{\Pi}_f\right\}$ provides the equation necessary to solve for K_f:

$$K_f = \left[\frac{\beta}{I_f}\frac{1+\rho}{(\rho+(1+\rho)\delta)}\right]^{\frac{1}{1-\beta}}\left[\frac{CE\left\{\tilde{D}_f^{1/\beta}\right\}}{\overline{p}}\right]^{\frac{\beta}{1-\beta}}H_s^{\alpha}L_s^{1-\alpha}.$$

It is interesting to compare this relation with the corresponding expression for in-house firms in (26). Treating $\overline{p}$ as given and looking at either a pure in-house equilibrium, $H_i = H$ and $L_i = H$, or a pure fragmented equilibrium, $H_s = H$ and $L_s = L$, the difference in the range of variations of the consumption good is determined by the difference in the expectation of the demand shock, $\tilde{D}$. If $\left[E\left\{\tilde{D}_f^{1/\beta}\right\}\right]^{\beta} \geq Q_i^*E\left\{\tilde{D}_i^*\right\}$ the range of variations in the fragmented equilibrium exceeds the range of variations in the in-house equilibrium.

4.3 Firms Summary

The only endogenous variables left to determine are the employment variables, i.e. H_i, L_i, H_s, and L_s. To solve for the employment variables, some long run steady state equilibrium conditions are necessary.

5 Equilibrium results

This section discusses some of the results that are possible to derive from the model. Results based on closed form solutions are complemented by figures. First the model is used to analyze what factors affect a potential firm owner's choice to start an in-house or a fragmented firm. Next the effect from shutdown threats on the skill premium is discussed. Finally the results are illustrated in a numerical example where the skill premium is graphed for various parameter values.

5.1 Steady State Equilibria

To solve for the steady state equilibrium it is necessary to determine H_i, L_i, H_s and L_s. Appendix C shows how to determine the type of equilibrium that will exist, using the steady state equilibrium conditions:

$$E\left\{\tilde{W}_{ih}\right\} = w_{sh} \qquad E\left\{\tilde{W}_{il}\right\} = w_{sl}. \tag{41}$$

The equilibrium conditions, which simply state that expected wage rates must be equal in in-house and specialized firms, are a bit simplified. While the proper conditions should be written in terms of lifetime utilities, simplifying them does not alter the results qualitatively, but rather simplifies the exposition.

From Appendix C it follows that the economy will be in a fragmented equilibrium only if ΔI_{if} is positive where ΔI_{if} is defined as:

$$\Delta I_{if} = \left[\frac{w_s}{Q_i^* w_i}\right]^{1/\beta} \left[\frac{1}{\left(\frac{m_x G_{ih}}{Q_i^*}\right)^{\alpha} \left(\frac{m_z G_{il}}{Q_i^*}\right)^{1-\alpha}}\right]^{\frac{1-\beta}{\beta}} I_i - I_f > 0. \tag{42}$$

If this condition is violated, the economy will be in an in-house equilibrium. The interpretation is quite simple. If the investment cost for fragmented firms is large relative to in-house firms, an in-house equilibrium is more likely, and vice versa. This follows from the relation between I_f and I_i in the expression for ΔI_{if}.

The m_x and m_z parameters determine the markup factor over marginal cost for specialized firms producing intermediate goods, i.e. producers of the X and Z goods. The parameters can be interpreted as the degree of competitiveness in the market for the X and Z good. A larger m implies less competitive markets which in turn implies higher prices of intermediates goods. An intuitive conjecture is that a higher degree of market power for specialized firms should decrease the likelihood of a fragmented equilibrium, which is also verified since increasing either of the m's decreases the first term on left hand side.

To investigate the impact of a mean preserving spread in demand, it is assumed that all workers are identical. This assumption sidesteps any distributional consideration and simplifies the expressions. To see this, note that if $\alpha = \gamma = 1/2$ and $\overline{\rho}_{ih} = \overline{\rho}_{il}$, it follows that $G_{ih} = G_{il}$ and $G_{ih}^{\alpha} G_{il}^{1-\alpha}/Q_i^*$ is independent of Δ. The only remaining term depending on the variation in demand, i.e. depending on Δ, is:

$$\frac{w_s}{Q_i^* w_i} = \frac{\left[E\left\{\tilde{D}_f^{1/\beta}\right\}\right]^{\beta}}{Q_i^* E\left\{\hat{D}_i^*\right\}}. \tag{43}$$

To see how the right hand side follows from the left hand side, see the definitions in Appendix C. To analyze the numerator, note that since

$$\frac{\partial E\left\{\tilde{D}_f^{1/\beta}\right\}}{\partial \Delta} = \frac{(1+\Delta/\beta)(1-\Delta)^{1/\beta} - (1-\Delta/\beta)(1+\Delta)^{1/\beta}}{2(1+1/\beta)\Delta^2}, \tag{44}$$

it follows that:

$$\frac{\partial \left[E\left\{D_f^{1/\beta}\right\}\right]^{\beta}}{\partial \Delta} > 0.$$

This relation tells that the value of a fragmented firm increases due to a mean preserving spread in demand. The rationale for this result follows from the demand function derived in Section 2.2. If a firm owner must commit to an employment choice before the demand shock is revealed, a mean preserving spread in demand does not affect the value of the firm. However, an owner of a fragmented firm has the option to choose the production level ex post the demand realization, and this extra option must increase, or at least not decrease, the expected profit and thereby the value of the firm.

Turning to the denominator, when $\beta > \Delta$ the denominator is simply 1. How-

Table 2: Default Parameter Values

$$
\begin{array}{lll}
\delta = 1/10 & \rho = 1 & I_i = I_f = 1 \\
\theta_{ih} = \theta_{il} = 1/2 & C = 1 & H = L = 1
\end{array}
$$

ever, expanding $Q_i^* E \left\{ \hat{D}_i^* \right\}$ assuming that $\beta \leq \Delta$ and using (23) and (22a) gives:

$$
Q_i^* E \left\{ \hat{D}_i^* \right\} \Big|_{\beta < \Delta} = \frac{\beta}{\Delta} \left[\frac{1+\Delta}{1+\beta} \right]^2 . \tag{45}
$$

The derivative of this expression with respect to Δ is clearly negative. The analysis of the numerator and the denominator implies that if workers are identical, a greater variation in demand implies that a fragmented equilibrium is more likely. This implies that the value derived from the option to purchase intermediate goods ex post the realization of the demand shock increases as the variation in demand increases, relative to the value of having the option to shut down the firm.
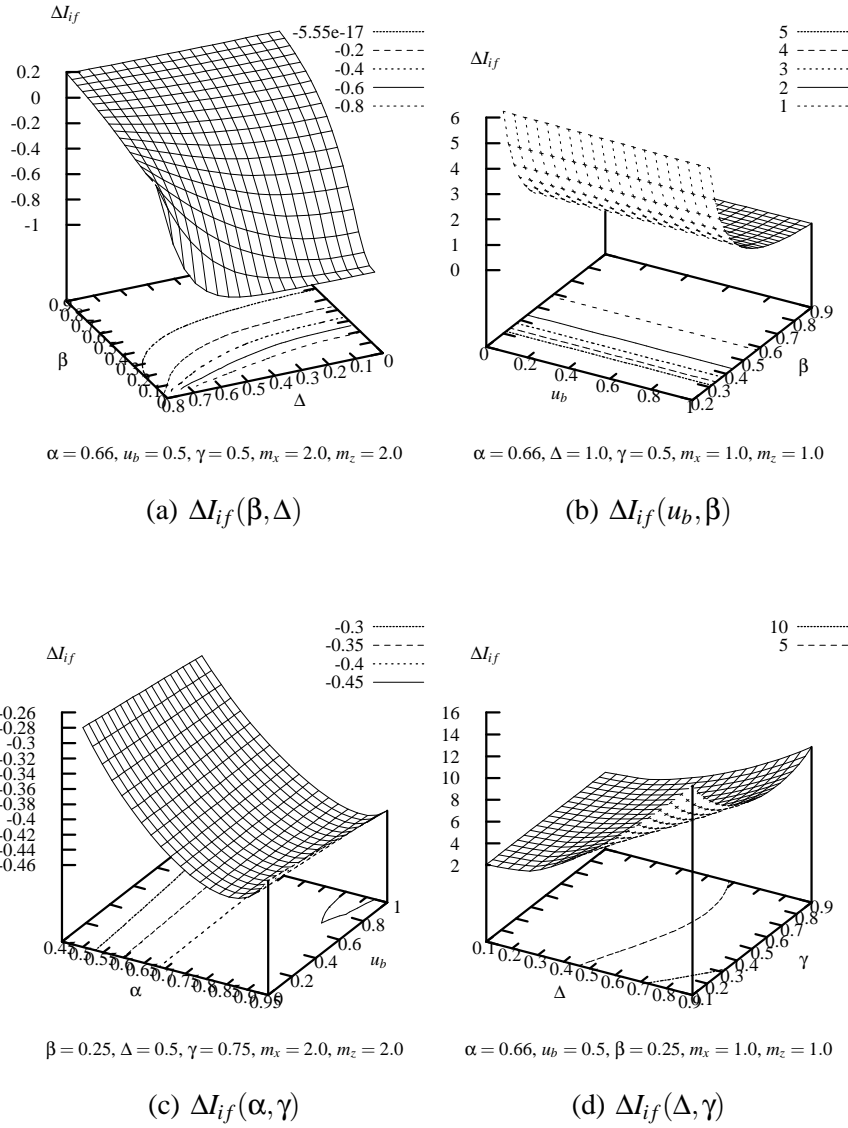
In Figure 3 relation (42) is graphed. If ΔI_{if} is greater than zero the equilibrium is a fragmented equilibrium; otherwise it is an in-house equilibrium. Figure 3(a) verifies that increasing the variation in demand pushes the economy towards a fragmented equilibrium.

Figures 3(a) and 3(b) indicate that β has an ambiguous effect on the type of equilibrium. Note that in Figure 3(a) the potential fragmented equilibrium is heavily distorted by non-competitive markets for intermediate goods; the markup over marginal cost is 100%. If Δ is small, increasing β pushes the economy towards a fragmented equilibrium. If Δ is close to unity, increasing β has a less clear effect.

A greater β decreases the frequency of shutdown threats, bringing the in-house equilibrium closer to a competitive equilibrium where firm owners maximize expected profits and cannot dismiss workers ex post the realization of the demand shock. A larger β also decreases the effect of the demand shock on revenues, thereby decreasing the value of choosing employment ex post the realization of the demand shock, an option only available for owners of fragmented firms. This implies that increasing β brings (43), i.e. the first bracketed term in (42), closer to unity from above.

The second effect from increasing β is that $\frac{G_{ih}}{Q_i^*}$ and $\frac{G_{il}}{Q_i^*}$ approach unity from above. Therefore, the term within the second brackets in (42) approaches $m_x^{-\alpha}$ $m_z^{\alpha-1}$ from below as β increases. However, due to the $(1-\beta)/\beta$ exponent the

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Figure 3: Equilibrium Type

$\alpha = 0.66, u_b = 0.5, \gamma = 0.5, m_x = 2.0, m_z = 2.0$

(a) $\Delta I_{if}(\beta, \Delta)$

$\alpha = 0.66, \Delta = 1.0, \gamma = 0.5, m_x = 1.0, m_z = 1.0$

(b) $\Delta I_{if}(u_b, \beta)$

$\beta = 0.25, \Delta = 0.5, \gamma = 0.75, m_x = 2.0, m_z = 2.0$

(c) $\Delta I_{if}(\alpha, \gamma)$

$\alpha = 0.66, u_b = 0.5, \beta = 0.25, m_x = 1.0, m_z = 1.0$

(d) $\Delta I_{if}(\Delta, \gamma)$

For $\Delta I_{if} < 0$ only in-house firms operate in the steady state equilibrium; otherwise only fragmented firms do. See Table 2 for default parameter values.

entire second term in (42) approaches unity from below as β increases. This tends to push the economy towards a fragmented equilibrium. Taken together, the effect

of β on the type of equilibrium is ambiguous.

An interesting hypothesis is that the more distorted the relative prices in the in-house scenario, the more likely that a fragmented equilibrium arises. A test for falsifying this hypothesis can be carried out by varying the unemployment replacement ratio, u_b. Increasing u_b decreases the surplus to bargain over and thereby brings the expected relative wages closer to the relative marginal revenue product of each factor.

Also, bear in mind that for $\alpha = \gamma$, relative wages are not distorted by bargaining between high-skill and low-skill workers, and that in the fragmented equilibrium the relative wages of high-skill and low-skill workers are not distorted. Figure 3(c) graphs ΔI_{if} varying α and u_b given that the bargaining power is $\gamma = 3/4$.

If the hypothesis that more distorted relative prices in the in-house scenario push the economy towards a fragmented equilibrium is correct, then ΔI_{if} should by minimized at $\alpha = \gamma$, since this minimizes the distortion in relative wages. Further, increasing u_b should decrease ΔI_{if}, because the surplus to bargain over decreases. The situation depicted in Figure 3(c) indeed shows that ΔI_{if} is minimized at $\alpha = \gamma$ and that increasing u_b decreases ΔI_{if}; hence, the hypothesis cannot be falsified by those tests.

A second test of the same hypothesis is found in 3(d) where ΔIf is graphed for combinations of Δ and γ. Besides once again verifying that as Δ increases so does the likelihood of a fragmented equilibrium, note that, as before, at $\gamma = \alpha = 2/3$ the likelihood of an in-house equilibrium is maximized. Further, the greater the Δ relative to β, the more distorted the relative prices and the less the likelihood of an in-house equilibrium.

It is tempting to fall back on the analysis in Acemoglu et al. (2001), where the degree of redistributive contracts that can be specified by low-skill workers is limited by the wage rate paid by firms hiring only high-skill workers. In this setting there are no outside firms hiring only high-skill workers, and even though a high degree of redistribution from high-skill and low-skill workers increases the value of starting a specialized firm producing the high-skill intermediate good, it decreases the value of starting a firm producing the low-skill intermediate good. It is therefore not straightforward to see why more distorted relative wages of high-skill and low-skill workers tend to decrease the likelihood of an in-house equilibrium.

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5.2 The Bargaining Effect

In Appendix C, it is shown that a steady state equilibrium will not exist with both in-house firms and fragmented firms. Therefore this section presents results by comparing the two possible steady state equilibria: one in which all firms are in-house firms, and one in which all firms are fragmented or specialized firms. Let H denote total employment of high-skill workers, $H \equiv H_i + H_s$, and let L denote total employment of low-skill workers, $L \equiv L_i + L_s$.

The Fragmented Skill Premium   If there are no in-house firms, it follows that every employed high-skill worker is employed by a specialized firm; $H_s = H$ and $H_i = 0$. Similarly, every employed low-skill worker is employed by a specialized firm; $L_s = L$ and $L_i = 0$.

The skill premium, $\omega_s = \frac{E\{W_{sh}\}}{E\{W_{sl}\}} = \frac{w_{sh}}{w_{sl}}$, is easily computed using (37) and (38):

$$\omega_s = \frac{\alpha}{1-\alpha} \times \frac{L}{H} \times \frac{m_z}{m_x}. \tag{46}$$

This is the standard Cobb-Douglas skill premium, augmented by a market competitiveness term. The first term corresponds to the standard Cobb-Douglas weights in the production technology. The second term is the standard Cobb-Douglas relative supply term. The last term corrects for differences in the markup over marginal cost for firms employing high-skill and low-skill workers.

The In-house Skill Premium   If there are no specialized firms, it follows that every employed high-skill worker is employed by an in-house firm; $H_i = H$ and $H_s = 0$. Similarly, every employed low-skill worker is employed by an in-house firm; $L_i = L$ and $L_s = 0$.

The skill premium, $\omega_i = \frac{E\{W_{ih}\}}{E\{W_{il}\}}$, is easily computed using (32a), (32b), (24a) and (24b):

$$\omega_i = \frac{\alpha}{1-\alpha} \times \frac{L}{H} \times \frac{Q_i^* + \frac{1-Q_i^*}{1-\beta}\frac{E\{\psi\check{R}_i\}}{E\{\alpha\hat{R}_i\}}}{Q_i^* + \frac{1-Q_i^*}{1-\beta}\frac{E\{(1-\psi)\check{R}_i\}}{E\{(1-\alpha)\hat{R}_i\}}}. \tag{47}$$

The three different terms are easily interpreted. The first term corresponds to the standard Cobb-Douglas weights in the production technology. The second term is

the standard Cobb-Douglas relative supply term. The third term is the novel term. Due to shutdown threats, wage bargaining is introduced into the model.

The bargaining term, G_i, augments the standard Cobb-Douglas skill premium and is defined as

$$
G_i = \frac{Q_i^* + \frac{1-Q_i^*}{1-\beta} \frac{E\{\psi \check{R}_i\}}{E\{\alpha \hat{R}_i\}}}{Q_i^* + \frac{1-Q_i^*}{1-\beta} \frac{E\{(1-\psi)\check{R}_i\}}{E\{(1-\alpha)\hat{R}_i\}}} \left(= \frac{G_{ih}}{G_{il}} \right),
\tag{48}
$$

where the expected revenue shares obtained by high-skill and low-skill workers are defined by (31a) and (31b). The expression in parentheses reconciles the bargaining term with the definitions in Appendix C.

Proposition 5.2.1 (The Skill Premium with Shutdown Threats) *The possibility for firm owners to shut down the firm if revenues are low creates a bargaining situation where low-skill workers in general increase their relative wage rate, compared to high-skill workers.*

Proof First note that in general there is no reason to expect the market for high-skill intermediate goods to be more or less competitive than the market for low-skill intermediate goods. Therefore, in general $m_x = m_z$. Further, a necessary and sufficient condition for high-skill workers to have a higher competitive, given the same supply of labor, wage is that $\alpha > 1/2 > 1 - \alpha$.

If $G_i < 1$, bargaining under shutdown threats in general decreases the skill premium. From a simple inspection of (48), it is clear that a necessary and sufficient condition is that:

$$
\frac{\alpha}{1-\alpha} > \frac{E\{\psi \check{R}_i\}}{E\{(1-\psi)\check{R}_i\}}
$$

By inspecting (31a) and (31b), it is immediately clear that with a replacement rate equal to unity, $u_b = 1$, this condition reduces to:

$$
\frac{\alpha}{1-\alpha} > \frac{\gamma}{1-\gamma}.
$$

While high-skill workers might be in a superior bargaining position, i.e. have a better outside option, there is no reason to assume that high-skill workers have a greater bargaining power, i.e. $\gamma > 1/2$. Therefore, bargaining under shutdown threats decreases the skill premium with full unemployment coverage.

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In the case of a less than 100 percent replacement rate, $u_b < 1$, it is still possible to prove the proposition under the assumption that $\overline{\rho}_{ih} = \overline{\rho}_{il}$ and $\gamma = 1/2$. If so, the denominators in (31a) and (32a) are identical and the first term in the numerators are also identical, while the sign of the second term differs. With $\alpha > 1/2$, it is easy to see that $E\left\{\psi\check{R}_i\right\}$ is less than $E\left\{(1-\psi)\check{R}_i\right\}$, which proves the proposition. ∎

The *bargaining power* parameter, γ, could be interpreted as capturing differences in the bargaining skills of high-skill and low-skill union representatives. It seems far fetched to assume any systematic differences in the bargaining skills across the parties. Hence, it seems reasonable to assume $\gamma = 1/2$. Given the vague definition of high-skill and low-skill, it appears to be difficult to assert anything specific about the relation between the markup factor in markets for specialized goods. The assumption that m_x equals m_z, therefore seems reasonable.

The assumption that $\overline{\rho}_{ih}$ equals $\overline{\rho}_{il}$ is more problematic. It seems reasonable that high-skill workers find new employment more easily than low-skill workers, implying that $\overline{\rho}_{ih}$ is greater than $\overline{\rho}_{il}$ (recall that $\overline{\rho} = \rho + \theta$), which intuitively should increase the relative wage rate for high-skill workers. To verify this intuitive claim, maintain the assumption that γ equals $1/2$. The ratio $E\left\{\psi\check{R}_i\right\}/E\left\{(1-\psi)\check{R}_i\right\}$ is computable given the expressions in (31a) and (31b). In order to make this ratio a bit simpler, let the unemployment replacement ratio be zero, $u_b = 0$. The ratio then becomes:

$$
\frac{E\left\{\psi\check{R}_i\right\}}{E\left\{(1-\psi)\check{R}_i\right\}}
=
\frac{\left[\overline{\rho}_{ih}\overline{\rho}_{il}+(1-Q_i^*)\overline{\rho}_{ih}\right]E\left\{\check{R}_i\right\}-Q_i^*(1-\beta)[\alpha\overline{\rho}_{il}-(1-\alpha)\overline{\rho}_{ih}]E\left\{\hat{R}_i\right\}}{\left[\overline{\rho}_{ih}\overline{\rho}_{il}+(1-Q_i^*)\overline{\rho}_{il}\right]E\left\{\check{R}_i\right\}+Q_i^*(1-\beta)[\alpha\overline{\rho}_{il}-(1-\alpha)\overline{\rho}_{ih}]E\left\{\hat{R}_i\right\}}.
$$

Now consider a change increasing θ_h and decreasing θ_l such that the product $\overline{\rho}_{ih}\overline{\rho}_{il}$ remains constant. This is clearly beneficial for high-skill workers relative to low-skill workers because the matching quality on the labor market for high-skill workers increases while the matching quality on the labor market for low-skill workers decreases. Intuitively, this should improve the bargaining position of high-skill workers relative to low-skill workers and thereby increase the bargained relative wage for high-skill workers; that is, G_i should increase. Inspecting the ratio above, this is clearly the case, because the numerator increases while the denominator decreases. This of course raises some concerns about the importance of the result in Proposition 5.2.1. It should however be noted that if workers have

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a high discount rate, the importance of the matching quality in the labor market is low, since $\bar{\rho}$ equals $\rho + \theta$ and θ never appears outside this sum.

The bargaining term, G_i, is plotted in Figure 4 for various parameter settings. A G_i less than unity implies a lower skill premium relative to the standard competitive economy and the fragmented equilibrium. The lower the G_i, the lower the skill premium. From Figure 4(a) it is evident that wage bargaining only decreases inequality if $\gamma < \alpha > 1/2$, meaning only if the superior productivity of high-skill workers is not matched by an at least equally superior bargaining power.

By inspecting Figure 4(b), it is clear that a higher β decreases the moderating effect from shutdown threats. The moderating effect on the skill premium from shutdown threats on the skill premium decays as β approaches Δ from below. The reason is that a higher β implies that shutdown threats occur less frequently, greater Q_i^*, and workers are more frequently paid their marginal revenue product. Figure 4(b) also verifies that increasing the unemployment benefit ratio, u_b decreases the moderating effect of wage bargaining.

Figure 4(c) verifies that shutdown threats are redistributive as long as $\alpha > \gamma$, i.e. as long as the superior marginal productivity of high-skill workers is not matched by an equally superior bargaining power of high-skill workers relative to low skill workers. It is clear from the figure that high-skill workers benefit from shutdown threats relative to low-skill workers only if $\gamma > \alpha$.
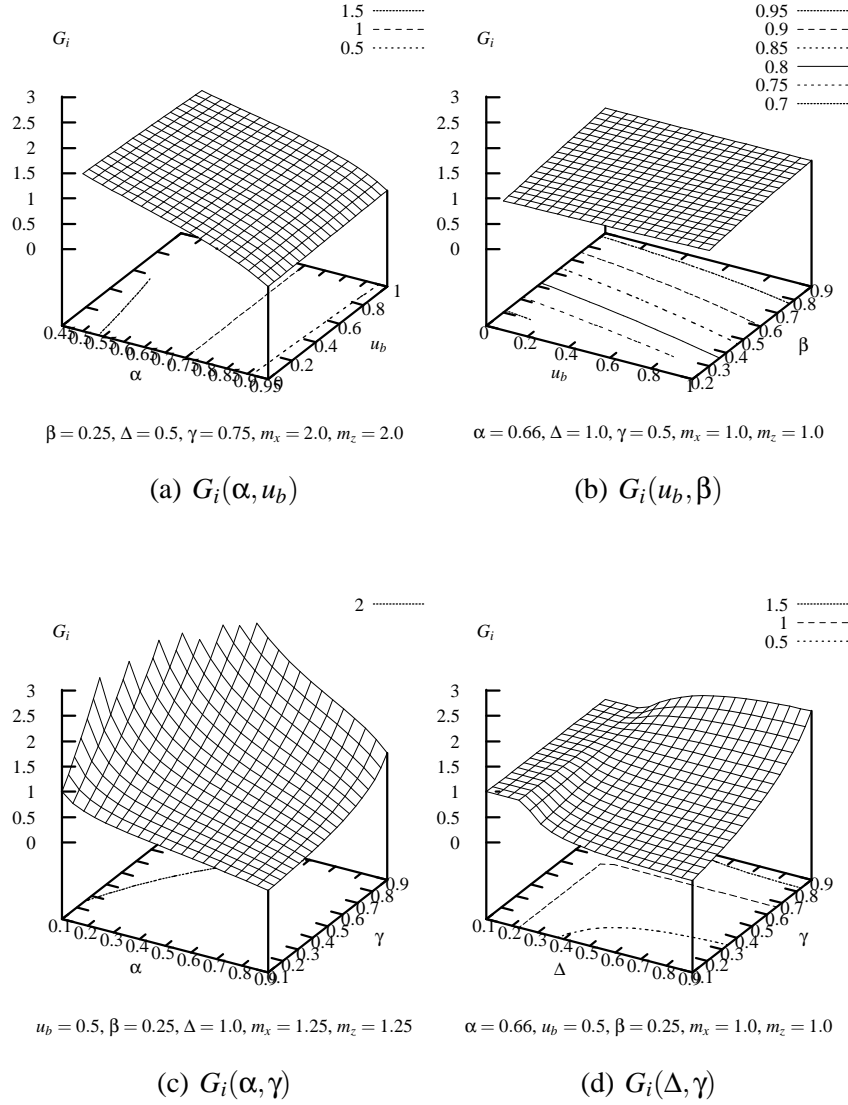
Figure 4(d) summarizes the results neatly. As long as demand shocks are small ($\Delta < \beta = 1/4$), shutdown threats do not affect the skill premium. With a more uncertain demand ($\Delta >> \beta$), shutdown threats decrease the skill premium more dramatically, unless high-skill worker bargaining power is sufficiently superior ($\gamma > \alpha$). In the latter case, demand uncertainty and shutdown threats magnify the skill premium, but this is unlikely unless there is some explicit reason as to why the bargaining power of high-skill workers should be greater than the bargaining power of low-skill workers.

From the results above it is clear that fragmentation in general increases the skill premium since low-skill workers never bargain over wages with high-skill workers. This hurts low-skill workers, relative to high-skill workers. In a fragmented economy low-skill workers can no longer compensate their inferior productivity via a relatively stronger bargaining position.

5.3 The Skill Premium

To exemplify the full results of the model, consider a scenario where the markets for intermediate goods are not perfectly competitive, such that $m_x = 2 > 1$ and

Figure 4: Bargaining Effect

$\beta = 0.25, \Delta = 0.5, \gamma = 0.75, m_x = 2.0, m_z = 2.0$

(a) $G_i(\alpha, u_b)$

$\alpha = 0.66, \Delta = 1.0, \gamma = 0.5, m_x = 1.0, m_z = 1.0$

(b) $G_i(u_b, \beta)$

$u_b = 0.5, \beta = 0.25, \Delta = 1.0, m_x = 1.25, m_z = 1.25$

(c) $G_i(\alpha, \gamma)$

$\alpha = 0.66, u_b = 0.5, \beta = 0.25, m_x = 1.0, m_z = 1.0$

(d) $G_i(\Delta, \gamma)$

If $G_i < 1$, the skill premium is lower in the in-house equilibrium, while for $G_i > 1$, the skill premium is higher in the in-house equilibrium. See Table 2 for default parameter values.

$m_z = 2 > 1$. Figures 5(a) – 5(d) depict the model's prediction of the skill premium

for different combinations of demand uncertainty, Δ, and degrees of market power, β.

Figure 5(a) illustrates the skill premium in a hypothetical fragmented equilibrium. In a fragmented equilibrium the skill premium is simply $\alpha/(1-\alpha) = 2$. Changing the variation in demand, Δ, or the degree of market power, β, does not alter the skill premium as both high-skill and low-skill worker wages are proportional to the marginal revenue product.

Figure 5(b) depicts the skill premium in a hypothetical in-house equilibrium. In this scenario the skill premium decreases as β/Δ increases. The reason is that as β/Δ increases, shutdown threats and thereby renegotiations of wages become less frequent.

Note that as long as $\beta > \Delta$, firm owners never exercise their right to shut down the firm and the skill premium is identical to the skill premium in the hypothetical fragmented equilibrium. However, as the Δ/β ratio increases, the skill premium decreases. At most the skill premium is about 37 percent lower than in the hypothetical fragmented equilibrium.

Figure 5(c) maps each combination of β and Δ with either an in-house or a fragmented equilibrium. Everywhere in the graph where $\Delta I_{if} < 0$, the economy is characterized by an in-house equilibrium, and otherwise by a fragmented equilibrium. As can be seen in 5(c), an in-house equilibrium is most likely for low values of Δ compared to β.
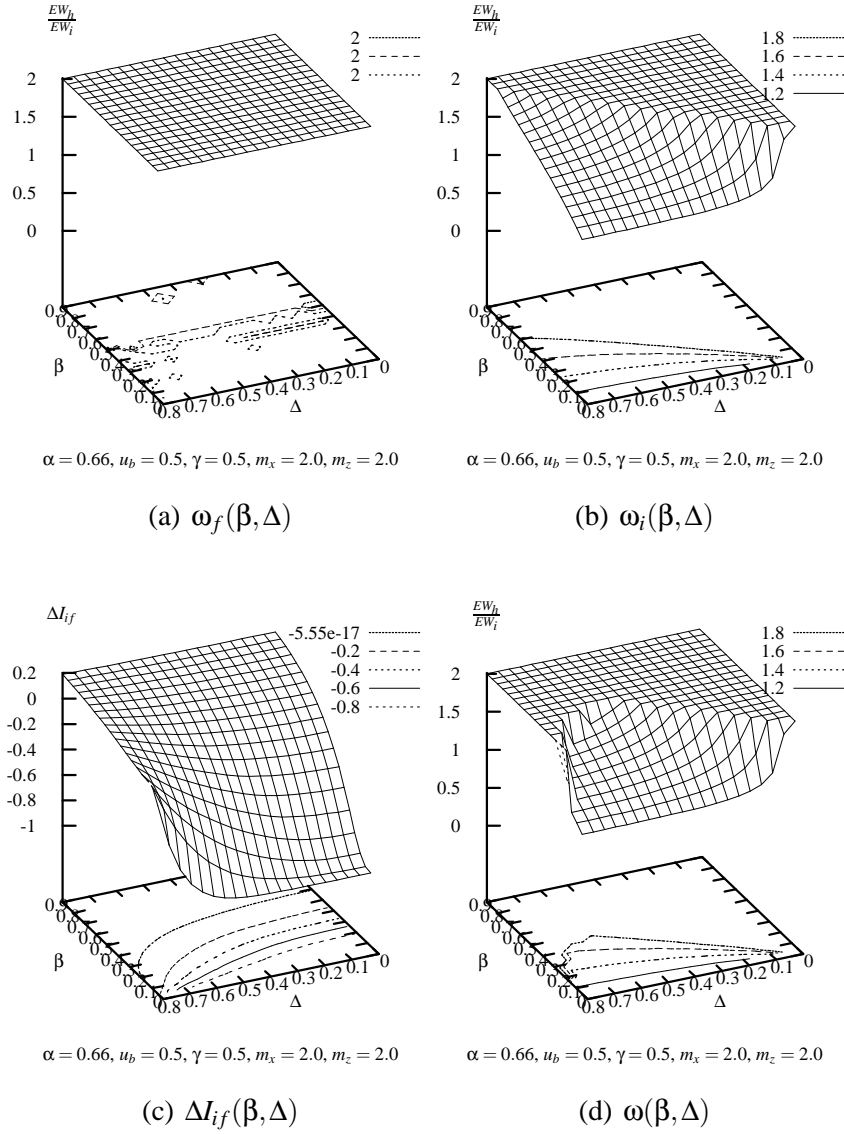
Combining 5(a) – 5(c) yields 5(d), which plots the skill premium taking into account the type of equilibrium. As can be seen in 5(d), the skill premium is clearly non-linear in β as well as in Δ. Taking into account the type of equilibrium hampers the potential for redistribution, since some equilibria where the in-house equilibrium redistributes, i.e. where $\beta < \Delta$, are discarded.

6 Conclusions

Relating the squeeze of wages for low-skill workers to outsourcing or linking wages to profits via bargains is nothing new. However, this paper spins those stories by considering domestic outsourcing or domestic contracting out and linking wages to shutdown threats.

Shutdown Threats Firm owners always have the option to default, i.e. to shut down the firm and sell its assets. If demand is lower than some endogenous threshold, firm owners can minimize losses by shutting down. In this case the losses for

Figure 5: Skill Premium

$\alpha = 0.66$, $u_b = 0.5$, $\gamma = 0.5$, $m_x = 2.0$, $m_z = 2.0$

(a) $\omega_f(\beta, \Delta)$

$\alpha = 0.66$, $u_b = 0.5$, $\gamma = 0.5$, $m_x = 2.0$, $m_z = 2.0$

(b) $\omega_i(\beta, \Delta)$

$\alpha = 0.66$, $u_b = 0.5$, $\gamma = 0.5$, $m_x = 2.0$, $m_z = 2.0$

(c) $\Delta I_{if}(\beta, \Delta)$

$\alpha = 0.66$, $u_b = 0.5$, $\gamma = 0.5$, $m_x = 2.0$, $m_z = 2.0$

(d) $\omega(\beta, \Delta)$

Figures 5(a) and 5(b) illustrate the skill premium in a fragmented equilibrium and an in-house equilibrium, respectively. The type of equilibrium is determined by ΔI_{if} illustrated in Figure 5(c). Figure 5(d) illustrates the skill premium, taking the type of equilibrium into account. See Table 2 for default parameter values.

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firm owners are limited to the depreciation of initial capital investments, since labor can be disposed without cost. Workers on the other hand always have the option to leave the firm and become unemployed, but the expected lifetime utility of being unemployed is inferior to the expected lifetime utility of being employed.

The firm owner's option to default on labor contracts leads to a bargaining situation. Firm owners threaten to shut down the firm if the realized profit rate is sufficiently low. Workers are reluctant to become unemployed and therefore negotiate new wage contracts to motivate the firm owner to keep the firm alive. Renegotiated wage rates are determined by the bargaining positions of high-skill versus low-skill workers. In the simple setting of the model, marginal productivity only matters indirectly via the outside options of high-skill and low-skill workers.

The superior marginal productivity of high-skill workers relative to low-skill workers is not matched by an equally superior bargaining power of high-skill workers relative to low-skill workers. Therefore low-skill workers benefit from bargaining relative to high-skill workers.

Another interesting consequence is the firm size effect. If owners have the option to shut down the firm, they are no longer inclined to face the full cost of low demand realizations. This asymmetry motivates firm owners to increase the size of the firm in response to greater demand uncertainty. Workers are paid higher wages if the firm owner does not threaten to shut down the firm, but shutdown threats become more frequent.

Fragmentation   Looking at the fragmentation process (i.e. outsourcing or contracting out) and taking into account wage bargains over losses, it is easy to see that fragmentation is likely to increase the skill premium.

In an economy with a low degree of fragmentation, each firm carries many tasks and requires a wide range of workers with different skills and skill levels. In the presence of shutdown threats, low-skill workers can increase their wage rate relative to high-skill workers via bargaining.

In an economy with a high degree of fragmentation, each firm carries out a much smaller set of tasks and hires a more homogenous group of workers. As more firms employ only high-skill *or* low-skill workers, the possibility for low-skill workers to compensate for low marginal productivity by bargaining with high-skill workers vanishes, and the skill premium increases.

The analysis shows that fragmentation is more likely to occur with greater variation in demand and as the market power of firms selling the consumption good declines, i.e. if consumer preferences for variety decreases. As would be

expected intuitively, when markets for intermediate goods become less competitive, the likelihood of fragmentation declines. The quantitative analysis in the paper also suggests that the more distorted the relative wages of high-skill and low-skill workers (i.e. the more the relative wage rates of high-skill and low-skill workers deviate from their relative marginal productivity), the more likely that a fragmented equilibrium arises. However, this remains a conjecture that could neither be proven nor falsified.

Appendices

A Record of Notation

Table 1 depicts the general logic for subscripts used to categorize different variables. Indices over a continuum are written in parentheses. All symbols are listed in Table 3. Random variables are marked as $\tilde{}$, $\hat{}$, or $\check{}$, depending on the information available. An upper case symbol is used for the stochastic variable while a lower case symbol is used to denote a particular realization of the corresponding random variable. Upper case letters are also used to denote aggregate quantities, while lower case letters are also used to denote micro quantities. Symbols marked by a superscripted $*$ are derived from an optimization problem.

B Profit Maximization

The shutdown condition in (20) is greatly simplified by using the inverse demand function in (2a) together with the production functions in (19) and (3b):

$$\left[\frac{C}{\overline{p}}\right]^{\beta} h_i^{\alpha(1-\beta)} l_i^{(1-\alpha)(1-\beta)} \underline{d}_i - w_{ih}h_i - w_{il}l_i = 0.$$

The objective for the in-house firm owner is therefore to maximize the expected profit rate $E\{\Pi\}$:

$$[1 - F(\underline{d}_i)]\left\{\left[\frac{C}{\overline{p}}\right]^{\beta} h_i^{\alpha(1-\beta)} l_i^{(1-\alpha)(1-\beta)} E\{\hat{D}_i\} - w_{ih}h_i - w_{il}l\right\} - \delta I_i, \quad \text{(49a)}$$

subject to the shutdown condition (solved for $\underline{d}_i$):

$$\underline{d}_i = \left[\frac{\overline{p}}{C}\right]^{\beta} \frac{w_{ih}h_i + w_{il}l}{h_i^{\alpha(1-\beta)} l_i^{(1-\alpha)(1-\beta)}}. \quad \text{(49b)}$$

The impact of $\underline{d}_i$ on the objective function is twofold. On the one hand, a higher $\underline{d}_i$ increases the probability, via $F(\underline{d}_i)$, that the firm owner will threaten shut down. On the other hand, increasing $\underline{d}_i$ increases the expected productivity of workers, via $E\{\hat{D}_i\}$, and thereby increases expected profits if the firm owner does not threaten to shut down the firm. The constraint guarantees that the firm manager is loyal to the firm owner by assuring that the value of the firm is maximized.

Table 3: List of Symbols

Symbol	Meaning
α	Cobb-Douglas exponent.
β	Representative agent's preference for variety.
G	Bargaining terms.
C	Consumption spending.
δ	Depreciation rate.
Δ	Parameterizes the variation in demand.
ΔI_{if}	Adjusted investment cost difference.
D	Stochastic demand parameter.
$F(\cdot)$	Cumulative Density Function.
γ	High-skill workers' relative bargaining strength.
H	High-skill employment.
I	Investment cost.
J	Lifetime utility, employed.
K	Range of firms.
L	Low-skill employment.
m	Markup over marginal cost.
N	Range of specialized firms.
ω	Relative wage (skill premium).
Π	Profit rate
P	Price of the conusumption good.
$\overline{p}$	Price index.
ψ	Bargaining outcome, high-skill workers' share.
Q	Probability of shutdown threat.
$\overline{\rho}$	$\rho + \theta$.
R	Firm revenue.
$\underline{d}$	Shutdown threat threshold.
θ	Labor market matching quality.
u_b	Unemployment benefit, fraction of expected income.
U	Lifetime utility, unemployed.
V	Value of a firm, post investment.
W	Wage rate.
wb	Wage bill.
x	Quantity of the X good.
y	Quantity of the Y good.
z	Quantity of the Z good.

B.1 The Unconstrained Solution

To solve the problem: First expand $1 - F(\underline{d}_i)$ using the definition in (1), then expand $E\{\hat{D}_i\}$ using the conditional expectation formula such that $E\{\hat{D}_i(\underline{d}_i)\} = 1/2 \times (1 + \Delta + \underline{d}_i)$, and finally replace $\underline{d}_i$ in the objective function, using the constraint. The objective function simplifies to

$$(1+\Delta)^2 \Theta^{1-\beta} - 2 \left[\frac{\overline{p}}{C}\right]^\beta (1+\Delta)\Phi + \left[\frac{\overline{p}}{C}\right]^{2\beta} \Phi^2 \Theta^{\beta-1},$$

where the auxiliary variables Θ and Φ are defined as:

$$\Theta = h_i^\alpha l_i^{1-\alpha} \qquad \Phi = w_{ih} h_i + w_{il} l_i.$$

The first order conditions with respect to h_i and l_i simplify to:

$$\alpha(1-\beta)(1+\Delta)^2 \Theta^{1-\beta} - \alpha(1-\beta)\left[\frac{\overline{p}}{C}\right]^{2\beta} \Phi^2 \Theta^{\beta-1}$$
$$= \tag{50a}$$
$$2\left[\left[\frac{\overline{p}}{C}\right]^\beta (1+\Delta) - \left[\frac{\overline{p}}{C}\right]^{2\beta} \Phi\Theta^{\beta-1}\right] w_{ih} h_i$$

$$(1-\alpha)(1-\beta)(1+\Delta)^2 \Theta^{1-\beta} - (1-\alpha)(1-\beta)\left[\frac{\overline{p}}{C}\right]^{2\beta} \Phi^2 \Theta^{\beta-1}$$
$$= \tag{50b}$$
$$2\left[\left[\frac{\overline{p}}{C}\right]^\beta (1+\Delta) - \left[\frac{\overline{p}}{C}\right]^{2\beta} \Phi\Theta^{\beta-1}\right] w_{il} l_i.$$

Dividing the first order conditions results in the familiar Cobb-Douglas mix of factors:

$$\frac{\alpha}{1-\alpha} = \frac{w_{ih}}{w_{il}} \frac{h_i}{l_i}.$$

This implies that firms minimize costs, whichever quantity the firm plan to produce, an intuitive result. The auxiliary variables simplify as:

$$\Theta = \left[\frac{1-\alpha}{\alpha} \frac{w_{ih}}{w_{il}}\right]^{1-\alpha} h_i \qquad \Phi = \frac{w_{ih} h_i}{\alpha}. \tag{51}$$

Eliminating h_i in the first order condition with respect to h_i using the Φ expression:

$$\left[\frac{\Theta^{1-\beta}}{\Phi}\right]^2 - \frac{2\left[\frac{\overline{p}}{C}\right]^\beta}{(1-\beta)(1+\Delta)} \frac{\Theta^{1-\beta}}{\Phi} = \frac{(1+\beta)\left[\frac{\overline{p}}{C}\right]^\beta}{(1-\beta)^2(1+\Delta)^2}.$$

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Solving the second order equation in $\Theta^{1-\beta}/\Phi$ gives two solutions:

$$\frac{\Theta^{1-\beta}}{\Phi} = \frac{1 \pm \beta}{1 + \Delta}\left[\frac{\overline{p}}{C}\right]^{\beta}. \tag{52}$$

Solving for h_i, l_i and $\underline{d}_i$ is trivial given the intermediate results above:

$$h_i^{\beta} = \left[\frac{\alpha}{w_{ih}}\right]^{1-(1-\alpha)(1-\beta)} \left[\frac{1-\alpha}{w_{ih}}\right]^{(1-\alpha)(1-\beta)} \frac{\Phi}{\Theta^{1-\beta}}$$

$$l_i^{\beta} = \left[\frac{\alpha}{w_{ih}}\right]^{\alpha(1-\beta)} \left[\frac{1-\alpha}{w_{ih}}\right]^{1-\alpha(1-\beta)} \frac{\Phi}{\Theta^{1-\beta}}$$

$$\underline{d}_i = \frac{(1-\beta)(1+\Delta)}{1 \pm \beta}.$$

To verify that $\underline{d}_i = (1-\beta)(1+\Delta)/(1+\beta)$ is a local maximum, note that the first order conditions imply that both Θ and Φ are linear in h_i. Therefore, the objective function and the constraint defining $\underline{d}_i$ can be written as

$$E\{\Pi\} = a_1 h_i^{1-\beta} - a_2 h_i + a_3 h_i^{1+\beta} - \delta I_i$$

$$\underline{d}_i = a_4 h_i^{\beta},$$

with $a_i > 0$.

Clearly, for $h_i = 0$ the objective function is $-\delta I_i$, while for sufficiently small choices of $h > 0$, the objective function is greater than $-\delta I_i$. However, the solution implying that $\underline{d}_i = 1 + \Delta$ implies that $1 - F(\underline{d}_i) = 0$. Again the objective function equals $-\delta I_i$.

Taken together, this implies that at $h_i = 0$, the objective function equals $-\delta I_i$, but increases as h_i increases. Eventually h_i equals h_1^* which implies that $\underline{d}_i = (1+\Delta)(1-\beta)/(1+\beta) < 1+\Delta$. Increasing h_i further decreases the value of the objective function until $h_i = h_2^*$ which implies that $\underline{d}_i = 1+\Delta$, where again the objective function equals $-\delta I_i$, since $1 - F(1+\Delta) = 0$.

Therefore the solution with

$$\underline{d}_i = \frac{(1-\beta)(1+\Delta)}{1+\beta}$$

$$h_i^{\beta} = (1-\beta)\frac{1+\Delta}{1+\beta}\left[\frac{C}{\overline{p}}\right]^{\beta}\left[\frac{\alpha}{w_{ih}}\right]^{1-(1-\alpha)(1-\beta)}\left[\frac{1-\alpha}{w_{il}}\right]^{(1-\alpha)(1-\beta)}$$

$$l_i^{\beta} = (1-\beta)\frac{1+\Delta}{1+\beta}\left[\frac{C}{\overline{p}}\right]^{\beta}\left[\frac{\alpha}{w_{ih}}\right]^{\alpha(1-\beta)}\left[\frac{1-\alpha}{w_{il}}\right]^{1-\alpha(1-\beta)},$$

is indeed a maximum.

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B.2 The Constrained Solution

Before accepting the solution derived above it is necessary to check that the cut-off value $\underline{d}_i$ is within the range of the realizations of the stochastic demand variable, $\tilde{D}_i$. That is, it is necessary to check that $1 - \Delta < \underline{d}_i < 1 + \Delta$, which in turn is equivalent to checking that:

$$0 \leq F(\underline{d}_i) \leq 1. \tag{53}$$

Given the definition of $F(d)$ in (1) and maximizing choice of $\underline{d}_i$ this implies checking that:

$$0 \leq \frac{(1-\beta)(1+\Delta)/(1+\beta) - (1-\Delta)}{2\Delta} \leq 1. \tag{54}$$

Note that this function is decreasing in β, for $0 \leq \beta \leq 1$ ($\Delta \leq 1$). For $\beta = 0$, $F(\underline{d}_i) = 1$, implying that the maximizing choice of $\underline{d}_i$ never violates the condition $F(\underline{d}_i) \leq 1$.

However, increasing β starting at $\beta = 0$ violates $F(\cdot) \geq 0$ at $\beta > \Delta$. $F(\cdot)$ is nothing but $1 - Q_i$. So for $\beta > \Delta$, the solution to an unconstrained problem dictates the firm owner to shut down the firm with a negative probability. This is of course the same as to say that the firm owner keeps the firm running with a probability greater than one.

The proper way to handle this problem would have been to solve the optimization problem adding the constraints $0 \leq F(\underline{d}_i) \leq 1$. The result would, given the discussion above, be that for $\beta \leq \Delta$ the unconstrained solution derived above would apply, while for $\beta > \Delta$, the first constraint would bind and the solution to the problem would be argmax of:

$$[1 - F(\underline{d}_i)] \quad \left\{ \left[\frac{C}{\overline{p}}\right]^{\beta} h_i^{\alpha(1-\beta)} l_i^{(1-\alpha)(1-\beta)} E\left\{\hat{D}_i\right\} - w_{ih}h_i - w_{il}l \right\} - \delta I_i$$

$$s.t. \quad F(\underline{d}_i) = 0.$$

Consequently, the first bracketed term equals unity while $F(\underline{d}_i) = 0$ implies that $\underline{d}_i = 1 - \Delta$, which in turn implies that $E\left\{\hat{D}_i\right\}$ equals $E\left\{\tilde{D}_i\right\}$. Therefore the solution to a firm owner's problem when $\beta > \Delta$ is:

$$\max_{h_i, l_i} \quad \left[\frac{C}{\overline{p}}\right]^{\beta} h_i^{\alpha(1-\beta)} l_i^{(1-\alpha)(1-\beta)} E\left\{\tilde{D}_i\right\} - w_{ih}h_i - w_{il}l - \delta I_i.$$

The solution to this problem is simpler. Straightforward use of the first order conditions for h_i and l_i implies:

$$\underline{d}_i = 1 - \Delta$$

$$h_i^{\beta} = (1-\beta) \left[\frac{C}{\overline{p}}\right]^{\beta} \left[\frac{\alpha}{w_{ih}}\right]^{1-(1-\alpha)(1-\beta)} \left[\frac{1-\alpha}{w_{ih}}\right]^{(1-\alpha)(1-\beta)}$$

$$l_i^{\beta} = (1-\beta) \left[\frac{C}{\overline{p}}\right]^{\beta} \left[\frac{\alpha}{w_{ih}}\right]^{\alpha(1-\beta)} \left[\frac{1-\alpha}{w_{ih}}\right]^{1-\alpha(1-\beta)}.$$

B.3 The Complete Solution

By combining the constrained and unconstrained solutions, the complete solution can be stated as:

$$\underline{d}_i^* = \begin{cases} \frac{(1-\beta)(1+\Delta)}{1+\beta} & \beta \leq \Delta \\ 1-\Delta & \beta > \Delta \end{cases}$$

$$h_i^{\beta} = (1-\beta) \; E\left\{\hat{D}_i\right\}\Bigg|_{\underline{d}_i^*} \left[\frac{C}{\overline{p}}\right]^{\beta} \left[\frac{\alpha}{w_{ih}}\right]^{1-(1-\alpha)(1-\beta)} \left[\frac{1-\alpha}{w_{il}}\right]^{(1-\alpha)(1-\beta)}$$

$$l_i^{\beta} = (1-\beta) \; E\left\{\hat{D}_i\right\}\Bigg|_{\underline{d}_i^*} \left[\frac{C}{\overline{p}}\right]^{\beta} \left[\frac{\alpha}{w_{ih}}\right]^{\alpha(1-\beta)} \left[\frac{1-\alpha}{w_{il}}\right]^{1-\alpha(1-\beta)}.$$

Note that for $\beta < \Delta$, firm owners choose the threshold level $\underline{d}_i = 1 - \Delta$ which is at the lowest realization of the demand variable. In this case, the firm owner is never inclined to shut down the firm ex post the realization of the demand variable, i.e. $Q_i = 1$. Further, expected demand $E\left\{\hat{D}_i\right\}$ evaluated at $\underline{d}_i = 1 - \Delta$ is simply $E\left\{\tilde{D}_i\right\}$. Therefore, for low values of β the firm owner never threatens to shut down the firm and the standard competitive results hold.

C Equilibrium Conditions

To solve for the steady state equilibrium it is necessary to determine H_i, L_i, H_s and L_s. To do this it is assumed that in the steady state equilibrium:

$$E\left\{\tilde{W}_{ih}\right\} = w_{sh} \qquad E\left\{\tilde{W}_{il}\right\} = w_{sl}. \tag{55}$$

The expected wage rates are rewritten as

$$E\left\{\tilde{W}_{ih}\right\} = \alpha w_i G_{ih} \frac{K_i}{H_i} \left[\frac{H_i^\alpha L_i^{1-\alpha}}{K_i}\right]^{1-\beta} \qquad E\left\{\tilde{W}_{il}\right\} = (1-\alpha)w_i G_{il} \frac{K_i}{L_i} \left[\frac{H_i^\alpha L_i^{1-\alpha}}{K_i}\right]^{1-\beta}$$

$$\tag{56a}$$

$$w_{sh} = \alpha \frac{w_s}{m_x} \frac{K_f}{H_s} \left[\frac{H_s^\alpha L_s^{1-\alpha}}{K_f}\right]^{1-\beta} \qquad w_{sl} = (1-\alpha)\frac{w_s}{m_z} \frac{K_f}{L_s} \left[\frac{H_s^\alpha L_s^{1-\alpha}}{K_f}\right]^{1-\beta},$$

where

$$w_i = (1-\beta)E\left\{\hat{D}_i^*\right\}\left[\frac{C}{\bar{p}}\right]^\beta \tag{57a}$$

$$w_s = (1-\beta)\left[E\left\{\tilde{D}_f^{1/\beta}\right\}\right]^\beta \left[\frac{C}{\bar{p}}\right]^\beta \tag{57b}$$

$$G_{ih} = Q_i^* + \frac{1-Q_i^*}{1-\beta}\frac{E\left\{\psi\check{R}_i\right\}}{E\left\{\alpha\hat{R}_i\right\}} \tag{57c}$$

$$G_{il} = Q_i^* + \frac{1-Q_i^*}{1-\beta}\frac{E\left\{(1-\psi)\check{R}_i\right\}}{E\left\{(1-\alpha)\hat{R}_i\right\}}. \tag{57d}$$

Solving the system in (55) for H_s and L_s implies:

$$H_s = \left[\frac{w_s}{w_i}\frac{1}{(m_x G_{ih})^{1-(1-\alpha)(1-\beta)}(m_z G_{il})^{(1-\alpha)(1-\beta)}}\right]^{1/\beta}\frac{K_f}{K_i}H_i \tag{57e}$$

$$L_s = \left[\frac{w_s}{w_i}\frac{1}{(m_x G_{ih})^{\alpha(1-\beta)}(m_z G_{il})^{1-\alpha(1-\beta)}}\right]^{1/\beta}\frac{K_f}{K_i}L_i. \tag{57f}$$

These expressions show that if the joint marginal revenue product for workers is higher in specialized firms than in in-house firms, then

$$w_s > w_i(m_x G_{ih})^{\alpha(1-\beta)}(m_z G_{il})^{(1-\alpha)(1-\beta)},$$

meaning that more workers are employed by specialized firms and vice versa.

In the steady state equilibrium, the value of owning a firm must equal the start-up cost. This condition is met for in-house firms only if (12) is satisfied. The expected profit rates for in-house firms not threatening to shut down are given by (25a), which rewritten using the definitions above gives:

$$E\left\{\hat{\Pi}_i\right\} = \frac{\beta w_i}{1-\beta}\left[\frac{H_i^\alpha L_i^{1-\alpha}}{K_i}\right]^{1-\beta} - \delta I_i.$$

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Since the value of owning a firm must equal the start-up cost, (12) must be satisfied. Solving for K_i:

$$K_i = \left[\frac{\beta}{1-\beta} \frac{1+\rho}{\delta+\rho+\delta\rho} \frac{Q_i^* w_i}{I_i} \right]^{\frac{1}{1-\beta}} H_i^\alpha L_i^{1-\alpha}. \qquad (58)$$

In order to investigate if in-house firms and fragmented firms can co-exist in the steady state equilibrium, H_s and L_s in the expected profit expression for fragmented firms, see (40) are first eliminated using (57e) and (57f). In the resulting expression, K_i is eliminated using (58). The resulting expected profit rate for fragmented firms is:

$$E\left\{\tilde{\Pi}_f\right\} = \frac{\rho+(1+\rho)\delta}{1+\rho} \left[\frac{w_s}{w_i}\right]^{1/\beta} \left[\frac{1}{(m_x G_{ih})^\alpha (m_z G_{il})^{1-\alpha}}\right]^{\frac{1-\beta}{\beta}} \frac{I_i}{Q_i^*} - \delta I_f.$$

As for in-house firms, in the steady state equilibrium the value of owning a fragmented firm must equal the start-up cost. That is, (12) must hold with $E\left\{\hat{\Pi}_i\right\}$ replaced by $E\left\{\tilde{\Pi}_f\right\}$, I_i replaced I_f, and Q_i replaced by 1, implying:

$$\left[\frac{w_s}{w_i}\right]^{1/\beta} \left[\frac{1}{(m_x G_{ih})^\alpha (m_z G_{il})^{1-\alpha}}\right]^{\frac{1-\beta}{\beta}} \frac{I_i}{Q_i^*} - I_f = 0. \qquad (59)$$

This relation depends only on parameters and exogenous variables, which in turn implies that a mixed equilibrium can occur only for a specific set of parameter values, with measure zero. That is, in the steady state equilibrium there exist only in-house firms or only fragmented firms, but not both.

If (59) is greater than zero, it is more profitable to start a fragmented firm than an in-house firm, while if (59) is less then zero then it is more profitable to start an in-house firm. As it turns out, the main determinant for which type of equilibrium to occur is the relation between fixed investment costs and marginal productivity. A higher investment cost for starting a fragmented firm, $I_f > I_i$, must be compensated by high marginal productivity, $w_s > w_i (m_x G_{ih})^{\alpha(1-\beta)} (m_z G_{il})^{(1-\alpha)(1-\beta)}$, corrected for the additional cost due to markup over marginal cost for goods purchased on the market.

It is not surprising that a mixed equilibrium will not exist, since there is no mechanism generating an interior equilibrium. Consumers do not care whether goods are produced by in-house or fragmented firms and in equilibrium the most cost efficient production method is used, taking into account that the expected wage rates paid by in-house and specialized firms cannot deviate.

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