UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

Master Degree Project in Finance

Commodity Futures Investing from a Swedish Pension Fund Perspective

Linn Nilsson and Tilda Wikström

Supervisor: Charles Nadeau
Master Degree Project No. 2016:131
Graduate School

Acknowledgements

We would like to express our deepest appreciation to our supervisor Charles Nadeau for his valuable guidance and encouragement throughout our thesis process. We are also sincerely grateful for the econometric advice and feedback of Reda Moursli, whose assistance has helped us bringing our thesis forward. Lastly we would like to gratefully acknowledge the help of Thomas Franzén, Olle Eriksson and Torbjörn Hamnmark at the AP-funds, who provided valuable insight about the AP-funds investment strategies.

Abstract

Our study examines if the Swedish General Pension funds (AP-funds) could benefit from investing in commodity futures derivatives, which they are currently prohibited from. The effect of adding commodity futures to the holdings of the AP-funds is examined during the period 2001 to 2015, with extended analyses on accumulated bull and bear periods. We conduct an introductory descriptive analysis of time-varying correlation between the respective asset classes in the AP-funds' portfolio and commodity futures. However, our primary analysis is based on two portfolio efficiency tests from adding commodity futures, the intersection and mean-variance spanning framework of Kan and Zhou (2012).

The results imply that the AP-funds have potential in risk reduction by extending their portfolio with commodity futures indices with regards to their asset mix. However, the test of a replicated AP-fund portfolio shows that their specific allocations inhibit diversification benefits as performance is only improved in bull periods when considering portfolio weights. Furthermore, because of an observed difference in diversification benefits in bull and bear periods, a tactical allocation strategy for the AP-funds, linked to the movements in the equity market, is advocated, where commodities can function as an actively managed risk instrument.

Key Words: commodity futures, pension funds, AP-funds, portfolio efficiency, intersection-test, mean-variance spanning test, DCC GARCH

Authors Contact Details
Tilda Wikström, e-mail: tilda.w@gmail.com
Linn Nilsson, email: linn.nilsson123@gmail.com

Table of Contents

Table of Tables

1 Introduction

The Swedish general pension funds, also referred to as AP-funds, are among few pension funds in the world prohibited from investing in derivatives with commodities as underlying asset. In financial literature, strategic benefits of commodity related derivatives in portfolios were first highlighted in the beginning of the millennium after influential US studies demonstrated slightly negative correlation between commodity derivatives and US bond and equity markets. It was then suggested that commodity derivatives could serve well as diversification tool in portfolio management, in particular for institutional investors in need of hedging alternatives. As a result, the value of commodity-index purchased by institutional investors increased more than tenfold from 2003 to 2008 in the US (CFTC 2008). Most common was to invest passively in commodity futures indices.

After a time of disappointing performance for commodity derivatives during the financial crisis in 2008, the trend of commodity investments for institutional investors subsided. Financial literature then shifted toward questioning if the speculation in commodity derivatives from institutional investors had distorted the market place, and led to increased correlation between US equity markets and commodities, implying deteriorated diversification possibilities. Studies have since attempted to clarify the role commodity related derivatives in portfolios and the effect of institutional investors' sudden entrance to the market. Research has been applied mainly on the US market to ascertain if the efficient frontier of a well-diversified investor's portfolio can be improved by the inclusion of commodity future derivatives. Our study makes use of a similar approach, only with the perspective being extended further, namely from a Swedish pension fund perspective.

The question of commodity investing is particularly interesting for pension funds, as the low yield environment combined with the set of regulations they are under hampers investment flexibility and in extension, their ability to achieve performance objectives. In Sweden, the pension scheme is managed by six AP-funds[1], who have the assignment of harmonizing fluctuations between pension contributions and pay-outs that arise due to demographic changes and/or the current economic situation. All funds have statutory requirements on how their investments are allowed to be carried out, among these is the restriction which prohibits commodity derivative investing. The restriction was implemented in 2000 and was foremost attributed to the fact that no foreign pension funds engaged in commodity investments. In 2012 and again in 2015 there were proposals of allowing for commodity investment as part of major pension reforms, however, these were rejected at both occasions and meant that no partial proposals passed. At both times, all AP-funds expressed positive toward the lifting of the restriction. With regard to this, and to the fact that the proportion of alternative investments in the funds' portfolios has increased in recent years, it has been suggested that the AP-funds have a need for increased investment opportunities. This naturally raises the question if commodity derivatives in fact can contribute to the AP-funds' investments, considering their ambiguous properties in portfolios.

[1] AP-funds one to four are responsible for managing the buffer of the income pension and are the funds referred to in this study.

For this reason, our study aims to investigate whether the portfolio efficiency of the AP-funds can be improved by the inclusion of commodity derivatives, an unexplored asset class from a Swedish pension fund perspective. Similar studies investigating diversification benefits of adding commodity futures derivatives to portfolios, often perform an initial correlation analysis between asset categories in the portfolio and commodity futures, and continue with quantifiable tests of the risky asset added to the portfolio. Therefore, we use a similar methodology, starting with an introductory analysis of time-varying correlation between the respective asset classes in the AP-portfolio and two commodity futures indices, using the Dynamic Conditional Correlation (DCC) model of Engle (2002). The primary analysis of possible diversification advantages of commodity derivatives however, is based on portfolio efficiency tests; the intersection and mean-variance spanning tests. The intersection test examines ex-post efficiency of the AP-fund portfolio by the inclusion of commodity futures between the years 2001 and 2015. The mean-variance spanning test is a more general test that examines the impact of extending the strategic asset mix of the AP-funds with commodity futures, and does not take the AP-funds' portfolio weights into account. By comparing the results from the intersection and spanning tests we are able to provide a comprehensive analysis of the diversification benefits of commodities for the AP-funds. The DCC model, the intersection and mean-variance spanning tests are frequently used in studies examining diversification possibilities of commodity futures in portfolios. Our study is however the first to our knowledge that contributes to this field of research from a Swedish pension fund perspective using this methodology.

1.1 Research Question

The research questions examined in this study is whether the first to fourth Swedish general pension funds can improve portfolio efficiency and strategic asset mix by adding commodity futures derivatives to their holdings, where portfolio efficiency is defined as improvement in risk adjusted return. The study is structured by two null hypotheses, the *first hypothesis* tests the ex-post efficiency of a replicated AP-portfolio based on the historical holdings of the AP-funds from 2001 to 2015. The *second hypothesis* examines the impact of extending the strategic asset mix of the AP-funds with commodity futures and does not, unlike the intersection test, take the AP-funds' portfolio weights into account. This is a test of whether the AP-funds by adding commodity futures to their asset mix can form a superior portfolio to their current, from a mean-variance perspective. Analyses of both tests are made for the period 2001 to 2015 as well as for accumulated bull and bear periods.

Table 1. Hypotheses

	Hypothesis One	
Test of the AP-funds' portfolio 2001-2015	H_0:	The replicated AP-portfolio shows no improvement in portfolio efficiency from investment in commodity futures
	H_1:	The replicated AP-portfolio shows improvement in portfolio efficiency from investment in commodity futures
	Hypothesis Two	
Test of the strategic asset mix of the AP- funds	H_0:	The strategic asset mix of the AP-funds is not improved by adding commodity futures
	H_1:	The strategic asset mix of the AP-funds is improved by adding commodity futures

Table 1. Hypothesis testing from adding commodity futures to the AP-funds' investments

2 Literature Review

Commodity investment as an alternative investment to stocks and bonds is not a new practise, however the magnitude of indirect commodity investment has increased with the number of commodity linked products (Greer, 2000). Attractive commodity investment instruments are commodity futures indices that do not require active management, an uncommon feature when it comes to alternative investment (Greer, 2000). As opposed to stocks and bonds, commodity futures are derivative securities rather than claims on corporations. Thus, commodity futures contracts are valued differently from stocks and bonds as they are linked to physical assets and not a stream of future cash flows (Gorton and Rouwenhorst, 2005).

The traditional view of commodity futures prices was established by Keynes (1930) and Hicks (1946) and the theory of normal backwardation. The theory states that speculators that buy commodity futures contracts provide insurance to producers that want to protect themselves from a fall in commodity prices. The price of the contract will be below the spot price that can be expected to persist at maturity of the futures contract. The backwardation of the futures price relative to the expected futures spot price will create a risk premium to speculators for bearing the risk of futures prices (Keynes, 1930). In short, the market is in *backwardation* when the spot price is above the futures price or similarly, when the nearby futures price is above the more distant futures prices. If an index is long a nearby futures contract and will roll into more distant contracts when the nearby contract is about to expire, there is a roll return[2] that will be positive (Jensen and Mercer, 2011). *Contango* is the opposite of backwardation and implies that if the spot price, or the nearby futures contract, is below the distant futures price, the market is in contango (Jensen and Mercer, 2011). Hence, when an index is long the nearby contract and roll into a new contract the roll return will be negative.

2.1 Commodity Futures in Portfolios

Among the first studies on incorporating commodities in an investment portfolio and the use of commodities as a diversification tool, was published by Greer (1978) and Bodie and Rosansky (1980). Both studies propose that a combination of commodity futures and stocks could enhance the risk and return profile of a portfolio. Thereafter numerous studies on commodity investment and the characteristics of commodities in traditional portfolios have been published. Empirical studies covering different sample periods confirm the improvement of risk-adjusted return from including a commodity futures index in a traditional portfolio (Becker and Finnerty, 2000; Gorton and Rouwenhorst, 2006; Worthington and Pahlavani, 2007; Hoevenaars et al. 2008; Bekaert and Wang, 2010; Bruno and Chincarini, 2010; Beckmann and Czudaj, 2013).

[2] Return of commodity futures is composed by three components; change in spot price, return on margin collateral, and roll return. An investor must deposit collateral when investing in commodity futures and the interest return from the collateral can be seen as a component of total return of commodities. Roll return is the profit or loss that arises when rolling an expiring futures contract into a new contract. (Jensen and Mercer, 2011)

As investments in commodity futures have increased, their function as diversification tool has further been examined (Stoll and Whaley, 2010). Gorton and Rouwenhorst (2006) made a comprehensive study covering a time period of 45 years and a large number of commodity futures and concluded that commodity futures have historically offered the same return and Sharpe ratio as equities, and at the same time, the return of commodity futures displays negative correlation to the return of stocks and bonds. Moreover, Gorton and Rouwenhorst (2006) confirm the findings of Weiser (2003) and Vrugt et al (2004) that commodity futures perform well during early stages of recessions, when stocks generally underperform and, in later stages of recessions commodity returns drop while equities generally are thriving. In 2015, Gorton and Rouwenhorst complemented their previous study to cover ten more years of data, consisting of a period of both global economic expansion and global financial crisis. Gorton and Rouwenhorst (2015) prove many of the conclusions of their original study, such as the return of commodity futures being similar to equities. The updated study, also concludes that the correlation between commodity futures and traditional investment products increases in recessions (Gorton and Rouwenburst, 2015). Furthermore, Greer (2007) finds that the appropriate allocation to commodities in a traditional portfolio should be ten to fifteen percent based on risk and return parameters, rather than the typical allocation by investors to commodities of around five percent.

In recent years, the introduction of commodity futures to portfolios has been analysed from a mean-variance perspective with statistical tests. The statistical significance of a shift in the efficient frontier has been analysed when the investment universe is extended by a commodity futures index by using the mean-variance intersection and spanning tests (Nijman and Swinkels, 2003; Kooli, 2006; Daskalaki and Skiadopoulos, 2011; Belousova and Dorfleitner, 2012; Huang and Zhong, 2013). These tests were first introduced by Huberman and Kendel (1987) and developed out of the mean-variance framework by Markowitz (1952). A portfolio is mean-variance efficient if it is not possible to form a portfolio with the same risk and higher return with the same set of assets (Markowitz, 1952). The intersection and mean-variance spanning tests are two related tests that are used in research of portfolio analysis for analyzing the effect of introducing a new risky asset to a set of assets. The economic significance of the spanning test has further been developed by Kan and Zhou (2012) who presented the step-down spanning test as a sequential test of a shift in the tangency portfolio[3] and the Global Minimum Variance (GMV)[4] portfolio from adding an additional asset to a set of assets. Berlousova and Dorflitner (2012) use the spanning test and the step-down test to examine commodity diversification from a Euro investor perspective and find that diversification opportunities persist also from a Euro perspective. Kooli (2006) uses a similar method and observes added value in terms of improved risk-return trade-offs for hedge funds from adding commodity futures.

The research of commodity futures investing in the context of pension funds is modest, however, it has been suggested that commodity futures have attributes which could be

[3] The tangency portfolio is defined as the portfolio that provides the highest amount of return for least amount of risk, namely the portfolio tangent to the efficient frontier.
[4] The Global Minimum Variance (GMV) portfolio is the minimum variance portfolio in the mean-variance framework.

beneficial in pension fund portfolios. Nijman and Swinkles (2003) were early to examine commodity investing in pension funds. Using the mean-variance framework of intersection and spanning, they were concerned whether the variance risk of pension schemes portfolios could be reduced by adding commodity investment. The study showed that for nominal pension schemes, the use of commodities in the portfolio can improve the risk-return profile when conditioning information on the macro economic environment, while for inflation indexed pension scheme they reduce the volatility on the funding ratio by more than 30 percent regardless conditions. The authors concluded that timing strategies are effective for both inflation-indexed and nominal pensions. Beenen (Futures Industry, 2005) evaluated one of Holland's largest pension funds PGGM's performance after the fund had allocated a small part of its holdings to the Goldman Sachs Commodity Index. He concluded that significant effects could be obtained with small measures, an example was that 50 percent of PGGM's return during the first quarter of 2005 was attributable to the 4 percent in commodity derivatives. The investment in the Goldman Sachs Commodity Index was motivated by its suitable characteristics, such as its skewness towards the energy sector. Although energy commodities in themselves have high volatility, PGGM found a reduction in overall volatility of their portfolio from the inclusion of energy commodities, (Beenen, 2005). Thus, commodity derivatives reduced the risk in the portfolio while not compensating on return.

Although many authors have proposed diversification benefits of commodity futures in traditional portfolios there is ambiguity in the literature. Erb and Harvey (2005 and 2006) challenge the conclusions of commodity returns being equal to equities. Their results show that the return of individual commodity futures contracts has been close to zero, and that a commodity futures portfolio can only have equity like returns if it is able to achieve a diversification return, or by skewing portfolio allocation towards commodity futures that are highly assured to have positive returns in the future. Historically, high returns of long-only positions in commodity portfolios are highly driven by the allocated weights to different commodities (Erb and Harvey, 2006). Furthermore, Erb and Harvey (2006) advocate tactical allocation in commodities with a long-short strategy depending on the contracts being in backwardation or contango. In 2015, after almost a decade of disappointing results of commodity futures, Erb and Harvey updated their study and attributed the bad performance of commodity futures in portfolios to lack of competence in commodity investing.

More recent studies investigate whether the role of commodity futures in portfolios has changed over time. Cao et al. (2010) examines recent data, 2003 to 2010, and find no evidence that adding commodities to a portfolio of global stocks and bonds enhance portfolio efficiency from a mean-variance perspective. Wolfgang and Wolff (2015) looked into subgroups of commodity indices, such as agriculture and livestock. Their results show that diversification benefits of adding commodities to traditional portfolios differs among different groups of commodities. Aggregate commodity indices, industrial and precious metals show positive diversification effects while agriculture and livestock do not. Hence the critically discussed commodity subgroups of food are not fundamental for the effect of commodity diversification. A study from 2015 by Daigler et al., finds that allocation of individual commodity futures, rather than commodity indices, in a traditional portfolio of bonds and stocks improves the return of the portfolio.

2.2 Correlation Implications in Literature

Diversification benefits of commodity futures are mainly attributed to its correlation with other assets. The less correlation between returns on assets included in a portfolio, the more effective is the reduction in risk. In early 2000, research suggested that commodity markets were partially separated from conventional financial markets, and from each other, implying diversification possibilities (Tang and Xiong, 2012). In the years following, commodities were recognized by institutional investors as a new investment opportunity, much based on the findings by Greer (2000), Gorton and Rouwenhorst (2006) and Erb and Harvey (2006). Their studies suggested that correlation between commodity futures derivatives and US equity and bond markets were slightly negative. Consequently, institutional investors in commodity related derivatives increased substantially, from $15 billion in 2003 to $200 billion in 2008 (CFTC 2008).

Greer (2000), Gorton and Rouwenhorst (2006) and Erb and Harvey (2006) based their recommendations on rolling window or constant correlation estimators. Recent studies problematize the use of such unconditional correlation estimates, and suggest that they are too restrictive to fully capture the time varying aspect of correlations, and in extension, diversification possibilities (Büyuksahin and Robe, 2014). The implication of this is that initial recommendations on diversification benefits for institutional investors could have been overestimated (Tang and Xiong, 2012; Tuysuz, 2013). Huang and Zhong (2010) who are concerned with time variation in diversification benefits of commodities along with two other asset types, base their findings on a correlation model that takes into account the time varying aspect, namely the Dynamic Conditional Correlation model (DCC) of Engle (2002). Their study concludes that diversification benefits for the asset classes examined alter significantly over time, which implies that diversification possibilities need to be evaluated frequently.

The DCC model enables observation of the response in correlation due to economic shocks and fluctuations. The model is an extension of the Constant Conditional Correlation (CCC) model introduced by Bollererlev (1990) and unlike its predecessor, can take into account the time-varying nature of correlation. DCC is widely used to measure correlation in time series returns as means to optimize portfolios, and to evaluate the change in diversification possibilities (Tuysuz, 2013). Many studies comparing methods for measuring time-varying correlation have highlighted the benefits and flexibility of DCC (Huang and Zhong, 2010; Silvennoinen and Teräsvirta, 2007) and its superiority to other dynamic correlation estimates.

Studies measuring correlation between commodity futures derivatives and equity markets during recent years are often interested in the financialization of commodity markets. (Basak and Pavlova, 2015; Cheng and Xiong 2013; Irwin and Sanders, 2011; Silvionnen and Thorp 2013; Tang and Xiong, 2012). Financialization is a concept referring to institutional investors potentially having a distorting effect on the market place by the speculation in commodity indices, with the consequence of increased correlation between equity markets and commodity markets. Tang and Xiong (2012) believe that this helps explain the increased price volatility of non-energy commodities around the time of the financial crisis in 2008 which in extension led to food-shortages in developing countries. Their study showed that correlation between oil and non-energy related commodity futures increased extensively after 2004, and that the trend was more distinct for commodities that traded on indices. Likewise, Basak and Pavlova (2015) find a stronger increasing trend in correlation to equity markets for commodity futures that trade on indices compared to those that do not. Li et al. (2011) make a comprehensive study using the DCC model and find that 32 equity markets out of 45 examined demonstrate an upward change in the long run correlation with commodity indices in the last decade, Sweden included. 43 of the 45 markets also had a sharp increase during the subprime mortgage crisis. The authors conclude from these findings that the deteriorating trend in diversification possibilities of commodity futures is long-run and global. Opponents argue that the increase in correlation during the time of the financial crisis is consistent with historical variations during different business cycles (Gorton and Rouwenhorst, 2015), and that structural changes between commodity markets and equity markets not are harmful but instead lead to decreased risk premiums (Irwin and Sanders, 2012). However, as Büyksahin and Robe (2012) state, in spite of increased correlation between commodity and equity markets, the evidence does not preclude that long-term diversification possibilities have vanished.

3 Theory Review

This section elaborates econometric frameworks used to conduct the analysis of adding commodity futures to the AP-funds' holdings. First, we present the Dynamic Conditional Correlation (DCC) model of Engle (2002) used to estimate time-varying correlation between each asset class in the AP-fund's portfolio and one of the commodity futures indices used (SPGSCI and BCOM). Secondly, we present the portfolio efficiency tests, intersection and mean-variance spanning tests of Kan and Zhou (2012).

3.1 Dynamic Conditional Correlation Model

The DCC model is a generalization of Bollerslev's (1990) Constant Conditional Correlation (CCC) model, and uses the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model (Bollerslev, 1986). We estimate GARCH(1,1) and DCC(1,1) models, meaning that the models only contain one lag for the squared innovation and the variance respectively. These are the simplest and most frequently used specification for GARCH and DCC models.

The model suggested by Engle (2002) is as follow:

(1) $$H_t = D_t R_t D_t$$

DCC defines the conditional covariance matrix as H_t in equation (1) [5], where D_t is the diagonal of the matrix of standard deviations estimated from a univariate GARCH(1,1) model for each time-series of returns, which is the first step in estimating our DCC(1,1). In a second step, R_t is the time-varying correlation matrix estimated using time-varying standard errors $\varepsilon_{i,t}$ from the first step.

For a GARCH(1,1) model of returns $r_{i,t}$

(2) $$r_{i,t} = u_{i,t} + \sqrt{h_{i,t}}\varepsilon_{i,t}$$

with the following conditional variance structure:

(3) $$h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}$$

where $h_{i,t}$ in (3)[6] is a matrix of conditional variances, $u_{i,t}$ in (2) is the conditional mean of $r_{i,t}$, $\sqrt{h_{i,t}}$ is the matrix of conditional standard deviations, and α_i and β_i in equation (3) give information on the dynamics of the volatility time series.

[5] In the CCC model (Bollerslev, 1990), the conditional correlation matrix is time invariant and $\boldsymbol{R_t} = \boldsymbol{R}$

[6] In equation (3) $\omega_i > 0, \alpha_i \geq 0, \beta_i \geq 0, and\ \alpha_i + \beta_i < 1$

Further specifications of the DCC model are:

(4) $$D_t = diag(\sqrt{h_{i,t}})$$

(5) $$R_t = diag(Q_t)^{-1} Q_t diag(Q_t)^{-1}$$

$Q_t = (q_{i,j,t})$ describe covariance between $r_{i,t}$ and $r_{j,t}$ at time t and is specified in equation (6).

(6) $$Q_t = (1 - a - b)\bar{Q} + a(\varepsilon_{t-1}\varepsilon'_{t-1}) + bQ_{t-1}$$

$\bar{Q}$ in equation (6)[7] is the N*N unconditional covariance matrix of standardized error terms. $Q_t = (q_{i,j,t})$ enables calculation of conditional correlation between assets i and j at time t with a standard correlation formula:

(7) $$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}$$

3.2 Intersection Test

The intersection test analyses the efficiency of a given portfolio from the introduction of additional assets (Gibbons et al., 1989; Jobson and Korkie, 1989; DeRoon and Nijman, 2001). Intersection occurs when the regression of an additional asset on a portfolio or assets have exactly one point in common and it implies that there is no benefit in terms of return from adding the new asset to the portfolio (DeRoon and Nijman, 2001). Equation (8) is the intersection regression equation of test assets on a benchmark portfolio, N is the number of test assets and K is the number of benchmark assets.

(8) $$R_{it} = \alpha_i + \beta \cdot R_{pt} + \varepsilon_t$$

The equation is constructed of R_{it} which is the excess return of test asset *i* at time *t*, α is an N dimensional vector of intercepts, β is an N*K dimensional matrix of slope coefficients, R_{pt} is the excess return of the benchmark portfolio *p* at time *t*, and ε_t is an N dimensional vector of error terms. Equation (9) is the null hypothesis that implies intersection of the test asset on the benchmark portfolio.

(9) $$H_0\colon \alpha = 0$$

The intercept is the additional risk-adjusted return that is available and a test of statistical significance is used to test whether the intercepts are jointly different from zero. Kan and Zhou (2012) present the F_1-test statistic in equation (10) for testing the intersection null hypothesis.

(10) $$F_1 - test = (T - K - 1)\left(\frac{|\bar{\Sigma}|}{|\hat{\Sigma}|} - 1\right) \sim F_{N,T-K-N}$$

$\hat{\Sigma}$ is the unconstrained estimate of the covariance matrix and $\bar{\Sigma}$ is the constrained estimate of the covariance matrix by imposing the constraint of all alphas being jointly equal to zero. A

[7] $a\ and\ b\ are\ non-negative\ coefficients\ subject\ to\ a + b < 1$

rejection of the intersection null hypothesis implies that the covariances of the constrained and unconstrained estimates are significantly different.

3.3 Mean-variance Spanning Test

The objective of the mean-variance spanning test is whether there is a shift in the mean-variance frontier from the introduction of new assets to an initial set of assets. Spanning occurs when a set of K risky assets has an identical minimum variance frontier as a larger set of N plus K risky assets, where K is denoted the number of benchmark assets, N is the number of test assets and T is the number of time-series observations. In this study we are using the notations of Kan and Zhou (2012) for defining the mean-variance spanning test. Equation (11) is the spanning regression equation and equation (12) is the matrix form of this equation. The equations are constituted by the excess return of the test assets projected on the excess return of the risky assets.

(11) $$R_{2t} = \alpha_t + \beta \cdot R_{1t} + \varepsilon_t$$

(12) $$Y = \mathrm{XB} + \mathrm{E}$$

Y is a vector of excess returns of the test assets with the dimensions T*N, X is a T*(K+1) matrix of excess returns of the benchmark assets, β is the coefficients vector $[\alpha, \beta]'$ with dimensions K+1 and E is the T*N dimensional matrix of error terms.

The spanning model is built on the assumptions that the expected value of the error term is zero $(\mathrm{E}(\varepsilon) = 0_{\mathrm{N}})$ and that the error terms must be uncorrelated $(\mathrm{E}(\varepsilon \mathrm{R}') = 0_{\mathrm{N*K}})$. Spanning is tested by the null hypothesis (13) which is a joint null hypothesis of the intercepts being equal to zero and deltas being equal to zero, which implies that the sum of the slopes for each regression, β, are equal to one.

(13) $$\mathrm{H0}: \alpha = 0, \delta = 1 - \beta = 0$$

When the joint null hypothesis (13) is rejected there is a significant improvement of the tangency portfolio $(\alpha \neq 0)$ and the GMV-portfolio $(\delta \neq 1 - \beta = 0)$ respectively. The test statistic of the null hypothesis (13) when the number of test assets are equal to one $(N = 1)$ is the F-test statistic in equation (14).[8]

(14) $$F - test = \left(\frac{1}{U} - 1\right)\left(\frac{T-K-1}{N}\right) \sim F_{2N,T-K-1}$$

Where U is the ratio of unconstrained and constrained estimators of variance, imposing the restriction of all alphas and deltas being jointly equal to zero.

[8] For $N > 1$ the test statistic becomes; $F - test = \left(\frac{1}{U^{\frac{1}{2}}} - 1\right)\left(\frac{T-K-N}{N}\right) \sim F_{2N,2(T-K-N)}$

3.3.1 Step-down Spanning Test

The step-down spanning test is a sequential test of spanning that, contrary to the previously mentioned test, provides information about what causes the rejection of the null hypothesis. Dividing the hypothesis of spanning into two, the first hypothesis tests the intersection and the second hypothesis tests the slope coefficients. The first null hypothesis is tested using the F_1–test statistic, which is the same as the previously mentioned test statistic in the intersection test.

Thereafter, the step-down spanning test examines the second null hypothesis of the delta being equal to zero, conditional on the constraint that the alphas are equal to zero. This is a test of whether the GMV-portfolio has zero weight in the test asset (Kan and Zhou, 2012). With one test asset (N=1), the second hypothesis is tested by the F_2-test statistic in equation (16).

(15) $$F_2 - test = (T-K)\left(\frac{|\tilde{\Sigma}|}{|\hat{\Sigma}|} - 1\right) \sim F_{N,T-K}$$

The F_2-test statistic tests the statistical difference in covariance of the constrained and unconstrained model, $\hat{\Sigma}$ is the unconstrained estimate of the covariance matrix and $\tilde{\Sigma}$ is the constrained estimate of the covariance matrix, by imposing the constraint of all alphas and deltas being jointly equal to zero.

If there is a rejection of the first test (F_1), it is known that the two tangency portfolios are statistically different and it implies that the test assets are proved to improve return. If the rejection is due to the second test (F_2) the two GMV-portfolios are statistically different and it implies that the level of risk is reduced by introduce the test asset (Belousova and Dorfleitner, 2012). The step-down procedure allows allocation of different significance levels to the two tests based on their relative economic significance. For the computations of the F-test statistics see appendix section 8.2, and the complete geometry of the spanning tests as well as a power analysis can be found in Kan and Zhou (2012).

4 Data and Methodology

4.1 The Dataset

A generic AP-fund's content is replicated to the extent possible with suitable indices. The portfolio is constructed based on a combination of AP-fund one to four's allocations. Assembling portfolio allocations is problematic, as the funds report portfolio holdings in differing ways. For instance, the third AP-fund report their holdings based on division into risk groups and not asset classes. Furthermore, the funds have in some cases changed reporting approach during the observed time period 2001 to 2015. For this reason, we believe that an attempt to compile our own division of asset classes would lead to consistency problems, therefore we rely on the consultancy firm McKinsey's division of the AP-funds' asset classes. From 2001 until today, they perform an annual evaluation of the funds' performance and report a portfolio combining the holdings of AP-fund one to four.

The correlation model DCC as well as the intersection and mean-variance spanning tests are performed on the 15-year period 2001 to 2015, for the reason that the Swedish pension scheme as it is today was introduced in 2001. Thus, index data for the period 1st of January 2001 to the 31st of December 2015 is retrieved from Bloomberg in weekly and monthly frequencies using last price. When a non-trading day occurs the most recent price is used as a substitute.

The correlation estimate DCC is performed on weekly data, implying 733 observations for each time series. Weekly data prices are retrieved on Wednesdays to eliminate a potential weekend effect. DCC is preferably conducted at higher frequencies to capture the changing nature of correlation over time. The intersection and mean-variance spanning tests are performed on monthly data, resulting in 180 observations for each index series. For both weekly and monthly data, returns on prices are calculated logarithmically, and excess returns are obtained by deducting the risk free rate. The risk-free rate is noted by a three-month Swedish Government T-bill and converted to weekly and monthly basis in accordance to the data. When measuring diversification possibilities of risky assets, economic research commonly employs an initial correlation analysis between asset categories and the risky asset, and continues with quantifiable tests of the risky asset added to the portfolio. The correlation estimate DCC and the portfolio efficiency tests, mean-variance spanning and intersection test, are non-related, therefore we find it appropriate to use the frequencies suitable for each respective test following previous literature.

The intersection and mean-variance spanning tests are accompanied by an analysis on accumulated bull and bear markets. Bull and bear periods are assigned by looking at the returns of the OMX Stockholm 30 index. Figure 1 (next page) illustrates prices of the OMX Stockholm 30 index during 2001 to 2015 to provide a visual view of the division between bull and bear periods.

Figure 1. OMX Stockholm 30

Figure 1. OMX Stockholm 30 prices 2001-2015. The red vertical lines mark division between bull and bear periods.

4.2 Selection of Indices

The analysis of the AP-funds' investments is implemented by replicating their portfolio by representable indices and thus, the choice of indices is central to our study. The selection of the most suitable indices originates from the benchmark indices used by the AP-funds to evaluate the funds' performance. Table 2 presents the division of asset classes, each representative index and the notations for the indices sometimes used in this study.

Table 2. Asset Classes and Representative Indices

AP-funds' Asset Class	Representative Index	Notation in this study
Foreign Equities	MSCI World Local Index	MXWO
Equities in Emerging Markets	MSCI Emerging market Local Currency Index	MXEM
Swedish Equities	OMX Stockholm Benchmark Index	SBX
Fixed Income Assets		
Swedish Fixed Income	Handelsbanken Sweden All Bonds Total Return Index	SWEBON
Foreign Fixed Income	JP Morgan Global Aggregate Bond Index	GLOBON
Alternative Investments	[Constructed]	ALT
Commodity Indices		
Commodity Index 1	Bloomberg Commodity Index	BCOM
Commodity Index 2	S&P Goldman Sachs Commodity Index	SPGSCI

Table 2. AP-fund's asset classes and their representative indices and notations in this study

The asset class 'Alternative investments' differs between AP-fund one to four but includes for instance investments in hedge funds, real estate, infrastructure and private equity. As pointed out by Thomas Franzén, head investor of the second AP-fund, replication of this asset class is difficult to achieve as there are no existing representative indices for some of the categories. Nonetheless, the predominant allocation within alternative investments is Swedish real estate and therefore, we find a Swedish real estate index most suitable to represent the asset class alternative investments. Furthermore, the real estate investments held by the AP-funds are mainly Swedish commercial and residential properties. Due to the absence of a representative Swedish real estate index reflecting commercial and residential properties, a real estate index was constructed to be used as a proxy for the real estate holdings of the AP-funds. The Swedish real estate companies Wallenstam and Hufvudstaden are selected to represent the holdings in residential and commercial real estate respectively. The stock prices of the real estate companies were unleveraged to adjust for debt, thereafter a weighted index from the AP-funds' holdings in commercial and residential real estate is comprised. A more detailed description of the formation of the real estate index is found in appendix section 8.3.1.

The selected indices to represent investment in commodity futures contracts are the aggregate commodity indices S&P Goldman Sachs Commodity Index (SPGSCI) and Bloomberg Commodity Index (BCOM, formerly the Dow Jones-UBS Commodity Index). SPGSCI and BCOM are commonly used commodity futures indices included in portfolios of institutional investors (Yau et al., 2007). Furthermore, the indices are frequently examined in previous research which increases comparability in this study. SPGSCI comprises of 24 commodity futures contracts weighted by world production, making it have a considerable high exposure to energy. BCOM comprises of 19 futures contracts, where any contract is constrained to a maximum weight of 15 percent and any sector is constrained to a maximum weight of 33 percent. In appendix section 8.3.2 the full compositions of the commodity indices are presented. The commodity indices are retrieved in excess returns meaning the collateral yield from collateralization of commodity investment is not included. According to Belousova and Dorfleitner (2012) the collateralization implies that it is possible for the total return index to have positive returns when the underlying commodity price declines and returns from direct investments in physical commodities are negative. For this reason we find the use of excess return of the representative indices appropriate for our study.

The AP-funds are to a large extent currency hedging their investments. To account for currency hedging, the global indices MSCI world and MSCI emerging markets are collected in local currency. Using local currency indices eliminates the currency effect as the returns are not affected by fluctuations in the exchange rates. The commodity indices SPGSCI and BCOM are collected in USD, justified by the predominant trade of these indices in USD. The global bond index is also collected in USD, because of the difficulty to find a representable global bond index in local currency. The Swedish equity index, the Swedish bond index and the Swedish real estate indices (Hufvudstaden and Wallenstam) are all collected in SEK.

4.3 Delimitations

The impact from commodity futures investing could differ between AP-fund one to four as they have differing portfolio weights. The decision of not performing tests separately for each AP-fund is motivated by the fact that we are mainly interested in the general idea of commodity investing for the Swedish income pension buffer. Furthermore, the decision only has an impact on the intersection test as this is the only test taking portfolio weights into consideration. The mean-variance spanning test takes into account the asset mix which in our study would not differ between the funds as the same representative indices would be used.

Moreover, we are unable to fully reflect the currency hedging of the AP funds. The funds have restrictions on currency exposure to a maximum of 40 percent and hence, the major part of their holdings is not exposed to exchange rate fluctuations. Therefore, we find it suitable to not reflect currency exposure in this study and focus on the return of the underlying assets.

4.4 Application of the Dynamic Conditional Correlation Model

DCC(1,1) is performed to measure correlation between each asset class in the AP-funds' portfolio to a commodity futures index, represented by SPGSCI and BCOM respectively. The estimation is performed in Stata on weekly excess return. Excess returns are defined as time series with weekly frequencies, after which the GARCH(1,1) parameters (ω_i, α_i, β_i, $u_{i,t}$) are estimated simultaneously using maximum likelihood. The variables are subject to $\omega_i > 0, \alpha_i \geq 0,\ \beta_i \geq 0, and\ \alpha_i + \beta_i < 1$. The model accounts for information in the financial time series, such as volatility clustering. For instance, if the volatility was high at time $t - 1$ it is more likely to be high at time t. In the second step of DCC(1,1), a and b are estimated simultaneously by maximizing the log likelihood. Here as well, the model captures information in the financial time series, so that if correlation is high at time $t - 1$ it is more likely to be high at time t. Because the equation is subject to $a + b < 1$ the correlation is mean revering, and fluctuates around the unconditional correlation. Further, correlations are calculated from dynamic variances using a standard correlation formula. Section 8.5 in appendix specifies the coding for the DCC GARCH estimation in Stata. The results from the DCC(1,1) model estimation are time-series of conditional correlation, and are visually displayed in figures 2 to 13 in appendix.

4.5 Application of the Intersection and Mean-Variance Spanning Test

The primary empirical analysis of this study is an intersection and spanning test of commodity futures on the AP-funds' investments. The analysis of the AP-funds' investments will be done in two steps, firstly an intersection test will be made with the replicated AP-portfolio as the benchmark portfolio and each commodity futures index as test assets. Equation (17) is the intersection regression equation of the AP-portfolio and the corresponding null hypothesis of intersection is specified in equation (18).

(16) $$R_{com} = \alpha_i + \beta \cdot R_{AP-portfolio} + \varepsilon_i$$

(17) $$H_0: \alpha = 0$$

The intersection hypothesis (18) from adding commodities to the AP-portfolio is a test of the intercepts from the regression equation being equal to zero. The regression is run on the excess returns of commodity futures and the excess return of the replicated AP-portfolio.

Secondly, a mean-variance spanning test of commodity futures on the strategic asset mix of the AP-funds is completed to evaluate the effect of commodity futures on the AP-funds current investment universe. Equation (19) is the spanning equation of a commodity index on the benchmark assets separately and equation (20) is the joint null hypothesis for spanning.

(18) $$R_{com} = \alpha + \beta_{MXWO} \cdot R_{MXWO} + \beta_{MXEM} \cdot R_{MXEM} + \beta_{SBX} \cdot R_{SBX} + \beta_{SWBON} \cdot R_{SWBON} + \beta_{GLOBON} \cdot R_{GLOBON} + \beta_{ALT} \cdot R_{ALT} + \varepsilon$$

(19) $$H_0: \alpha = 0, \delta = 1 - \beta = 1 - \beta_{MSCIWO} - \beta_{MSCIEM} - \beta_{SBX} - \beta_{SWBON} - \beta_{GLOBON} - \beta_{ALT} = 0$$

The joint hypothesis (12) states that the indices representing the current set of asset classes that the AP-funds invest in span the commodity index. Spanning implies that the efficient frontier of the strategic asset mix is not improved by adding commodity futures and hence, there is no improvement of the AP-funds investment opportunity when adding commodity futures. The joint null hypothesis of spanning, alphas equal to zero and slope coefficients equal to one, is also tested separately by the step-down spanning test.

(20) H_0^1: $\alpha = 0$

H_0^2: $\delta = 1 - \beta = 1 - \beta_{MXWO} - \beta_{MXEM} - \beta_{SBX} - \beta_{SWBON} - \beta_{JPGLOB} - \beta_{ALT} = 0 \qquad conditional\ on\ \alpha = 0$

The step-down null hypothesis (13) extends the spanning analysis, a rejection of the first null hypothesis is interpreted as enhanced return and a rejection of the second null hypothesis is interpreted as a reduction in risk. The improvement in portfolio efficiency is determined from the p-values of the test statistics where a lower p-value means higher diversification benefits and therefore, the rejection of the null hypothesis will be analysed according to different significance levels. The intersection and spanning tests are performed in MATLAB for which Kan and Zhou (2012) have provided a code to complete the tests, the code can be found in appendix section 8.6.

5 Empirical Results and Analysis

Results are presented in the following order; firstly, the time-varying correlation results from the DCC model are presented. Secondly, the results from the intersection test of two different commodity futures indices on the replicated AP-portfolio are presented. Thirdly, the spanning results of the two different commodity futures indices on the strategic asset mix are presented.

5.1 Dynamic Conditional Correlation Results and Analysis

Figures 2 to 13 in the appendix display the results of time varying correlation between the two commodity indices SPGSCI and BCOM to each representative index, using the DCC model. Unconditional correlation for the period January 1st 2001 to December 30th 2015 is displayed with a green horizontal line, and bull and bear periods are marked with red vertical lines to visualise correlation behaviour in changing economic climates. A fitted trend line for the time varying correlation is displayed with a pink line. Note that the range of the y-axis differs for each figure. Table 13 in appendix section 8.4 summarizes the descriptive statistics of the DCC correlation estimation.

Based on the results of the DCC model, we can see that correlation alters considerably over time. Figures 2 to 7 display correlation between SPGSCI and all asset classes in the AP-funds' portfolio. Common to all asset classes except for Swedish bonds is that correlation has had an increasing trend during the observed time period. All asset classes have a correlation near zero at the beginning of the observed period. World equities and emerging market equities follow a similar trend to each other, with increasing correlation and in 2015 reaching a correlation around just under/over 0.4 respectively. Swedish equities have a weaker increasing trend than that of world and emerging market equities, however during some periods the correlation peaks and reaches correlation estimates in level with world- and emerging market equities. These findings are consistent with previous research, that correlation between equity- and commodity markets have increased world-wide (Li et al., 2011; Tang and Xiong 2012). Correlation between alternative investment and SPGSCI also displays a weak increasing trend during the observed time period.

Correlation between global bonds and SPGSCI has a weak increasing trend, which appears to level off during 2015. This result is consistent with previous research, that correlation between the US fixed income market and commodity futures has increased. (Li et al., 2011). However, except for two peaks in 2003 and late 2008, correlation can still be considered relatively low for most of the period between global bonds and SPGSCI. Swedish bonds (SWBON) distinguish itself with a negative correlation to SPGSCI during most of the period, in addition to a decreasing trend. The heavy fluctuations displayed in the beginning of the period seem to have levelled off since the financial in 2008. Correlation between Swedish bonds and SPGSCI is during one occasions -0.45, which is by far the lowest correlation found compared with all other classes.

Figures 8 to 13 display conditional correlations between the BCOM commodity index and each asset class separately. As in the case of SPGSCI, all correlation estimates for the different asset classes lies initially at around zero and show similar dynamic correlation patterns as with SPGSCI, with slightly differing trends. World- and emerging market equities have stronger trend coefficients compared to SPGSCI during the time period examined. Swedish equities display a weaker trend coefficient than that of world equities and emerging markets, it is however more steep than for SPGSCI. Global bonds and real estate also have an increasing correlation with a higher trend coefficient than that of SPGSCI.

The correlation results for both SPGSCI and BCOM confirm results from previous studies, that commodity futures as diversification tool has deteriorated, if solely viewing individual correlation between different asset classes and commodity futures indices. Considering most previous research has been dedicated to US equity markets it is especially interesting to see correlation results from remaining asset categories, namely the Swedish equity market, global bonds, alternative investments and Swedish bonds. World and emerging market equities display a considerable change in correlation during the 15 year time period observed. The Swedish equity market is affected as well but not to the same extent. These findings aligns well with the results from the study made by Li et al. (2012) in which they found that 32 out of 45 equity markets examined displayed a long-run upward trend in correlation after the most recent financial crisis, Sweden included. The findings are also in line with Belousova and Dorfleitner (2012) who suggest that diversification possibilities differ between equity markets.

Nonetheless, diversification benefits for a portfolio cannot be evaluated solely based on correlation estimates between individual assets categories. From the perspective of the AP-funds we can conclude that commodity futures offer best diversification possibilities toward Swedish bonds if only taking correlation into consideration. Although commodity futures have had increasing correlation trends with the remaining asset categories, correlation between commodity futures and global bonds can still be considered low, especially in recent years. In light of the portfolio consisting a third of fixed income (Swedish and global bonds) and for some years as much as 39 percent, the use of commodities as diversification tool bodes well despite the results from the equity markets.

5.2 Intersection Test of the AP-portfolio

The intersection test examines the ex-post efficiency of the AP-funds' portfolio 2001 to 2015 from adding a commodity futures index. Table 3 (next page) summarizes the results from the intersection regression of the SPGSCI and BCOM commodity index on the replicated AP-portfolio.

Table 3. Intersection test of the AP-portfolio

The table presents two sets of intercept testing on two commodity indices, SPGSCI and BCOM, using the AP-portfolio as benchmark portfolio. F_1 tests the null hypothesis for intersection of the portfolio. Results are presented for the whole observation period, 2001 to 2015, as well as for accumulated bull and bear periods.

Commodity Index	$\hat{\alpha}$	**F_1-test**	**p-value**
	Entire Period Jan 2001 - Dec 2015		
BCOM	-0,005	2,354	0,127
SPGSCI	-0,008	2,647	0,106
	Bull Period		
BCOM	-0,007	3,250	0,074 ***
SPGSCI	-0,007	1,338	0,250
	Bear Period		
BCOM	-0,010	1,665	0,202
SPGSCI	-0,015	1,907	0,172

***Significant at 1% significance level, **Significant at 5% significance level, ***Significant at 10% significance level**

Table 3. Summary of regression results from the intersection test of the AP-portfolio

The intersection test of the AP-portfolio shows no significance for either BCOM or SPGSCI when tested over the entire observation period. In bull periods the null hypothesis for BCOM is rejected (significant at ten percent), thus there is evidence of diversification benefits for the BCOM commodity futures index in bull periods. In bear periods there is no significant improvement of the AP-portfolio from adding the BCOM and SPGSCI commodity indices respectively. These results contradict Nijman and Swinkles (2003) and Beenen (2005) that proposed positive diversification effects from commodity futures in pension fund portfolios.

The BCOM index is significant in bull periods but not in bear periods. This implies decreased diversification possibilities during bear periods, and suggests that correlation between the return on the portfolio and BCOM increases in bear markets. Nijman and Swinkels (2001) conclude that the effects from commodities in pension fund portfolios vary in different time periods which are consistent with our findings for BCOM, but cannot be confirmed for SPGSCI. The variations in improvement between time periods from adding the BCOM commodity index to the AP-portfolio requires a tactical allocation strategy with frequent rebalancing linked to the price movements in equity markets, as suggested by Belousova and Dorfleitner (2012) when they examine commodity diversification from the perspective of a Euro investor.

For the whole observation period and for bear periods, there is no distinct difference between the p-values of the BCOM and the SPGSCI. For bull periods however, there is a distinct difference. This implies that in bull periods the BCOM commodity index enhances the performance of the AP-portfolio more than the SPGSCI, while the difference between the indices is not as distinct for the whole observation period and for bear periods. The SPGSCI is biased towards the energy sector and Belousova and Dorfleitner (2012) conclude that energy commodities have higher volatility in bull periods compared to other commodity futures which can be an explanation for weaker diversification benefits of the SPGSCI index in bull periods. On the other hand, the Dutch pension fund PGGM which was successful in investing in commodity futures, had a preference to the energy sector in their allocations. The reduction in volatility of the PGGM portfolio from adding energy commodities was explained by an increase in roll return (the return received when going into a new contract at a lower price when an old contract expires) from commodities in periods when financial assets underperform. Moreover, SPGSCI displays lower p-values for the overall period compared to bull and bear periods separately, meaning that a strategic allocation works better for diversification purposes with the SPGSCI index. Noteworthy is that PGGM's performance measure from commodities was done while the correlations between commodities and equities were lower compared to today.

5.3 Mean-variance Spanning Test

The mean-variance spanning test examines diversification possibilities of commodity futures on the strategic asset mix of the AP-funds. Table 4 summarizes the results from the mean-variance spanning regressions of SPGSCI and BCOM on the AP-funds' asset mix.

Table 4. Mean-Variance Spanning test on SPGSCI and BCOM

The table presents two sets of mean-variance spanning tests on two different commodity futures indices (SPGSCI and BCOM), using the MSCI World, MSCI Emerging Markets, OMX Stockholm Benchmark Index, Handelsbanken All Bonds, JP Morgan Global Bonds and an Alternative Investment index as benchmark assets. First is an F-test of the joint null hypothesis for spanning. The second test is a step-down spanning test where F_1 tests $\alpha=0$ and F_2 tests $\delta=0$ conditional on $\alpha=0$. Results are presented for the whole observation period, 2001 to 2015, as well as for accumulated bull and bear periods.

					Step-Down Test			
Commodity Index	$\hat{\alpha}$	$\hat{\delta}$	**F-test**	**p-value**	**F_1-test**	**p-value**	**F_2-test**	**p-value**
				Entire Period Jan 2001 - Dec 2015				
BCOM	-0,003	1,655	9,942	0,000 *	0,988	0,322	18,897	0,000 *
SPGSCI	-0,003	2,295	7,707	0,001 *	0,442	0,507	15,021	0,000 *
				Bull Period				
BCOM	-0,001	1,111	3,312	0,040 **	0,032	0,857	6,649	0,011
SPGSCI	0,003	1,866	3,223	0,044 **	0,269	0,605	6,216	0,014
				Bear Period				
BCOM	-0,012	2,723	7,573	0,001 *	3,819	0,056 ***	10,775	0,002 *
SPGSCI	-0,016	3,433	5,181	0,009 *	2,806	0,100 ***	7,316	0,009 *

***Significant at 1% significance level, **Significant at 5% significance level, ***Significant at 10% significance level**

Table 4. Summary of regression results from the spanning test of the asset mix of the AP-funds

From the perspective of statistical significance, commodities contribute to diversification of the set of assets included in the AP-funds' portfolios. The combined spanning test (F-test) proves diversification benefits for the whole period as well as for bull and bear periods individually. The results of this test indicate that diversification benefits function for the strategic asset mix of the Swedish AP-funds. The positive diversification results are in line with the findings of Becker and Finnerty (2000), Gorton and Rouwenhorst (2006 and 2015), and Belousova and Dorfleitner (2012) but contradicts the results of Daskalaki and Skiadopoulos (2011) and Cao et al. (2010). Differing results compared to Daskalaki and Skiadopoulos (2011) can be due to the use of individual commodities in their study instead of a commodity index. Cao et al. (2010) examined a shorter time period 2003-2010 and thus, a period influenced by the financial crisis.

The F-test shows that the rejection of the joint spanning hypothesis is most significant for the overall period as well as for bear periods, where spanning is rejected at one percent significance while in bull periods the rejection is at five percent significance. Less significant results in bull markets were also reported by Belousova and Dorfleitner (2012), but contradict Gorton and Rouwenhorst (2015) who suggest that diversification possibilities are inferior in bear periods compared to bull. Hence, decreased diversification benefits in bear periods are not clear for the asset mix of the Swedish AP-funds.

Furthermore, the step-down spanning test reveals more information about the rejection of spanning. The results of this test display that that the driver behind the rejections is statistical significance of the GMV-portfolio (F_2-test). The shift of the GMV-portfolio is statistically significant for the period as a whole as well as in bear periods and is interpreted as a reduction in risk from adding commodities to the strategic asset mix. This result has been confirmed by Beenen (2005) who reports that the added value of commodities for pension funds lies in its power to reduce the overall risk without complementing on expected overall return.

The intercept null hypothesis (F_1), which is a test of the tangency portfolio, is solely rejected in bear periods. An intercept rejection is interpreted as improvement in the ability to enhance return. This result is interesting since financial assets usually underperform in bear periods and hence investors seek to improve the risk-return profile of their investments. In bear periods both the tangency portfolio and the GMV-portfolio is proved to be enhanced by adding commodities to the portfolio asset mix of the AP-funds. The variations in diversification between bull and bear periods suggest that the benefits of commodity diversification can be enhanced by implementing a tactical allocation strategy linked to the movements in the equity market. Tactical allocation to commodities as opposed to strategic allocation have also been suggested by Erb and Harvey (2006) and Belousova and Dorfleitner (2012).

The results show that overall the observed p-values are lower for bear than for bull periods. This observation points toward the conclusion made by Gorton and Rouwenburst (2015) that correlation between commodity futures and traditional asset classes increases in recessions, meaning that diversification benefits decreases. However, this was not the case for the intersection test of the replicated AP-portfolio where the return of the portfolio was enhanced by adding commodity futures in bull periods. This implies that the replicated AP-portfolio is not located closely to either the tangency portfolio or the GMV-portfolio in this case.

The intersection test of the AP-portfolio and the spanning test of the set of asset categories that the AP-funds are currently investing in provide differing results. The determination of the strategic asset mix is an important investment decision of pension funds as their investments cover long-time horizons. Observed in the AP-portfolio is an ongoing reduction of the allocation to fixed income and an increase in allocation to risk bearing assets, such as equity, which increases the need of diversification (Beenen, 2005). Despite the fact that the intersection test shows no increase in return from adding commodity futures to the replicated AP-fund portfolio, the spanning test reveals that commodity futures could enhance the performance of the strategic asset mix of the AP-funds.

6 Conclusion

Our study examines if the AP-funds can benefit from investing in commodity futures, something which they are currently prohibited from. We find this question of research interesting due to the AP-funds positive attitude toward a lifting of the commodity investment restriction, in addition to the increased need for investment flexibility in the current low-yield environment. As other studies concerned with diversification benefits of commodity investing, we perform an introductory correlation analysis, but base our primary findings on portfolio efficiency tests. We make use of the Dynamic Conditional Correlation (DCC) model of Engle (2002) to measure time-varying correlation between asset classes in the AP-fund portfolio to commodity futures indices, and use the intersection and mean-variance framework of Kan and Zhou (2012) to measure portfolio efficiency when including commodity futures to the AP-fund portfolio. Our observation period is January 2001 to December 2015.

The DCC correlation estimation show that for all asset classes in the AP-funds' portfolio except Swedish bonds, diversification possibilities have deteriorated, as the trend in correlation between the asset classes respectively and commodity indices has increased. However, the fund has considerable holdings in fixed income to which correlation is low, for both Swedish and global bonds meaning there are still possibilities for diversification benefits.

The mean-variance spanning test and intersection test provide mixed results, and confirm the ambiguity of the conditional correlations. The return of the replicated AP-portfolio is not enhanced by the inclusion of a commodity index except for the BCOM commodity index in bull periods. Nevertheless, the mean-variance spanning test of the strategic asset mix of the AP-funds shows diversification benefits from adding commodity futures. The results display that there is mainly a reduction in risk from adding commodities to the asset mix of the AP-funds. Moreover, the results from adding commodities is more significant in the mean-variance spanning test compared to the intersection test, suggesting that based on the specific allocations of the AP-funds, the benefits of commodity futures investing is limited.

Improved efficiency of the AP-portfolio is achieved for the BCOM commodity index in bull periods. Hence, potential commodity investing by the AP-funds should be to aggregated commodity futures within different sectors, rather than commodity investments with bias towards the energy sector. Moreover, the investment method should be tactical rather than strategical and the strategy should be linked to the movements in the equity market since there is an observed difference between bull and bear periods from adding the BCOM index to the replicated AP-portfolio. This implies that commodities can work as an actively managed risk instrument for the AP-funds.

Although previous research on diversification benefits of commodities in portfolios of stocks and bonds show to some extent diverging outcomes, our results imply what has been suggested in previous literature that inclusion of commodities in a portfolio reduces the risk rather than improves the return. Our study contributes of this field of research, by drawing conclusions on commodity futures role in the AP-funds' portfolio and thus, a portfolio with holdings of Swedish assets.

6.1 Further research

This study opens up for further research particularly as it indicates that commodity futures have risk reducing properties that can be beneficial in the AP-funds' portfolios. To further examine our findings, it would be interesting to see the results from an optimization of the AP-funds' portfolio weights with respect to the allocation restrictions that the AP-funds are subjected to, as to see how an optimal portfolio including commodity futures could look like. Another aspect for further research is the different subgroups of commodities and their different diversification properties. Instead of analysing aggregate commodity indices the impact of commodity subgroups on the AP-funds' investments can be examined.

7 References

7.1 Journal Articles

Basak, S. and Pavlova, A., 2014. ‘A model of financialization of commodities’ *Available at SSRN 2201600*

Becker, K.G. and Finnerty, J.E., 2000. ’Indexed commodity futures and the risk and return of institutional portfolios’

Beckmann, J, and Czudaj, R. 2013. ’Gold as an inflation hedge in a time-varying coefficient framework’ *The North American Journal of Economics and Finance,* 24, 208-222.

Bekaert, G. and Wang, X., 2010. ’Inflation risk and the inflation risk premium’ *Economic Policy,* vol. 25, no. 64, pp. 755-806.

Belousova, J. and Dorfleitner, G., 2012. ’On the diversification benefits of commodities from the perspective of euro investors.’ *Journal of banking and finance.*

Bhardwaj, G., Gorton, G. and Rouwenhorst, G., 2015. ’Facts and Fantasies about Commodity Futures Ten Years Later’ *National Bureau of Economic Research,* No. w21243.

Bodie, Z. and Rosansky, V.I., 1980. ‘Risk and return in commodity futures’ *Financial Analysts Journal* vol. 36, no. 3, pp. 27-39.

Bollerslev, T, 1986. ‘Generalized autoregressive conditional heteroskedasticity.’ *Journal of econometrics* vol 31, no. 3, pp. 307-327.

Bollerslev, T., 1990. ‘Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model.’ *The review of economics and statistics*, pp. 498-505.

Bruno, S. and Chincarini, L. 2010. ‘A historical examination of optimal real return portfolios for non-US investors.’ *Review of Financial Economics* vol. 19, no. 4, pp. 161-178.

Büyükşahin, B. and Robe, M. A., 2014. ‘Speculators, commodities and cross-market linkages.’ *Journal of International Money and Finance*, vol. 42, pp. 38-70.

Cheng, I. and Xiong, W., 2013. ‘The financialization of commodity markets’. *National Bureau of Economic Research,* No. w19642

Cao, B., Jayasuriya, S. and Shambora, W., 2010. Holding a commodity index fund in a globally diversified portfolio: a placebo effect? *Economics Bulletin,* vol. 30, pp. 1842–1851.

Daigler, R.T., Dupoyet, B. and You, L., 2015. ’Spicing up a portfolio with Commodity futures: Still a Good Recipe?’

Daskalaki, C. and Skiadopoulos, G., 2011. ‘Should investors include commodities in their portfolios after all? New evidence.’ *Journal of Banking and Finance,* vol. 35, pp. 2606– 2626.

De Roon, F. A. and T. E. Nijman, 2001. 'Testing for mean-variance spanning: A survey.' *Journal of Empirical Finance,* vol. 8, pp. 111–15.

Engle, R., 2002. 'Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models.' *Journal of Business & Economic Statistics*, vol. 20, no. 3, pp. 339-350.

Erb, C.B. and Harvey, C.R., 2005. 'The tactical and strategic value of commodity futures' *National Bureau of Economic Research*, No. w11222.

Erb, C.B., and Harvey C.R.., 2006. 'The strategic and tactical value of commodity futures' *Financial Analysts Journal,* vol. 62, no. 2, pp.69-97.

Erb, C.B., and Harvey C.R..,, 2015. 'Conquering Misperceptions about Commodity Futures Investing' *Available at SSRN.*

Gibbons, M.R.., Ross, S.A. and Shanken, J. 1989. 'A test of the efficiency of a given portfolio.' *Econometrica: Journal of the Econometric Society*, pp. 1121-1152.

Gorton, G.B., and Rouwenhorst, K.G., 2005. 'A note on Erb and Harvey (2005)'

Gorton, G.B., and Rouwenhorst, K.G., 2006. 'Facts and fantasies about commodity futures' *Financial Analysts Journal,* vol. 62, no. 2, pp. 47-68.

Greer, R.J., 1978. 'Conservative Commodities: A Key Inflation Hedge.' *Journal of Portfolio Management,* vol. 4, pp. 26-29.

Greer, R.J., 2000. 'The Nature of Commodity Index Returns.' *Journal of Alternative Investments*, vol. 3 no. 1, pp. 45–52.

Greer, R.J., 2007. 'The Role of Commodities in Investment Portfolios.' *CFA Institute, Conference Proceedings Quarterly* December, pp. 35-44.

Hicks, C., 1946. 'Value and Capital' *Mathematical Appendix*

Hoevenaars, R.P., Molenaar, R.D., Schotman, P.C., and Steenkamp, T.B., 2008. 'Strategic asset allocation with liabilities: Beyond stocks and bonds.' *Journal of Economic Dynamics and Control,* vol. 32, no. 9, pp. 2939-2970.

Huang, J.Z., and Zhong, Z.K., 2013. 'Time variation in diversification benefits of commodity, REITs, and TIPS.' *The Journal of Real Estate Finance and Economics*, vol. 46, no. 1, pp. 152-192.

Huberman, G. and Kandel, S., 1987. 'Mean-variance spanning.' *Journal of Finance.* Vol. 42, pp. 873–888.

Irwin, S.H., and Sanders, D.R., 2011. "Index funds, financialization, and commodity futures markets." *Applied Economic Perspectives and Policy*

Jensen, G. R., and Mercer, J. M., 2011. 'Commodities as an investment.' *The Research Foundation of CFA Institute Literature Review.*

Jobson, J.D., and Korkie, B., 1989. 'A performance interpretation of multivariate tests of asset set intersection, spanning, and mean-variance efficiency.' *Journal of Financial and Quantitative Analysis* vol. 24, no. 2, pp.185-204.

Kan, R, and Zhou, G., 2012. 'Tests of mean-variance spanning.' *Annals of economics and finance* 13(1), 139-187.

Keynes, J.M., 1930. 'A Treatise on Money' New York: Macmillan & Co, 2.

Kooli, M., 2006. 'Funds of funds and the diversification effect.' *Funds of hedge funds—performance, assessment, diversification, and statistical properties*, pp. 239-250.

Li, X., Zhang, B., and Du, Z., 2011. 'Correlation in commodity futures and equity markets around the world: Long-run trend and short-run fluctuation.'

Markowitz, H., 1952. 'Portfolio selection.' *The Journal of Finance,* vol. 7, pp. 77–9.

Nijman, T. Swinkles, L., 2003. 'Strategic and Tactical Allocation to Commodities for Retirement Saving Schemes' *The Handbook of Commodity Investing* (2008), pp 522-546.

Silvennoinen, A., and Teräsvirta, T., 2009. 'Multivariate GARCH models.' *Handbook of financial time series,* pp. 201-229. Springer Berlin Heidelberg.

Silvennoinen, A, and Thorp, S., 2013. 'Financialization, crisis and commodity correlation dynamics.' *Journal of International Financial Markets, Institutions and Money*, vol. 24, pp. 42-65.

Stoll, Hans R., and Robert E. Whaley., 2010. 'Commodity index investing and commodity futures prices.' *Journal of Applied Finance,* vol. 20, no. 1.

Tang, K. Xiong, W., 2012. 'Index Investment and the Financialization of Commodities' *Financial Analyst Journal,* vol. 68, no. 5, pp. 54-74.

Tuysuz, S., 2013. 'Conditional Correlations between Stock Index, Investment Grade Yield, High Yield and Commodities (Gold and Oil) during Stable and Crisis Periods.' *International Journal of Economics and Finance* vol .5, no. 9, pp. 28.

Vrugt, E. B., Bauer, R., Molenaar, R., and Steenkamp, T., 2004. 'Dynamic Commodity Timing Strategies' *Universiteit van Maastricht*

Weiser, S., 2003. 'The strategic case for commodities in portfolio diversification' *Commodities now*, pp. 7-11.

Wolfgang, B. and Wolff, D., 2015. 'Do commodities add value in multi-asset portfolios? An out-of-sample analysis for different investment strategies' *Journal of Banking & Finance,* vol. 60, pp. 1-20.

Worthington, A.C., and Pahlavani, M., 2007. ’Gold investment as an inflationary hedge: Cointegration evidence with allowance for endogenous structural breaks.’ *Applied Financial Economics Letters*, vol. 3, no. 4, pp. 259-262.

Yau, J. K., Schneeweis, T., Robinson, T. R., and Weiss, L. R.., 2007. ‘Chapter 8: Alternative Investments Portfolio Management’ *CFA Institute Investment Books*, vol. *2007* no. 4, pp.477-578.

7.2 Newspaper Articles

Beenen, J., 2005 ‘Commodity Investing, A Pension Fund Perspective’ *Futures Industry* October.
Available from: < http://www.trendfollowing.com/whitepaper/commodity-investing.pdf >
[3 February 2016].

7.3 Company Reports

Commodity Future Trading Comission (CFTC) 2008, *Staff Report on Commodity Wrap Dealers & Index Traders with Commission Recommendations*

McKinsey, 2015, *Evaluation report of the AP-funds*

8. Appendix

8.1. Descriptive Statistics

Table 5. Descriptive Statistics

	Annual Mean %	Annual volatility %	Sharpe ratio	Skewness	Excess Kurtosis	Range	Min Return %	Max Return %	No of pos. Returns	No of neg. Returns
Entire Period										
Benchmark Assets										
MXWO	-0,30	14,77	-0,02	-0,95	1,73	27,79	-18,27	9,52	103	77
MXEM	5,20	18,40	0,28	-0,91	2,84	38,16	-25,72	12,44	96	84
SBX	3,78	20,41	0,19	-0,65	1,54	38,67	-19,98	18,69	103	77
SWBON	2,62	2,69	0,97	0,25	0,13	4,39	-1,56	2,83	110	70
GLOBON	3,11	5,66	0,55	0,01	0,83	10,28	-4,04	6,23	103	77
REES	14,83	24,75	0,60	0,44	0,28	37,19	-15,23	21,95	92	88
Commodity Indices										
BCOM	-4,36	17,17	-0,25	-0,82	2,79	36,46	-24,29	12,17	91	89
SPGSCI	-5,71	23,98	-0,24	-0,77	2,15	51,38	-33,49	17,90	94	86
AP-funds										
Replicated Portfolio	2,95	9,88	0,30	-0,70	1,43	19,13	-11,01	8,12	107	73
Bear Market										
Benchmark Assets										
MXWO	-22,01	18,89	-1,17	-0,40	0,13	30,70	-18,27	7,53	22	39
MXEM	-14,13	23,88	-0,59	-0,72	1,50	40,16	-25,72	12,44	26	35
SBX	7,86	27,17	0,29	-0,12	-0,34	22,81	-19,98	14,44	25	36
SWBON	-23,62	2,69	-8,76	0,34	0,67	7,64	-1,41	2,83	33	28
GLOBON	3,69	6,35	0,58	0,34	0,97	20,32	-3,67	6,23	35	26
REES	-14,65	24,03	-0,61	0,49	0,03	24,70	-13,64	16,65	19	42
Commodity Indices										
BCOM	-16,07	20,52	-0,78	-0,88	2,71	36,46	-24,29	11,06	24	37
SPGSCI	-22,47	29,58	-0,76	-0,76	1,62	39,50	-33,49	12,17	25	36
AP-funds										
Replicated Portfolio	-12,19	12,74	-0,96	-0,20	-0,09	17,05	-11,03	6,02	23	38
Bull Market										
Benchmark Assets										
MXWO	12,85	10,53	1,22	-0,46	1,06	17,79	-8,27	9,52	81	38
MXEM	16,58	14,04	1,18	-0,11	0,01	20,03	-8,68	11,34	70	49
SBX	21,57	13,97	1,54	0,17	2,69	30,21	-11,51	18,69	78	41
SWBON	2,58	2,54	1,01	-0,11	-0,44	3,45	-1,56	1,89	77	42
GLOBON	2,81	5,30	0,53	-0,29	1,13	8,60	-4,04	4,56	68	51
REES	33,32	23,39	1,42	0,69	0,50	37,19	-15,23	21,95	73	46
Commodities										
BCOM	2,20	14,90	0,15	-0,40	1,12	28,24	-16,07	12,17	67	52
SPGSCI	1,22	20,18	0,06	-0,34	0,65	32,56	-14,67	17,90	69	50
AP-funds										
Replicated Portfolio	11,43	7,04	1,62	-0,03	1,27	12,70	-4,58	8,12	84	35

Table 5. Descriprive statistics of the excess returns of the AP-funds' asset classes represented by indicies, and of the aggregated AP-portfolio.

8.2. Computations of F-test Statistics

The test statistics used to test the null hypothesis of intersection and spanning are the F-test, F_1-test and F_2-test statistics and the computations of these are described below. The derivations of the test statistics requires the definitions of the constants $\hat{a}, \hat{b}$ and $\hat{c}$, which determines the location of the minimum variance frontier of the N+K set of assets, and $\hat{a}_1$, $\hat{b}_1$ and $\hat{c}_1$ which determines the location of the K benchmark assets. T is the number of time series observations, K is the number of benchmark assets and N is the number of test assets.

For the benchmark assets and test assets combined:

$$\hat{a} = \hat{\mu}' * \hat{V}_i^{-1} * \hat{\mu}$$

$$\hat{c} = 1_{N+K}' * \hat{V}^{-1} * 1_{N+K}$$

$$\hat{d} = \hat{a} * \hat{c} - \hat{b}^2$$

$$\hat{b} = \hat{\mu}'\hat{V}^{-1}1_{N+K}$$

Where $\hat{\mu}$ and $\hat{V}_i^{-1}$ is the mean and covariance of the returns of the benchmark assets and test assets combined (N+K).

For the benchmark assets:

$$\hat{a}_1 = \hat{\mu}_1' * \hat{V}_{11}^{-1} * \hat{\mu}_1$$

$$\hat{c}_1 = 1_K' * \hat{V}_{11}^{-1} * 1_K$$

$$\hat{d}_1 = \hat{a}_1 * \hat{c}_1 - \hat{b}_1^2$$

$$\hat{b}_1 = \hat{\mu}'_1\hat{V}_{11}^{-1}1_K$$

Where $\hat{\mu}_1$ and $\hat{V}_{11}^{-1}$ is the mean and covariance of the return of the benchmark assets (K). These are the calculations of the mean and covariance of returns:

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} R_t$$

$$\hat{V} = \frac{1}{T}\sum_{t=1}^{T}(R_t - \hat{\mu})(R_t - \hat{\mu})'$$

The F-test statstic of the mean-variance spanning test is equation (14).

$$(14) \quad F-test = \left(\frac{1}{U} - 1\right)\left(\frac{T-K-1}{N}\right) = \left(\frac{T-K-1}{N}\right)\left[\left(\frac{\hat{c}}{\hat{c}_1}\right)\left(\frac{1+\frac{\hat{d}}{\hat{c}}}{1+\frac{\hat{d}_1}{\hat{c}_1}}\right) - 1\right] \sim F_{2N,T-K-1}$$

Where the ratios $\left(\frac{\hat{c}}{\hat{c}_1}\right)$ and $\left(\frac{1+\frac{\hat{d}}{\hat{c}}}{1+\frac{\hat{d}_1}{\hat{c}_1}}\right)$ are always greater than or equal to one, since the ex-post frontier of the K+N assets dominates or is identical to the ex-post frontier of the K benchmark assets. Under the null hypotheisis the minimum-variance frontiers are identical which means that $\left(\frac{\hat{c}}{\hat{c}_1}\right)$ and $\left(\frac{1+\frac{\hat{d}}{\hat{c}}}{1+\frac{\hat{d}_1}{\hat{c}_1}}\right)$ will be equal to one and the F-test will be close to zero.

Equation (10) is the F_1-test statistic that tests the intersection null hypotheisis, $\alpha = 0$, and it is used in the intersection test and the step-down spanning test.

$$(10) \quad F_1 - test = (T - K - 1)\left(\frac{|\bar{\Sigma}|}{|\hat{\Sigma}|} - 1\right) = (T - K - 1)\left(\frac{\hat{a} - \hat{a}_1}{\hat{a}_1 - 1}\right) \sim F_{N,T-K-N}$$

Where $\hat{\Sigma}$ is the unconstrained estimate of the covariance matrix and $\bar{\Sigma}$ is the constrained estimate of the covariance matrix by imposing the constraint of all alphas being jointly equal to zero.

The F_2-test statistic in equation (15) tests the second null hypotheisis of the step-down spanning test, $\delta = 0$.

$$(15) \quad F_2 - test = (T - K)\left(\frac{|\tilde{\Sigma}|}{|\hat{\Sigma}|} - 1\right) = \left[\left(\frac{\hat{c} - \hat{d}}{\hat{c}_1 - \hat{d}_1}\right)\left(\frac{1 + \hat{a}_1}{1 + \hat{a}}\right) - 1\right] \sim F_{N,T-K}$$

Where $\hat{\Sigma}$ is the unconstrained estimate of the covariance matrix and $\tilde{\Sigma}$ is the constrained estimate of the covariance matrix, by imposing the constraint of all alphas and deltas being jointly equal to zero.

8.3. Replication of AP-portfolio

Table 6. Bull and Bear periods

Bear Periods (61 observations)	Bull Periods (119 observations)
January 1st 2001 - April 30th 2003	May 1st 2003 - June 27th 2007
June 28th 2007 - March 25th 2009	March 26th 2009 - January 28th 2015
January 29th 2015 - December 30th 2015	

Table 6. Specifies the dates of the division into bull and bear periods

Table 7. McKinsey Division of the AP-funds' Asset Classes

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Swedish Equities	19	16	17	17	17	17	15	14	16	17	14	13	14	13	13
Foreign Equities	38	39	39	39	40	37	37	35	33	31	29	29	30	30	26
Fixed Income Assets*	38	39	38	39	38	39	38	38	35	34	37	35	32	33	32
Equities in Emerging Markets	1	2	2	2	2	3	5	4	6	7	7	8	7	7	6
Alternative Investments	3	4	3	3	3	4	5	8	9	10	11	14	15	18	21
Other*	0	0	1	1	0	1	1	2	1	1	1	2	2	0	1

Table 7. Portfolio weights in percentages from McKinseys annual evaluation performance of the AP-funds 2001 to 2015. The weights are the combined holdings of AP-funds one to four.

Table 8. Division of the AP-funds' Asset Classes

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Swedish Equities	20	16	18	17	17	17	15	15	17	18	16	14	16	12	15
Foreign Equities	38	39	39	39	40	37	37	35	33	31	29	29	30	30	26
Fixed Income Assets*	38	39	38	39	38	39	38	38	35	34	37	35	32	33	32
Foreign Fixed Income	*14*	*20*	*18*	*19*	*20*	*19*	*21*	*20*	*20*	*17*	*18*	*16*	*15*	*20*	*20*
Swedish Fixed Income	*24*	*19*	*20*	*20*	*18*	*20*	*17*	*18*	*15*	*17*	*19*	*19*	*17*	*13*	*12*
Equities in Emerging Markets	1	2	2	2	2	3	5	4	6	7	7	8	7	7	6
Alternative Investments	3	4	3	3	3	4	5	8	9	10	11	14	15	18	21

Table 8. Portfolio weights in percentages used in our study for a combined portfolio of AP-fund one to four 2001 to 2015. The holdings of the AP-funds combined are based on McKinsey's division of asset classes with exception of the asset class "other" (see table 7) which is excluded from our study in consultancy with the head investor of the second AP-fund Thomas Franzén. Fixed Income Assets were examined in each of AP-fund one to four's annual reports 2001-2015 to see the proportions of foreign and Swedish fixed income respectively, after which annual average weights for all funds combined were used for the purpose of our study.

Table 9. Asset Classes and Representative Indices

AP-funds' Asset Class	Representative Index	Ticker	Notation in this study
Swedish Equities	OMX Stockholm Benchmark Index	SBX	SBX
Foreign Equities	MSCI World Local Index	MSDLWI	MXWO
Fixed Income Assets			
Foreign Fixed Income	JP Morgan Global Aggregate Bond Index	JGAGGUSD	GLOBON
Swedish Fixed Income	Handelsbanken Sweden All Bonds Total Return Index	HMSCTTR	SWEBON
Equities in Emerging Markets	MSCI Emerging market Local Currency Index	MSELEGF	MXEM
Alternative Investments	[Constructed]	-	ALT
Commodity Indices			
Commodity Index 1	S&P Goldman Sachs Commodity Excess Return Index	SPGSCIR	SPGSCI
Commodity Index 2	Bloomberg Commodity Index	BCOM	BCOM

Table 9 The AP- funds' asset classes, the representative indices, tickers, and notations in this study. The table also presents the two commodity indices which are used to test if comomdities can improve performance of the AP-funds' portfolio, their ticker as well as their notations in this study.

8.3.1 Alternative Investments Index Construction

As motivated in section **4.2 Selection of Indices**, a Swedish real estate index is found suitable to represent the asset class Alternative Investments. Due to the absence of a representable Swedish real estate index reflecting commercial and residential properties in weekly frequencies, a real estate index was constructed based on the real estate holdings of the AP funds. Weighing between commercial real estate- and private housing holdings were made for each AP-fund one to four by looking at respective annual reports in the observed period. Thereafter weights were accumulated to a total weighing between the two types of real estate investments for all A P-funds. The total results of the weights can be found in table 10.

They Swedish property firms Castellum and Wallenstam were used to represent commercial real estate holdings and private housing holdings respectively. Weekly and monthly last price data were retrieved in Bloomberg in the period 1st of January 2001 to the 31st of December 2015. Beta and leverage ratios were retrieved from Bloomberg as well, Beta in weekly and monthly frequency while leverage ratio could be retrieved quarterly at the highest frequency.

In order to obtain the value of properties on the Swedish market, and not the value of the property firms, the companies' prices were unleveraged, using the formulas below:

(22) $$\beta_U = \frac{\beta_L}{1+[(1-tax\ rate)*Debt/Equity]}$$

(23) $$Unlevered\ stock\ price = levered\ stock\ price * \beta_U$$

Based on unleveraged prices, logarithmic returns were calculated for each of the time series. Thereafter each return series was weighted according to table 10, after which they were assembled and the final real estate index was obtained.

Table 10. Proportion Commercial/Residential Real Estate

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Commercial	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	95%	85%	85%	80%	82%
Residential	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	5%	15%	15%	20%	16%

Table 10. Presents proportion in commercial and residential real estate for AP-funds one to four combined, based on information in their separate annual reports 2001-2015.

8.3.2 Composition of the Commodity Futures Indices

Table 11. SPGSCI and BCOM Index Compositions

		S&P Goldman Sachs Commidity Indes (%)	Bloomberg Commodity Index (%)
Energy	Natural Gas	3,20	8,50
	WTI Crude Oil	23,00	7,50
	Brent Crude Oil	20,40	7,50
	ULS Diesel	-	3,80
	Unleaded Gasoline	5,30	3,80
	Heating Oil	5,20	-
	Gas Oil	5,80	-
Grains	Corn	4,20	7,40
	Soybeans	3,00	5,70
	Wheat	3,50	3,30
	Soybean Oil	-	2,80
	Soybean Meal	-	2,80
	HRW Wheat	0,90	1,20
Industrial Metals	Copper	3,90	7,60
	Aluminium	3	4,60
	Zinc	0,90	2,50
	Nickel	0,60	2,40
	Lead	0,70	-
Precious Metals	Gold	3,20	11,40
	Silver	0,40	4,20
Softs	Sugar	1,60	4
	Coffee	0,90	2,30
	Cotton	1,20	1,50
	Cocoa	0,50	-
Livestock	Live Cattle	4,80	3,60
	Feeder Cattle	1,60	-
	Lean Hogs	2,30	2,00
Weighting Methodology		World production quantity, with market liquidity inclusion thresholds	1/3 world production value and 2/3 market liquidity
Contracts Used		Front month	Front month
Number of individual commodities tracked		24	20
Rebalancing frequency		Does not rebalance based on changes in prices	Annual
Index Reconstruction		Annual	Annual
Sector Weighting constraints		None	Related commodity groups: max 33%; Single commodity: max 15%, min2 %; Single commodity and its derivatives: max 25%
Roll Period		Five business days	Five business days

Table 11. Display composition of SPGSCI and BCOM respectively

8.4. Correlation Descriptives

Table 12. Unconditional Correlation

	MXWO	MXEM	SBX	SWBON	JPGLOB	ALT	BCOM	SPGSCI
MXWO	1							
	0,0000							
MXEM	0,7872*	1						
	0,0000							
SBX	0,8438*	0,7293*	1					
	0,0000	0,0000						
SWBON	-0,4142*	-0,3789*	-0,3815*	1				
	0,0000	0,0000	0,0000					
JPGLOB	-0,1062*	0,0045	-0,1623*	0,4576*	1			
	0,0029	0,8992	0,0000	0,0000				
ALT	0,3699*	0,3393*	0,4153*	-0,1359*	-0,0517	1		
	0,0000	0,0000	0,0000	0,0000	0,1487			
BCOM	0,3686*	0,4026*	0,2390*	-0,2092*	0,1851*	0,1232*	1	
	0,0000	0,0000	0,0000	0,0000	0,0000	0,0006		
SPGSCI	0,3197*	0,3433*	0,1807*	-0,1927*	0,1485*	0,0842**	0,9073*	1
	0,0000	0,0000	0,0000	0,0000	0,0000	0,184	0,0000	

Table 12 Displays unconditional correlation between the asset classes included in the AP-funds portfolio and the two commodity futures indices (SPGSCI and BCOM commodity indices). (*) denotes statistial significance at a 1 percent significance level and (**) denotes statistical significance at a 5 percent significance level

Table 13. Dynamic Conditional Correlation Descriptive Statistics

	Conditional Correlation Descriptives				Dynamic Correlation Series Descriptives		
	Trend	**Std. Err.**	**z**	**P> I z I**	**Min**	**Max**	**Range**
SPGSCI							
MXWO	0,3289	0,0592	5,56	0,0000	-0,1106	0,5234	0,634
MXEM	0,3638	0,0565	6,44	0,0000	0,0101	0,5408	0,5307
SBX	0,2458	0,0622	3,95	0,0000	-0,1668	0,414	0,5808
SWBON	-0,1792	0,0607	-2,95	0,0030	-0,4531	0,2653	0,7184
JPGLOB	0,0834	0,0613	1,36	0,1730	-0,0517	0,3913	0,4431
ALT	0,1358	0,0627	2,17	0,0300	-0,1016	0,2944	0,396
BCOM							
MXWO	0,3615	0,0561	6,44	0,0000	-0,006	0,5434	0,5494
MXEM	0,4099	0,0530	7,73	0,0000	0,0144	0,563	0,5487
SBX	0,2771	0,0596	4,65	0,0000	-0,0666	0,4425	0,5092
SWBON	-0,1685	0,0600	-2,81	0,0050	-0,4681	0,1771	0,6452
JPGLOB	0,1489	0,0598	2,49	0,0130	-0,0234	0,3449	0,3683
ALT	0,1711	0,0614	2,79	0,0050	-0,077	0,3357	0,4127

Table 13 Displays DCC descriptive statistics and is based on the resulting series of dynamic conditional correlation

8.5 Dynamic Conditional Correlation Stata Code

```
// tsset time
tsset Date, daily delta(7)

// Unconditional correlation
pwcorr MXWO MXEM SBX SWBON JPGLOB REES BCOM SPGSCI, sig star(1)

// Conditional Correlation
// Predict univariate GARCH
mgarch dcc (MXWO MXEM SBX SWBON JPGLOB ALT BCOM SPGSCI =, noconstant), arch(1)
garch(1)

// Creating variables for dynamic variance
predict H*, variance dynamic(td(30dec2015))

// Creating variables for dynamic correlation
generate
rMXWOSPGSCI=H_SPGSCI_MXWO/((sqrt(H_SPGSCI_SPGSCI))*(sqrt(H_MXWO_MXWO)))
```

The last step is performed on BCOM to all different asset classes in the AP-funds portfolio and on SPGSCI to all different asset classes in the AP-funds portfolio.

8.6 Intersection and Mean-Variance Spanning Matlab Code

The matlab code was provided by R. Kan and G. Zhou to perform their spanning tests and has been modified to fit the data used in this thesis.

```
%   This Matlab program computes the step-down spanning test.
%   Input:
%   R1: TxK matrix of returns on benchmark assets
%   R2: TxN matrix of returns on test assets
%   Output:
%   Ftest: Joint test of alpha=0_N, delta=0_N
%   Ftest1: Test of alpha=0_N,
%   Ftest2: Test of delta=0_N conditional on alpha=0_N
%   pval: p-value of Ftest
%   pval1: p-value of Ftest1
%   pval2: p-value of Ftest2
%   alpha: sample estimate of alpha
%   delta: sample estimate of delta
%
function [Ftest,Ftest1,Ftest2,pval,pval1,pval2,alpha,delta] = stepdown(R1,R2)
[T,K] = size(R1);
N = size(R2,2);
mu1 = mean(R1)';
V11i = inv(cov(R1,1));
a1 = mu1'*V11i*mu1;
b1 = sum(V11i*mu1);
c1 = sum(sum(V11i));
d1 = a1*c1-b1^2;
G = [1+a1 b1; b1 c1];
R = [R1 R2];
mu = mean(R)';
Vi = inv(cov(R,1));
a = mu'*Vi*mu;
b = sum(Vi*mu);
c = sum(sum(Vi));
d = a*c-b^2;
%
%   Compute \hat\alpha and \hat\delta
A = [1 zeros(1,K); 0 -ones(1,K)];
C = [zeros(1,N); -ones(1,N)];
X = [ones(T,1) R1];
B = X\R2;
Theta = A*B-C;
e = R2-X*B;
Sigma = cov(e,1);
H = Theta*inv(Sigma)*Theta';
lam = eig(H*inv(G));
%
%   Compute the three test statistics
Ui = prod(1+lam);
if N==1
   Ftest = (T-K-1)*(Ui-1)/2;
else
   Ftest = (T-K-N)*(sqrt(Ui)-1)/N;
end
Ftest1 = (T-K-N)/N*(a-a1)/(1+a1);
Ftest2 = (T-K-N+1)/N*((c+d)*(1+a1)/((c1+d1)*(1+a))-1);
%
%   Compute the p-values
if nargout>3
   if N==1
      pval = 1-fcdf(Ftest,2,T-K-1);
   else
      pval = 1-fcdf(Ftest,2*N,2*(T-K-1));
   end
   pval1 = 1-fcdf(Ftest1,N,T-K-N);
   pval2 = 1-fcdf(Ftest2,N,T-K-N+1);
end
if nargout>6
   alpha = Theta(1,:)';
   delta = Theta(2,:)';
end
```

8.7 SPGSCI Dynamic Correlation to the AP-funds'Asset Classes

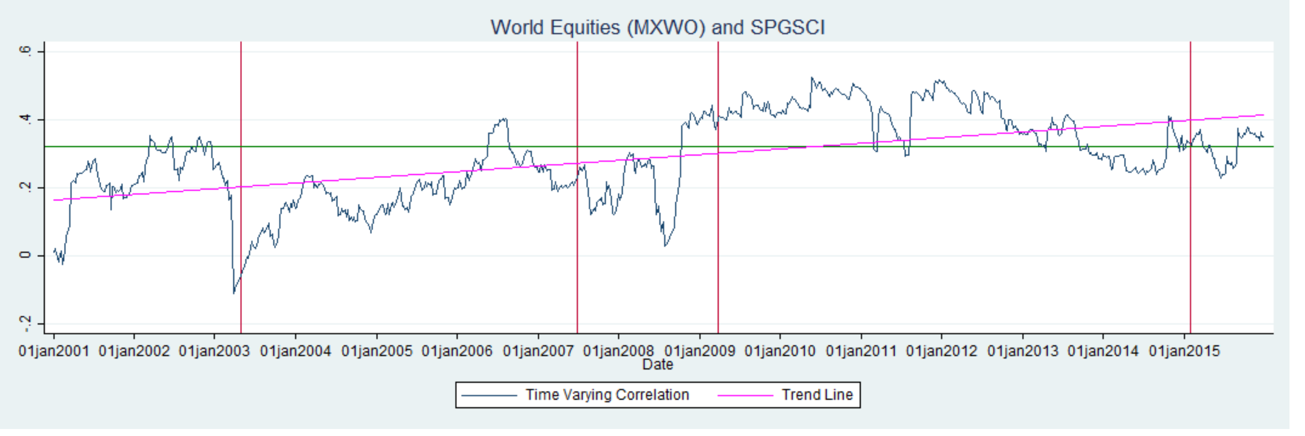

Figure 2 Display dynamic conditional correlation between world equities and a commodity futures derivative, represented by MSCI World Local Index and S&P Goldman Sachs Commodity Index

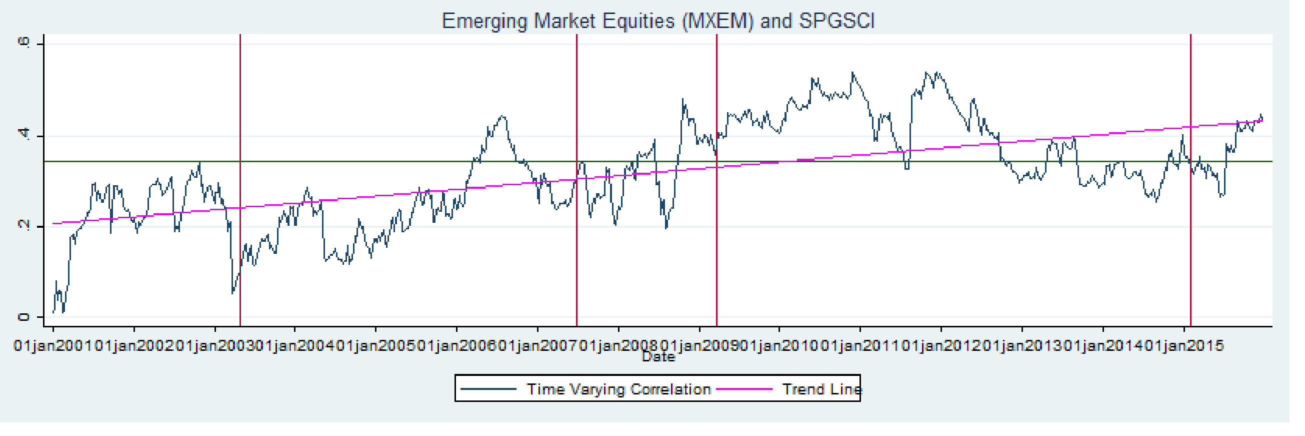

Figure 3 Display dynamic conditional correlation between emerging market equities and a commodity futures derivative, represented by MSCI Emerging Market Local Currency Index and S&P Goldman Sachs Commodity Index

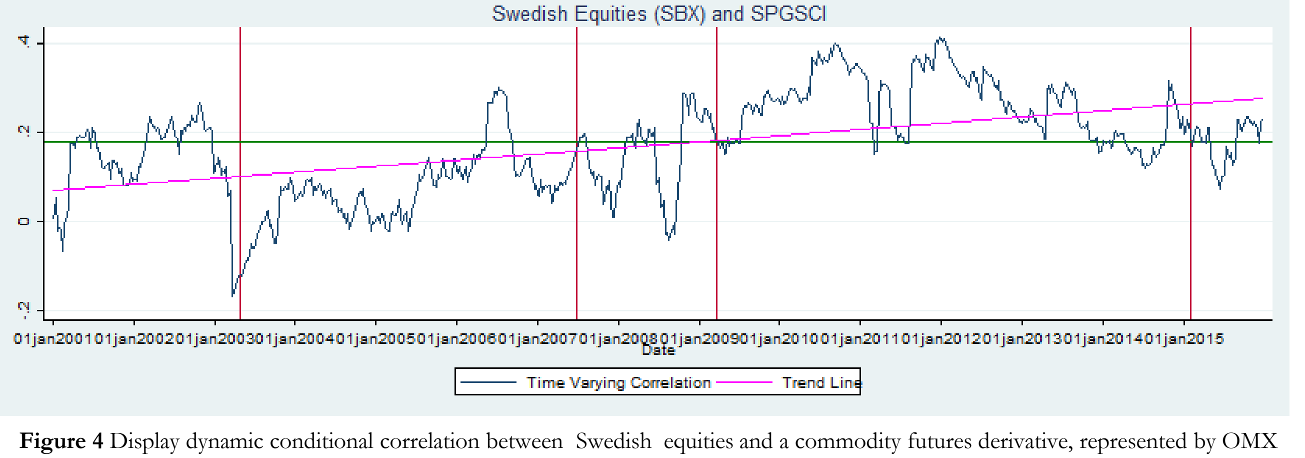

Figure 4 Display dynamic conditional correlation between Swedish equities and a commodity futures derivative, represented by OMX Stockholm Benchmark Index and S&P Goldman Sachs Commodity Index

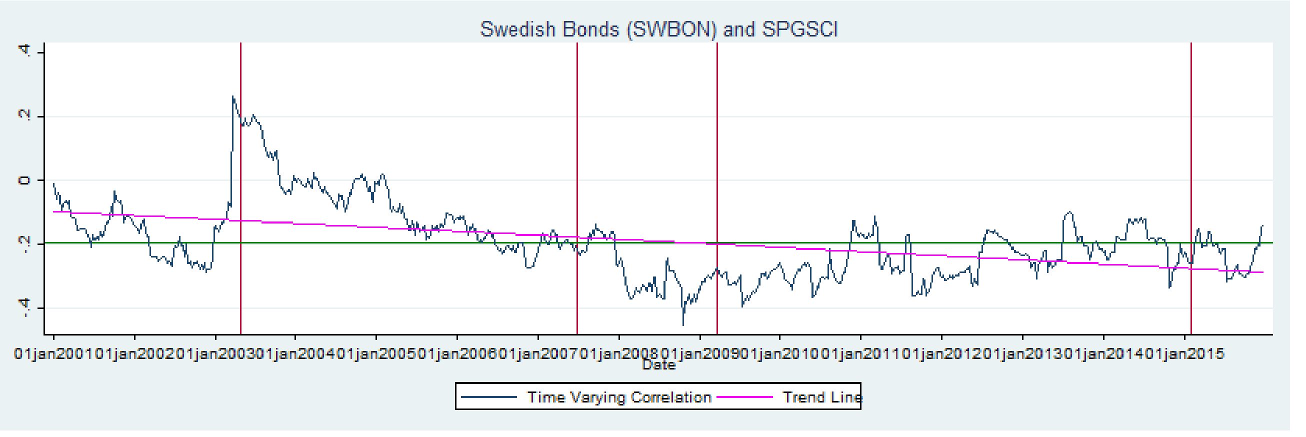

Figure 5 Display dynami conditional correlation between Swedish bonds and a commodity futures derivative, represented by Handelsbanken Sweden All Bonds Total Return Index and S&P Goldman Sachs Commodity Index

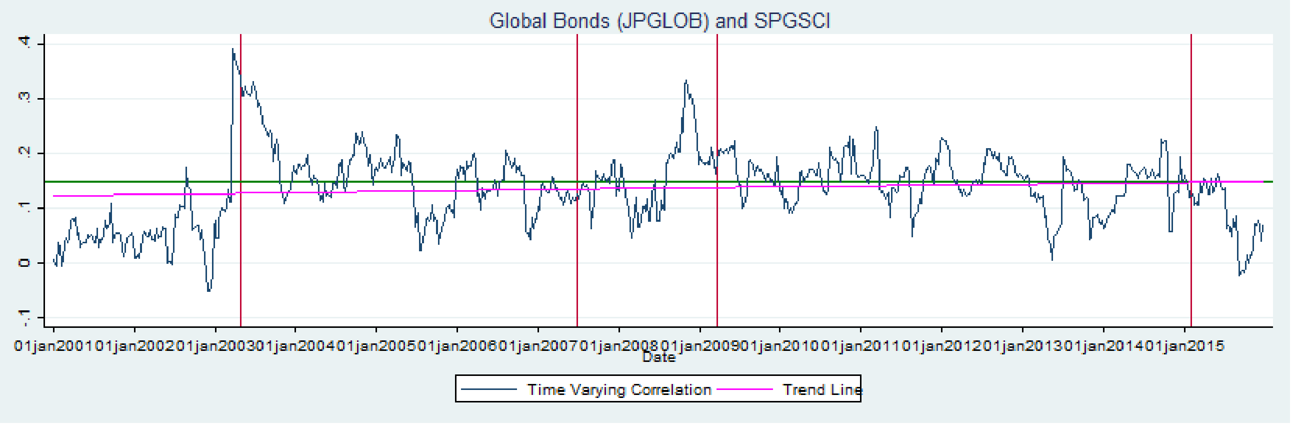

Figure 6 Display dynamic conditional correlation between global bonds and a commodity futures derivative, represented by JP Morgan Global Aggregate Bond Index and S&P Goldman Sachs Commodity Index

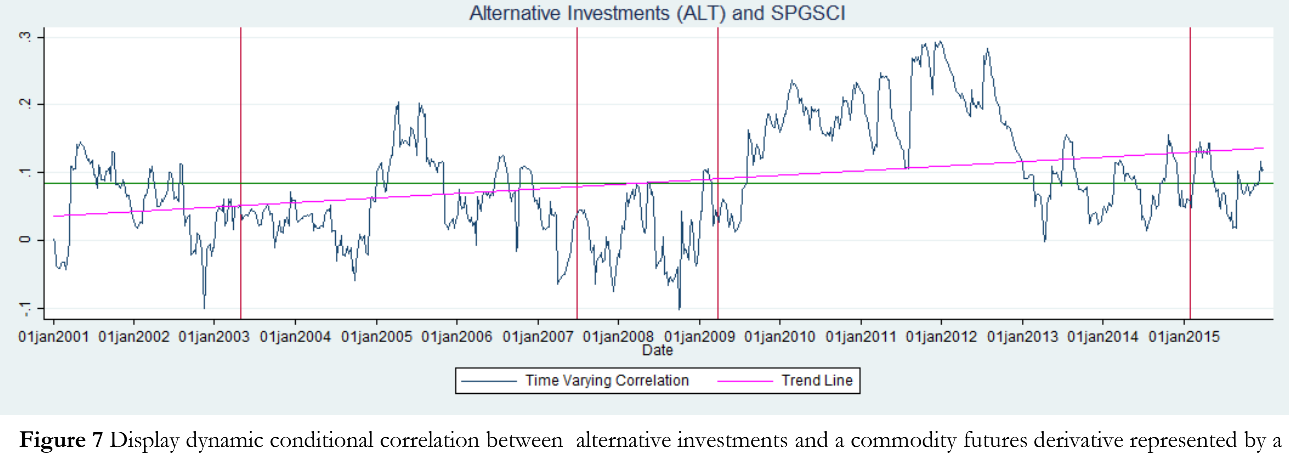

Figure 7 Display dynamic conditional correlation between alternative investments and a commodity futures derivative represented by a constructed index based on Swedish real estate and S&P Goldman Sachs Commodity Index

8.8 BCOM Dynamic Correlation to the AP-funds'Asset Classes

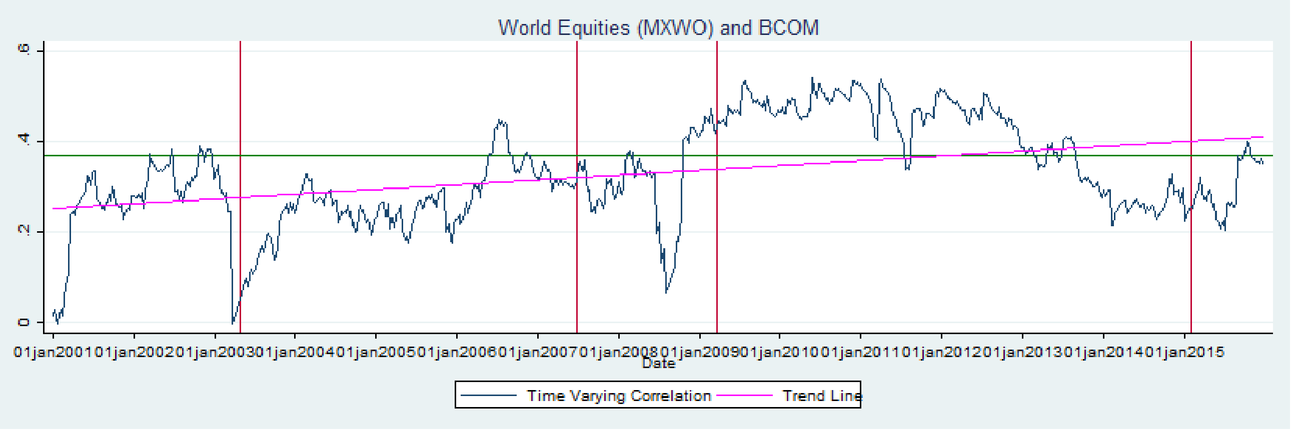

Figure 8 Display dynamic conditional correlation between world equities and a commodity futures derivative , represented by MSCI World Local Index and Bloomberg Commodity Index

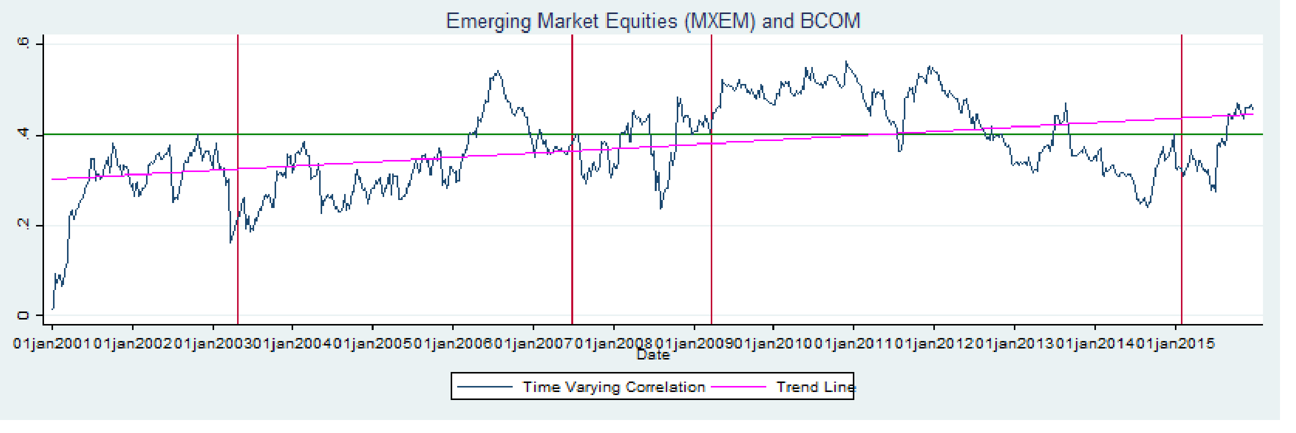

Figure 9 Display dynamic conditional correlation between emerging market equities and a commodity futures derivative, represented by MSCI Emerging Market Local Currency Index and Bloomberg Commodity Index

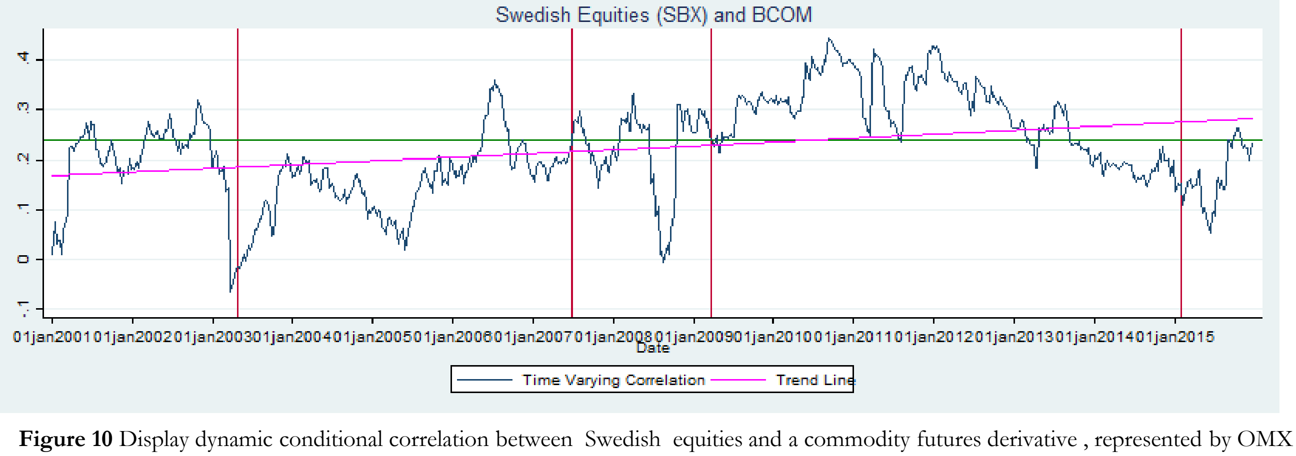

Figure 10 Display dynamic conditional correlation between Swedish equities and a commodity futures derivative , represented by OMX Stockholm Benchmark Index and Bloomberg Commodity Index

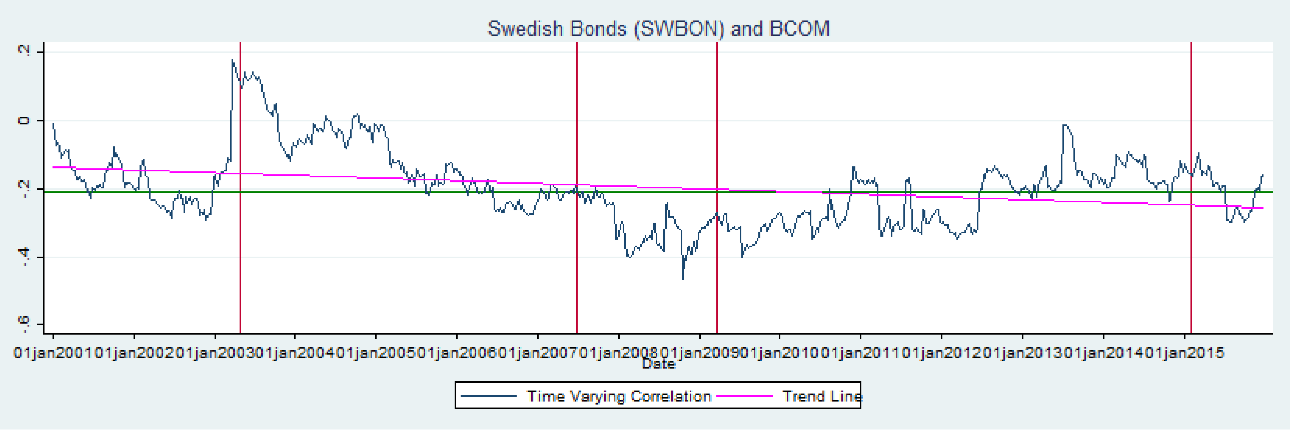

Figure 11 Display dynamic conditional correlation between Swedish bonds and a commodity futures derivative represented by Handelsbanken Sweden All Bonds Total Return Index and Bloomberg Commodity Index

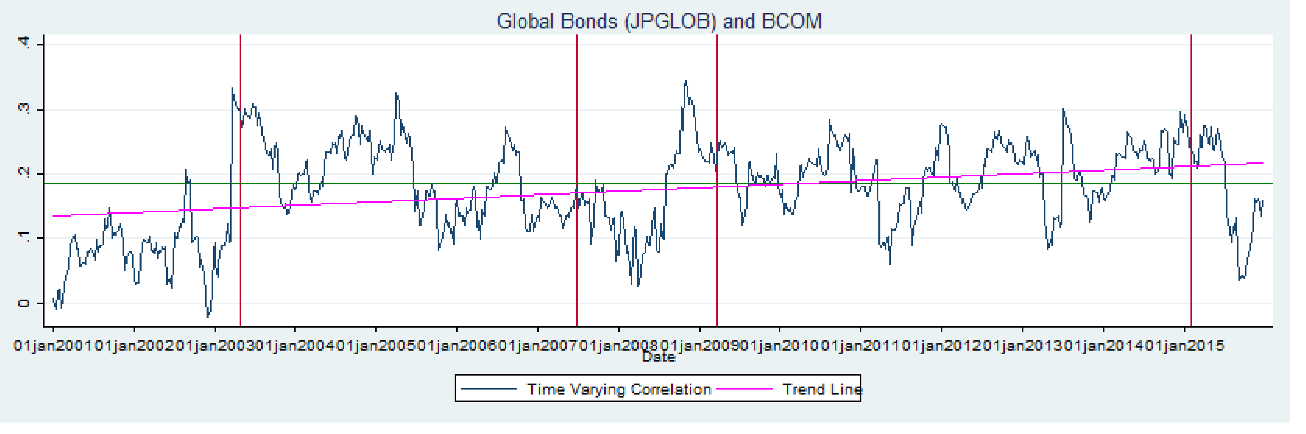

Figure 12 Display dynamic conditional correlation between global bonds and a commodity futures derivative represented JP Morgan Global Aggregate Bond Index and Bloomberg Commodity Index

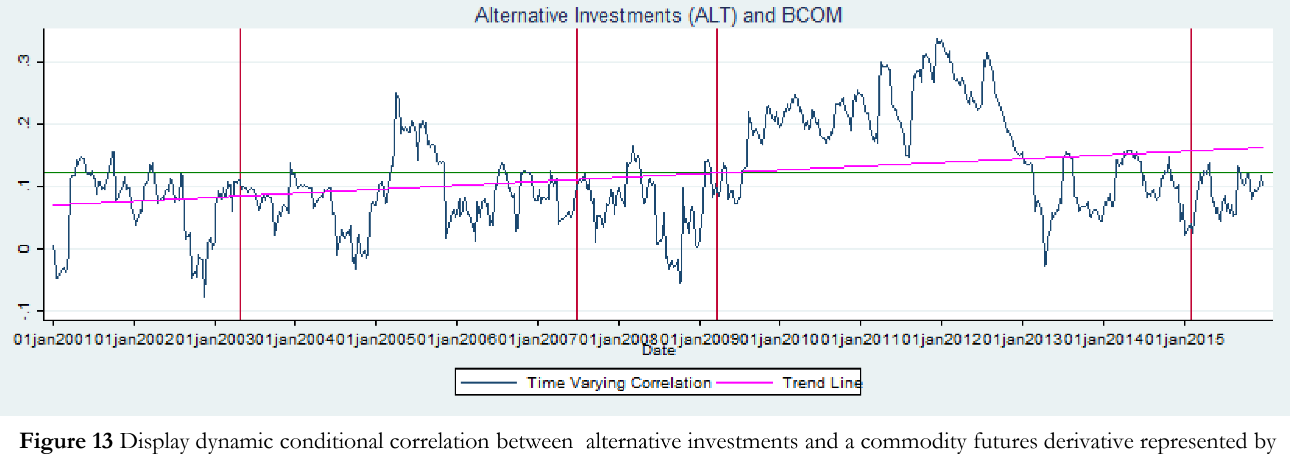

Figure 13 Display dynamic conditional correlation between alternative investments and a commodity futures derivative represented by a constructed index based on Swedish real estate and Bloomberg Commodity Index