Ödsmål. Kville sn, Bohuslän

Hällristning
Fiskare från
bronsåldern

Rock carving
Bronze age
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First Results of a New Numerical Model for Baltic Water Levels.

by

Artur Svansson

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1. Introduction.

Many years ago the author started work on water level problems (Svansson 1959). The Skagerrak, the Kattegatt, the Danish straits and the Baltic were treated as canals. Water levels and water transports were computed by numerical integration, the sectioning being mostly copied from Neumann (1941). Later the method was improved by using e.g. a better sectioning (Svansson 1966) of some parts of the system.

The old numerical system was an explicit one allowing timesteps of approximately 15 minutes. The system presented here is an implicit one aiming at the use of large timesteps, e.g. 24 hours. While it seems possible to get a stable implicit model with the old system of equations, it is not quite clear if the introduction of the new damping term tested here upsets the stability or not. The results presented in this paper, however, do not seem to be influenced by such a possible instability.

2. The system of equations.

$$\begin{cases} \dfrac{\partial U}{\partial t} = -gA\dfrac{\partial h}{\partial x} + b\tau_W - b\tau_B + \dfrac{\partial}{\partial x} K \dfrac{\partial U}{\partial x} \\ \\ \dfrac{\partial h}{\partial t} = -\dfrac{1}{b}\dfrac{\partial U}{\partial x} \end{cases}$$

3. Notations:

U	=	Transport of water	m^3/s
g	=	Acceleration of gravity	m/s^2
A	=	Cross sction area	m^2
b	=	Width of sction	m
h	=	Variation of water level	m
q	=	Density	t/m^3
τ'	=	Turbulent stress component	t/m and s^2
τ	=	τ/q	m^2/s^2
n	=	Index of timestep	
j	=	Index of sction	

In this paper the windstress $\tau_W = \tau$ and the bottom stress $\tau_B = \dfrac{\beta}{b}.U$. In all the computations so far, however, $\beta = 0$.

The introduction of the term $\frac{\partial}{\partial x} K \frac{\partial u}{\partial x}$ is an experiment. As pointed out in Svansson (1968) the tests with a damping term of the type βU did not fully come up to expectations, nor did the trial with the scheme of Platzman (1963). The K:s used were determined in the following way: The mean salinity from surface to bottom was determined from Granquist (1938) and Anon. (1933) for the period April-October (See Fig. 2). Using the assumption that $Z S = A K' \frac{\partial S}{\partial x}$ (Boicourt 1969) is valid (Z = fresh water supply in m^3/s during the same period of the year taken from Brogmus 1952) the K':s were determined for every section (Fig. 3). They are presented in Table 1. The idea is now to assume that $K = k A K'$, where k is independent of the section but adjustable in order to give the best identity with nature.

4. The Numerical System. is presented as an Appendix.

5. Results.

The reason for publishing results already is that one of the first tests is rather interesting. The water levels of Smögen (approximately section 3:2) were introduced as boundary condition $h_{1/2}$ at the open end of canal 3. No wind was applied; k = 0.001 was found to be a good choice. Fig. 4 shows the result of h_G compared with the measured levels at Landsort. The agreement is rather good and could probably be made better by working more carefully:

1) Checking the stability. Hints have been given by H. Sundquist (personal communication). The possible instability comes from the open end, section 3:0, but seems to be filtered away in the Danish straits.
2) Applying the real boundary condition at section 3:0 and not the levels at Smögen (3:2).
3) Studying the new damping term extensively.
4) Introducing branching also in the Danish straits as was done in Svansson (1966).
5) Applying the actual winds.

6. Discussion.

Fig 5 shows the variations of a) the atmospheric pressure in Stockholm, b) the water level in Smögen c) the water level in Landsort and d) the surface salinity in Kattegatt (approximately section 3:12) during the period concerned. While the level of Smögen approximately follows the variations in atmospheric pressure the salinity of Kattegatt approximately follows the level of Landsort.

The explanation might be that the large scale variations of the level of the Baltic (Hela 1944 has shown that the levels of Landsort can be used rather well to represent the whole Baltic) are mostly a function of the level at the mouth, but the Kattegatt and the Danish straits filter and damp a large amount of the signal. The variation of the Kattegatt surface salinity which is due to horizontal movements should then probably follow the variations of the Baltic. As Hela (1944) shows transports due to level variations might often amount to 200 000 m^3/s in the Danish straits and if this goes on for e.g. 10 days it corresponds to a movement of approximately 75 km, if the water moves homogenously, but 150 km if only the 15 m deep surface layer takes part in the transport. Fig. 6 shows that also the surface salinities at the Bornö station (approximately section 3:2) are influenced by the same phenomenon. This relation does not, however, appear that clearly.

Fig. 7 shows the events at a well known occasion, e.g. the large inflow of salt water to the Baltic 1951 (Wyrtki 1954, Fonselius 1962).

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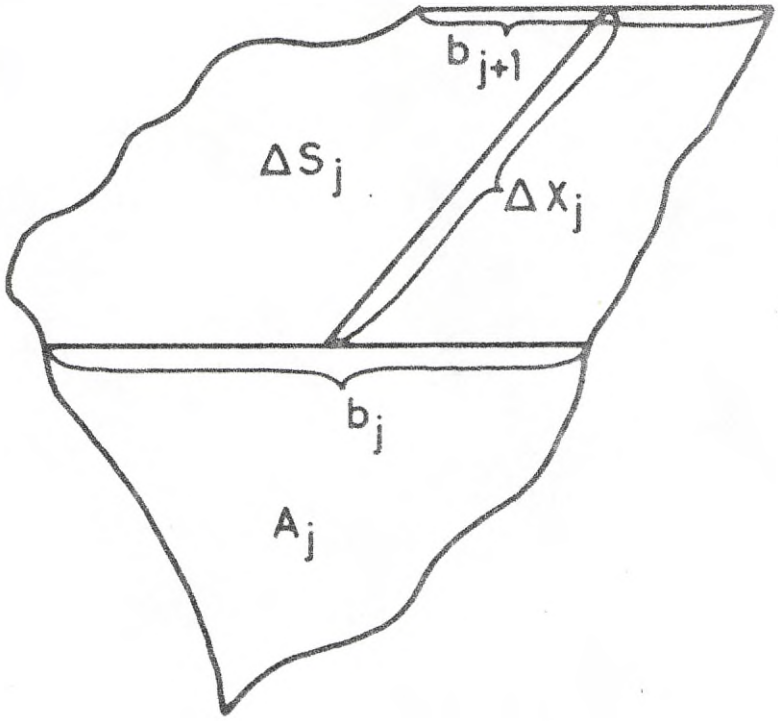
Table 1.

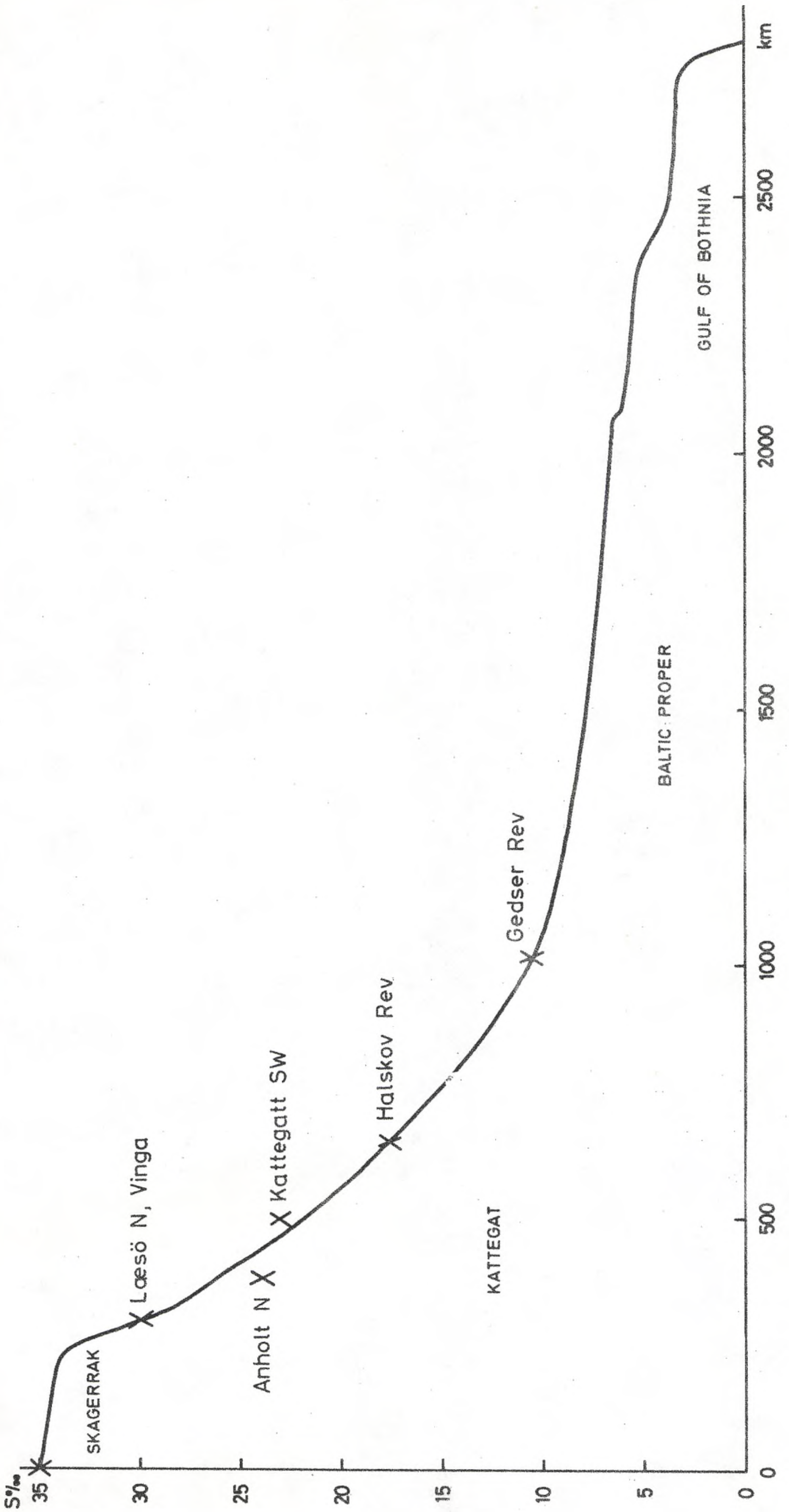
K′ in m^2/s

Branch	1	2	3
Section			
0	0	0	4300
1	169	628	4300
2	79	406	2552
3	573	376	3091
4	641	637	4281
5	737	1677	5005
6	2243	1645	5379
7	4600	2280	5485
8	4703		3804
9	2468		4483
10	479		5236
11	211		6749
12	145		9424
13	146		16410
14	227		13880
15	519		10590
16	1545		34210
17	1704		38395
18	2243		31580
19	746		62230
20	964		51690
21	1000		22180
22			56380
23			12640
24			16420
25			28490
26			37020
27			9005
28			12810
29			17720
30			33600
31			22230
32			10305
33			5090
34			4535
35			2120
36			1345
37			1760
38			2000



b j+1
ΔS j
ΔX j
b j
A j

Fig. 2



S‰
35
30
25
20
15
10
5
0
0
500
1000
1500
2000
2500
km
SKAGERRAK
Læsö N, Vinga
Anholt N
Kattegatt SW
KATTEGAT
Halskov Rev
Gedser Rev
BALTIC PROPER
GULF OF BOTHNIA

Fig. 3

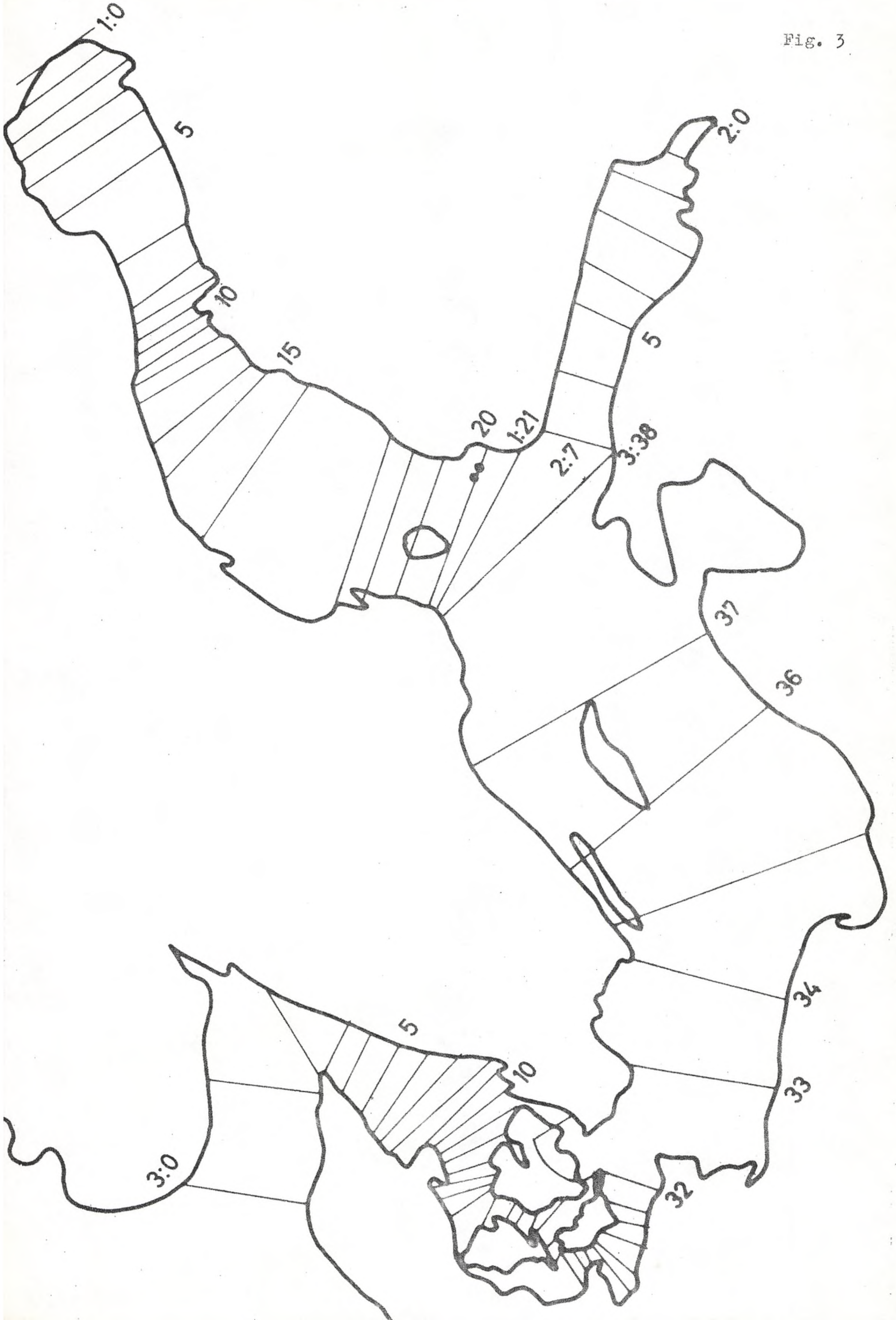
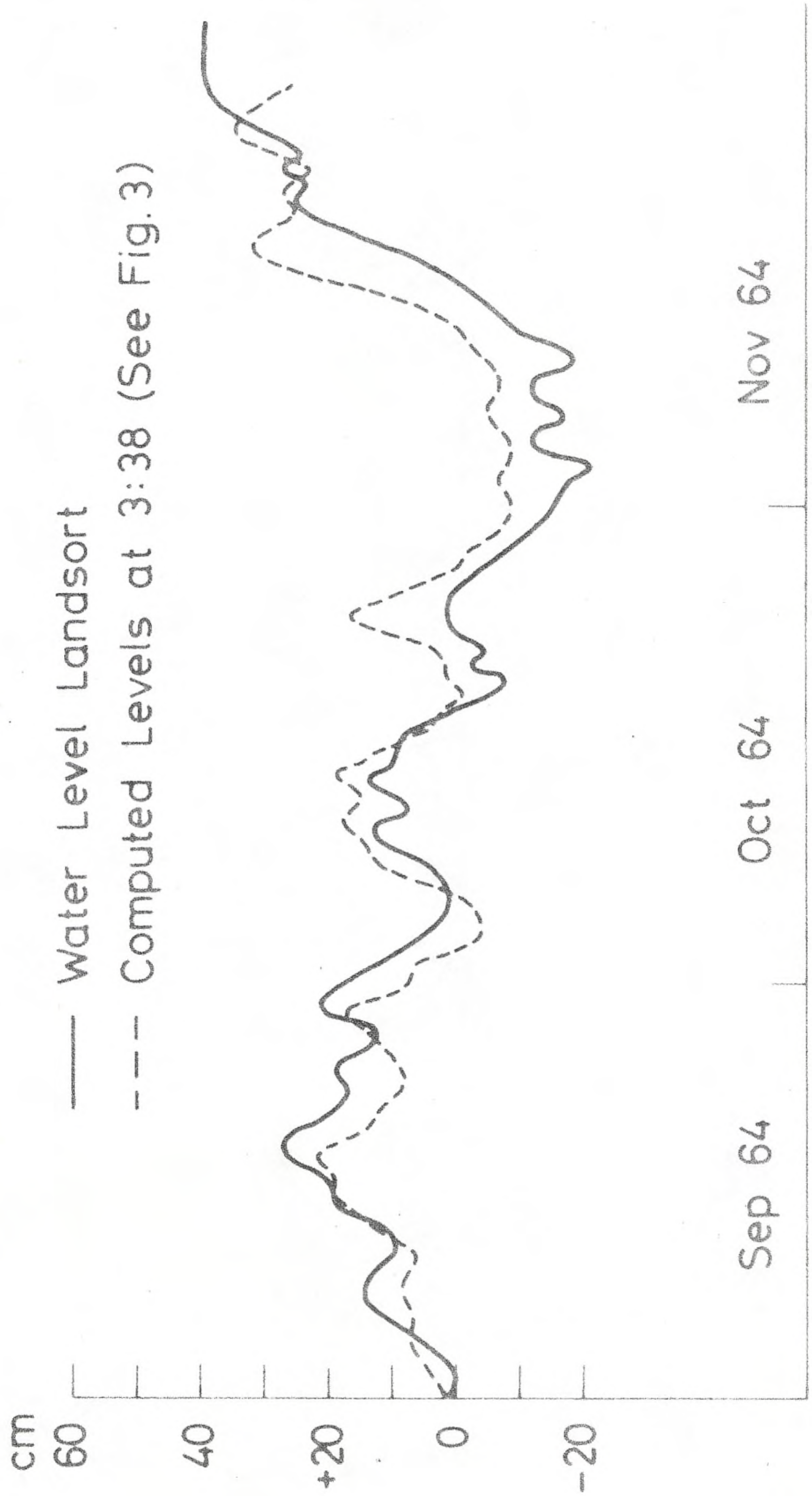


Fig. 4



— Water Level Landsort
- - - Computed Levels at 3:38 (See Fig. 3)
cm
60
40
+20
0
-20
Sep 64
Oct 64
Nov 64

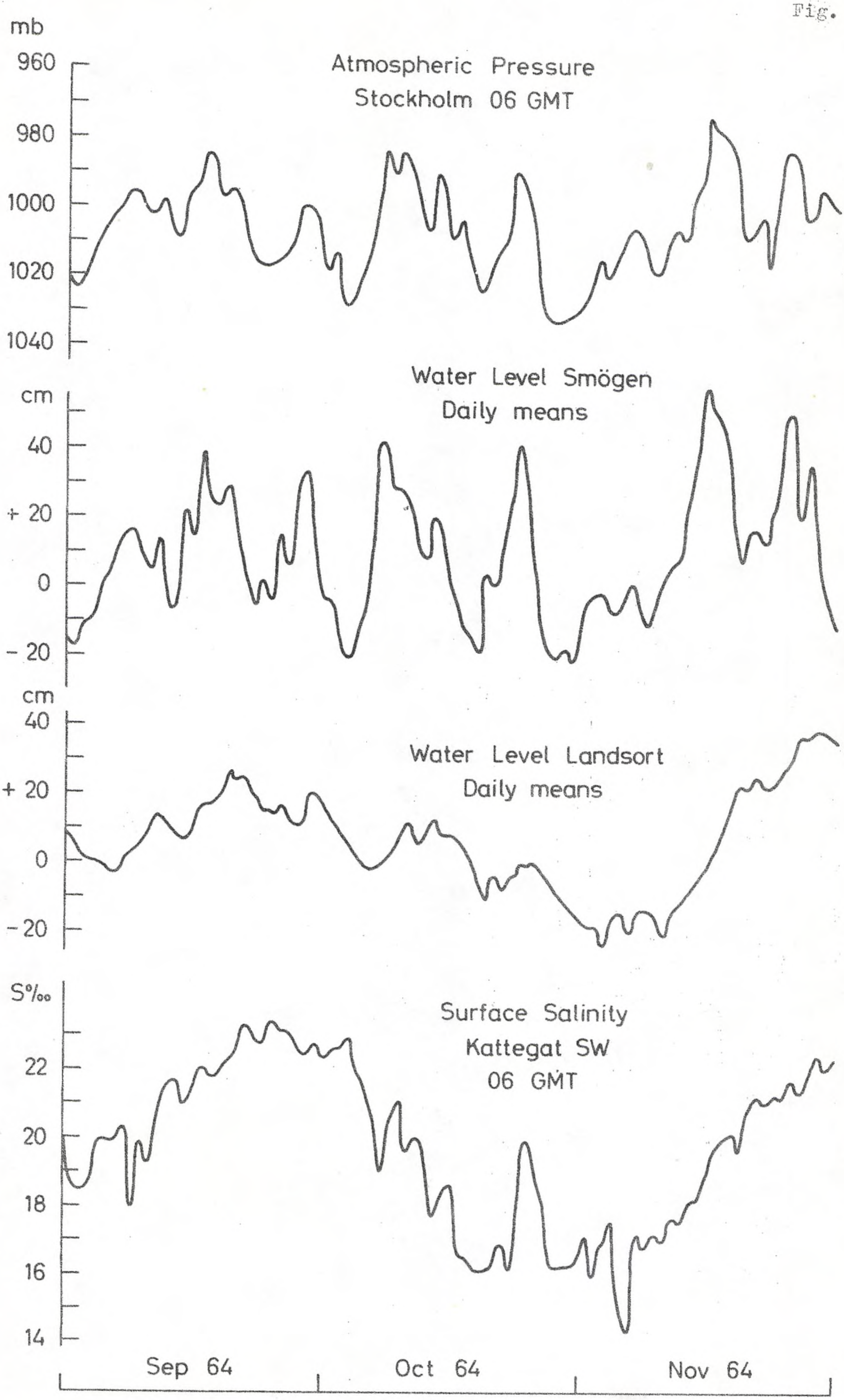
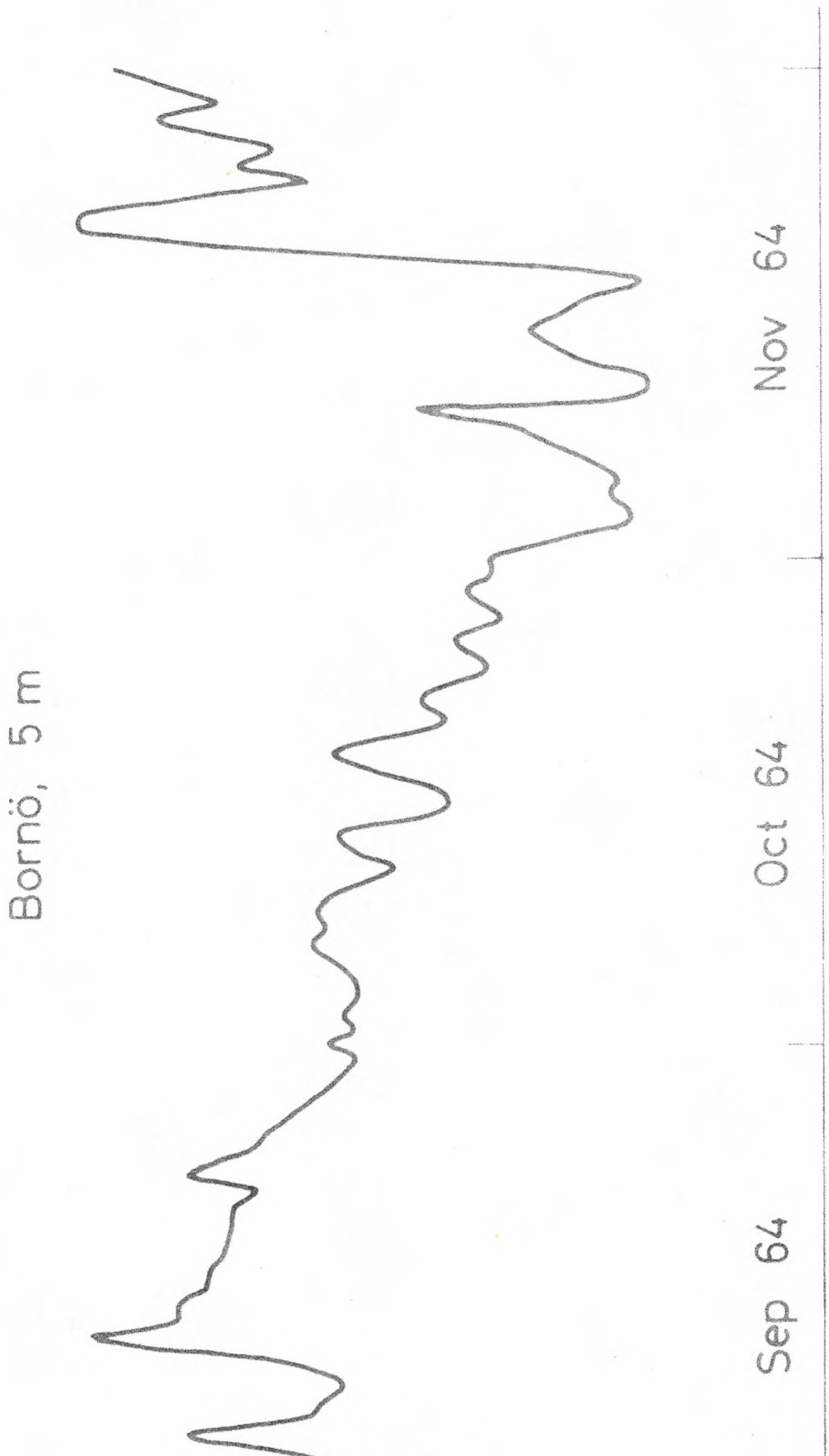
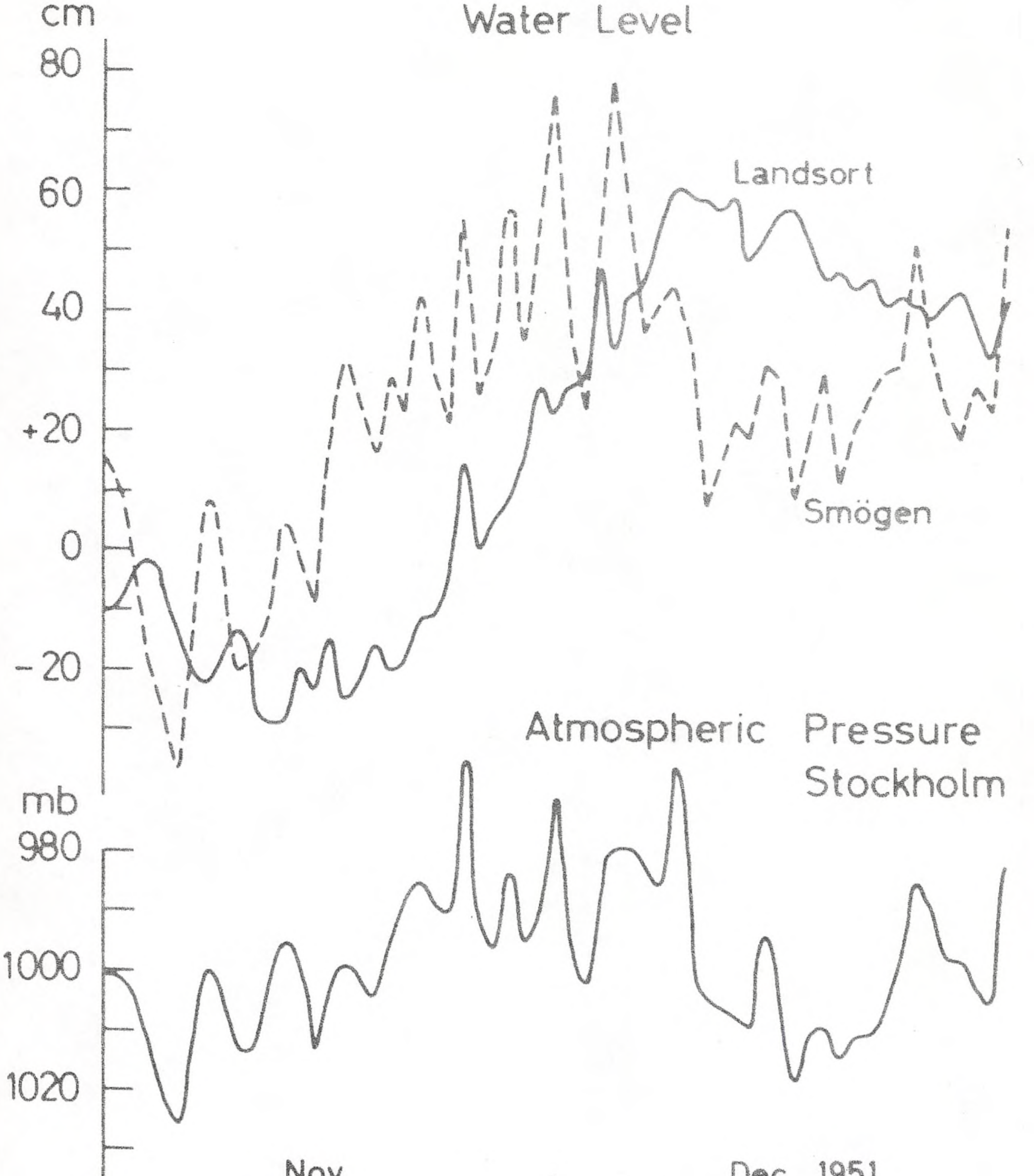

mb
960
980
1000
1020
1040
Atmospheric Pressure
Stockholm 06 GMT
cm
40
+ 20
0
- 20
Water Level Smögen
Daily means
cm
40
+ 20
0
- 20
Water Level Landsort
Daily means
S‰
22
20
18
16
14
Surface Salinity
Kattegat SW
06 GMT
Sep 64
Oct 64
Nov 64

Fig. 6




cm
Water Level
80
60
40
+20
0
- 20
Landsort
Smögen
Atmospheric Pressure
Stockholm
mb
980
1000
1020
Nov
Dec 1951

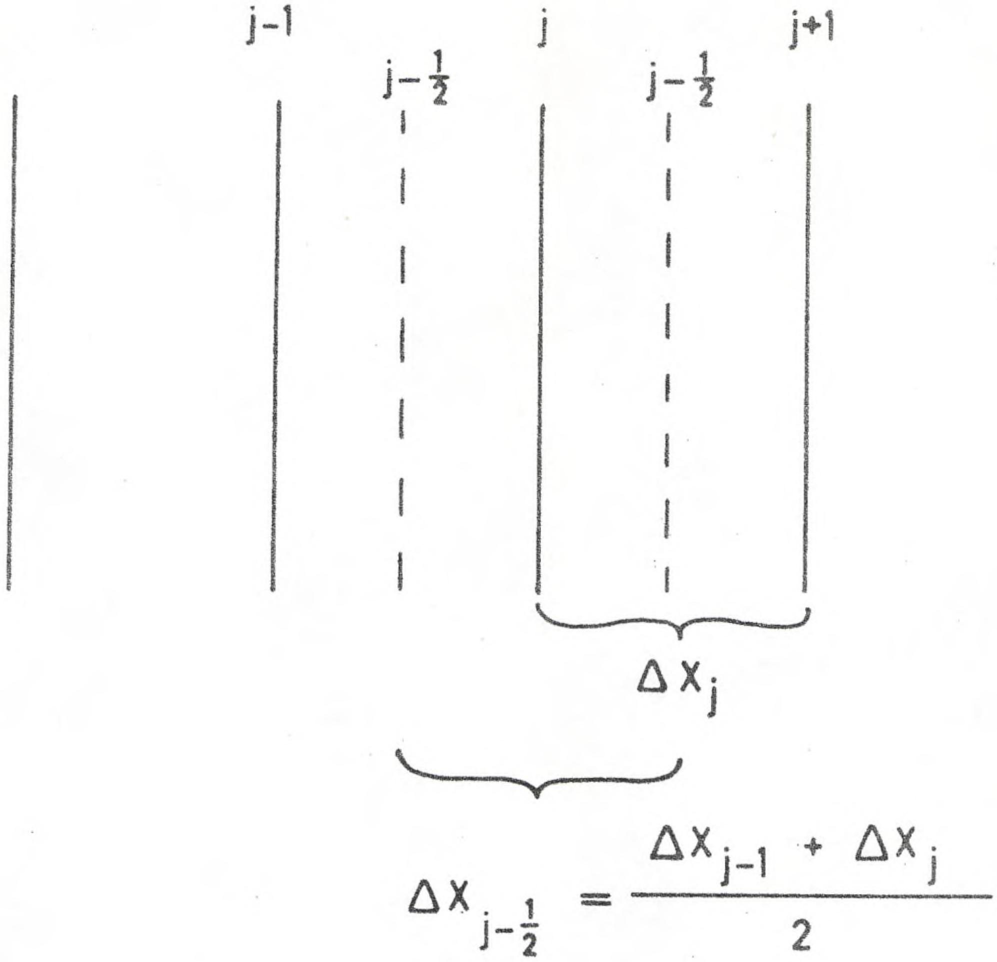

j-1
j
j+1
j-½
j-½
ΔX_j
ΔX_{j-½} = (ΔX_{j-1} + ΔX_j) / 2

Fig. 8

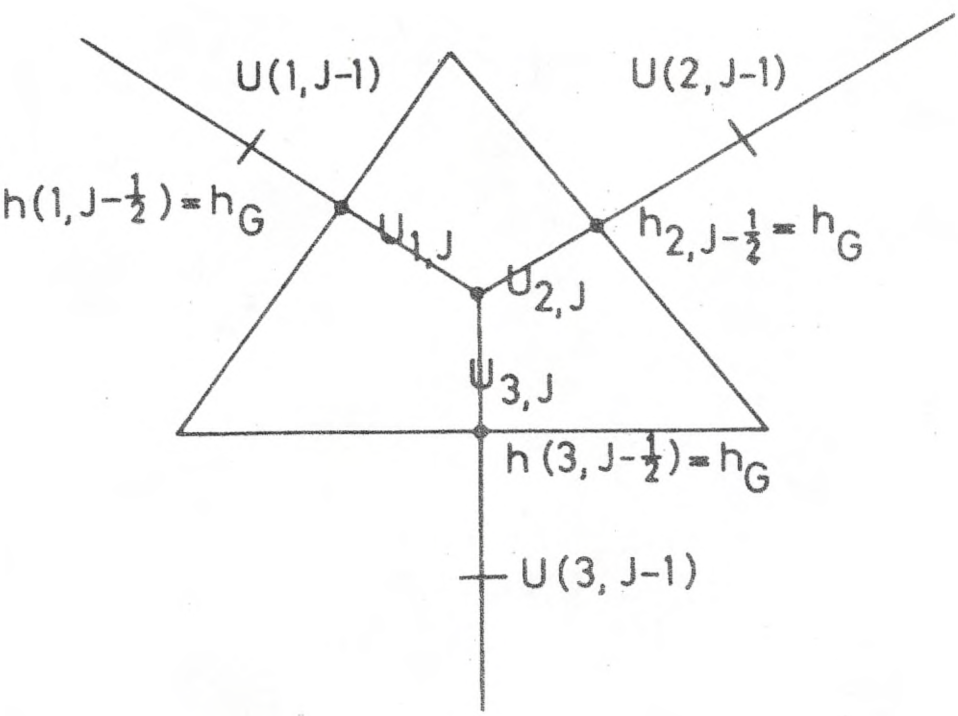

U(1,J-1)
U(2,J-1)
h(1,J-½)=h_G
U_{1,J}
h_{2,J-½}= h_G
U_{2,J}
U_{3,J}
h (3,J-½)=h_G
U(3,J-1)

Fig. 9

$$\frac{U_j^{n+1} - U_j^{n}}{\Delta t} = - g \cdot A_j \cdot \frac{h_{j+\frac{1}{2}}^{n+1} + h_{j+\frac{1}{2}}^{n} - h_{j-\frac{1}{2}}^{n+1} - h_{j-\frac{1}{2}}^{n}}{\Delta x_{j-1} + \Delta x_j} - \beta_j \cdot \frac{U_j^{n+1} + U_j^{n}}{2} +$$

$$+ \frac{K_{j+\frac{1}{2}} \dfrac{U_{j+1}^{n+1} + U_{j+1}^{n} - U_j^{n+1} - U_j^{n}}{2 \cdot \Delta x_j} - K_{j-\frac{1}{2}} \dfrac{U_j^{n+1} + U_j^{n} - U_{j-1}^{n+1} - U_{j-1}^{n}}{2 \cdot \Delta x_{j-1}}}{\Delta x_{j-\frac{1}{2}}} +$$

$$+ b_j \, \tau_j^{n+\frac{1}{2}} ;$$

(See Fig. 8)

$$U_j^{n+1} - U_j^n = -\frac{g A_j \Delta t}{\Delta X_{j-1} + \Delta X_j} \left(h_{j+\frac{1}{2}}^{n+1} + h_{j+\frac{1}{2}}^{n} - h_{j-\frac{1}{2}}^{n+1} - h_{j-\frac{1}{2}}^{n} \right) - \beta_j \cdot \Delta t \frac{U_j^{n+1} + U_j^n}{2}$$

$$+ \frac{K_{j+\frac{1}{2}} \cdot \Delta t}{2 \cdot \Delta X_j \cdot \Delta X_{j-\frac{1}{2}}} \cdot U_{j+1}^{n+1} - \left(\frac{K_{j+\frac{1}{2}} \cdot \Delta t}{2 \cdot \Delta X_j \cdot \Delta X_{j-\frac{1}{2}}} + \frac{K_{j-\frac{1}{2}} \Delta t}{2 \cdot \Delta X_{j-1} \cdot \Delta X_{j-\frac{1}{2}}} \right) \cdot U_j^{n+1}$$

$$+ \frac{K_{j-\frac{1}{2}} \cdot \Delta t}{2 \Delta X_{j-1} \cdot \Delta X_{j-\frac{1}{2}}} \cdot U_{j-1}^{n+1} + \frac{K_{j+\frac{1}{2}} \Delta t}{2 \Delta X_j \cdot \Delta X_{j-\frac{1}{2}}} \cdot U_{j+1}^{n} - \left(\frac{K_{j+\frac{1}{2}} \Delta t}{2 \Delta X_j \Delta X_{j-\frac{1}{2}}} + \frac{K_{j-\frac{1}{2}} \Delta t}{2 \Delta X_{j-1} \cdot \Delta_{j-\frac{1}{2}}} \right) \cdot U_j^n$$

$$+ \frac{K_{j-\frac{1}{2}} \Delta t}{2 \Delta X_{j-1} \Delta X_{j-\frac{1}{2}}} U_{j-1}^{n} + b_j \Delta t \cdot \tau_j^{n+\frac{1}{2}} \;;$$

$$h_{j-\frac{1}{2}}^{n+1} = h_{j-\frac{1}{2}}^{n} - \frac{\Delta t}{\Delta x_{j-1}(b_{j-1}+b_j)} \left(U_j^{n+1} - U_{j-1}^{n+1} + U_j^{n} - U_{j-1}^{n}\right) ;$$

$$\frac{g \cdot A_j \cdot \Delta t}{\Delta x_{j-1} + \Delta x_j} = m_j \quad ; \frac{\Delta t}{\Delta x_{j-1}\,(b_{j-1}+b_j)} = \ell_{j-1} \, ; \; \frac{\Delta t}{\Delta x_j\,(b_j+b_{j+1})} = \ell_j \, ;$$

$$m_j h_{j-\frac{1}{2}}^{n+1} = m_j h_{j-\frac{1}{2}}^{n} - m_j \ell_{j-1} (U_j^{n} - U_{j-1}^{n}) - m_j \ell_{j-1} (U_j^{n+1} - U_{j-1}^{n+1});$$

$$m_j h_{j+\frac{1}{2}}^{n+1} = m_j h_{j+\frac{1}{2}}^{n} - m_j \ell_j (U_{j+1}^{n} - U_j^{n}) - m_j \ell_j (U_{j+1}^{n+1} - U_j^{n+1});$$

$$\frac{K_{j+\frac{1}{2}}\,\Delta t}{2\,\Delta x_j \cdot \Delta x_{j-\frac{1}{2}}} = r_j \quad ; \quad \frac{K_{j-\frac{1}{2}}\,\Delta t}{2\,\Delta x_{j-1}\,\Delta x_{j-\frac{1}{2}}} = s_j$$

$$U_j^{n+1} - U_j^{n} = -m_j h_{j+\frac{1}{2}}^{n} + m_j \ell_j U_{j+1}^{n} - m_j \ell_j U_j^{n} + m_j \ell_j U_{j+1}^{n+1} - m_j \ell_j U_j^{n+1} - m_j h_{j+\frac{1}{2}}^{n}$$

$$+ m_j h_{j-\frac{1}{2}}^{n} - m_j \ell_{j-1} U_j^{n} + m_j \ell_{j-1} U_{j-1}^{n} - m_j \ell_{j-1} U_j^{n+1} + m_j \ell_{j-1} U_{j-1}^{n+1} + m_j h_{j-\frac{1}{2}}^{n}$$

$$- \frac{\beta_j \Delta t}{2} U_j^{n+1} - \frac{\beta_j \Delta t}{2} U_j^{n} + r_j U_{j+1}^{n+1} - (r_j + s_j) \cdot U_j^{n+1} + s_j U_{j-1}^{n+1} +$$

$$+ r_j U_{j+1}^{n} - (r_j + s_j) U_j^{n} + s_j U_{j-1}^{n} + b_j \Delta t \, T_j^{n+\frac{1}{2}} ;$$

$$-U_{j+1}^{n+1}\underbrace{\left(m_j l_j + r_j\right)}_{P_j} + U_j^{n+1}\underbrace{\left(1+m_j l_j + m_j l_{j-1} + \frac{\beta_j \Delta t}{2} + r_j + s_j\right)}_{Q_j} - U_{j-1}^{n+1}\underbrace{\left(m_j l_{j-1} + s_j\right)}_{C_j}$$

$$= D_j^n$$

$$D_j^n \equiv U_{j+1}^n\left(m_j l_j + r_j\right) - U_j^n\left(-1 + m_j l_j + m_j l_{j-1} + \frac{\beta_j \Delta t}{2} + r_j + s_j\right)$$

$$+ U_{j-1}^n\left(m_j l_{j-1} + s_j\right) - 2m_j\left(h_{j+\frac{1}{2}}^n - h_{j-\frac{1}{2}}^n\right) + b_j \Delta t\, \tau_j^{n+\frac{1}{2}} ;$$

or

$$-U_{j+1}^{n+1} \cdot P_j + U_j^{n+1} Q_j - U_{j-1}^{n+1} C_j = D_j^n$$

$$D_j^n \equiv U_{j+1}^n P_j - U_j^n\left(Q_j - 2\right) + U_{j-1}^n C_j - 2m_j\left(h_{j+\frac{1}{2}}^n - h_{j-\frac{1}{2}}^n\right) + b_j \Delta t\, \tau_j^{n+\frac{1}{2}} ;$$

Richtmyer and Morton (1967):

$$U_j^{n+1} = E_j U_{j+1}^{n+1} + F_j$$

where

$$E_j = \frac{P_j}{Q_j - C_j E_{j-1}}$$

$$F_j = \frac{D_j^n + P_j F_{j-1}}{Q_j - C_j E_{j-1}}$$

Branch 1 and 2:

$$U_0 = 0 \quad \because E_0 = 0 \;, \quad F_0 = 0$$

Branch 3: $\frac{H^{n+1} - H^n}{\Delta t}$ given

$$\frac{U_0^{n+1} + U_0^n - U_1^{n+1} - U_1^n}{2\, b_{j+\frac{1}{2}}\, \Delta X_0} = \frac{H_{\frac{1}{2}}^{n+1} - H_{\frac{1}{2}}^n}{\Delta t}$$

$$U_0^{n+1} = U_1^{n+1} + \underbrace{2 b_{j+\frac{1}{2}} \Delta X_0 \frac{H^{n+1} - H^n}{\Delta t} - U_0^n + U_1^n}_{F_0}$$

$$E_0 = 1 \;,$$

The Branching Point (See Fig. 9

NB. Algol notations):

$$U(L,J-1)=E(L,J-1)*U(L,J)+F(L,J-1)$$

$$h_G^{n+1}=h_G^{n}-\ell(L,J-1)*(U^{n}(L,J)-U^{n}(L,J-1))$$

$$-\ell(L,J-1)*(U^{n+1}(L,J)-E(L,J-1)*U^{n+1}(L,J)$$

$$-F(L,J-1));$$

or

$$h_G^{n+1}=\overbrace{h_G^{n}-\ell(L,J-1)*(U^{n}(L,J)-U^{n}(L,J-1)-F(L,J-1))}^{T(L)}$$

$$-U^{n+1}(L,J)*\underbrace{\ell(L,J-1)*(1-E(L,J-1))}_{LE(L)};$$

Therefore from the system

$$\begin{cases} h_G^{n+1} = T(L) - U^{n+1}(L,J) * LE(L) & \text{3 ekv:s} \\ U^{n+1}(1,J) + U^{n+1}(2,J) + U^{n+1}(3,J) = 0 \end{cases}$$

all $U^{n+1}(L,J)$ and also h_G^{n+1} can be determined

Then

$$U_{J-1} = E_{J-1} U_J + F_{J-1} \qquad \text{etc backwards.}$$

Finally

$$h^{n+1}_{j+\frac{1}{2}} = h^{n}_{j+\frac{1}{2}} - \ell_j \left(U^{n}_{j+1} - U^{n}_{j} + U^{n+1}_{j+1} - U^{n+1}_{j} \right)$$